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Heterogenous Beliefs and Employee Stock Options

Abstract

This paper uses a market valuation model to explore why firms grant employee stock options. When insider managers and outside investors have different opinions about the future prospects of the firm, employee stock options can be used to capture future investor overvaluation and to save employee compensation costs. Options can enhance the stock value for existing shareholders if the difference in opinion is highly volatile. The equilibrium option grant is positively correlated with both the perception error of investors, and the volatility of this error, as well as the correlation between investors' error and firm fundamental value. The model provides implications on the cross-sectional differences in option grants, and these implications can be examined empirically.

1 Introduction

An employee stock option is an agreement between a firm and its employees under which the employees can buy a specified number of shares of stock at a specified price. Over the past decade, the use of employee stock options has been rising dramatically, and options have become a large part of employee compensation. For example, here we quote Schwarzbach of Aon Consulting in *Financial Times* ("US Stock Options" by Kerry Townsend, Nov 17, 2000):

> Technology companies were able to attract a lot of talent by offering less than competitive salaries for executives. They could hire managers who would normally make $150,000, and would only pay them $70,000 with lots of stock options.

Apparently, the managers in this case are paid the remaining $80,000 in stock options. Why do firms prefer options to cash compensation? How do options affect the value of external shareholder's stocks? This paper examines these issues in a general equilibrium setting.

The model is based on the assumption that investors and firm insiders may have different opinions about the firm's future profitability. A difference in opinion implies that the market value of the firm may be different from the fundamental value perceived by the insider manager.[1] For example, if investors are more optimistic than the manager, the share price will be higher than the firm value per share. In this case, a rational manager would want to sell extra shares to investors. However, asymmetric information models such as Myers and Majluf (1984) show that an equity offering is generally a signal of market overvaluation and that the share price will decrease as a consequence. Empirical works by Loughran and Ritter (1995), Spiess and Affleck-Graves (1995) find that market reactions to seasoned equity offerings are negative. Therefore, the firm will incur an information cost if it sells equity directly.

Suppose the manager believes that investors' estimation of the firm value will vary in the future. Sometimes investors will overestimate the firm value and sometimes investors

[1] Here, fundamental value is the firm value perceived by the managers; it may or may not be the "true" value of the firm.

will underestimate it. Thus, from the manager's perspective, it is always possible that the market price will be higher than the fundamental value. The manager can either wait and issue new equity whenever investors are over-optimistic in the future, or the manager can use employee stock options to capture future investor overvaluation immediately. To understand the intuition, note that options by contract design will be exercised only when share price is above the strike price. The firm issues new shares to employees when options are exercised and employees can sell these new shares to investors. When new shares are issued, the firm receives cash proceeds from the exercise of options and employees get the difference between the market and strike prices. If employees exercise their options when the share price is high, the firm effectively sells over-valued equity to outside investors. Therefore, optimistic investors overpay for shares at the time of option exercise and effectively subsidize the firm by compensating employees.

This model generates a number of interesting implications. As long as investors' misperception is highly volatile, it might be optimal for firms to grant employee stock options even when they are currently undervalued. However, firms are more likely to grant options when they are overvalued at the time of option grant. Because employee stock options are generally issued at-the-money, high market valuation at the time of option grant sets a high strike price for options. This benefits the firm by keeping a larger part of new share issue proceeds during option exercise. On the other hand, both employee risk aversion and the volatility of firm fundamental value reduce the firm's incentive to grant options. In the equilibrium, the number of options granted is positively correlated with both the perception error of investors, and the volatility of this error, as well as the correlation between investors' error and firm fundamental value. The fundamental volatility reduces the equilibrium option grant.

Financial constraints introduce an interesting pecking order in terms of financing choices by the firm. That is, options are used first to fill small budget shortfalls, and equities are used next to raise large amount of money. Financing with options is more efficient because it has a lower information cost. In addition, because options bring the proceeds of selling future equity to the present, they are more effective when firms are moderately undervalued at the financing time.

There is already a large literature on employee stock options. Suggested motivations for broad-based stock options include incentive (Kedia and Mozumdor (2002)), liquidity constraint (Core and Guay (2001)), employee retention (Ittner, Lambert and Larker (2001), Oyer (2001)) and employee sorting (Lazear (2001), Oyer and Schaefer (2001)). This paper argues that options can be used to sell overvalued equity indirectly. Thus it complements this literature and provides a new rationale for broad-based employee stock options.

This paper is also related to several papers studying rational behavior in an irrational world. Stein (1996) studies a rational manager's capital budgeting choice when investors do not value stock correctly. Shleifer and Vishny (2001) model acquisitions as driven by market valuations. In their model, investors who buy overvalued shares of a merged firm are subsidizing the original shareholders of both the bidder firm and the target firm. In my model, the same investors are subsidizing both employees holding stock options and the original shareholders.

The paper is organized as follows: Section 2 describes the base case model. Section 3 solves the equilibrium and presents the main results for the model. The extension of the base case model to allow for equity offering, tax effects and financial constraints is presented in Section 4. Section 5 concludes the paper.

2 The Model

Consider a firm in a two-period economy, with time indexed by $t = 0, 1, 2$. The economy is populated with investors, a manager and employees. At time 0, the firm has liquid assets in place, A, and a positive net-present-value project. This project requires a number of employees to develop and market it, and generates an uncertain revenue of z_2 at time 2. The firm is liquidated to all shareholders after the revenue is generated. The project is human capital intensive, and the only cost of the project is employee compensation. Employees are homogeneous and risk averse. There is a large pool of employees from which the firm can hire.[2] Employees can earn an aggregate cash compensation of C_A payable at time 1 if they work elsewhere. Because the pool of available employees is large, the firm holds all the bargaining power. Hence the firm can either pay employees C_A in cash, or a combination of cash and stock options which provide employees the same expected utility as fixed compensation. For a sketch of the time line, see Figure 1. The interest rate is normalized to zero.

2.1 Beliefs about future revenue

The key assumption of the model is that investors and the manager have different beliefs about future revenue. This is modeled as follows: At time 1, the manager believes that the project revenue at time 2, z_2, has a normal distribution with mean z_1 and variance σ_1^2. z_1 is a random variable at time 0. The manager believes that z_1 is normally distributed with mean μ_z and variance σ_0^2. By iterating conditional expectation, it is easy to show that the manager's belief about z_2 at time 0 is that z_2 is normally distributed with mean μ_z and variance $\sigma_0^2 + \sigma_1^2$.

The investors' belief on z_2 at time 1 is different from the manager's. They believe that z_2 is normally distributed with mean w and variance σ_1^2. Note that investors and the manager have different mean estimates for z_2 but the same variance estimate. The same variance is a simplifying assumption but it does not affect the results, because in most of following analysis variance σ_1^2 does not enter the calculation. Let $d_1 = w - z_1$. Then, d_1 measures the difference between investors' belief and manager's belief. When d_1 is positive, investors are more op-

[2]This assumption gives all the bargaining power to the firm, so that the firm can extract all the gain from the project. If the number of employees qualified to work on the project is limited, then employees are likely to extract a share of the gain. To model this would require an explicit model on the bargaining game between the firm and its employees, and that is beyond the scope of this paper. However, our results are not sensitive to this assumption because as long as employees do not get all the surplus from the project, any action to increase the surplus is beneficial to the firm as well.

timistic than the manager, and when d_1 is negative, investors are more pessimistic. At time 0, w is a random variable and investors believe its probability density is also normal. Instead of specifying the mean and variance for w, we define parameters for the distribution of d_1. Because both w and z_1 are normal random variables at time 0, d_1 is also a random variable with normal density. Let the mean of d_1 be μ_d and the variance of d_1 be σ_d. Furthermore, let the correlation between d_1 and z_1 be ρ. That is,

$$\begin{bmatrix} z_1 \\ d_1 \end{bmatrix} \sim n\left(\begin{bmatrix} \mu_z \\ \mu_d \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \rho\sigma_0\sigma_d \\ \rho\sigma_0\sigma_d & \sigma_d^2 \end{bmatrix} \right). \tag{1}$$

Then it is easy to see that the mean and variance of w are $\mu_z + \mu_d$ and $\sigma_0^2 + 2\rho\sigma_0\sigma_d + \sigma_d^2$. By iterative conditional expectation, investors expect z_2 is distributed with mean $\mu_z + \mu_d$ and variance $\sigma_0^2 + 2\rho\sigma_0\sigma_d + \sigma_d^2 + \sigma_1^2$.

Investors' beliefs are common knowledge to everyone. The manager knows his own beliefs, but investors do not know, or consider, the manager's expectation of z_2. Investors are only concerned about their own projection of the future project revenue. As will become clear in the next section, investors here can be considered as a representative agent who determines the firm share price. There may be some investors who hold the same belief as the manager and some who hold different beliefs. As long as the belief of the representative investor is different from the manager's, the current model is valid.

In the majority of this paper, it is stated that the manager has the correct belief about future prospects of the firm and investors have wrong beliefs. In this sense, the manager is rational and investors are irrational. It does not affect any of the results if the reverse is true. That is, investors are perfectly rational and have the correct assessment of the profitability of the firm, while the manager is irrational.

Employees are assumed to sell stocks immediately after option exercise.[3] Because of this, employees' beliefs about the project outcome do not affect the model outcome. However,

[3]This assumption is made to make the computation of employee option value tractable. However, because most frequent option users do not pay any dividend, it is optimal to sell stocks after exercising non-expiring

employees understand the belief system of investors and they are able to correctly assess the distribution of future share price. From this distribution, employees can calculate the expected payoff of their options.

2.2 Stock and share price

The firm's stock is traded in the market at time 0 and time 1. The firm is liquidated to all shareholders at time 2, after the realization of revenue z_2. At time 0, the number of shares outstanding is normalized to 1. Stocks are held by investors only, and investors are assumed to have the following demand curve:

$$D_0 = \frac{1}{\gamma_0}(F_0^I - P_0), \tag{2}$$

$$D_1 = \frac{1}{\gamma_1}(F_1^I - P_1), \tag{3}$$

where D_0 and D_1 are investor demands at time 0 and time 1, γ_0 and γ_1 denote the slopes of the downward-sloping demand curves at time 0 and 1 respectively. F_0^I and F_1^I are the expected liquidation value of one unit of stock based on investors' expectation at the two dates, while P_0 and P_1 are share prices. This type of demand function can be derived if investors are assumed to have exponential utility and fundamental values are normally distributed. Kyle and Xiong (2001) apply the same demand function for long term investors in their model. For empirical evidence on the shape of stock demand curves, see Shleifer (1986), Kaul, Mehrotra and Morck (2000). Furthermore, γ_0 is assumed to be greater than γ_1. That is, investors require a larger discount at time 0 than at time 1. This is a reasonable assumption because the variance on project revenue at time 0 is greater than the variance at time 1. In the case of exponential utility and normality, it can be shown that γ is positively correlated with the variance of the fundamental value.

options. Actually, most employees sell stocks immediately following option exercise (Heath, Huddart and Lang (1999)).

Share prices are determined by the market clearing condition. If the firm does not grant options or engage in any equity offering, the supply is 1. Otherwise, the supply is 1 plus the number of options exercised or the number of shares offered. We will focus on options in this section and address equity offerings in Section 4.

2.3 Options

The manager might grant stock options to employees at time 0. These options mature at time 1 and they have strike price K. Employees have perfect knowledge of investors' revenue expectations and demand curves. At time 1, employees decide whether to exercise the options, and share prices are then set by the market clearing condition. The share supply increases if options are exercised and remains constant otherwise. Because employees are rational and have all the necessary information to compute the price impact of option exercise, they will choose to exercise their options only if the share price after exercise is higher than the strike price. In addition, it is assumed that options are held by a diverse group of employees, and that their competition in exercising options would cause either all options to be exercised or none to be exercised. This rules out the scenario where part of the options may be exercised.

Suppose that the firm pays employees with x options and cash amount C. First, the condition for option exercise at time 1 needs to be determined. If options are exercised, share price P_1^e is determined from the following market clearing condition,

$$\frac{1}{\gamma_1}(F_1^{Ie} - P_1^e) = 1 + x \tag{4}$$

where F_1^{Ie} is the expected fundamental value of one share of stock given a share increase of x, in particular,

$$F_1^{Ie} = \frac{A+w+xK-C}{1+x}. \tag{5}$$

In Equation (5), the expected liquidation value of the whole firm is the sum of initial asset A, (investors') expected project revenue w, and the cash inflow from option exercise xK, less the cash compensation paid to employees C. Because of option exercise, there is a dilution effect and the total number of shares outstanding becomes $1+x$. Hence the value for each share is simply the firm value divided by the number of shares.

Substitute F_1^{Ie} in Equation (4) and the share price after option exercise can be solved,

$$P_1^e = \frac{A-C+w+xK}{1+x} - \gamma_1(1+x) \tag{6}$$

Employees exercise options only when they are in-the-money. Thus the condition for option exercise is $P_1^e \geq K$, or equivalently,

$$w \geq K - A + C + \gamma_1(1+x)^2 \tag{7}$$

The expected value of each option at time 0 is then the expected option payoff over the random variable w. Let $Q = K - A + C + \gamma_1(1+x)^2$. Recall that w is normally distributed with mean $\mu_w = \mu_z + \mu_d$ and variance $\sigma_w^2 = \sigma_0^2 + \sigma_d^2 + 2\rho\sigma_0\sigma_d$. Hence the expected option value can be rewritten as

$$\begin{aligned} E_0[(P_1^e - K)^+] &= \int [\tfrac{A-C+w+xK}{1+x} - \gamma_1(1+x) - K]^+ f(w)\, dw \\ &= \tfrac{1}{1+x} \int [w - Q]^+ f(w)\, dw \\ &= \tfrac{1}{1+x} [\sigma_w n(G) + (\mu_w - Q) N(G)] \end{aligned} \tag{8}$$

where $G = \frac{\mu_w - Q}{\sigma_w}$, $n(\cdot)$ and $N(\cdot)$ are the probability density function and cumulative probability function of the standard normal distribution. The last equation uses the result of integrating normal random variable in the Appendix.

Employees are risk averse, and they have all the information to compute this expected value of options. Instead of assuming an explicit utility function for employees, we follow a simple approach to denote the cash equivalent value of options as the expected option value multiplied by a discount factor η, $(0 < \eta \leq 1)$. This η captures the risk aversion of employees. If η is 1, employees are risk neutral. If η is small, then employees are risk averse and the cash value equivalent to the variable option income is low.

2.4 Share value at time 0

The share value at time 0 is the discounted expected liquidation value at time 2. When options are exercised, this value is $\frac{A-C+xK+z_2}{1+x}$. When options are not exercised, this value is $A-C+z_2$. The expected liquidation value of one share of stock is thus

$$E_0^j[V_2] = E_0^j\left[\frac{A-C+xK+z_2}{1+x} I(w \geq Q) + (A-C+z_2)I(w < Q)\right], j \in \{M, I\}. \tag{9}$$

The expectation is different for the manager and investors because they have different beliefs about z_2.

For the manager, the belief at time 0 is that z_2 is normally distributed with mean μ_z and variance $\sigma_0^2 + \sigma_1^2$. It can be shown that the covariance between z_2 and w, $\sigma_{z_2 w}$, is $\sigma_0^2 + \rho\sigma_0\sigma_d$. Applying the integration results for the normal distribution (see the Appendix), the expected value of one share from the perspective of the manager becomes

$$E_0^M[V_2] = A - C + \mu_z + \frac{x}{1+x}\left[(K - A + C - \mu_z)N(G) - \frac{\sigma_{z_2 w}}{\sigma_w} n(G)\right]. \tag{10}$$

On the other hand, investors project that the mean of z_2 is $\mu_z + \mu_d$. Their belief on the variance of z_2 is $\sigma_w^2 + \sigma_1^2$. Thus the expected value of one share to investors is

$$E_0^I[V_2] = A - C + \mu_z + \mu_d + \frac{x}{1+x}\left[(K - A + C - \mu_z - \mu_d)N(G) - \frac{\sigma_{z_2 w}}{\sigma_w} n(G)\right]. \tag{11}$$

2.5 The manager's optimization problem

The manager attempts to maximize the expected liquidation value for long-term shareholders at time 0. This is equivalent to maximizing the per-share liquidation value of the stock. Suppose the manager owns a certain number of stocks at time 0 and is restricted from trading at time 1. In this case, the exercise of employee stock options will cause a dilution in the manager's ownership of the company. The manager's self-interest leads to maximizing the manager's own value at liquidation time.

The simplest scenario is considered first. In this scenario, the firm is not cash constrained, that is, $A \geq C_A$. The strategy of paying employees C_A and pursuing the project would enhance the firm value by an expected amount of $E_0^M(z_2) - C_A$. Now the manager might grant x options as part of the compensation package and reduce the cash payment to C. Hence the manager's optimization problem becomes

$$\max_{C,x} E_0^M[V_2] \tag{12}$$

subject to

$$C + \eta x E_0[(P_1^e - K)^+] \geq C_A \tag{13}$$

$$C \geq 0, x \geq 0. \tag{14}$$

Equation (13) is the participation constraint of the employees. The cash salary and options must be high enough to be equivalent to employees' outside opportunities, C_A.[4]

3 General results

3.1 Predetermined option strike price

First, consider the scenario where the option strike price is predetermined when the manager grants options. In this case investors do not anticipate that the firm will issue options at time 0. Hence, given that investors' projected revenue mean is $\mu_z + \mu_d$ and that the demand curve slope is γ_0, the stock price without the option grants would be

$$P_0^{no} = (A - C + \mu_z + \mu_d) - \gamma_0. \tag{15}$$

[4]It does not affect the results qualitatively if the participation constraint is changed to a traditional expected utility type.

This price is derived from the expected fundamental value of a share from investors' perspective, with the total supply of shares normalized to 1. To conform to the common practice of issuing at-the-money options,[5] the strike price K is set to equal P_0^{no}.

The equilibrium is where the manager optimizes his expected value per share (Equation (12)) with the choices of C and x subject to the employee participation constraint (Equation (13)), and the stock market clears. The equilibrium is solved numerically by finding solutions to the first order conditions of the optimization problem (Equation (12)). For more details, see the Appendix.

3.1.1 When to grant options

The first result is that options can improve shareholder values under certain circumstances. This can be seen from Figures 2 and 3. These two figures show the thresholds of investor misperception and future volatility of misperception (μ_d, σ_d), when options are granted in the equilibrium. Because the manager maximizes shareholder value and he has the choice of whether to grant options, observing options in the equilibrium indicates that employee stock options enhance firm value in these cases.

The main intuition is that options enable the firm to sell overvalued equities at time 1. To see this, note that the contract design of options indicates that options are exercised only if the time 1 stock price is higher than the strike price. From the manager's perspective, investors are sometime optimistic and sometime pessimistic. Since when investors are over-optimistic, stock prices are going to be high. The conditional probability of investors being over-optimistic is high when stock prices are higher than the strike. Hence, option exercise is

[5]In practice, almost all employee stock options are issued at-the-money. Employee stock options that are not at-the-money are less attractive to the firm for various reasons. The in-the-money options (i.e., discount options) usually carry a tax disadvantage. Hall and Murphy (2000) show that out-of-the-money options (i.e., premium options) are very costly because the cash equivalent values of out-of-the-money options are substantially low to risk-averse employees.

more likely to coincide with overvalued stocks. Unlike a direct equity offering, where the firm receives the proceeds, employees actually obtain the proceeds of option exercise and the firm receives the amount up to the strike price. However, since employees are expected to break even in expected utility with or without options, the firm retains a part of the option exercise proceeds through reduction of the employees' cash salaries. The firm does not receive all of the proceeds from the indirect equity sale through option exercise because employees are risk averse and they consider the varying option income less valuable than a fixed wage. A number of parameters affect the firm's willingness to grant options in equilibrium. These are illustrated in Figures 2 and 3. One natural question is why the firm does not issue equity when investors are overoptimistic. This question is addressed in the next section. For now, issuing options is the only strategy available to the firm.

Figure 2 considers the base setup in Section 2. The manager might not agree with investors, reflected in a non-zero value of μ_d at time 0 and time 1. The manager also anticipates a variation of this future difference given by σ_d. Figure 2 depicts the regions in the graph of μ_d and σ_d where options are granted in equilibrium. Panel (a) is the base case with the following set of parameters: $A = 70, C_A = 70, \mu_z = 100, \sigma_0 = \sigma_1 = 20, \eta = 0.9, \gamma_0 = 8, \gamma_1 = 4, \rho = 0$. The project yields an average profit of 30. For a summary of all the parameters used in the model, see Table 1.

The first observation is that options are granted when investors are more optimistic than the manager, or when the volatility of investor misperception is high. When investors' projection on firm profitability is higher than the manager's, equity carries a high valuation and it is logical for the firm to issue options to sell more shares. However, when investors underestimate the firm's profitability, both at the current time and in the future, granting options might still be optimal if the volatility of investors' misperception is sufficiently high. For example, in the base case, if the investors' expectation is ten below the manager's, the firm still issues options as long as the standard deviation of future misperception is higher than 28.5. The intuition here is that the volatility more than offsets the lower projection. Call options by design capture only the positive half of the distribution. Large volatility in investor misperception enlarges the positive half of the price distribution and thus increases the attractiveness of options to the firm. This result has an interesting practical implication. The manager can choose to grant options even if he is not sure whether the firm is currently overvalued or undervalued. As long as the manager is sure that investors' projections on firm profitability are highly volatile, he can be confident that options will enhance shareholder value.

Panel (b) of Figure 2 shows the effect of changing employee risk-aversion parameter η. Note that the further η is from one, the more risk averse the employees are. Because employees get the same utility with or without options, higher risk aversion results in the firm obtaining a smaller part of the option exercise proceeds. This can be inferred in Figure 2, where high risk aversion $(\eta = 0.85)$ pushes the option threshold northeast (higher relative investor projection or greater volatility of misperception).

Panel (c) illustrates the effect of the correlation between fundamental value and investors' misperception, and Panel (d) addresses fundamental volatility. Both correlation and fundamental volatility affect the option-issuing decision by changing the relative importance of misperception volatility in determining the option value. Negative correlation implies that investors tend to be pessimistic when firm fundamental values are strong. This creates a damping effect on the share price and discourages the firm from granting options. Positive correlation has the opposite effect. Thus in Panel (c), the threshold line for positive correlation lies below the threshold line for negative correlation, indicating that a firm grants options under a wider range of parameters when the correlation between fundamental value and investors' misperception is positive.

Fundamental volatility affects the firm's option granting decision negatively. Both fundamental volatility and misperception volatility contribute to the volatility of share prices at time 1. However, only the misperception volatility is beneficial to the manager. Option exercise or, equivalently, new stock issue, benefits the firm when investors overestimate the firm profitability at time 1. Hence, holding the misperception volatility constant while increasing fundamental volatility actually reduces the relative weight of investor misperception in share prices. Therefore, it is not surprising that high fundamental volatility makes the firm more selective in granting options.

In the scenario of Figure 2, the manager holds a different belief from investors and this belief does not change from time 0 to time 1. This is similar to the “random walk” assumption from the efficient market literature.[6] The manager may believe that the current price reflects all the information available to investors and, without further information, investors should hold the same view about the firm.

On the other hand, the manager might believe that investors' beliefs are “mean reverting,” that is, $\mu_d = 0$. In other words, the manager expects that, on average, investors' perception is the same as the manager's in the future (at time 1). However, the manager still expects a certain level of variation of investors' estimate. In addition, the mean estimate of z_2 at time 0 may be different between investors and the manager. Let d_0 be this difference. In this case, the current misperception of the investors serves only to determine the strike price of options.[7]

Figure 3 illustrates the results under “mean-reverting” investors' beliefs. Note that the threshold lines never cross the $\sigma_d = 0$ axis. This implies that some positive misperception volatility is required for the firm to consider granting options. This is not true in Figure 2 because with μ_d positive, the manager already expects investors to overvalue the firm at time 1. Hence, positive volatility of investor misperception is not a necessary condition for the firm's choice to grant options there.

Furthermore, it is optimal for the firm to grant options when the firm is undervalued ($d_0 < 0$) at the time of option grant. Actually, because the current undervaluation does not affect future investor expectation, d_0 has a less pronounced impact on option grant decision in Figure 3 than it has in Figure 2. The fact that d_0 still has some impact in Figure 3 is caused by

[6] See Fama (1970, 1991) for detailed reviews.

[7] In this case, investors do not follow iterative expectation. Nevertheless, this case is studied because it is common belief that prices will come back to fundamental values in the long run.

the option strike price, which is determined by d_0. The effects of risk aversion, correlation between fundamental value and misperception, and fundamental volatility are about the same as in Figure 2. The intuitions are the same as well.

3.1.2 Comparative statistics

The comparative statistics of equilibrium option grant with respect to variables of interest are shown in Figure 4. Panel (a) shows the relationship between option grant and current investor perception. Current investor perception affects the current valuation of the equity, and thus it also influences the relationship between equilibrium option grant and current market valuation. Two scenarios are considered, persistent difference of beliefs ($\mu_d = d_0$) and long-run convergence of beliefs ($\mu_d = 0$). In both scenarios, when investors move from undervaluing the project to slightly overvaluing it, the optimal option grant increases rapidly, but when investors overvalue the firm by a large margin, the optimal option grant position continues to increase, but at a much slower rate. Note that increasing the number of option grants always reduces the probability that options will be in-the-money because of the market impact of options. Initially, when the number of outstanding options is small, the value increase from the number of options can fully offset the reduction in per-unit option value. When the number of options is large, the incremental effect of an extra option can no longer offset the per-unit value reduction. Hence the equilibrium option grant has this concave shape with respect to investors' misperception.

There is some difference between the two scenarios. When the mean misperception stays the same, the firm starts to grant options when μ_d is around -4 and increases the number of options granted rapidly as μ_d increases. When beliefs are converging ($\mu_d = 0$), the lowest d_0 for firm to grant options is -11. However, the number of option grants is less than it is in the other case, when d_0 becomes positive. This is caused by the difference in μ_d. Positive μ_d implies that investors are likely to overvalue the firm at time 1 and therefore the firm wants to issue more options to take advantage of that.

Panel (b) shows the comparative statistics between option grant and correlation between fundamental and misperceived values. There is a positive relationship. The intuition is the same as we have discussed previously. Positive correlation between fundamental value and misperception creates an amplifying effect between fundamental value and share price, while negative correlation has a damping effect on share price. When the firm fundamental value and misperception move in the same direction, the share price varies more and the misperception part of the share price is high when the price is high. Because high share price coincides with the time of option exercise, options are able to capture more of mispricing when the correlation between fundamental value and misperception is positive. Therefore, in the equilibrium, the number of option grant and correlation between fundamental and misperceived values is thus positive.

The effect of fundamental volatility on equilibrium option grant is shown in Panel (c). The trend is negative and option grant drops to zero after the fundamental volatility become too large. On the other hand, the volatility of misperception has the opposite effect, as shown in Panel (d). Both of these effects are driven by the same reason. Options are granted to sell overvalued equity indirectly. The key feature of the option contract is that it becomes an equity offering when the share price is higher than the strike price. If shares are overvalued when share price is higher than the strike price, then options are effective as a method to sell overvalued equity. The two volatility measures contribute to the relative probability of whether shares are overvalued during option exercise. High misperception volatility increases the overvalue probability while high fundamental volatility decreases this probability. This is why equilibrium option grant decreases with fundamental volatility but increases with misperception volatility.

3.1.3 Price effect

In the analysis so far, investors have a misperception μ_d on the firm profitability. Not anticipating option grant, a share price is available from the demand equation (2). The manager makes the option grant decision setting the strike price to the existing share price. Because the manager optimizes the stock value of shareholders from his own projection, obviously the firm value is enhanced from the manager's perspective. One interesting question is what the share price is after investors observe that the firm is granting options. In other words, what is the announcement effect of option grant in this equilibrium? To answer this question, first calculate the fundamental value of one share from the perspective of investors, $E_0^I[V_2]$, using Equation (11). Then, compute the share price after option grant using the same demand equation. The difference in share values is the announcement effect in this equilibrium.

Figure 5 illustrates the announcement effect for the scenario of persistent belief difference. As can be seen from the figure, there is virtually no announcement effect from the investors' perspective. Remember that investors believe that the project revenue has mean $\mu_z + \mu_d$. To them, stocks are always fairly valued and there is no merit in issuing options to try to capture future overvaluation. Hence the announcement effect is almost nonexistent.

3.2 Endogenous strike price

In the previous section, the option strike price is predetermined, which corresponds to the scenario that investors do not anticipate the firm to grant options. Suppose investors fully anticipate the firm to grant options and set share price accordingly at time 0, then the option strike price will be determined endogenously. The equilibrium is constructed as follows: first the manager decides to grant x at-the-money options and pay cash compensation C, then investors determine the share value after option grant and the share price from market clearing condition and, finally, the option strike price must be equal to the share price at time 0. The equilibrium is solved by adding the constraint on strike price to the maximization program:

$$K = E_0^I[V_0] - \gamma_0. \tag{16}$$

Figure 6 shows the comparative statistics of equilibrium option grant with respect to mean misperception (μ_d), correlation between fundamental value and investor misperception (ρ), fundamental volatility (σ_0), and volatility of misperception (σ_d). The results are very similar to the results when the strike price is predetermined as in Figure 4. This is related to the previous result of virtually no price effect. The share value according to investors remains almost the same before and after option grant. Hence the endogenously determined strike prices do not differ too much from the predetermined strike prices. The comparative statistics are then similar.

Before we proceed to extend this model, it is helpful to summarize the results so far:

1. When the manager perceives the firm profitability differently from investors, it might be optimal for the manager to grant employee stock options to capture future overvaluation.

2. As long as the future volatility of investor misperception is large enough, granting options is optimal even when the firm is currently undervalued.

3. The decision to grant options is positively correlated with current and future misvaluation, volatility of misperception, and correlation between fundamental value and misperception, but negatively correlated with fundamental volatility and employee risk aversion.

4. The equilibrium number of grant options is positively correlated with current and future misvaluation, volatility of misperception, and correlation between fundamental value and misperception, but negatively correlated with fundamental volatility.

4 Extensions

4.1 Equity offering

In the base model, the manager is not allowed to issue equity to capture the overvaluation of the equity. A natural question is whether allowing the firm to issue equity changes the results. The answer to this question depends on when an equity offering might occur and the cost of an equity offering. In this model, because trading occurs at times 0 and 1, it seems logical to allow the firm to sell new shares at these two time points.

First, consider the case of equity offering at time 1. The sequence of events at time 1 can be modeled as follows: z_1 and d_1 are realized, the firm decides how many new shares to sell to the market, investors determine the fundamental value of each share and their demand, market clears by equating supply and demand, and share price is determined. If there is no information cost from the equity offering, that is, the new equity does not change investors' projection on firm profitability, $z_1 + d_1$, then options are no longer needed to capture the market overvaluation at time 1. The main rationale for the use of options in this model is to sell overvalued equity at time 1 indirectly. If a direct sale does not reduce the overvaluation d_1, then direct equity offering would be more efficient to the firm.

However, both theory and empirical evidence argue that seasoned equity offerings do carry negative information cost. For example, Loughran and Ritter (1995), and Spiess and Affleck-Graves (1995) found that market reactions to seasoned equity offerings are negative. The asymmetric information models such as Myers and Majluf (1984) show that equity offering is generally a bad signal and thus new equities are discounted by the market. Actually, in the current setup, because the manager observes d_1 before deciding whether to issue new shares and he will only do so if d_1 is positive, equity offering is a signal for $d_1 > 0$. Thus it is reasonable to investors to adjust downward d_1 after they see new shares coming to market. We model this as

$$d_1'(y) = d_1 - \beta y \tag{17}$$

where y is the number of new shares offered ($y \geq 0$) and β is the rate that investors adjust their beliefs, ($\beta > 0$). Note that option exercise is conditioned only on the stock price being higher than the strike price, and it does not reveal anything about the manager's belief. Hence the current assumption of no information cost on option exercise is valid.[8]

[8] In practice, employees exercise their options for various reasons and thus may not necessarily reflect private information about market valuation. See Heath, Huddart and Lang (1999) for a discussion of factors that influence employee option exercise.

To solve the optimal option grant and cash compensation for the manager (Equation 12), given that the firm can judiciously issue new equity at a later time, is extremely complicated. Here we are only going to show that the firm might still want to use options when it has the choice to sell new shares with some information cost. The purpose is to show that there are some strategies with non-zero amount of options and cash compensation that satisfy employee participation constraint and produce higher shareholder values.

At time 1, z_1 and d_1 are realized and option amount x, option strike K, and cash compensation C are already set. The manager needs to decide the optimal amount of shares to issue. Suppose the number is y and share price is P_1. Then, the fundamental value of one share to investors is

$$F_1^I = \frac{A-C+z_1+d_1'(y)+xKI(e)+yP_1}{1+xI(e)+y}, \tag{18}$$

where $I(e)$ is 1 for option exercise and 0 for no option exercise. Using the demand function of investors (Equation 3) with the supply of all shares $1+xI(e)+y$, the market clearing condition is

$$\frac{1}{\gamma_1}\left(\frac{A-C+z_1+d_1'(y)+xKI(e)+yP_1}{1+xI(e)+y}-P_1\right) = 1+xI(e)+y. \tag{19}$$

Thus, P_1 can be solved as a function of y and the manager attempts to choose y to maximize the share value from his perspective,

$$\max_y F_1^M = \frac{A-C+z_1+xKI(e)+yP_1}{1+xI(e)+y}. \tag{20}$$

Let y^* be the solution to Equation (20). In this way, y^* is the optimal number of new shares to issue, given a realization of z_1 and d_1. The option payoff is simply

$$V^o = (P_1-K)^+ I(e) \tag{21}$$

where option exercise $I(e)$ is determined by new shares and share price at time 1.

The values $E_0^M(V_2)$, $E_0^I(V_2)$, and $E_0(V^o)$ are expectations of F_1^M, F_1^I and V^o at time 0. Because there are no close-form solutions for these expectations, we use Monte Carlo integral to compute these expectations. In particular, 2000 draws of z_1 and d_1 are generated given their mean values and variance-covariance matrix.[9] For each draw, optimal y^* is found, and share values according to the manager, investors, and option value are computed. Then, these values are averaged over all 2000 draws and generate $E_0^M(V_2)$, $E_0^I(V_2)$, and $E_0(V^o)$.

In Table 2, a number of feasible strategies with different option grant levels x, cash compensation C and their corresponding share values are presented. These strategies are found using the following method: fix x and C, compute the expectations using Monte Carlo integral, check to see if employee participation constraint is satisfied, that is,

$$C + \eta x E_0(V^o) \geq C_A, \tag{22}$$

finally adjust C until this condition is just satisfied. As shown from the table, a small amount of option grant x generates higher shareholder value than no option grant, while a large amount of option grant generates lower shareholder value. Hence, as long as future equity offering has some informational cost, the result that moderate option grant can enhance shareholder value does not change.

Because options are used mainly to capture future share overvaluation, allowing equity offering at time 0 does not change the result at all. For example, when the volatility of misperception is large enough, the manager may grant options even if stock is undervalued at time 0. Clearly, no rational manager would choose to offer equity if he thinks the stock is undervalued and the firm is not financially constrained. We address this issue of financial constraint and financing choices between equity and option in Section 4.3.

4.2 Tax treatment of options

Another issue that is omitted in the base model is tax. Since most of employee stock options are nonqualified stock options, these options have interesting tax implications for both the firm and employees. When nonqualified stock options are exercised, the difference between share

[9]The draws are generated by using 1000 random draws and 1000 antithetic values.

price and option strike price is counted as ordinary employee compensation and subject to personal income tax in the year of option exercise. On the other hand, the firm can deduct this amount as cost and reduce its tax liability. Hence, the tax treatment of employee stock options appears to make them more favorable to the firm and less so to employees. However, since employees always break even, the net tax effect on the firm's willingness to grant options is not clear.[10]

In this section, we will show that the main result of this paper–employee stock options can enhance shareholder value–does not change after considering tax issues. Consider the following scenario: the firm is required to pay taxes on its profit at a rate of τ_c; employees are taxed at a rate of τ_p for income above C_A and not taxed for income less than C_A. The personal tax schedule is an oversimplified one but it captures the main spirit of the personal income tax system: that is, large extra income may bump one into a high tax bracket and be subject to higher tax rate. For the firm, the cost is the cash compensation C plus the deduction for option exercise.

Given realization of z_1 and d_1 at time 1, the firm's tax liability and share price are closely related. Investors perceive the share value as

$$F_1^I = \frac{A-C+E_1^I(z_2)+xKI(e)-E_1^I(tax)}{1+xI(e)}, \tag{23}$$

where $I(e)$ is 1 for option exercise and 0 for no option exercise, $E_1^I(z_2)$ is investors' expectation of project revenue, and $E_1^I(tax)$ is the expected tax liability. Since the firm is liquidated at time 2, there is no tax carryover in this model. The firm only needs to pay tax on its profit–the amount of z_2 over cash compensation C and option exercise deduction $xI(e)(P_1-K)^+$. Hence, the expected tax liability is simply

$$E_1^I(tax) = E_1^I\{\tau_c[z_2 - C - xI(e)(P_1-K)^+]^+\}. \tag{24}$$

[10]For an empirical analysis of tax impact of employee stock options, see Graham, Lang and Shackelford (2002).

Investors believe that z_2 is normally distributed with mean $z_1 + d_1$ and variance σ_1^2. Using the integration result in the Appendix, the expected tax liability can be shown as

$$E_1^I(tax) = \tau_c \sigma_1 [n(J) + JN(J)] \tag{25}$$

where

$$J = \frac{z_1 + d_1 - C - xI(e)(P_1 - K)^+}{\sigma_1}. \tag{26}$$

Now that the share value to investors is expressed in term of share price P_1 and other known parameters at time 1, the share price P_1 can be solved using the investor demand function and market clearing condition,

$$\frac{1}{\gamma_1}\left\{\frac{A - C + E_1^I(z_2) + xKI(e) - E_1^I(tax)}{1 + xI(e)} - P_1\right\} = 1 + xI(e). \tag{27}$$

Solving Equation (27) for P_1 and option exercise decision, the share value to the manager can be computed,

$$F_1^M = \frac{A - C + E_1^M(z_2) + xKI(e) - E_1^M(tax)}{1 + xI(e)}. \tag{28}$$

Note that both the expected project revenue $E_1^M(z_2)$ and expected tax liability $E_1^M(tax)$ are different because the manager's belief about z_2 is normal distribution with mean z_1 and variance σ_1^2. The employee after-tax compensation is

$$Comp_1 = C + \eta\{x(P_1 - K)I(e) - \tau_p[C + x(P_1 - K)I(e) - C_A]^+\} \tag{29}$$

where $\tau_p[C + x(P_1 - K)I(e) - C_A]^+$ is the personal tax liability caused by option exercise, and η is the risk aversion discount factor and is applied on the variable income.

The share values perceived by investors and the manager, respectively, and the expected after-tax compensation for employees at time 0 are the expectation over their corresponding time 1 values. Again, because the expectations can not be solved analytically, they are computed using Monte Carlo integration. The same method is used to find feasible strategies given different option grant amount that satisfy employees' participation constraint. Table 3 illustrates such feasible strategies. Note that the share value perceived by the manager increases with option grant and peaks at around $x = 0.5$. This shows that the tax effect does not change the main result of the paper qualitatively.

4.3 Financial constraints

In the analysis so far, the firm is not financially constrained as its liquid asset A is enough to pay for employee salary C_A. In this section, the scenario of financial constraint is discussed, that is, $A < C_A$. Because employee stock options reduce the cash salary of employees, they are useful in helping firms with financial constraint. The firm is also allowed to issue equity at time 0 so that the questions of when to grant options and when to issue equity can be addressed. To keep the problem manageable, the firm is not allowed to issue equity at time 1.

At time 0, given liquid asset A, the firm decides to issue y_0 amount of new equity and x amount of options to employees. Investors compute the value of each share and provide their demand for stocks. The market clearing condition at time 0 yields the share price P_0. Equilibrium occurs when (1) the manager maximizes the stock value of shareholders, (2) the market clears, (3) employees are paid with the same expected utility, and (4) the liquid asset plus proceeds from the equity offer is enough to pay employee cash salary.

Consider the situation at time 1 first. Suppose that the firm has granted x options to employees and issued y_0 in equity. If the share price at time 0 was P_0, then the firm has liquid asset $A + y_0 P_0$. The option strike price K is also equal to P_0 because options granted at-the-money. The number of shares outstanding is therefore $1 + y_0$ when options are not exercised and $1 + y_0 + x$ when options are exercised. The share value according to investors is then

$$F_1^I = \frac{A + y_0 K - C + E_1^I(z_2) + xKI(e)}{1 + xI(e) + y_0}. \tag{30}$$

This value is then used in determining the demand curve of investors, and market clearing condition is used for setting the price P_1. The derivation of expected option value is very similar to the one in the base model, and we will only state the results here.

$$E_0[V^o] = \frac{\sigma_w}{1 + y_0 + x}[n(G') + G'N(G')] \tag{31}$$

where w is the investors' expected revenue at time 1, μ_w and σ_w^2 are the mean and variance for w and derived in the previous section, $G' = \frac{\mu_w - Q'}{\sigma_w}$ and $Q' = -A + C + K + \gamma_1(1 + y_0 + x)^2$. Options are exercised only when $w > Q'$.

Given this result for time 1, the share value at time 0 can be expressed as

$$\begin{aligned} E_0^j[V_2] &= E_0^j\left\{\tfrac{A-C+(y_0+x)K+z_2}{1+y_0+x}I(w \geq Q')\right. \\ &+ \left.\tfrac{A-C+y_0K+z_2}{1+y_0}I(w < Q')\right\}, j \in \{I, M\}. \end{aligned} \tag{32}$$

The manager believes that z_2 has mean μ_z and variance $\sigma_0^2 + \sigma_1^2$. Investors believe that the mean of z_2 is different. In the base model, this mean is $\mu_z + \mu_d$. Here, equity offering is allowed. Because, as argued before, equity offering is likely to have an information cost which reduces investors' optimism on the project, investors' belief on the mean of z_2 is set as $\mu_z + \mu_d - \beta y_0$. Using the integration results in the Appendix, it can be shown that

$$E_0^M(V_2) = \tfrac{S+y_0K}{1+y_0} + \tfrac{x}{(1+y_0+x)(1+y_0)}[(K-S)N(G') - \tfrac{\sigma_{z_2w}}{\sigma_w}n(G')] \tag{33}$$

$$\begin{aligned} E_0^I(V_2) &= \tfrac{S+y_0K+\mu_d-\beta y_0}{1+y_0} + \tfrac{x}{(1+y_0+x)(1+y_0)}[(K-S-\mu_d+\beta y_0)N(G') \\ &- \tfrac{\sigma_{z_2w}}{\sigma_w}n(G')] \end{aligned} \tag{34}$$

where $S = A - C + \mu_z$. The market clearing condition at time 0 is

$$\tfrac{1}{\gamma_0}(E_0^I(V_2) - P_0) = 1 + y_0 \tag{35}$$

and $P_0 = K$. Now the equilibrium condition can be written as the following constrained optimization problem:

$$\max_{x,C,y_0} E_0^M(V_2) \tag{36}$$

subject to

$$C + \eta x E_0(V^o) \geq C_A \tag{37}$$

$$\tfrac{1}{\gamma_0}(E_0^I(V_2) - K) = 1 + y_0 \tag{38}$$

$$A + y_0K \geq C. \tag{39}$$

The first constraint (37) is the participation constraint of employees, the second (38) is the market clearing condition and the third (39) is the liquidity constraint.

Table 4 presents numerical results for five different cases ranging from severe investor undervaluation to extreme overvaluation.[11] In Panel A, investors underestimate the project value by a large margin, and they are also likely to underestimate in the future; in this case, equity is the only viable method of financing. Options are not used because their values are too low given that the time 1 price is likely to be low. The last column in the table shows the difference between the manager's valuation of one share $E_0^M(V2)$ and initial liquid asset A. Without any financing cost or gain, this value should be the net present value of the project (30). As can be seen in Panel A, when the firm faces financial constraint, this value is less than NPV of the project because of the cost of equity financing.

Panel B presents the scenario for moderate undervaluation by investors. In this case, options are used initially to finance a project when the firm needs only a small amount of financing. As the amount of financing needed increases, equities are used much more aggressively while options increase only slowly.

The results for fair value case are presented in Panel C. Note that, when the firm is not financially constrained, options are used by the firm but no equity is issued. This is the case of the base model. Options are used to capture future overvaluation and can enhance shareholder value. The value $E_0^M(V_2) - A$ is higher than the project NPV. As the firm requires more and more funds, options are used as the first choice of financing and equities are issued only when options cannot satisfy the financing need.

Panel D and Panel E show the results for the scenarios where investors overestimate the project profitability. In both scenarios, options are granted when the firm has enough cash to pay employees. Equity is not issued in the moderate overvaluation case, and a small amount of equity is issued in the extreme case. Without financial constraints, the setup is the same with the base model with the addition of equity offering at time 0. The fact that options are used in these scenarios illustrates that the main results developed in Section 3 are robust to allowing equity offering at time 0.

[11] The numerical optimization is done using Quasi-Newton method with nonlinear constraints.

As more funding is needed for operation, options are again the first choice to cover the initial financial requirement. Equity offering is increased only after the cost saving provided by options is exhausted. Hence, a pecking order of financing choices emerges for the firm: use options first and equity second. This result is not surprising, given the result that options can enhance shareholder value under a wide range of conditions ranging from undervaluation to overvaluation. On the other hand, equity can be beneficial only in the case of high overvaluation. It is expected that employee stock options will be used as the first choice for financing.

5 Conclusion

This paper proposes a new rationale for employee stock options, based on differences in perception between the insider manager and investors. When the manager anticipates the stock to be possibly overvalued, he might grant employee stock options to save cash compensation costs. If employees exercise their options when share prices are high, the firm effectively sells overvalued equity through option exercise to outside investors. Therefore, options may enhance the stock value for current shareholders.

The model generates a number of interesting implications. Firms are more likely to grant options when they are overvalued both currently and in the future, or when volatility of investors' misperception is high. The equilibrium number of options granted is positively correlated with current and future misvaluation, volatility of misperception, and correlation between fundamental value and misperception, but negatively correlated with fundamental volatility. Future research can test these implications empirically. Another extension is to study other corporate decisions in a world with heterogeneous beliefs. It would be interesting to model corporate governance and agency problems when investors and managers hold different beliefs and these beliefs change frequently.

6 Appendix

6.1 Integration results for normal distribution

Suppose the distribution of u is normal with mean μ_u and variance σ_u^2. Then the expected value of $(u-a)^+$ is

$$E[(u-a)^+] = \int_a^\infty (u-a)\, f(u)\, du = \sigma_u n(\tfrac{\mu_u - a}{\sigma_u}) + (\mu_u - a) N(\tfrac{\mu_u - a}{\sigma_u}). \tag{40}$$

where $n(\cdot)$ and $N(\cdot)$ are the probability density function and cumulative probability function of the standard normal distribution.

Suppose u and v are jointly normal as

$$\begin{bmatrix} u \\ v \end{bmatrix} \sim n\left(\begin{bmatrix} \mu_u \\ \mu_v \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix} \right). \tag{41}$$

Then the expectation of u given that v is greater than a is

$$E[uI(v \geq a)] = \mu_u N(\tfrac{\mu_v - a}{\sigma_v}) + \tfrac{\sigma_{uv}}{\sigma_v} n(\tfrac{\mu_v - a}{\sigma_v}). \tag{42}$$

The expectation of u given that v is less than a is

$$E[uI(v \leq a)] = \mu_u N(\tfrac{a - \mu_v}{\sigma_v}) - \tfrac{\sigma_{uv}}{\sigma_v} n(\tfrac{\mu_v - a}{\sigma_v}). \tag{43}$$

6.2 Optimization with predetermined option strike price

Because it is never optimal for the manager to pay employees more than C_A, the manager's optimization problem when the option strike price is predetermined can be written as,

$$\max_{C,x} E_0^M[V_2] = A - C + \mu_z + \tfrac{x}{1+x} \left[(K - A + C - \mu) N(G) - \tfrac{\sigma_{(z_2 w)}}{\sigma_w} n(G) \right] \tag{44}$$

subject to

$$C + \eta \tfrac{x}{1+x} [\sigma_w n(G) + (\mu_w - Q) N(G)] = C_A. \tag{45}$$

Then the Lagrangean is

$$\mathcal{L} = E_0^M[V_2] + \xi\{C + \eta\frac{x}{1+x}[\sigma_w n(G) + (\mu_w - Q)N(G)] - C_A\}. \qquad (46)$$

The first order conditions are

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial x} &= \frac{x}{1+x}\left[-n(G)\frac{2(1+x)\gamma_1(\mu_w-Q)\sigma_{z_2w}}{\sigma_w^3} - n(G)\frac{2(1+x)\gamma_1 H}{\sigma_w}\right] \\
&+ (\tfrac{1}{1+x} - \tfrac{x}{(1+x)^2})\left[-n(G)\frac{\sigma_{z_2w}}{\sigma_w} + N(G)H\right] \\
&+ \xi\left\{-2x\gamma_1\eta N(G) + \eta(\tfrac{1}{1+x} - \tfrac{1}{(1+x)^2})[n(G)\sigma_w + (\mu_w - Q)N(G)]\right\} \\
&= 0 \qquad (47) \\
\frac{\partial\mathcal{L}}{\partial C} &= -1 + \xi(1 - \tfrac{x\eta}{1+x}N(G)) \\
&+ \tfrac{x}{1+x}[-n(g)\frac{(\mu_w-Q)\sigma_{z_2w}}{\sigma_w^3} - n(G)\frac{H}{\sigma_w} + N(G)] = 0 \qquad (48) \\
\frac{\partial\mathcal{L}}{\partial\xi} &= C + \eta\tfrac{x}{1+x}[\sigma_w n(G) + (\mu_w - Q)N(G)] - C_A = 0 \qquad (49)
\end{aligned}$$

where

$$\begin{aligned}
G &= \tfrac{\mu_w - Q}{\sigma_w}, & H &= K - A + C - \mu_z, \\
\mu_w &= \mu_z + \mu_d, & \sigma_w^2 &= \sigma_0^2 + \sigma_d^2 + 2\rho\sigma_0\sigma_d, \\
\sigma_{z_2w} &= \sigma_0^2 + \rho\sigma_0\sigma_d.
\end{aligned}$$

These first order conditions are solved numerically for C, x and ξ and the result is checked to ensure that the solution is indeed a maximum point.

6.3 Optimization with endogenous option strike price

When the option strike price is determined endogenously, the strike price equals to the price that investors are willing to pay for shares. Hence the optimization problem needs to have the additional condition on strike price:

$$K = E_0^I[V_2] - \gamma_0 \qquad (50)$$

The Lagrangean is

$$\mathcal{L} = E_0^M[V_2] + \xi\{C + \eta\frac{x}{1+x}[\sigma_w n(G) + (\mu_w - Q)N(G)] - C_A\} + \xi_1(K - E_0^I[V_2] + \gamma_0). \qquad (51)$$

The first order conditions are

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x} &= \frac{x}{1+x}\left[-n(G)\frac{2(1+x)\gamma_1(\mu_w-Q)\sigma_{z_2w}}{\sigma_w^3}-n(G)\frac{2(1+x)\gamma_1 H}{\sigma_w}\right] \\
&+ \left(\frac{1}{1+x}-\frac{x}{(1+x)^2}\right)\left[-n(G)\frac{\sigma_{z_2w}}{\sigma_w}+N(G)H\right] \\
&+ \xi\left\{-2x\gamma_1\eta N(G)+\eta\left(\frac{1}{1+x}-\frac{1}{(1+x)^2}\right)[n(G)\sigma_w+(\mu_w-Q)N(G)]\right\} \\
&+ \xi_1\{-\frac{x}{1+x}\left[-n(G)\frac{2(1+x)\gamma_1(\mu_w-Q)\sigma_{z_2w}}{\sigma_w^3}-n(G)\frac{2(1+x)\gamma_1(H-\mu_d)}{\sigma_w}\right] \\
&+ \left(-\frac{1}{1+x}+\frac{x}{(1+x)^2}\right)\left[-n(G)\frac{\sigma_{z_2w}}{\sigma_w}+N(G)(H-\mu_d)\right]\}=0 \qquad (52)\\
\frac{\partial \mathcal{L}}{\partial C} &= -1+\xi\left(1-\frac{x\eta}{1+x}N(G)\right) \\
&+ \frac{x}{1+x}\left[-n(G)\frac{(\mu_w-Q)\sigma_{z_2w}}{\sigma_w^3}-n(G)\frac{H}{\sigma_w}+N(G)\right] \\
&+ \xi_1\left\{1-\frac{x}{1+x}\left[-n(G)\frac{(\mu_w-Q)\sigma_{z_2w}}{\sigma_w^3}-n(G)\frac{H-\mu_d}{\sigma_w}+N(G)\right]\right\}=0 \qquad (53)\\
\frac{\partial \mathcal{L}}{\partial K} &= -\frac{x\eta\xi}{1+x}N(G)+\frac{x}{1+x}\left[-n(G)\frac{(\mu_w-Q)\sigma_{z_2w}}{\sigma_w^3}-n(G)\frac{H}{\sigma_w}+N(G)\right] \\
&+ \xi_1\left\{1-\frac{x}{1+x}\left[-n(G)\frac{(\mu_w-Q)\sigma_{z_2w}}{\sigma_w^3}-n(G)\frac{H-\mu_d}{\sigma_w}+N(G)\right]\right\}=0 \qquad (54)\\
\frac{\partial \mathcal{L}}{\partial \xi} &= C+\eta\frac{x}{1+x}[\sigma_w n(G)+(\mu_w-Q)N(G)]-C_A=0 \qquad (55)\\
\frac{\partial \mathcal{L}}{\partial \xi_1} &= H+\gamma_0-\mu_d-\frac{x}{1+x}\left[-n(G)\frac{\sigma_{z_2w}}{\sigma_w}+(H-\mu_d)N(G)\right]=0 \qquad (56)
\end{aligned}
$$

These first order conditions are solved numerically for C, x, K, ξ and ξ_1. The result is checked to ensure that the solution is indeed a maximum point.

7 References

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Table 1: Parameters used in the model.

Parameter	Explanation
A	Liquid asset that the firm has at time 0.
C_A	Cash compensation that employees can get by working elsewhere.
z_2	Revenue realization of the project at time 2.
z_1	The mean of the distribution on z_2 at time 1 according to the manager's belief.
w	The mean of the distribution on z_2 at time 1 according to investors' belief.
d_1	The difference between w and z_1, ($d_1 = w - z_1$).
σ_1^2	The variance of z_2 at time 1.
μ_z	The mean of the distribution on z_1 at time 0.
μ_d	The mean of the distribution on d_1 at time 0.
σ_0^2	The variance of the distribution on z_1 at time 0.
σ_d^2	The variance of the distribution on d_1 at time 0.
ρ	Correlation between z_1 and d_1.
γ_0	The slope of investors' demand curve at time 0.
γ_1	The slope of investors' demand curve at time 1.
d_0	The difference on the mean of z_2 at time 0 between investors' belief and the manager's belief.
β	The information cost of equity offering.
τ_c	Corporate tax rate.
τ_p	Personal tax rate.

Table 2: Feasible strategies when firm can issue new equity.

This table presents the feasible strategies when firm has the choice to sell new shares at time 1. Option grant (x) and cash compensation (C) with the corresponding total employee compensation (C^T), share value to investors ($E_0^I(V_2)$), and share value to the manager ($E_0^M(V_2)$) are reported. The parameters used are $A = 70$, $C_A = 70$, $\mu_z = 100$, $\sigma_0 = 20$, $\sigma_1 = 20$, $\eta = 0.9$, $\gamma_0 = 8$, $\gamma_1 = 4$, $\rho = 0$, $\beta = 10$, $K = 112$, $d_0 = \mu_d = 20$.

x	C	C^T	$E_0^I(V_2)$	$E_0^M(V_2)$
0	70.00	70.00	116.60	102.26
0.1	69.23	70.02	116.47	102.56
0.2	68.78	70.04	116.14	102.75
0.3	68.52	70.05	115.83	102.81
0.4	68.50	70.03	115.49	102.72
0.5	68.72	70.07	115.06	102.40
0.6	69.12	70.09	114.67	102.02
0.7	69.65	70.06	114.30	101.58
0.8	70.10	70.10	114.13	101.28
0.9	70.00	70.00	114.47	101.48

Table 3: Feasible strategies with tax.

This table presents the feasible strategies when firm can deduct the difference between share price and option strike and employees need to pay taxes for compensation over C_A. Option grant (x) and cash compensation (C) with the corresponding total employee compensation (C^T), share value to investors ($E_0^I(V_2)$), and share value to the manager ($E_0^M(V_2)$) are reported. The parameters used are $A = 70$, $C_A = 70$, $\mu_z = 100$, $\sigma_0 = 20$, $\sigma_1 = 20$, $\eta = 0.9$, $\gamma_0 = 8$, $\gamma_1 = 4$, $\rho = 0$, $K = 112$, $d_0 = \mu_d = 20$, $\tau_c = \tau_p = 0.3$.

x	C	C^T	$E_0^I(V_2)$	$E_0^M(V_2)$
0	70.00	70.00	104.62	90.36
0.1	69.52	70.02	104.29	90.99
0.2	69.13	70.04	103.95	91.43
0.3	68.82	70.04	103.62	91.71
0.4	68.59	70.03	103.31	91.89
0.5	68.50	70.07	102.97	91.93
0.6	68.43	70.06	102.69	91.91
0.7	68.37	70.02	102.50	91.84
0.8	68.46	70.07	102.29	91.67
0.9	68.53	70.06	102.10	91.52

Table 4: Equilibrium strategies under financial constraint.

This table presents the equilibrium strategies when firm faces financial constraint. Firm can use both options and equity. The available liquid asset (A), option grant (x), cash compensation (C), new equity grant (y_0), share price P_0, and share value to the manager over liquid asset ($E_0^M(V_2) - A$) are reported. The parameters used are $C_A = 70$, $\mu_z = 100$, $\sigma_0 = 20$, $\sigma_1 = 20$, $\eta = 0.9$, $\gamma_0 = 8$, $\gamma_1 = 4$, $\rho = 0$.

Panel A. $d_0 = \mu_d = -20$					
A	x	C	y_0	P_0	$E_0^M(V_2) - A$
70.0	0.000	70.00	0.000	72.00	30.00
67.5	0.000	70.00	0.036	68.54	28.98
65.0	0.000	70.00	0.077	64.95	27.85
62.5	0.000	70.00	0.123	61.19	26.58
60.0	0.000	70.00	0.175	57.21	25.12
57.5	0.000	70.00	0.236	52.91	23.39
55.0	0.067	69.25	0.292	48.76	21.28
Panel B. $d_0 = \mu_d = -5$					
A	x	C	y_0	P_0	$E_0^M(V_2) - A$
70.0	0.000	70.00	0.000	87.00	30.00
67.5	0.112	68.79	0.015	84.24	29.66
65.0	0.117	68.74	0.046	80.92	29.21
62.5	0.125	68.66	0.080	77.51	28.69
60.0	0.136	68.57	0.116	73.99	28.09
57.5	0.149	68.45	0.156	70.36	27.39
55.0	0.167	68.29	0.200	66.56	26.55

Table 4 (continued).

Panel C. $d_0 = \mu_d = 0$					
A	x	C	y_0	P_0	$E_0^M(V_2) - A$
70.0	0.183	68.12	0.000	92.17	30.17
67.5	0.262	67.50	0.000	89.65	30.15
65.0	0.276	67.41	0.028	86.39	29.85
62.5	0.271	67.45	0.060	83.02	29.49
60.0	0.267	67.48	0.094	79.56	29.07
57.5	0.265	67.49	0.132	75.99	28.57
55.0	0.266	67.49	0.173	72.28	27.97
Panel D. $d_0 = \mu_d = 5$					
A	x	C	y_0	P_0	$E_0^M(V_2) - A$
70.0	0.373	66.77	0.000	97.03	30.78
67.5	0.373	66.77	0.000	94.53	30.78
65.0	0.412	66.56	0.017	91.51	30.65
62.5	0.401	66.62	0.047	88.21	30.42
60.0	0.390	66.68	0.079	84.83	30.14
57.5	0.380	66.74	0.114	81.35	29.79
55.0	0.371	66.79	0.152	77.75	29.37
Panel E. $d_0 = \mu_d = 20$					
A	x	C	y_0	P_0	$E_0^M(V_2) - A$
70.0	0.661	65.59	0.055	109.9	33.72
67.5	0.661	65.59	0.055	107.4	33.72
65.0	0.661	65.59	0.055	104.9	33.72
62.5	0.661	65.59	0.055	102.4	33.72
60.0	0.661	65.59	0.056	99.86	33.72
57.5	0.648	65.62	0.084	96.59	33.70
55.0	0.634	65.65	0.114	93.24	33.63

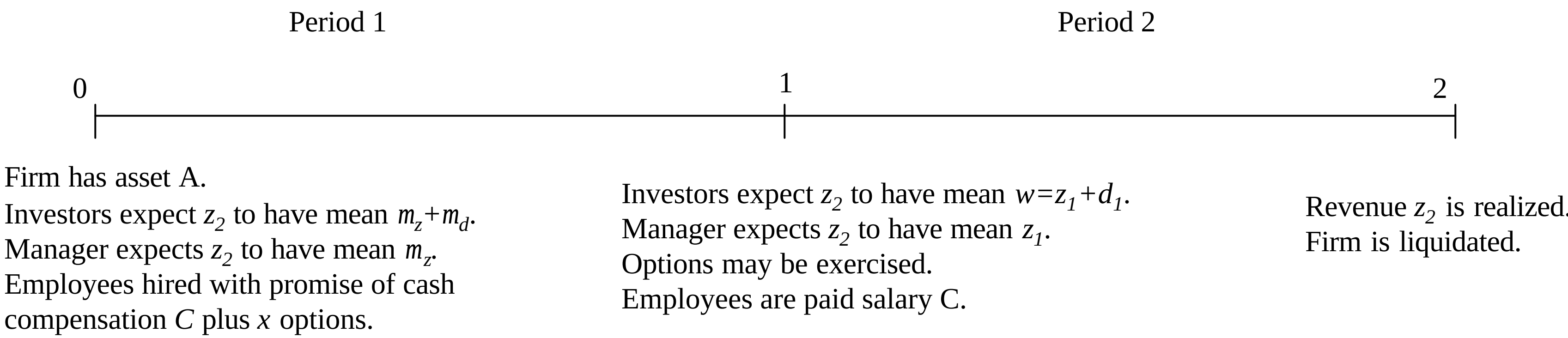
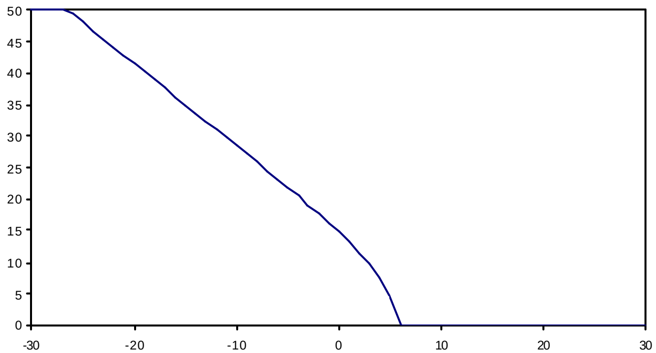
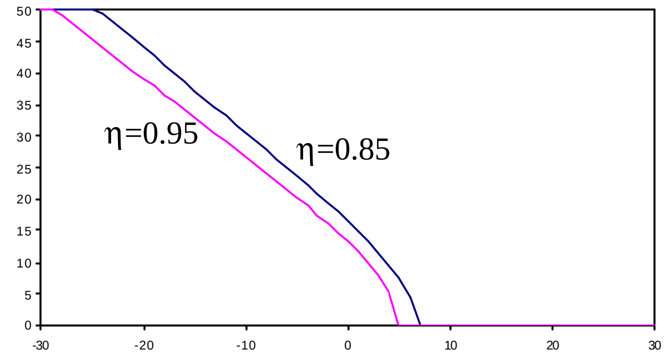


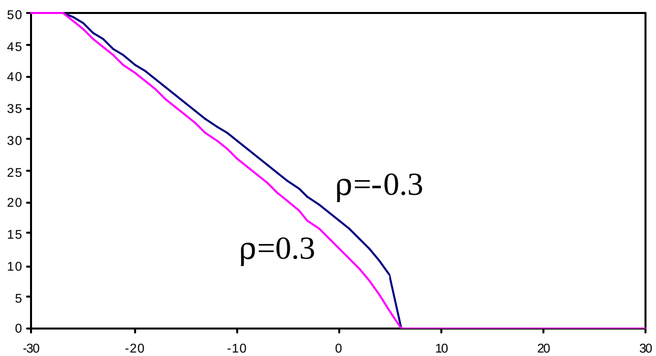
Figure 1. Time line of the model.



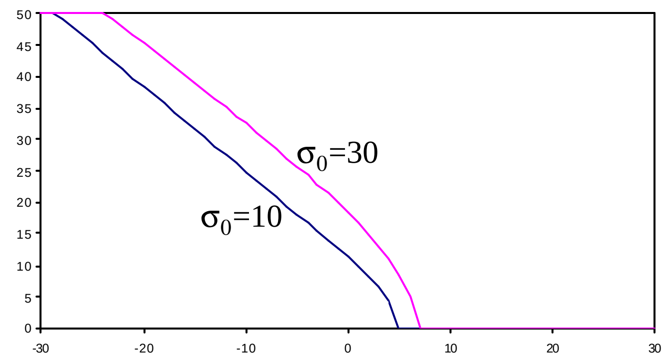
(a) Base case.



(b) Changing risk aversion (η).

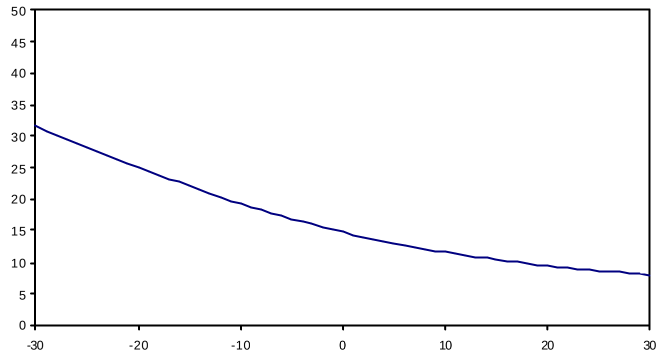


(c) Changing correlation (ρ).

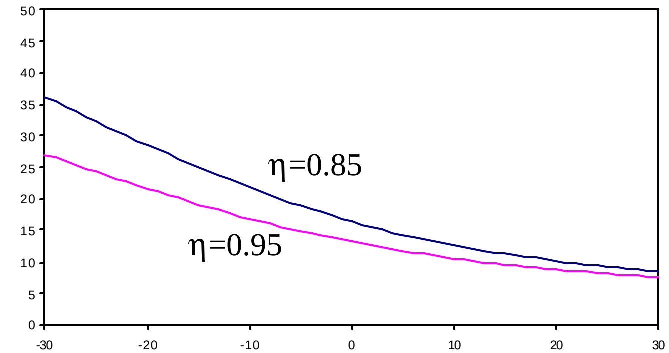


(d) Changing fundamental volatility (σ_0).

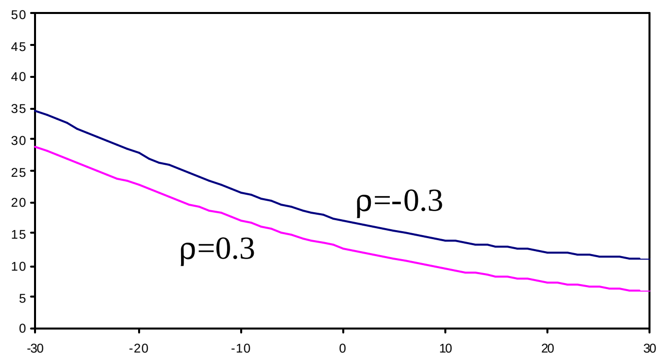
Figure 2. Regions where options are issued in the equilibrium. The lines are thresholds for the firm to grant options. Northeast of the lines are regions where options are issued in the equilibrium. Southwest of lines are regions where options are not issued. The horizontal axis is investors' misperception (m_d). The vertical axis is volatility of future investor misperception (σ_d). The parameters used in the base case (a) are $A=70$, $C_A=70$, $m_z=100$, $s_0=20$, $s_1=20$, $h=0.9$, $g_0=8$, $g_1=4$, $r=0$. Option strike price K is determined exogenously as $m_z+m_d-g_0$. Only one of the parameters is changed in other panels.



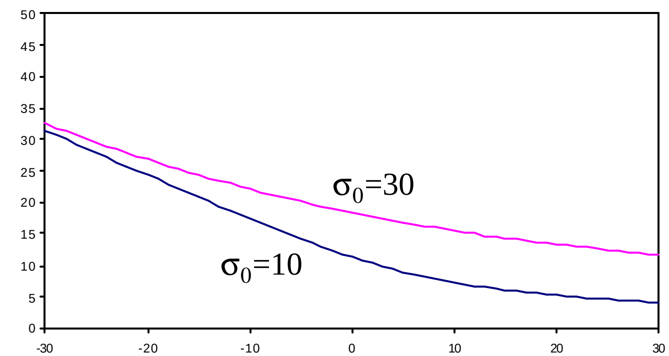
(a) Base case.



(b) Changing risk aversion (η).

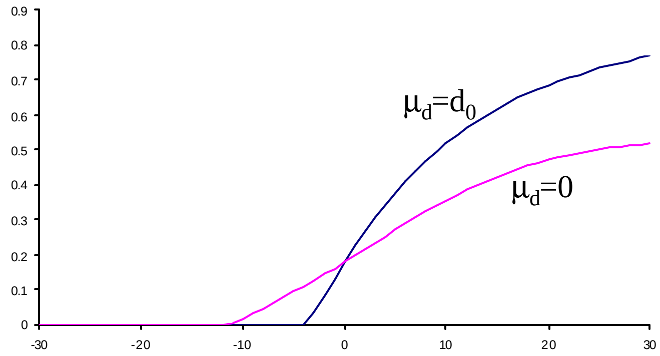


(c) Changing correlation (ρ).

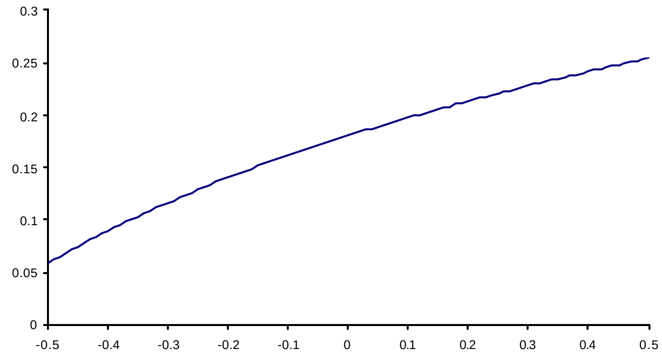


(d) Changing fundamental volatility (σ_0).

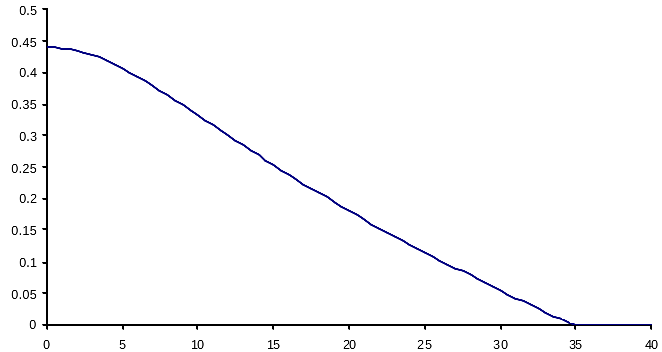
Figure 3. Regions where options are issued in the equilibrium when investors' belief is mean reverting. The lines are thresholds for the firm to grant options. Northeast of the lines are regions where options are issued in the equilibrium. Southwest of lines are regions where options are not issued. The horizontal axis is current investors' misperception (d_0). The vertical axis is volatility of future investor misperception (σ_d). The mean of future investor misperception (μ_d) is equal to 0. The parameters used in the base case (a) are $A=70$, $C_A=70$, $m_z=100$, $s_0=20$, $s_1=20$, $h=0.9$, $g_0=8$, $g_1=4$, $r=0$. Option strike price K is determined exogenously as $m_z+d_0-g_0$. Only one of the parameters is changed in other panels.



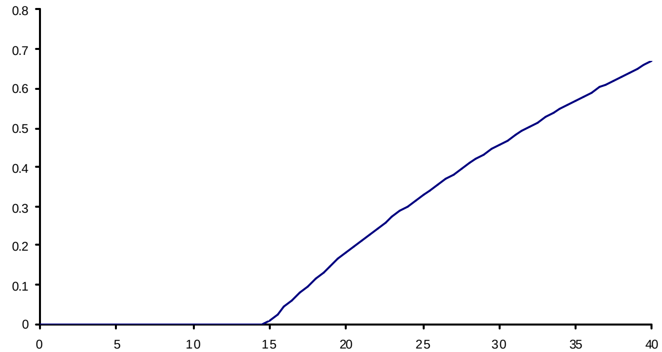
(a) Misperception (d_0).



(b) Correlation (ρ).



(c) Fundamental volatility (σ_0).



(d) Volatility of misperception (s_d).

Figure 4. Comparative statistics of equilibrium option grant with respect to variables. The vertical axis is equilibrium option grant (x^*). The horizontal axis is the variable of interest. The base set of parameters are $A=70$, $C_A=70$, $m_z=100$, $s_0=20$, $s_1=20$, $h=0.9$, $g_0=8$, $g_1=4$, $d_0=m_d=0$, $r=0$. Option strike price K is determined exogenously as $m_z+d_0-g_0$. Only one of the parameters is changed in each panel.

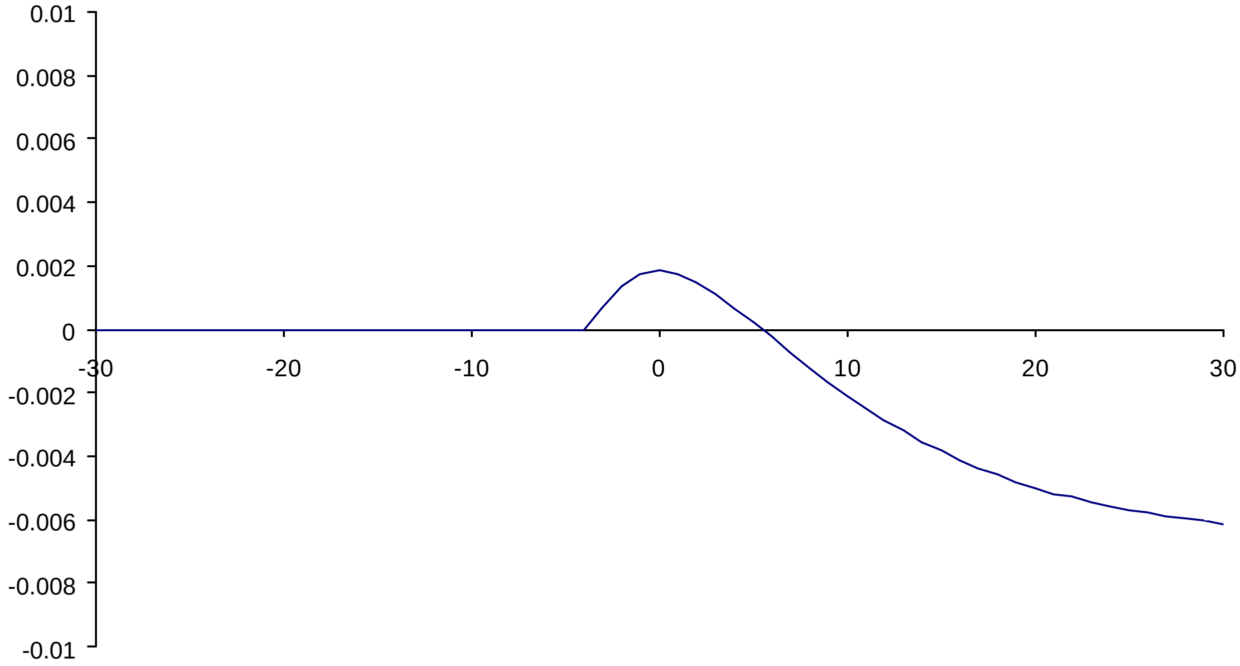
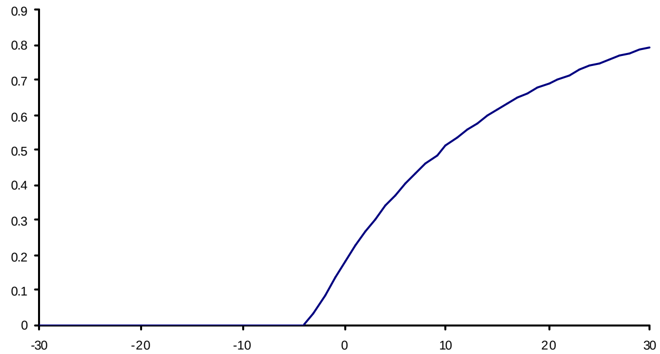
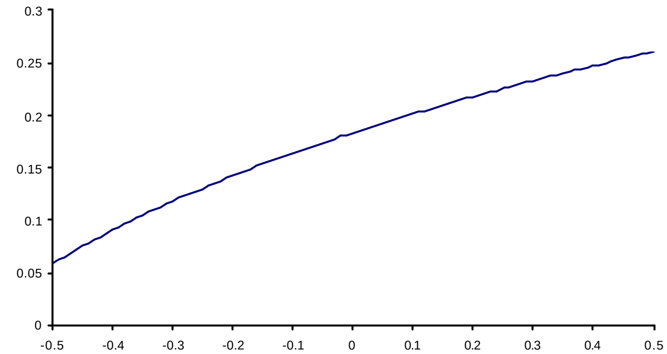


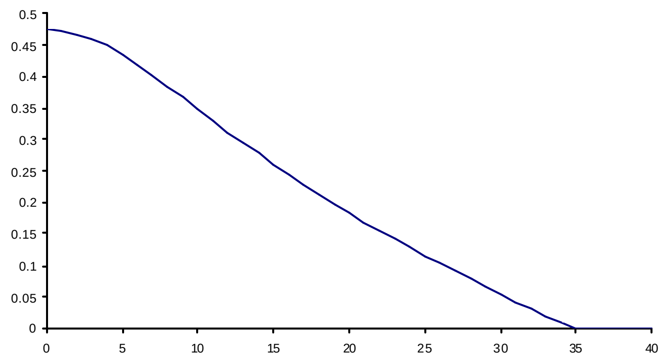
Figure 5. Price effect of option grant. The vertical axis is the change of share prices before and after option grant normalized by the share price before option grant. The horizontal axis is d_0 and m_d. The base set of parameters are $A=70$, $C_A=70$, $m_z=100$, $s_0=20$, $s_1=20$, $h=0.9$, $g_0=8$, $g_1=4$, $r=0$. Option strike price K is determined exogenously as $m_z+d_0-g_0$.



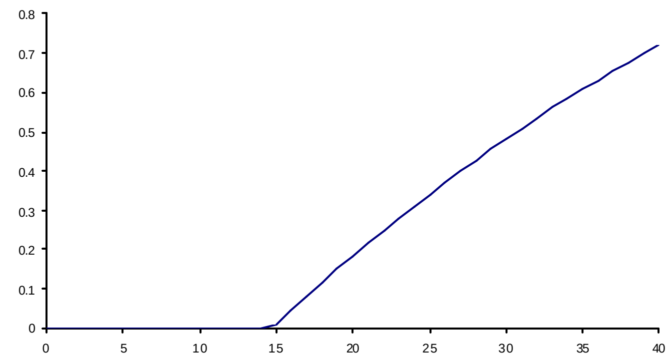
(a) Misperception (μ_d).



(b) Correlation (ρ).



(c) Fundamental volatility (σ_0).



(d) Volatility of misperception (s_d).

Figure 6. Comparative statistics of equilibrium option grant with respect to variables. The vertical axis is equilibrium option grant (x^*). The horizontal axis is the variable of interest. The base set of parameters are $A=70$, $C_A=70$, $m_z=100$, $s_0=20$, $s_1=20$, $h=0.9$, $g_0=8$, $g_1=4$, $d_0=m_d=0$, $r=0$. Option strike price K is determined endogenously as $E_0^I[V_2]$ $-g_0$. Only one of the parameters is changed in each panel.