Determination of the optic axis and optical properties of absorbing uniaxial crystals by reflection perpendicularincidence ellipsometry on wedge samples

Rasheed M.A. Azzam

University of New Orleans, razzam@uno.edu
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Given an arbitrarily cut uniaxial crystal wedge, a procedure is described using reflection perpendicular-incidence ellipsometry (PIE) for (1) locating the optic axis, and (2) determining the ordinary \((N_o)\) and extraordinary \((N_e)\) complex refractive indices. The optic axis is located by finding the principal directions of the two wedge faces and subsequently solving three spherical triangles. \(N_o\) and \(N_e\) are determined by two complex ratios of principal reflection coefficients (of light normally incident on and linearly polarized along the principal directions of each face) as measured by PIE. The solution for \(N_o\) and \(N_e\) is explicit but requires finding the roots of a sixth-degree algebraic equation in \(N_o\).

I. Introduction

Consider the normal-incidence reflection of a polarized monochromatic (or quasi-monochromatic) light beam by an arbitrary surface of a uniaxial absorbing crystal. Let \(n\) be the index of refraction of the transparent medium of incidence, \(N_o (= n_o - jk_o)\) and \(N_e (= n_o - jk_e)\) be the ordinary and extraordinary complex refractive indices of the crystal, and let \(\theta\) be the angle between the optic axis and the surface. In Fig. 1, the principal plane, which is defined by the surface normal and the optic axis, is assumed to be in the plane of the page. When the incident light is linearly polarized with its electric vector vibrating either parallel \((p)\) or perpendicular \((s)\) to the principal plane, the reflected light is similarly linearly \(p\) and \(s\) polarized. Incident states other than the \(p\) and \(s\) linear eigenpolarizations are changed upon normal-incidence reflection.

If we resolve the electric vectors \(E_i\) and \(E_r\) of the incident and reflected waves into components parallel and perpendicular to the principal plane of complex amplitudes \((E_{ip}, E_{is})\) and \((E_{rp}, E_{rs})\), respectively, we can write

\[
E_{rp} = R_{pp}E_{ip}, \quad E_{rs} = R_{ss}E_{is}.
\]

In Eqs. (1), \(R_{pp}\) and \(R_{ss}\) are the principal reflection coefficients [the diagonal elements of the diagonal reflection matrix \(R = (R_{ij}), i,j = p,s\)] and are given by

\[
R_{pp} = \frac{n - N_p}{n + N_p}, \quad R_{ss} = \frac{n - N_e}{n + N_e},
\]

where

\[
N_p^{-2} = N_o^{-2}\sin^2\theta + N_e^{-2}\cos^2\theta.
\]

The ratio of principal reflection coefficients,

\[
\rho = \frac{R_{pp}}{R_{ss}},
\]

can be measured by perpendicular-incidence ellipsometry\(^2\) (PIE) using instrumentation that operates on null\(^3,4\) or photometric\(^5,6\) principles. If Eqs. (2) are substituted into Eq. (4), we obtain

\[
\rho = \frac{(n - N_p)(n + N_o)}{(n + N_p)(n - N_o)},
\]

Equation (5) can be solved for \(N_p\) in terms of \(N_o\) and \(\rho\); this gives

\[
N_p = n - \frac{nqN_o}{n + qN_o},
\]

where

\[
q = (1 - \rho)/(1 + \rho).
\]

If we eliminate \(N_p\) between Eqs. (3) and (6), we obtain

\[
N_o^2 = \sec^2\theta[(n + qN_o)/(n^2q + nN_o)]^2 - N_e^{-2}\tan^2\theta.
\]

One of the simplest methods\(^2\) to determine the complex refractive indices \(N_o\) and \(N_e\) of a uniaxial...
crystal is to measure two ratios of principal reflection coefficients \( p_1 \) and \( p_2 \) with the crystal immersed in two different ambients (e.g., air and a suitable liquid) of refractive indices \( n_1 \) and \( n_2 \). Equating the two values of \( N_p \) that result from using Eq. (6) twice yields a quadratic in \( N_o \),

\[
(q_1 q_2 - q_2 q_1)N_o^2 + (q_1 q_2 (n_1^2 - n_2^2)N_o + n_1 n_2 (q_1 q_2 - q_2 q_1) = 0,
\]

that can be solved for \( N_o \). Equation (8) then gives \( N_e \).

The procedure requires that the direction of the optic axis (hence the angle \( \theta \)) be known (e.g., the optic axis is parallel to the surface under measurement; \( \theta = 0 \) and \( N_p = N_e \)) and that two ambient media be used. The latter requirement can present difficulty in spectroscopic applications, as the ambient must remain transparent over the spectral region used. Furthermore, an inert ambient may not always be readily available. Under these circumstances, it is worthwhile to consider another method in which only one ambient (e.g., vacuum or air) is used and PIE measurements on two crystal surfaces are made. Such a method is described in the following section.

II. Determination of \( N_o \) and \( N_e \) from PIE Measurements on Two Crystal Surfaces

For simplicity, and without loss of generality, we take \( n = 1 \). This means that \( N_o, N_e, \) (and \( N_p \)) are normalized with respect to any ambient refractive index \( n \), so that the ratios \( N_o/n \) and \( N_e/n \) are being measured.

PIE measurements on two different and arbitrary crystal surfaces in the same ambient yield two ratios of principal reflection coefficients \( p_1 \) and \( p_2 \). The corresponding \( q_1 \) and \( q_2 \) are determined by Eq. (7). If Eq. (8) is used twice (with \( n = 1 \), corresponding to the two measurements, and the right-hand sides are equated, we get

\[
\sec^2 \theta_1 [(1 + q_1 N_o)/(q_1 + N_o)]^2 - N_o^{-2} \tan^2 \theta_1 = \sec^2 \theta_2 [(1 + q_2 N_o)/(q_2 + N_o)]^2 - N_o^{-2} \tan^2 \theta_2.
\]

Equation (10) can be put in the form of a sixth-degree equation in \( N_o \) as follows:

\[
A_6 N_o^6 + A_5 N_o^5 + A_4 N_o^4 + A_3 N_o^3 + A_2 N_o^2 + A_1 N_o + A_0 = 0,
\]

where

\[
\begin{align*}
A_0 &= -\mu^2 \tau \\
A_1 &= -2 \mu \sigma \tau \\
A_2 &= (q_1 q_2 - q_2 q_1) - \tau (\sigma^2 + 2 \mu) \\
A_3 &= 2 \mu (q_1 q_2 - q_2 q_1) - 2 \sigma \tau \\
A_4 &= (\sigma^2 - \mu - \tau) \\
A_5 &= 2 \mu (q_1 q_2 - q_2 q_1) \\
A_6 &= \sigma^2 + 2 \mu - \tau \\
\mu &= q_1 q_2 \\
\sigma &= q_1 + q_2 \\
p &= 1 + q_1 q_2 \\
q_1 &= \sec \theta_1, i = 1, 2 \\
\tau &= (\tan^2 \theta_1 - \tan^2 \theta_2)
\end{align*}
\]

Equation (11), with the coefficients \( A_k \) evaluated using Eqs. (12) and (13), can be solved for \( N_o \). The condition that \( N_o \) is confined to the fourth quadrant of the complex plane may be sufficient to exclude all but the correct root of six roots. If necessary, an additional (crude) intensity-reflectance measurement of \( s \)-polarized light on one of the two faces can be used for the unambiguous determination of \( N_o \) with the help of the second of Eqs. (2). Once \( N_o \) has been determined, \( N_e \) can be obtained using Eq. (8) with \( (\theta, q) = (\theta_1, q_1) \) or \( (\theta_2, q_2) \).

The foregoing procedure for the determination of \( N_o \) and \( N_e \) using PIE measurements on two crystal surfaces in the same ambient is as explicit as that described in Sec. I using PIE measurements on one crystal surface in two ambient. However, the two-ambient method leads to a simple quadratic equation in \( N_o \) in contrast with a sixth-degree equation in the two-surface method.

III. Determination of the Optic Axis of a Uniaxial Crystal Wedge

The measurements that are required to determine \( N_o \) and \( N_e \) by the method of Sec. II can be made on a sample in the form of a wedge (or prism). This is advantageous because then only one and the same sample is used, and the two planar wedge surfaces can be prepared under similar conditions. The other alternative would be two surfaces on two different samples of the same crystal.

Furthermore, by determining the principal directions of the two adjacent side surfaces of the wedge, the orientation of (the otherwise unknown) optic axis can be fixed using spherical trigonometry, as will be shown in this section.

The principal directions of an absorbing uniaxial crystal surface are unique orthogonal directions in the...
surface along which normally incident light must be linearly polarized so that the reflected light will have the same linear polarization. These directions are denoted by \( p \), which is the line of intersection of the surface with the principal plane (see Sec. I), and \( s \) which is, of course, orthogonal to \( p \). These directions can be easily located experimentally (1) from the fact that they are directions of maximum and minimum reflectance for incident linearly polarized light of variable azimuth, or (2) from obtaining extinction along these directions if an arrangement with crossed linear polarizers is used. Because of the latter property, the principal directions are also often called extinction directions. For measurements on the two faces of an arbitrarily cut wedge, the \( p \) directions, \( p_1 \) and \( p_2 \), can be specifically identified as the directions of unequal reflectances. Reflectances in the \( e \) and \( s \) directions are the same, because they are determined by the same refractive index \( N_0 \) [see the second of Eqs. (2)].

Figure 2(a) shows the geometry involved with the wedge of two faces 1 and 2. \( E \) is the edge of the wedge, and \( p_1 \) and \( p_2 \) are principal directions in faces 1 and 2 (parallel to the principal plane), determined as explained in the preceding paragraph. \( p_1 \) and \( p_2 \) intersect \( E \) at a common point or origin \( O \). The wedge angle is \( \omega \) and the angles between \( p_1 \) and \( p_2 \) and \( E \) are \( \alpha_1 \) and \( \alpha_2 \). The optic axis \( O'A \) is uniquely determined as the line of intersection of the two principal planes \( P_1 \) and \( P_2 \) that are drawn through lines \( p_1 \) and \( p_2 \) perpendicular to faces 1 and 2 of the wedge.

To determine the angles \( \theta_1 \) and \( \theta_2 \) between the optic axis and the wedge faces \( \angle AOp_1, \angle AOp_2 \), and the angle \( \gamma \) between the optic axis and the wedge edge \( \angle AOE \), spherical trigonometry is applied to the spherical-triangle equivalent to Fig. 2(a) shown in Fig. 2(b) (with \( O \) as the center of the sphere). Additional angles \( \beta_1 \) and \( \beta_2 \) and \( \epsilon \) are defined in Fig. 2(b). The first spherical triangle to be solved is \( E\beta_1\beta_2 \) with two known sides \( \alpha_1 \) and \( \alpha_2 \) and known included angle \( \omega \). From Napier’s analogies, the unknown angles \( \beta_1 \) and \( \beta_2 \) are determined as follows:

\[
\begin{align*}
\tan(\beta_1 + \beta_2) &= \cot\omega \cos(\alpha_1 - \alpha_2) / \cos(\alpha_1 + \alpha_2) \] \\
\tan(\beta_1 - \beta_2) &= \cot\omega \sin(\alpha_1 - \alpha_2) / \sin(\alpha_1 + \alpha_2).
\end{align*}
\]

The law of sines\(^9\) is then used to obtain \( \epsilon \) (the angle between the wedge-face principal directions \( p_1 \) and \( p_2 \)):

\[
\sin \epsilon = \sin \omega \sin \alpha_1 \sin \beta_2.
\]

The second spherical triangle to be solved is \( A\beta_1\beta_2 \) with two known angles \( 90^\circ - \beta_1 \) and \( 90^\circ - \beta_2 \) and known included side \( \epsilon \). Napier’s analogies determine the unknown angles (sides) \( \theta_1 \) and \( \theta_2 ):

\[
\begin{align*}
\tan(\theta_1 + \theta_2) &= \tan \omega [\cos(\beta_1 - \beta_2) / \sin(\beta_1 + \beta_2)], \\
\tan(\theta_1 - \theta_2) &= \tan \omega [\sin(\beta_1 - \beta_2) / \cos(\beta_1 + \beta_2)].
\end{align*}
\]

Finally, the angle \( \gamma \) between the optic axis and wedge edge is derived by solving the right-angled triangle \( E\beta_1A \):

\[
\cos \gamma = \cos \alpha_1 \cos \theta_1.
\]
References

7. A single polarizer, which also acts as an analyzer, can be used if a $\pi/4$ Faraday rotator is placed between the polarizer and the surface. See D. W. Stevens, Surf. Sci. 96, (1980), in press.

4,063,800 20 Dec. 1977 (Cl. 350–184)
*Zoom lens for a projector.*

A complex zoom projection lens is described covering a focal-length range of 1.76:1 at f/1.0. The front cemented doublet and the rear eight-element double-Gauss lens are fixed, while the intermediate cemented doublet negative component is movable along the axis. Good aberration correction is claimed over a field angle that ranges from $\pm 12.2^\circ$ to $\pm 6.9^\circ$ at this high aperture.

R.K.

4,063,801 20 Dec. 1977 (Cl. 350–216)
*Telephoto type objective.*

Two examples are given of 280-mm f/3.5 telephoto lenses that are focused by a movement of the rear negative member. The front positive member consists of an airspaced triplet, while the rear member consists of two negative components that move at different rates for focusing.

R.K.

4,148,587 10 Apr. 1979 (Cl. 356–356)
*Laser gauge for measuring changes in the surface contour of a moving part.*

A visible laser beam is focused on the surface of a rotating cylinder or laterally moving plate. The speckle pattern formed by light scattered directly back through the focusing lens is detected by two detectors separated by a few millimeters to determine the speckle-rate variation and the speckle-size variation. The signals are analyzed by a correlator that develops the cross-correlation function between the two signals and determines when they are coincident. The focus lens is moved to peak the focus and the correlation signal.

E.D.P.

4,182,995 8 Jan. 1980 (Cl. 331–94.5 H)
*Laser diode with thermal conducting, current confining film.*

A diode laser is constructed of an n-type InP layer and a p-type InP layer that sandwich a p-type InGaASP layer. A stripe metal electrode deposited on top of the p-type InP is covered with a Ge film that covers the rest of the p-type InP as well. This Ge film acts as a good heat conductor and forms a blocking junction with the p-type InP region so as to confine the current across the p-n junction away from the sides of the structure.

E.D.P.

4,190,811 26 Feb. 1980 (Cl. 331–94.5 M)
*Laser controlled optical switching in semiconductors.*

A pulsed CO$_2$ IR laser beam of 200-nsec time duration and 10-MW/cm$^2$ intensity is reflected off a slab of polycrystalline n-type Ge at Brewster's angle. When a pulsed ruby laser with a 2-nsec pulse duration illuminates the sample surface with intensity in the 15-MW/cm$^2$ range, the free-carrier density is increased rapidly so that the plasma-reflection effect occurs at 10.5 $\mu$m, thus turning on the reflected beam. Variations of this approach produce ultrashort IR pulses of $\mu$s pace duration.

E.D.P.