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R. M.A. Azzam
University of New Orleans, razzam@uno.edu

A. Alsamman
University of New Orleans, aalsamma@uno.edu

Quasi index matching for minimum reflectance at a dielectric–conductor interface for obliquely incident *p*- and *s*-polarized light

R. M. A. Azzam* and A. Alsamman

Department of Electrical Engineering, University of New Orleans, New Orleans, Louisiana 70148, USA

*Corresponding author: razzam@uno.edu

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Conditions for reducing the reflectance of a dielectric–conductor interface for p- and s-polarized light to a minimum at any angle of incidence ϕ are determined. The refractive indices of a transparent immersion medium (liquid) that achieve minimum reflectance at normal incidence, $\phi = 0$, and at $\phi = 45°$ are independent of polarization. These indices provide sufficient data to determine the real and imaginary parts of the complex refractive index of an absorbing substrate. Reflection at a dielectric–Au interface at 500 nm wavelength is considered as an example. It is shown that the lowest possible reflectance is attained for p-polarized light at $\phi = 45°$ and that the associated p-reflection phase shift is also minimum at that angle. For $\phi \geq 65°$ the lowest reflectance of p-polarized light occurs when the ambient is vacuum or air. However, this lowest reflectance at the air–Au interface is not a true minimum in a mathematical sense.

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1. Introduction

Consider the reflection of monochromatic s- and p-polarized light at an angle of incidence ϕ by the planar interface between a transparent medium of incidence (ambient) of refractive index n_0 and an absorbing medium of refraction (substrate) of complex refractive index $N_1 = n_1 - jk_1$, Fig. 1. Both media are assumed to be linear, homogeneous, and optically isotropic. The amplitude and phase changes that accompany reflection are determined by the absolute value and angle of the Fresnel complex-amplitude reflection coefficient for each polarization [1–3]:

$$r_s = \frac{n_0 \cos\phi - (\varepsilon_1 - n_0^2 \sin^2\phi)^{1/2}}{n_0 \cos\phi + (\varepsilon_1 - n_0^2 \sin^2\phi)^{1/2}}, \tag{1}$$

$$r_p = \frac{\varepsilon_1 \cos\phi - n_0(\varepsilon_1 - n_0^2 \sin^2\phi)^{1/2}}{\varepsilon_1 \cos\phi + n_0(\varepsilon_1 - n_0^2 \sin^2\phi)^{1/2}}, \tag{2}$$

$$\varepsilon_1 = N_1^2 = (n_1 - jk_1)^2 = (n_1^2 - k_1^2) - j2n_1k_1. \tag{3}$$

For given (n_0, N_1) it is well known that the amplitude reflectances $|r_s|$ and $|r_p|$ as functions of ϕ are minimum at normal incidence, $\phi = 0$, and at the pseudo-Brewster angle [4–6], respectively.

The minima that $|r_s|$ and $|r_p|$ exhibit as functions of n_0, when light is reflected at a dielectric–conductor interface at a given angle of incidence ϕ, are the subject of this paper. A complex reflection coefficient is expressed in polar form as

$$r_\nu = |r_\nu| \exp(j\delta_\nu), \qquad v = p, s. \tag{4}$$

By taking the first derivative of the natural logarithm of both sides of Eq. (4), we obtain

$$r'_\nu / r_\nu = |r_\nu|' / |r_\nu| + j\delta'_\nu, \tag{5}$$

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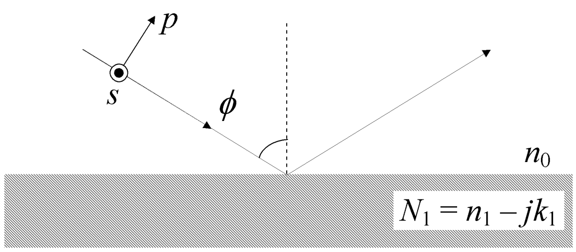

Fig. 1. Reflection of p- and s-polarized light at a dielectric–conductor interface at an angle of incidence ϕ.

where the prime indicates differentiation with respect to n_0. From Eq. (5), the condition of minimum reflectance $|r_\nu|' = 0$ can be put in its simplest form as

$$\mathrm{Re}(r'_\nu/r_\nu) = 0. \tag{6}$$

For given N_1 and ϕ, the refractive indices $n_0 = n_{0p}$, n_{0s} at which the p and s reflectances are minimum are determined by solving Eq. (6) for $\nu = p, s$. The associated minimum reflectances $|r_s|^2{}_{\min}$, $|r_p|^2{}_{\min}$ and corresponding phase shifts δ_s, δ_p at the minimum are calculated next from Eqs. (1) and (2).

The paper is organized as follows. In Section 2 we begin with the simple case of normal incidence. Conditions of minimum reflectance for p- and s-polarized light at oblique incidence are discussed next in Section 3. A minimum-reflectance liquid-immersion method for the explicit determination of the complex refractive index $N_1 = n_1 - jk_1$ of an absorbing substrate is described in Section 4. Section 5 presents specific results for the reflection of light of wavelength $\lambda = 500$ nm by a dielectric–Au interface. Finally, Section 6 gives a brief summary of the paper.

2. Condition of Minimum Reflectance at Normal Incidence

At normal incidence, $\phi = 0$, Eqs. (1) and (2) reduce to

$$r_s = -r_p = (n_0 - N_1)/(n_0 + N_1). \tag{7}$$

The difference in sign of r_p and r_s at $\phi = 0$ is in accord with the Nebraska–Muller conventions [7,8]. From Eq. (7) we obtain

$$r_s{}'/r_s = 2N_1/(n_0{}^2 - N_1{}^2). \tag{8}$$

From Eqs. (6) and (8) and $N_1 = n_1 - jk_1$ the condition of minimum reflectance at normal incidence becomes

$$n_0 = \left|N_1\right| = (n_1{}^2 + k_1{}^2)^{1/2}. \tag{9}$$

Equation (9) represents a quasi index-matching condition at a dielectric–conductor interface. The trivial case of $k_1 = 0$, $n_0 = n_1$ corresponds to a vanishing optical interface with zero reflectance at all angles. Substitution of $n_0 = |N_1|$ and $N_1 = |N_1|\exp(-j\theta_1)$ into Eq. (7) yields

$$r_s = -r_p = j\tan(\theta_1/2), \qquad \theta_1 = \arctan(k_1/n_1). \tag{10}$$

The minimum intensity reflectance at normal incidence is given by

$$\begin{aligned} |r_s|^2{}_{\min} &= |r_p|^2{}_{\min} = \tan^2(\theta_1/2) \\ &= (|N_1| - n_1)/(|N_1| + n_1). \end{aligned} \tag{11}$$

The first of Eqs. (10) indicates that under the conditions of minimum reflectance at normal incidence the associated reflection phase shifts for the s and p polarizations are quarter wave:

$$\delta_s = \pi/2, \qquad \delta_p = -\pi/2. \tag{12}$$

3. Conditions of Minimum Reflectance for s- and p-Polarized Light at Oblique Incidence

Equation (1) is of the form

$$\begin{aligned} r_s &= (A - B)/(A + B), \qquad A = n_0\cos\phi, \\ B &= (\varepsilon_1 - n_0^2\sin^2\phi)^{1/2}. \end{aligned} \tag{13}$$

Differentiation of Eq. (13) gives

$$r'_s/r_s = 2(A'B - AB')/(A^2 - B^2). \tag{14}$$

If A and B and their derivatives (with respect to n_0) A' and B' are substituted into Eq. (14), the vaule of $\varepsilon_0 = n_{0s}^2$ that achieves minimum reflectance for s-polarized light at oblique incidence is found using Eq. (6) to satisfy a cubic equation:

$$a_3\varepsilon_0^3 + a_2\varepsilon_0^2 + a_1\varepsilon_0 + a_0 = 0. \tag{15}$$

The coefficients of Eq. (15) are determined by $N_1 = n_1 - jk_1$ and ϕ as follows:

$$\begin{aligned} a_0 &= (n_1^2 + k_1^2)^4, \quad a_1 = 0, \\ a_2 &= -(1 + 2\sin^2\phi)(n_1^2 + k_1^2)^2, \\ a_3 &= 2\sin^2\phi(n_1^2 - k_1^2). \end{aligned} \tag{16}$$

Reaching Eq. (15) involves several steps and uses the fact that, if $\mathrm{Re}(F) = 0$, then $\mathrm{Im}(1/F)^2 = 0$. At normal incidence, $\phi = 0$, Eqs. (15) and (16) give $\varepsilon_0 = (-a_0/a_2)^{1/2} = (n_1^2 + k_1^2)$, $n_0 = |N_1|$, in agreement with Eq. (9). Another special case is that of an absorbing substrate with $n_1 = k_1$. The latter condition makes $a_3 = 0$ and reduces Eq. (15) to a quadratic equation with an explicit solution

$$n_{0s} = \sqrt{2}n_1/(1 + 2\sin^2\phi)^{1/4}. \tag{17}$$

An analysis similar to that presented above that starts with Eq. (2) leads to the following condition of minimum reflectance for p-polarized light at oblique incidence:

$$\mathrm{Im}\left[\frac{(\varepsilon_1-\varepsilon_0\sin^2\phi)(\varepsilon_1{}^2\cos^2\phi-\varepsilon_1\varepsilon_0+\varepsilon_0{}^2\sin^2\phi)^2}{\varepsilon_1{}^2(\varepsilon_1-2\varepsilon_0\sin^2\phi)^2}\right]=0. \tag{18}$$

For given $\varepsilon_1 = N_1^2 = (n_1 - jk_1)^2$ and ϕ, Eq. (18) is solved for $\varepsilon_0 = n_{0p}^2$ as its only unknown.

At $\phi = 45°$, the Abelès condition [9], $r_p = r_s^2$, is satisfied and the value of n_0 for minimum reflectance must be the same ($n_{0p} = n_{0s}$) independent of polarization. This is also readily verified since Eq. (18) is reduced to Eq. (15) at $\phi = 45°$.

4. Minimum-Reflectance Liquid-Immersion Method for Determination of Substrate Complex Refractive Index

The complex refractive index of an absorbing substrate $N_1 = n_1 - jk_1$ can be determined by using a liquid immersion ambient medium and varying its refractive index n_0 to achieve minimum reflectance at normal incidence, $\phi = 0$, and at $\phi = 45°$. Optical liquids, whose refractive index can be changed from 1.30 to 2.30 in steps of 0.002, are available commercially [10]. If the values of n_0 for minimum reflectance at these two angles are denoted by $n_0(0)$, $n_0(45)$, respectively, then we obtain

$$n_1{}^2 + k_1{}^2 = n_0{}^2(0), \tag{19}$$

$$n_1{}^2 - k_1{}^2 = \frac{n_0{}^4(0)}{n_0{}^2(45)}\left(2 - \frac{n_0{}^4(0)}{n_0{}^4(45)}\right). \tag{20}$$

Equations (19) and (20) follow from Eq. (9) and Eqs. (15) and (16), respectively. It is evident that n_1 and k_1 are determined by taking the sum and difference of Eqs. (19) and (20).

The method described above has the advantages of being independent of light polarization and of not requiring any absolute reflectance measurement. However, it is restricted by the availability of variable-index immersion liquids of known refractive indices that satisfy Eq. (19) and can work only for a limited range of absorbing substrates. The detection of a broad minimum is also challenging. The results of Section 5 for light reflection at a dielectric–Au interface provide a test case for the method. Other reflectance methods for measuring n_1 and k_1 have been published [11–14].

5. Reflection at a Dielectric–Au Interface

The optical constants of Au at $\lambda = 500\,\mathrm{nm}$ are $n_1 = 0.8472$ and $k_1 = 1.8775$ as is obtained by interpolation using the data of Lynch and Hunter [15]. Figure 2 shows a family of curves of $|r_s|^2$ versus n_0 at angles of incidence $\phi = 0$ and $\phi = 20°$ to $80°$ in steps of 5°. It is apparent that the ambient refractive index for minimum reflectance $n_0 = n_{0s}$ decreases monotonically as ϕ increases from normal to grazing incidence, whereas the minimum reflectance itself $|r_s|^2{}_{\min}$ increases as ϕ increases. This is shown separately in Fig. 3.

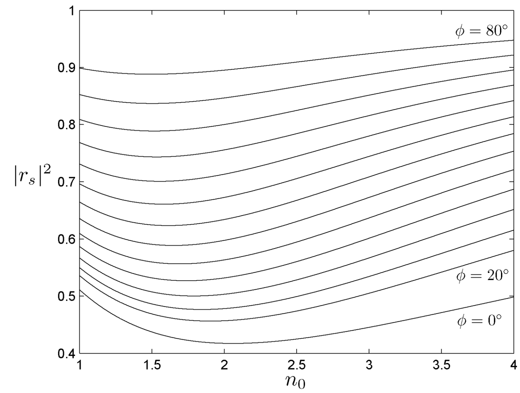

Fig. 2. Intensity reflectance $|r_s|^2$ versus ambient refractive index n_0 for reflection of s-polarized light ($\lambda = 500\,\mathrm{nm}$) at a dielectric–Au interface at fixed angles of incidence $\phi = 0$ and $\phi = 20°$ to $80°$ in steps of 5°.

At $\phi = 0$ and $\phi = 45°$, $n_0(0) = 2.0598$, $n_0(45) = 1.6506$ are obtained from Eqs. (9) and (15), respectively. (The subscript s of n_0 is dropped since the condition of minimum reflectance is independent of polarization at these two angles.) If these refractive indices are substituted into Eqs. (19) and (20), the optical constants of Au ($n_1 = 0.8472$, $k_1 = 1.8775$) are retrieved, which confirms the method described in Section 4.

Figure 4 shows the corresponding family of curves of $|r_p|^2$ versus n_0 for the p polarization at the same set of angles $\phi = 0$ and $\phi = 20°$ to $80°$ in steps of 5°. It is interesting to note that for $\phi \geq 65°$ the reflectance minimum appears at values of $n_0 < 1$, which is acceptable mathematically but not physically. This means that for $\phi \geq 65°$ the lowest reflectance of p-polarized light occurs if $n_0 = 1$, i.e., when the ambient medium is vacuum or air. However, this lowest reflectance itself is not a true minimum in a mathematical sense.

In Fig. 5 the ambient refractive index for minimum reflectance $n_0 = n_{0p}$ and the minimum reflectance $|r_p|^2_{\min}$ are plotted as functions of ϕ. Note that n_{0p}

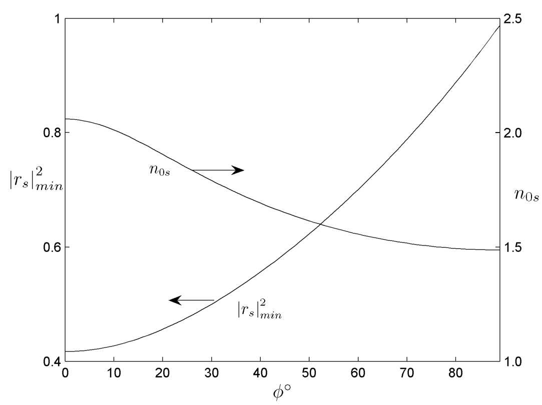

Fig. 3. Ambient refractive index, $n_0 = n_{0s}$, for minimum reflectance of s-polarized light at a dielectric–Au interface ($\lambda = 500\,\mathrm{nm}$) and the associated minimum reflectance $|r_s|^2{}_{\min}$ plotted versus the angle of incidence ϕ.

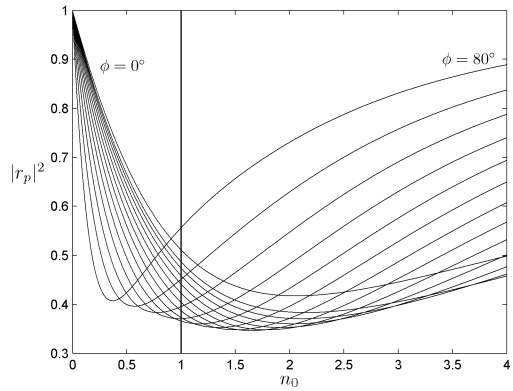

Fig. 4. Intensity reflectance $|r_p|^2$ versus ambient refractive index n_0 for reflection of p-polarized light ($\lambda = 500$ nm) at a dielectric–Au interface at fixed angles of incidence $\phi = 0$ and $\phi = 20°$ to $80°$ in steps of 5°.

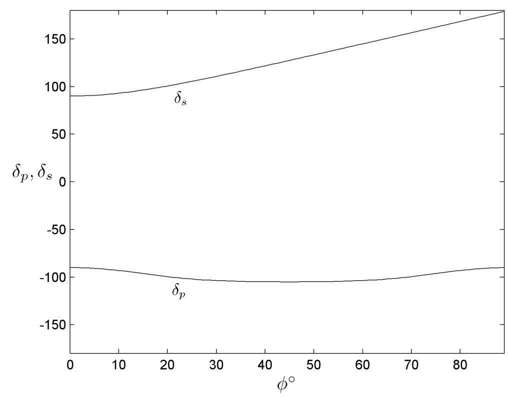

Fig. 6. Phase shifts δ_s, δ_p of s- and p-polarized light ($\lambda = 500$ nm) under conditions of minimum reflectance at a dielectric–Au interface versus angle of incidence ϕ.

initially increases from $n_{0p}(0) = 2.0598$ at $\phi = 0$ (point N) to a local maximum (point M) at $\phi \approx 19°[n_{0p}(19°) = 2.1532]$, then drops to 1 (point O) at $\phi = 64.635°$. [The precise angle of incidence at point O is obtained by solving Eq. (18) for ϕ with $\varepsilon_0 = 1$.] Because values of $n_0 < 1$ are nonphysical, continuation of the n_{0p}-versus-ϕ curve for $\phi \geq 65°$ is represented by a dashed line in Fig. 5. Figure 5 also shows that the minimum reflectance $|r_p|^2{}_{\min}$ has a global minimum ($|r_p|^2{}_{\min} = 0.3464$) at $\phi = 45°$ and is symmetric with respect to ϕ around this angle.

For completeness, the phase shifts δ_s, δ_p under conditions of minimum reflectance are also determined. Figure 6 shows δ_s, δ_p plotted as functions of ϕ. Whereas δ_s increases monotonically from $+90°$ to $180°$ as ϕ increases from 0 to 90°, δ_p starts at $-90°$ at $\phi = 0$, decreases to a minimum of $-105.054°$ at $\phi = 45°$ then increases back to $-90°$ in the limit of grazing incidence, $\phi = 90°$. Furthermore, δ_p is a symmetric function of ϕ around $\phi = 45°$. At the center of symmetry, $\phi = 45°$, $\delta_p = -105.054° + 360° = 254.946°$, $\delta_s = 127.473°$ so that the Abelès phase condition [9], $\delta_p = 2\delta_s$, is satisfied.

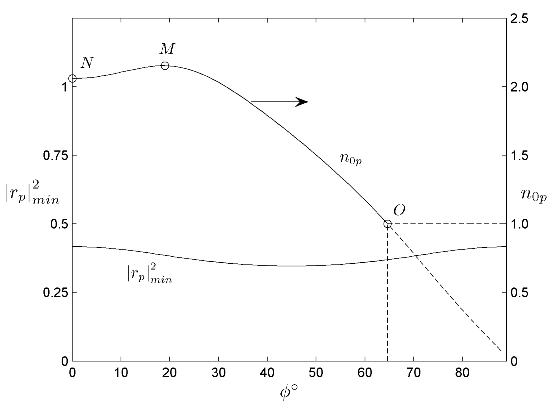

Fig. 5. Ambient refractive index, $n_0 = n_{0p}$, for minimum reflectance of p-polarized light at a dielectric–Au interface ($\lambda = 500$ nm) and the associated minimum reflectance $|r_p|^2{}_{\min}$ plotted versus angle of incidence ϕ.

6. Summary

For a given conducting substrate, the refractive indices of a transparent medium of incidence (ambient) that reduce the reflectance of a dielectric–conductor interface to a minimum for p- and s-polarized light at oblique incidence are determined, and the results are given by Eqs. (15) and (18). The condition of minimum reflectance at normal incidence, $\phi = 0$, and at $\phi = 45°$, is independent of polarization. The refractive indices of an immersion liquid for minimum reflectance at these two angles provide sufficient data to determine the real and imaginary parts of the complex refractive index of an absorbing substrate, as described in Section 4. The analysis of Sections 2–4 is applied to light reflection at a dielectric–Au interface at 500 nm wavelength, and the results are presented in Section 5. It is shown that the lowest possible interface reflectance is obtained for p-polarized light at $\phi = 45°$ angle of incidence and that the associated p reflection phase shift is also at a minimum at the same angle. For $\phi \geq 65°$ the lowest reflectance of p-polarized light occurs if the ambient medium is vacuum or air. However, this lowest reflectance at the air–Au interface is not a true minimum in a mathematical sense.

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