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Analytical Estimation of Value at Risk Under Thick Tails and Fast Volatility Updating

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ANALYTICAL ESTIMATION OF VALUE AT RISK UNDER THICK TAILS AND FAST VOLATILITY UPDATING

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ABSTRACT

Despite its recent advent, value at risk (VaR) became the most widely used technique for measuring future expected risk for both financial and non-financial institutions. VaR, the measure of the worst expected loss over a given horizon at a given confidence level, depends crucially on the distributional aspects of trading revenues. Existing VaR models do not capture adequately some empirical aspects of financial data such as the tail thickness, which is vital in VaR calculations. Tail thickness in financial variables results basically from stochastic volatility and event risk (jumps). Those two sources are not totally separated; under event risk, volatility updates faster than under normal market conditions. Generally, tail thickness is associated with hyper volatility updating.

Existing VaR literature accounts partially for tail thickness either by including stochastic volatility or by including jump diffusion, but not both. Additionally, this literature does not account for fast updating of volatility associated with tail thickness.

This dissertation fills the gap by developing analytical VaR models account for the total (maximum) tail thickness and the associated fast volatility updating. Those aspects are achieved by assuming that trading revenues are evolving according to a mixed *non-affine* stochastic volatility-jump diffusion process. The mixture of stochastic volatility and jumps diffusion accounts for the maximum tail thickness, whereas the *non-affine* structure of stochastic volatility captures the fast volatility updating. The *non-affine* structure assumes that volatility dynamics are non-linearly related to the square root of

current volatility rather than the traditional linear (*affine*) relationship. VaR estimates are obtained by deriving the conditional characteristic function, and then inverting it numerically via the Fourier Inversion technique to infer the cumulative distribution function.

The application of the developed VaR models on a sample that contains six U.S banks during the period 1995-2002 shows that VaR models based on the non-affine stochastic volatility and jump diffusion process produce more reliable VaR estimates compared with the banks' own VaR models. The developed VaR models could significantly predict the losses that those banks incurred during the Russian crisis and the near collapse of the LTCM in 1998 when the banks' VaR models fail.

CHAPTER ONE: INTRODUCTION

1.1 Overview

Value at Risk or VaR is a quantile measure[1] to quantify the risk for financial institutions. It measures the *market risk* of a financial firm's "book"-- the list of positions in various instruments that expose the firm to financial risk. Roughly speaking, VaR measures the worst expected loss over a given horizon under normal market conditions at a given level of confidence.

In a loose form, VaR calculation models make the following statement: "We are c percent certain that we will not lose more than V dollars in the next N days". The variable V is the VaR of the portfolio. It is a function of two parameters: a time period (horizon) N and a confidence level c. Thus, when we calculate VaR for a portfolio of a financial institution, we calculate the expected loss in the portfolio's market value over a given horizon such as one day or two weeks (N) that is exceeded with a small probability, say, 1 percent (1-c).

Accordingly, the quality of VaR for a portfolio depends on its distributional assumption and its valuation model. Assumptions about distribution specify what the VaR model assumes about the distribution of trading revenues --profits and losses (P&L)-

[1] Other measures to quantify risk, in addition to quantile measures, include standard deviation, interquartile range, and lower partial moments or shortfall measures.

of the financial firm's portfolio. It specifies as well what the model assumes about the distribution of the underlying market risk sources upon which the portfolio's value depends. The valuation model specifies how VaR relates the portfolio's value to different shocks in the market risk sources, or the relationship between the return on the portfolio and returns on the instruments included in the portfolio.

The so-called normal approach to VaR assumes that all risk factors in the market are normally distributed and the portfolio is a linear function of those normally distributed risks, which implies that the P&L distribution for the portfolio is also normally distributed. Under such assumptions, VaR calculations become easy to handle. VaR becomes a multiple of the portfolio standard deviation, where the standard deviation is a linear function of individual volatilities and covariances of underlying market risk factors. So all we need to calculate VaR is the variance covariance matrix and information about sizes of individual positions to determine the portfolio standard deviation. Then, by multiplying this standard deviation by a confidence level parameter and scale variable reflecting the size of the portfolio, we obtain a VaR number.

The normality assumption gives "normal VaR" great advantages such as tractability and informativity. Informativity includes translatability across confidence levels and holding periods and informativity about expected tail losses, [Dowd (1998) and Jorion (2001)].

Unfortunately, the normal approach to VaR does not always fit actual financial institutions' portfolios or actual finance data. On one hand, portfolio returns may be nonlinear in risk factors, as in the case when the portfolio contains positions in options or fixed-income instruments. On the other hand, normality is a very strong assumption for finance data, since many empirical studies in time series data, Bollerslev (1987) for example, show that the rate of return of percentage change of many financial variables are not normally distributed. It tends to be skewed and leptokurtic.

In response to non-linearity, authors use the Taylor series expansion or linear approximation for returns to these instruments, and then use the linear approximation to work out VaR. The simplest approximation produces the so-called delta-normal approach. Delta-normal, by definition, is the first order Taylor series expansion of a portfolio's value with respect to stock returns. Delta-normal restores the linear normality and makes VaR estimation easy. However, such a simple approximation produces an inaccurate VaR estimate. Wilson (1996) argues that delta-normal approach would produce reliable estimates for VaR for a small holding period or / and when the portfolio has few option positions (close to normality).

To get a more accurate VaR for non-linear positions (options and fixed income), some literature including Wilson (1994, 1996), Jamshidian and Zhu, (1996), and Zangari (1996 a, b) use the quadratic model or the second Taylor series expansion known as delta--gamma approach. Jamshidian and Zhu, (1996), Zangari (1996 a, b), and Fallon (1996) report that their delta-gamma approaches improve VaR estimates tremendously compared with delta-normal.   Gamma, the first derivative of delta with respect to stock returns, measures the curvature of the relationship between the portfolio value and the underlying market variable. A non-zero gamma implies skewness in the distribution of P&L of the portfolio. When gamma is positive (negative), changes in the portfolio are positively (negatively) skewed.

The normality assumption of risk factors or P&L of the portfolio would certainly affect VaR estimates since it depends crucially on the distribution of the tail. Thus if we have a thicker (thinner) tail compared with the normal distribution, then VaR estimates based on normality would be under-- (over--) estimated. Zangari (1996 c) discusses VaR as a technique of risk management under departure from normality. He shows that VaR calculated under normality assumption underestimates risk since the observed distribution of many financial return series have tails that are flatter than those implied by conditional normal distribution.

Generally speaking, with deviation from normality assumption, there are two main methods to construct VaR: parametric approach (analytical approach) and the non-parametric approach (the simulation approach). In the parametric approach, an alternative distribution is explicitly assumed instead of the normal distribution, and based on this assumed distribution; a formula to describe the confidence interval is analytically derived. The variance-covariance approaches that include the distribution of portfolio return method, the delta normal valuation method and delta-gamma method fall under the parametric approach.

In the non-parametric approach, no particular distribution assumption is needed. VaR is calculated from the standard theory of order statistics, as in Kupic (1995), or from Monte Carlo simulations. Under this approach, VaR is deduced from multiple runs that might be representative of the possible market price outcomes. Chapter Two below provides full description of VaR, VaR methods, recent developments on VaR methods, and strengths and weaknesses of VaR analysis.

By definition, VaR estimates depend on the distribution of the left tail. VaR parametric approaches that deviate from normality assumption aim basically at modifying for kurtosis. A probability distribution with fat tails has a greater probability mass out in the tails of the distribution, where large price movement occurs compared with the normal distribution. Accordingly, VaR estimates tend to under-- or over-- estimate risk if normal distribution is assumed, Zangari (1996c).

Fat tails in finance data come from stochastic volatility of the underlying market risk factor (like stock returns). The size of kurtosis depends on the correlation between volatility of returns and returns themselves (Duffie and Pan 1997), Das and Sundaram (1999) and Lewis (2000). However, Bollerlev (1987) reports that actual market data are typically found to have more leptokurtic compared with what would come from stochastic volatility alone.

Another wave of literature including Bates (1996), Duffie and Pan (1997), Das and Sundaram (1999), Lewis (2001), and Airoldi (2001) see that the other source of fat tails in the distribution of financial data comes from jumps in the underlying risk factors. New evidence from Chenov, Gallant, Ghysels and Tauchen (1999), Chourdakis (2000), Lewis (2002), and Chacko and Viceira (2003) find that stochastic volatility and jumps are both significant in explaining the dynamics of stock returns for different frequencies. Chenov, Gallant, Ghysels and Tauchen (1999) find that non-affine jumps and stochastic volatility are significant in finance time series data.

To account for kurtosis in the distribution of underlying risk factors in VaR estimates, authors use different methods. One method used is to assume an alternative distribution that accounts for tail thickness. Fong and Vasicek (1997) assume that changes in the portfolio value follow gamma distribution. Rachev and Mittnik (2000), Khindanova, Rachev, and Schwartz (2001), Rachev, Schwartz and Khindanova, (2002) use stable Paretian models for modeling the distribution of the P&L in the portfolio. Another method used to achieve kurtosis is to use a mixture of two normal distributions as in Zangari (1996 d, e),Venkataraman (1997), and Hull and White (1998a).

Duffie and Pan (1997 and 2001) and Gibson (2001), on the other hand, adopt event risk (jumps) as a source of tail thickness. Duffie and Pan (1997) and El-Jahel, Perraudin, and Sellin (1999) use stochastic volatility to achieve kurtosis in the underlying risk factors for a portfolio with derivatives' positions and they include the first four moments in their estimation. Levin and Tchernitser (2001) account for tail fatness by assuming time varying volatility and Levy Processes (stochastic volatility). Li (1999) also includes skewness and kurtosis explicitly in VaR estimates. Many other VaR methods utilize the extreme value theory (EVT) to account for tail dynamics and thickness[2]. Chapter Two summarizes the literature of VaR techniques under non-normality assumption.

One crucial issue in estimating VaR is estimating the current volatility of underlying risk factors[3] or the change in the portfolio returns. VaR normal models assume volatility to be constant, since it is estimated under the assumption of normal distribution. Hull and White (1998b) report that incorporating volatility updating into VaR models improves substantially VaR estimates. Generally, VaR models use the common estimation techniques for estimating volatility including historical volatility,

[2] For a survey of such literature see Embrechts, Kluppelberg and Mikosch (1997) and Embrechts (2002).
[3] When the portfolio depends on several positions, VaR estimates need estimation for the variance-covariance matrix and the correlation matrix. And with increase of the number of positions this process becomes more cumbersome.

ARCH/GARCH volatilities, implied volatility from option pricing, moving average volatilities and extreme value theory (EVT).

Berkowitz and O'Brien (2002) find that VaR models depend on GARCH estimation for volatility bound actual daily losses in banks' portfolio closer than internal VaR models used by banks. However, they find that even VaR models that depend on GARCH models could not describe actual portfolio dynamics during the Russian crisis and the near collapse of the LTCM in August and September 1998.

Eberlein, Kallsen and Kristen (2001) test for the impact of using different methods of estimating current volatility in VaR models and find that the estimation technique substantially affects VaR estimates. Chapter Two gives detailed explanations for volatility estimation methods used in VaR context.

1.2 Motivation

This research is motivated by the suggestion by Bates (1996), Duffie and Pan (1997), Das and Sundaram (1999), Lewis (2001), and Airoldi (2001) that full kurtosis can not be captured by stochastic volatility alone or jumps alone. It is also motivated by new evidence from Chenov, Gallant, Ghysels and Tauchen (1999), Chourdakis (2000), Lewis (2002), and Chacko and Viceira (2003) who find that stochastic volatility and jumps are both significant in explaining the dynamics of stock returns. Gibson (2001) and Lewis (2002) suggest that modeling of financial data should combine both, time varying volatility and event risk. Both Gibson (2001) and Lewis (2002) consider this task very important and very challenging, especially in modeling risk factor's dynamics.

The other motivation comes from the suggestion of Airoldi (2001) that fat tails are associated with faster updating of volatility; the thicker the tail, the faster the volatility dynamics. According to Airoldi (2001), jumps and extreme events cause hyper-growth in volatility. Thus, the assumed linear relation between volatility dynamics (updating in volatility) and volatility levels would no longer hold during jumps and extreme events (fat tails), hence a non-linear representation of volatility dynamics becomes necessary. Additionally, if we accept the non-linear dynamics of volatility updating, Hsieh's (1993) suggestion becomes essential. Hsieh (1993) suggests that when non-linear dynamics emerge in finance data, conditional densities provide better description of short-term movements compared with unconditional densities, which has very important implication in risk management.

1.3 The Purpose of the Work

The purpose of this dissertation is to provide an approximate analytical estimate of VaR for a class of non-normally distributed portfolio's P&L that allows for the maximum tail thickness and fast volatility updating.

To obtain such non-normal distribution and nonlinear relationship, the dissertation assumes that trading revenues on financial institution's portfolio is evolving according to a mix of a non-affine stochastic volatility-jump diffusion model (NASVJ) in a form mentioned by Chacko and Viceira (2003). Such a model allows for both sources of tail thickness in financial data. It allows as well, through the non-affine stochastic volatility specification, for fast updating in volatility that usually combines thick tails and extreme events, as Airoldi (2001) suggests. In this non-affine stochastic volatility model, volatility grows according to a non-linear square relation with its current level.

A VaR estimate based on such distribution should be able to predict the losses that banks experienced during the stock market crash in October 1987. It should be able to explain also the large bank losses occur in September 1998 during the Russian crisis and the near collapse of the LTCM, what internal banks' VaR models failed to capture, as reported by Berkowitz and O'Brien (2002).

With certain restrictions on the set of parameters of the NASVJ, as shown later, five well-known processes emerge: the diffusion process with no jumps, jump diffusion process, stochastic volatility process, non-affine stochastic volatility, and stochastic volatility with jumps. Those processes are well-known in finance literature. However, it received limited use in VaR literature. A limited number of works including El-Jahel, Perraudin, and Sellin (1999) and Duffie and Pan (2001) use those processes. To my knowledge, this is the first work that combines stochastic volatility, jumps, and non-affine structure of volatility in deriving an analytical solution for VaR.

1.4 Methodology

To derive an analytical estimate for VaR under thick tails and fast volatility updating, the dissertation assumes that trading revenues of the bank follow a non-affine mixed stochastic volatility with jump process. An explicit form of VaR is derived by obtaining the conditional distribution function of trading revenues. The necessity of using the conditional distribution rather than the unconditional one comes from Hsieh's (1993) concern that conditional densities provide a better description of asset price movements in the presence of non-linear dynamics.

Deriving the conditional density crucially depends on the transformation approach developed by Shephard (1991) and utilized by Duffie, Pan, and Singleton (2000), and Chacko and Das (2002) for affine processes. According to this transformation method, the conditional probability function (cpf) is derived from the conditional characteristic function (ccf). When the conditional probability function (cpf) becomes known, it becomes straightforward to calculate an estimate for VaR based on the statistical definition of VaR as:

$$\Pr\ \left[\Delta P(N) < -VaR\right] = F\left[\Delta P(-VaR)\right] = \int_{-\infty}^{-VaR} f\left(\Delta P(x)\right)dx = 1\text{-}c \qquad (1\text{-}1)$$

Where $F[\Delta P(.)]$ is the cumulative distribution function, (cdf) of trading revenues, ΔP, and $f(\Delta P(.))$ is the probability density function of (pdf) of ΔP.

Thus the first task would be to derive the conditional characteristic function, and then by the standard Fourier-inversion, we obtain the cumulative density function from which we can directly calculate VaR. The dissertation derives an analytical VaR for the mixed non-affine stochastic volatility --jumps and for the five special cases of the process mentioned above based on the same procedure.

This methodology of deriving the conditional probability distribution is applicable directly for affine processes. As a matter of fact, it is applicable only for affine processes as shown in Shephard (1991), Duffie, Pan, and Singleton (2000), and Chacko and Das (2002. For non-affine processes, we need first to transform the non-affine processes to affine processes, mostly by using some of the perturbation methods of Kevorkian and Cole (1981), and then perform the Fourier-inversion.

The procedure of deriving VaR estimate from the characteristic function has been used before. El-Jahel, Perraudin, and Sellin (1999) and Duffie and Pan (2001) show how to replace the simulation step in VaR with a stochastic volatility process for derivative portfolio through Delta-Gamma approach. Duffie and Pan (2001) apply similar principle on a jump diffusion process. Cardenas, Fruchard, Koehler, and Michel (1997) also replace a Delta-Gamma approach for option's portfolio with analytical solution for VaR for a pure diffusion model with no jumps or stochastic volatility. Chapter Four describes this method in details, and derives VaR estimates and characteristic functions for the NASVJ process and other processes.

As mentioned earlier, a crucial issue in estimating VaR is the estimation of current volatility. Current volatility will be estimated from historical data by estimating the assumed processes for stock returns using spectral GMM estimator of Chacko and Viceira (2003). Spectral GMM estimator depends on the empirical characteristic function, where the unobserved variable (volatility, for example) can be integrated out. Singleton (2001) and Jiang and Knight (2000) estimate continuous time processes through the characteristic function. Note that volatility in the jump diffusion process would be constant.

Empirically, the dissertation applies the analytical VaR model developed in this context on a sample of six largest banks' trading portfolios in the United States. In the empirical part, the dissertation proceeds through comparing among different VaR estimates based on different processes with a performance criterion of actual trading profits and losses for those banks during the period 1995: Q1-2002: Q3. The compared models include internal VaR estimate, the Geometric Brownian Motion VaR model (GBM VaR), the stochastic volatility model VaR (SV-VaR), the jump diffusion VaR, (JD VaR), the stochastic Volatility with jump diffusion VaR (SVJ VaR), the non-affine stochastic volatility VaR (NASV VaR), and non-affine stochastic volatility with jump diffusion VaR (NASVJ VaR).

1.5 Data

This part of the dissertation uses quarterly data published by the largest US banks, for a sample starting from 1995:Q1-2002: Q3. Banks start to publish their internal VaR calculations almost with the beginning of 1995, Jorion (2002). The probability distribution for profit and losses is not published on daily or weekly bases. It is only published on quarterly and yearly bases.

For their "1-day" VaRs, banks measure only price fluctuation risk, and they ignore expected returns. Berkowitz and O'Brien (2002) use the banks' 1-day trading revenues (unpublished P&L), which include fee income from trading and possibly trading position net interest income for some banks, as well as gains and losses on positions. This is, of course, inconsistent with the risk being measured in their VaR models. Accordingly, trading revenues are modified using the moving average method of Jorion (2002) to extract unexpected trading revenues from the published trading revenues. As mentioned before, the expected trading revenues component includes fees income and interest rate income. Banks' internal VaR models ignore those expected components from trading revenues when calculating VaR.

On a quarterly basis, there is an item of "trading revenue" reported in the bank's call reports, which includes position's gains and losses and fees and expenses incidental to trading. The call reports also report separately interest income and expense on trading positions. These trading revenue components are available in the banks' quarterly (10Q) and annual (10K) reports filed with the SEC. The sample period is chosen to include the third quarter of 1998 where banks experienced large losses because of the Russian crisis and the near collapse of the LTCM. Chapter Three provides a fully detailed explanation for the data and its statistical proprieties.

The dissertation proceeds as follows: Chapter Two provides a detailed explanation of various risk facing financial institutions, detailed definition of VaR, methods of VaR, developments of VaR, comparisons among VaR methodologies, and VaR weaknesses.

The third chapter introduces the data and manipulations for the data. The chapter explains the data sample, sources of the data, data statistics. It also tests for the degree of accuracy of banks' VaR models in capturing actual losses for banks during the period 1995-2002. The test for accuracy of VaR models follows Berkowitz and O'Brien (2002) and Jorion (2002). The results of those tests show that banks' VaR are very conservative compared with actual losses occurred during the third quarter of 1998. Those results indicate the need for new VaR models that account for tail thickness and fast volatility updating.

Chapter Four explains in detail the methodology deriving VaR estimates. It reviews the general methodology and reviews the literature concerning the characteristic function and the use of the Fourier inversion theorem and its uses in economics and finance. In Chapter Four also, the dissertation introduces the derivation of the models, the characteristic functions, and the Fourier inversion for the processes.

The fifth chapter applies VaR solutions developed in Chapter Four on the trading revenues of the banks in the sample. Additionally, the different estimate of VaRs based on the proposed models will be compared with each other to a reference point of the distribution of the actual losses that banks incurred in the last eight years. This includes the period where banks incurred massive losses during the Russian crisis and the near collapse of the LTCM, where most of VaR models fail.

CHAPTER TWO: REVIEW LITERATURE

Two methodologies of VaR measures have evolved during the last decade. One obtained by deriving an analytical valuation of VaR, which depends on certain parameter estimates, called the parametric approach. The other derives VaR by repeatedly performing several simulation steps, including historical and Monte Carlo simulations. This chapter reviews the concepts of VaR, VaR measurement techniques, developments of VaR techniques, and the strengths and weaknesses of such techniques.

2.1 The Need for Unified and Quantified Risk Measure

Modern finance theory emphasizes risk as a major determinant of return. Merton (1980) argues that expected return has a linear relationship with risk. According to his view, it is easier to measure risk – defined as volatility of return – than to measure expected return itself. Merton's idea sounds intuitively clear when applied to an equity portfolio replicating an index. However, it is questionable when applied to a globally diversified portfolio with positions traded in completely segmented markets, as in the case of trading portfolios of financial institutions.

Taking a look at a trading portfolio of a financial institution like JP Morgan Chase & Co. or CitiCorp as it is published in their reports suggests the following two observations: The great diversity of trading positions, and the difficulty of and the need for combining all risk classes underlying those positions in one single measure. The trading portfolio of JP Morgan Chase & Co. contains, among others, positions in U.S and foreign governments securities, corporate securities, and derivative securities of interest rates, foreign exchange rates, bonds, equities, and commodity contracts. Each of those positions is associated with a different source of risk and a different risk calculation method, as follows.

The major risk underlying U.S Government securities is the interest rate risk. Duration is a simple measure for estimating volatility in the prices of those securities and their relationship with interest rates. By estimating convexity and sensitivity parameters in the case of non-parallel shift in the term structure of interest rates, duration can be modified to account for nonlinearity in the relationship between interest rates and government security prices. A more complicated analysis may involve a parametric quantifying of the corresponding risk factors affecting the term structure of interest rates (see for example Golub and Tilman [2000] and Fabozzi, [2000]). For foreign government securities, the foreign exchange rate is considered a distinctive risk factor in determining returns. Foreign exchange rates have their own risk indices, spreads, and volatilities. Recently, target zones have been added to several currencies, (Winer [1997]).

Risk factors underlying corporate securities are dependent on the type of securities held in the portfolio. For corporate bonds, in addition to the risk associated with the interest rate, default is a major source of risk that correlates with other indices in the economy. One way to quantify this risk is to use credit rating (Golub and Tilman [2000]).

For equities, volatility of returns is a straightforward measure of risk. Many models used in finance like the CAPM and the APT utilizes the correlation between the asset and the risk factor as a risk measure. For example, in the CAPM, where the only risk factor is the market, the measure of risk (Beta) is the covariance between the market and stock returns.

Risk factors underlying derivative securities are essentially the same as those associated with the underlying assets of those derivatives. However, measuring the risk associated with each derivative position depends on the shape of the relationship between the derivative price and the underlying asset price. Forwards and futures prices have a linear relationship with the prices of the underlying assets. Accordingly, the risk associated with those contracts can be expressed as a linear transformation of the risk associated with the underlying asset.

For options, where the relationship between the price of the option and the price of the underlying asset is non-linear, risk can be measured by the Greeks, delta, gamma, theta, vega and rho. Each of these letters reflects the rate of change in the price of the option when only one of the parameters change, like the price of the underlying asset, time to maturity, volatility and interest rate, (see for example Hull [2002]). To measure the risk associated with interest rate and currency swaps we need a model of default and recovery, which is necessary for measuring credit risk derivatives, (Saunders [1999]).

Risk is not limited to the above sources. The risk sources mentioned above are coming basically from the market; the risk of unexpected changes in prices or rates. Credit risk is another type of market risk (Duffie and Pan [1997]). Financial institutions are subject to other types of risk like operation risk legal risk and liquidity risk. However, for trading activities, market risk, the risk of unexpected changes in prices or rates, might be the most important source of risk affecting the value of the trading portfolio. Jorion (2001) and Crouhy, Galai, and Mark (2001) introduce a comprehensive explanation of risk sources affecting financial institutions' portfolios and propose risk management tools for each risk factor.

Even if we focus our attention on the market risk, we still see that each position in the financial institution's portfolio, in the case of JP Morgan Chase & Co., is related to a different market risk, requiring a different measurement technique. The measurement technique is unique to that risk factor and not applicable to other factors. Clearly, those measures are not additive, or even comparable. Thus all of those risk measures cannot give a simple answer to the basic question regarding the level of risk that a portfolio faces over a specific trading horizon.

Linsmeier and Pearson (1999) note that those complications were raised after 1973, the year that witnessed the collapse of the Bretton Woods system of fixed exchange rates and the publication of the Black-Scholes option pricing formula. Those events resulted in considerably high volatility in exchange rates, interest rates, and commodity prices, and a proliferation of derivative instruments as useful tools for hedging risks that emerge from market rates and prices dynamics. With those changes, financial institutions increased their positions in financial derivatives for both hedging and speculative purposes, a move that made the extent of market risk less obvious. Ultimately, this led to an increased demand for quantitative measures of market risks for trading portfolios. A different rationale is provided by Jorion (2001) and Crouhy, Galai and Mark (2001) who argue that the recent financial distresses worldwide were the primary motive behind the search for a simple risk measurement tool.

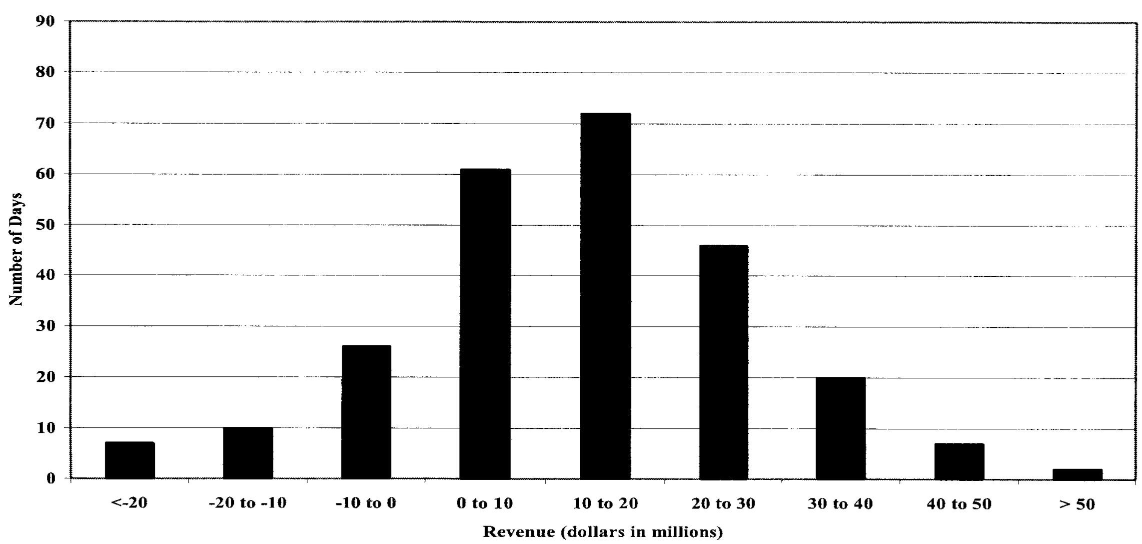
One way to avoid these above complexities is to look at the probability of a given loss to occur during a specified time horizon *N*, a straightforward statistical measure. List all possible outcomes in each of the *N* periods and for each outcome, assign a probability for the set of lower values to occur. If we know the probability distribution of the profit and loss of a certain portfolio, then we can identify the probability of incur losses exceeds each event (the left tail of the distribution). When we face the question, " what is the maximum loss we may suffer over some time horizon, say *N* days?" we answer, " we are c% confident that we are not losing more than $V of our wealth over this time horizon." Notice here that $V is the event (loss) that would be exceeded by (1-c)%. Equivalently, we can say that there is a (1-c)% probability of losing more than $V of our wealth over the next *N* trading horizon. Our answer still implies that there is a possibility of losing all of our wealth but with a minimal probability. However, the probability of losing everything depends on our portfolio risk as defined by the standard deviation of the profits and losses assuming normal distribution.

For example, given a time horizon of 100 trading days, and a confidence level of 95%, our losses will exceed \$$V_1$ only in 5 trading days. Here N = 1 day and c = 95%. For 99% confidence level, we will incur losses exceeding \$$V_2$ only in one day out of 100 trading days or 2-3 days during a trading year, where $|V_1| < |V_2|$. This means additionally that, for the next trading day, we are 95% sure that our losses will not exceed \$$V_1$, that is we are 99% sure that our losses for the next trading day will not exceed \$$V_2$.

2.2 The Concept of VaR

Before going to the formal definition of VaR, let us apply the above loose definition to the distribution of the profit and losse (P&L) of the trading accounts for Bank of America. Figure (2.1) below represents the histogram for the daily trading revenues for Bank of America during the period Oct.1st 2001 to Sep. 30th 2002, which represents 251 trading days. If we assume that the histogram represents the daily P&L of the trading portfolio under the normal market conditions, then we can make our previous statements as follows.

Figure 2.1
Histogram of Daily Market Risk –Related Results
For Bank of America, 12 Months Ended September 30, 2002.



Source: Bank of America Quarterly Report (Q10), September 2002.

We are 97.2% confident that losses of Bank of America will not exceed \$20 millions over the next trading day. That means there is only 2.8% probability that Bank of America will incur losses exceeds \$20 millions during the next trading day. The 2.8%

is the percentage of the number of days that the Bank incur losses more than $20 millions to the total number of trading days. Additionally, we can say that at a 95% confidence level, the losses of Bank of America will not exceed $14.5 millions over the next trading day. Notice that 5% of the trading days is 12.5 days. We see from the histogram that there are 17 days (6.8% of total trading days) the bank suffered from losses exceeding $10 millions and 7 days (2.8% of the total trading days) the bank suffered from losses exceeding $20 millions. Interpolating, we find that the amount of losses that will be exceeded by 5% of the trading days is $14.5 millions. The number $14.5 millions is VaR at 95% confidence level over one trading-day time horizon.

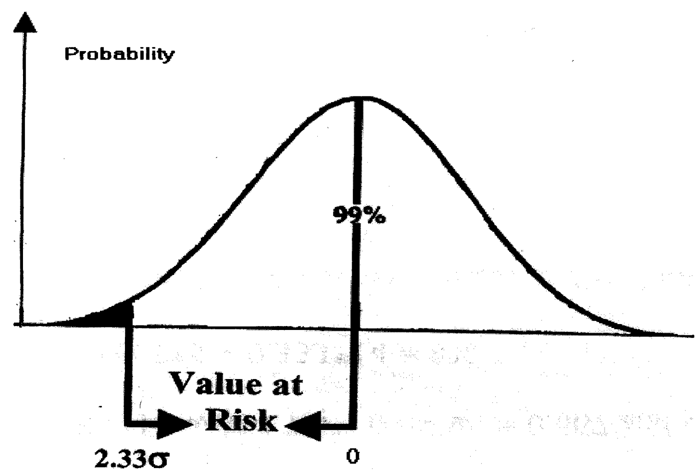
Accordingly, we can define *Value at Risk* or VaR as a quantile measure[4] to quantify the risk for financial institutions. It measures the *market risk* of a financial firm's "book", the list of positions in various instruments that expose the firm to financial risk. Roughly speaking, VaR measures the worst expected loss over a given horizon under normal market conditions at a given level of confidence, Duffie and Pan (1997).

As indicated in the loose form definition, VaR calculation models make the following statement: "We are c percent certain that we will not lose more than V dollars in the next N days". The variable V is the VaR of the portfolio. It is a function of two parameters: a time period (horizon), N, and a confidence level c. Thus, when we calculate VaR for a portfolio of a financial institution we calculate the expected loss in the portfolio's market value over a given horizon such as one day or two weeks, N, that is exceeded with a small probability say 1 percent, 1-c. Statistically speaking, value at risk

[4] Other measures to quantify risk, in addition to quantile measures, include standard deviation, interquartile range, and lower partial moments or shortfall measures.

measure-in this case- is the 0.01 critical value of the probability distribution of changes in market value as shown in figure 2.2 below.

Figure 2.2
0.01 Critical Value of Probability Distribution of Changes in Market Value Assuming Normality



Accordingly, VaR is defined as the upper limit of the one-sided confidence interval:

$$\Pr\,[\Delta P(N) < -\mathrm{VaR}] = 1-c \tag{2-1}$$

Where c is the confidence level, and $\Delta P(N) = \Delta P_t(N)$ is the relative change in the portfolio value (P&L) over the time horizon N. $\Delta P_t(N) = P(t+N) - P_t$. $P(t+N)$ is the natural logarithm of the portfolio value at time $t + N$ and P_t is the natural logarithm of the portfolio value at time t.

Statistically, equation (2-1) means that VaR values are obtained directly from the probability distribution of P&L as follows,

$$\Pr\ \left[\Delta P(N) < -VaR\right] = F\left[\Delta P(-VaR)\right] = \int_{-\infty}^{-VaR} f\left(\Delta P(x)\right)dx = 1\text{-}c \qquad (2-2)$$

Where $F[\Delta P(.)]$ is the cumulative distribution function, (cdf) of the trading revenues, ΔP, and $f(\Delta P(.))$ is the probability density function of (pdf) of ΔP.

2.3 VaR Conversion across Time Horizons and Confidence Levels

At a confidence level of 99%, a one day VaR is a one day loss that is expected to be exceeded in only one trading day out of 100 trading days. A 99% two weeks VaR is a two weeks loss that will be exceeded roughly once every 4 years. By the same argument, a 99% three months VaR is a three months loss that will be exceeded once every 25

$$\text{VaR}_{2\text{ weeks}} = \text{VaR}_{1\text{ day}}\sqrt{10} \approx 3.16\ \text{VaR}_{1\text{ day}}$$

$$\text{VaR}_{3\text{ Months}} = \text{VaR}_{1\text{ day}}\sqrt{63} \approx 7.93\ \text{VaR}_{1\text{ day}}$$

$$\text{VaR}_{3\text{ Months}} = \text{VaR}_{2\text{ weeks}}\ \frac{\sqrt{63}}{\sqrt{10}} \approx 2.51\ \text{VaR}_{2\text{ weeks}}$$

years. Here we need to recognize that:

Where we assume that the two weeks period contains 10 trading days and the 3 months period contains roughly 63 trading days. So, in absolute value, $\text{VaR}_{1\text{ day}} < \text{VaR}_{2\text{ weeks}} < \text{VaR}_{3\text{ Months}}$, and so on.

For confidence level, if we assume normality, and constant volatility, then a 99% confidence level VaR can be expressed as a 95% confidence level VaR as follows:

$$\text{VaR}_{99\%} = \text{VaR}_{95\%}\ \frac{2.326}{1.645} \approx 1.41\ \text{VaR}_{95\%}$$

Where 2.326 and 1.645 (known as α's) are the standard normal deviate corresponding to a 99% and 95% confidence level. In the same way, the three-months 99% confidence level VaR can be expressed as a one day, two weeks or any horizon length VaR at any confidence level as follows:

$$\mathrm{VaR}_{99\%\,,3\,\mathrm{Months}} = \mathrm{VaR}_{95\%,1\,\mathrm{day}} \; \frac{2.326\sqrt{63}}{1.645} \approx 11.19 \quad \mathrm{VaR}_{95\%,1\,\mathrm{day}}$$

$$\mathrm{VaR}_{99\%\,,3\,\mathrm{Months}} = \mathrm{VaR}_{95\%,\;2\,\mathrm{weeks}} \; \frac{2.326\sqrt{63}}{1.645\sqrt{10}} \approx 3.54 \quad \mathrm{VaR}_{95\%,\;2\,\mathrm{weeks}}$$

Worth to say that the above conversion method is mostly convenient and applicable for trading revenues that are normally distributed, where we assume that the standard deviation of trading revenues is constant.

2.4 VaR Users and VaR in Regulations

In spite of its recent advent as a risk management tool, the use of VaR has blown up very quickly. VaR was used first by major financial firms in the late 1980's. Linsmeier and Pearson (1999) suggest that the releasing of JP Morgan to its RiskMetrics™ system in 1994 was the reason behind this tremendous use of VaR in spite of its relatively new age. VaR is now used by most of derivatives' dealers, even small financial firms, non-financial corporations, institutional investors and central banks. Linsmeier and Pearson (1999) report results of some surveys about VaR use by different types of institutions in their notes section. For more about the developments of using VaR see Jorion (2001) and Crouhy, Galai and Mark (2001).

Regulators, on the other hand, become increasingly interested in VaR as measure for risk management, especially after the consecutive financial disasters during the late

1980's and 1990's. Basle Committee on Banking Supervision allows banks to calculate their capital requirements for market risk based on their own VaR models. The Securities and Exchange Commissions requires U.S. companies to disclose quantitative measures of market risk, with VaR listed as one of three possible market risk disclosure measures. For detailed analysis of VaR in regulations, see Jorion (2001) and Crouhy, Galai and Mark (2001). Khindanova and Rachev (2000) introduce very summarized and conclusive review for the developments of VaR in use and VaR in Regulations.

2.5 Components of VaR Measures

In order to implement VaR measures, we should have a histogram like the one in figure 2.1, or the components of equation 2-2, namely, the cdf, or the pdf that will enable us to draw a figure as the one in figure 2.2. The histogram in figure 2.1 represents basically the distribution function. To draw a figure like 2.1 or 2.2 we need to collect data regarding the current portfolio positions or the trading revenues, P&L, for a specified time interval, which represents the holding period or the trading horizon. We need also to build a model to predict the distribution of the P&L of the portfolio as a function of the market parameters. The last step means that we need to infer the cdf or the pdf. As we will see later, VaR methodologies are exclusively different in the ways of constructing pdf's.

More formally, VaR measure for any trading portfolio depends on its quality on the following components:

1- the distribution assumption

2- volatility and covariance estimates

3- the window length of the data used for parameter estimates

4- time horizon or the holding period

5- confidence level

In addition to the above components which assumes the mean of return zero, some VaR measures as Jackson, Maude and Perraudin (1997) incorporate the mean of return in VaR analysis. However, their results show no proof for any certain differences when the mean of returns assumed to be zero.

2.5.1 The Distribution Assumption

As mentioned earlier, VaR methodologies are different basically because of the way they construct the pdf. Traditionally, VaR techniques approximate the cdf's in three broad methods: the parametric method or analytical models, historical simulation or the empirical based methods and the Monte Carlo simulation or stochastic simulation method. The historical simulation and the Monte Carlo simulation are usually called the non-parametric approaches. Section 2.6 deals with VaR methods in details.

2.5.2 Volatility and Covariance Estimates

Estimating current volatility is essential in VaR models. When the portfolio includes several positions, VaR estimates need estimation for the variance-covariance matrix and the correlation matrix, as we will see in the variance covariance VaR method. However, with increasing the number of positions this process becomes more cumbersome.

VaR methods apply different models to estimate current volatility. Traditionally, VaR estimates use constant volatility models to predict current volatility. But we know since Engle (1982) that volatility in financial data is not constant and it is moving over time. Hull and White (1998b) report that incorporating volatility updating into VaR models improves substantially VaR estimates.

To account for volatility updating, researchers in VaR use different models. JP Morgan (1995) uses a uniform exponential weighting volatility model. Lawrence and Robinson (1995) use what is know as asset-specific exponential weighting volatility model. Jackson, Maude and Perraudin (1997) show that certain parameters' value in volatility estimate (within the exponential weighting volatility models) leads to higher tail probabilities. They show that there is a trade off between the degree of approximating time-varying volatility and VaR predictions. Hendricks (1996) shows that there is a negative relationship between the size of the parameters of the exponential weighting volatility model and the variability of VaR measurements. For more about uniform and asset-specific exponential weighting volatility models see Dowd (1998), Jorion (2001) and Hull (2002).

Other models of time-varying volatility used in VaR context include the ARCH models of Engle (1982), GARCH models of Bollerslev (1986), EGARCH models of Nelson (1991), extreme value theorem EVT as in Bradley and Taqqu (2001), and implied volatilities from option prices as in Duffie and Pan (2001). Berkowitz and O'Brien (2002) find that VaR models that depend on GARCH estimation for volatility bound the actual daily losses in banks' portfolios closer than internal VaR models used by banks.

Eberlein, Kallsen and Kristen (2001) test the impact of using different methods of estimating current volatility in VaR models and find that the estimation technique affects substantially VaR estimates. For different techniques for estimating volatility in VaR and its effect in VaR estimates see in addition to Eberlein, Kallsen and Kristen (2001), Duffie and Pan (1997), Dowd (1998), Khindanova and Rachev (2000), Gencay, Selcuk, and Ulugulyagci (2001) and Bradley and Taqqu (2001), Jorion (2001) and Hull (2002).

VaR estimates for portfolios that contain many positions needs to estimate also the correlation matrix beside the covariance matrix. Beder (1995) finds that VaR are sensitive to correlation assumptions. She calculated VaR for a portfolio under different assumptions of the correlation matrix. Her results show significant differences of VaR estimates under those different assumptions of correlation matrix.

2.5.3 The Window Length

The window length is the length of the data sample (the observation period) used for VaR estimation. The window length choice is related to sampling issues and availability of databases. The Basle Committee suggests the 250-day (one-year) window length.

Beder (1995) estimates VaR applying the historical simulation method for 100-day and 250-day window lengths. Beder shows that VaR values increase with the expanded observation intervals. Hendricks (1996) calculates VaR measures using the parametric approach with equally weighted volatility models and the historical simulation approach. Hendricks (1996) uses different window lengths ranging from 50 days to 1250 days. He reports that VaR measures become more stable for longer observation periods. Jackson, Mauder and Perraudin (1997) reach to the same conclusion of Hendricks (1996) by computing parametric and simulation VaRs for 1-day and 10-day time horizons using window lengths from three to 24 months. They concluded that VaR predictions based on longer data windows are more stable and reliable.

2.5.4 The Holding Period and Confidence Level

The holding period or the time horizon in VaR analysis takes any trading interval. In regulations it varies from one day to two weeks (10 trading days). The choice of the length of the holding period depends on liquidity of assets and frequency of trading transactions. A basic assumption in all VaR calculations is the portfolio composition remains unchanged over the holding period. The Basle Committee recommends 10-day holding period. RiskMetrics uses 1 day. Most of financial institutions use 1 day holding period and report VaR estimates at daily basis in their quarterly and annually reports. Long holding periods are usually recommended for portfolios with illiquid instruments.

Beder (1995) investigates the impact of time horizon on VaR estimations. She calculates VaR for three hypothetical portfolios applying four different approaches for time horizons of 1-day and 10-day. For all VaR calculations with one exception Beder (1995) reports larger VaR estimates for longer time horizons.

The confidence level, on the other hand, reflects some internal measures of the financial institution. RiskMetrics uses 95% confidence level. The Basle Committee uses 99% confidence level. Jorion (2001) explains the Committee choice of 99% confidence level as a reflection of the tradeoff between the desire of regulators to ensure a safe and sound financial system and the adverse effect of capital requirements. Dowd (1998) and Jorion (2001) discuss the internal and external rationales for financial institutions to choose certain parameters (confidence level and holding periods).

2.5.5 Basle Committee Parameters

As mentioned above, Basle Committee requires financial institutions to calculate its VaR models at a 99% confidence level and 10-business-days horizon. The resulting VaR then multiplied by a safety factor, K, equal 3 to provide the minimum capital requirement for regulatory purposes. Khindanova and Rachev (2000) report that financial institutions think that 10-day time horizon is inadequate for frequently traded instruments and is restrictive for illiquid assets. Jorion (2001) suggests that the choice of the 10-trading-day period reflects the trade off between the cost of frequent monitoring and the benefits of early detection of potential problems.

According to Basle criteria, a 2 weeks loss value more than VaR would occur once every 4 years. Jorion (2001) links between the confidence level and the safety factor choice. That is with low probability of failure to occur once every 4 years, the safety factor provides near absolute insurance against bankruptcy. Additionally, Jorion (2001) explains that the safety factor accounts form model risk. Model risk includes underestimation of VaR numbers because of inaccuracy of VaR components' choice.

Stahl (1997) justifies the choice of the safety factor to be 3 based on Chebyshev's inequality. Other authors link the choice of the number 3 to the kurtosis of the normal distribution, but this sounds a little unjustifiable.

2.6 VaR Methods

Section 2.5.1 above differentiates among VaR methodologies based on the way they construct the pdf. This section deals with different VaR approaches and the recent advancements to VaR methodologies.

2.6.1 Parametric Methods

In the parametric method, trading revenues (P&L) are characterized by assumed parametric distribution, as normal distribution, Gamma distribution, Student t-distribution or any other distribution or a mixture of any set of distributions. Other parametric methods include a linearization and quadratic approximation of portfolios' value. VaR estimates derived by this method appear as a function of the parameters of the assumed distribution, and that is why it is called parametric.

Parametric methods are subject to some kind of inaccuracy. This inaccuracy arises from the fact that the distribution assumption might be incorrect. For example, if the VaR method assumes the P&L to have a multivariate normal distribution (as frequently assumed) whereas the actual data exhibit excess kurtosis, resulted VaR estimate underestimates the maximum potential losses at high confidence levels.

Under the parametric method with normality assumption, two VaR approaches are usually used: the variance covariance approach, and the Greeks approach. Both of those approaches reserve the normality assumption of risk factors underlying portfolio returns. While the variance covariance approach assumes a linear relationship between portfolio returns and the underlying risk factors, the Greeks approach deals with non-linear cases as in the case of option portfolios and fixed income portfolios.

2.6.1.1 Variance Covariance

The most popular analytical approach for deriving VaR is the variance covariance method. This approach assumes that all risk factors in the market are normally distributed and the portfolio is a linear function of those normally distributed risks. This implies that the P&L distribution for the portfolio is also normally distributed. Under such assumptions, VaR calculations become easy to handle. VaR basically is a multiple of the portfolio standard deviation, and the standard deviation is a linear function of individual volatilities and covariances of the underlying market risk factors. So, all we need to calculate VaR is the variance covariance matrix and information about the sizes of individual positions to determine the portfolio standard deviation. Then, multiply this standard deviation by a confidence level parameter and scale variable reflecting the size of the portfolio, as follows in matrix notations, Dowd (1998) and Jorion (2001):

$$VaR_P = -\alpha\sigma_P P = -\alpha[\boldsymbol{w}\Sigma\boldsymbol{w}^{\boldsymbol{T}}]^{1/2} P = [\mathbf{VaR}{*}\rho{*}\ \mathbf{VaR}^{\boldsymbol{T}}] \quad (2\text{-}3)$$

Where α is the standard normal deviate corresponding to a specific confidence level. σ_P is the standard deviation of the portfolio, P the scale variable reflecting the size of the portfolio. $\boldsymbol{w}$ represents n x 1 vector of sizes of individual positions in the portfolio. ρ is the correlation matrix of positions held in the portfolio and **VaR** represents n x 1 vector of individual positions' VaRs (undiversified VaRs), where VaR for individual positions is $-\alpha\sigma W$ and W is a scaling variable reflects the initial value of individual positions.

The normality assumption gives VaR estimate great advantages such as tractability and informativity. Informativity includes translatability across confidence levels and holing periods (section 2.3 above) and informativity about expected tail losses, Dowd (1998) and Jorion (2001).

Unfortunately, the variance covariance approach to VaR does not always fit actual financial institutions' portfolios. Portfolio returns may be nonlinear in risk factors, as in the case when the portfolio contains positions in options or fixed-income instruments. In response to non-linearity, authors use the Taylor series expansion or linear approximation for returns to these instruments, and then use the linear approximation to work out the VaR estimate. This kind of approximation includes the delta-normal method and delta – gamma approximation (Greeks). Delta-normal is the first order Taylor series expansion of the portfolio value, and delta – gamma approximation is the second order Taylor series expansion.

2.6.1.2 The Greeks Methods

The Greeks methods come basically from the sensitivity parameters of option prices to different risk factors underlying an option portfolio. Assume the value of the portfolio depends on the price of the underlying asset, and time to expiration, such that $P = P(S,t)$. Taking Taylor series expansion or alternatively applying Ito's Lemma to get the process for dynamics in the portfolio value, we get:

$$dP = \frac{\partial P}{\partial S} dS + \frac{1}{2}\frac{\partial^2 P}{\partial S^2} dS^2 + \frac{\partial P}{\partial t} dt + \frac{\partial^2 P}{\partial S \partial t} dSdt \tag{2-4}$$

Note that all higher order of change in t goes to zero, thus the last term in the right-hand-side of (2-4) goes to zero. The first part in the right hand side of (2-4) is delta of the portfolio, Δ, which defined as the change of the portfolio value with respect to the change in the price of the underlying asset. The second term is gamma of the portfolio, Γ, which is the second derivative of the value of the portfolio with respect to the underlying asset price (first derivative of delta to underlying asset price). The third term, known as theta, Θ, is the change of the value of the portfolio with respect to time to expiration.

$$dP = \Delta dS + \frac{1}{2}\Gamma dS^2 + \Theta dt \tag{2-5}$$

The simplest approximation produces the so-called delta-normal approach. Delta-normal by definition is the first order Taylor series expansion of a portfolio's value with respect to stock returns. In this case we ignore the second and the third argument in the right hand side of (2-5). If we ignore Γ and Θ then the potential loss in the value of the portfolio is then computed as $dP = \Delta dS$ which involves potential change in prices. This relationship is linear and the worst loss in P is associated with an extreme value of S. When the distribution of *dS* is normal the portfolio VaR, VaR_P, will be a product of the sensitivity parameter Δ and the VaR_S of the underlying asset as follows:

$$\text{VaR}_P = |\Delta|\ \text{VaR}_S = |\Delta| \times \alpha\sigma S_0 \tag{2-6}$$

For fixed income securities the underlying asset (risk factor) is the yield, and the first order Taylor series expansion will be a function of the yield, and the constant in this case is the modified duration, as follows:

$$dP = -DPdy \tag{2-7}$$

Where D is the modified duration and y is the yield. And the portfolio VaR in this case will be:

$$dP = -DP \times \alpha\sigma \tag{2-8}$$

Where σdy is volatility of changes in the level of yield. Of course with this procedure, we assume that changes in yield are normally distributed.

Delta-normal restores the linear normality and makes VaR estimation easy. However, such a simple approximation produces inaccurate VaR estimate. Wilson (1996) argues that delta-normal approach would produce reliable estimate for VaR for small holding periods or/and when the portfolio has few option positions (close to normality).

To get more accurate VaR for non-linear positions (options and fixed income), some literature including Wilson (1994, 1996), Pritsker (1996), Jamshidian and Zhu, (1996), and Zangari (1996 a, b) use the quadratic model or the second Taylor series expansion known as delta- gamma approaches. In terms of equation (2-5), we add the second term that includes Γ. Then the change in the portfolio value that includes positions in option would be $dP = \Delta dS + \frac{1}{2}\Gamma dS^2$. Gamma is the first derivative of delta with respect to underlying asset price. It measures the curvature of the relationship between the portfolio value and the underlying market variable. A non-zero gamma implies skewness in the distribution of P&L of the portfolio. When gamma is positive (negative), changes in the portfolio is positively (negatively) skewed.

For a portfolio with positions in fixed income securities the relationship between the value of the portfolio and the yield will be as follows:

$$dP = -DPdy + \frac{1}{2}CVdy^2 \tag{2-9}$$

Where the second-order coefficient C is the convexity parameter and it is equivalent to Γ.

The task now is to deal with dS and dS^2 for option portfolios or dy and dy^2 for fixed income portfolios. Pritsker (1996) deals with those terms by taking the variance for the both sides of the quadratic approximation (2-5). This is what is called *delta-gamma-delta* method. If dS is normally distributed then all odd moments will be zero. And with the assumption that dS and dS^2 are jointly normally distributed then dP is normally distributed and then VaR can be calculated directly.

Pritsker (1996) use another method to infer VaR estimates based on delta gamma approximation. In his method known as *delta-gamma-Monte Carlo*, Pritsker (1996) in the first place creates a random simulation of the risk factor *S*. Then he uses the Taylor approximation to create simulated movement in the option value. VaR in this case can be calculated from the empirical distribution of the portfolio value. Other modifications of delta-gamma methods include delta-gamma-minimization of Wilson (1994) and Fallon (1996).

Zangari (1996 a, b), Fallon (1996) and Pritsker (1996) improve over delta-gamma method by using Cornish-Fisher expansion which accounts for skewness. In this method, α , the standard normal deviate corresponding to a specified confidence level is replaced by $\alpha' = \alpha - \frac{1}{6}(\alpha^2 - 1)\gamma$, where γ is the skewness parameter. For more about Cornish-Fisher expansion see Hull (2002).

Zangari (1996 a, b), and Pritsker (1996) include another modifications for delta-gamma approach, the delta-gamma-Johnson. This method chooses a distribution function for *dP* and estimates its parameters to match the first four moments of the delta-gamma approximation.

Jamshidian and Zhu, (1996), Zangari (1996 a, b) and Fallon (1996) report that delta-gamma approaches improved VaR estimates tremendously compared with delta-normal. For more about detailed analysis of the developments on delta normal approach see Khindanova and Rachev (2000).

The Greeks methods still assume normality for the risk factors. Unfortunately, normality is very strong assumption for finance data, since many empirical studies in time series data, Bollerslev (1987) for example, show that the rate of return (the percentage change) of many financial variables is not normally distributed. It tends to be skewed and leptokurtic.

The normality assumption of risk factors or P&L of the portfolio would certainly affect VaR estimates since it depends crucially on the distribution of the tail. Thus, if we have thicker (thinner) tail compared with the normal distribution, then VaR estimates based on normality would be under- (over-) estimated. Zangari (1996 c) discusses VaR as a technique of risk management under departure from normality. He shows that VaR calculated under normality assumption underestimates risk since the observed distribution of many financial return series have tails that are flatter than those implied by conditional normal distribution.

With deviation from normality assumption, we still can use the parametric (analytical) approach to construct VaR. However, many researchers use non-parametric approach (the simulation approach) to account for non-normality. In the parametric approach, an alternative distribution is explicitly assumed instead of the normal distribution, and based on this assumed distribution; a formula to describe the confidence interval is analytically derived.

In the non-parametric approach, no particular distribution assumption is needed. VaR is calculated from the standard theory of order statistics, as in Kupic (1995), or from Monte Carlo simulations, where VaR is deduced from multiple runs that might be representative of the possible market price outcomes. In some cases, actual historical return distribution is used by bootstrapping rather than simulations.

2.6.1.3 Analytical VaR Approaches with Deviation from Normality

Generally, the parametric approach for VaR estimates with non-normal distribution classes aims at modifying for kurtosis since VaR estimates depend on the distribution of the left tail. A probability distribution with fat tails has greater probability mass out in the tails of the distribution, where large price movement occurs compared with the normal distribution. Accordingly, VaR estimates tend to under or over estimate risk if the normal distribution is assumed, Zangari (1996c).

Fat tails in actual finance data comes from stochastic volatility of the underlying market factor (like stock returns). The size of the "tail fatness" depends on the correlation between volatility of returns and returns themselves (Duffie and Pan 1997), Das and Sundaram (1999) and Lewis (2000). However, Bollerlev (1987) reports that actual market data are typically found to have flatter tails compared with that would come from stochastic volatility alone.

Another source of fat tails in the distribution of financial data comes from price jumps, Bates (1996), Duffie and Pan (1997), Das and Sundaram (1999), Lewis (2001) and Airoldi (2001). New evidences from Chenov, Gallant, Ghysels and Tauchen (1999), Chourdakis (2000), Lewis (2002) and Chacko and Viceira (2003) find that stochastic volatility and jumps are both significant in explaining the dynamics of stock returns for different frequencies. Chenov, Gallant, Ghysels and Tauchen (1999) find that non-affine jumps and stochastic volatility are significant in finance time series data.

To account for Kurtosis "tail fatness" in the distributions of underlying risk factors in VaR estimates, authors use different methods. Fong and Vasicek (1997) assume that the changes in the portfolio value follow gamma distribution. Additionally, they assume that the risk factors underlying the portfolio returns are jointly normally distributed. Fong and Vasicek (1997) use basically the delta- gamma approximation with allowing for skewness parameter (not kurtosis) to appear in VaR. Fong and Vasicek (1997) argue that their model accounts for thick tails in spite that the forth moment does not appear explicitly in VaR estimate. Worth to say that delta-gamma approximation that Fong and Vasicek (1997) use differs from Cornish-Fisher approximation of Zangari (1996 a, b), Fallon (1996) and Pritsker (1996).

Rachev and Mittnik (2000), Khindanova, Rachev, and Schwartz (2001), Rachev, Schwartz and Khindanova, (2002) use stable Paretian models for modeling the distribution of the P&L in the portfolio. Stable Paretian models allow for both skewness and thick tails. The above set of papers apply VaR measure derived from the stable Paretian distribution on eight international financial series including S&P 500. They conclude that stable models produce VaR estimates with higher values compared to normal distribution.

Zangari (1996 d & e),Venkataraman (1997), and Hull and White (1998a) use a mixture of two normal distributions to achieve tail thickness for their VaR estimates. Duffie and Pan (1997 and 2001) and Gibson (2001) adopts event risk (jumps) as s source of tail thickness. Duffie and Pan (1997) and El-Jahel, Perraudin, and Sellin (1999) use stochastic volatility to achieve kurtosis in the underlying risk factors for a portfolio with derivatives' positions. El-Jahel, Perraudin, and Sellin (1999) include the first four moments in their VaR estimation. Levin and Tchernitser (2001) account for tail fatness by assuming time varying volatility and Levy Processes. Li (1999), on the other hand, includes skewness and kurtosis explicitly in VaR estimates. Another wave of literature utilizes the extreme value theory (EVT) to account for tail dynamics and thickness.

2.6.2 Non-Parametric Approaches

As we mentioned earlier, a basic component on VaR estimate is the probability density function. Non-parametric approaches infer the P&L distribution directly from the standard theory of order statistics, with no need for pre-assumption about the distribution. The following two sections, examine the two non-parametric approaches and the recent advances on those approaches.

2.6.2.1 Historical Simulation

This method is the simplest method to construct VaR. It makes no pre-assumptions about the distribution of trading revenues. Instead, the distribution is constructed from the historical behavior of the portfolio P&L. The current portfolio is subjected to actual changes in the market factors during the past. The only assumption that is made in this method is that the past trends of profits and losses will continue in the future. One advantage of this method that it is free from any estimating inaccuracy, Khindanova and Rachev (2000).

However, it has a problem of the trade of between the length of the historical data used for the simulation process and the irrelevancy. It is important to use as long historical data as we can to take in to account any rare event happened in the past that lead to heavy losses. In the same time, the further we go into the past for data, the less relevant this information is to today’s market, Wiener (1997). Wiener (1997) points out to another problem of the historical simulation that is it is not applicable for technical trading strategies develop on the basis of historical data. That is because we can not use the same data used to construct VaR for the calibration of technical trading strategies.

To overcome the problem of using older data Duffie and Pan (1997), Show (1997), and Jorion (2001) suggest the bootstrapped historical simulation method to generate returns of risk factors by bootstrapping from historical observations. By this approach, volatilities and correlations are updated rather than estimated from old data.

Butler and Schachter (1996, 1998) suggest combining the historical simulation with the kernel estimation. They perform this combination in three steps: at the beginning they approximate the pdf and the cdf of portfolio returns. After that, they approximate the distribution of the order statistics corresponding to the confidence level. In the last step, they estimate VaR using the first and the second moments of the pdf for the order statistics determined by (1-c)th quantile. This modification allows to estimate the precision of VaR estimate and to construct confidence intervals around them.

Boudoukh, Richardson, and Whitelaw (1998) enter different modification to the historical simulation method. They combine the historical simulation method with exponential smoothing. Basically, this method attaches exponentially declining weights to historical portfolio returns starting from current time and going back. The future returns are obtained from past returns and sorted in increasing order (regular historical approach). After that, VaR estimate is computed from the empirical density function. This modification is know as hybrid approach, and it also ties to overcome the problem of using past information by accentuates the most recent observations. One advantage of this method is that it takes into account time varying volatility and it is suitable for fat-tailed series.

Boudoukh, Richardson, and Whitelaw (1998) perform the hybrid approach to a different financial series and compared the results with exponential smoothing and plain vanilla historical simulations. Results for 99% confidence level VaR, applied to the S&P 500, show a reduction in the absolute error estimation ranging from 30%-40% compared with the exponential smoothing method. The improvement over the historical simulation was less obvious. The absolute error reduced by 14%-28% compared with the historical simulation. Boudoukh, Richardson, and Whitelaw (1998) report that hybrid approach works better for exchange rate portfolios and heavy-tailed series.

2.6.2.2 Monte Carlo Simulation

The Monte *Carlo* method is one of the most popular methods among sophisticated users, Wiener (1997). It has a number of similarities to the historical simulation. The most significant difference is that Monte Carlo methods apply the simulation using observed changes in the market factors over the last N periods to generate N hypothetical portfolio P&L. However, this method simulates the behavior of risk factors and asset prices by generating random price paths. Monte Carlo simulations provide N possible portfolio values on a given future date (t + N). The VaR value can be determined from the distribution of simulated portfolio values.

To perform Monte Carlo approach, we first specify stochastic processes and process parameters that we think it is the best to capture risk factors dynamics. Then simulate the hypothetical price trajectories for all risk factors. Hypothetical price changes are obtained by simulations drawn from the specified distribution. At the end, we obtain asset prices at time t + N from the simulated price trajectories and compute the portfolio value and VaR. One major disadvantage of this method is that it takes long time to converge.

The modifications for the Monte Carlo method were aiming basically at improving the accuracy of VaR estimates without much additional timing cost. VaR estimates by Monte Carlo methods are subject to a tradeoff between accuracy and time of convergence. Wiener (1997) reports that to increase the precision by a factor of 10 you must perform 100 times more simulations.

Owen and Tavella (1997) use Paskov and Traub (1995) quasi-Monte Carlo idea to obtain faster valuation financial derivatives and apply it to VaR estimates. Owen and Tavella (1997) application of the quasi-Monte Carlo method noticeably improve the convergence of the simulation compared with the traditional Monte Carlo method with the same level of accuracy.

Pritsker (1996) make two improvements on Monte Carlo methods. The first is grid Monte Carlo approach, and the other one is the modified grid Monte Carlo. In the first improvement, he forms a grid of changes in the risk factors. Then he computes the portfolio values at each node of the gird. The possible realizations of the grid are obtained from random drawings from pre-decided models. Portfolio values for new draws are approximated by interpolating portfolio values at adjacent grid points. However, this method is subject to a problem Pritsker (1996) call it dimensionality problem.

To alleviate the dimensionality problem in the grid Monte Carlo approach, Pritsker (1996) tries to lower the dimension grid and combine them with a linear Taylor approximation. This modification assumes that changes in the portfolio value are caused by two types of factors, linear and non-linear. Each type of those changes subject to different estimation method. Changes in the value according to linear factors are evaluated by linear approximation. Changes in the portfolio values that are resulted from non-linear factors are estimated through grid points. Pritsker (1996) find that the modified gird Monte Carlo method entails the level of accuracy close to delta-gamma Monte-Carlo approach. However, delta-gamma Monte-Carlo requires less computational time. Gibson and Pritsker (2000) extend the modified gird method to portfolios contain positions in fixed income securities.

2.6.3 Comparison Among Different Methods

Application of different VaR methods provides different VaR estimates. The resulted estimate from different VaR methodologies sometimes not even close. Generally speaking the choice of the method depends on the composition of the portfolio. VaR methods always compared among each other regarding several considerations. The first is the ability to capture risk factors underlying portfolio returns, with the assumption that the portfolio includes positions in options and fixed income securities. The second consideration is the easiness of implementation. The third is how quick in terms of time computations can be implemented. Other considerations that usually take into account are the easiness to explain to senior management. Table 2.1 includes a summary of comparisons among different traditional methods. However, the most important comparison among different methods includes the tradeoff between the accuracy of the estimate and the cost of implementing computations in terms of time and the easiness of implementation.

The variance covariance method becomes burdensome when the number of positions in the portfolio increases, because this method requires a computation for the variance covariance matrix and the correlation matrix of all positions. In implementing the Monte Carlo simulation method, we face a tradeoff between accuracy and computation time. One advantage for the Monte Carlo simulation is that it can be performed under alternative assumptions, which is not available for other methods. However, the Monte Carlo simulation method subjects to the model risk and measurement error. Monte Carlo simulation method models risk factors according to a pre-assumed stochastic process and this pre-assumed stochastic process might be incorrect. Measurement errors involve in all VaR estimates, because the true parameters should be estimated.

Dowd (1998), Linsmeier and Pearson (1999), and Jorion (2001) provide a detailed explanation for traditional VaR approaches. Pritsker (1997) and Pearson and Simthson (2000) investigate the developments on VaR methodologies to improve the speed of computations and accuracy.

, Pearson and Simthson (2000) provide Figure 2.3 for comparison. The figure shows that the full Monte Carlo method is the most accurate and the most time consuming method. Delta normal on the other side is the least accurate and the fastest to implement. We notice also that delta-gamma Monte Carlo outperforms the modified gird Monte Carlo. In the same time, delta-gamma-delta outperforms delta-gamma-minimization. Roughly speaking, delta-gamma Monte Carlo is the best among the quadratic approximation methods in terms of balancing between accuracy and speed of computation

Table (2.1)
Comparisons of Traditional VaR

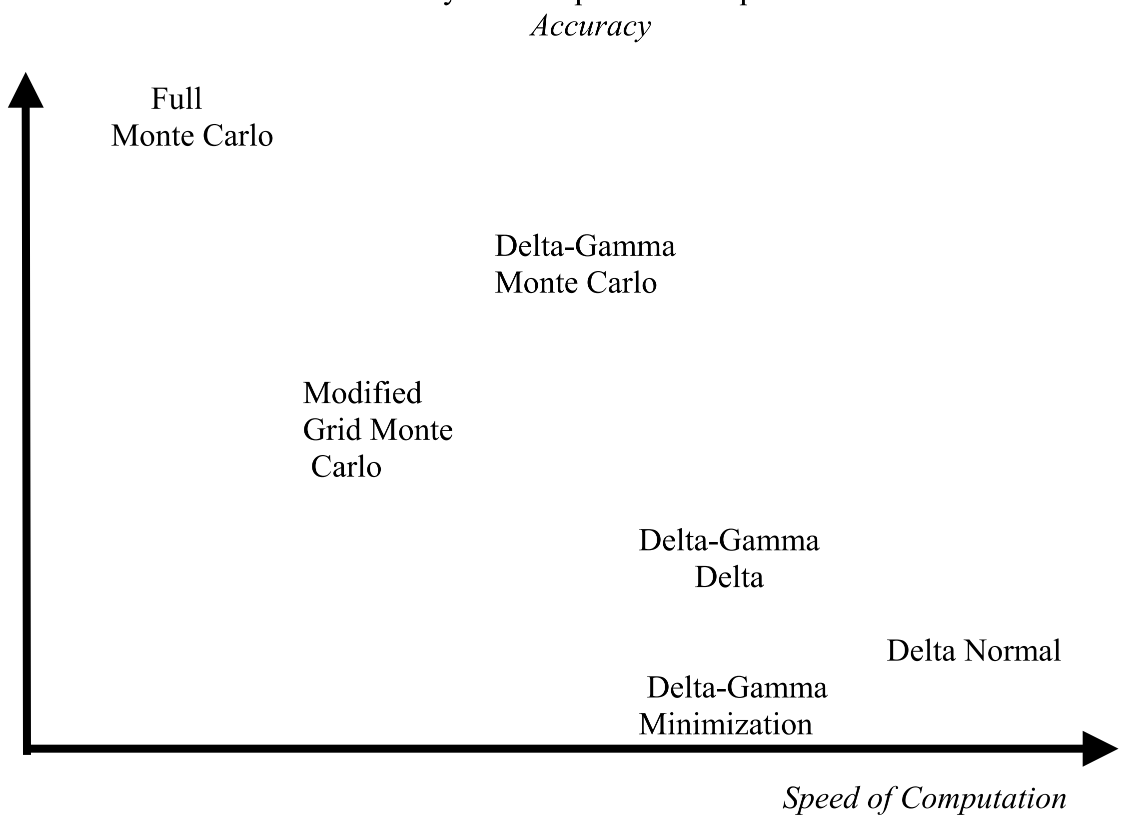
Method Factor	**Variance-Covariance**	**Quadratic Approximations**[(1)]	**Historical Simulation**	**Monte Carlo Simulation**
Ability to capture risk Factors for position with nonlinear dependence on risk factors	Ineffective at all	Can handle it, but the accuracy diminishes with their existence. VaR value understated.	Effective	Effective
Assumptions	Impose normal distribution assumption	Impose normal distribution assumption	Past trends continue in the future	Impose stochastic model for risk factors
Accounting for thick tails	No	No	Yes, if the past data imply it	Yes, if the modeled risk imply it
Accuracy	Inaccurate with thick tails data and when recent past is anomalous[(2)]	Inaccurate with thick tails data and when recent past is anomalous	Future might have extreme events, or the opposite	Misleading when the past is anomalous
Easiness of implementation	Yes, when the number of positions in the portfolio is limited, difficult with large number of positions	Yes, with the availability of data and few number of position	Yes, with the availability of data	Yes, with complex software
Quickness of computation	Relatively quick, depending on the number of positions	Relatively quick, depending on the number of positions	Relatively quick	Time consuming, tradeoff between computation time and accuracy
Easy to explain to senior management	No	No	Yes	No
Performed under different assumption	No	No	No	Yes

(1) Includes delta normal and delta-gamma
(2) Different correlations and volatility may be used to account for that

2.6.4 VaR Drawbacks

There are three major weaknesses for VaR methodologies: the implementation risk, inability to be applied during transition periods and the assumption of constant portfolio components during the time horizon (holding period). Implementation risk means that implementation of the same model by different users produces different VaR estimates. Marshall and Siegel (1997) conduct a study of implementation risk. They compare VaR results obtained by several risk management developers using JP Morgan's RiskMetrics approach. Marshal and Siegel (1997) find that different systems do not

Figure 2.3
Comparisons of VaR Methods Advancements,
Accuracy versus Speed of Computation



Source: Pearson and Simthson (2000), VaR: The State of Play.

produce the same VaR estimates for the same model and identical portfolios. They explain the varying in estimates by the sensitivity of VaR models to users' assumptions. The degree of variation in VaR numbers was associated with the portfolio composition. Nonlinear securities entail larger discrepancy in VaR results than linear securities. In order to take into account implementation risk, it is advisable to accompany VaR computations for nonlinear portfolios with sensitivity analysis to underlying assumptions.

Additionally, VaR measures generally reflect observed risks and they are not useful in transition periods characterized by structural changes, additional risks, contracted liquidity of assets, and broken correlations across assets and across markets.

Finally, VaR methodologies assume that trading positions are unchanged during time horizon. In fact, financial institutions change its portfolio composition frequently. Therefore, extrapolation of a VaR for a certain time horizon to longer time periods might be inappropriate. Duffie and Pan (1997) point out that if intra-period position size is stochastic, then VaR measures obtained under the assumption of constant position sizes, should be multiplied by a certain factor.

2.7 Back Testing

The Basle Standard requires financial institutions to perform back testing for its internal VaR models. Back testing is a posterior procedure in which the financial institution checks how often actual losses have exceeded the level predicted by VaR. A financial institution that conduct daily VaR over a 99% confidence level should not observe more than 1% cases of losses exceeds VaR. For 250 days trading period, the financial institution should notice that actual losses during that period exceeded VaR estimate only by 3 times.

Basle Standard computes the market risk capital requirement at time t by:

$$C_t = A_t \times Max\left\{\frac{1}{60}\sum_{i=1}^{60} VaR_{t-i}, VaR\right\} + SR_t \qquad (2\text{-}10)$$

Where C_t is the market risk capital requirement at time t. A_t is a multiplication factor ranging between three and four. SR_t is the capital specific risk.

The capital specific risk is part of the market risk. According to the new capital requirements market risk is classified into general market risk and specific risk. The general market risk is the risk from changes in overall market factors like equity prices, commodity prices, exchange rates and interest rates. Specific risk is the risk from changes in prices of assets for non-market reasons.

In equation (2-10), the value of A_t depends on the accuracy of the internal VaR model during the past periods, say past trading year. Basle Standard divides the number of violations (the number of times when actual daily losses exceeded VaR estimates during the last 250 days) into three zones: the green zone, the yellow zone and the red zone. The green zone applies when there are four or less violations, or violations happened in 1.6% of the time. In this case A takes a value of 3. The yellow zone applies if the number of violations is between five and nine (violations happened 2% to 3.6% of the time). In this case A takes a value between 3 and 4. The red zone applies when there are more than nine violations (more than 4 % of the time) and A takes a value of four. If A is in the red zone, the whole model should be revised.

Wiener (1997) argue that this procedure prevents banks from setting low levels of VaR, and with enough capital reserve safety, banks may only set upper bound level for

VaR rather than precise value. For more details regarding back testing and Basle Standard, see Jorion (2001).

2.8 Stress Testing (Scenario Analysis)

As we said earlier, VaR is a single, summary, statistical measure of normal market risk. According to this definition, VaR is just a normal loss that occurs according to normal market conditions. But when VaR is exceeded, the question becomes by how large this normal loss can be exceeded?

Stress testing attempts to answer this question. Stress testing is not one of VaR methods. It is rather a general method that performs a set of scenario analyses to investigate the effect of extreme market events on portfolio returns. There is no standard way to apply stress testing, and no standard set of scenarios to consider. The process depends crucially on the judgment and experience of the risk manager.

Stress testing often begins with a set of hypothetical extreme market scenarios. These scenarios might be created from stylized extreme scenarios, such as a movement of the market rates and prices by large amounts expressed in numbers of standard deviation moves. That might come from actual extreme events. For equity portfolio for example, the scenarios might be based upon the extreme movements of the U.S. equity prices that happened on October 19, 1987 when the S&P 500 moved by 22.3 standard deviations. Or for a less extreme case when the S&P 500 moved by 6.8 standard deviations on January 8, 1988.

For fixed income securities portfolio, we can use the changes in US dollar interest rates and bond prices experienced during the winter and spring of 1994. For foreign exchange rate we can use the changes in some of the European exchange rates that occurred in September 1992, or the dramatic changes in the exchange rates of several East Asian countries during the summer and fall of 1997. After developing a set of scenarios, the next step is to determine the effect on the prices of all instruments in the portfolio, and the impact on portfolio value. This requires knowledge of the covariances and correlations among different positions in the portfolio.

The are two well-known types of stress testing, the worst case scenario analysis (WCS) of Boudoukh, Richardson and Whitelaw (1997) and the factor-based interest rate scenarios of Frye (1997). Boudoukh, Richardson and Whitelaw (1997) reports that WCS result exceeds VaR estimates and WCS can be used beside VaR estimates. Frye's (1997) method is suitable to portfolios of bonds and interest rate positions. It generates the scenarios based on changes in the yield curve and factors underlying it. For details about the scenario analysis and the modifications on it see Khindanova and Rachev (2000) and Jorion (2001).

2.9 Conclusion

VaR techniques essentially model a prediction of the maximum expected loss for a given portfolio during a given time period. In VaR, losses are approximated by the left tail quantile in portfolio returns distribution. So VaR estimates basically depends on the distribution of the tail of the P&L of the portfolio. As explained in section 2.6.1.3, empirical observations exhibit fat tails and excess kurtosis for the distribution of the underlying risk factors, time varying volatility and discontinuities in the data sample path. Khindanova and Rachev (2000) conclude that neither the traditional methods of VaR nor its improvements do give satisfactory and unified estimation for VaR that captures the above properties of real financial data.

The variance covariance method and the Greeks approximations do not cope with the observed thick tail aspect with financial data. The historical method is not reliable in estimating low quantiles. Monte Carlo simulation, on the other hand, depends on the stochastic modeling (distribution assumption) of risk factors, which is usually assumed to be normally or log-normally distributed.

Berkowitz and O'Brien (2002) test the accuracy of the daily VaR models used by the largest six U.S banks in predicting actual daily losses during the period January 1998 to March 2000. They find that non of the used VaR models could predict the daily P&L in an acceptable manner. Berkowitz and O'Brien (2002) report that Banks' VaR have no significant ability in predicting Banks' losses during the periods of the Russian crises and the near collapse of the LTCM in August and September 1998. Jorion (2002), on the other hand, test the informativity of VaR disclosures in quarterly and annually reports for the largest eight U.S. Banks. He reports that published VaR estimates disclose the actual quarterly P&L for those banks. However, he makes the same point of the inability of those VaR estimates to predict losses during the third and the forth quarter of 1998.

The witness from Berkowitz and O'Brien (2002) and Jorion (2002) is as follows: Berkowitz and O'Brien (2002) report their results for the largest six U.S banks without naming the banks. But we would know from the quarterly reports what are the major banks that used VaR models during that period and what VaR methods they used. For example Bank of New York reports that they use Monte Carlo simulations. Bankers Trust use proprietary simulations, JP Morgan Chase & Co. use historical simulations, CitiCorp covariance matrix, and Bank of America describe its method as "sophisticated techniques". These methods are disclosed in the quarterly reports (Jorion 2002 report them in table 1). So, According to Berkowitz and O'Brien (2002) non of those methods could predict the Banks' losses especially in the second half of 1998.

Jorion (2002) is more optimistic compared with Berkowitz and O'Brien (2002). Jorion (2002) deduced his results based on a regression analysis between the unpredictable P&L and daily volatility. For me, Jorion's results are questionable since he takes quarterly VaR over a 99% confidence level. According to this VaR specification, we expect to see that a three months loss exceeding VaR occurs once every 25 years, and any violations should be tested along 25 years or 100 quarters period. However, no data available with that long.

The conclusion from those works is that non of the VaR methods could predict the actual banks' losses even with inclusion of time varying volatility. Berkowitz and O'Brien (2002) find that VaR estimates that GARCH models for volatility bound actual daily losses in banks' portfolios closer than internal VaR models. However, even those models could not describe actual portfolio dynamics during the periods of the Russian crises and the near collapse of the LTCM in August and September 1998.

Generally speaking, accounting for thick tails may improve the performance of VaR estimates. However, VaR estimates that deviate from normality assumption in an attempt to account for thick tails (section 2.6.1.3) do not consider for the following aspects:

1. The full kurtosis cannot be captured totally by time varying volatility alone or jumps (event risk) alone. In Gibson (2001 p.2) language "what is needed is a model that can combine time varying volatility with event risk" and Gibson (2001) considers this task is challenging. Similarly, Lewis (2002) reports that

mixing the stochastic volatility with jumps gives much better fit to both actual stock price distribution with their wide tails and smile patterns. However, he considers this as a tough task but it deserves trying for the benefit of getting more accurate modeling of financial data. Additionally, Chenov, Gallant, Ghysels and Tauchen (1999), Chourdakis (2000), and Chacko and Viceira (2003) find that both stochastic volatility and jumps are significant in their estimations for mixed stock returns processes. Those studies even employ a non-affine stochastic process and jump-diffusion process.

2. As Airoldi (2001) suggests, with fat tails, volatility grows and updates faster. The thicker the tail, the faster the dynamics of volatility. Thus jumps and extreme events cause hyper growth in volatility. Chenov, Gallant, Ghysels and Tauchen (1999) report similar evidences in their non affine estimations. Accordingly, the assumed linear relation between volatility dynamics (updating in volatility) and volatility levels would no longer holds during jumps and extreme events (fat tails) hence a non-linear representation of volatility dynamics becomes necessary.
3. When non-linear dynamics involve (even in the case of linear dynamics), point 2 above, conditional densities provide better description of short term asset price movements compared with the unconditional densities, which has very important implication in risk management, (Hsieh 1993). Hence, the dependence of VaR models on the unconditional densities would not give the best estimate for VaR when we assume nonlinear dynamics (or even linear). And we know that there are strong evidences of nonlinear dynamics in short

term movements of asset returns structure, see Hsieh (1989 and 1991), LeBaron (1988), and Scheinkman and LeBaron (1989).

4. Kurtosis has non-flat term structure that changes over time and this deepens the dependence of VaR estimates on the time horizon. As a matter of fact if we assume that volatility is time varying this means that the fatness of tail would be time varying, Duffie and Pan (1997), and Das and Sundaram (1999). And this may create some covariance between the confidence level and the VaR horizon.

Any efficient VaR estimates should take the above four considerations into account, and this is the task of the following chapters of this dissertation.

CHAPTER THREE: DATA ANALYSIS

The purpose of this part is to analyze the data set used for back testing of the proposed VaR models under thick tails developed in this study. In addition to describing the data set and data sources and the data manipulation, we performed some back testing for the internal VaR models used by the banks compared with the actual trading revenues using Berkowitz and O'Brien (2002) and Jorion (2002) methodologies. The analysis aims at shedding more light on the need for different kind of VaR models that accounts for fat tails, stochastic volatility and jumps in underlying risk factors. Here, we are deducting those features through focusing on VaR violations by actual losses incurred in the trading portfolios of a sample of six U.S banks.

3.1 Selecting the Sample

Because of its recent advent, VaR data available publicly are very limited. Before 1995, only few banks disclosed VaR in its quarterly and annually publications. After June 1998[5], the number of financial institutions disclosing VaR numbers in its public reports increased tremendously. The Basle Committee on Banking Supervision Accord 1999

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[5] In its market disclosure rule FRR No. 48, effective, June 1998, the U.S. Securities and Exchange Commission (SEC) require all large U.S. publicly traded corporations to report quantitative information about their market risk in financial reports filed with the SEC. The SEC requires the corporation to choose among three methods for quantitative market risk disclosure, tabular presentation, sensitivity analysis and VaR. For more details, see Linsmeier and Pearson (1997).

(BCBS 1999) reports that 86 percent of financial institutions that Basle Committee surveyed disclose VaR in their public financial reports[6]. Work that uses actual trading revenues (P&L) of financial institutions portfolios and the associated internal VaR estimates are really limited. Berkowitz and O'Brien (2002) and Jorion (2002) are pioneers in this area. Berkowitz and O'Brien (2002) check the accuracy of VaR model used by banks using a sample of six largest U.S. dealer banks with daily trading revenues and daily VaR estimates over a two years period spanning from January 1998 to March 2000. Berkowitz and O'Brien (2002) use private internal data that is not available at all for anyone else. They even do not include the names of the six banks that included in the sample.

Jorion (2002) on the other hand, try to check how informative value at risk numbers disclosed in the quarterly and annually (10-Q and 10-K) reports. He uses a sample contains the largest eight U.S. banks. Jorion uses quarterly trading revenues and daily VaR estimates published on a quarterly basis over almost 6 years period, spanning from 1995: Q1-2000: Q3. Jorion's sample includes Bank of America, Bank of New York, Bankers Trust, Chase Manhattan, CitiCorp, First Chicago, JP Morgan and Nations Bank. Except for Nations Bank that includes only 14 observations, Jorion's (2002) sample includes 23 observations for each bank.

The criterion for inclusion a bank in the analysis is the availability of VaR estimates, assuming that trading revenues are available too. All banks that Jorion (2002) include in the sample started to disclose VaR since December 1994. However, Jorion

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[6] Most of small and regional banks are exposed to the interest rate risk, thus they report sensitivity analysis to shocks in interest rate rather than reporting VaR.

(2002) derives his data set from the SEC website (the only public available source), where the quarterly and annually reports are available beginning of December 1993.

For the purpose of testing VaR models under thick tails developed in this dissertation, this study uses the same bank sample that Jorion (2002) uses but for a time period extended from 1995:Q1–2002: Q3. In terms of banks number, the sample includes six banks rather than eight banks. The reason is that some banks witnessed merging activities during the sample period. During the sample period, Nations Bank merged into Bank of America and Chase Manhattan and JP Morgan Merged into JP Morgan Chase. Accordingly, banks used in this dissertation for back testing are Bank of America, Bank of New York, Bankers Trust, Bank One/First Chicago, CitiCorp, and JP Morgan Chase. The data collecting from the quarterly (10Q) and annually (10K) reports available in the companies filling in the SEC website.

Figure 3-1 below, shows example of Bank of New York VaR disclosure. Bank of New York illustrates that they use VaR as one of three measures to manage trading risk. The bank measures VaR at the 99 percent confidence level over one day. Bank of New York also tells that he uses Monte Carlo Simulation approach to calculate VaR. The breakdown by type of risk indicates that this is a diversified portfolio. Bank of New York reports the average, high, low of the quarter, the average, high, low of the report period, and end-of-period VaR across categories of market risk and for the portfolio as a whole. By September 2002, all banks included in the sample (except Bank of America) disclose VaR in this manner. Other banks' disclosures include some back testing through a histogram of P&L compared with VaR of a graph. Some banks (JP Morgan Chase) report their stress testing results.

Table 3-1 summarizes the characteristic of VaR estimates disclosed in companies filling in the SEC. Table 3-1 shows that all banks in the sample started to announce VaR numbers at December 1994. VaR estimate is available until September 2002 (the most recent Q10) except for Bankers trust, which is available only for December 2001. Table 3-1 shows also that banks in the sample are diversified in using VaR method.

Figure (3-1)
Excerpts from the Quarterly Report of Bank of New York, September 2002
Dealing with Value at Risk

The Company manages trading risk through a system of position limits, a value at risk (VAR) methodology, based on a Monte Carlo simulation, stop loss advisory triggers, and other market sensitivity measures. Risk is monitored and reported to senior management by an independent unit on a daily basis. The VAR methodology captures, based on certain assumptions, the potential overnight pre-tax dollar loss from adverse changes in fair values of all trading positions. The calculation assumes a one day holding period for most instruments, utilizes a 99% confidence level, and incorporates the non-linear characteristics of options. As the VAR methodology does not evaluate risk attributable to extraordinary financial, economic or other occurrences, the risk assessment process includes a number of stress scenarios based upon the risk factors in the portfolio and management's assessment of market conditions. Additional stress scenarios based upon historic market events are also tested.

The following table indicates the calculated VAR amounts for the trading portfolio for the periods indicated. During these periods, the daily trading loss did not exceed the calculated VAR amounts on any given day.

(In millions)	3rd Quarter 2002			Year-to-Date 2002			
	Average	Minimum	Maximum	Average	Minimum	Maximum	9/30/02
Interest Rate	$4.1	$2.6	$5.8	$4.7	$2.6	$9.2	$4.1
Foreign Exchange	1.1	0.6	2.5	1.2	0.6	3.8	0.9
Equity	0.1	-	0.9	-	-	1.1	-
Diversification	(1.4)	NM	NM	(1.7)	NM	NM	(1.2)
Overall Portfolio	3.9	2.5	5.7	4.2	2.5	8.3	3.8

(In millions)	3rd Quarter 2001			Year-to-Date 2001			
	Average	Minimum	Maximum	Average	Minimum	Maximum	9/30/01
Interest Rate	$5.6	$3.8	$7.7	$4.9	$2.6	$7.7	$5.4
Foreign Exchange	1.6	0.7	3.1	1.3	0.6	3.1	2.3
Equity	-	-	-	-	-	0.3	-
Diversification	(2.1)	NM	NM	(2.0)	NM	NM	(3.1)
Overall Portfolio	5.1	3.4	7.1	4.2	2.3	7.1	4.6

NM - Because the minimum and maximum may occur on different days for different risk components, it is not meaningful to compute a portfolio diversification effect.

Table (3-1)
Description of VaR Disclosure of 6 U. S. Commercial Banks

Bank	**VaR Method Used**	**Reporting Details as of Sep. 2002**	**First VaR Disclosure**	**Reported Average Since**	**Reported End-Quarter**	**VaR Confidence Level**
Bank of America	Historical Simulation	Average, Min, Max, End	Dec. 1994	1998:3	1995:4-1998:3	95%, 97.5% and 99% since 98:3
Bank of New York	Monte Carlo Simulation	Average, Min, Max and End	Dec.1994	1996:4	1997:4	95% until 1997:4 99% since 1998:1
Bankers Trust	Proprietary * Simulation	Average, Min, Max and End	Dec.1994	1998:3	1998:3	99% **
Bank One / First Chicago	Statistical Model ***	Average, Min, Max and End	Dec.1994	1994:1	1997:4	%99.87 until 98:2, 99% 97:4 and after 98:2
CitiCorp	Covariance Matrix	Average, Min, Max and End	Dec.1994	1997:4	1997:4	97.7% until 97:3 and 99% since 97:4
JP Morgan Chase & Co.	Historical Simulation	Average, Min, Max and End	Dec.1994	1994:4	1997:4	97% until 95:3, 95% until 97:3, 99% after 97:4

* Bankers Trust stopped publishing VaR disclosure in Dec. 2001. The VaR method as Reported in 10K 2001.
** Bankers Trust Report VaR at 10 days horizon until 1999:Q3, after that VaR reported at 1 day 99%.
***Bank One reports that VaR is calculated based on a statistical model applicable to cash and derivative positions, including options. However, they mentioned that VaR calculated at 2.33 standard divination from the mean.

3.2 Trading Revenues

Quarterly trading revenues are collected from banks' quarterly reports (10-Q) and annual reports (10-K), along with the banks' VaR disclosures. Quarterly trading revenues are available in the SEC fillings beginning of March 1993. However, because banks begin disclosing VaR data in 1994, trading revenue data are collected from 1994 to the third quarter of 2002.

Most banks report "total trading-related revenues," which includes (1) direct trading revenues, (2) fees, and (3) interest revenues on trading assets net of the costs to fund trading positions. Fees and net interest earned on assets and liabilities are more stable over time than are trading revenues. When banks construct VaR they exclude those stable items from trading revenues and they calculate VaR for trading revenues abstracted from fees and interest revenues, see Jorion (2002) and Berkowitz and O'Brien (2002). To abstract the expected component from of trading revenues -to construct a measure of unexpected trading revenues- I follow Jorion (2002) methodology. Jorion (2002) correct for the expected component of the trading revenue by measuring the unexpected component as the difference between the quarterly trading revenue and its moving average over the previous four quarter. He reasons the use of the previous four quarters as a benchmark due to the fact that trading revenues are not seasonal. Accordingly, the unexpected trading revenues are defined as

$$R_{t+1} - E[R_{t+1}] = R_{t+1} - \frac{1}{4}\sum_{i=1}^{4} R_{t+1-i} \qquad (3\text{-}1)$$

Jorion (2002) concludes that this transformation should produce an expected value of unexpected trading revenues indistinguishable from zero. Additionally this transformation imposes a non-autocorrelation among trading revenues. However, with this transformation, we expect to see some kind of autocorrelation. A quarter with large negative trading revenues will pull down the estimate of expected trading revenues $E[R_{t+1}]$ for the next three quarters which will result in positive values for unexpected trading revenues for the next three quarters, Jorion (2002).

Empirically, as Table 3-2 and Figure 3-2 show that real data proves the improvement of unexpected trading revenues over the reported trading revenues. The new aspects of unexpected trading revenues (zero expected value and zero autocorrelation) are necessary for a certain types of back testing of VaR models, like the one Jorion (2002) uses.

Table 3-2 reports the average and standard deviation of quarterly trading related revenues in millions of dollars. It shows in the same time some statistics on the measure of unexpected component of trading revenues. The average trading revenues differs widely across banks, ranging from almost 40 millions for Bank One/First Chicago to 746 millions for JP Morgan Chase. The standard deviation is ranging widely among the banks in the sample. It is ranging from 24 million for Bank of New York to 784 million for JP Morgan Chase.

Table 3-2 show that the mean of unexpected trading revenues is far less than the mean of trading revenues. The average unexpected trading revenues ranging from –9 millions for Bankers Trust to 69 millions for CitiCorp. The null hypothesis that the mean of unexpected trading revenues is indistinguishable from zero could not be rejected for four banks and could not be accepted for two banks (Bank of New York and CitiCorp).

Table (3-2)
Descriptive Statistics of the Trading Revenues and the Unexpected Trading Revenues in the Banks Sample

Bank	**Trading Revenues**				**Unexpected Trading Revenues**			
	Average* **	**Std. Deviation**	**ρ(1)**(1)	**Jarque-Bera**(2)	**Average**	**Std. Deviation**	**ρ(1)**(1)	**Jarque-Bera**(2)
Bank of America	227.90	249.42	0.37**	3.61	11.98	220.39	0.011	8.25**
Bank of New York	44.97	24.29	0.85***	1.60	4.79**	12.18	0.293*	0.62
Bankers Trust	87.3	196.97	0.18	8.56**	-9.02	196.47	-0.05	16.82***
Bank One / First Chicago	39.55	33.34	0.04	8.33**	4.51	34.53	-0.02	1.19
CitiCorp	619.94	262.65	0.86***	2.62	69.44***	138.67	-0.081	0.29
JP Morgan Chase & Co.	764.03	784.22	0.46***	35.48***	10.12	700.48	0.162	150.69***

*** Significant at 1%.

** Significant at 5%.

* Significant at 1%

(1) First order autocorrelation.

(2) Test for normality, H0: the null hypothesis is the trading revenues are normally distributed. H1: the alternative hypothesis, the distribution does not follow normality.

This is different from the mean of trading revenues that is statistically different from zero at high significance level for all banks.

Table 3-2 shows also, that unexpected trading revenues satisfy the condition of zero autocorrelation. The first order autocorrelation, $\rho(1)$, for unexpected trading revenues is indistinguishable from zero for five banks, with weak degree of significancy (10%) for Bank of New York. The case is not the same for the related-trading revenues, where the first order and other higher orders autocorrelation are significant for most of the banks in the sample.

From Table 3-2, we can see that the transformation used to deduct the unexpected trading revenue almost keeps the same distribution assumptions of trading revenues. Jarque-Bera test for normality indicates that we can not reject the normality assumption for three banks. This analysis is almost the same for both the related-trading revenues and unexpected trading revenues.

Figure 3-2 shows that both unexpected trading revenues and the related trading revenues took the same path over time. This reflects the fact that all dynamics in related - trading revenues comes basically from the dynamics in unexpected trading revenues. Which approximately means that related-trading revenue is just a scalar transform of the unexpected component.

Figure 3-2 deducts also a very volatile upward trend in the banks' trading-related revenues and unexpected trading revenues. Volatility was very high, with some banks' big losses during the third quarter of 1998. The period of turmoil in financial markets, beginning with the Russian default and the near collapse of Long-Term Capital

Figure (3-2)
Unexpected and Reported Trading Revenues for the Banks' Sample

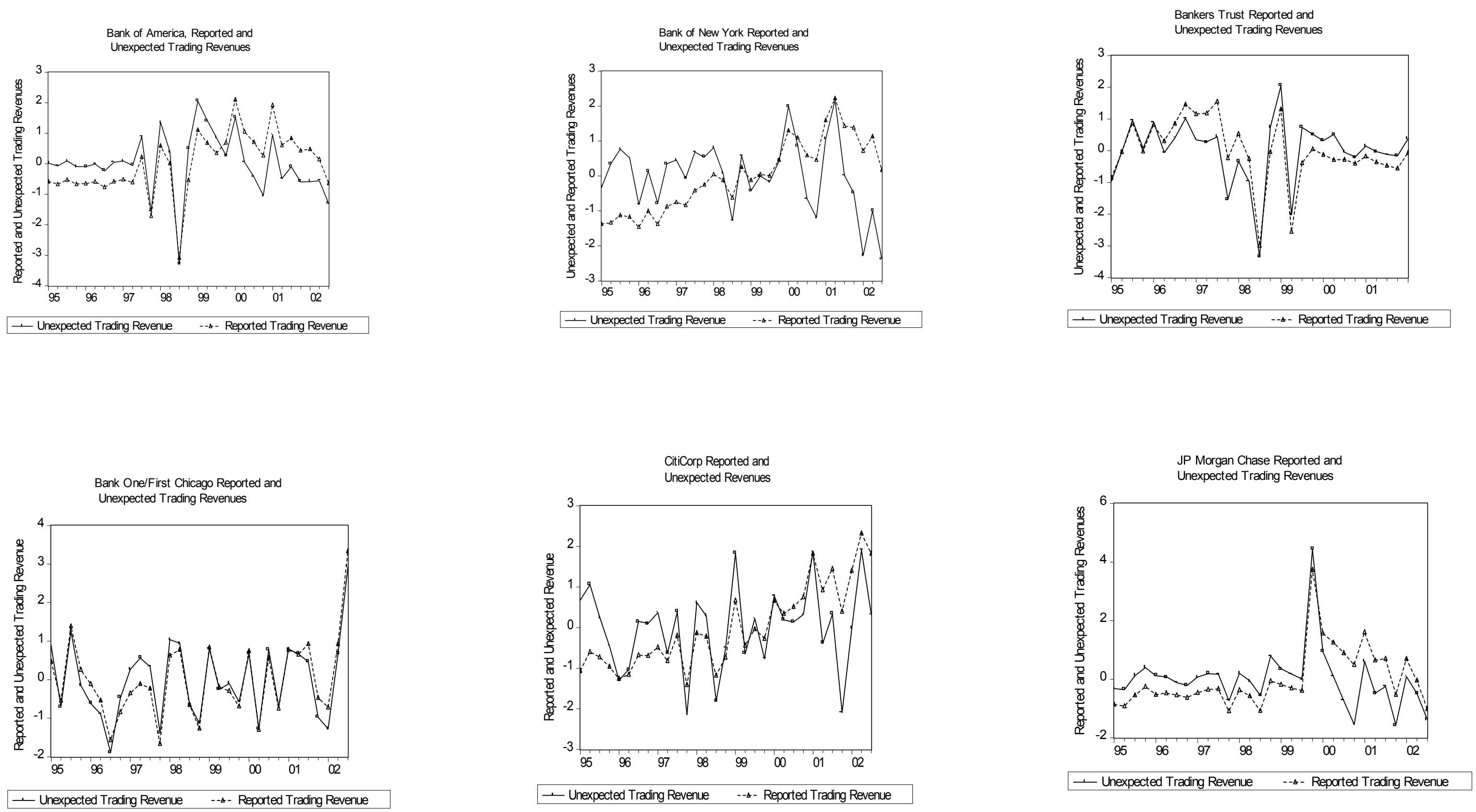
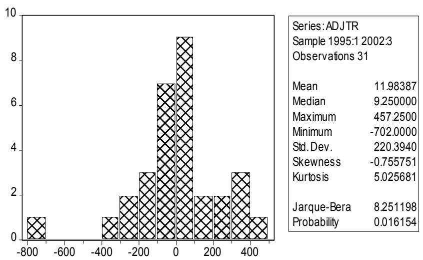
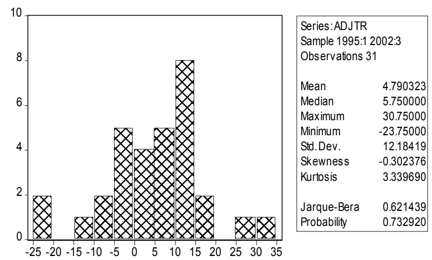


Figure (3-3)
The Distributions of Unexpected Trading Revenues (P&L) For Banks in the Sample during the Period 1995Q1-2002Q3[(1)]

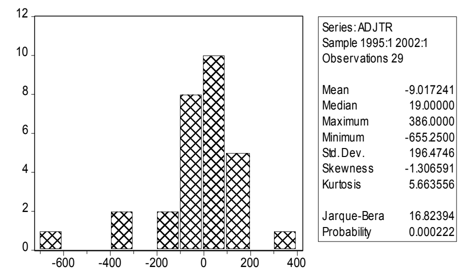
Bank of America



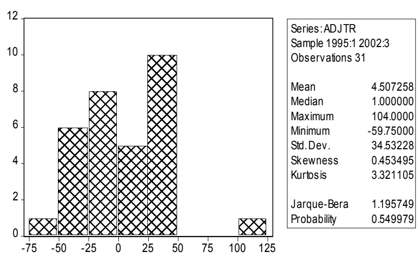
Bank of New York



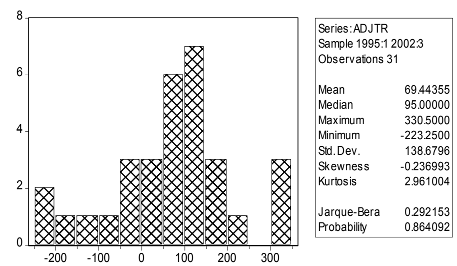
Bankers Trust



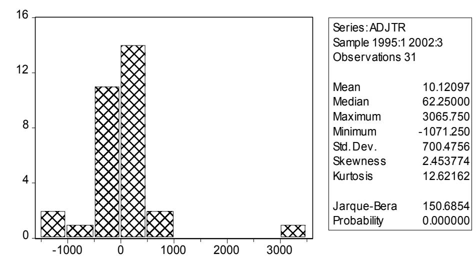
Bank One/First Chicago



CitiCorp



JP Morgan Chase



(1) Bankers Trust Sample Ended 2002Q1

Management (LTCM). Financial markets were severely disrupted when it appeared that LTCM might collapse, due to the huge size of LTCM's positions.

Figure 3-3 on the other hand, shows the detailed statistical characteristics of unexpected trading revenues. Figure 3-3 represents histograms of unexpected trading revenues for all banks in the sample with additional statistical characteristics like skewness and kurtosis. Consistent with financial data series, unexpected trading revenues for banks in the sample are negatively skewed and five banks exhibit kurtosis exceeds 3. However, negative skewness is not significant in many cases. Actually test for normality (Jarque-Bera) rejects normality assumption only in three cases.

3.3 Value at Risk

A normal practice, as reported in Figure 3-1, financial institutions measure VaR over a short period horizon, one day to two trading weeks, assuming the current positions are fixed over that horizon. However, we know that financial institutions change positions actively during the trading horizon. Additionally, the actual losses may be less than VaR estimate if management takes corrective actions when losses start to appear. With long holding periods, like quarterly horizons, the assumption of fixed positions in the portfolio during the holding period becomes more problematic.

From the definition of VaR, the maximum loss that will be exceeded with a small percentage over the next N days, back testing implies, Ideally, matching VaR_t (VaR estimate at time t) with the TR_{t+N} (subsequent trading revenue at time t+N). With N = three months, we have to match the VaR estimate at the end of quarter t with the t+1 quarter trading revenues.

As Table 3-1 shows, banks began disclosing annual VaR numbers in 10-K forms in 1994. However, because the SEC's market disclosure rule FFR-48 was effective on January 1, 1998, some banks start to disclose VaR estimates in 1998, see Table 3-1.

VaR data for trading activities were collected from the 10-K and 10-Q reports. As mentioned earlier, the standard is to use the end of the quarter VaR to match it with the subsequent quarter trading revenues. As Table 3-1 indicates, most of the banks started to publish end of quarter VaR in the 1997 10-K. Bank of America published the end of the quarter VaR during the period March 1995 to March 1998, and then they started to publish the average only. However, we can deduce the end of quarter VaR for recent quarters from daily VaR graphs. Bankers Trust also started to publish end of quarter VaR since March 1998.

Because of those differences, the following decision rule is used in collecting VaR estimates:

1) If quarterly VaR data are available
    a) End-of-quarter VaR data for the previous quarter when available.
    b) Average VaR for the previous quarter.
2) If quarterly data are not available, I resort to annual data.
    a) VaR reported at the end of the previous year.
    b) Average VaR for the previous year.

This approach selects the VaR number that is the closest to the beginning of the subsequent quarter over which trading revenue is measured.

Additionally, banks report VaR on daily basis in their reports while publish trading revenues on quarterly basis. More over, in previous periods, some banks used to disclose VaR at confidence level different than 99% (see Table 3-1). Accordingly, we need to convert the daily VaR at different confidence levels to quarterly VaR at 99% confidence level. For that, we used the conversion rules described in section 3 of chapter two.

3.4 The Accuracy of Banks' Disclosed VaR

The aim of this section is to test for the ability of VaR models developed by banks to predict future losses. It is very important to evaluate banks VaR models to know the need for developing more accurate VaR models accounting more for financial data aspects. As shown in chapter 2, if we define the quarterly P&L distribution by R_t , so that at the end of each quarter t the bank forecasts the next quarter P&L (R_{t+1}) using VaR technique. The forecast is the amount VaR_t that is calculated at the end of t quarter such that $\Pr[R_{t+1} < \mathrm{VaR}_t] = 1\text{-}c$ over the next quarter. Here c = 99%, so the model predicts a lower bound on losses not to be exceeded with 99 percent confidence. Using the definition of unexpected trading revenues from equation (3-1)

$$\Pr[R_{t+1} - \mathrm{E}(R_{t+1}) < \mathrm{VaR}_t] = 1\text{-}c \qquad (3\text{-}2)$$

Where $\mathrm{E}(R_{t+1})$ as defined in equation (3-1). Here we use VaR_t instead of VaR_{t+1} just to match the notation with our data set. To investigate whether VaR accurately predict future losses we use the usual back testing method or forecast evaluation as in Berkowitz and O'Brien (2002) where the number of violations or realizations beyond VaR are counted. Or we can use VaR based volatility regressions used by Jorion (2002).

3.4.1 Forecast Evaluation

This approach basically performed by comparing the targeted violation rate 1-c with the observed violations. As we know a 99% confidence level quarterly VaR predicts to observe one quarter loss exceeding VaR once in 25 years (100 quarters) trading period. However, we can not apply this principle easily, since we have only 31 quarter of data. Accordingly, we expect to see this violation only 0.31 quarters during the sample period. The 0.31 trading quarters are equivalent to 20 trading days, assuming the quarter contains 63 trading days. As long as we do not have the daily P&L we can not use this daily definition. Thus the best would be to analyze the number of violations during the sample periods. Figure 3-3 that represents the histograms for unexpected trading revenues can help in that to see the number of quarters that VaR has been exceeded by. Or the number of quarter can be transformed into percentage to see the percentage of violations if it exceeds 1% of the sample.

The analysis is clearer in Figure 3-4, which shows unexpected trading revenues bounded by VaR from below and absolute VaR from above. Figure 3-4 roughly shows that there are 4 banks that had violations more than once during the sample period. Violations mean times where actual losses exceed VaR. Figure 3-4 shows that VaR could bind actual trading revenues totally for two cases, Bank of New York and Bank One/First Chicago. For the other 4 banks, VaR could not bind actual losses regardless of actual profits. Figure 3-4 shows that violations are not limited to the third quarter of 1998, the period that witnessed the Russian crises and the near collapse of the LTCM. Actually, 3 banks out of those 4 banks witness violations after the third quarter of year 2000 in addition to the period of the Asian crisis.

Table 3-3 gives more details of those violations. The first column shows the 99th percentile losses. The 99th percentile loss represents 2.326 standard deviations below the mean assuming normal distribution. For four banks in the sample, the average VaR lies outside the 99^{th} percentile loss. The numbers in column 3 are the banks' mean VaR but standarized with unexpected trading revenues' standard deviation. A bank with a standarized average VaR less in absolute value than 2.326 means that the mean of VaR lies outside the 99^{th} percentile and we expect to see violation in this bank. Again column 3 emphasizes The conclusion that 4 banks have mean VaR lies outside the 99^{th} percentile and 2 banks their average VaR lies within the 99^{th} percentile. Accordingly we expect to observe violations in 4 banks.

Column 4 reports the number of violations (Losses went beyond VaR) in each bank of the four banks that have violations. The number of violations is relatively big. If we compare it with the 1% percent target, we conclude that VaR estimates were very conservatives. VaR is exceeded with more than 6 % in two banks and with more 20% in the other two banks.

The last four columns in Table 3-3 show the mean size of violations and the maximum violation for each bank. The violation size represents the difference between the actual loss and VaR estimate. The largest violation in terms of mean happened in Bank of America. The mean violation for Bank of America is 1.47 standard deviation. The violations in columns 6 and 8 are bigger than expected value conditional on exceeding the quantile for the normal distribution. The expected value conditional on exceeding the quantile (known also as expected shortfall, tail conditional expectation, conditional loss or tail loss) for normal distribution is 2.667. For normal distribution, the

Figure (3-4)
Internal VaR Estimates and Unexpected Trading Revenues P&L For Banks in the Sample during the Period 1995Q1-2002Q3

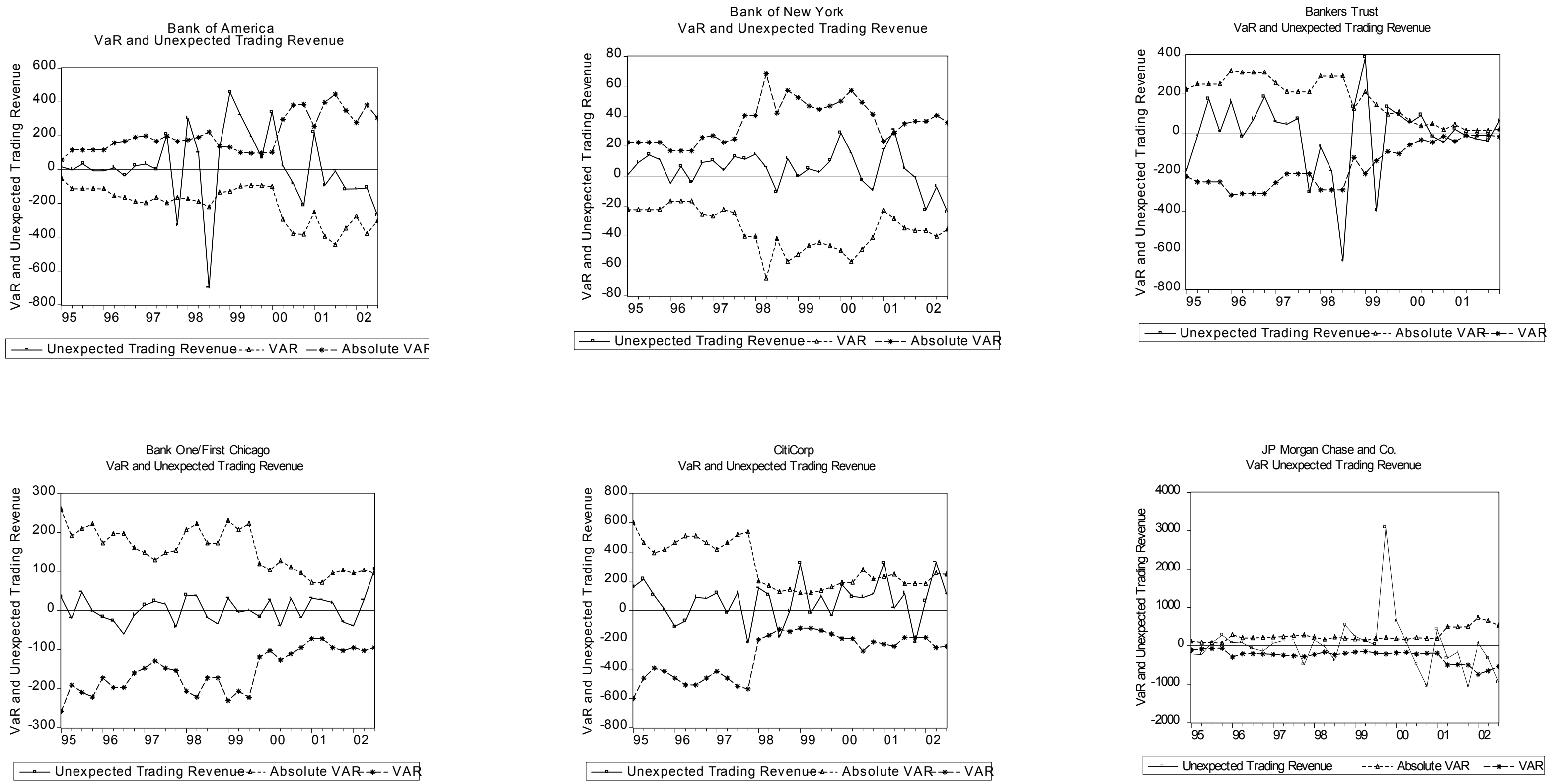


Table (3-3)
VaR Violation Statistics

Bank	99th Percentile(1)	Mean VaR $Millions	Mean VaR(3) In Stds	No. of Violations	Mean(2) Size of Violations $ Millions	Mean(3) Size of Violations in Stds	Max Violation $Millions	Max Violation Std.
Bank of America	-512.63	-213.48	-0. 948	2	-320.9	-1.47	-479.76	-3.19
Bank of New York	-28.33	-35.70	-2.93	0	NA(4)	NA	NA	NA
Bankers Trust	-456.99	-169.24	-0.86	7	-113.61	-0.58	-364.75	-1.86
Bank One / First Chicago	-80.32	-155.1	-4.49	0	NA	NA	NA	NA
CitiCorp	-322.55	-297.85	-2.15	2	-42.35	-0.31	-51.00	-0.37
JP Morgan Chase & Co.	-1629.32	-274.38	-0.39	8	-342.06	-0.49	-864.83	-1.23

(1) Calculated as the multiplication of the standard deviation of the unexpected trading revenues and the 99th percentile of normal deviate. This is by definition -2.236 standard deviation below the mean.
(2) The amount of loss exceeding VaR, which equal to average loss – VaR.
(3) Normalized in standard deviation.
(4) NA: Not Applicable

tail conditional distribution exceeds the normal deviate of 99% confidence level by 0.341 standard deviations which is close to violations in 3 banks but it is less than the maximum violations in the four banks that have such violations. Berkowitz and O'Brien (2002) report relatively higher violations in their daily samples and VaR estimates.

Generally, based on quarterly data and disclosed VaR, and using the expected forecast method, we report the following conclusion. Disclosed banks' VaR data do not accurately measure the maximum expected loss for the banks. So there is a need to find more accurate measure. As Berkowitz and O'Brien (2002) conclude, there is a need for VaR estimates that takes into account tail fatness, time varying volatility, and high correlation among positions especially during the times of collapse and crisis. As reported by Airoldi (2002) volatility updates faster and correlation increase among asset returns during periods of collapses.

3.4.2 VaR Based Volatility Method

Jorion (2002) uses different method to test for the informativeness of VaR disclosure in the banks' quarterly and annual reports. Jorion's (2002) method retrieves volatility of the trading revenues from VaR estimate and then looking for a certain distributional relationship between volatility and the expectations of trading revenues.

The first step in this method is to retrieve trading revenue's volatility from VaR estimates. As we know, for normally distributed risk factors, VaR measure is the forecasted volatility s_t multiplied by the standard normal deviate α for selected confidence level. $\alpha = 2.326$ for one tailed confidence level of 99%. Then

$$\mathrm{VaR} = \alpha\, s_t \qquad (3\text{-}2)$$

With normally distributed risk factors, the expected absolute value $|R_{t+1}|$ is

$$E(|R_{t+1}|) = \sqrt{\frac{2}{\pi}} \times s_t = 0.8 \times s_t \qquad (3\text{-}3)$$

Equation 3-3 shows that the expected absolute value of trading revenue is linearly increasing with the forecasted volatility of trading revenues that are proportional to VaR. Jorion (2002) claims that this setup is valid for a wide class of trading revenues provided that the conditional distribution of trading revenues is fixed and symmetric. However, the coefficient of this relationship depends on the type of the distribution. For example, with the six degrees of freedom Student-t distribution (typical distribution for daily financial time series) this coefficient is 0.74.

Jorion (2002) tests the predictive power of banks quarterly VaR disclosure by estimating an equation of the form:

$$|R_{t+1}| = a + b\sigma_t + \varepsilon_{t+1} \qquad (3\text{-}4)$$

Where R_{t+1} is the trading revenues for quarter t+1, and σ_t is the forecasted volatility of quarterly trading revenue inferred from VaR at quarter t. As we know, trading revenues are published quarterly as a summation of daily trading revenues. Whereas VaR is disclosed quarterly based on one-day horizon. Jorion (2002) uses the conversion method explained in chapter 2 section 3, assuming that the quarter includes 63 trading days, as the following:

$$\sigma_t = s_t \sqrt{63} \qquad (3\text{-}5)$$

According to this analysis, VaR is informative about future trading revenues risk if b in equation 3-4 is significantly positive. However, the accuracy of this estimate and the closeness of b estimation to its theoretical value of 0.8 depend on several assumptions. The first factor is the error in measuring σ_t. This approximation assumes that trading revenues daily volatility is almost constant and does not move quickly across trading days in the quarter. The other source of error in σ_t estimation is that VaR models assume that composition of the trading portfolio is constant over the trading horizon that VaR calculated for. This also might create error in trading revenues themselves.

An important assumption to apply equation 3-3 is that the R_{t+1} has a mean zero and is normally distributed. Additional important assumption is that trading revenues are not autocorrelated. Then R_{t+1} should be the unexpected component of trading revenues. As mentioned earlier, the reported trading revenues contain expected components like fees and interest rate. Even banks do not take this expected component into account when calculate VaR.

As showed above in equation 3-1, this expected component is factored out by taking the unexpected trading revenue as a difference between the latest trading revenue and the average over the previous four quarters. Unlike reported trading revenues, unexpected trading revenues are closer to have zero expected mean and zero autocorrelation as in table 3-2 and figure 3-2. This transformation could overcome the last problem. By replacing trading revenues with unexpected trading revenues in equation 3-1, equation 3-4 can be rewritten for the purpose of estimation in the following fashion:

$$\left|R_{t+1} - E\left[R_{t+1}\right]\right| = a + b\sigma_t + \varepsilon_{t+1} \qquad (3 - 6)$$

The same criterion still applies. If banks' VaR is informative, b should be positively significant from zero and the regression R^2 should be high implying that σ_t captures much of the variations in unexpected trading revenues. As we know, the regression will produce meaningful results only if σ_t varies across the quarters. Unfortunately, using annual VaR-based volatility data instead of quarterly data, should reduce the accuracy of the risk forecast because in the regressions four quarterly trading revenue observations correspond to the same value of the VaR-based volatility.

Before testing for the accuracy of disclosed VaR, it is important to see the correlation structure among unexpected trading revenues for the banks in the sample. Table 3-4 reports relatively high correlation among unexpected trading revenues in the sample. It exceeds 60% between certain banks.

To test for the accuracy of disclosed VaR, equation 3-6 is estimated first for Table 3-5 reports the results of the six banks specific time-series regressions of unexpected trading revenues of VaR based volatility. Generally speaking, regression results did not report any form of informativeness of disclosed VaR about future risk expectations. For OLS results, only Bankers Trust shows the assumed positive relationship between unexpected trading revenues and VaR based volatility. However, the relationship seems very weak as indicated by OLS regression. As the table shows, the confidence level of rejecting H0 (the slope coefficient equal zero) for Bankers Trust is less than 95% (t-statistic is significant at 10%).

Table (3-4)
The Correlation Matrix of Unexpected Trading Revenues for Banks in the Sample

	B.A	B.NY	B.T	B.ONE	CITI	JP
B.A	1.00	0.41	0.56	0.43	0.63	0.38
B.NY	0.41	1.00	0.16	0.39	0.21	0.28
B.T	0.56	0.16	1.00	0.09	0.46	0.27
B.ONE	0.43	0.39	0.09	1.00	0.52	0.09
CITI	0.63	0.21	0.46	0.52	1.00	0.12
JP	0.38	0.28	0.27	0.09	0.12	1.00

* BA stands for Bank of America, B.NY stands for Bank of New York, B.T stands for Bankers Trust, B. ONE stands for Bank One, CITI stands for CitiCorp, and JP stands for JP Morgan Chase and Co.

However, this confidence level improved with using SUR regression. The b coefficient for Bankers Trust is significant at 95% confidence level under VaR regression. Although the joint test for all coefficients in SUR model significantly different than zero, the joint test for slops only is not significant. Those results are different than the results reported by Jorion (2002). Jorion reports positive significant result for Bank of America, JP Morgan Chase and Bank One/First Chicago. One reason may explain this difference in results is the different data set. Jorion's (2002) estimation does not include recent quarters where many banks witnessed high violations of their VaR estimates.

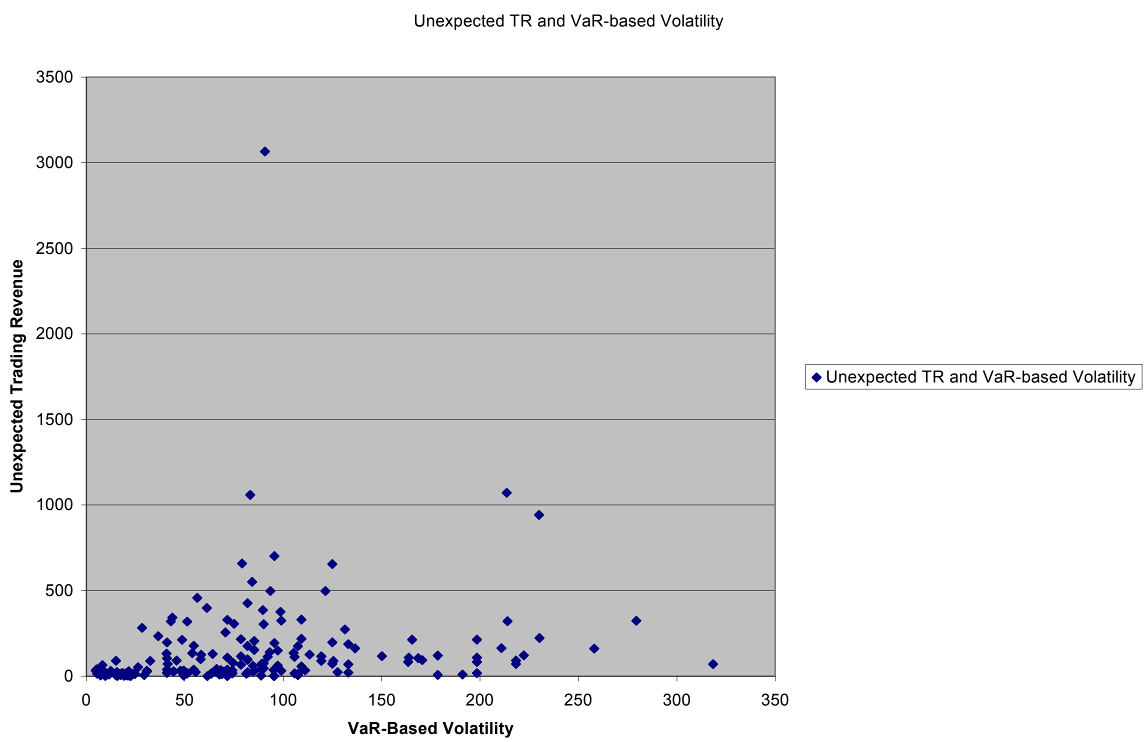
Table (3-5)
Bank-Specific Regression of Absolute Value of Quarter t+1 Unexpected Trading Revenues on Quarter t VaR-Based Volatility

$$\left[R_{i,t+1} - E\left[R_{i,t+1}\right]\right] = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

Bank	**Period No. of Observations**	**OLS**			**SUR**	
		Constant	**Slope**	**R^2**	**Constant**	**Slope**
		(t-Statistic)			(t-statistic)	
Bank of America	95,Q1-02,Q3 31	161.452 (2.488)	-0.153 (-0.239)	0.002	143.103 (3.131)	0.0188 (0.0456)
Bank of New York	95,Q1-02,Q3 31	10.024 (2.478)	0.026 (0.104)	0.0004	10.272 (2.930)	-0.013 (-0.063)
Bankers Trust	95,Q1-02,Q1 29	55.9639 (1.1745)	1.009* (1.840)	0.111	77.175 (2.165)	0.717** (2.065)
Bank One / First Chicago	95,Q1-02,Q3 31	38.168 (3.502)	-0.145 (-0.937)	0.030	23.870 (3.104)	0.030 (0.284)
CitiCorp	95,Q1-02,Q3 31	135.343 (3.725)	-0.077 (-0.305)	0.003	103.295 (3.518)	0.119 (0.619)
JP Morgan Chase & Co.	95,Q1-02,Q3 31	338.177 (1.697)	0.484 (0.324)	0.004	347.007 (1.663)	0.285 (0.165)
Joint test of all Parameters =0						368 P<0.001

- * Significant at 10%.
- ** Significant at 5%.
- Number of observation for SUR is 29. Another SUR performed without including Bankers Trust with 31 observation for each bank. The results are qualitatively the same.
- The joint test for the slopes = 0 is not significant.

Figure (3-5)
Absolute Unexpected Trading Revenues in
Quarter t+1 and VaR-Based Volatility in Quarter t:
Pool Sample



In contrast to single bank data that do not show the significant positive relationship between absolute unexpected trading revenues and VaR based volatility, figure 3-5 shows this relationship. Figure 3-5 displays the pool sample data. From figure 3-5 we can notice that higher VaR based volatility is associated with greater variationf in unexpected trading revenues.

Table 3-6 reports the results of a cross-sectional estimation of equation 3-6, with a sample of 185 observations. The pool sample OLS results (with and without intercept) report significant relationship of coefficient of VaR based volatility. Tests applied on

Table (3-6)
Pooled Regression of Absolute Value of Quarter t+1 Unexpected Trading Revenues on Quarter t VaR-Based Volatility

$$\left[R_{i,t+1} - E\left[R_{i,t+1}\right]\right] = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

Pooled Method	**Period**	**Constant**	**Slope**
	Observations	(t-Statistic)	
Pooled Sample OLS without Intercept	95,Q1-02,Q3 184	NA	1.452** (7.213)
Pooled Sample OLS	95,Q1-02,Q3 184	59.077 (1.740)	0.986** (2.945)
Pooled GLS Cross weighting	95,Q1-02,Q3 184	12.221 (1.243)	0.788** (5.883)
Pooled SUR	95,Q1-02,Q1 184	30.624 (4.718)	0.577 (6.517)
Fixed Effect Pool OLS	95,Q1-02,Q3 184	NA	0.274 (0.704)
Fixed Effect Pool GLS	95,Q1-02,Q3 184	NA	-0.036 (-0.314)
Fixed Effect Pool SUR	95,Q1-02,Q3 184	NA	0.155 (1.596)
GLS Random Effect Pool	95,Q1-02,Q3 184	103.929 (1.830)	0.434 1.159

- * Significant at 5%.
- ** Significant at 1%.
- NA: I did not report the fixed effect intercept.

single bank's data reject both heteroskedasticity and serialcorrelations[7], but the case is different for pooled data. To correct for heteroskedasticity and cross-sectional correlation in the pool data a cross section weights Generalized least Squares (GLS) method is applied. The results are reported in raw 3 in table 3-6. I checked for the random and fixed effects in the pool data, but the results do not show distinctive results for either random or fixed effect, so I report the results for both of them. However, it is insignificant.

[7] LM test, White and ARCH tests applied to test for heteroskedasticity and all show that there is no heteroskedasticity on a single bank level. Serial correlation for single banks is also rejected using LM test except for CitiCorp. The result are different for pooled data, figure 3-5 might give some ideas about heteroskedasticity.

Pool regression results give some indications for the informativeness of VaR models used by banks to predict extreme losses. However, bank-specific regression do not show such a relationship. Maybe with longer data set we can discover such relation. In conclusion, VaR based volatility method (like the forecast evaluation method) do not give distinctive proof for the accuracy of banks VaR models. As table 3-1 indicates banks used different VaR techniques, which means that those VaR techniques do not usually give accurate estimation for the maximum expected loss at a certain confidence level. As Berkowitz and O'Brien (2002) suggest any VaR model that takes into account tail fatness and time varying volatility would be more appropriate for capturing banks' losses especially in times of crisis and collapse.

3.5 Trading Portfolios

Trading portfolios' sizes are necessary for calculating VaR. For the purpose of back testing of VaR models developed in this dissertation, I collect trading portfolios from the 10Qs and 10Ks starting of December 31, 1994. One thing we should note here is that changes in trading portfolios do not reflect trading profit and losses. Trading portfolios are defined in the 10Qs and 10Ks as the trading assets and liabilities and derivative positions, and this is modified daily. Derivative positions widely exceed trading assets and liabilities for equities and bonds. Trading portfolios in all banks in the sample exceeds $1 billions (Thus they are all required by the SEC to disclose a measure for market risk). Table 3-7 reports the mean and the standard deviation of trading portfolios for the set of banks included in the sample.

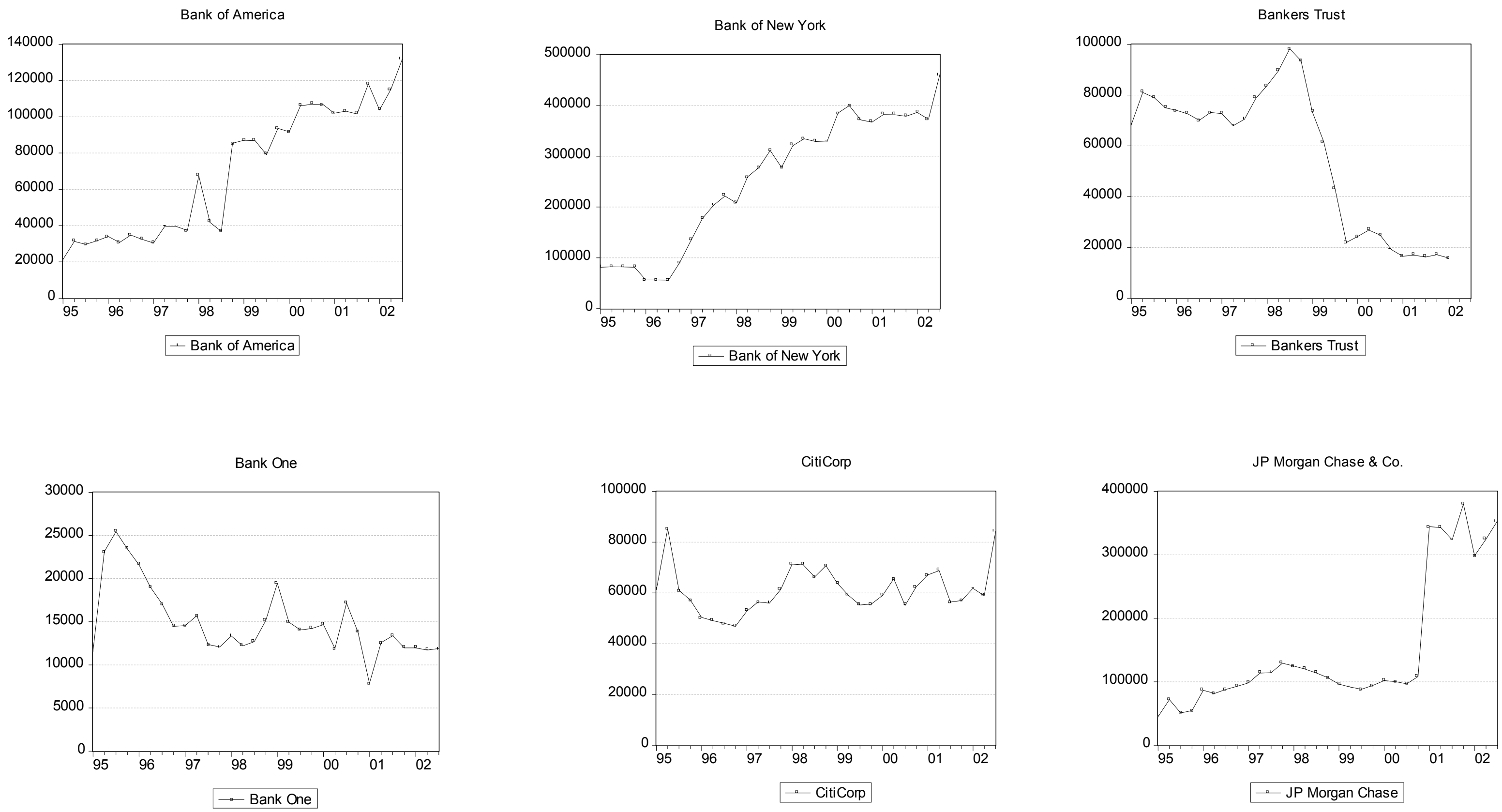
Table (3-7)
The Mean and Standard Deviation of the Trading Portfolios
(Trading Assets and Liabilities including Derivatives)

Bank	**Trading Portfolio ($Millions)**	
	Average	**Std. Deviation**
Bank of America	71,776.53	36780.34
Bank of New York	253,283.55	129551.95
Bankers Trust	56064.62	28437.87
Bank One /First Chicago	14989.47	3972.09
CitiCorp	61516.94	9247.01
JP Morgan Chase & Co.	156,735.34	111974.83

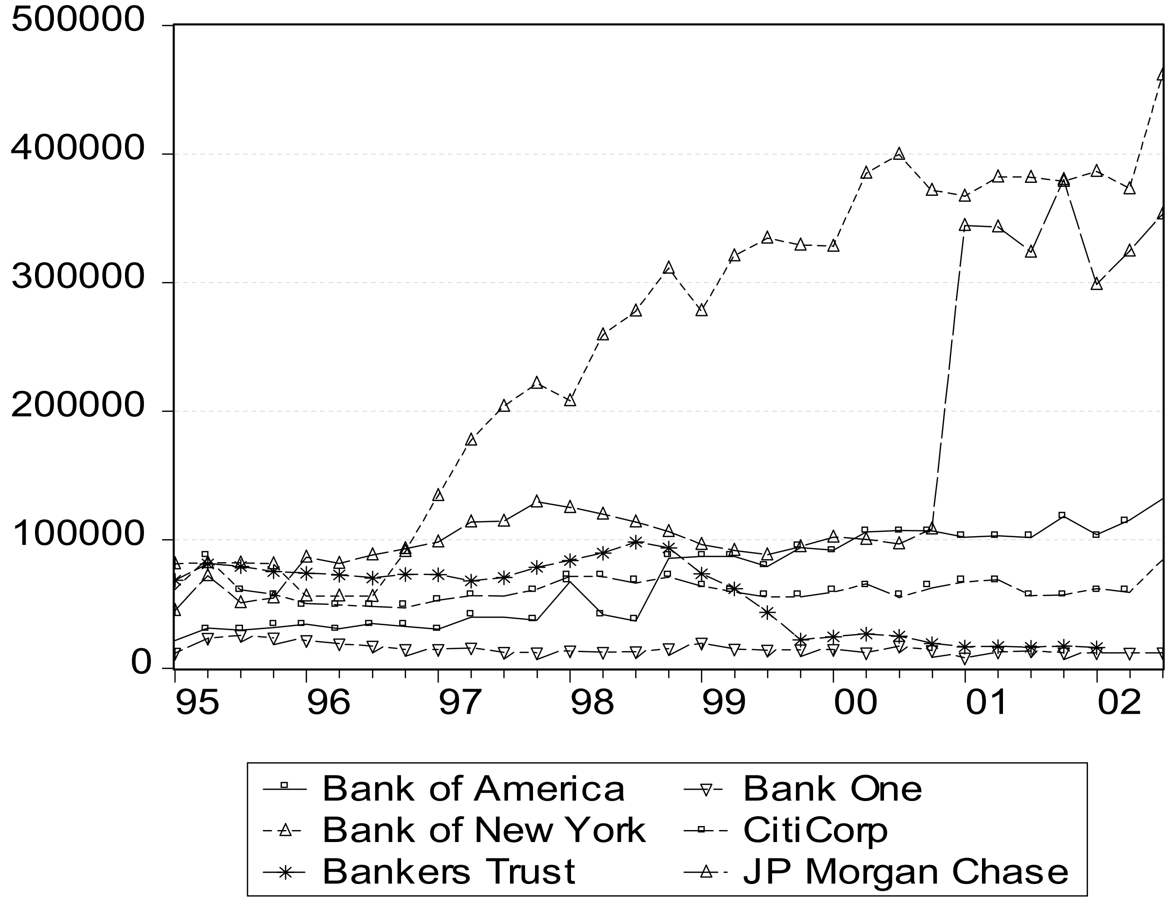
From table 3-7, we see that all banks in the sample have average trading assets and liabilities including derivatives of more than $14 billions. The lowest trading portfolio is for Bank One/ First Chicago and the highest is for Bank of New York. Figure 3- 6 shows the movement of trading portfolios as measured of trading asset and liabilities including derivative securities over time. Figure 3-6 shows that Bank of America, Bank of New York and JP Morgan Chase's trading portfolios are upward trending. Bankers Trust and Bank One show downward trending over the sample period, and CitiCorp shows a flat trend in its trading portfolio. Figure 3-7 below compares trading portfolios over time, where we see that Bank of New York maintains the highest trading portfolio over time, and Bank One has the lowest and the most stable portfolio over the sample period. The portfolio trending and levels explains significant part of the trending and the levels of VaR estimates as we will see later.

(Figure 3-7)

Trading Portfolios Sizes for Individual Banks in The Sample during The Period 1994:Q4-2002Q: Q3



(Figure 3-8)
Trading Portfolio Sizes for All Banks over The Sample Period.



CHAPTER FOUR: METHODOLOGY

As mentioned in Chapter three, VaR estimates that banks announce or use internally can not capture losses experienced by banks during the Russian Crisis and the near collapse of the LTCM. Additionally, as in chapter two, VaR models that account for tail thickness do not account for many aspects of financial data. Table 4-1 summarizes the literature that investigate VaR under the assumption of thick tails distribution, more explanations are available the literature review, chapter 2.

Table (4-1)
Summary of the Literature Deals with VaR under Thick Tails Distribution

The Paper	**Sources of Kurtosis**	**VaR Method**
Zangari(1996), Venkataraman (1997), and Hull and White (1998)	Mixture of two distributions	VaR percentile
Duffie and Pan (2001, *F&S*) and Gibson (2001)	Event risk (jumps)	Delta-Gamma Approach
Levin and Tchernitser (2001)	Gamma – Variance Model/ Maximum Entropy Approach	Delta-Gamma Approach
Li (1999)	Includes skewness and kurtosis explicitly in VaR	VaR percentile, (Cornish-Fisher Expansion).
El-Jahel, Perraudin and Sellin (1999)	Stochastic Volatility	Delta-Gamma Approach

For example non of the models above account for full thickness of the tail distribution, by taking a model that combine both jumps and stochastic volatility, the two sources of kurtosis in financial data, see Gibson (2001) and Lewis (2002). As mentioned earlier, both jumps and stochastic volatility are significant in financial data as in Chenov, Gallant, Ghysels and Tauchen (1999), Chourdakis (2000), and Chacko and Viceria(2003). Additionally, non of the models deals with fast volatility during crashes and collapse as Airoldi (2001) suggests.

4.1 The Assumed Processes for P&L

To account for those gaps in the previous literature, we derive an approximate analytical estimate of VaR for a class of non-normally distributed market risk factors and non-normally distributed portfolio's P&L that allows for the maximum tail thickness and fast volatility updating. To obtain such non-normal distribution, it is assumed that the P&L and the underlying market risk factors are evolving according to a mix of a non-affine stochastic volatility-jump diffusion model (NASVJ) in the form used by Chacko and Viceira (2003). According to this specification, trading revenues are assumed to evolve according to the following process:

$$\begin{aligned} \frac{dP_t}{P_t} &= \mu dt + \sqrt{v_t}\, dW_P + [\exp(J_u - 1]dN_u(\lambda_u) + [\exp(-J_d - 1]dN_d(\lambda_d) \\ dv_t &= \kappa(\theta - v_t)dt + \sigma v_t^{\gamma/2} dW_v \end{aligned} \tag{4-1}$$

Where W_P, W_v are Wiener processes with instantaneous correlation ρ, μ is the expected return on the banks portfolio or trading revenues, v_t is the instantaneous variance of the portfolio, $J_u, J_d > 0$, are stochastic jump magnitudes. $\lambda_u, \lambda_d > 0$ are constants the determine jump frequencies. Hence, $[\exp(J_u - 1]dN_u(\lambda_u)$ is a positive jump and $[\exp(-J_d - 1]dN_d(\lambda_d)$ is a downward jump. Here, the upward and downward jumps are asymmetric. $\kappa, \theta, \sigma, \gamma$ are constants. κ represents the speed of adjustment, θ is the long term mean, and σ is volatility of the instantaneous volatility. When γ takes value greater than 1, $(\gamma > 1)$, stochastic volatility is described to be non-affine stochastic volatility process, with $(\gamma = 1)$, stochastic volatility process would be described as affine process. Following Chacko and Das (2002) and Chacko and Viceira (2003), we assume here that jumps magnitudes are determined by draws from exponential distribution rather than a Poisson distribution, with the densities

$$f(J_i) = \frac{1}{\eta_i}\exp\left(-\frac{J_i}{\eta_i}\right) \qquad (4 - 2)$$

Where $i = u, d$.

The stochastic differential equation in 4-1 (even if we disregard stochastic volatility) is a mixed of normal process, Geometric Brownian Motion, (GBM), and Poisson-Exponential process in the jump part. This mixture result in an unknown conditional density function for P_t. Additionally, with discretely sample data, it is difficult to know which returns have a discontinuous component in it and which one does not. The matter becomes worse when we add stochastic volatility, since it is unobservable stochastic variable, and also the density function is unknown (even with jumps exclusion).

Jumps are regularly used in finance to capture discontinuous behavior in asset pricing. Merton (1976) was one of the first who use jump diffusion processes in finance. Returns discontinuities typically exhibit themselves in discretely sampled data in the form of excess kurtosis. So, one part of kurtosis in trading returns can be captured by jump diffusions. The other part of kurtosis is captured by the instantaneous correlation between the Brownian Motions of portfolio returns and volatility of returns on the portfolio ρ. Notice here that dynamics in volatility are non-linear of the squared volatility, which makes volatility updating faster.

Thus a process in such form allows for the complete kurtosis in trading revenues (portfolio returns). The non-affine stochastic process gives faster updating in volatility compared with affine volatility stochastic processes, which might explains the case of hyper volatility that arises with thick tails, Airoldi (2001). A VaR estimate based on such distribution should be able to predict losses that banks experience during market crashes. The cases where internal banks' VaR models and GARCH based VaR models failed to capture, as reported by Berkowitz and O'Brien (2002).

With certain restrictions on the parameters of the NASVJ, five well-known processes are emerged. If we suppress $\lambda_u, \lambda_d = 0$ and $v_t = \sigma$, ($dv = 0$), we get the popular Geometric Brownian Motion process (GBM). On the other hand of we restore jump intensities $\lambda_u, \lambda_d > 0$, with the assumption that $v_t = \sigma$ ($dv = 0$) we would be left with a jump diffusion process with asymmetric jumps (JD). If we suppress the jump frequencies ($\lambda_u, \lambda_d = 0$) and $\gamma = 1$, we get the square root process (SV) as in Heston (1993). With $\lambda_u, \lambda_d > 0$ and $\gamma = 1$, we get a mixed stochastic volatility process with jumps (SVJ). Finally, if we restrict γ to be greater that 1 ($\gamma > 1$) and restrict jump intensities to equal zero ($\lambda_u, \lambda_d = 0$), we get the non-affine stochastic volatility (NASV). Table 4-2 summarizes those restrictions and the resulted special cases processes.

Table (4-2)
Restrictions on the NASVJ and the Resulted Special Cases

Restrictions	**Resulted Process**	**Notes**
$\lambda_u, \lambda_d = 0$ and $v_t = \sigma$,	$\frac{dP_t}{P_t} = \mu dt + \sqrt{v_t} dW_P$	Log-normal Distribution, zero skewness and kurtosis
$\lambda_u, \lambda_d > 0$, and $v_t = \sigma$	Jump diffusion	Part of skewness and kurtosis is captured
$\lambda_u, \lambda_d = 0$ and $\gamma = 1$	Square Root process	Part of skewness and kurtosis is captured
$\lambda_u, \lambda_d > 0$ and $\gamma = 1$	Mixed stochastic volatility process	The whole skewness and kurtosis is captured
$\lambda_u, \lambda_d = 0$ and $\gamma > 1$	Non-affine stochastic volatility	Part of skewness and kurtosis is captured, but volatility updates faster.

Those processes are well known in finance literature. However, it got limited use in VaR literature. Cardenas, Fruchard, Koehler and Michel (1997) and Rouvinez (1997) derive an analytical estimate for VaR assuming that mark to market evolves according to a multidimensional GBM process. El-Jahel, Perraudin and Sellin (1999) derive VaR estimate for portfolio contains derivative securities based on stochastic volatility model. Duffie and Pan (2001) derive an analytical delta-gamma VaR estimate for jump diffusion process. On the other hand, Levin and Tchernitser (2001) derive VaR model under different levy processes one of which Ornestein Uhlenbeck process.

4.2 The General Methodology

VaR deals with the probability of loss. Thus the first task is to identify the distribution of the left tail. To derive an explicit solution of VaR, the dissertation assumes that trading revenues' of the bank follow NASVJ and the other special cases processes summarized in Table 4-2. Unfortunately, the probability distribution function (pdf) and the cumulative distribution function for those processes are unknown. So the first step is to derive the conditional distribution function for those processes. The necessity of using the conditional distribution rather than unconditional one comes from Hsieh's (1993) concern that conditional densities provide better description of asset price movements in the presence of non-linear dynamics.

As mentioned earlier, the cdf for those processes is unknown. However, we can solve explicitly for the characteristic function (cf). Epps (1993) defines the characteristic function for a random variable X at some real number ω as the center of mass of the distribution of ωX wrapped around the unit circle in the complex plane. The characteristic function can be represented by:

$$\phi(\omega) = E[e^{i\omega X} \int_{-\infty}^{\infty} e^{i\omega u} dF(x) \qquad (4\text{-}3)$$

The cf uniquely defines the distribution function, cdf through the Fourier inversion theorem of the cf that gives the cdf. According to the Fourier transforms theorems if:

$$\begin{aligned} \phi(\omega) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(u) e^{i\omega u} du \\ f(x) &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(\omega) e^{-i\omega x} d\omega \end{aligned} \qquad (4\text{-}4)$$

The function $\phi(\omega)$ is called the Fourier transform of $f(x)$, and the $f(x)$ is the inverse Fourier transform of $\phi(\omega)$. The constants preceding the integral sign in equation system 4-4 can be any constant different from zero as long as their product is $\frac{1}{2\pi}$. In our case $\phi(\omega)$ is the characteristic function and $f(x)$ is the probability density function.

However, in our analysis we are in need for the cdf rather than the pdf. For more about the characteristic function and the Fourier transformation theorems see Sturat and Ord (1994: chapter 4), and Pollock (2000: chapter 13).

Inferring the distribution function from the characteristic function has a long history in statistics. Imhof (1961) and Bohman (1961) start this effort. Bohman (1970, 1972, 1975,1980), Davies (1973) and Schorr (1975) originally propose algorithms to invert numerically characteristic function to obtain cumulative distribution function. Bohman (1975) gives five methods for numerical inversion of cf to get the cdf. However his five methods are applied for random variables with zero mean and unit variance. Waller, Turnbull and Hardin (1995) and Waller (1995) apply the first method of Bohman (1975) which is the least accurate among the five methods at some known distributions and they conclude that the numerical inverse of the characteristic function distinguish the distribution function.

Using the characteristic function to infer the distribution function in economics starts with Shephard (1991). He derives the Fourier theorem version to infer the cumulative distribution function and he extends it to multivariate random variables. Additionally, Shephard (1991) establishs the condition of this inversion theorem. Shephard (1991) setup the cdf function as an inversion of the characteristic function according to the following equation:

$$F(x) = \frac{1}{2} - \frac{1}{2\pi} \int_0^\infty \mathrm{Re}\left[\frac{\phi(\omega)e^{i\omega x}}{i\omega}\right] d\omega \qquad (4\text{-}5)$$

Where Re[.] is the real part of the imaginary root.

After that the theory of inversion used extensively in both option pricing and estimation. In option pricing, Stein and Stein (1991) use the inversion method for the moment generating function rather than the characteristic function. However, the moment generating function does not uniquely identify the distribution as Waller, Turnbull and Hardin (1995) and Waller (1995) indicate. Heston (1993) uses the Fourier theorem in deriving option pricing under stochastic volatility. Bates (1996 a, b) use the same methodology for deriving option pricing under jumps. Schobel and Zhu (1999) apply the inversion method in option pricing stochastic volatility with Ornstein-Ublenbeck process. Bakshi, Cao and Chen (1997) and Scott (1997) use the Fourier inversion to calculate option prices under different stochastic process with jumps in pricing and evaluating the performance of different option pricing models. Benhamou (1999), on the other hand, uses the fast Fourier transform (discrete algorithm) to evaluate discrete Asian options.

Many authors try to generalize the Fourier inversion of the characteristic function for pricing certain type of derivatives. Duffie, Pan and Singleton (2000) generalize the Fourier transform method for pricing assets with affine jump diffusion returns. Chacko and Das (2002) generalize this approach for pricing interest rate derivatives. Lewis (2000, 2001, 2002) introduce a general derivation for a fundamental transform method of the characteristic function depends on the payoff of the option and the type of the stochastic process of the underlying asset. In his work, Lewis could specify the imaginary strips where the fundamental transform method applies.

Using the characteristic function for the purpose of estimation starts to appear recently in economics and finance as a substitute of the unknown distribution function. Most of the work in using the characteristic function in estimation uses the generalizations of Duffie, Pan and Singleton (2000) and Chacko and Das (2002). Jiang and Knight (2002), Knight and Yu (2002), and Knight, Satchel and Yu (2002) use the empirical characteristic function to develop a maximum likelihood estimator (MLE) for continuous time processes. Yu (1999) uses the empirical characteristic function to estimate a jump diffusion model. Singleton (2001) uses this method in developing EMM estimation of affine processes. Finally Chacko and Viceira (2003) use the conditional characteristic function to estimate the NASVJ and its special cases for the S&P 500 data.

Using the inversion of the characteristic function to obtain the cumulative distribution function in VaR context is very limited. Cardenas, Fruchard, Koehler and Michel (1997) assume that the mark to market follow multidimensional pure Ito's process. In their work they derive the characteristic function and then they numerically invert it with fast Fourier inversion method. Using Taylor expansion, they expand trading revenues to include vega rather than delta and gamma only. After they numerically obtain the cumulative distribution, they include this estimation into delta-gamma approach for option portfolios with analytical solution for VaR for a pure diffusion model. The source of skewness and kurtosis in their model is gamma.

El-Jahel, Perraudin, and Sellin (1999) use the characteristic function to derive numerically the third and the fourth moments for stochastic volatility process and include them in the delta-gamma approach to calculate VaR for derivative securities. El-Jahel, Perraudin, and Sellin (1999) do not invert the characteristic function to obtain the cumulative distribution but instead they calculate the higher moments of the stochastic volatility model numerically and include them in the delta gamma method.

Duffie and Pan (2001) use a jump diffusion model with return jumps at Poisson arrival. They infer the tail distribution by deriving the characteristic function and then invert it numerically via Fourier inversion theorem. Duffie and Pan (2001) in their method replace the simulation step in VaR with a jump diffusion model for option portfolio through delta-gamma approach.

This work differ from the above three papers in four aspects:

1- We do not use the delta-gamma or any Greeks. Instead, we derive analytical solution for VaR that is a function of the tail distribution (the tail percentile) and its deviates. Then we estimate tail percentile using the numerical inversion of the conditional characteristic function. After that we substitute the percentile deviate into VaR equation to get VaR estimate.
2- This work assumes diffusion processes that contain both stochastic volatility and jumps simultaneously to capture the full skewness and kurtosis.
3- This dissertation adopts a non-affine stochastic volatility model that allows for fast updating of stochastic volatility.
4- In the back testing part, this dissertation applies its theoretical VaR estimate on a real sample of portfolio trading returns, in a try to mimic actual trading losses of those actual portfolios during high volatility periods and market crashes. This method of testing has not been applied by any of the papers who produced analytical VaR estimates. Most of the papers apply the VaR estimate on a hypothetical portfolios contain one option position written on the S&P 500.

Table 4-3 summarizes the differences between this work and other work that use the characteristic function. The comparison includes those who use the Fourier inversion like Cardenas, Fruchard, Koehler and Michel (1997) and Duffie and Pan (2001). Or those how use the characteristic function to derive the moments rather than the distribution like

El-Jahel, Perraudin, and Sellin (1999). As a matter of fact, Perraudin, and Sellin (1999) methodology in calculating VaR is a bit different from the method used here. Their methodology does not use the Fourier inversion, and Fourier inversion is crucial in this work.

In deriving an analytical VaR for the NASVJ and the special cases (SV, JD, SVJ, and NASV) we proceed according to the following steps:

1- Deriving an analytical VaR estimate as a function of the distribution deviate α associated with a specific confidence level (99%).

2- Derive the cumulative distribution function for each process, to get α, and this can be obtained by:

   a- Deriving the conditional characteristic function for each process

   b- Invert the characteristic function according to equation 4 –5 for each characteristic function

   c- Obtain the deviate α that is associated with the 99% confidence level and substitute is in the derived analytical solution for VaR.

The methodology of deriving the conditional probability distribution mentioned above is applicable for affine processes. As a matter of fact, it is applicable only for affine processes as Shephard (1991), Duffie, Pan and Singleton (2000) and Chacko and Das (2002) show. For non-affine process, we need first to transform the non-affine processes to affine process, mostly by using some kind of Taylor series expansion and then perform the Fourier-inversion.

Table (4-3)
Summary of the Existing Literature Using the Characteristic Function in VaR Estimate

Paper **Indicator**	**Cardenas *et.al*.(1997)**	**El-Jahel *et.al* (1999)**	**Duffie and Pan (2001)**	**The Proposed Work**
Source of Dynamics	Mark to market in Options Portfolio	Underlying Assets in Options Portfolio	Underlying Assets in Options Portfolio	The whole trading revenues (P&L)
Source of Tail thickness	Ito's Process for stochastic Volatility, Vega Term	Square Root Stochastic Volatility	Jumps	Non affine Stochastic Volatility and Jumps
Inferring The Distribution	Inverting The Characteristic Function	Including higher moments into VaR	Inverting The Characteristic Function	Inverting The Characteristic Function
Volatility Estimate	Assumed Parameters	Assumed Volatility Parameters	Pre-Exist Data	Historical Volatility
VaR Method	Delta-Gamma Approximation	Delta-Gamma Approximation	Delta-Gamma Approximation	VaR Percentile

Schobel and Zhu (1999) argue that applying the inversion formula and solving the integral in equation 4-5 is not a trivial issue. Even they consider the numerical solution for such integral of real value is also not trivial. In Schobel and Zhu (1999, p. 28) language “Numerical integration of these real valued probability integrals is not trivial. Heston (1993) as well as Bakshi, Cao and Chen (1997) did not report their numerical procedures in detail. Bates (1996) evaluated the integrals in his formula using Gaussian quadrature software and obtained sufficiently accurate results except for extreme and implausible jump parameters”.

Scott (1997) compared the Fourier inversion method with Monte Carlo simulations and finite difference methods and report that the Fourier inversion method is superior with respect to accuracy and computing time. Schobel and Zhu (1999) results support Scott (1997) results in the superiority of Fourier inversion method to Monte Carlo simulations. As mentioned earlier, Bohman (1961, 1970, 1972, 1975, and 1980) invent five accurate methods to solve this integral numerically. Bohman (1975) summarizes those methods and test those methods in terms of accuracy and time consuming.

4.3 Deriving VaR Estimator

In this section, the formal derivations of VaR for the NASVJ and all other special cases including the GBM are introduced. As stated earlier, the derivation procedure starts with the confidence level (α). After that, we derive the conditional characteristic function for each process, which is known with a closed form. Finally, using the Fourier inversion theorem introduced in equation 4-5, we infer the cumulative distribution function (cdf) for each process.

Through the derivation, the following notations will be used,

P_t: The portfolio size at time t.

τ: The holding period or time horizon over which the portfolio held without change in position. τ is the holding period over which we calculate VaR.

$\Delta P_{t+\tau}$: The profit and the loss of the bank or the trading revenue over the period τ.

c: The confidence level (99% for example).

α: The deviate associated with the confidence level, 2.326 for 99% confidence level of a standard normal distribution. We use α_{sn} for standard normal distribution.

p_t : $\ln P_t$

$\Delta p_{t+\tau}$: $\Delta \ln P_{t+\tau}$, and it represents return on the portfolio over τ holding period.

v_t: The volatility at time t, v without subscript is used when the volatility is constant. When volatility is constant $v = \sigma_0^2$ and $\sqrt{v} = \sigma_0$.

4.3.1 Value at Risk (VaR)

Value at risk (VaR) is defined as the maximum expected loss in the market value that would be exceeded by a small probability (1-c)% over a defined trading time horizon τ. if we assume that P_t is the value of the portfolio at time t and ΔP_t is the profit and loss (P&L) of the bank or the trading revenues over the time horizon τ, then the $Pr\,(-\Delta P_{t+\tau} \geq \text{VaR}_\tau) = (1\text{-c})\%$. Expressed in terms of the percent point function $Pr\,(-\Delta P_{t+\tau} \geq G(1-c)) = (1-c)\%$, where $\text{VaR} = G(1-c) = G(F(-\text{VaR}))$.

Following Jorion (2001) in constructing VaR, assume that there is a lowest value of the portfolio donated P^* at a confidence level of c%, which means that there is a probability of (1-c)% that the portfolio value would be less than P^*. Assume that the lowest level of the portfolio is associated with a return on the portfolio of ΔP^*. Where

$\Delta P^* = \ln\left[\frac{P^*_{t+\tau}}{P_t}\right]$. Then:

$$\Pr\left(\Delta \ln P_{t+\tau} \le \ln\left[\frac{P^*_{t+\tau}}{P_t}\right]\right) = (1-c)\% \tag{4-6}$$

Notice here that P^*satisfies the definition of VaR. Then:

$$\Pr\left(\Delta \ln P_{t+\tau} \le \ln\left[1 - \frac{VaR_\tau}{P_t}\right]\right) = (1-c)\% \tag{4-7}$$

4.3.1.1 The GBM Case (Log-Normal Case)

Assume that the P&L (trading revenues) follow a Geometric Brownian Motion process of the form:

$$\frac{dP_t}{P_t} = \mu_P dt + \sqrt{v}dW_P \tag{4-8}$$

Where μ_P is the expected return on the portfolio. Then by Ito's Lemma

$$dp_t = d\ln P_t = \left(\mu_P - \frac{1}{2}v\right)dt + \sqrt{v}dW_P \tag{4-9}$$

So, $\Delta \ln P_t \sim N\left[\left(\mu_P - \frac{1}{2}v\right)\tau, \sigma_0\sqrt{\tau}\right]$

In case of standard normal equation (4-7) becomes

$$\Pr\left(\Delta \ln P_{t+\tau} \le \frac{\ln\left[1 - \frac{VaR_\tau}{P_t}\right] - \left(\mu_P - \frac{1}{2}v\right)\tau}{\sigma_0\sqrt{\tau}}\right) = (1-c)\% \tag{4-10}$$

From equation (4-10) we recognize that

$$G(1-c)=\alpha_{sn}=\frac{\ln\left[1-\dfrac{VaR_{\tau}}{P_t}\right]-\left(\mu_P-\dfrac{1}{2}v\right)\tau}{\sigma_0\sqrt{\tau}} \tag{4-11}$$

For equation (4-11) we can obtain α_{sn} from the standard normal cumulative distribution tables. For confidence level of c = 99%, α_{sn} = -2.326. Accordingly,

$$VaR_{\tau}=P_t\left[1-e^{\left(\mu_P-\frac{1}{2}v\right)\tau+\alpha_{sn}\sigma_0\sqrt{\tau}}\right] \tag{4-12}$$

4.3.1.2 VaR under Thick Tails

Since VaR is related to the distribution of the tail of the P&L of the bank, in this part, we are looking to get an estimate for the VaR under different processes that allow for thick tails. We start with generalizing a model for the non-affine stochastic volatility process with jump (NASVJ). Then we find stochastic Volatility process (SV), the jump diffusion process (JD), a mixed process of stochastic volatility and jump (SVJD), non-affine stochastic volatility process (NASV), all as special cases. The stochastic volatility process and the jump diffusion process allow for skewness and excess kurtosis, the mixed process allows for the maximum tail thickness. Whereas the non-affine structure (including the general model) allow for hyper volatility updating as in the cases of stock market crashes. Airoldi (2001) argues that in case of thick tails, volatility updates faster than linear relation with past volatility, as in the case of the near collapse of the LTCM.

Under those processes, the probability of getting trading revenues under certain level, like, α can be written as

$$\Pr\left(\Delta\ln P_{t+\tau}\leq\alpha\right)=(1-c)\% \tag{4-13}$$

From equation (4-7) above $\ln\left[1-\frac{VaR_\tau}{P_t}\right]=(G(1-c))=\alpha$, accordingly, we find

VaR_τ to be:

$$VaR_\tau = P_t\left[1-e^{\alpha}\right] \tag{4-14}$$

The task now is to infer α associated with the confidence level, i.e. deriving the cumulative distribution function. As mentioned earlier, because the cdf is unknown for those processes, deriving the cdf requires first deriving the conditional characteristic function (ccf) for each process and then using the Fourier inversion theorem.

4.3.2 The Conditional Characteristic Function of Non-normal Processes

Deriving the ccf will be implemented according to the following steps:

1. Deriving the Kolomogorov Backward Equation (KBE) or Fokker-Plank Forward Equation (F-PFE), two names for same equation. The KBE or the F-PFE is a partial differential equation with a known solution form. The conditional characteristic function is the solution for that equation. And this whole procedure is known as Feynman-Kac Formula.
2. To solve KBE we conjecture a solution for the characteristic function and substitute this conjecture into the KBE.
3. When substituting the conjecture into the KBE, we get two ordinary differential equations (ODE) of the form of Raccati equations.
4. Solving those two Raccati equations gives the parameters of the characteristic function.

4.3.2.1 Deriving the ccf for the NASVJ Process

Assuming that the trading revenues evolve according to NASVJ, means that the P&L are evolving according to equation (4-1) in section 4-1 above:

$$\frac{dP_t}{P_t} = \mu\ dt + \sqrt{v_t}\,dW_P + [\exp(J_u - 1]dN_u(\lambda_u) + [\exp(-J_d - 1]dN_d(\lambda_d)$$
$$dv_t = \kappa(\theta - v_t)dt + \sigma v_t^{\gamma/2}\,dW_v$$

Parameters are as defined in section 4.1 above. Applying Ito's Lemma, to get the log transformation gives the following

$$\begin{aligned} dp_t &= d\ln P_t = (\mu_P - \frac{1}{2}v_t)dt + \sqrt{v_t}\,dW_P + J_u dN_u(\lambda_u) - J_d dN_d(\lambda_d) \\ dv_t &= \kappa(\theta - v_t)dt + \sigma v_t^{\gamma/2}\,dW_v \end{aligned} \tag{4-15}$$

Donate the ccf by $\phi(\omega, p_t, v_t, \tau)$, the Feynmann-Kac formula, implies that $E[d\phi(\omega, p_t, v_t, \tau)] = 0$. This is the same as Heston (1993) assumption that $\phi(\omega, p_t, v_t, \tau)$ is martingale. The solution for the partial differential equation (pde) $E[d\phi(\omega, p_t, v_t, \tau)] = 0$ is the characteristic function $\phi(\omega, p_t, v_t, \tau)$ with the terminal condition $\phi(\omega, p_t, v_t, 0) = e^{i\omega p}$. ω is a real valued dummy variable and *i* is the imaginary root, where $i = \sqrt{-1}$.

By Feynmann-Kac formula:

$$\begin{aligned} &\frac{\partial\phi}{\partial p}(\mu_P - \frac{1}{2}v_t) + \frac{\partial\phi}{\partial v_t}\kappa(\theta - v_t) + \frac{1}{2}\frac{\partial^2\phi}{\partial p^2}v_t + \frac{1}{2}\frac{\partial^2\phi}{\partial v^2}\sigma^2 v_t^{\gamma} + \\ &\frac{\partial^2\phi}{\partial v\partial p}\sigma v_t^{\frac{\gamma+1}{2}}\rho_{pv} - \frac{\partial\phi}{\partial\tau} + \lambda_u E[\phi(\omega, p_t + J_u, v_t, \tau) - \phi(\omega, p_t, v_t, \tau)] \\ &+ \lambda_d E[\phi(\omega, p_t - J_d, v_t, \tau) - \phi(\omega, p_t, v_t, \tau)] = 0 \end{aligned} \tag{4-16}$$

Equation 4-16 is known as Kolomogrov Backward Equation (KBE) or Fokker - Plank Forward Equation and the conditional characteristic function

$\phi(\omega, p_t, v_t, \tau)$ is the solution for this equation. The last two terms concerning jumps in 4-16 are very familiar in the jump literature. Kushner (1967) and Gihman and Skorohod (1972) provide a full derivation to such terms. Merton (1971,1976) Ahn and Thompson (1988), Duffie, Pan and Singleton (2000), Singleton (2001), Chacko and Das (2002) and Chacko and Viceira (2003) use Kushner (1967) and Gihman and Skorohod (1972).

To solve for the conditional characteristic function explicitly, we guess a functional form for the characteristic function of the form

$$\phi(\omega, p_t, v_t, \tau) = e^{i\omega p + A(\tau) + B(\tau)} \tag{4-17}$$

with the terminal condition $\phi(\omega, p_t, v_t, 0) = e^{i\omega p}$

Substituting the conjecture in equation (4-17) and its derivatives in the KBE, equation (4-16) we get:

$$\begin{aligned} &i\omega(\mu_P - \frac{1}{2}v_t) + A(\tau)\kappa(\theta - v_t) + \frac{1}{2}(i\omega)^2 v_t + \frac{1}{2}A^2(\tau)\sigma^2 v_t^\gamma + \\ &i\omega A(\tau)\sigma v_t^{\frac{\gamma+1}{2}} \rho_{pv} - \left[\frac{dA(\tau)}{d\tau}v_t + \frac{dB(\tau)}{d\tau}\right] + \lambda_u E\left[e^{i\omega J_u} - 1\right] + \lambda_d E\left[e^{i\omega J_d} - 1\right] = 0 \end{aligned} \tag{4-18}$$

The term (4-18) contains $v_t^\gamma, v_t^{\frac{\gamma+1}{2}} + \lambda_u E\left[e^{i\omega J_u} - 1\right]$and $\lambda_d E\left[e^{i\omega J_d} - 1\right]$ which are non-linear. For the terms $\lambda_u E\left[e^{i\omega J_u} - 1\right]$and $\lambda_d E\left[e^{i\omega J_d} - 1\right]$ we apply the moment generating function (mgf) method for the exponential distribution as reported in Rose and Smith (2002, p.142) for $E\left[e^{i\omega J_u}\right]$and $E\left[e^{i\omega J_d}\right]$, which gives:

$$E\left[e^{i\omega J_u}\right] = \frac{1}{1\text{-}i\omega\eta_u} \tag{4-19}$$

$$E\left[e^{-i\omega J_d}\right] = \frac{1}{1 + i\omega\eta_d} \tag{4-20}$$

Chacko and Viceria (2003) adopt the transformation above for those non-linear terms. Another way is to follow Chacko and Das (2002), by applying the mgf method for the whole terms $E\left[e^{i\omega J_u}-1\right]$ and $E\left[e^{i\omega J_d}-1\right]$:

$$E\left[e^{i\omega J_u}-1\right]=-\frac{\omega\eta_u}{i+\omega\eta_u} \tag{4-21}$$

$$E\left[e^{-i\omega J_d}-1\right]=\frac{\omega\eta_d}{i-\omega\eta_d} \tag{4-22}$$

Here we follow Chacko and Viceria (2003) by adopting the transformation shown in (4-19) and (4-20). For the terms v_t^{γ} and $v_t^{\frac{\gamma+1}{2}}$, we apply some kind of perturbation method to linearize the parameters in v_t. This approximation is basically a Taylor series expansion around the unconditional mean volatility process θ as follows:

$$v_t^{\gamma}\approx\theta^{\gamma}(1-\gamma)+\gamma\theta^{\gamma-1}v_t \tag{4-23}$$

$$v_t^{\frac{\gamma+1}{2}}\approx\theta^{\frac{\gamma+1}{2}}\left(\frac{1-\gamma}{2}\right)+\left(\frac{1+\gamma}{2}\right)\theta^{\frac{\gamma-1}{2}}v_t \tag{4-24}$$

Substituting the terms (4-19), (4-20), (4-23), and (4-24) into (4-18) and rearranging,

$$\begin{aligned}&\left[\left(i\omega\mu_P+\frac{\lambda_u}{1-i\omega\eta_u}+\frac{\lambda_d}{1+i\omega\eta_d}-(\lambda_u+\lambda_d)\right)+\left(\frac{1}{2}i\omega(i\omega-1)v_t\right)\right]+\\&\left[\left(\kappa\theta+i\omega\sigma\rho\theta^{\frac{\gamma+1}{2}}\left(\frac{1-\gamma}{2}\right)\right)+\left(i\omega\sigma\rho\left(\frac{1+\gamma}{2}\right)\theta^{\frac{\gamma-1}{2}}-\kappa\right)v_t\right]A(\tau)+\\&\left[\left(\frac{1}{2}\sigma^2\theta^{\gamma}(1-\gamma)\right)+\left(\frac{1}{2}\sigma^2\gamma\theta^{\gamma-1}\right)v_t\right]A^2(\tau)-\frac{dA(\tau)}{d\tau}v_t+\frac{dB(\tau)}{d\tau}=0\end{aligned} \tag{4-25}$$

Redefining 4-25 as

$$c+aA(\tau)+\frac{1}{2}bA^2(\tau)-\frac{dA(\tau)}{d\tau}v_t+\frac{dB(\tau)}{d\tau}=0 \tag{4-26}$$

Each parameter of *c*, *a*, and *b* consists two parts, the first part is independent and the second part depends on the state variable v_t. We can rewrite those parameters in the following formats:

$$a = a_1 + a_2 v_t \tag{4-27}$$

$$b = b_1 + b_2 v_t \tag{4-28}$$

$$c = c_1 + c_2 v_t \tag{4-29}$$

Where, $a_1 = \kappa\theta + i\omega\sigma\rho\theta^{\frac{\gamma+1}{2}}\left(\frac{1-\gamma}{2}\right)$,

$a_2 = i\omega\sigma\rho\left(\frac{1+\gamma}{2}\right)\theta^{\frac{\gamma-1}{2}} - \kappa$,

$b_1 = \sigma^2\theta^{\gamma}(1-\gamma)$,

$b_2 = \sigma^2\gamma\theta^{\gamma-1}$,

$c_1 = i\omega\mu_P + \frac{\lambda_u}{1-i\omega\eta_u} + \frac{\lambda_d}{1+i\omega\eta_d} - (\lambda_u + \lambda_d)$ and $c_2 = \frac{1}{2}i\omega(i\omega - 1)$

Now, substituting a_1, a_2, b_1, b_2, c_1, and c_2 into 4-26

$$(c_1 + c_2 v_t) + (a_1 + a_2 v_t)A(\tau) + \left(\frac{b_1}{2} + \frac{b_2}{2}v_2\right)A^2(\tau) - \frac{dA(\tau)}{d\tau}v_t + \frac{dB(\tau)}{d\tau} = 0 \tag{4-30}$$

(4-30) can be written in the following fashion:

$$-\left(c_1 + a_1 A(\tau) + \frac{1}{2}b_1 A^2(\tau) - \frac{dB(\tau)}{d\tau}\right) = \left(c_2 + a_2 A(\tau) + \frac{1}{2}b_2 A^2(\tau) - \frac{dA(\tau)}{d\tau}\right)v_t \tag{4-31}$$

As in Ingersoll (1987, chapter 18), the equality in (4-31) can only hold if both sides of the equation are zeros, because the state variable v_t is stochastic. This leads to the following ordinary differential equations

$$c_1 + a_1 A(\tau) + \frac{1}{2} b_1 A^2(\tau) = \frac{dB(\tau)}{d\tau} \tag{4-32}$$

$$c_2 + a_2 A(\tau) + \frac{1}{2} b_2 A^2(\tau) = \frac{dA(\tau)}{d\tau} \tag{4-33}$$

With the boundary conditions A(0)=0, and B(0)=0.

Equations (4-32) and (4-33) are the well know imaginary Raccati equations, Duffie, Pan and Singleton (2000), Liu (2001) and Duffie, and Schachermayer (2002) introduce a general solution for such kind of equations. However, the solution for equation (4-33) is given by:

$$A(\tau, \omega) = \frac{2}{b_2}\left[\frac{u_1 u_2 e^{u_1 \tau} - u_1 u_2 e^{u_2 \tau}}{u_1 e^{u_2 \tau} - u_2 e^{u_1 \tau}}\right], \tag{4-34}$$

Where

$$u_1 = a_2 + \sqrt{a_2^2 - 2 b_2 c_2}$$
$$u_2 = a_2 - \sqrt{a_2^2 - 2 b_2 c_2}$$

The solution for $B(\tau)$ in (4-32) is given by:

$$B(\tau) = \int_0^\tau \left(c_1 + a_1 A(\tau) + \frac{1}{2} b_1 A^2(\tau) - \frac{dB(\tau)}{d\tau} \right) du \tag{4-35}$$

Where *u* here is the integral dummy. From (4-33),

$A^2(\tau) = \frac{2}{b_2}\frac{dA(\tau)}{d\tau} - \frac{2a_2}{b_2} A(\tau) - \frac{2c_2}{b_2}$, Accordingly, we can rearrange the terms inside the integral and (4-35) can be written as

$$B(\tau) = \int_0^\tau \left[\frac{b_1}{b_2}\frac{dA(\tau)}{du} + \left(a_1 - \frac{b_1 a_2}{b_2} \right) A(\tau) + \left(c_1 - \frac{b_1 c_2}{b_2} \right) \right] du \tag{4-36}$$

Distributing the integral through out the expressions, we get:

$$B(\tau) = \int_0^\tau \frac{b_1}{b_2} dA(\tau) + \int_0^\tau \left(a_1 - \frac{b_1 a_2}{b_2} \right) A(\tau) du + \int_0^\tau \left(c_1 - \frac{b_1 c_2}{b_2} \right) du \tag{4-37}$$

Since $A(0) = 0$, then integrating the first and the third terms:

$$B(\tau) = \frac{b_1}{b_2} \left[A(\tau) \right] + \left(a_1 - \frac{b_1 a_2}{b_2} \right) \int_0^\tau A(\tau) du + \left(c_1 - \frac{b_1 c_2}{b_2} \right) \tau \tag{4-38}$$

$$\int_0^\tau A(\tau) du = \frac{2}{b_2} \ln \left[\frac{u_2 - u_1}{u_2 e^{u_1 \tau} - u_1 e^{u_2 \tau}} \right],$$

Where u_1, and u_2 as defined above.

Accordingly, the solution for the ODE (4-32) is given by:

$$B(\tau) = \frac{b_1}{b_2} \left[A(\tau) \right] + \left(c_1 - \frac{b_1 c_2}{b_2} \right) \tau + \left(a_1 - \frac{b_1 a_2}{b_2} \right) \frac{2}{b_2} \ln \left[\frac{u_2 - u_1}{u_2 e^{u_1 \tau} - u_1 e^{u_2 \tau}} \right] \tag{4-39}$$

To get the value of the characteristic function for the NASVJ process, we substitute the values of a_1, a_2, b_1 b_2, c_1 and c_2 in $A(\tau)$ equation (4-34) and $B(\tau)$ equation (39), in the conjectured form of the characteristic function in (4-17).

After substituting the values of a_1, a_2, b_1 b_2, c_1 and c_2 in $A(\tau)$ and $B(\tau)$ then we find that

$$A(\tau, \omega) = \frac{2}{\sigma^2 \gamma \theta^{\gamma - 1}} \left[\frac{u_1 u_2 e^{u_1 \tau} - u_1 u_2 e^{u_2 \tau}}{u_1 e^{u_2 \tau} - u_2 e^{u_1 \tau}} \right], \tag{4-40}$$

$$B(\tau,\omega)=\frac{\theta}{\gamma}(1-\gamma)[A(\tau)]+\left(i\omega\mu_P\tau+\frac{\lambda_u\tau}{1-i\omega\eta_u}+\frac{\lambda_d\tau}{1+i\omega\eta_d}-(\lambda_u+\lambda_d)\tau\right)-$$

$$\frac{\theta}{\gamma}(\omega-\gamma\omega)\left(-\frac{1}{2}\omega-\frac{1}{2}i\right)\tau$$

$$+\left[\left(\kappa\theta+i\omega\sigma\rho\theta^{\frac{\gamma+1}{2}}\left(\frac{1-\gamma}{2}\right)\right)-\frac{\theta}{\gamma}(1-\gamma)\left(\frac{1}{\sqrt{\theta}}\sqrt{\theta^{\gamma}}\left(\frac{1}{2}i\rho\sigma\omega+\frac{1}{2}i\rho\sigma\gamma\omega\right)-\kappa\right)\right]$$

$$\frac{2}{\sigma^2\gamma\theta^{\gamma-1}}\ln\left[\frac{u_2-u_1}{u_2e^{u_1\tau}-u_1e^{u_2\tau}}\right] \tag{4-41}$$

Where

$$u_1=i\omega\sigma\rho\left(\frac{1+\gamma}{2}\right)\theta^{\frac{\gamma-1}{2}}-\kappa+\sqrt{\left(i\omega\sigma\rho\left(\frac{1+\gamma}{2}\right)\theta^{\frac{\gamma-1}{2}}-\kappa\right)^2-i\omega\sigma^2\gamma\theta^{\gamma-1}(i\omega-1)}$$

$$u_2=i\omega\sigma\rho\left(\frac{1+\gamma}{2}\right)\theta^{\frac{\gamma-1}{2}}-\kappa-\sqrt{\left(i\omega\sigma\rho\left(\frac{1+\gamma}{2}\right)\theta^{\frac{\gamma-1}{2}}-\kappa\right)^2-i\omega\sigma^2\gamma\theta^{\gamma-1}(i\omega-1)}$$

4.3.2.2 Deriving the ccf for the Special Cases Processes

Depending on the restrictions we impose on the parameters in the process, we can derive the characteristic function for the special processes. All we need to do, is to adjust the values for a_1, a_2, b_1, b_2, c_1, and c_2 taking into account the restrictions on the parameters estimate. Restrictions are shown in table 4-2.

As mentioned above, cdfs for such processes are not known in a closed form, but the characteristic function (cf) are known. The cf has 1 to 1 correspondence with the cdf,

$$\Pr(P\le\alpha)=\frac{1}{2}-\frac{1}{2\pi}\int_0^{\infty}\mathrm{Re}\left[\frac{\phi(\omega,\tau)e^{i\omega\alpha}}{i\omega}\right]d\omega$$

and the Fourier inversion of the cf gives the cdf, as indicated in section 4.2 above according to equation

Where Pr(Δp≤α) is the probability of the portfolio value to be less than or equal α where α any quantile in the distribution. So by solving the integral in (4-5) we can get a value for α at any confidence level and then substitute it in (4-14) to get VaR under thick tails distributions, represented by the above processes. Unfortunately, the integral in the (real part) above does not have closed form solution, and the only way is to solve it numerically. Solving the integral above numerically is not a trivial step as Schobel and Zhu (1999). Most of papers that use this method did not report how the solve this integral. Using this method of obtaining the cdf is the most efficient method in terms of accuracy and time compared with the Monte Carlo simulations and finite difference methods, Scott (1997). As mention earlier, a great credit for inventing methods to solve this integral goes to Bohman work listed earlier especially Bohman (1975).

4.4 Estimating Volatility of Trading Revenues

As mentioned earlier, estimation of current volatility is a crucial issue in estimating VaR. As explained in section 2.5.2 of chapter 2, VaR literature use different models for estimating current volatility including constant volatility models, exponential weighting volatility model, ARCH, GARCH, EGARCH models and implied volatility from option pricing. VaR models that use stochastic volatility as a risk factor including Cardenas, Fruchard, Koehler and Michel (1997) and El-Jahel, Perraudin, and Sellin (1999) pre-assumed the volatility process parameters rather than estimate them.

We can use the characteristic function in estimating the diffusion processes above to get the current volatility and all continuous time processes. Recently, authors including Yu (1999), Singleton (2001), Jiang and Knight (2002), Knight and Yu (2002), Knight, Satchel and Yu (2002), and Chacko and Viceira (2003) start to use the characteristic function for estimation continuous time processes because it is known in closed form for many continuous time processes. A great advantage of those methods depend on the characteristic function that they do not need to discretize the data. Chacko and Viceira invented a GMM estimator (they called it Spectral GMM) in which they integrate the unobserved variable (volatility in this case) in estimating the continuous time processes.

In our case, we already derived the characteristic functions which is the most challenging part in the estimation processes. With the existence of volatility and non zero correlation between the portfolio trading revenues and the portfolio revenue the portfolio revenue is Markovian. Thus the conditional characteristic function that we calculated is conditional only on the previous portfolio value. Accordingly, the characteristic function do not condition on the entire path of portfolio value, but on the portfolio value in the previous period.

For Chacko and Viceira's (2003) estimation method, volatility is latent variable and it is unobservable. Thus, to implement their GMM method, we have to integrate volatility out. In this case, we still condition on the previous portfolio level, but the portfolio trading revenues is no longer Markovian. It is only Markovian if it is associated with the volatility process through the correlation between volatility and returns on the portfolio. When we integrate volatility out, the previous portfolio level will not reflect the entire path of the portfolio level, because we lost the knowledge of volatility. With this necessary modification, our estimation loses some efficiency but the estimation timing improves tremendously. For more details see Chacko and Viceira (2003).

CHAPTER FIVE: APPLICATION OF VaR MODELS UNDER THICK TAILS

This chapter applies VaR models under the six different stochastic processes derived in chapter four on the data sample discussed in chapter three. In each section, we compare VaR estimate based on each process with unexpected trading profits and losses, and disclosed banks' VaR estimate. Additionally, we compare VaR model derived based on different processes to explore the effect of jumps, stochastic volatility and non-affine stochastic volatility on VaR estimate. This comparison considered is as a form of back testing, where the two back testing methodologies explained in chapter three sections 3.4.1 and 3.4.2 are used.

5.1 The GBM VaR Model (Log-Normal Case)

As indicated in chapter four earlier, VaR based on the GBM process takes the form shown in equation 4-12.

$$VaR_\tau = P_t\left[1 - e^{(\mu - \frac{1}{2}v)\tau + \alpha\sigma_0\sqrt{\tau}}\right]$$

Where α represents the normal deviate. At a 99% confidence level, α = -2.326. The parameters μ and σ_0 are estimated based on the method discussed in chapter four using the actual banks' sample unexpected trading revenues. Appendix A-1 shows parameter estimates for the GBM process for the sample's banks unexpected trading revenues. With knowing the parameters μ and σ_0 and the size of the trading portfolio we can calculate

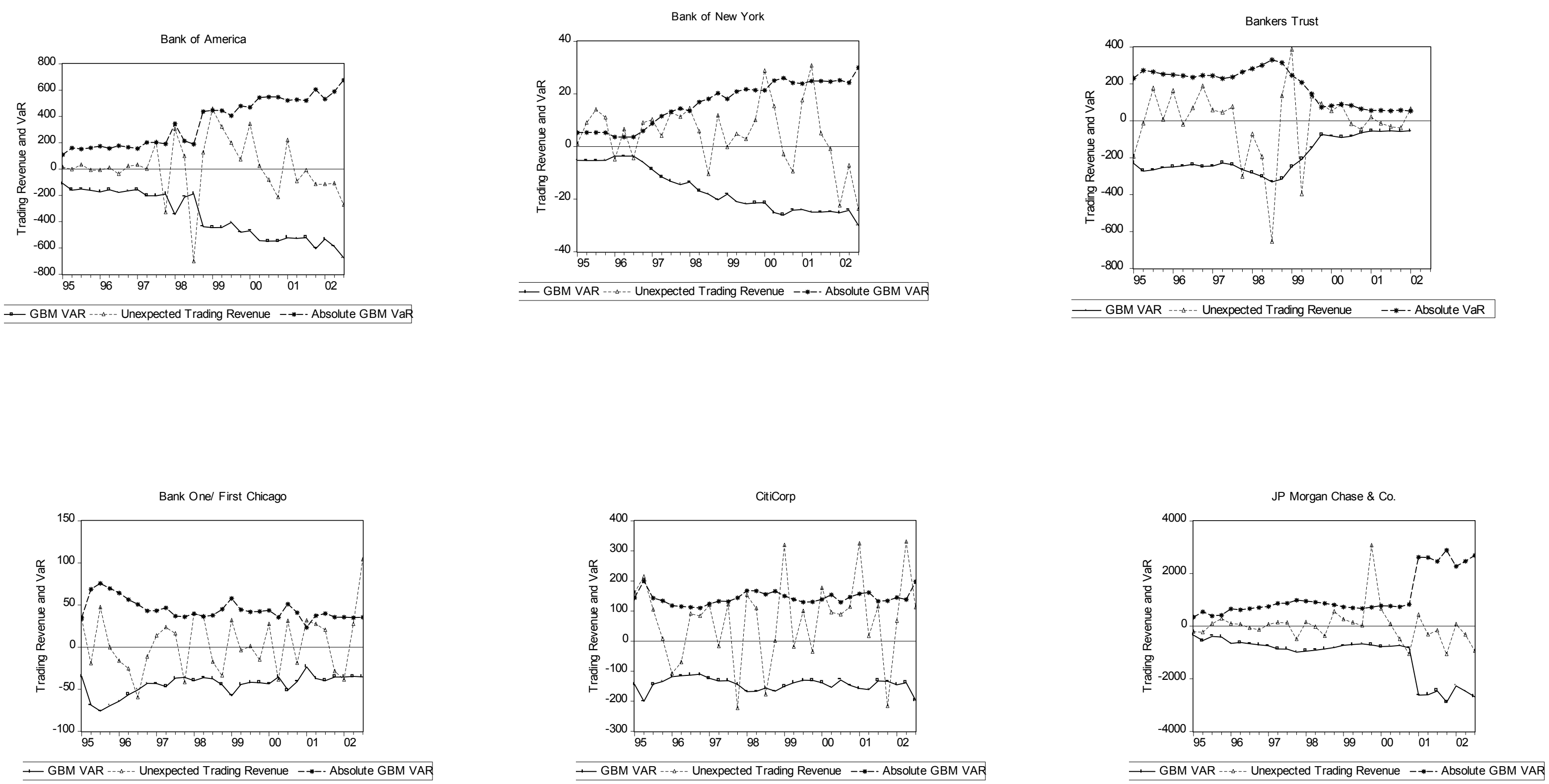
GBM VaR. Figure 5-1 gives GBM VaR estimate compared with unexpected trading revenues for the banks in the sample during the period 1995: Q1-2002: Q3.

5.1.1 Forecast Evaluation Back Testing

This section uses the back testing methodology described in section 3.4.1 to evaluate the performance of GBM VaR and compare it with the performance of the disclosed banks' VaR.

Figure 5-1 is similar to figure 3-4, in that it shows the number of violations under GBM VaR. Violations are the numbers of times that actual trading losses exceed GBM VaR. As mentioned earlier in chapter three, a 99% confidence level quarterly VaR means that a quarter losses can exceed VaR estimate once every 25 years. For our 31 quarters sample, a 99% confidence level quarterly VaR should be exceeded only 0.31 times during the sample period. In our sample period, figure 5-1 shows that all banks in the sample witness at least one violation during the sample period. If we compare this result with banks VaR estimate in figure 3-4, we find that violations under GBM VaR occur more frequent across the banks in the sample. For banks' VaR there are violations only in four banks in the sample as figure 3-4 indicates. However, the size and the number of violation under GBM VaR are different. Figure 5-1 shows that violations for GBM VaR in four banks of the sample occur also during the last quarter of 1997 and the third quarter of 1998, the periods of the Asian crisis, Russian debt crisis and the near collapse of the LTCM. Except for JP Morgan Chase. GBM VaR could not bind trading losses accurately. The violation occurs after the crisis periods of 1997 and 1998. Table 5-1 gives a detailed look at the number and sizes of those violations in terms of dollar value and standard deviations.

Figure (5-1)
Unexpected Trading Revenues and GBM VaR For the Banks in the Sample During the Period 1995:Q1-2002: Q3.



*For Bankers Trust the analysis is ended by 2003:Q1.

Table (5-1)
GBM VaR Violation Statistics

Bank	Mean VaR $Millions (test-stat)	Mean VaR – Minus 99th Percentile(1) (Test-stats)(2)	Mean VaR(3) In Stds	No. of Violations	Mean(4) Size of Violations $ Millions (test-stat)(5)	Mean Size of Violations in Stds(6)	Max Violation $Millions	Max Violation Std.
Bank of America	376.31	-136.32*** (-4.09)	0.67	2	325.39*** (3.46)	1.48	513.2	2.329
Bank of New York	16.22	-12.11*** (-8.43)	1.39	2	1.25*** (10.29)	0.09***	1.35	0.11
Bankers Trust	183.60	-273.39*** (-15.45)	0.96	3	189.73*** (6.86)	0.94	331.04	1.65
Bank One / First Chicago	44.94	-35.38*** (-16.581)	1.29	4	5.50*** (10.97)	0.16***	8.95	0.26
CitiCorp	144.62	-177.93*** (-44.85)	1.04	3	61.27*** (4.66)	0.44	82.29	0.59
JP Morgan Chase & Co.	1220.83	-408.49** (-2.67)	1.70	1	234.09	0.33	234.09	0.33

(1) Calculated as the difference between the mean VaR in millions and the 99th percentile. The 99th percentile is multiplication of the standard deviation of unexpected trading revenues and the 99th percentile of normal deviate, the numbers are provided in table 3-3. This is by definition -2.236 standard deviation below the mean.
(2) H0: the absolute Mean of GBM VaR equals the absolute 99th percentile.
(3) Normalized by the standard deviation of unexpected trading revenues.
(4) The amount of loss exceeding VaR, which is equal to average loss – VaR.
(5) H0: Mean violations = 0
(6) H0: Mean violations in terms of standard deviation is different than the expected value conditional on exceeding the 99th percentile (0.341 standard deviations) for normal distribution. H0: |mean violation of GBM| - 0.341= 0.
(7) *** significant at 1% , ** significant at 5% and * is significant at 10%.

The first column in table 5-1 shows the difference between the mean of GBM VaR and the 99th percentile losses. As indicated earlier, the 99th percentile loss represents 2.326 standard deviations below the mean assuming normal distribution. For the six banks in the sample, the average GBM VaR significantly lies outside the 99th percentile loss (absolute value of GBM VaR is significantly less than the absolute value of the 99th percentile loss). The numbers in column 3 show this fact in a clear way. It tabulates the mean GBM VaR standarized with unexpected trading revenues standard deviation. A bank with a standarized average GBM VaR less in absolute value than 2.326 means that the mean of GBM VaR lies outside the 99th percentile and we expect to see violations in this bank. Unlike table 3-3, in table 5-1, we see that both Bank of New York and Bank One/ First Chicago are now falling outside the 99th percentile loss, which means that we expect to see violations in those two banks also.

Column 4 reports the number of violations (Losses went beyond GBM VaR) in each bank of the sample. The number of violations is relatively big. If we compare it with the 1% percent target, we conclude that GBM VaR estimates are relatively conservative. GBM VaR is exceeded with more than 6 % for two banks and with more than 9% in other three banks.

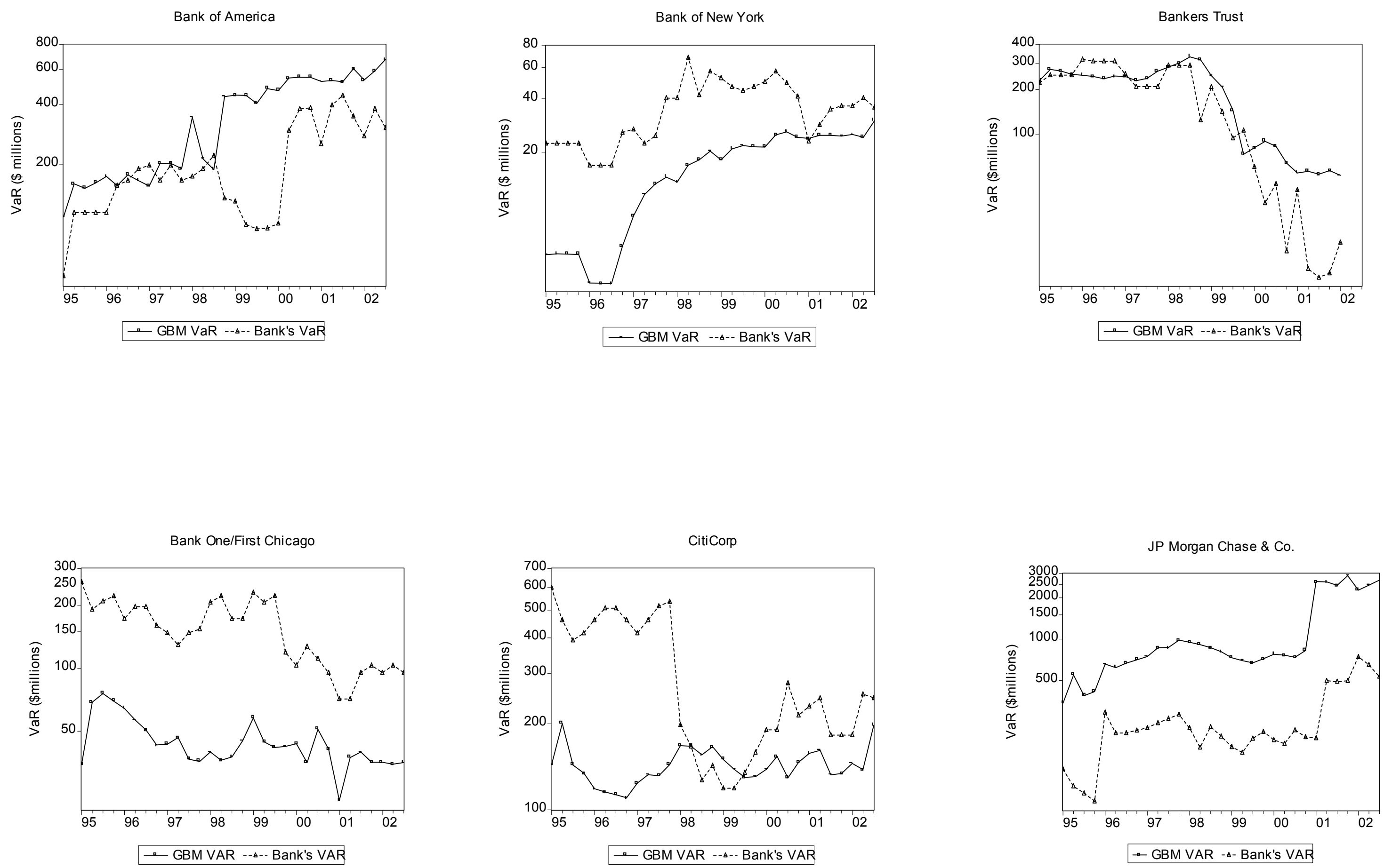
The last four columns in table 5-1 show the mean size of violations and the maximum violation in each bank. Violation size represents the difference between the actual loss and GBM VaR estimate. In the six banks, we see that the mean of violations is significantly different from zero. The largest violation in terms of mean (in dollar value and measured in standard deviation) happened in Bank of America. The mean violation in Bank of America is 1.48 standard deviation, however, it does not differ from zero significantly. The mean violations in 3 banks of the six are bigger than the expected value conditional on exceeding the 99th percentile (0.341 standard deviations) for normal distribution, but the test statistics shows that those mean violations do not differ significantly from 0.341 in those 3 banks. For other two banks (Bank of New York and Bank One) the mean violation is significantly less than 0.341 standard deviation. The case is similar when we look at the last column, which shows the maximum violations of GBM VaR for banks' sample.

Figure 5-2 and table 5-2, on the other hand, compare the disclosed banks' VaR and the GBM VaR estimate. Figure 5-2 shows that the absolute value of GBM VaR is lower than the absolute banks' VaR in 3 cases, (Bank of New York, Bank One and CitiCorp), and higher in one case (JP Morgan Chase and Co.). Generally, the GBM VaR and disclosed banks' VaR takes the same trend in all cases. However, banks' VaR estimates are more volatile than GBM VaR for the reason that GBM VaR assumes constant volatility.

In one case (JP Morgan Chase), GBM VaR is higher than the Bank's VaR for all portfolio positions, and during the whole sample period. For Bank One, GBM VaR is less than the Bank's VaR during the entire sample period. For Bank of New York, GBM VaR is less than the Bank's VaR for most of the quarters. Actually, the disclosed VaR of Bank of New York equals GBM VaR in the third quarter of year 2000. For Bank of America, GBM VaR and the Bank's VaR are started to be close until the first quarter of 1998, when the GBM VaR jumped and the Bank's VaR plunged down. GBM VaR for Bank of

Figure (5-2)

The Absolute Value of The GBM VaR versus The Absolute Value of The Disclosed Banks' VaR (Logarithmic Scaling)



America increased suddenly because of the increase in the trading portfolio for the Bank from $37 billions in June 1998 to $84.5 billions in September 1998. However, the Bank's VaR witnesses a similar jump in December 1999. As we see in figure 2, the two VaR measures for Bankers Trust take the same path during the whole period. They start the downward sloping at September 1998. CitiCorp VaR starts very high compared with GBM VaR measure then it drops down towards GBM VaR, but it continues to be higher after the first quarter of 1999.

Table (5-2)
GBM VaR versus Banks' Disclosed VaR
($Millions)

Bank	**Mean VaR Difference**(1) (test-stats)(2)	**No. of Violations GBM VaR**	**No. of Violations Banks' VaR**	**Mean**(3) **Size of Violations Difference** (test-stat)(4)	**Max Violation Difference**
Bank of America	157.72*** (6.29)	2	2	4.5 (0.03)	33.44
Bank of New York	-20.11*** (-10.28)	2	0	1.25*** (10.23)	1.49
Bankers Trust	16.26* (1.37)	3	7	76.13 (1.11)	-33.71
Bank One / First Chicago	- 105.61*** (-12.99)	4	0	5.50*** (10.97)	8.95
CitiCorp	-143.48*** (-5.28)	3	2	18.92* (1.64)	31.29
JP Morgan Chase & Co.	941.24*** (7.32)	1	8	-107.97*** (-3.82)	-630.74

(1) Calculated as the difference between the absolute value of GBM VaR and the absolute value of Bank's Disclosed VaR.
(2) H0: The difference between mean VaRs equals zero.
(3) The difference between the absolute value of mean sizes of violation of GBM VaR and the absolute value of mean sizes of bank's VaR. Positive sign means that GBM VaR violations are bigger than the Banks' VaR violations.
(4) H0: The difference of the mean violations equal to zero.
(5) *** significant at 1% , ** significant at 5% and * is significant at 10%

Table 5-2 analyzes closely the differences between the GBM VaR estimates and the disclosed banks' VaR. The first column of the table shows the differences between the absolute mean of GBM VaR and the absolute value of the banks' VaR. Positive number means that GBM VaR is higher than the bank's VaR, and negative numbers imply the opposite. From table 5-2, we see that GBM VaR is significantly higher in 3 cases (Bank of America, Bankers Trust and JP Morgan Chase and Co.). In the other three cases, the mean GBM VaR appears to be significantly less than the banks' VaR disclosed by the Bank of New York, Bank One and CitiCorp. And that is why we see some violations under GBM VaR estimates in those banks.

Because the GBM VaR is significantly less in three banks, we see that those banks witness higher number of violations under GBM VaR compared with the disclosed banks' VaR. GBM VaR is violated twice for Bank of New York and Bank One, compared with now violations under their own VaR estimates. For CitiCorp, the number of violations increases to 4 under GBM VaR estimates compared with 3 violations under its own VaR measure. Bankers Trust and JP Morgan Chase and Co. that have high number of violations under their own VaR measures, the number of violations decreases significantly under GBM VaR estimates in those banks.

The last two columns in table 5-2 compare the mean violations and the maximum violations respectively between the two VaR measures. The mean violation difference is calculated as the difference between the absolute mean violation under GBM VaR and the absolute mean violation under the banks' disclosed VaR. The positive numbers for the first five banks in the sample means that the absolute mean violation under GBM VaR is higher than the absolute mean violation under the banks' VaR, and the opposite for JP Morgan Chase. Statistically speaking, the means of violations under GBM VaR are higher than the means of violations under the banks' VaR for only 3 banks. For the other two banks the means of violations are statistically insignificant. JP Morgan own VaR estimates seemed to be very conservative, the high number of violations and the sizes of violations are extremely higher than the GBM VaR calculations.

The last column shows the difference between the maximum violations under GBM VaR estimates and the disclosed banks' VaR estimate. The results are almost the same as the means of violations comparison except for Bankers Trust, where the maximum violation under the Bank's VaR is higher than the maximum violation under the GBM VaR.

From the analysis above, we see that GBM VaR estimates produce less desired results compared with disclosed banks' VaR. It produces higher number as well as higher sizes of violations in most of the cases. In some cases GBM VaR produces more favorable results compared with some banks results, but that is can not be generated.

5.1.2 Volatility Method

The first step in this method is to retrieve trading revenue's volatility from GBM VaR estimates according to equation 3-2, and then apply the regression formula in equation 3-6 that describes the absolute value of unexpected trading revenues as a linear function of volatility.

$$\left| R_{t+1} - E\left[R_{t+1} \right] \right| = a + b\sigma_t + \varepsilon_{t+1}$$

As mentioned earlier, GBM VaR reflects the future trading revenues risk if the parameter b is significantly positive. For the purpose of back testing of GBM VaR model, equation 3-6 is first estimated for each bank individually using a univariate

Ordinary Least Square (OLS) and Seemingly Unrelated Regression (SUR). The SUR method is used to account for the correlation among of unexpected trading revenues among banks as reported in table 3-4.

Table (5-3)
Bank-Specific Regression of Absolute Value of Quarter t+1 Unexpected Trading Revenues on Quarter t GBM VaR-Based Volatility

$$\left[R_{i,t+1} - E\left[R_{i,t+1}\right]\right] = a_i + b_i \sigma_{i,t} + \varepsilon_{i,t+1}$$

Bank	**Period No. of Observations**	**OLS**			**SUR**	
		Constant	**Slope**	**R^2**	**Constant**	**Slope**
		(t-Statistic)			(t-statistic)	
Bank of America	95,Q1-02,Q3 31	90.136 (1.381)	0.188 (0.985)	0.032	101.072 (1.844)	0.291 (1.844)
Bank of New York	95,Q1-02,Q3 31	5.5465 (1.864)	0.687* (1.831)	0.104	5.673 (2.075)	0.672501 (1.984**)
Bankers Trust	95,Q1-02,Q1 29	-8.935 (-0.164)	0.843*** (2.843)	0.230	36.980 (0.875)	1.108*** (2. 684)
Bank One / First Chicago	95,Q1-02,Q3 31	40.581 (2.979)	-0.629 (-0.918)	0.028	39.538 (3.088)	-0.573 (-0.891)
CitiCorp	95,Q1-02,Q3 31	5.057 (0.046)	1.949 (1.105)	0.201	27.385 (0.282)	1.582 (1.020)
JP Morgan Chase & Co.	95,Q1-02,Q3 31	295.372 (1.619)	0.199 (0.653)	0.015	260.942 (1.489)	0.2679 (0.916)
Joint test of Slope Parameters =0						16.5 P= 0.011

- * **Significant at 1%.
- ** Significant at 5%.
- * Significant at 10%.

Table 5-3 reports the results of the six banks specific time-series regressions of unexpected trading revenues of GBM VaR based volatility. For OLS results, Bank of New York and Bankers Trust show the assumed positive relationship between unexpected trading revenues and VaR based volatility. Those results improve over the result in table 3-5 which shows a weak significant positive value of b only for Bankers trust. The OLS results in table 5-3 strongly suggest that b takes a positive value in case of Bankers Trust, and Bank of New York.

Those significant positive values for b improve by using SUR regression for both banks, and b coefficient becomes statistically significant at 1% level. Additionally, the joint-test of all slopes in SUR model is significantly different from zero. And this is additional improvement over the results in table 3-5 where the joint test for slopes is insignificantly different from zero.

Figure 5-3 displays the pool sample data. The figure shows that higher VaR based volatility is associated with greater variation in unexpected trading revenues for the cross sectional and time series data. Accordingly, pool estimation for equation 3-6 is performed to back test on the whole data set.

Figure (5-3)
Absolute Unexpected Trading Revenues in Quarter t+1 and GBM VaR-Based Volatility in Quarter t: Pool Sample

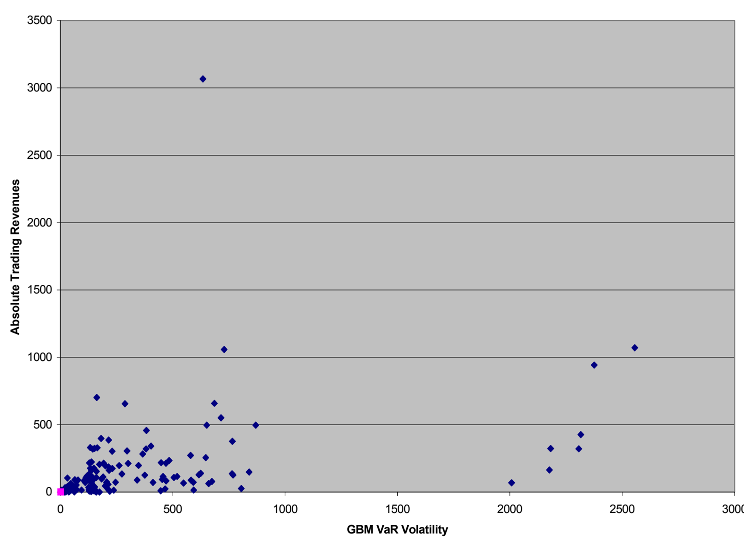


Table 5-4 reports the results of a cross-sectional estimation of equation 3-6, with a sample of 184 observations. The pool sample OLS results (with and without intercept) report significant relationship of coefficient of VaR based volatility. To correct for heteroskedasticity and cross-sectional correlation in the pool data across section weights Generalized Least Squares (GLS) method is applied. The results are reported in raw 3 in table 5-4, which still shows significant positive results for the coefficient b. Unlike the case for the disclosed bank's VaR, the fixed effect and random effect pool regression are both showing significant positive b coefficient for the GBM VaR based volatility.

Table (5-4)
Pooled Regression of Absolute Value of Quarter t+1 Unexpected Trading Revenues on Quarter t GBM VaR-Based Volatility

$$\left[R_{i,t+1} - E\left[R_{i,t+1}\right]\right] = a_i + b_i\sigma_{i,t} + \varepsilon_{i,t+1}$$

Pooled Method	**Period**	**Constant**	**Slope**
	Observations	(t-Statistic)	
Pooled Sample OLS without Intercept	95,Q1-02,Q3 186	NA	0.338*** (3.088)
Pooled Sample OLS	95,Q1-02,Q3 186	3.088 (2.496)	0.297*** (2.720)
Pooled GLS Cross weighting	95,Q1-02,Q3 186	52.592 (2.076)	0.498*** (5.477)
Pooled SUR	95,Q1-02,Q1 186	21.049 (2.043)	0.296*** (121.614)
Fixed Effect Pool OLS	95,Q1-02,Q3 186	AUR	0.308*** (2.764)
Fixed Effect Pool GLS	95,Q1-02,Q3 186	AUR	0.519*** (5.642)
Fixed Effect Pool SUR	95,Q1-02,Q3 186	AUR	0.308*** (235.600)
GLS Random Effect Pool	95,Q1-02,Q3 186	71.366 (6.256)	0.242** (2.276)

- ** Significant at 5%.
- *** Significant at 1%.
- AUR: The fixed effect and random effect intercepts are not reported here, but it is available upon request (AUR).

In summary, volatility back testing shows that GBM VaR gives better result in compared with the disclosed banks' VaR, since it gives more significant results for the related coefficient. The pool regression results indicate that GBM VaR gives some informativeness about future risk, but the specific OLS and SUR regressions could not provide the same power of anticipating future risk in each single bank.

5.2 VaR Models Under Thick Tails

As derived in chapter four earlier, VaR based on processes allow for thick tails takes the form

$$VaR_{\tau} = P_t\left[1 - e^{\alpha}\right]$$

Where α represents the distribution deviate at 99% confidence level. We obtain α parameter by numerically inverted the characteristic function of each process using equation 4-5. Thus, the processes' parameters are estimated and then substituted in the inversion formula of the characteristic function (equation 4-5). As explained in chapter 4, the estimation method proposed by Chacko and Viceira (2003) for estimating continuous time processes is used to estimate parameters. The parameters are estimated based on actual banks' data of unexpected trading revenues.

Tables A-1 to A-6 in appendix show parameters' estimate for those processes. One important notice on the parameter estimate is that the correlation coefficient between the stochastic volatility process and the trading revenue process ρ is significant in some cases, which imply skewness and thick tails distribution. On the other hand, the inclusion of jump diffusion with the stochastic volatility could in some cases capture part of the skewness and kurtosis, through reducing the estimated value of the correlation coefficient

ρ., Tables A-4 and A-6 show additionally that for certain banks in the sample, both the stochastic volatility parameters and the jump diffusion parameters are significant.

Another important feature we deduce from the estimate that in certain cases, the estimate for the updating volatility parameter, γ, seems to be significantly exceeding 1, a required condition for non-affine stochastic volatility. Which means that volatility could be updating faster than what the square root process assumes.

After obtaining α, equation 4-5 is applied for specific portfolio sizes. For each bank in the sample, within every single process, α has been calculated 31 times (except for Bankers Trust, where it has been calculated 29 times). Across the five thick tail processes, α has been calculated 155 times for each bank, and almost 775 times for all banks across all processes. In a clear way, α (and hence a VaR estimate) have been calculated for the six banks in the sample across the five thick tail processes for the period 1995:Q1 – 2002:Q3. The total VaR estimates that have been calculated for the thick tail process are 775 times in addition to 184 GBM VaR. VaR numbers in the following analysis are presented through graphs and averages rather than tables because of the large number entries.

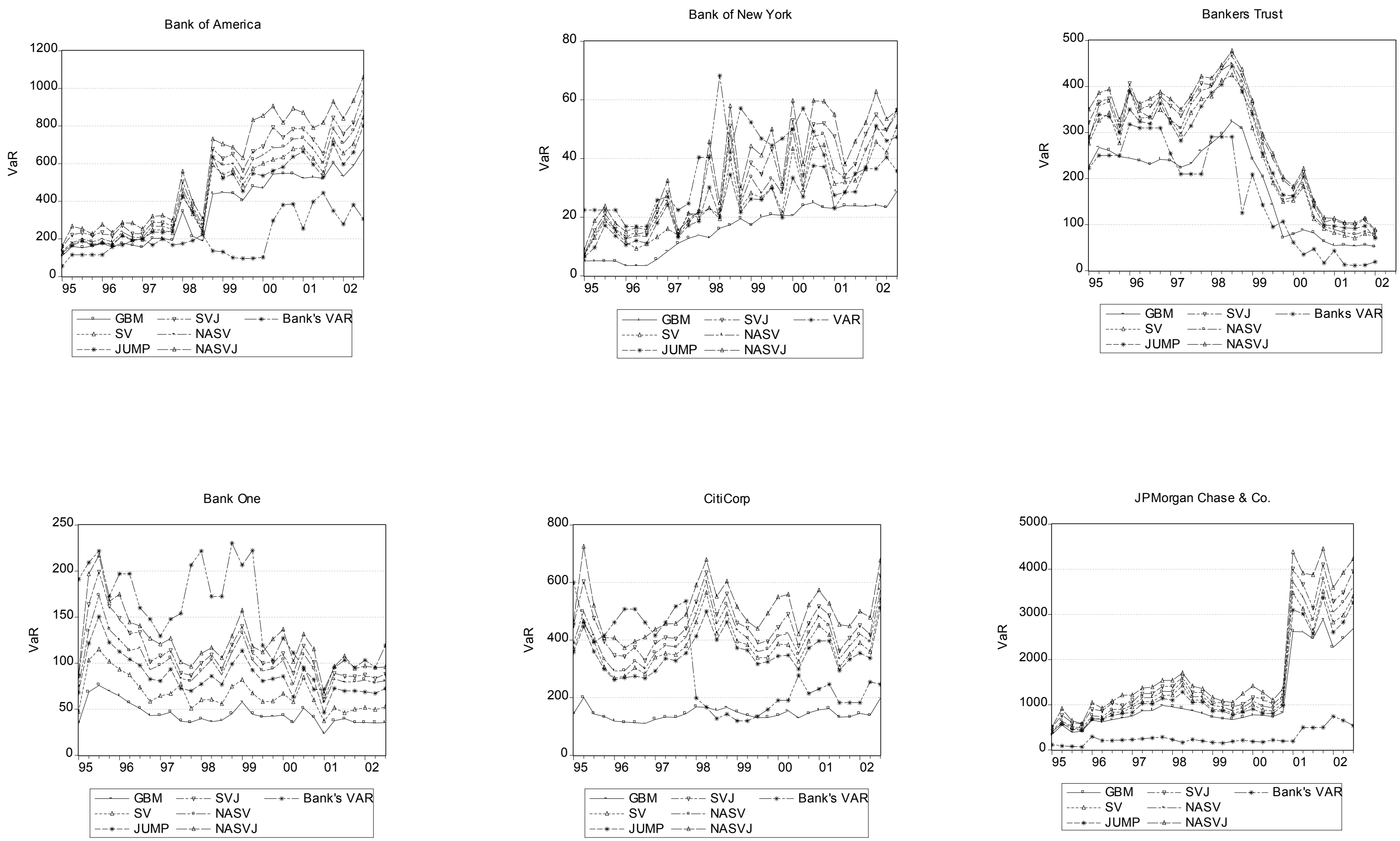
Parameters' estimate is also used to estimate volatility updating in the five thick tail processes (SV, SVJ, NASV, and NASVJ) that assume stochastic volatility. We need the estimated parameters to obtain the updated volatility level based on equations 4-59 to 4-62 in chapter 4 also α in the cases where volatility is changing. Basically, this might be the reason for seeing that thick tail processes in general produce a more volatile VaR estimate compared with the GBM VaR.

Figure 5-4 plots VaR estimates for all banks, for all processes during the study period in addition to the disclosed banks' VaR. As seen from the figure different processes yield different levels of VaR numbers. Roughly speaking, we can say that the GBM process produces the lowest VaR and the non–affine stochastic volatility with process yields the highest VaR. Among those two high and low VaR estimates, other processes produce mixed results of VaR sizes. Additionally, we can see that all processes provide more volatile VaR estimates compared with the GBM VaR method.

Table 5-5 on the other hand, shows the mean and the standard deviation of the absolute value of VaR estimates derived based on the assumptions of thick tails and faster volatility updates compared with the GBM VaR and disclosed banks' VaR. The first note that we deduce from table 5-5 is that the mean and the standard deviation of all thick tails and updating volatility VaR models exceed the GBM VaR mean and standard deviation, and this is one of the basic assumptions of the analysis. Regarding the proposed models, except for Bankers Trust and Bank One, VaR estimates are coming in the following acceding order, GBM VaR, SV-VaR, JD VaR, NASV VaR, and NASVJ VaR. In the other two cases, the mean of JD VaR exceeds SV-VaR.

Regarding the banks' disclosed VaR, table 5-5 indicates that the disclosed VaR measures of Bank of New York and Bank One have exceeded all the proposed VaR measures (except NASVJ VaR for Bank of New York). The last note from table 5-5 is that the proposed VaR models, started from the GBM VaR to the NASVJ VaR have exceeded with a big difference the disclosed VaR by JP Morgan Chase. Actually, we conclude that JP Morgan Chase's VaR estimates are very conservative since it suffers 8 violations during the sample period.

Figure (5-4)
Derived VaR measures Compared to the Bank's Disclosed VaR, 1995:Q1-2002:3



Volatility of VaR estimates takes almost the same path as the mean with some exceptions. Among the developed models, the NASVJ is the most volatile and the GBM VaR is the least volatile. In between those two extremes, the volatility of the processes (SV-VaR, JD VaR, SVJ VaR and NASV VaR) models, the volatility direction among those processes is less obvious. Banks' VaR seems to be more volatile compared with all other developed VaR methods in three cases, Bank of New York, Bank One and CitiCorp.

5.2.1 The Stochastic Volatility Process-Based VaR (SV-VaR)

The basic assumption for using stochastic volatility is that stochastic volatility is a source of tail thickness in financial data. As mentioned earlier, the parameter that determines tail thickness of the distribution is the correlation coefficient between the stochastic volatility process and the trading revenue process, ρ. From Table A-2, we see that ρ is significant in some cases. This implies that the distribution of trading revenues in those cases exhibit some kind of deviation from the tail thickness of normal distribution. Hence they should result in VaR estimates different from the VaR estimate based on the GBM process.

The estimated parameters in table A-2 are used to estimate α parameter through equation 4–5, and to estimate volatility dynamics for the purpose of computing VaR. We calculated 184 stochastic volatility based VaR (SV-VaR) for the six banks during the period 1995:Q1 – 2002:Q3. Figure 5-5 plots SV-VaR estimates as upper and lower bounds for unexpected trading revenues for the six banks in the sample.

Two issues should be noted from figure 5-5, if it is compared with figure 5-1 and figure 3-4 that pertains to the GBM VaR and disclosed banks' VaR respectively.

Table (5-5)
Means and Variances for Different VaR Estimates

Bank	**Banks'VaR** Mean St. D	**GBM-VaR** Mean St. D	**SV-VaR** Mean St. D	**JD VaR** Mean St. D	**SVJ VaR** Mean St. D	**NASV-VaR** Mean St. D	**NASVJ** Mean St. D
Bank of America	218.58 107.73	376.31 185.58	457.53 228.45	438.22 213.07	536.26 262.49	491.75 244.82	603.75 296.85
Bank of New York	36.33 13.62	16.22 8.00	26.86 12.11	25.84 11.23	33.59 14.67	30.28 13.38	37.52 16.51
Bankers Trust	167.34 114.04	183.60 95.26	254.08 123.37	258.84 115.39	285.78 124.93	267.78 129.04	303.28 121.38
Bank One / First Chicago	150.55 49.91	44.94 11.88	66.90 18.37	86.85 21.04	110.35 28.91	99.83 23.28	123.90 31.91
CitiCorp	288. 10 142.62	144.62 22.09	383.44 74.02	357.32 66.24	452.12 81.75	411.90 77.23	507.48 88.91
JP Morgan Chase & Co.	279.58 174.36	1220.83 852.09	1499.00 1051.43	1416.01 994.58	1765.04 1215.99	1610.44 1124.94	1954.87 1339.00

* An F-test rejects the null hypothesis that the means of all VaR models are equal

The first issue is that the number of violations (losses beyond VaR) reduces significantly based on SV-VaR model. Violations appear only in 3 banks rather than in 4 banks as in the case of the disclosed bank's VaR estimate and in all six banks according to GBM VaR estimate. Additionally, the maximum number of violations is two under SV-VaR, rather a maximum of 8 violations as seen from table 3-4 pertain to bank VaR and 4 violations in all six banks according in GBM VaR estimate.

The other issue that we should note in figure 5-5 (and figure 5-4) compared with figure 5-1is that SV-VaR is more volatile compared to the GBM VaR. This issue can be seen from table 5-5 that tabulates the standard deviation of VaR according to each measure. One reason is that GBM VaR assumes constant volatility whereas, by assumption, the SV-VaR model assumes that volatility is changing over time.

Table 5-6 below shed more lights on the performance of SV-VaR for all six banks in the sample compared with GBM-VaR and banks' VaR. The first column in table 5-6 is identical to that in table 5-1, in that, it gives the difference between the mean of 99% confidence level SV-VaR and the 99th percentile of the banks' unexpected trading revenues distribution. Five out of six banks shows that the SV-VaR mean falls outside the 99th percentile. However, only two of those five banks appear to be significantly falling outside the 99th percentile. The other one case (CitiCorp) shows that the mean of SV-VaR falls significantly inside the 99th percentile.

The second two columns in table 5-6 compare the mean of absolute SV-VaR with the mean of absolute banks' VaR and GBM VaR. The mean of SV-VaR estimate appears to be significantly higher than the GBM VaR estimate in all cases and this is what we expect from using stochastic volatility in VaR. We expect stochastic volatility to produce

Figure (5-5)

Stochastic Volatility VaR Compared With Unexpected Trading Revenues, 1995:1-2002:3

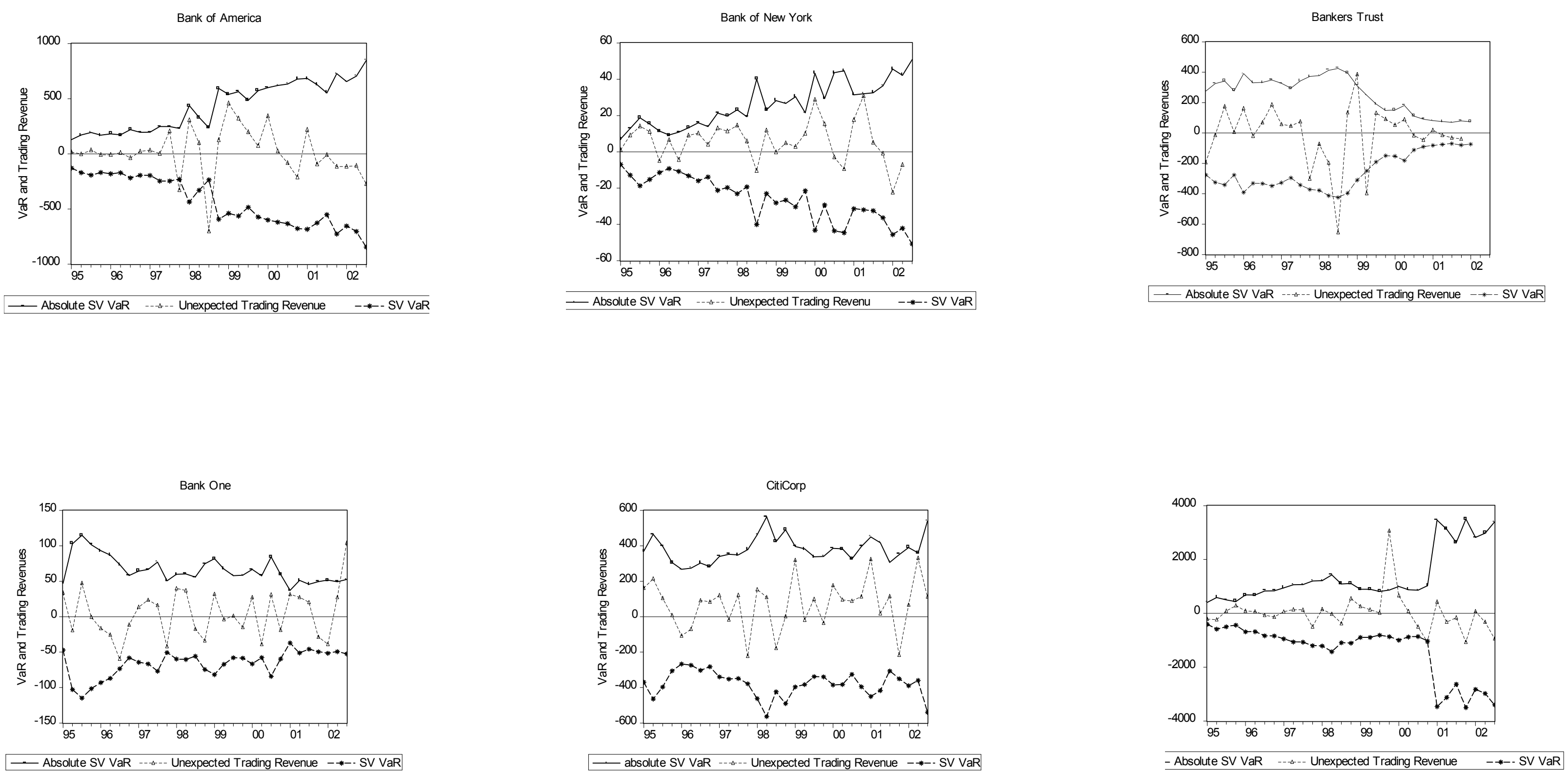


Table (5-6)
Stochastic Volatility VaR Violation Statistics

Bank	Difference from 99 percentile(1) (test-stat)(2)	Mean Difference Compared with Banks' VaR (test stats)(3)	Mean Difference Compared with GBM VaR (test-stats)(4)	No. of Violations	SV-VaR Mean(5) Size of Violations (test-stat)(6)	Max SV-VaR Violation	Mean Violation Compared with GBM VaR(7) (test-stat)(8)	Mean Violation Compared with Bank VaR(9) (test-stat)(10)
Bank of America	-55.1* (-1.34)	238.95*** (7.31)	81.23*** (9.01)	2	82.287*** (3.06)	466.56	-43.10 (-0.22)	-38.61 (-0.23)
Bank of New York	-1.47 (-0.67)	-9.47*** (-3.90)	10.64*** (9.33)	0	0	0	-1.25 NA	0
Bankers Trust	-202.91*** (-8.86)	86.74*** (6.97)	70.48*** (6.99)	2	189.95*** (9.28)	230.89	0.21 (0.01)	76.34 (1.02)
Bank One / First Chicago	-13.42*** (-4.06)	-83.65*** (-10.59)	21.96*** (13.76)	0	0	0	-5.5 (NA)	0
CitiCorp	60.89*** (4.58)	95.34** (2.84)	238.82*** (23.46)	0	0	0	-61.27 NA	-42.35 NA
JP Morgan Chase & Co.	-130.32 (-0.69)	1219.42*** (7.40)	278.18*** (4.81)	1	29.60	29.60	-204.49 NA	-312.46*** (-4.09)

(1) Calculated as the difference between the absolute SV-VaR and absolute value of the 99th percentile. The 99th percentile is reported in table 3-3.
(2) H0: The absolute Mean SV-VaR minus absolute 99th percentile=0.
(3) The difference between the mean absolute SV-VaR and the mean absolute banks' VaR is zero.
(4) The difference between the mean absolute SV-VaR and the mean absolute GBM VaR is zero.
(5) The amount of loss exceeding SV-VaR, which equal to average loss minus SV-VaR.
(6) H0: Mean violation = 0.
(7) Defined as absolute mean violation of SV-VaR minus absolute mean violation GBM VaR.
(8) H0: the difference between the mean violation of SV-VaR and GBM VaR equal to zero.
(9) Defined as absolute mean violation SV-VaR minus absolute mean violations banks' VaR.
(10) H0: the difference between the mean violation of SV-VaR and banks' VaR equal to zero.
(11) *** Significant at 1%, ** significant at 5% and * is significant at 10%.

higher VaR measure because it is one source of tail thickness in financial data. By comparing SV-VaR with banks' own VaR, we see that SV-VaR is significantly higher in 4 cases and significantly less in two cases. The two cases where the banks' VaR is higher are Bank of New York and Bank One. Those banks that have not experienced any violations according to their own VaR estimates, which means that they have originally representative VaR models.

Table 5-6 shows also the number of violations within SV-VaR estimate. Compared to tables 3-3 and 5-1, we see that SV-VaR has performed better compared with the banks' VaR and GBM VaR. The numbers of violations in each bank and maximum number of violations in all banks are decreased also. Under SV-VaR, violations occur only in three banks with a maximum of two violations, compared with violations in 4 banks with a maximum of 8 violations according under the banks' VaR and violations in all banks under GBM VaR with a maximum of 4 violations. With SV-VaR violations happen in Bank of America, Bankers Trust and JP Morgan Chase.

Under SV-VaR model, the number of violations stays the same for Bank of America as in GBM-VaR and bank's VaR estimates. The mean size of violation decreases noticeably under SV-VaR compared with the GBM VaR and the Bank's own calculated VaR. The mean size of violation decreases by $43.11 millions and $38.61 millions for GBM VaR and the Bank's own VaR respectively. Even with this sizable reduction in the mean of violation, those differences seem to be statistically indistinguishable from zero. On the other hand, if we take the differences in the maximum violations, we notice the same reduction, where the maximum violation drops from \$479.76 millions and \$513.2 millions under the Bank's VaR and GBM VaR respectively to \$466.6 millions.

For Bankers Trust, the number of violations decreases from 7 violations and 3 violations according to the Bank's VaR and GBM VaR measures respectively to only 2 violations under SV-VaR measure. According to the mean size of violation, it increases under SV-VaR model rather than decreases as expected. The reason behind this negative difference refers to the high number of violations under the Bank's own VaR and GBM VaR. However, this increase in the mean size of violations under SV-VaR is statistically insignificant, and it is relatively small in dollar size. If we look at the maximum size of violation occurs under SV-VaR compared with the Bank's own VaR and GBM VaR we see a different picture. The maximum violation decreases significantly from \$364.75 millions and \$324.3 under the Bank's VaR and GBM VaR respectively to only \$230.9 million under SV-VaR.

For JP Morgan Chase, the number of violations decreases from 8 violations under the Bank's VaR to 1 violation under GBM VaR, and this violation continues to appear under SV-VaR measure. However, the size of this violation decreases from \$234.1millions according to GBM VaR to \$29.6 millions under the SV-VaR. In case of CitiCorp the number of violations decreases from 2 violations and 3 violations according to CitiCorp's VaR and GBM VaR calculations respectively to no violations at all. Bank of New York and Bank One show no violations under SV-VaR model compared to two violations under the banks' own VaR and GBM VaR calculations.

In conclusion, the inclusion of SV process could improve the VaR performance of compared with banks' VaR and GBM VaR measures. In terms of VaR size, stochastic volatility produces higher VaR in all cases compared with GBM process and it also produces higher VaR measures in 4 cases compared with the bank's VaR. As a result of the higher VaR measure based on stochastic volatility, the number and size of violations are decreased numerically compared with GBM VaR and banks' VaR, however, the results are not so robust statistically.

5.2.2 The Jump Diffusion Process-Based VaR (JD-VaR)

Like in case of stochastic volatility VaR, the basic assumption for using jump diffusion is that jumps are the other source of tail thickness in financial data beside stochastic volatility. From Table A-3, we see that jump sizes and jump intensities (η_u, η_d and λ_u, λ_d) are significant in certain cases. This implies that the distribution of trading revenues in those cases exhibit some kind of tail thickness compared with normal distribution. Hence they should result in VaR estimates higher than VaR estimates based on GBM process.

The estimated parameters in the table A-3 are used to estimate α parameter through equation 4–5. 184 jump diffusion based VaR (JD VaR) are calculated for the six banks during the period 1995:Q1 – 2002:Q3. Figure 5-6 plots JD VaR estimates as an upper and lower bounds for unexpected trading revenues for the six banks in the sample. As in figure 5-5 belongs to SV-VaR, two issues should be noted from figure 5-6, if it is compared with figure 5-1 and figure 3-4 that pertains to the GBM VaR and disclosed banks' VaR respectively.

The first issue, like in case of SV-VaR, the number of violations (losses beyond VaR) reduces significantly based on JD VaR model. Violations appear only in 3 banks rather than in 4 banks as in case of the disclosed bank's VaR estimate and in all the six banks according to GBM VaR estimate. Additionally, the maximum number of violations is only two violations, rather than a maximum of 8 violations as in table 3-4 pertain to banks' VaR and 4 violations in all six banks according to GBM VaR estimate. Those results are identical to the results associated with using the stochastic volatility in VaR estimate, however, the sizes of those violations in each bank are pretty much different.

The other issue that we should note in figure 5-6 (and figure 5-4) compared with figure 5-1is that JD VaR is more volatile compared to the GBM VaR. However, it is less volatile compared with SV-VaR estimates. And this is assured almost by table 5-5, which shows the standard deviation of VaR estimates under different stochastic processes.

Table 5-7 below addresses the differences in the mean between JD VaR estimates, the banks' VaR estimates, the GBM VaR and the SV-VaR. The first column in the table shows the difference between the mean of JD VaR and the 99th percentile of the unexpected trading revenues assuming normality. Negative numbers show that JD VaR falls outside the 99th percentile of unexpected trading revenues. Table 5-7 shows that average of JD VaR estimates for four banks fall outside the 99th percentile. However, two of them (Bank of America and Bankers Trust) appear to be significantly falls outside the 99th percentile. On the other hand, averages of JD VaR estimates for Bank One and JP Morgan are significantly fall within the 99th percentile of unexpected trading revenues.

According to the mean difference between JD VaR estimates and banks' VaR estimates, the absolute mean of JD VaR exceeds significantly the absolute mean of bank's VaR in four cases (Bank of America, Bankers Trust, CitiCorp and JP Morgan

Chase). On the other hand, the absolute mean of banks' VaR exceeds the absolute mean of JD VaR significantly in the other two cases (Bank of New York and Bank One).

Compared with GBM VaR, the effect of inclusion jump diffusion in VaR models is obvious. The absolute mean of JD VaR estimates exceeds significantly the absolute mean of GBM VaR estimates in all cases. For the SV-VaR the case is a little different. The absolute mean of JD VaR estimates appear to be significantly less than the mean of SV-VaR estimate in 4 banks (Bank of America, Bank of New York, CitiCorp and JP Morgan).

For Bankers Trust and Bank One, results are flipped over. The absolute mean of JD VaR estimates appears to be significantly higher only in case of Bank One. In conclusion, we can say that the absolute mean of SV-VaR is higher than the absolute mean JD VaR in 4 cases, equal it in one case and appear to be significantly less in one case. Based on that, we may conclude that the SV produces higher VaR estimate than the jump diffusion. That is may be the case because SV affects VaR through two avenues, the correlation issue (tail thickness) and the changing in volatility.

Table 5-8 below provides the number, size, maximum size of violations and mean size of violations comparison. Banks that do not have violations under JD VaR estimates are excluded from the table. As in case of SV-VaR, the number of violations drops significantly. Only three banks suffer violations under JD VaR rather than 4 as in the case of banks own VaR or in all banks as in case of GBM VaR. The number of violations in each bank under the JD VaR is again less than the number of violations that appears under the banks own VaR and GBM VaR. As a matter of fact, the number of violations

Figure (5-6)
JD VaR and Unexpected Trading Revenues, 1995:1-2002:3

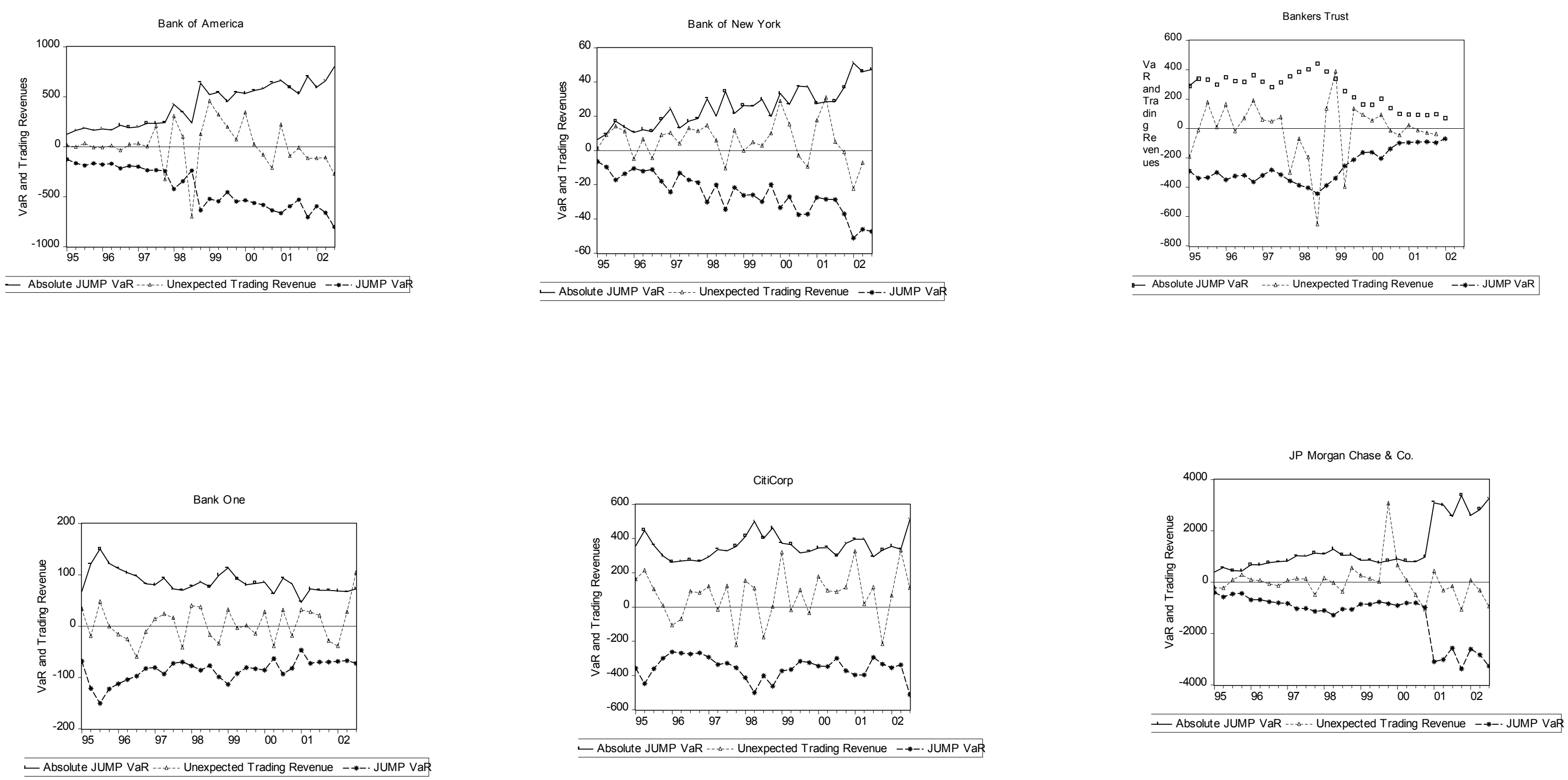


Table (5-7)
JD VaR Absolute Mean Differences Compared with
Banks' VaR, GBM VaR and SV-VaR

Bank	**Difference from 99 percentile**(1) (test-stat)(2)	**JD VaR Mean-Banks' VaR Mean**(3) (test stats)(4)	**JD VaR Mean-GBM VaR Mean**(5) (test stats)(6)	**JD VaR Mean-SV-VaR VaR Mean**(7) (test stats)(8)
Bank of America	-74.41** (-1.94)	219.64*** (7.20)	61.91*** (7.84)	-19.32*** (-5.24)
Bank of New York	-2.49 (-1.23)	-10.49*** (-4.34)	9.62*** (8.44)	-1.03** (-1.84)
Bankers Trust	-198.15*** (-9.25)	91.49*** (7.52)	75.23*** (8.44)	4.75 (0.53)
Bank One / First Chicago	6.53** (1.73)	-63.70*** (-8.39)	41.91*** (21.37)	19.95*** (12.49)
CitiCorp	34.77*** (2.92)	69.22** (2.14)	212.70*** (24.55)	-26.12*** (-5.14)
JP Morgan Chase & Co.	-213.31 (-1.19)	1136.43*** (7.36)	195.19*** (3.90)	-82.99** (-1.76)

(1) The difference between the mean of JD VaR and the normal 99th percentile of unexpected trading revenus.
(2) H0: the absolute Mean JD VaR minus the absolute 99th percentile equals zero.
(3) Absolute mean JD VaR minus absolute mean Bank's VaR.
(4) H0: absolute mean difference equals zero.
(5) Absolute mean JD VaR minus absolute mean GBM VaR.
(6) H0: absolute mean difference equals zero.
(7) Absolute mean JD VaR minus absolute mean SV-VaR.
(8) H0: absolute mean difference equals zero.
(9)*** significant at 1% , ** significant at 5% and * is significant at 10%.

in each bank under JD VaR estimates is identical to the number of violations in each bank under SV-VaR estimates.

For Bank of America, the number of violations appears to stay the same under the four VaR models including the JD VaR. The mean size of violation under the JD VaR decreases in case of Bank of America compared with the Bank's own VaR, the GBM VaR and the SV-VaR. However, this reduction in the mean size of violation is statistically insignificant in the three cases in Bank of America, which means that we can not reject the hypothesis that the mean size of violations under the four VaR models (under comparison) are equal. If we compare the maximum violations under JD VaR estimates compared with the Bank's own VaR, the GBM VaR and SV-VaR respectively in Bank of America, we find that the maximum violation decreases by $15.4 millions, $48.9 millions and $2.21 millions.

As in the case of SV-VaR, the number of violations decreases for Bankers Trust from 7 violations and 3 violations under the Bank's own VaR and GBM VaR to 2 violations under the JD VaR. Compared with Bankers Trust's own VaR, the mean size of violation increases by $63.43 millions under the JD VaR. This increase is attributed to the large number of VaR violations under the Bank's own VaR that contains small violations that reduced the mean of violations under the Bank's own VaR and then disappeared under the JD VaR. Comparing the mean size of violations under JD VaR for Bankers Trust with the GBM VaR and SV-VaR, we find that the mean of violations under the JD VaR is less. However, the statistical test shows that the mean of VaR violations for Bankers Trust does not differ significantly from the mean violations under the Bank's own VaR, the GBM VaR and the SV-VaR.

For JP Morgan Chase and Co. the number of violations decreases form 8 violations under the Bank's VaR to one violation under the GBM VaR, SV-VaR and JD VaR. Compared with the Bank's VaR and the GBM VaR, the mean violation decreases by $246.2 millions and $156.2 millions respectively. In comparison with the SV-VaR, the mean of violations increases by almost $48.3 millions, which means that the SV-VaR performs better than the JD VaR in the case of JP Morgan.

Table (5-8)
JD VaR Violation Statistics Compared with Banks' VaR, GBM VaR and SV-VaR, for The Banks that Have Violations (Means of Violations in Absolute Value)

Bank	**No. of Violations in JD VaR**	**JD VaR Mean(1) Size of Violations** (test-stat)(2)	**Max JD VaR Violation** (Millions)	**Mean Violation Compared with Bank VaR(3)** (test-stat) (4)	**Mean Violation Compared with GBM VaR(5)** (test-stat) (6)	**Mean Violation Compared with SV-VaR(7)** (test-stat) (8)
Bank of America	2	275.59*** (2.92)	464.36	-45.32 (-0.26)	-49.80 (-0.26)	-6.70 (-0.04)
Bankers Trust	2	177.04*** (10.35)	211.25	63.43 (0.81)	-12.70 (-0.14)	-12.91 (-0.34)
JP Morgan Chase & Co.	1	77.86	77.86	-264.20*** (-4.03)	-156.23	48.26

(1) The absolute mean of JD VaR violations.
(2) H0: The absolute Mean SV- VaR =0.
(3) Absolute mean of JD VaR minus absolute mean of Banks' VaR.
(4) H0: The difference between the mean of absolute JD VaR and the mean of absolute banks' VaR is zero.
(5) Absolute mean of JD VaR minus absolute mean of GBM VaR.
(6) H0: The difference between the mean absolute JD-VaR and the mean absolute GBM VaR is zero.
(7) Absolute mean of JD VaR minus absolute value of SV-VaR.
(8) H0: The difference between the mean absolute JD-VaR and the mean absolute SV-VaR is zero
(9) *** significant at 1% , ** significant at 5% and * is significant at 10%.

5.2.3 Stochastic Volatility-Jump Diffusion Based VaR Model (SVJ VaR)

This model combines both stochastic volatility and jump diffusion processes to obtain the maximum tail thickness of the P&L distribution. Accordingly, with this stochastic volatility jump diffusion process we expected to find VaR estimates that exceed SV-VaR estimates and JD VaR estimates. Table A-4 does not show a lot of significant numbers that relate to the stochastic volatility, jump sizes and jump intensities (ρ, η_u, η_d and λ_u, λ_d). This may be because of the short period of the estimation. However, as mentioned earlier (in chapter 2) many of the studies find that both jumps and stochastic volatility are significant in financial data in general.

The estimated parameters in the table A-4 are used to estimate α parameter through equation 4–5. 184 stochastic volatility-jump diffusion based VaR (SVJ VaR) are calculated for the six banks during the period 1995:Q1 – 2002:Q3. Figure 5-7 plots the SVJ VaR estimates as an upper and lower bounds for unexpected trading revenues for the six banks in the sample. As in figures 5-5 and 5-6 belongs to the SV-VaR and JD VaR respectively, some issues can be distinguished from figure 5-7 if it is compared with GBM VaR and disclosed banks' VaR in one hand or the SV-VaR and the JD VaR on the other hand.

The first issue is that number of violations under the SVJ VaR is less than the number of violations under banks' VaR and GBM VaR and this is very obvious. In spite that figure 5-7 looks like figures 5-5 and 5-6, but we can see that the one violation that appears under the SV-VaR and the JD VaR in JP Morgan has disappears under the SVJ VaR. Accordingly, we have two violations in each of Bank of America and Bankers

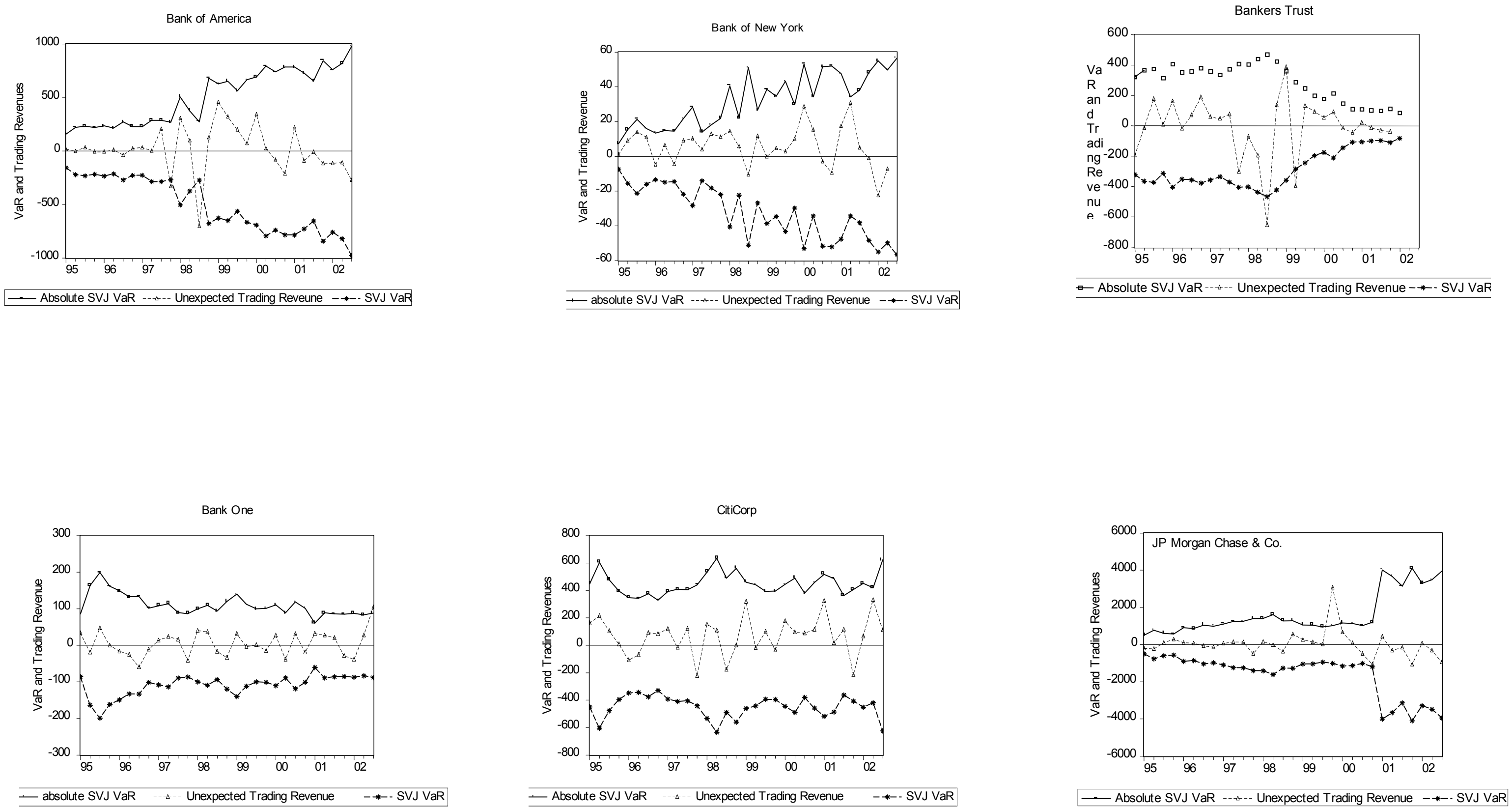
Trust. Even one of the violations in Bank of America is very small that even hard to recognize from the figure. An immediate result that can we deduce from figure 5-7 is that the SVJ VaR improves over both the SV-VaR and the JD VaR, especially in case of JP Morgan Chase.

Table 5-9 below addresses the differences in the average between SVJ VaR estimates in one hand, and the banks' VaR estimates, GBM VaR, SV-VaR and JD VaR estimates. As in table 5-7, the first column in table 5-9 shows the difference between the mean of SVJ estimate and the 99^{th} percentile of unexpected trading revenues assuming normality. As mentioned before, negative numbers show that SVJ VaR falls outside the 99^{th} percentile of unexpected trading revenues. Table 5-9 shows that only in one case (Bankers Trust) the absolute mean of SVJ VaR falls significantly outside the 99^{th} percentile. Of the other five cases, the mean of SVJ VaR models in 3 banks (Bank of New York, Bank One and CitiCorp) falls significantly inside the 99^{th} percentile. The other two cases (Bank of America and JP Morgan Chase) the positive differences between the absolute mean of SVJ VaR estimates for those banks and the 99^{th} percentile of unexpected trading revenues are statistically insignificant.

Regarding the mean difference, table 5-9 shows that the mean of SVJ VaR estimates significantly exceeds the mean of banks' VaR estimates in four cases (Bank of America, Bankers Trust, CitiCorp and JP Morgan Chase). On the other hand, the absolute mean of banks' VaR exceeds the absolute mean of SVJ VaR in the other two cases, only the mean for Bank One's VaR exceeds the SVJ VaR significantly.

The discussions of the SV-VaR and JD VaR shows that both (SV-VaR and JD VaR) improve significantly over the GBM VaR. The previous analysis shows also that

Figure (5-7)
Stochastic Volatility Jump VaR Models and Unexpected Trading Revenues, 1995:1-2002:Q3



both SV-VaR estimates and JD VaR estimates are significantly higher than the GBM VaR. Additionally, the violations in terms of size and number decrease significantly with the inclusion of the stochastic volatility alone or jump diffusion alone into VaR analysis. Accordingly, we expect to see that the SVJ VaR produces higher estimates compared with the GBM VaR and that is clear from the table 5-9, where the absolute mean of SVJ estimates is significantly higher than the mean of GBM VaR estimates in all cases. The important issue here is whether the inclusion of the mixed stochastic volatility jump diffusion into VaR analysis improves over the SV-VaR alone and the JD VaR alone. Table 5-9 shows explicitly that the absolute mean of SVJ VaR estimates is significantly higher than the absolute mean of SV-VaR and JD VaR estimates in the six banks. With such results, we become sure that the SVJ VaR utilizes the maximum tail thickness in the data and produces the highest VaR estimate compared with the SV-VaR and the JD VaR.

In conclusion, we see that the absolute mean of SVJ VaR estimates is undoubtedly higher than the absolute mean of the GBM VaR, SV-VaR, JD VaR and in some case (not all) the banks' own VaR.

Table 5-10 provides the number, the size, the maximum size of violations and the mean size of violations comparison. Under the case of SVJ VaR, the number of violations drops significantly. JP Morgan Chase who experienced 8 violations under the Bank's own VaR appear now to have no violations under the SVJ VaR. Bank of America and Bankers Trust are the only banks who still suffer violations under SVJ VaR. The number of violations does not change it stays two violations, as in the cases of SV-VaR and JD VaR. However, the mean sizes of violation in those two banks are different though.

Table (5-9)
SVJ VaR Absolute Mean Differences Compared with Banks' VaR, GBM VaR, SV-VaR and JD VaR

Bank	**Difference from 99 percentile(1)** (test-stat)(2)	**SVJ VaR Mean-Banks' VaR Mean(3)** (test stats)(4)	**SVJ VaR Mean-GBM VaR Mean(5)** (test stats)(6)	**SVJ VaR Mean-SV-VaR VaR Mean(7)** (test stats)(8)	**SVJ VaR Mean JD-VaR VaR Mean(9)** (test stats)(10)
Bank of America	23.63 (0.50)	317.68*** (8.35)	159.95*** (11.14)	78.72*** (12.15)	98.04*** (9.93)
Bank of New York	5.26** (1.99)	-2.74 (-1.04)	17.37*** (10.77)	6.72*** (10.41)	7.75*** (9.85)
Bankers Trust	-171.21*** (-7.38)	118.43*** (9.29)	102.17*** (10.07)	31.69*** (3.60)	26.94*** (2.90)
Bank One / First Chicago	30.03*** (5.78)	-40.20*** (-5.34)	65.41*** (19.94)	43.45*** (17.18)	23.50*** (11.81)
CitiCorp	129.57*** (8.82)	164.02*** (4.88)	307.50*** (27.16)	68.68*** (11.72)	94.80*** (15.41)
JP Morgan Chase & Co.	135.72 (0.62)	1485.45*** (7.65)	544.21*** (6.73)	266.03*** (4.59)	349.02*** (5.51)

(1) The difference between the mean JD VaR and the 99th percentile of the data assuming normality.
(2) H0: absolute Mean JD VaR minus the absolute 99th percentile equals zero.
(3) Absolute mean SVJ VaR minus absolute mean Bank's VaR.
(4) H0: absolute mean difference equals zero.
(5) Absolute mean SVJ VaR minus absolute mean GBM VaR.
(6) H0: absolute mean difference equals zero.
(7) Absolute mean SVJ VaR minus absolute mean SV-VaR.
(8) H0: absolute mean difference equals zero.
(9) Absolute mean SVJ VaR minus absolute mean JD VaR.
(10) H0: absolute mean difference equals zero.
(11) *** significant at 1% , ** significant at 5% and * is significant at 10%.

Numerically speaking, the mean size of violation under the SVJ VaR is less than the mean size of violation under all comparable VaR models in the table. Only one case in Bankers Trust appears that the SVJ VaR violation is higher than the Bank's own VaR model. Statistically speaking, we could not reject the null hypothesis that the mean size of violations under the SVJ VaR model is the same as the mean sizes of violations under the banks' VaR, GBM VaR, SV-VaR and JD VaR. In JP Morgan Chase where there are no violations registered under the SVJ VaR we could see one case where a test statistic show a significant difference from the Bank's VaR estimate.

If we compare the maximum violations under the SVJ VaR model with the Bank's own VaR, the GBM VaR the SV-VaR and the JD VaR respectively in Bank of America, we find that the maximum violations decrease by \$50.0 millions, \$83.5 millions, \$36.8 millions and \$34.6 millions respectively. For Bankers Trust, we see that the maximum violation under the SVJ VaR decreases by \$178.0 millions, \$144.4 millions, \$44.20 millions and \$24.6 millions compared with the Bank's own VaR, the GBM VaR, the SV-VaR and the JD VaR respectively. Generally speaking, we find that the SVJ VaR improves as expected over the SV-VaR and JD VaR estimates.

Table (5-10)
JD VaR Violation Statistics Compared with Banks' VaR, GBM VaR, SV-VaR and JD VaR, for The Banks that Have Violations (Means of Violations in Absolute Value)

Bank	**No. of Violations in JD VaR**	**SVJ VaR Mean(1) Size of Violations** (test-stat)(2)	**Max SVJ VaR Violation** (Millions)	**Mean Violation Compared with Bank VaR(3)** (test-stat) (4)	**Mean Violation Compared with GBM VaR(5)** (test-stat) (6)	**Mean Violation Compared with SV-VaR(7)** (test-stat) (8)	**Mean Violation Compared with JD-VaR(9)** (test-stat) (10)
Bank of America	2	245.23*** (2.66)	429.74	-75.68 (-0.44)	-80.16 (-0.43)	-37.06 (-0.20)	-30.36 (-0.16)
Bankers Trust	2	149.32*** (7.99)	186.70	35.71 (0.47)	-40.42 (-0.46)	-40.63 (-1.03)	-27.72 (-0.77)
JP Morgan Chase & Co.	0	0	0	-342.06*** (-4.13)	-234.09	-29.60	-77.86

(1) The absolute mean of SVJ VaR violations.
(2) H0: The absolute Mean SVJ VaR =0.
(3) Absolute mean of SVJ VaR minus absolute mean of Banks' VaR.
(4) H0: The difference between the mean absolute SVJ VaR and the mean absolute banks' VaR is zero.
(5) Absolute mean of SVJ VaR minus absolute mean of GBM VaR.
(6) H0: The difference between the mean absolute SVJ VaR and the mean absolute GBM VaR is zero.
(7) Absolute mean of SVJ VaR minus absolute value of SV-VaR.
(8) H0: The difference between the mean absolute SVJ VaR and the mean absolute SV-VaR is zero.
(9) Absolute mean SVJ VaR minus absolute value of JD VaR.
(10) H0: The difference between the mean absolute SVJ VaR and the mean absolute SV-VaR is zero.
(11) *** significant at 1%, ** significant at 5% and * is significant at 10%.

5.2.4 The Non-Affine Stochastic Volatility VaR Models (NASV VaR and NASVJ VaR)

The basic assumption behind including the non-affine stochastic volatility is the assumption that volatility updates faster during crashes and collapses of financial markets. As mentioned earlier, volatility is a crucial factor in constructing VaR estimates. Faster updating of volatility affects positively VaR estimates, Hull and White (1998b). This section deals basically with two VaR models developed in this dissertation; the non-affine stochastic volatility VaR model and the non-affine stochastic volatility jump diffusion VaR model. The later model considered the general model that compromises all other VaR models developed in chapter four as special cases.

As mentioned above, the inclusion of non-affine stochastic volatility would lead to higher VaR estimate because volatility updates according to non-affine specification rather than the square root assumption that usually adopted. The basic parameter that controls this faster updating is γ in equation 4-1. Tables A-5 and A-6 give estimation for γ in equations 4-55 and 4-1 respectively. The basic assumption for non-affine stochastic volatility is that γ should be significantly greater than one. However, actual data did not give significant indication for that.

The estimated parameters in the tables A-5 and A-6 are used to estimate α parameter through equation 4–5 with the characteristic functions of those processes as shown in equations 4-18, 4-37, 4-43, 4-56 and 4-57. By calculating α for the NASV and NASVJ we could calculate 184 non-affine stochastic volatility based VaR (NASV VaR) and 184 non-affine stochastic volatility with jump based VaR (NASVJ VaR) for the six banks during the period 1995:Q1 – 2002:Q3. Figures 5-8 and 5-9 plot the NASV VaR and the NASVJ VaR estimates respectively as an upper and lower bounds for unexpected trading revenues for the six banks in the sample. Actually, the analysis should focus here on how the NASV VaR improves over the SV-VaR and how the NASVJ VaR improves over SVJ VaR.

If figure 5-8 compared with figure 5-5 and figure 5-9 compared with figure 5-7 we should note that VaR estimates in 5-8 and 5-9 are higher and may be more volatile compared with VaR estimates in figure 5-5 and 5-7 respectively. This is because of the faster updating in volatility assumption. If we compare figure 5-8 with figure 5-5 that belongs to the SV-VaR we will find that the only one violation in JP Morgan that appears under the SV-VaR is vanished under the NASVJ. This explicitly means that the non-affine stochastic volatility has improved over the square root (affine) stochastic volatility. The matter is less obvious for figures5-9 and 5-7. But in all cases, VaR estimates in figure 5-9 (NASVJ VaR) should be at least as much as VaR estimates in figure 5-7 (NASV VaR).

As mentioned above the comparison between SVJ VaR and NASVJ VaR is less obvious because they have the same number of violations in each bank. As figures 5-8 and 5-9 show, Bank of America and Bankers Trust are the only banks that still have violations under the NASV VaR and NASVJ VaR. As we noted before those banks still have the same number of violations as under the SVJ VaR which is two violations in each bank. However, as indicated below (table 5-12), even with the same number of violations in those two banks, the mean sizes of violations and the maximum violation sizes are not the same, at least numerically.

Figure (5-8)

Non-Affine Stochastic Volatility with Jump VaR and Unexpected Trading Revenues, 1995:Q1-2002: Q3

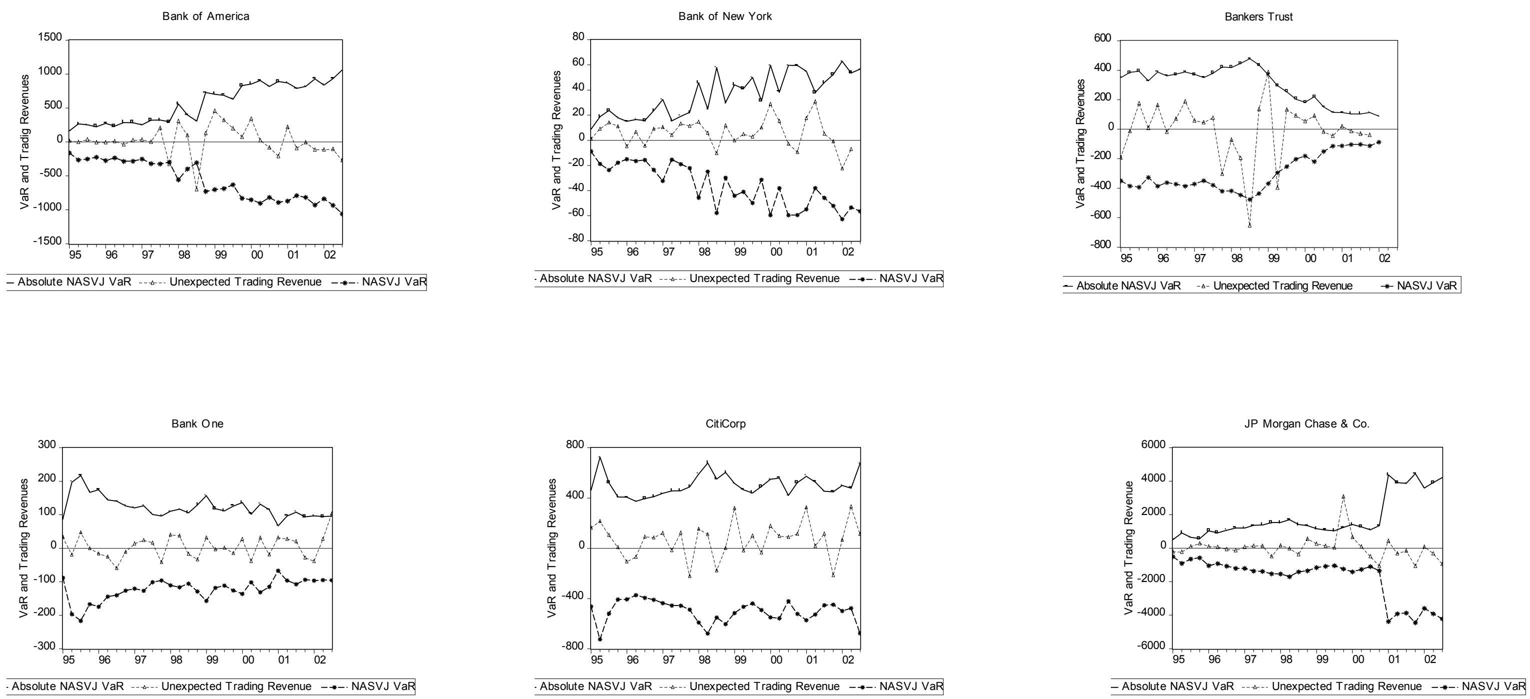


Figure (5-9)
Non-Affine Stochastic Volatility with Jump VaR and Unexpected Trading Revenues, 1995:Q1-2002: Q3

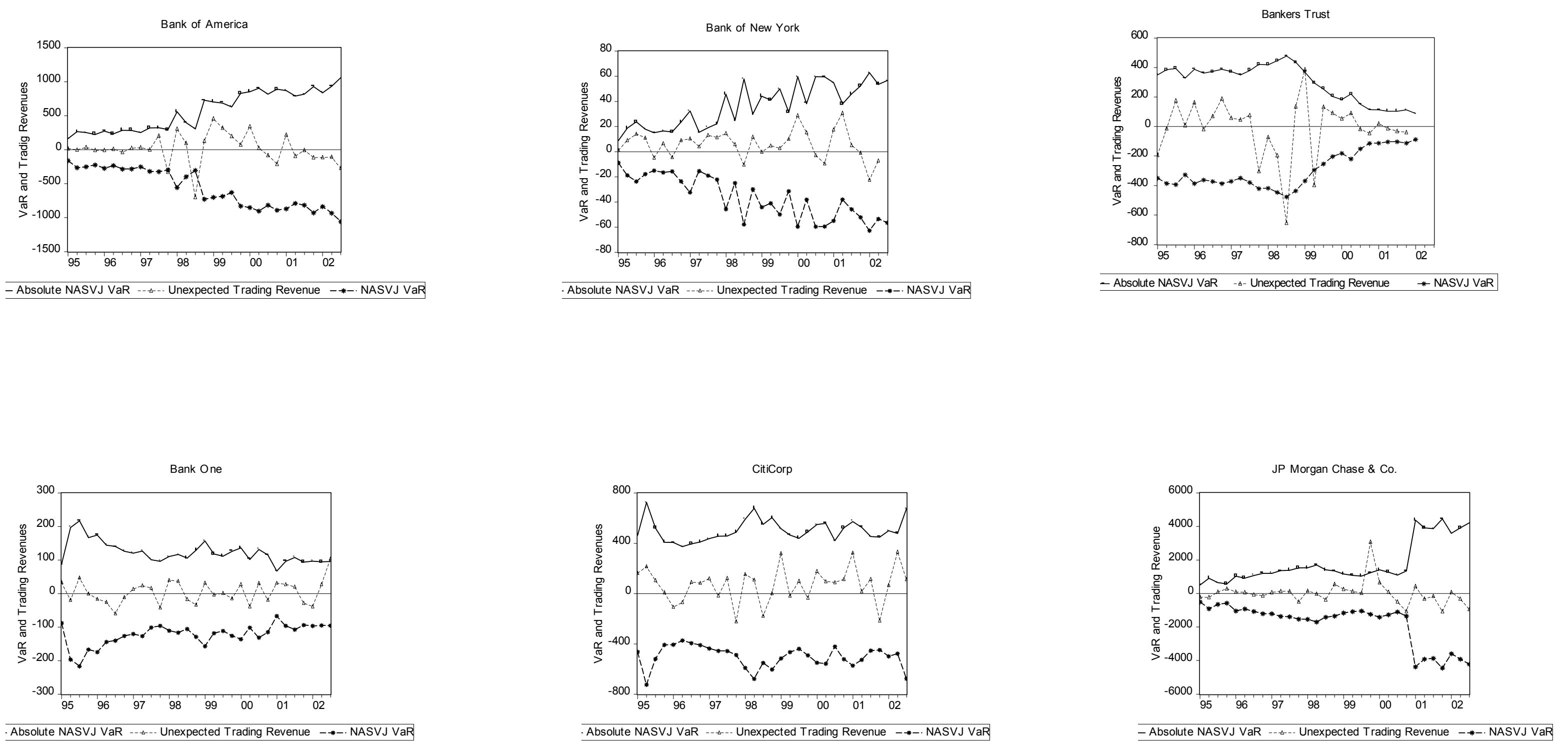


Table 5-11 below addresses the effect of including the non-affine volatility component in the SV-VaR and the SVJ VaR. The table below compared the means of NASV VaR and NASVJ VaR estimates with the means of SV-VaR and SVJ VaR estimates. As in previous tables, the first two columns in table 5-11 show the differences between the means of the NASV and NASVJ estimates and the 99th percentile of unexpected trading revenues assuming normality. Negative numbers show that those VaR means falls outside the 99th percentile of unexpected trading revenues.

To get sense of the effect of non-affine stochastic volatility, the best way is to compare the first column of table 5-11 with the first column of table 5-6, and the second column of table 5-11 with the first column of table 5-9. The first column in table 5-11 indicates that there are three banks fall inside the 99th percentile under the NASV VaR, two of them significantly fall inside this percentile. This is one improvement over the SV VaR where only the mean of SV-VaR in CitiCorp falls inside the 99th percentile. Additionally, some of the banks that significantly fall outside the 99th percentile are no longer significant under the NASV. The picture is also clearer with the NASVJ VaR model, where the mean of NASVJ VaR for 5 banks (Bankers Trust is the only exclusion) is now significantly fall inside the 99th percentile compared with only 3 banks that are significantly falling within the 99th percentile under the SVJ VaR.

Table 5-11 also gives another avenue to see how the non-affine structure increases VaR estimates through comparing the means of VaR estimates with and without the non-affine stochastic component. The third and fifth columns in table 5-11 explore more the effect of the non-affine structure of stochastic volatility on VaR estimates. In those

Table (5-11)
NASV VaR and NASVJ VaR Absolute Mean Differences Compared with SV-VaR, and SVJ VaR.

Bank	**Mean NASV VaR Difference from 99 percentile**[(1)] (test-stat)[(2)]	**Mean NASVJ VaR Difference from 99 percentile**[(3)] (test-stat)[(4)]	NASV VaR Mean Minus SV- VaR Mean[(5)] (test stats)[(6)]	NASV VaR Mean Minus SVJ VaR Mean[(7)] (test stats)[(6)]	**NASVJ VaR Mean Minus SVJ VaR Mean**[(8)] (test stats)[(6)]	**NASVJ VaR Mean Minus NASV VaR Mean**[(9)] (test stats)[(6)]
Bank of America	-20.88 (-0.47)	91.12** (1.71)	34.22** (12.91)	-44.51 (-13.84)	67.49*** (9.494)	112.00*** (11.06)
Bank of New York	1.95 (0.81)	9.19*** (3.10)	3.42** (22.84)	-3.31*** (-13.41)	3.93*** (16.55)	7.24*** (5.34)
Bankers Trust	-189.12*** (-7.89)	-153.71*** (-6.82)	13.67* (1.52)	-18.02** (-1.99)	17.50*** (3.87)	35.52*** (6.56)
Bank One / First Chicago	19.51*** (4.67)	43.58*** (7.60)	32.93*** (18.11)	-10.52*** (-6.14)	13.55*** (7.25)	24.06*** (10.55)
CitiCorp	89.35*** (6.44)	184.93*** (11.58)	28.46*** (5.84)	-40.2*** (-6.86)	55.37*** (8.11)	95.58*** (12.63)
JP Morgan Chase & Co.	-18.88 (0.09)	325.55* (1.35)	111.43** (2.25)	-154.60*** (-2.87)	189.83*** (3.11)	344.4343*** (5.12)

(1) The difference between the mean NASV VaR and the 99^{th} percentile of the data assuming normality.
(2) H0: the absolute mean of NASV VaR minus the absolute 99^{th} percentile equals zero.
(3) The difference between the mean NASVJ VaR and the 99^{th} percentile of the data assuming normality.
(4) H0: the absolute mean of NASVJ VaR minus the absolute 99^{th} percentile equals zero.
(5) Absolute mean of NASV VaR minus absolute mean of SV-VaR.
(6) H0: absolute mean difference equals zero.
(7) Absolute mean of NASV VaR minus absolute mean of SVJ VaR.
(8) Absolute mean of NASVJ VaR minus absolute mean of SVJ VaR.
(9) Absolute mean of NASVJ VaR minus absolute mean of NASV VaR.
(10) *** significant at 1%, ** significant at 5% and * is significant at 10%.

two columns the mean of NASV VaR and NASVJ VaR estimates are compared with the means of SV-VaR and SVJ VaR estimates respectively. As mentioned before positive values imply that the absolute means of NASV VaR and NASVJ VaR estimates are higher than the absolute means of SV-VaR and SVJ VaR estimates respectively. Easily, we can see that the means of NASV VaR and NASVJ VaR exceed significantly the means of SV-VaR and SVJ VaR respectively in all banks included in the sample. Those results are undoubtedly emphasizes the role of the non-affine structure of stochastic volatility in improving VaR results.

Columns 4 and 6 in the table compare the effect of non-affine structure of stochastic volatility with the jump component. The non-affine structure of volatility and the assumed jump diffusion of trading revenues both improve the estimate of VaR measures. Although those two components have the same effect on VaR, the source of increment to VaR is different under each of those components. Non-affine stochastic volatility increases VaR because it gives higher estimate of volatility that affects VaR positively. The jump diffusion affects VaR estimate on the other hand, by contributing to the tail thickness of the distribution, which is the core idea of VaR estimates.

Table 5-11 shows that the jump diffusion assumption contributes more in increasing VaR estimates. Column 4 shows in all cases that the absolute mean of NASV VaR is significantly less that the absolute mean of SVJ VaR. On the other hand, column 6 indicates that the absolute mean of the NASVJ VaR is significantly higher than the absolute mean of the NASV VaR. This means that VaR models that include jump structures have significantly higher mean than VaR models that include the non-affine structure. However, we need to be careful here, not to conclude that jump diffusion contributes to (distribution's tail thickness) and hence VaR more than what stochastic volatility (the other distribution's tail thickness source) contributes. The conclusion here is that the jump as a tail thickness source contributes to VaR estimates more than the non-affine structure which is volatility increasing source (rather than a tail thickness source).

Table 5-12 below explores the effect of the non-affine stochastic structure through changes in the violation sizes. The table provides the number of violations, the mean size of violations and the difference in the mean size of violations between the NASV VaR and NASVJ VaR in one side and the SV-VaR and the SVJ VaR in the other side.

As mentioned above, under the NASV VaR, SVJ VaR, and NASVJ VaR, Bank of America and Bankers Trust witness two violations in each bank. In addition to those two banks, JP Morgan Chase (excluded from table 5-12) witnesses one violation under SV-VaR which directly give the sense that NASV VaR perform better than SV-VaR in binding banks' losses.

The first issue that we see from table 5-12 is that the mean violation under the NASVJ VaR is less than the mean violations under the NASV VaR. This means that jumps improve VaR estimates better than the non-affine structure. However, as the last column in the table shows, those differences in violations' mean are statistically insignificant.

On the other hand, we can see that the inclusion of the non-affine structure to the stochastic volatility reduces the mean size of violations in the NASV VaR and NASVJ VaR compared with the SV-VaR and SVJ-VaR, only numerically. If we check for how significant those difference in the mean of violations under those different VaR models, we find that non-of those differences is significant. The same is applicable to the

Table (5-12)
NASV VaR and NASVJ Violation Statistics Compared with SV-VaR and SVJ VaR, for The Banks that Have Violations under the NASV and NASVJ VaRs, (Means of Violations in Absolute Value)

Bank	**No. of Violations in NASV VaR**	**NASV VaR Mean(1) Size of Violations** (test-stat)(2)	**No. of Violations in NASVJ VaR**	**NASVJ VaR Mean(3) Size of Violations** (test-stat)(4)	**NASV Violation Minus SV-VaR(5)** (test-stat)(6)	**NASV Violation Minus SVJ VaR(7)** (test-stat)(8)	**NASVJ Violation Minus SVJ-VaR(9)** (test-stat)(10)	**NASVJ Violation Minus NASV VaR(11)** (test-stat)(12)
Bank of America	2	263.94 *** (2.86)	2	213.60*** (2.35)	-18.34 (-0.10)	18.71 (0.10)	-31.63 (-0.17)	-50.34 (-0.28)
Bankers Trust	2	164.55 *** (8.17)	2	140.38*** (7.44)	-25.40 (-0.63)	15.23 (0.39)	-8.93 (-0.24)	-24.17 (-0.62)

(1) The absolute mean of NASV VaR violations.
(2) H0: The absolute Mean NASV VaR =0.
(3) The absolute mean of NASVJ VaR violations.
(4) H0: The absolute Mean NASVJ VaR =0.
(5) Absolute mean NASV VaR minus absolute mean of SV-VaR.
(6) H0: The difference between the mean absolute NASV VaR and the mean absolute SV-VaR is zero.
(7) Absolute mean NASV VaR minus absolute mean of SVJ VaR.
(8) H0: The difference between the mean absolute NASV VaR and the absolute mean SVJ VaR is zero.
(9) Absolute mean NASVJ VaR minus absolute value of SVJ VaR.
(10) H0: The difference between the mean absolute NASVJ VaR and the absolute mean SVJ VaR is zero.
(11) Absolute mean NASVJ VaR minus absolute mean of NASV.
(12) H0: The difference between the mean absolute NASVJ VaR and the absolute mean NASV VaR is zero.
(13) *** significant at 1% , ** significant at 5% and * is significant at 10%.

comparisons of jump component and non-affine structure that shows that mean violations are less under VaR models that include jumps. However, as mentioned before, those differences are statistically insignificant.

Comparing the maximum violations under the NASV VaR and NASVJ VaR models with the SV VaR and SVJ VaR models respectively should give clearer picture. In case of Bank of America, the maximum violation under NASV VaR decreases by \$18.0 millions compared with SV VaR, and increases by \$18. 8 millions compared with SVJ VaR. The same direction of changing in the maximum violation appears in Bankers Trust, where the maximum violation decreases under the NASV VaR by \$26.1 compared with the SV-VaR and increases by \$18.1 millions compared with the SVJ VaR.

The maximum violation under the NASVJ VaR in Bank of America decreases by \$34.1 millions compared with the maximum violation under the SVJ VaR, but it decreases by almost \$53 millions compared with the NASV VaR. The same trend in changing maximum VaR violation is noticed in Bankers Trust, where the maximum violation under NASVJ VaR decreases by \$8.6 millions compared with the SVJ VaR and by more than triple of this amount, under the NASV VaR.

All of the above results in this section indicate that the inclusion of the non-affine structure to the volatility improves VaR estimates compared with the affine square root structure of volatility. But the inclusion of jump diffusion has more significant impact on VaR estimation, since jumps affect directly tail thickness of the distribution and hence VaR estimate, whereas the non-affine structure affects volatility magnitude which also affect VaR positively.

5.3. Conclusion

The empirical investigation and estimation of the proposed VaR models on this dissertation through applying it to the data sample yields the following conclusions:

1- As Assumed, the stochastic volatility and jump models, as two sources of tail thickness of the distribution, could produce higher estimates for VaR. VaR estimates under the stochastic volatility and jump diffusion model appear to be significantly higher than the GBM VaR estimate (the plain vanilla VaR model) almost in all cases.

2- The inclusion of both stochastic volatility and jumps in VaR estimate could improve the estimate of VaR compared with the VaR models that include the stochastic volatility alone or the jump diffusion alone. This could be a way to prove that stochastic volatility and jumps are tail thickness sources in financial data as many theoretical and empirical studies show.

3- The results of comparisons of SV-VaR model and JD VaR model give some how mixed results. In some cases the SV-VaR estimates appeared to be higher, in other cases, JD VaR estimates to be higher. The general impression might be that the SV gives higher VaR estimates compared with jump diffusion models. However, this conclusion can not be distinctive because volatility varying it self could produce higher levels of volatility and hence higher VaR estimate.

4- Non–affine component of stochastic volatility seems also to improve VaR estimates. This is because of the faster updating of volatility that might produce higher volatility levels and hence higher VaR estimates. In spite the fact that the non-affine structure improves VaR estimates, it has contributes less compared with jumps that affect the tail thickness rather than volatility as in the case of the non-affine structure. Researches should explore more the non-affine structure of stochastic volatility in finance literature. The non-affine structure appears to be very suitable in modeling volatility during collapses and crashes where volatility updates vary fast. Thus I think that finance people should focus of such volatility models.

5- VaR models developed in this dissertation could produce VaR estimate closer to the actual unexpected trading revenues of the banks in the sample. The number of violations and the sizes of violation decreased significantly under the proposed VaR models, which means that the proposed VaR models could bind the actual bank losses better than the banks' VaR in some cases. Generally speaking that NASVJ VaR produced the highest VaR estimates among all the proposed models, and of course the GBM VaR produces the lowest. The second highest VaR estimate comes with the association of stochastic volatility with jumps, the NASV VaR model produces roughly the third highest VaR estimate.

6- All the developed VaR models fail to bind the losses for two banks, Bank of America and Bankers Trust during the late 1997 and 1998, in spite that the sizes of violations decrease noticeably under the proposed models.

7- Some banks appear to have very low and conservative VaR estimates that is even less that the GBM VaR, others have relatively higher VaR estimates that exceeds the estimates of all proposed models. The banks with conservative

VaR estimates witnesses more frequent violation (losses went beyond VaR) during the analysis period. On the other hand Banks with high VaR estimates do not suffer any violations of its models during the analysis period.

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APPENDIXES

Parameter Estimate of the Processes

Table (A-1)
Parameter Estimate for GBM Model*

$$\frac{dP_t}{P} = \mu dt + \sqrt{v_t}\, dW_{P,t}$$

Bank	**Obs.** No.	μ (S. E)	***v*** (S. E)
Bank of America	94:Q1-02:Q3 35	-0.093 (0.096)	4.397 (2.462)
Bank of New York	94:Q1-02:Q3 35	0.030 (0.044)	0.063 (0.048)
Bankers Trust	94,Q1-02,Q1 33	0.119 (0.103)	2.927 (2.722)
Bank One/First Chicago	94:Q1-02:Q3 35	0.422 (0.641)	2.653 (0.966)
CitiCorp	94:Q1-02:Q3 35	1.083 (0.848)	2.256 (2.033)
JP Morgan Chase	94:Q1-02:Q3 35	0.743 (1.006)	6.726 (3.328)

* Estimate parameters numbers are multiplied by 1000.

Table (A-2)
Parameter Estimate for Stochastic Volatility Model*

$$\frac{dP_t}{P} = \mu dt + \sqrt{v_t}\, dW_{P,t}$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma\sqrt{v_t}\, dW_{v,t}$$

$$Corr(dW_{P,t}, dW_{v,t}) = \rho$$

Bank	**Obs.** No.	μ (S. E)	σ (S. E)	θ (S. E)	κ (S. E)	ρ (S. E)
Bank of America	94:Q1-02:Q3 35	-0.071 (0.041)	4.757 (3.661)	0.067 (0.116)	0.277 (0.198)	-0.678 (0.031)
Bank of New York	94:Q1-02:Q3 35	0.052 (0.022)	1.199 (0.865)	0.031 (0.062)	0.135 (0.287)	-0.433 (0.389)
Bankers Trust	94,Q1-02,Q1 33	0.131 (0.018)	3.227 (2.143)	0.011 (0.006)	0.136 (0.251)	-0.586 (0.198)
Bank One/First Chicago	94:Q1-02:Q3 35	0.575 (0.284)	2.258 (1.061)	0.020 (0.012)	0.217 (0.189)	0.201 (0.236)
CitiCorp	94:Q1-02:Q3 35	0.972 (0.664)	4.127 (4.054)	0.051 (0.053)	0.566 (0.611)	-0.489 (0.277)
JP Morgan Chase	94:Q1-02:Q3 35	0.883 (0.755)	9.935 (5.217)	0.184 (0.058)	0.693 (0.469)	0.266 (0.432)

* Estimate parameters numbers are multiplied by 1000 except for the correlation coefficient ρ.

Table (A-3)
Parameter Estimate for Jump-Diffusion Model*

$$\frac{dP_t}{P_t} = \mu dt + \sigma dW_{P,t} + [\exp(J_u) - 1]dN_u(\lambda_u) - [\exp(J_d) - 1]dN_d(\lambda_d)$$

Bank	**Obs.** No.	μ (S. E)	σ (S. E)	η_u (S. E)	λ_u (S. E)	η_d (S. E)	λ_d (S. E)
Bank of America	94:Q1-02:Q3 35	-0.089 (0.058)	4.397 (2.462)	0.008 (0.013)	1.231 (1.385)	0.041 (0.016)	2.016 (0.953)
Bank of New York	94:Q1-02:Q3 35	0.028 (0.026)	0.063 (0.048)	0.012 (0.009)	0.882 (0.241)	0.074 (0.058)	1.284 (0.691)
Bankers Trust	94,Q1-02,Q1 33	0.114 (0.094)	2.927 (2.722)	0.024 (0.016)	0.632 (0.915)	0.088 (0.060)	0.944 (0.830)
Bank One/First Chicago	94:Q1-02:Q3 35	0.399 (0.422)	2.653 (0.966)	0.031 (0.029)	0.446 (1.125)	0.053 (0.044)	1.373 (1.521)
CitiCorp	94:Q1-02:Q3 35	1.001 (0.766)	2.256 (2.033)	0.017 (0.019)	0.523 (0.213)	0.080 (0.076)	2.026 (3.115)
JP Morgan Chase	94:Q1-02:Q3 35	0.684 (0.953)	6.726 (4.428)	0.048 (0.011)	0.961 (0.807)	0.082 (0.107)	1.363 (2.171)

* Estimate parameters numbers are multiplied by 1000.

Table (A-4)
Parameter Estimate for Stochastic Volatility, Jump-Diffusion Model*

$$\frac{dP_t}{P_t} = \mu dt + \sqrt{v_t} dW_{P,t} + [\exp(J_u) - 1] dN_u(\lambda_u) - [\exp(J_d) - 1] dN_d(\lambda_d)$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma \sqrt{v_t} dW_{v,t}$$

Bank	μ (S. E)	κ (S. E)	θ (S. E)	σ (S. E)	ρ (S. E)	η_u (S. E)	λ_u (S. E)	η_d (S. E)	λ_d (S. E)
Bank of America	-0.112 (0.096)	0.213 (0.209)	0.061 (0.108)	4.625 (4.326)	-0.241 (0.017)	0.010 (0.004)	1.046 (1.247)	0.040 (0.017)	2.008 (1.628)
Bank of New York	0.064 (0.059)	0.213 (0.215)	0.024 (0.058)	1.031 (0.847)	-0.131 (0.243)	0.011 (0.013)	0.625 (0.323)	0.055 (0.141)	1.310 (0.991)
Bankers Trust	0.162 (0.133)	0.331 (0.624)	0.020 (0.018)	2.773 (2.554)	-0.201 (0.137)	0.026 (0.014)	0.547 (0.857)	0.076 (0.058)	0.874 (0.602)
Bank One	0.690 (0.333)	0.219 (0.128)	0.012 (0.027)	3.055 (2.136)	0.009 (0.012)	0.038 (0.211)	1.224 (1.264)	0.044 (0.056)	1.879 (1.488)
CitiCorp	0.948 (0.739)	0.486 (0.441)	0.048 (0.055)	4.008 (4.106)	-0.163 (0.236)	0.019 (0.027)	0.511 (0.384)	0.065 (0.153)	2.166 (3.045)
JP Morgan Chase	0.944 (0.962)	0.681 (0.545)	0.167 (0.096)	7.328 (4.619)	0.113 (0.362)	0.054 (0.023)	0.755 (0.821)	0.096 (0.367)	1.328 (2.112)

* Estimate parameters numbers are multiplied by 1000 except for ρ.

Table (A-5)
Parameter Estimate for Non-affine Stochastic Volatility Model*

$$\frac{dP_t}{P} = \mu dt + \sqrt{v_t}\, dW_{P,t}$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma v_t^{\frac{\gamma}{2}} dW_{v,t}$$

$$Corr(dW_{P,t}, dW_{v,t}) = \rho$$

Bank	**Obs.** No.	μ (S. E)	κ (S. E)	θ (S. E)	σ (S. E)	γ (S.E)	ρ (S. E)
Bank of America	94:Q1-02:Q3 35	-0.073 (0.054)	0.258 (0.164)	0.061 (0.135)	3.696 (2.931)	1.128 (0.489)	-0.531 (0.631)
Bank of New York	94:Q1-02:Q3 35	0.048 (0.020)	0.104 (0.221)	0.022 (0.056)	1.125 (0.743)	1.101 (0.662)	-0.504 (0.326)
Bankers Trust	94,Q1-02,Q1 33	0.127 (0.011)	0.122 (0.230)	0.005 (0.002)	2.883 (1.938)	2.370 (2.602)	-0.311 (0.166)
Bank One/First Chicago	94:Q1-02:Q3 35	0.516 (0.291)	0.187 (0.111)	0.016 (0.011)	2.161 (0.967)	1.991 (0.438)	0.124 (0.212)
CitiCorp	94:Q1-02:Q3 35	0.899 (0.618)	0.511 (0.586)	0.049 (0.050)	3.814 (3.610)	1.583 (1.714)	-0.367 (0.463)
JP Morgan Chase	94:Q1-02:Q3 35	0.857 (0.671)	0.644 (0.407)	0.167 (0.066)	9.278 (6.681)	1.335 (0.932)	0.217 (0.292)

* Estimate parameters numbers are multiplied by 1000 except for γ and ρ.

Table (A-6)
Parameter Estimate for Non-affine Stochastic Volatility, Jump-Diffusion Model*

$$\frac{dP_t}{P_t} = \mu dt + \sqrt{v_t}\, dW_{P,t} + [\exp(J_u) - 1]dN_u(\lambda_u) - [\exp(J_d) - 1]dN_d(\lambda_d)$$

$$dv_t = \kappa(\theta - v_t)dt + \sigma v_t^{\frac{\gamma}{2}} dW_{v,t}$$

Bank	μ (S. E)	κ (S. E)	θ (S. E)	σ (S. E)	ρ (S. E)	γ (S.E)	η_u (S. E)	λ_u (S. E)	η_d (S. E)	λ_d (S. E)
Bank of America	-0.136 (0.110)	0.256 (0.241)	0.060 (0.041)	4.414 (4.228)	-0.451 (0.026)	1.116 (1.454)	0.017 (0.108)	1.112 (1.004)	0.039 (0.018)	2.354 (1.929)
Bank of New York	0.098 (0.072)	0.300 (0.143)	0.027 (0.050)	1.102 (0.832)	-0.201 (0.252)	1.646 (2.088)	0.016 (0.064)	0.852 (0.428)	0.031 (0.019)	1.564 (1.352)
Bankers Trust	0.221 (0.151)	0.414 (0.802)	0.023 (0.020)	2.728 (2.623)	-0.338 (0.325)	1.626 (1.021)	0.023 (0.113)	0.544 (0.818)	0.069 (0.065)	0.885 (0.618)
Bank One/First Chicago	0.711 (0.328)	0.288 (0.155)	0.017 (0.024)	3.221 (2.432)	0.014 (0.018)	1.868 (0.324)	0.029 (0.109)	1.381 (1.098)	0.071 (0.039)	1.924 (1.631)
CitiCorp	0.955 (0.689)	0.499 (0.442)	0.042 (0.054)	4.057 (4.326)	-0.231 (0.182)	1.479 (2.700)	0.034 (0.025)	0.771 (0.828)	0.045 (0.043)	1.968 (2.102)
JP Morgan Chase	0.858 (0.822)	0.573 (0.474)	0.18 (0.091)	6.581 (4.588)	0.116 (0.069)	1.311 (1.698)	0.041 (0.022)	0.722 (0.686)	0.073 (0.047)	1.753 (2.277)

* Estimate parameters numbers are multiplied by 1000 except for γ and ρ.

VITA

Ahmad Ali Telfah was born on August 24, 1969 in Irbid Jordan. He achieved his Bachelor and Masters degrees in Economics from Yarmouk University-Irbid Jordan in 1991 and 1995 respectively.

In 1992, Ahmad worked in Central Bank of Jordan as an Economic Researcher until 1998, when the Central Bank of Jordan offered him a scholar ship to study the Ph.D. in Financial Economics at University of New Orleans, majoring in Investment, corporate finance and financial institutions and monetary policy. Ahmad now is an assistant professor at the School of Business at Rutgers University. He teaches investments, derivative securities, and fixed income securities. He lives now in Philadelphia, Pennsylvania. Ahmad is married and a father for a daughter and a son.

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CANDIDATE: Ahmad A. Telfah

MAJOR FIELD: Financial Economics

TITLE OF DISSERTATION: "Value at Risk under Thick Tails and Fast Volatility Updating"

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