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Abstract

We introduce increasing returns to scale into an otherwise standard New Keynesian model with capital, and study the determinacy and E-stability of Taylor-type interest rate rules. With very mild increasing returns supported by empirical research, the conventional wisdom regarding the design of interest rate rules can be overturned. In particular, the "Taylor principle" no longer guarantees either determinacy or E-stability of the rational expectations equilibrium.

Keywords: increasing returns, indeterminacy, E-stability, Taylor principle

JEL Classifications: E32, E52

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1 Introduction

It is well-known that self-fulfilling expectations may cause business fluctuations if there are certain types of coordination failures in the markets. In the real business cycle (RBC) literature, researchers emphasize the importance of increasing returns in generating such fluctuations (Farmer and Guo, 1994). Increasing returns are usually originated from externalities or monopolistic competitions. Coordination failures also have important implications for economic agents who do not possess rational expectations and try to learn about the economic structure adaptively. With increasing returns, the rational expectations equilibrium (REE) may not be "expectationally-stable" (E-stable) under learning (Duffy and Xiao, 2003). The recent literature of monetary policy design, however, emphasizes the role of interest rate policies in either facilitating or restraining fluctuations caused by self-fulfilling expectations or E-instability. It is believed that if an interest rate policy is properly designed, it leads the economy to a determinate (free from self-fulfilling fluctuations) and E-stable REE (Clarida, et al., 2000, Bullard and Mitra, 2002). Determinacy and E-stability have undoubtedly become two crucial criteria in evaluating monetary policies (Evans and Honkapohja, 2003).

Interestingly, when selecting the proper interest rate rules to prevent excess volatilities, researchers prefer to condition on an economic environment that is free from any market coordination failures. In other words, the possibility that *both* sources of indeterminacy and E-instability exist in the economy has been largely neglected. For example, there are extensive studies of the potential benefits and risks associated with Taylor-type interest rate rules. Yet when specifying the economic environment for these studies, researchers seem to ignore the possibility of increasing returns, which are known to cause indeterminacy and E-instability. The workhorse for this area – the New Keynesian model, has monopolistic competitions, staggered prices, but constant returns to scale. Since increasing returns are widely believed to occur in monopolistically competitive economies, one naturally wonders how robust the current findings are if the assumption of constant returns to scale does not hold. Indeed, to some researchers, one is "required" to postulate increasing returns in a monopolistic competition framework, since it is the "only way to account for the absence of significant pure profits in the United States economy" (Rotemberg and Woodford, 1995). Therefore, incorporating increasing returns into the designing of interest rate rules seems the next logical step to take in extending this research.

In this paper, we propose a first step towards such an extension. We introduce increasing returns to scale into an otherwise standard New Keynesian model with capital, and study the determinacy and E-stability of Taylor-type interest rate rules, as in Bullard and Mitra (2002). Bullard and Mitra's important finding is that if the interest rate rule follows the so-called *Taylor principle*, the REE of the model is mostly likely to be both determinate and E-stable. The Taylor principle asserts that the monetary authority must adjust the short-term interest rate more than one-for-one with changes in inflation. Our research question is: when there are increasing returns in the economy, how must the interest rate rules be changed to achieve a stable macroeconomic equilibrium? Does the Taylor principle still guarantee the determinacy and E-stability of the REE?

Our major findings are as follows. We re-examine the determinacy and E-stability of REE under four variants of the Taylor rule studied by Bullard and Mitra (2002). The four variants are: 1. the contemporaneous data rule, 2. the lagged data rule, 3. the forward expectations rule, and 4. the contemporaneous expectations rule. Bullard and Mitra (2002) find that in most cases the Taylor principle is sufficient to guarantee both determinacy and E-stability. Moreover, with rule 1 and rule 4 a determinate REE is always E-stable and vice versa. We find that with small increasing returns that are consistent with empirical estimates, these findings no longer hold. In particular, the Taylor Principle cannot guarantee either determinacy or E-stability in any of the four rules. In some cases, a less than one-for-one response of the interest rate to inflation can lead to determinacy and E-stability. The policy implications are clear. To rule out indeterminacy and E-instability, it is critical for the monetary authority to identify the level of increasing returns – given a certain level of increasing returns, a distinct set of parameters for the interest rate rule will maintain the determinacy and E-stability of the REE.

The assumption of increasing returns to scale is widely considered in the business cycle and growth literature.[1] A major problem of models that possess indeterminate equilibria is that the

[1] See, among others, Hornstein (1993), Benhabib and Farmer (1994), Roternberg and Woodford (1995), and Benhabib and Wen (2004).

required increasing returns are too high to live up to empirical tests. In empirical studies, the earlier work of Hall (1990) is known to have over-estimated the degree of increasing returns (larger than 1.5). More recent research find mild but significant levels of increasing returns in the US economy. For example, Basu and Fernald (1994 and 1997) conclude that the returns to scale is between 1.03 and 1.09. Laitner and Stolyarov (2004) use stock market data to estimate the returns to scale and obtain values between 1.09 and 1.11. In our model, the required level of increasing returns to generate indeterminacy is as low as 1.05.

In general, this paper adds to a series of other research that study the limitations of the Taylor Principle as a criterion to design interest rate rules. Gali et al. (2004), for example, find that the existence of rule-of-thumb consumers will render the REE indeterminate when the Taylor principle holds. Fair (2003) argues that the Taylor principle cannot guarantee determinacy if aggregate demand responds to nominal interest rates and inflation has a negative effect on consumption. Benhabib et al. (2001) find that the Taylor principle does not necessarily lead to determinate REE when there is zero bound on nominal rates. All these works focus on the determinacy of the REE. We have not seen any papers that challenge the role of the Taylor principle in maintaining the E-stability of the REE.

In the literature, the baseline New Keynesian model ignores endogenous variations in capital, on the ground that capital fluctuations do not correlate much with output at the business cycle frequency (McCallum and Nelson, 1999). However, a number of researchers have recently pointed out that certain topics can only be studied when capital is allowed to vary endogenously.[2] In our context, increasing returns in capital are known to have non-trivial effects on the determinacy of the equilibrium. For example, Benhabib (1998) illustrates that self-fulfilling expectations about future *investment* returns are important in generating indeterminate equilibrium. Grandmont et al. (1998) show that the capital-labor substitutability affects the robustness of sunspot equilibrium. Moreover, with endogenous capital, our model becomes a natural extension of Farmer and Guo (1994), which

[2] Gali et al. (2004) show that endogenous capital is required for rule-of-thumb consumers to make a difference in system dynamics. Christiano et al. (2001) use investment adjustment costs to generate hump-shaped response of output to a monetary shock. Edge (2000) shows that investment adjustment with a time-to-build technology helps generating a liquidity effect.

ensures that the same mechanism that causes indeterminacy in their paper still exists in the New Keynesian framework. We therefore incorporate capital into the model in this study. We introduce capital in a standard way, as in Gali et al. (2004). In our analysis, we compare a constant-return version of our model with Bullard and Mitra (2002)'s labor-only model to make sure that introducing capital alone does not alter the determinacy and E-stability of the REE. All changes in the REE properties are caused by incorporating increasing returns to scale.

The rest of the paper is organized as follows. Section 2 lays out the micro-founded model framework and derives the equilibrium conditions. Section 3 discusses the methodology and calibration of the model. Section 4 presents the results. Section 5 concludes.

2 A New Keynesian Model with Capital and Increasing Returns

This is a standard New Keynesian model with capital, except for the novel assumption of increasing returns in production. We abstract from any exogenous processes, such as productivity shocks or demand shocks, in order to minimize the number of equations to manipulate when deriving the conditions for E-stability analytically. Adding or leaving those processes do not change the properties of the equilibrium.

2.1 Households

The economy is composed of a large number of infinitely-lived consumers. Each of them consumes a final good C_t, and supplies labor N_t. Savings can be held in the form of real money balances $\frac{M_t}{P_t}$, bonds B_t, and capital K_t. Consumers seek to maximize life-time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t [\frac{C_t^{1-\sigma}}{1-\sigma} + \gamma \frac{(M_t/P_t)^{1-b}}{1-b} - v \frac{N_t^{1+\chi}}{1+\chi}],$$

where $\sigma, \gamma, b, v, \chi > 0$ and $0 < \beta < 1$, subject to a budget constraint

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} + I_t = \frac{W_t}{P_t}N_t + \frac{M_{t-1}}{P_t} + \frac{R_t}{P_t}K_t + (1+i_{t-1})\frac{B_{t-1}}{P_t} + D_t \tag{1}$$

and the capital accumulation equation

$$K_{t+1} = (1-\delta)K_t + \phi(\frac{I_t}{K_t})K_t. \tag{2}$$

Hence, the consumers receive real labor income $(W_t/P_t)N_t$, and real capital rental income $(R_t/P_t)K_t$. B_{t-1} is the quantity of riskless one-period bonds carried over from period $t-1$ which pay out interests at a nominal rate of $1+i_{t-1}$. D_t are dividends from ownership of firms. M_{t-1}/P_t are real money holdings carried over from period $t-1$. The consumers spend their income on consumption C_t, new money holdings M_t/P_t, new bond purchases B_t/P_t, and new investment I_t. Capital adjustment costs are introduced through the term $\phi(I_t/K_t)K_t$, which determines the change in capital stock induced by investment spending I_t. We assume $\phi' > 0$, and $\phi'' \leq 0$, with $\phi'(\delta) = 1$ and $\phi(\delta) = \delta$ as in Gali et al. (2004).

The first order conditions for the consumer's problem can be written as

$$\begin{aligned}
vN_t^{\chi} &= C_t^{-\sigma}\frac{W_t}{P_t}, && (3)\\
C_t^{-\sigma} &= \gamma(\frac{M_t}{P_t})^{-b} + \beta E_t C_{t+1}^{-\sigma}\frac{P_t}{P_{t+1}}, && (4)\\
1 &= \beta E_t(\frac{C_{t+1}}{C_t})^{-\sigma}\frac{P_t}{P_{t+1}}(1+i_t), && (5)\\
Q_t &= \beta E_t(\frac{C_{t+1}}{C_t})^{-\sigma}\{\frac{R_{t+1}}{P_{t+1}} + Q_{t+1}[(1-\delta) + \phi_{t+1} - \frac{I_{t+1}}{K_{t+1}}\phi'_{t+1}]\}, && (6)
\end{aligned}$$

where $\phi_{t+1} = \phi(I_{t+1}/K_{t+1})$ and $\phi'_{t+1} = \phi'(I_{t+1}/K_{t+1})$, respectively. Q_t is the real shadow value of capital, i.e., Tobin's Q. This is defined as

$$Q_t = \frac{1}{\phi'(\frac{I_t}{K_t})}. \tag{7}$$

Given our assumption about ϕ, the elasticity of the investment-capital ratio with respect to Q is $-\frac{1}{\phi''(\delta)\delta} = \eta$.

2.2 Firms

There exists a continuum of monopolistically competitive firms producing differentiated intermediate goods. The latter are used as inputs by a perfectly competitive firm producing a single final good.

2.2.1 Final Goods Producers

The final goods are produced by a representative, perfectly competitive firm with a constant returns to scale technology

$$Y_t = (\int_0^1 Y_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{8}$$

where y_{jt} is the quantity of intermediate goods j used as an input, and $\varepsilon > 1$ governs the price elasticity of individual goods. Profit maximization yields the demand schedule

$$Y_{jt} = (\frac{P_{jt}}{P_t})^{-\varepsilon} Y_t, \tag{9}$$

which, when plugged back into (8), yields

$$P_t = (\int_0^1 P_{jt}^{1-\varepsilon} dj)^{\frac{1}{1-\varepsilon}}. \tag{10}$$

2.2.2 Intermediate Goods Producers

The intermediate goods market features a large number of monopolistic competitive firms. The production function of a typical intermediate goods firm is

$$Y_{jt} = (K_{jt}^{\alpha} N_{jt}^{1-\alpha})^{\theta}, \theta > 0, \tag{11}$$

where K_{jt} and N_{jt} represent the capital and labor services hired by firm j. The parameter θ measures the level of returns to scale. When $\theta = 1$, the production technology reduces to the constant-return

Cobb-Douglas production function. When $\theta > 1$, the intermediate goods firm has increasing returns to scale.

The firms' real marginal costs φ_{jt} is derived by minimizing costs:

$$\varphi_{jt} = \frac{1}{(1-\alpha)\theta}\frac{W_t}{P_t}\frac{N_{jt}}{Y_{jt}} = \frac{1}{\alpha\theta}\frac{R_t}{P_t}\frac{K_{jt}}{Y_{jt}}, \tag{12}$$

which in turn implies the optimality condition

$$\frac{K_{jt}}{N_{jt}} = \frac{\alpha}{1-\alpha}\frac{W_t}{R_t}. \tag{13}$$

Note that when there are constant returns to scale, (12) and (13) imply that the real marginal costs φ_t^c are given by

$$\varphi_t^c = \frac{(1-\alpha)^{\alpha-1}}{\alpha^\alpha} R_t^a W_t^{1-\alpha}, \tag{14}$$

which is equalized across all firms since there is no j in the expression. When there are increasing or decreasing returns to scale, a firm's real marginal costs are associated with its production levels. In this case we can define the average level of marginal costs as

$$\varphi_t = \frac{1}{(1-\alpha)\theta}\frac{W_t}{P_t}\frac{N_t}{Y_t} = \frac{1}{\alpha\theta}\frac{R_t}{P_t}\frac{K_t}{Y_t}. \tag{15}$$

Using (12), (13), and the demand schedule, we can relate the real marginal costs of a firm φ_{jt} to the average level of marginal costs φ_t as

$$\varphi_{jt} = \varphi_t(\frac{P_{jt}}{P_t})^{\frac{\varepsilon(\theta-1)}{\theta}}. \tag{16}$$

Intermediate firms set nominal prices in a staggered fashion, according to the stochastic time dependent rule proposed by Calvo (1983). Each firm resets its price with probability $1-\omega$ each period, independent of the time elapsed since the last price adjustment. A firm resetting its price in period t seeks to maximize

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i (\frac{C_{t+i}}{C_t})^{-\sigma} (\frac{P_t^*}{P_{t+i}} Y_{jt+i} - \varphi_{jt+i} Y_{jt+i}), \tag{17}$$

where P_t^* represents the (common) optimal price chosen by firms resetting prices at time t.

Finally, the equation describing the dynamics for the aggregate price level is

$$P_t = [\omega P_t^{1-\varepsilon} + (1-\omega) P_t^{*1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \tag{18}$$

2.3 Monetary Authority

The central bank sets the nominal interest rate i_t every period according to a simple linear rule contingent on information about output and inflation. Following Bullard and Mitra (2002), we consider four variants of the interest rate rule. The first variant is called the "contemporaneous data rule":

$$i_t = \phi_\pi \pi_t + \phi_y y_t, \tag{19}$$

where $\phi_\pi \geq 0$ and $\phi_y \geq 0$. This is the original Taylor rule that conditions the interest rate on current output and inflation rate.[3] Since current data for output and inflation may not be available at time t, some suggest a "lagged data rule":

$$i_t = \phi_\pi \pi_{t-1} + \phi_y y_{t-1}. \tag{20}$$

The third rule is called "forward expectations rule":

$$i_t = \phi_\pi E_t \pi_{t+1} + \phi_y E_t y_{t+1}, \tag{21}$$

[3] In the standard New Keynesian model without capital, the interest rate rule conditions on output "gaps" rather than on output levels. It should be noted that the properties of the REE will not change whatsoever if output gaps are replaced by output levels in those models. We use output levels because output gaps cannot be easily derived in a model with endogenous capital.

where central bankers use the market's expectations about the future to set the interest rate rule. The fourth rule is called the "contemporaneous expectations rule":

$$i_t = \phi_\pi E_{t-1}\pi_t + \phi_y E_{t-1}y_t, \tag{22}$$

where the underlined assumption is that the market does not have current data and attempts to use past data to estimate today's output and inflation.

2.4 Equilibrium and Reduced Linear Systems

The following conditions clear the factor and goods markets: $N_t = \int_0^1 N_{jt}dj$, $K_t = \int_0^1 K_{jt}dj$, $Y_t = \int_0^1 Y_{jt}dj$ and $C_t + I_t = Y_t$.

We need to derive the linearized versions of the key optimality conditions in order to conduct our analysis. We use lower case letters to denote linearized variables. There are six non-dynamic equations and four dynamic equations. The first equation is the linearized version of the labor supply schedule (3):

$$\chi n_t + \sigma c_t = w_t - p_t. \tag{23}$$

The second equation is the linearized version of (7), which defines Tobin's Q:

$$x_t - k_t = \eta q_t, \tag{24}$$

where, to avoid confusion with the nominal interest rate, we have denoted investment by the letter x_t. The third and fourth equations are the linearized versions of (12). We are interested in the average level of marginal costs, which are given by

$$\varphi_t = n_t + (w_t - p_t) - y_t, \tag{25}$$

$$\varphi_t = k_t + (r_t - p_t) - y_t. \tag{26}$$

The fifth equation is the linearized production function

$$y_t = \alpha\theta k_t + (1-\alpha)\theta n_t. \tag{27}$$

The sixth equation is the market clearing condition

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}x_t, \tag{28}$$

where C, I and Y are steady state levels of consumption, investment and output. The first dynamic equation is Phillips curve, which is derived by solving the firm's dynamic price-setting problem and combining it with (18). The equation is given by

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\kappa}{1+A}\varphi_t, \tag{29}$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$ and $A = \frac{\varepsilon(1-\theta)}{\theta}$.

The second dynamic equation is the linearized version of (6), which describes the evolution of Tobin's Q:

$$q_t = \beta E_t q_{t+1} + [1-\beta(1-\delta)]E_t(r_{t+1} - p_{t+1}) - (i_t - E_t\pi_{t+1}). \tag{30}$$

The third dynamic equation is the Euler equation (5), which can be linearized as

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{t+1}). \tag{31}$$

The last dynamic equation is the capital accumulation equation (2), which is linearized as

$$k_{t+1} = (1-\delta)k_t + \delta x_t. \tag{32}$$

Finally, we add the interest rate rule and use the non-dynamic equations to substitute out seven variables q_t, $w_t - p_t$, $r_t - p_t$, x_t, i_t, φ_t, and y_t. The system becomes a four dimensional linear difference equation system consisting of $z_t = (c_t, n_t, k_t, \pi_t)$:

$$z_{t+1} = Jz_t. \tag{33}$$

3 Methodology and Calibration

3.1 General Methodology

Next, we examine the determinacy and E-stability of four variants of the Taylor-type interest rate rules. For each variant, the determinacy of the REE is decided by computing the eigenvalues of the system (33). Since there is only one predetermined variable k_t, an REE is determinate if the number of explosive roots is three and the number of stable roots is one. If the number of stable roots are bigger than one, we have an indeterminate REE. If there is no stable root, the system is explosive.[4]

To study adaptive learning, we re-write the system as

$$\begin{aligned} b_z z_t + b_k k_t &= d_k E_t k_{t+1} + d_z E_t z_{t+1}, & (34) \\ k_{t+1} &= e_z z_t + e_k k_t, & (35) \end{aligned}$$

where the second equation is derived from the capital accumulation equation that does not involve any expectations and does not need to be learned. We assume agents have the perceived law of motion (PLM)

$$z_t = a + \psi k_t,$$

which is in the same form as the MSV solution under REE.[5] The parameter vectors a and ψ will have to be learned. Given this PLM, we calculate the forward expectation of z_t as

$$E_t z_{t+1} = a + \psi E_t k_{t+1} = a + \psi E_t(e_z z_t + e_k k_t) = a + \psi e_z z_t + \psi e_k k_t.$$

[4] With the lagged data rule, the interest rate rule itself is a dynamic equation with a state variable i_{t-1}. In that case we require two stable roots to yield determinacy.

[5] With the lagged data rule and the contemporaneous expectations rule, the PLM will be slightly different since agents do not possess knowledge of current data. See the appendix for details.

Parameters	Values	Description
β	0.99	Discount factor
α	0.33	Capital share
δ	0.025	Depreciation rate
$\varepsilon/(\varepsilon-1)$	1.05	Level of markup
ω	0.75	Fraction of firms leaving prices unchanged
η	1	Elasticity of investment to Tobin's Q
σ	1	Risk aversion
χ	1	Inverse of labor supply elasticity

Table 1: Calibration

Plugging this expression into (34), we obtain the T-mapping from $(a, \psi)'$ to combinations of the true parameters of the model. The model is E-stable if $\frac{d}{d\tau}(a, \psi) = T(a, \psi) - (a, \psi)$ have eigenvalues less than 0. We derive the specific E-stability conditions for each interest rate rule in the appendix.

3.2 Benchmark Calibration

The system (33) has four dynamic equations and four variables. We cannot obtain analytical solutions for either determinacy or E-stability. We therefore rely on numerical simulations to study the properties of the equilibrium. Table 1 summarizes the values we used for the benchmark calibration.

Most parameters are chosen to conform with parameters used in the literature. For example, the discount factor is set at 0.99, the depreciation rate is set at 0.025, and the capital share in production is set at 1/3. The steady state mark-up is set at a mild level of 1.05, which implicitly defines a value for the elasticity of substitution across intermediate goods, ε. The inverse of the elasticity of labor supply, χ, is set to 1. The curvature of the utility function σ is set to 1 so that the utility is in logarithm form. The fraction of firms that keep their prices unchanged, ω, is given a value of 0.75, which corresponds to an average price duration of about one year. The elasticity of investment with respect to Tobin's Q, η, is set to 1, following King and Watson (1996).

The weights for inflation and output in the interest rate rule, ϕ_y and ϕ_π, and the level of increasing returns θ are left open so we can experiment with different values.

4 Determinacy and E-stability of Interest Rate Rules

In this section, we study the determinacy and E-stability of REEs under different interest rate rules. Since the results for the four variants of the Taylor rule bear some similarities, our strategy is to closely examine the results for the contemporaneous rule, and then go over the results for the other three variants briefly. To simplify exposition, we use the term “stable REE” to refer to an REE that is both determinate and E-stable, and the term “active policy” to refer to an interest rate rule that responds more than one-for-one to changes in inflation.

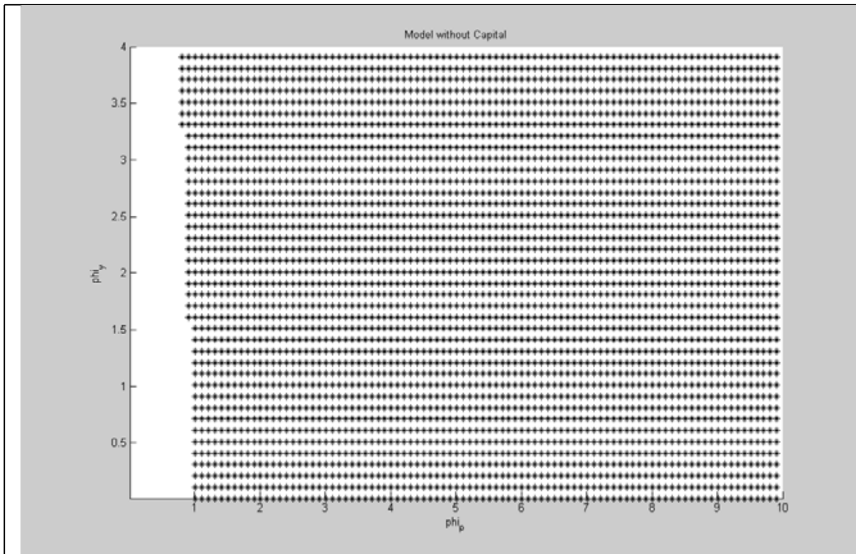
4.1 Contemporaneous Data Rule

In this section we consider the interest rate rule (19).

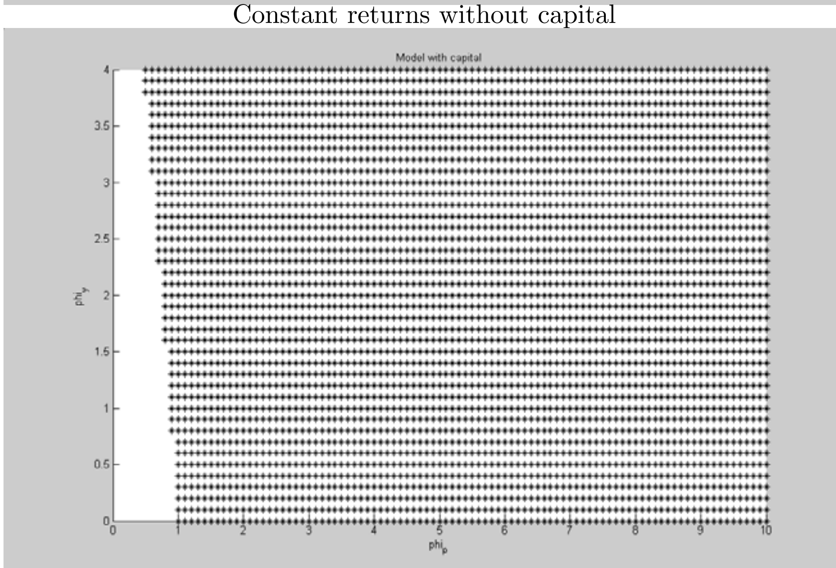
A standard New Keynesian model does not have endogenous capital. Therefore our first question is whether or not adding capital alone will change the properties of the equilibrium. To answer this question, we do a side-by-side comparison of a model with capital and a model without. The latter is a special case of the model in section 2 and is essentially the same as in Bullard and Mitra (2002). In both cases, the production function has constant returns to scale, and we keep all other parameters identical when necessary. We vary the policy weights for output and inflation and examine the properties of the REE for each combination of the parameters. The results are presented in Figure 1. We use a dark-colored star “*” to indicate that an REE is both determinate E-stable, a square to indicate that an REE is determinate but not E-stable, and a light-colored circle “o” to indicate that an REE is explosive.[6] We left indeterminacy areas blank.

The top panel of Figure 1 shows the REE properties of the model without capital. Not surprisingly, the results are identical to those of Bullard and Mitra (2002). When the policy weight for inflation is larger than 1, the REE is always determinate and E-stable. The Taylor principle therefore guarantees the uniqueness and stability of the REE. The lower panel of Figure 1 shows the results for the model with capital. We note that the stability area nearly coincides with that

[6] When the system is indeterminate, we do not examine the E-stability of the equilibrium, as in Bullard and Mitra (2002).



Constant returns without capital



Constant returns with capital

Figure 1: Properties of the REE with the contemporaneous data rule and constant returns. The areas of determinacy and E-stability are marked dark. The areas of indeterminacy are left blank.

of the top panel. Most of the determinate and E-stable regions require an inflation weight higher than 1. When the inflation weight goes below 1, the required output weight must adjust upwards. Moreover, a determinate REE must also be E-stable, and vice versa, since there is no region denoted by squares or circles. The Taylor principle undoubtedly still guarantees stability in this case. We hence conclude that adding capital alone basically does not change the equilibrium properties of the model.[7]

Next, we examine the effect of increasing returns to scale. As a first step, we fix the policy parameters for output and inflation to be 1.5 and 0.5, as originally proposed by Taylor, and increase the level of θ to see if the REE properties will change. We find that when θ is between 1 and 1.05, the REE remains determinate and E-stable. But when θ rises to 1.06, the system becomes indeterminate and E-unstable. This is a first hint that the Taylor Principle might not lead to stable equilibria with increasing returns.

To examine the issue more closely, we next study how the policy parameters ϕ_y and ϕ_π affect the outcomes when increasing returns exist. We fix the level of increasing returns to be 1.09. We choose this number for the benchmark experiment because it is the lower bound of the recent value estimated by Laitner and Stolyarov (2004), and is the upper bound estimated by Basu and Fernald (1994). Other values will be examined shortly. The results are presented in Figure 2.

The results are striking. With moderate increasing returns, the Taylor Principle no longer guarantees stability: the area of indeterminacy and the area of determinacy and E-stability almost exactly reversed when compared with the constant-return case. While the area of determinacy and the area of E-stability still coincide, this area requires policy weights for inflation that are mostly less than one. Contrary to previous studies, this suggests that an *inactive* monetary policy is appropriate in terms of stabilizing the equilibrium.

One naturally wonders how the area of stability shifted from the right to the left as the level of increasing returns changes. Next we plot a series of three graphs in Figure 3 to show the transition

[7] Dupor (2001) show that with endogenous capital, an active interest rate rule will render the REE indeterminate. Carlstrom and Fuerst (2000), however, point out that Dupor's findings are largely due to the timing conventions in the continuous time model he used. Our results confirm that Carlstrom and Fuerst are correct.

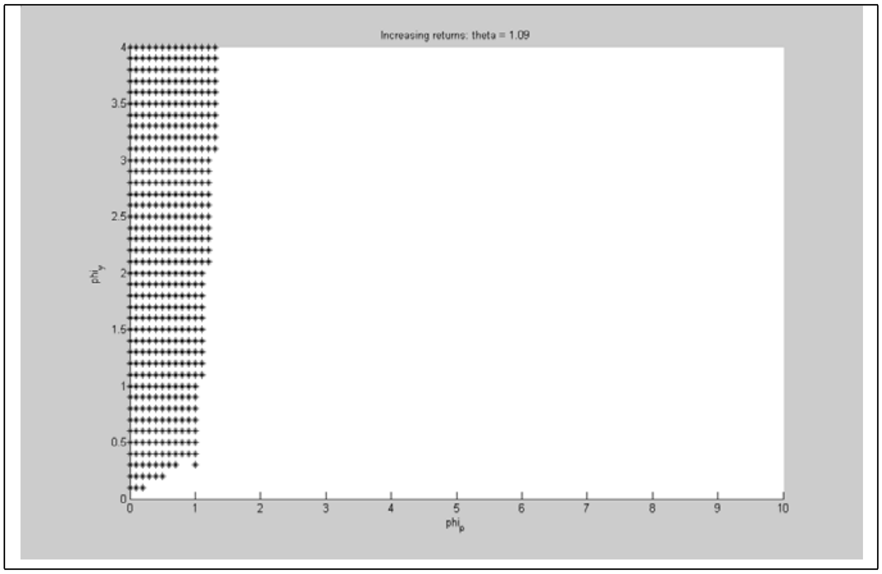
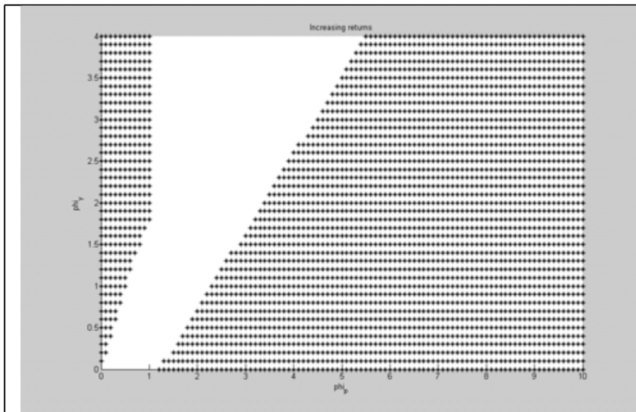


Figure 2: Properties of the REE with the contemporaneous data rule and increasing returns. The area of determinacy and E-stability is marked dark. The area of indeterminacy is left blank.

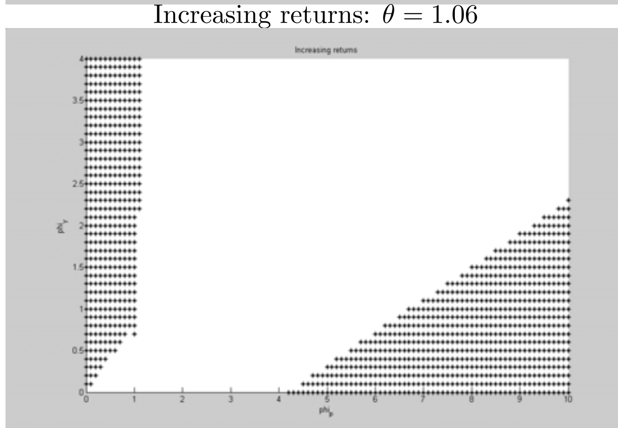
process. The level of returns to scale starts from 1.06 and increases at an increment of 0.01 in these graphs. We can clearly see that as θ increases, an area of indeterminacy and E-instability is created and gradually expands to the right and wipes off the stability areas on the right. In the mean time, a stable area occurs on the left and slowly expands. The E-stability and determinacy areas always coincide with each other, as in the case of constant returns (there is no area of squares).

In our analysis, we find that the level of markups, denoted by $\frac{\varepsilon}{\varepsilon-1}$, significant affects the required levels of increasing returns to generate indeterminacy. In our benchmark study, we set the markup level to be 1.05. It turns out that if we lower the markup level, the REE is more likely to become indeterminate. We show this finding in Table 2, where all results are obtained by setting the policy weight for output to 0.5 and for inflation to 1.5. When the level of markup is 1.03, for example, an increasing return of 1.04 is enough to generate indeterminacy. When the level of markup is 1.11, the required level of increasing returns is 1.12. This suggests that if an economy has small markups, it is more likely for the REE to be unstable.

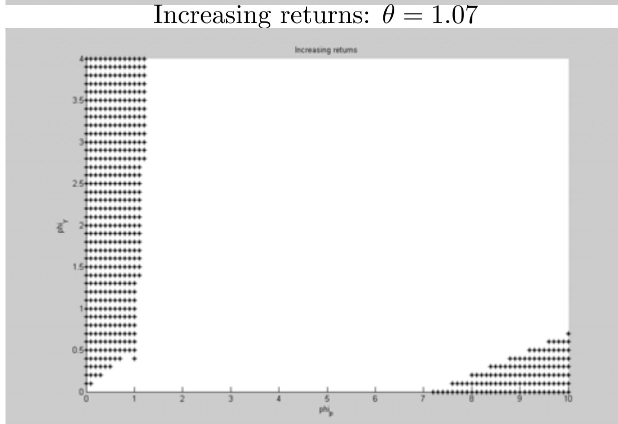
The series of results have important implications for policy making. First, it is no longer safe to implement the rule-of-thumb principle of reacting "more than one-for-one" to changes in the inflation rates. As Figure 2 shows, when increasing returns are at a moderate level, the Taylor



Increasing returns: $\theta = 1.06$



Increasing returns: $\theta = 1.07$



Increasing returns: $\theta = 1.08$

Figure 3: REE properties as returns to scale increase from 1.06 to 1.08. The areas of determinacy and E-stability are marked dark. The areas of indeterminacy are left blank.

Markup	**Lowest increasing returns leading to indeterminacy**
1.01	1.02
1.03	1.04
1.05	1.06
1.08	1.09
1.11	1.12
1.17	1.18

Table 2: Table Caption

principle will exactly lead to an unstable equilibrium. Second, the designing of policy rules should condition heavily on the status (level of increasing returns) of the economy. The combinations of policy parameters that lead to determinate and E-stable vary as the level of increasing returns vary. When θ is 1.06 (top panel of Figure 3), a strong response to inflation combined with a weak response to output will almost always guarantee stability, but when θ is 1.09 (Figure 2), such a policy always leads to instability.

In the next three sections we show that similar results hold for the other three variants of the Taylor rule.

4.2 Forward Expectations Rule

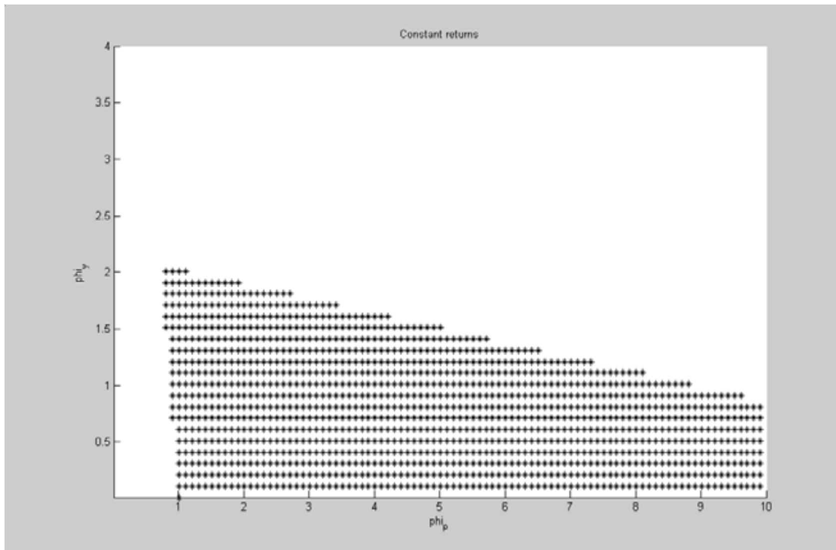
We now turn to the interest rate rule (21).

Just as in the previous section, a first experiment shows that when $\theta = 1.06$, the Taylor-suggested policy weights 1.5 for inflation and 0.5 for output no longer guarantee stability. We therefore make a side-by-side comparison of two different REEs, one with constant returns, and the other with increasing returns ($\theta = 1.09$). The results are presented in Figure 4.

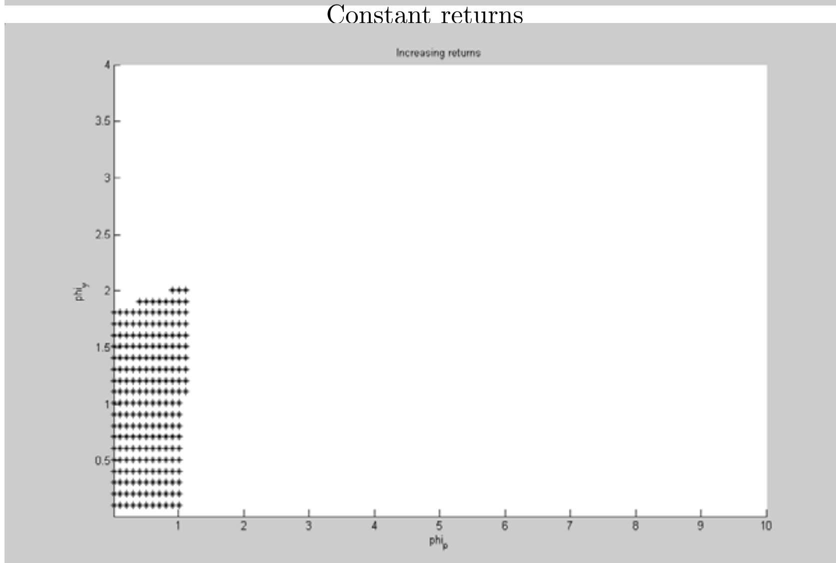
The top panel of Figure 4 displays the results for the case of constant returns to scale. The plot is again almost identical to the no-capital case studied by Bullard and Mitra (2002). While in general the stability area is smaller than the contemporaneous data case, a more than one-for-one response to inflation combined with a moderate response to output still guarantee the determinacy and E-stability of the REE. The lower panel of Figure 4 displays the results for the increasing returns case. The conclusion is again reversed. With increasing returns, a less than one-for-one response to inflation is required to obtain determinacy and E-stability of the REE. The smaller stability area compared with the contemporaneous data case shows that an expectation-based rule is in general less desirable.

4.3 Lagged Data Rule

We next examine the rule (20). We present the results in Figure 5.

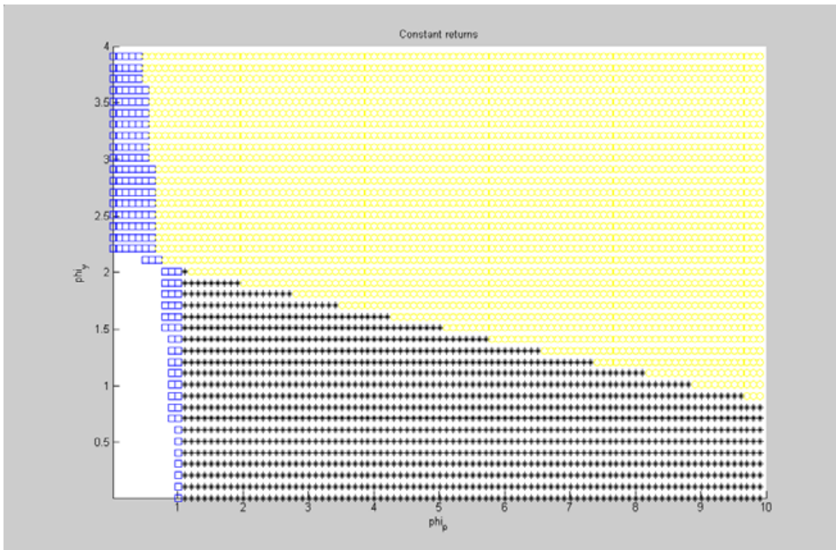


Constant returns

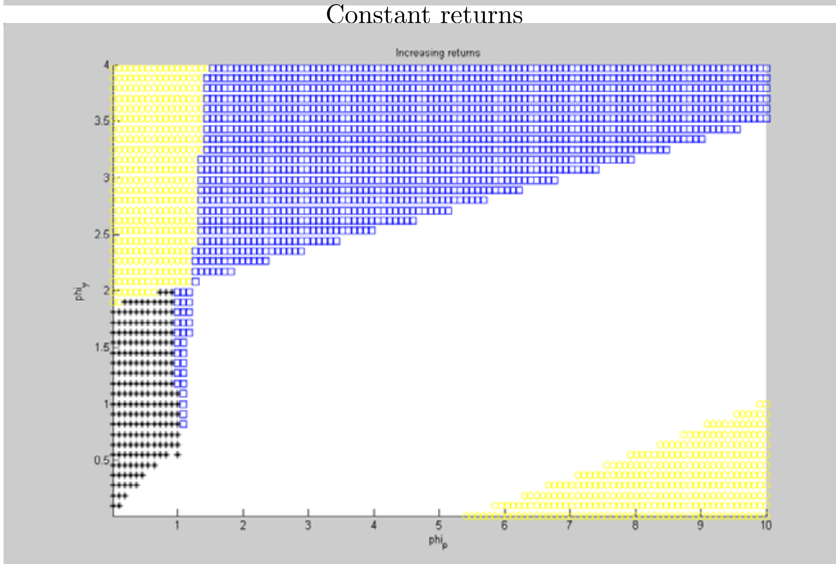


Increasing returns: $\theta = 1.09$

Figure 4: Properties of the REE with the forward expectations rule. The areas of determinacy and E-stability are marked with dark stars. The areas of indeterminacy are left blank.



Constant returns



Increasing returns

Figure 5: Properties of the REE with the lagged data rule. The areas of determinacy and E-stability are marked with dark stars. The Indeterminate areas are left blank. The determinate but E-unstable areas are denoted by squares. The explosive areas are marked by light circles.

The top panel of Figure 5 shows the results for the constant returns to scale economy. With a lagged data rule, it is no longer true that a determinate REE is always E-stable. Instead, two new areas are introduced. The areas denoted by squares represent determinate equilibria that are not E-stable. The areas denoted by light circles represent REEs that are explosive. While stability seems harder to achieve, it is still true that the Taylor principle basically guarantees determinacy and E-stability, as long as the weight for output is mild enough. The lower panel shows the results for the increasing returns economy. As before, the small area of determinacy and E-stability violates the Taylor principle and requires a less than one-for-one response to inflation. Active response to inflation leads to either indeterminacy or explosive REEs.

4.4 Contemporaneous Expectations Rule

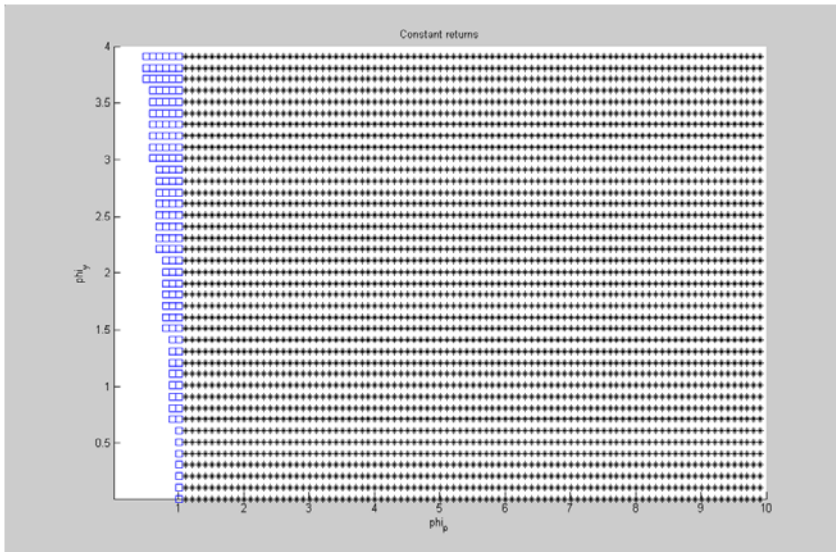
Lastly, we examine the economy with the rule (22). The results are presented in Figure 6.

Bullard and Mitra (2002) believe that the contemporaneous expectations rule is both practical and desirable – practical because current data on output and inflation are generally not available but can be estimated to form expectations, and desirable because it guarantees stability when the policy weight for inflation is larger than 1. This can be seen from the top panel of Figure 6. The large area of stability resides to right of the area where ϕ_π is equal to 1. However, as we introduce increase returns, the conclusion no longer holds. As shown in the lower panel of Figure 6, if we increase the level of θ to 1.09, the area of stability switches to the left, just as in the previous cases we studied. Now an active response to inflation will only lead to indeterminate or E-unstable REEs.

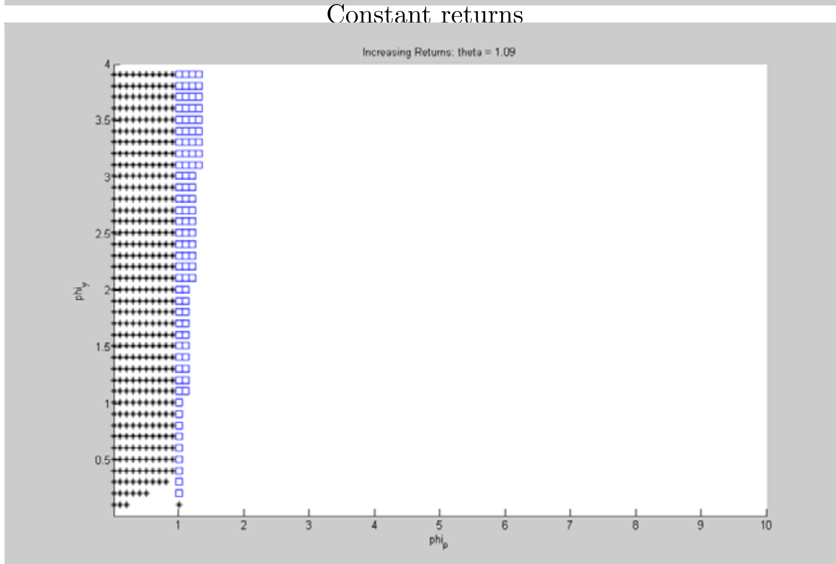
4.5 Discussion

When increasing returns are introduced, implementing the Taylor principle often leads to indeterminacy and E-instability. What explains this puzzling result? The key is to understand the role of increasing returns in generating self-fulfilling business cycles.

When Benhabib (1998) first explains the intuition of indeterminacy, he uses the example of sunspot-driven investment booms. When agents expect higher investment returns, they increase



Constant returns



Increasing returns: $\theta = 1.09$

Figure 6: Properties of the REE with the contemporaneous expectations rule. The areas of determinacy and E-stability are marked with dark stars. The Indeterminate areas are left blank. The determinate but E-unstable areas are denoted by squares. The explosive areas are marked by light circles.

investment and accumulate more capital. But with constant returns, the return of investment (marginal product of capital) decreases with more capital accumulation, and the expectations of higher returns will never be self-fulfilled. When increasing returns are high enough, however, more capital will actually *increase* the return of investment and fulfill the earlier expectations. In our context, this implies that with constant returns, we have the standard increasing marginal cost curve; but with increasing returns, the firms operates on the part of the marginal cost curve that *decreases* with the level of inputs.

The rest of the intuition is straightforward. In our model, the monetary authority's job is to dampen any fluctuations driven by inflation expectations. When consumers expect higher inflations, the monetary authority responds by raising the nominal interest rate more than one-for-one with the expected inflation rate. As a result, the real interest rate will rise, which in turn will curb the rise in aggregate demand. With lower demand and a standard marginal cost curve, firms will cut their prices – an action that goes against the earlier expectations of high inflation. This is why the Taylor principle leads to a determinate equilibrium with constant returns to scale. If the firms operate on the decreasing part of the marginal cost curve, on the other hand, lower demand will actually lead them to *increase* prices, which exactly fulfills the consumers' earlier expectations about high inflation rates. This is why the Taylor principle leads to indeterminacy in the increasing returns case.

5 Conclusion

This paper incorporates increasing returns into an otherwise standard New Keynesian model with capital. Within this framework, we re-examine the determinacy and E-stability of REE under four variants of the Taylor rule studied by Bullard and Mitra (2002). While Bullard and Mitra (2002) find that in most cases the Taylor principle is sufficient to guarantee both determinacy and E-stability, we find that with small increasing returns that are consistent with empirical estimates, these findings no longer hold. In particular, some levels of increasing returns require a less-than-one-for-one response of the interest rate rule to inflation to obtain determinacy and E-stability.

The results in this paper suggest that designing the interest rule is much more complicated than simply following a rule of thumb. In our context, a successful interest rule must condition on the level of returns to scale of the economy. There is no reason to believe that the returns to scale of the economy is constant over time. For example, when arguing about the existence of a "new economy," some researchers point out that the widespread usage of IT technology generates additional externality effect that gives rise to increasing returns. Our results suggest that the monetary authority may well be required to adjust its policy with such changes to ensure market stability.

This paper suggests that the types of interest rate rules that can maintain the stability of the REE are different when there are market failures in the economy. Given this result, opportunities now exist for us to study other implications of increasing returns for monetary policy making. In particular, we wonder what effect increasing returns will have when the monetary authority designs its interest rate rules by minimizing a cost function, either with discretion or with commitment. We leave this for future research.

6 Appendix

In this section we derive the E-stability conditions for all four variants of the interest rate rules. We re-write the system as

$$b_z z_t + b_k k_t = d_k E_t k_{t+1} + d_z E_t z_{t+1}, \tag{36}$$

$$k_{t+1} = e_z z_t + e_k k_t. \tag{37}$$

The second equation is derived from the capital accumulation equation that does not involve any expectations.

6.1 Contemporaneous Data and Forward Expectations Rules

With the contemporaneous data rule and the forward expectations rule, the information sets available for the learning agents are the same, therefore the E-stability conditions are similar. We assume agents have the perceived law of motion (PLM)

$$z_t = a + \psi k_t,$$

which is in the same form as the MSV solution under REE. The parameter vectors a and ψ will have to be learned. Given this PLM, we calculate the forward expectation of z_t as

$$E_t z_{t+1} = a + \psi E_t k_{t+1} = a + \psi E_t(e_z z_t + e_k k_t) = a + \psi e_z z_t + \psi e_k k_t.$$

Plugging this into (36), we get

$$z_t = (I - me_z)^{-1} b_z^{-1} d_z a + (I - me_z)^{-1}(me_k - b_z^{-1} b_k),$$

where $m = b_z^{-1} d_k + b_z^{-1} d_z \psi$. Therefore we obtain the T-mappings:

$$\begin{aligned} T(a) &= (I - me_z)^{-1} b_z^{-1} d_z a, \\ T(\psi) &= (I - me_z)^{-1}(me_k - b_z^{-1} b_k). \end{aligned}$$

The REE solution consists of values $\overline{a} = T(\overline{a})$ and $\overline{\psi} = T(\overline{\psi})$. The E-stability of $(\overline{a}, \overline{\psi})$ is governed by the local asymptotic stability of the matrix differential equation:

$$\frac{d}{d\tau}(a, \psi) = T(\alpha, \psi) - (a, \psi).$$

The conditions for expectational stability of the REE solutions are addressed in Evans and Honkapohja (2001, section 10.3). These conditions are that the eigenvalues of the matrices $DT(a)$ and $DT(\psi)$

all have real parts less than unity. The relevant matrices are:

$$
\begin{aligned}
DT(a) &= (I - me_z)^{-1} b_z^{-1} d_z, \\
DT(\varphi) &= e_k' \otimes N b_z^{-1} d_z - (e_z N m e_k)' \otimes N(-b_z^{-1} d_z) + (e_z N b_z^{-1} b_k)' \otimes N(-b_z^{-1} d_z),
\end{aligned}
$$

where $N = (I - me_z)^{-1}$ and a and ψ are evaluated at the steady state values.

6.2 Lagged Data Rule

With the lagged data rule

$$
i_t = \phi_y y_{t-1} + \phi_\pi \pi_{t-1},
$$

the implicit assumption is that the agents do not possess knowledge of current data. Therefore the perceived law of motion must be different. If we plug the interest rate rule into the set of equilibrium conditions, the system becomes

$$
\begin{aligned}
z_t &= F E_t k_{t+1} + G E_t z_{t+1} + H k_{t-1} + L z_{t-1}, \\
k_t &= e_z z_{t-1} + e_k k_{t-1}.
\end{aligned}
$$

The PLM of the agents is

$$
z_t = a + \gamma z_{t-1} + \psi k_{t-1}.
$$

Given this PLM, the T-mapping of parameters are derived as

$$
\begin{aligned}
T(a) &= F e_z a + G(\gamma a + a), \\
T(\gamma) &= F e_z \gamma + F e_k e_z + L + G(\gamma^2 + \psi e_z), \\
T(\psi) &= F e_z \psi + F e_k^2 + H + G(\gamma \psi + \psi e_k).
\end{aligned}
$$

The key matrices that determine the E-stability property of the REE are

$$\begin{aligned} DT(a) &= Fe_z + G(\gamma + I), \\ DT(\gamma) &= \gamma' \otimes G + I \otimes (G\gamma + Fe_z), \\ DT(\psi) &= e'_k \otimes G + I \otimes (Fe_z + G\gamma). \end{aligned}$$

6.3 Contemporaneous Expectations Rule

With the contemporaneous expectations rule

$$i_t = \phi_y E_{t-1} y_t + \phi_\pi E_{t-1} \pi_t,$$

our implicit assumption about agents' information set is that they do not possess knowledge of current data, and have to use past data to estimate today's output and inflation. We can substitute out the variable y_t and re-write the interest rate rule as

$$i_t = f_k E_{t-1} k_t + f_z E_{t-1} z_t.$$

The system can be re-written as

$$\begin{aligned} gi_t + b_z z_t + b_k k_t &= d_k E_t k_{t+1} + d_z E_t z_{t+1}, \\ k_t &= e_k k_{t-1} + e_z z_{t-1}. \end{aligned}$$

Plugging the PLM

$$z_t = a + \psi k_{t-1} + \gamma z_{t-1}$$

into the system, the system becomes

$$\begin{aligned} z_t &= FE_{t-1}k_{t+1} + GE_{t-1}z_{t+1} + Hk_{t-1} + Lz_{t-1} + Ma, \\ k_t &= e_z z_{t-1} + e_k k_{t-1}. \end{aligned}$$

Following the similar procedures, we derive the critical matrices as

$$\begin{aligned} DT(a) &= Fe_z + G(\gamma + I) + M, \\ DT(\gamma) &= \gamma' \otimes G + I \otimes (G\gamma + Fe_z - b_z^{-1} g f_z), \\ DT(\psi) &= e_k' \otimes G + I \otimes (Fe_z - b_z^{-1} g f_z + G\gamma). \end{aligned}$$

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