Constraint on the optical constants of a transparent film on an absorbing substrate for inversion of the ratio of complex p and s reflection coefficients at a given angle of incidence

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Constraint on the optical constants of a transparent film on an absorbing substrate for inversion of the ratio of complex \( p \) and \( s \) reflection coefficients at a given angle of incidence

R. M. A. Azzam and M. A. Habli

An absorbing substrate of complex refractive index \( n_2 - jk_2 \) can be coated by a transparent thin film of refractive index \( n_1 \) and normalized thickness \( \xi \) so that the ratio of complex reflection coefficients for the \( p \) and \( s \) polarizations of the film-covered substrate \( \rho \) is the inverse of that of the film-free substrate \( \rho \) at a given angle of incidence \( \phi \). A pair of parallel (metallic) mirrors, one uncoated and the other coated with a \( p \)-inverting layer, causes a beam displacement without change of polarization and with a certain net reflectance (insertion loss) \( R_i \). In this paper the constraint on \( n_1, n_2, k_2 \) for \( p \) inversion (\( \rho = 1 \)) is represented by a family of constant \( -n_1 \) contours in the \( n_2 - k_2 \) plane at \( \phi = 45, 60, \) and \( 75^\circ \). Along each solution curve, \( \xi \) and \( R_i \) are also plotted vs \( n_2 \) at constant \( n_1 \). Analysis of the effect of small errors of incidence angles, film refractive index, and thickness is presented for two specific designs using Al mirrors at 650 and 950 nm.

I. Introduction

At a given wavelength \( \lambda \) and angle of incidence \( \phi \), the change of the state of polarization of light on reflection from an uncoated absorbing (e.g., metallic) substrate is determined by the ratio

\[
\rho = \frac{R_p}{R_s}
\]

of complex reflection coefficients \( R_p \) and \( R_s \) for the linear polarizations parallel \( p \) and perpendicular \( s \) to the plane of incidence. If the substrate is coated by a transparent thin film, the ratio is changed to

\[
\rho = \frac{R_p}{R_s} \tag{2}
\]

By a suitable choice of film refractive index \( N_1 \) and thickness \( d \), it is possible to make

\[
\rho = 1/\rho \tag{3}
\]

i.e., \( \rho \) can be inverted as has been shown recently. In functional form, the condition of \( \rho \) inversion can be expressed as

\[
\rho(\phi, N_2)\rho(\phi, N_1, \xi, N_0) = 1, \tag{4}
\]

where \( N_2 = n_2 - jk_2 \) is the substrate complex refractive index and \( \xi \) is the film thickness as a fraction of the thickness period \( D \):

\[
\xi = d/D \tag{5}
\]

\[
D = (\lambda/2N_0)(N_1^2 - \sin^2 \phi)^{-1/2} \tag{6}
\]

\( N_1 \) and \( N_2 \) are normalized with respect to the refractive index \( N_0 \) of the transparent medium of incidence (usually air, \( N_0 = 1 \)).

If a \( p \)-inverting layer is applied to one of the two parallel (metallic) mirrors of a beam displacer (Fig. 1), the state of polarization of light is preserved after two reflections. The insertion loss of such a device is given by the net two-bounce intensity reflectance \( R_i \), which is the same for the \( p \) and \( s \) polarizations. Deviation from the exact condition of \( \rho \) inversion is determined by the deviation of the ratio of net complex \( p \) and \( s \) reflection coefficients

\[
\rho_n = \frac{\rho}{\rho_p} \tag{7}
\]

from 1. This can be broken down to separate magnitude and phase errors:

\[
\text{mag. error} = |\rho_n| - 1; \text{phase error} = \arg \rho_n \tag{8}
\]

In Ref. 1 Eq. (4) was considered as a constraint on \( \phi, \xi, N_1 \) for two specific values of \( N_2 \), namely, 1.212–j6.924 (Al at 0.6328 \( \mu \)m) and 9.5–j73 (Ag at 10.6 \( \mu \)m).

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In this paper we consider \( \rho \) inversion at a given angle of incidence over a wide range of substrates, corresponding to a rectangle of size 10 \( \times \) 20 in the \( n_2k_2 \) plane, with the constraint on optical constants being represented by contours of constant \( N_1 \) in that plane. The variation of the associated normalized film thickness \( \xi \) and net reflectance \( \mathcal{R} \) of the polarization-preserving parallel-mirror beam displacer (PPPMBD) with one coated mirror is determined along each \( N_1 = \) constant contour.

Finally, we conclude by an error analysis for two monochromatic designs using Al mirrors to show the effects of small errors of incidence angles, film refractive index, and thickness on the performance of such a system.

It should be noted that PPPMBD in which both mirrors are coated was considered earlier by Azzam\(^2\) and Azzam and Khan.\(^3\)

II. Constraint on Optical Constants for \( \rho \) Inversion

Equation (4) is complex and equivalent to two real constraints. For a given \( \phi \), \( \xi \) can be separated from Eq. (4) leading to a constraint on the optical constants \( N_1 = n_1 \) and \( N_2 = n_2 - jk_2 \) of the form

\[
f(n_1, n_2, k_2) = 0. \tag{9}
\]

This separation-of-variables method is explained in Ref. 1 and is not repeated here. Equation (9) is represented by constant-\( n_1 \) contours in the \( n_2k_2 \) plane. To generate one such contour, \( n_1 \) is assigned a fixed value, \( n_2 \) is scanned over a certain range, and for each \( n_2 \) Eq. (9) is solved for \( k_2 \) as its only unknown by numerical iteration. Only \( n_1 \) values of \( >1 \) are taken (i.e., the film is assumed to be more optically dense than the medium of incidence), and we restrict ourselves to a rectangle of 10 \( \times \) 20 size in the \( n_2k_2 \) plane (i.e., \( 0 < n_2 \leq 10, 0 < k_2 < 20 \)). The \( n_1 \) values are selected to generate reasonably well-spaced constant-\( n_1 \) contours in the \( n_2k_2 \) plane.

Figure 2 shows a family of equi-\( n_1 \) contours in the \( n_2k_2 \) plane for \( n_1 \) values (marked by each curve) from 1.2 to 1.9 representing the constraint on the film (\( n_1 \)) and substrate (\( n_2k_2 \)) optical constants for \( \rho \) inversion at \( \phi = 45^\circ \). All contours start from a common point \( (n_2, k_2) = (1,0) \). Those for \( n_1 \leq 1.59 \) terminate on the \( n_2 \) axis, and those for \( n_1 \geq 1.63 \) terminate on the \( k_2 \) axis. Figure 2 suggests that a film with refractive index in the narrow interval 1.59 < \( n_1 < 1.63 \) inverts \( \rho \) of substrates represented by points in a large part of the first quadrant of the infinite \( n_2k_2 \) plane.

Figure 3 shows the variation of the normalized film thickness \( \xi \) required for \( \rho \) inversion as a function of \( n_2 \) along each constant-\( n_1 \) contour of Fig. 2. Notice that \( \xi \to 0 \) as \( n_2 \to 1 \) and that \( \xi \) increases monotonically (notwithstanding the down-reading scale) toward \( \xi = \frac{1}{2} \) as \( n_2 \) increases above 1. \( \xi = \frac{1}{2} \) corresponds to a quarterwave optical thickness at oblique incidence; this is the required thickness of the \( \rho \)-inverting layer when the substrate is transparent (\( k_2 = 0 \)).

Figure 4 shows the two-bounce reflectance \( \mathcal{R} \) of the PPPMBD plotted vs \( n_2 \) with \( n_1 \) as a parameter along each constant-\( n_1 \) contour of Fig. 2. It is evident that beam displacement with polarization preservation on double reflection, realized using a single-layer coating on one mirror, is accompanied by some insertion loss. For the range of \( n_2k_2 \) in Fig. 2, an insertion loss of \( <3 \) dB (i.e., \( \mathcal{R} > 0.5 \)) requires a film with refractive index \( n_1 > 1.6 \) and a substrate with extinction coefficient \( k_2 \geq 3 \).
Figures 5–10 show additional results for angles of incidence of 60 and 75°, respectively. Except for a small compression of the range of n₁ at higher angles, the families of equi-n₁ contours in the n₂,k₂ plane for φ = 45, 60, and 75° are quite similar. The same applies to the contours of ζ vs n₂ at constant n₁ of Figs. 3, 6, and 9. Only the family of contours of R vs n₂ with an n₁ constant at 75°, Fig. 10, is significantly different from the corresponding families at 45 and 60° of Figs. 4 and 7.

III. Error Analysis

As specific examples, Table I lists two designs of PPPMBD using Al mirrors at wavelengths of 650 and 950 nm in the visible and near IR with incidence angles of 45 and 60°, respectively. The optical constants n₂,k₂ of Al are obtained from Ordal et al. The refractive index n₁ of the transparent ρ-inerting layer on mirror 2 is calculated as described in Ref. 1. The required indices of ~1.8 and 1.6 can be realized by coating materials such as Sc₂O₃ and LaF₃ as given by Arndt et al. and Pulker, respectively. The two-bounce net reflectance R of ~66 and 71% corresponds to an insertion loss of ~2 and 1.5 dB, respectively, which is acceptable.

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Table I. Refractive index $n_1$ and Thickness $d$ of a Transparent Film Coating on an Al Substrate of Complex Refractive-index $n_2$-$k_2$ Required to Invert $\rho$ at an Angle of Incidence $\phi$

<table>
<thead>
<tr>
<th>$\lambda$ (nm)</th>
<th>$n_2$</th>
<th>$k_2$</th>
<th>$n_1$</th>
<th>$d$ (nm)</th>
<th>$\phi$ (deg)</th>
<th>$\mathcal{R}$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td>1.240</td>
<td>6.600</td>
<td>1.792047</td>
<td>95.59</td>
<td>45</td>
<td>65.9044</td>
</tr>
<tr>
<td>950</td>
<td>1.750</td>
<td>8.500</td>
<td>1.616028</td>
<td>172.07</td>
<td>60</td>
<td>70.8287</td>
</tr>
</tbody>
</table>

$^a$ $\lambda$ is the wavelength of light, and $\mathcal{R}$ is the net polarization-independent reflectance after two reflections at the same range $\phi$ from a pair of parallel Al mirrors, one of which is uncoated, and the second is coated with the $\rho$-inverting layer. Such a system functions as a polarization-preserving beam displacer.

Table II. Magnitude and Phase Errors that Result when Errors of Incidence Angles $\Delta\phi_1$, $\Delta\phi_2$, Film Refractive-Index $\Delta n_1$, and Film Thickness $\Delta d$ are Introduced one at a time in the Parallel-Mirror Beam Displacers whose Characteristics are Specified in Table I

<table>
<thead>
<tr>
<th>$\lambda$ (nm)</th>
<th>$\Delta n_1 = 0.01$</th>
<th>$\Delta d = 1$ (nm)</th>
<th>$\Delta \phi_1 = 0.1$ (deg)</th>
<th>$\Delta \phi_2 = 0.1$ (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td>2.52E-3</td>
<td>0.6859</td>
<td>3.06E-3</td>
<td>1.2561</td>
</tr>
<tr>
<td>950</td>
<td>4.60E-3</td>
<td>1.8772</td>
<td>2.15E-3</td>
<td>1.5260</td>
</tr>
</tbody>
</table>

$^a$ The absolute values of the errors are indicated. $E-3$ is an abbreviated notation for $\times 10^{-3}$.
Table II lists the magnitude and phase errors, Eqs. (8), caused by introducing one at a time angle-of-incidence errors $\Delta \phi_1, \Delta \phi_2$ of $0.1^\circ$, film-refractive-index error $\Delta n_1 = 0.01$, and film-thickness error $\Delta d = 1$ nm. The first mirror is assumed to be the uncoated one (see Fig. 1) where the angle of incidence is $\phi_1$. These results indicate that the designs are reasonably tolerant to small angle errors (phase error $<0.25^\circ$, mag. error $<3.2 \times 10^{-4}$) but that the film refractive index and thickness should be tightly controlled to within 0.01 and 1 nm, respectively, to avoid appreciable polarization errors.

We wish to thank A. El-Saba for his assistance. M. A. Habli is presently at the Department of Electrical Engineering and the Center for Applied Optics, University of Alabama in Huntsville.

Reference


7. It is clear from Figs. 2, 5, and 8 and 4, 7, and 10, respectively, that higher net reflectances of PPPMBD are achieved with substrates, such as Ag, with $n_2 < 1$ and $k_2 \gg 1$.