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Abstract

We analyze the welfare impact of entrepreneur mobility in a two-country model. Increasing returns in production yield multiple equilibria that are stable under adaptive learning. Governments compete for the mobile resource by setting income taxes. We show that large welfare gains can arise from noncooperative taxation. If expectational barriers prevent the realization of high output equilibria, tax competition can sufficiently perturb expectations so that high steady states become attainable. Once in a high production regime, governments may institute cooperative tax increases or reductions so as to bring the economy to the global joint optimum without disturbing the regime.

Key words: competition for mobile factors, overlapping generations, multiple equilibria, bifurcations.

JEL codes: H2, F2, D83.

1 Introduction

International tax competition has attracted much interest in recent literature. At issue is the allocation of mobile tax bases, the location of which may be affected by strategic policy choices (in particular, tax reductions) of governments eager to attract them. The fear is that such unilateral and aggressive tax policies could prove harmful since public services might have to be cut as tax revenues dwindle.

Previous theoretical work has largely supported the above viewpoint. The Nash equilibrium of the tax competition game has been shown to be inferior to the hypothetical joint optimum attained from tax cooperation, and international tax coordination is usually suggested as the remedy for the potential welfare loss from tax competition. The voluminous literature that supports this view is surveyed in Wilson (1999).

There are, however, forces that can counterbalance the standard inefficiency argument against tax competition. Persson and Tabellini (1992) have shown, for example, that societies can find ways of adapting their internal political systems so as to prevent the slide toward unacceptably low levels of public spending. Edwards and Keen (1996) have observed that public decision makers may be self-serving and that, in such cases, tax competition may provide a useful constraint against unproductive public expenditures. Wilson and Wildasin (2004) survey these and other approaches toward modeling potential benefits from tax competition.

In this paper, we suggest that there are still additional circumstances in which international tax competition can be positively helpful. Our argument centers on the role that increasing returns, expectations, and learning dynamics play in determining the outcome of the tax competition game. While the study of nonconvexities, multiple equilibria, and learning have received much attention in recent macroeconomic literature, the implications of these phenomena in microeconomic policy models remain less known.

Our goal here is to demonstrate the effects of evolutionary expectations and learning in an overlapping generations model of tax competition that possesses multiple equilibria (some with high and some with low levels of output and well-being). Consideration of the time-adjustment of the economy following a policy change allows us to identify new positive effects that arise from international tax competition. First, when there are multiple equilibria, we show that tax competition can yield large (discrete) jumps in well-being, thus overturning the standard argument against noncooperative tax setting. In particular, tax competition can be much better than tax coordination if the effect of such coordination is to maintain a low productivity steady state. Second, tax competition can serve as a means of breaking a low-expectations trap that prevents a high output - high welfare equilibrium from being realized. And finally, once a high output production regime has been established (perhaps through tax competition), carefully chosen cooperative tax changes may be instituted so as to bring the economy from a Nash equilibrium to the global joint optimum without disturbing the newly attained high output regime. Thus, while expectational dynamics may cause stagnation at a low equilibrium trap, they can also support cooperative taxation of mobile factors.

What is also interesting is that depending on the importance of increasing returns, the cooperative tax reforms under the high output regime may include tax increases (as in the standard tax competition argument) or *tax reductions*. In other words, the Nash equilibrium in taxes, while always worse in welfare terms than the joint optimum, may involve taxes that are lower or higher than at the cooperative welfare maximum. The case in which unilaterally optimal taxes are higher than jointly optimal occurs when increasing returns are sufficiently strong.

We employ a symmetric two-country version of the overlapping generations model of social increasing returns due to Evans and Honkapohja (1995, 2001) to derive our results.[1] Overlapping generations models are natural vehicles for studying learning because they provide a clean example of one-step forward looking behavior. In this type of models, individuals pursue their interests given a forecast future reflected in expectations and expectations are adjusted based on observed history of the economy, thus leading to a dynamic process in which individuals alter their behaviors as they learn more about

[1] An overlapping-generations model of tax competition was also used by Wildasin and Wilson (1996) who analyzed land-value maximizing taxation under imperfect resident mobility across jurisdictions.

the economy. A rational expectations equilibrium is eventually attained as the outcome of the learning process.[2] The Evans and Honkapohja model is particularly attractive in that it yields a tractable example of an overlapping generations model with multiple equilibria. The learning dynamics in this model can be represented in terms of a single state variable and the effects of policy can be illustrated by simple diagrams. The model also builds a microeconomic foundation for the increasing returns in employment that can lead to multiple equilibria.

In our two-country extension of the Evans and Honkapohja model, all individuals are taken to be potentially mobile in their first period of life. During this time period individuals work and save so as to finance retirement in the second period. Income earned in either country is taxed according to the source principle, and the tax revenues are spent to supply publicly provided goods and services for the retired. We deviate from the standard tax competition models by treating the mobile individuals as household-producers (or entrepreneurs) whose labor is by nature mental, entrepreneurial, effort rather than physical work. These individuals do not exchange labor for a market wage but can, instead, set up shop and offer their services in either country depending on the available return. Skilled professionals (IT services, consulting, entertainment, design, arts, etc.) perhaps serve as a reasonable example. Contrary to the standard tax competition model in which aggregate capital is mobile and in fixed supply, our framework contains several mobile "human capital" factors, each in endogenous supply by entrepreneurs who respond to return opportunities in two markets. Just as mobile capital in the standard tax competition model is allocated so as to equalize returns between markets, the entrepreneurs in the present model respond to tax incentives and adjust the allocation of entrepreneurial services in the two locations. Optimal allocation of individual effort is characterized by equality of the real returns to it.[3]

Tax competition that results in lower taxation combined with international mobility of entrepreneurs can yield strong incentives to expand output. In our model, this effect is magnified by increasing social returns in a certain range of aggregate entrepreneurial effort. In particular, we assume that while effort by each individual (firm) is subject to decreasing returns in each location, external gains in productivity are reaped if the aggregate activity in a location exceeds a minimum threshold level. It is the interaction of these positive productivity externalities and the decreasing returns to individual effort that gives raise to multiple potential equilibria. Given this multiplicity, endogenous movements from one steady state to another can take place, and we are particularly interested in showing that tax competition can be a source of strongly favorable bifurcations in equilibria.[4]

[2] For an exhaustive discussion of adaptive learning, see Evans and Honkapohja (2001).

[3] Devereux, Lockwood and Redoano (DLR) (2002) have analyzed location decisions of mobile firms when countries compete in corporate taxation and financial capital is internationally mobile. The individual producers of our model are analogous to the mobile entrepreneurs of DLR, a difference being that we allow individuals to operate in both locations if doing so is profitable.

[4] The benefits from tax competition that we highlight arise from an expansion in aggregate entrepreneurial effort and are different from agglomeration effects in core-periphery models (see Baldwin and Krugman (2004)).

Learning dynamics allow us to classify potential equilibria into those that are stable under adaptive learning and those that are unstable. Stable equilibria are approached via an expectational adjustment process along which individual entrepreneurs observe the economy, adjust their forecasts for the future, and learn about the equilibrium values of the model variables. Since unstable equilibria cannot be approached by such small, gradual, steps, an unstable steady state that separates a high output equilibrium from a low steady state forms an expectational barrier that cannot be easily overcome. Only discrete changes in policy or other exogenous disturbances of sufficient size can perturb the prevailing expectations so as to cause an upward jump in the performance of the economy. We show that tax competition can serve in this welfare improving role.

2 Model

In this section, we expand the Evans and Honkapohja (1995, 2001) overlapping generations model to include two symmetric countries, H(ome) and F(oreign).

2.1 Production Technology

At any point in time, both countries H and F are the birthplace of a fixed number (K) individuals (entrepreneurs) who live for two time periods. Because of the assumed symmetry of the two economies, we discuss the model from the point of view of a representative individual born in country H in the beginning of time period t. Unless otherwise noted, all definitions and equations possess analogous counterparts that apply in country F.

In their first period of life, entrepreneurs invest in effort and produce a private consumption commodity which is sold to the currently retired. Given an entrepreneur-specific fixed factor ("firm"), the output of each individual is equal to

$$f(n_j, N_j) = n_j^{\alpha}\Psi(N_j), \quad j = H, F\,, \tag{1}$$

in the two locations. Here n_j refers to individual effort invested in either location (H or F), while N_j denotes the total supply of entrepreneurship in either country. We assume that $\alpha < 1$ so that decreasing returns to effort prevail given a fixed value of $\Psi(.)$. For simplicity, the production function is assumed to be the same in each location, i.e., f does not depend directly on j.[5]

Increasing external returns to entrepreneurship are represented by the function Ψ in (1). This function is taken to be increasing in N_j; thus, the larger the total supply of entrepreneurial effort in country j, the higher the productivity of each firm in that country. A particular functional form for social returns has been suggested by Evans and Honkapohja (1995). According to this specification,

$$\Psi(N_j) = \max\left[\widehat{I}, I_j\right]^{\beta}, \quad \widehat{I} > 0, \quad \beta \geq 1, \tag{2}$$

[5] We assume that all entrepreneurs can produce in both locations without additional (firm-specific or common) costs. Such costs could be included in the model without material changes in the results.

$$I_j \equiv \frac{\lambda N_j}{1 + a\lambda N_j}, \quad \lambda \in (0,1), \quad a > 0, \tag{3}$$

where $j = H, F$. By (2) and (3), an indicator of total entrepreneurial activity in a location, I_j, must exceed an exogenous threshold value, $\widehat{I}$, before external productivity gains can be felt. If I_j is larger than $\widehat{I}$ then, according to (2), $\Psi(N_j) = I_j^\beta$ in all production functions (1). Otherwise, there are no social returns and the production functions (1) just include the multiplying constant $\widehat{I}^\beta$.

The measure of entrepreneurial activity, I_j, reflects the sharing of experiences and ideas that naturally takes place when firms operate in proximity to each other. We assume that new ideas are created and broadcast at a uniform rate and that, for each particular firm, the fraction λ of all ideas is suitable to be applied. If it takes a time units to absorb a suitable new idea, then the total time required to receive and apply an idea equals $a + (\lambda N_j)^{-1}$. Per unit of time, therefore, the total number of usable ideas that any firm receives is I_j, as specified in (3). This quantity of usable ideas enters the firm-specific production functions as specified in (1) and (2). Because I_j is increasing in the total entrepreneurial activity at a location, external productivity gains increase with aggregate effort. There is, however, an upper bound for these gains: by (3), I_j approaches $1/a$ as N_j becomes very large.

2.2 Overlapping Generations

Individuals derive well-being from private consumption and from access to public services. The utility function of a representative individual born in country H at the beginning of time period t is taken to be

$$W_H = U(c_{H,t+1}) - V(n_{Ht} + n_{Ft}) + \mu \mathcal{U}(G_{H,t+1}). \tag{4}$$

In (4), $c_{H,t+1}$ denotes private consumption in retirement and μ reflects the importance of publicly provided benefits, $G_{H,t+1}$. Disutility of effort is represented by the function V. The utility functions U and $\mathcal{U}$ are assumed to be increasing and concave, while V is taken to be increasing and convex.[6]

National governments finance public consumption by appropriating a fraction τ_j of output in the country in each period. Accordingly, we have

$$\tau_j Y_{jt} = G_{jt}, \quad j = H, F, \tag{5}$$

where Y_{jt} equals the total (per capita) output in country j in period t and τ_j defines the national tax rate on entrepreneurial returns.[7] The subsequent (anticipated) budget

[6] The specification (4) implies that only public services provided by one's home country can be used when retired. This precludes the motivation to migrate so as to attain access to public benefits in the other country.

[7] In country H, $Y_{Ht} = f(n_{Ht}, N_{Ht}) + f(n^*_{Ht}, N_{Ht})$ and $N_{Ht} = K(n_{Ht} + n^*_{Ht})$, where n^*_{Ht} equals the entrepreneurial effort invested by Foreign entrepreneurs in H in time period t. Analogous definitions apply in country F.

constraints that apply to all individuals born in H at the beginning of time period t are

$$(1-\tau_H)p_t f(n_{Ht}, N_{Ht}) + (1-\tau_F)p_t f(n_{Ft}, N_{Ft}) = m_t, \tag{6}$$

$$p^e_{t+1} c_{H,t+1} = m_t. \tag{7}$$

In (6) and (7), p_t and p^e_{t+1} stand for the current and the anticipated future world price of private consumption (in money), respectively. Equation (6) defines the after-taxes income, m_t (measured in money), that each entrepreneur plans to spend in retirement in time period $t+1$ subject to the budget constraint (7). For simplicity, we assume that all (identical) individuals have identical price forecasts p^e_{t+1}. Money is taken to be the only means of transferring purchasing power from one time period to the next, and the two countries are assumed to have a common currency. The world stock of money remains always constant.

Entrepreneurs choose the quantity of effort in each location by maximizing (4) subject to the budget constraints (6) and (7). In this choice, N_j and G_j are treated as given, whereby first-order conditions for an interior optimum take the form[8]

$$\frac{V'(n_{Ht}+n_{Ft})}{U'(c_{H,t+1})} = (1-\tau_j) f'_1(n_{jt}, N_{jt}) \frac{p_t}{p^e_{t+1}}, \quad j = H, F. \tag{8}$$

Accordingly, the optimal n_{Ht} and n_{Ft} are such that an entrepreneur's marginal rate of substitution between effort and future consumption (on the left-hand side of (8)) is equal to the expected real return (in consumption) to such effort in both locations. Entrepreneurial activity is thus allocated so that the anticipated real returns to effort are equalized.

The world market for private consumption clears in every time period so that the world (per capita) consumption, $C^W_t = c_{Ht} + c_{Ft}$, is equal to the world (per capita) output, i.e.,

$$C^W_t = (1-\tau_H)Y_{Ht} + (1-\tau_F)Y_{Ft} = Y_t, \quad \forall t. \tag{9}$$

Market clearing also requires that the nominal savings of the young generation equal the world stock of money. If we set the constant world money stock equal to M, then $C^W_t = M/p_t$ for all t and therefore

$$\frac{p_t}{p^e_{t+1}} = \frac{(1-\tau_H)Y^e_{H,t+1} + (1-\tau_F)Y^e_{F,t+1}}{(1-\tau_H)Y_{Ht} + (1-\tau_F)Y_{Ft}} = \frac{Y^e_{t+1}}{Y_t}. \tag{10}$$

Substituting the price ratio (10) into the first order conditions (8) and the corresponding equations for individuals born in country F we obtain four *offer curve equations* that express the allocation of entrepreneurial effort across the two countries in time period t, n_t, as a function of the anticipated future amount of effort, n^e_{t+1}.[9] For the entrepreneurs

[8] Notation: f'_1 denotes the partial derivative of the function f with respect to its first argument.

[9] We formulate expectations and learning in a simple way using the level of employment rather than prices. Agents know that, by (1), Y^e_{t+1} is a function of future employment n^e_{t+1} which for brevity is assumed to be the same for all agents. By introducing price expectations, these assumptions could be relaxed without altering our results.

born in H, these equations require

$$\frac{V'(n_{Ht}+n_{Ft})}{U'(c^e_{H,t+1})} = (1-\tau_j) f_1'(n_{jt}, N_{jt}) \frac{Y^e_{t+1}}{Y_t}, \ j = H, F, \tag{11}$$

where

$$c^e_{H,t+1} = \frac{\left[(1-\tau_H) f(n_{Ht}, N_{Ht}) + (1-\tau_F) f(n_{Ft}, N_{Ft})\right] Y^e_{t+1}}{Y_t}. \tag{12}$$

Equations (11)-(12) together with the corresponding offer curves of individuals born in F determine the evolution of entrepreneurial effort in both countries over time.

The rational expectations (perfect foresight) equilibria are identified by the additional condition that expectations are correct, i.e., $n^e_{t+1} = n_{t+1}$. In the following section we adopt specific functional forms for the utility functions U, $\mathcal{U}$ and V, and illustrate the typical configurations of perfect foresight equilibria within the present model.

2.3 Symmetric Equilibria

We only consider symmetric equilibria at which $\tau_H = \tau_F = \tau$. Furthermore, for the purpose of illustrations, we set

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \qquad V(n) = n, \qquad \mathcal{U}(G) = \frac{G^{1-\sigma}}{1-\sigma}, \quad 0 < \sigma < 1. \tag{13}$$

Substituting these utility functions into (11) and taking into account that, due to symmetry, each entrepreneur devotes equal amounts of effort to both locations (whereby $Y_{jt} = 2f(n_{jt}, N_{jt})$ and $N_{jt} = 2Kn_{jt}$, $j = H, F$) we obtain the offer curve equation

$$n_t = 2^{-\sigma} \alpha (1-\tau)^{1-\sigma} (n^e_{t+1})^{\alpha(1-\sigma)} \max\left[\widehat{I}, \frac{\lambda N_{t+1}}{1 + a\lambda N_{t+1}}\right]^{\beta(1-\sigma)} \equiv F(\tau, n^e_{t+1}). \tag{14}$$

Conditional on τ and n^e_{t+1}, this offer curve determines the amount of entrepreneurial effort that all individuals invest in each country and subscript references to H and F have accordingly been dropped.

FIGURE 1: Offer Curves and Steady States.

Some typical offer curves derived from (14) are depicted in Figure 1. The concave segments of the offer curves obtain when, in comparison to the individual decreasing returns, the externality gains in the production technology (1) are sufficiently small. Along the first concave segment, to the left of the kink on each curve, there are no externalities at all (in this region, $I_j < \widehat{I}$; the kink occurs when $I_j = \widehat{I}$). Above the critical value $n_{\widehat{I}}$ at which externalities become operative (i.e., $I_j(n_{\widehat{I}}) = \widehat{I}$), a convex segment appears when increasing returns are sufficiently strong. At still higher levels of employment offer curves eventually turn concave because the positive externality effect is bounded from above. This yields the second concave region on the offer curves in Figure 1.[10]

Perfect foresight steady states are given in Figure 1 as the intersections of the offer curves with the 45-degree line (at these equilibria, $n_{t+1}^e = n_{t+1} = n_t$).[11] As shown in the figure, three alternatives exist as to the interior steady states. (In addition, the autarky equilibrium $n_t = 0$ always exists.) First, there may be a unique interior steady state to the left of the kink (equilibrium n_{Low} on the offer curve F). At such an equilibrium, young generations work relatively little, and output and consumption are low. Second, a unique steady state may occur to the right of the kink (equilibrium n_{High}^{NE} on F_{High}^{NE}). At n_{High}^{NE}, positive externalities are present and output and consumption are therefore high. The third possibility is that there are *multiple interior steady states* as illustrated by n'_{Low}, n_U, and n'_{High} along F'. At n'_{Low}, the realized output and consumption are much lower than at the high equilibrium n'_{High}. It is easy to see that welfare is predictably affected: *for any given level of taxation, welfare is an increasing function of n across steady states* so that all individuals are better off at n'_{High} than at n'_{Low}.

2.4 Learning Dynamics

We introduce dynamic adjustment paths toward rational expectations steady states using the adaptive learning approach. The basic idea is to begin with a particular forecast value of future effort, n_{t+1}^e. Given n_{t+1}^e, individuals choose their preferred level current effort, n_t, as described above. The resulting n_t defines the temporary equilibrium that corresponds to the initial expectations, n_{t+1}^e. If the realized temporary equilibrium in time period t differs from what was previously forecast for this time period (i.e., a rational expectations equilibrium was not attained), then individuals are assumed to revise their expectations. Such a revision yields the subsequent, improved, forecast, n_{t+2}^e, which in turn defines a new temporary equilibrium in time period $t+1$. If the observed expectational errors diminish over time as forecasts are updated and behavior adjusts, a rational expectations steady state is eventually attained. Such an equilibrium is called *stable under adaptive learning*. Equilibria that are unstable under adaptive learning cannot be approached along these sorts of adaptive learning paths.

The temporary equilibrium that corresponds to a given n_{t+1}^e can be read off the appropriate offer curve (14) (as illustrated in Figure 1), i.e.,

$$n_t = F(\tau, n_{t+1}^e). \tag{15}$$

We combine equation (15) with a simple description of expectational adjustments:

$$n_{t+1}^e = n_t^e + \frac{\kappa}{t}([F(\tau, n_t^e)] - n_t^e), \quad \kappa > 0. \tag{16}$$

According to this learning rule, individuals revise their expectations by an amount that is proportional to the previously observed forecast error. The proportionality factor

[10] Appendix C contains a detailed discussion of the concave and convex segments of the offer curve.

[11] For simplicity, we do not consider other "non-fundamental" rational expectations equilibria that exist in this model (e.g., sunspot equilibria).

κ/t is known as the gain parameter, and it determines the extent to which forecast errors are taken into account. If we set $\kappa = 1$ and select appropriate initial conditions, the forecast n^e_{t+1} is equal to the average of past values of n_t; in this case, individuals estimate the future supply of entrepreneurial effort by updating the sample mean of previous observations.[12] Equation (16) makes it clear that expectations depend on tax policy: the amount by which expectations are updated is a function of τ and thus any change in τ will have an impact on expectational dynamics.

Equations (15) and (16) define the dynamic adjustment paths toward rational expectations steady states. Proposition 1 of Evans and Honkapohja (1995, p. 225) can be extended so as to identify the stability properties of these types of equilibria. In particular, the interior steady states at which an offer curve cuts the 45°-line from above (e.g., n'_{Low} and n'_{High} along offer curve F') are stable under learning, whereas equilibria at which the 45°-line cuts the offer curve from below are unstable (e.g., n_U on F'). This means that for all initial expectations that fall between zero and n_U, learning dynamics converge to the low equilibrium at n'_{Low}, whereas for all n^e_{t+1} larger than n_U, the stable final equilibrium is n'_{High}. All unique interior steady states (such as n_{Low} and n^{NE}_{High}) are necessarily stable. It is clear from Figure 1 that, excluding unusual circumstances, unstable steady states, when they exist, will be located between two steady states that are stable under adaptive learning.

3 Gains from Tax Competition

In this section we analyze the consequences of cooperative and noncooperative tax policies for the steady state equilibria. We place particular emphasis on the role of multiple equilibria and expectations as these are the source of our sometimes counterintuitive results. To establish a connection to previous tax competition models we first discuss tax policy under the low productivity regime where externalities are not operative.

3.1 Standard Results in the Low Regime

Suppose that a steady state occurs at n_{Low} on offer curve F in Figure 1. Let the common tax rate at n_{Low} be τ^{opt}_{Low} and suppose this tax rate is locally jointly optimal. By this we mean that τ^{opt}_{Low} maximizes the joint welfare of the two countries, H and F, given that the level of entrepreneurial effort is too low for productive externalities to arise.

Individual well-being as a function of the common tax rate near n_{Low} is depicted by the curve labeled W_{Low} in Figure 2.

FIGURE 2: Taxes and Welfare

Curve W_{Low} has an inverted U-shape if public services have a positive weight in preferences, i.e., $\mu > 0$ in (4). This condition guarantees that individuals prefer some positive

[12] This formulation for learning about steady states is common in the recent literature. See Chapter 11 of Evans and Honkapohja (2001).

tax rate and public benefits to zero taxation with no public benefits (Appendix D gives formal arguments).

Let $W^j(\tau_H, \tau_F)$, $j = H, F$, denote steady state welfare for given tax rates (τ_H, τ_F). At the joint optimum n_{Low} both countries perceive an unilateral incentive to reduce taxation if per capita welfare is decreasing in the domestic tax rate $(\partial W^j(\tau_H, \tau_F)/\partial \tau^j < 0$, $j = H, F$, when $\tau_H = \tau_F = \tau_{Low}^{opt})$. This is typically the case as we show in Appendix E. If each country follows its unilateralist impulse at n_{Low} and lowers its tax rate, the symmetric steady state moves left from n_{Low} along W_{Low}.[13] That the final Nash equilibrium at n_{Low}^{NE} is worse in welfare terms than the joint optimum at n_{Low} is the *standard argument against international tax competition*. Furthermore, the relative location of n_{Low}^{NE} and n_{Low} on W_{Low} yields the *standard policy recommendation*: the two countries should cooperate and move toward higher taxation until the joint optimum at n_{Low} is re-established.

A particular numerical example of the usual tax competition argument is obtained by choosing the parameter values $\alpha = 0.9$, $\sigma = 0.6$, $\widehat{I} = 0.5$ and $\mu = 1$ in (2), (3) and (13). The jointly optimal tax rate, the Nash tax rate, and the corresponding levels of well-being are given in Table 1.[14]

TABLE 1: Standard Tax Competition in the Low Regime
Parameters: $\alpha = 0.9,\ \sigma = 0.6,\ \widehat{I} = 0.5,\ \mu = 1.$

	Nash Equilibrium	Joint Optimum
τ	0.14	0.35
W	0.86	1.53

3.2 Switching Production Regimes

The example in Table 1 is special in that neither competitive nor cooperative changes in taxes alter the prevailing production regime. That is, both at the joint optimum at n_{Low} and at the Nash equilibrium at n_{Low}^{NE}, there is no role for social returns in individual production functions. In Figure 2, this constancy of the production regime means that both the initial and the new steady states lie on the same welfare curve, namely W_{Low}. In Figure 1, this means that both n_{Low} and n_{Low}^{NE} occur on offer curves (such as F and F_{Low}^{NE}, respectively) that intersect the 45-degree line along the first concave segment of each respective curve. The offer curve that yields the Nash equilibrium, F_{Low}^{NE} say, lies above F because, given any positive n_{t+1}^e, all individuals invest more effort in production when taxes are lower (see equation (14)).

[13] For simplicity, we assume that individual learning is fast compared to the pace at which tax reductions take place. This guarantees that the temporary equilibria that are observed as the economies adjust toward a new symmetric steady state occur near the W_{Low} curve, which is then a reasonable approximation to adjustment paths.

[14] The Mathematica programs that were used to develop the numerical examples in this paper are available from the authors upon request. Note that, as long as we remain in the low production regime where externalities are not observed (as in Table 1), it is not necessary to specify the values of the parameters a, λ, and K that define the externality function $\Psi(.)$.

Outcomes that are significantly different can occur for alternative parameter values. If the positive response to reduced taxation is sufficiently large, the offer curve that yields the Nash equilibrium can reach a position such as depicted by curve F_{High}^{NE} in Figure 1. In such a case, following a period of adjustment during which expectations consistently point toward expansion and all individuals continuously increase production, a steady state is attained at n_{High}^{NE} on F_{High}^{NE}. The switch from a low joint optimum at n_{Low} on offer curve F to a Nash equilibrium at n_{High}^{NE} on F_{High}^{NE} is not a smooth local perturbation near an initial steady state (such as in Table 1) but involves a move from one production regime to another (a *bifurcation*): on F_{High}^{NE} the low productivity steady state near n_{low} no longer exists, and a high output production regime near n_{High}^{NE}, along the second concave segment of the offer curve, has appeared.

The discrete improvement in individual well-being that accompanies the movement from n_{Low} to n_{High}^{NE} is shown in Figure 2. The Nash equilibrium now occurs on curve W_{High} which, depending on the strength of external productivity gains, can be located much above W_{Low}. This means that, irrespective of the standard arguments against tax competition, there are cases in which tax competition can play a positive role. In particular, *when the supply of a mobile resource is highly responsive to tax reductions due to external returns, tax competition can push the competing economies well beyond their customary levels of performance*. Thus, tax competition need not always be a "race to the bottom"; outcomes that are worse can persist if coordinated policy of higher taxation ends up maintaining a low productivity regime.[15]

Table 2 gives a numerical example illustrating the switch in the production regime. (We postpone the discussion regarding the systematic differences between the parameter values in Tables 1 and 2 to Section 4 below.)

TABLE 2: Switching Production Regimes

Parameters: $\alpha = 0.5,\ \beta = 2.5,\ a = 0.2,\ \lambda = 0.02,$
$\widehat{I} = 0.97,\ K = 300,\ \sigma = 0.25,\ \mu = 2.5.$

	Low Joint Optimum	High Nash Equilibrium	High Joint Optimum
τ	0.50	0.43	0.44
W	1.55	206.72	206.74

Of course, bifurcation gains from tax competition cannot exist unless multiple production regimes can occur. In the present model, the source of the potential multiplicity is the externality function $\Psi(.)$ that includes an externality threshold. Depending on the height of this threshold and the amount of aggregate effort invested by individuals either a low or a high productivity regime is attained.[16]

[15] Bhagwati (2002) has argued that the race-to-the-bottom nature of tax competition is little supported by empirical evidence.

[16] Technological complementarities can also create multiple production regimes. See Honkapohja and Turunen-Red (2002).

Even when multiple production regimes are present, there is an additional consideration that can prevent the favorable regime switch. At issue is the location of the low productivity Nash equilibrium that generally exists on curve W_{Low} in Figure 2. If, as shown in Figure 2, this low Nash equilibrium occurs sufficiently near the joint optimum at n_{Low}, then, despite tax competition that lowers taxes, there is no regime switch; the tax reductions in this case are not large enough to cause a jump from curve W_{Low} to W_{High} because the high productivity steady state does not exist near n_{Low} (in Figure 1, while the offer curve pivots up to a position shown by F^{NE}_{Low}, the Nash equilibrium is still found on the first concave segment of the offer curve where no social externalities exist). If, on the other hand, the low Nash equilibrium lies suitably far to the left from the joint optimum on W_{Low} (along the dotted segment of W_{Low} in Figure 2), then a process of tax reductions that approaches this Nash equilibrium necessarily involves a jump to the high equilibrium regime. This is because the low productivity steady state ceases to exist as the common tax rate is reduced below the cut-off value τ_{cut} in Figure 2 (in Figure 1, the low steady state disappears as the offer curve pivots up and approaches a position such as illustrated by curve F^{NE}_{Hig}). Below τ_{cut}, the competitive tax reduction and learning process converges to the high productivity Nash equilibrium n^{NE}_{High} on W_{High} and a discrete jump in well-being, such as illustrated by the numerical example in Table 2, must exist.

The *cut-off tax,* τ_{cut}, is the lowest common tax rate at which the low productivity equilibrium exists. By (2) and (3) and given all parameter values, τ_{cut} can be solved from the equation

$$\widehat{I} = \frac{2\lambda K n(\tau)}{1 + 2a\lambda K n(\tau)}, \tag{17}$$

where $n(\tau)$ is the steady state solution for effort in the low productivity regime as obtained from the offer curve equation (14), i.e.,

$$n(\tau) = 2^{-\frac{\sigma}{z}} \alpha^{\frac{1}{z}} (1-\tau)^{\frac{1-\sigma}{z}} \widehat{I}^{\frac{\beta(1-\sigma)}{z}}, \quad z \equiv 1 - \alpha(1-\sigma). \tag{18}$$

For the parameter values in Table 2 for which a regime switch does exist, the cut-off tax equals $\tau_{cut} = 0.495$ and the low Nash equilibrium to the left of it occurs at $\tau^{NE}_{Low} = 0.48$. To illustrate the alternative possibility, we can expand our first example in Table 1 by augmenting that parameter set by the following: $\beta = 2.5$, $a = 0.2$, $\lambda = 0.02$ and $K = 100$. Then, $\tau_{cut} = 0.11$ which is smaller than the low Nash equilibrium tax $\tau^{NE}_{Low} = 0.14$; for these parameter values a tax competition process that begins from the low joint optimum at $\tau^{NE}_{Low} = 0.35$ can only yield a local deterioration in welfare.

While sufficient conditions characterizing the relative location of the cut-off tax and the low productivity Nash equilibrium are not available, we can point out circumstances in which a bifurcation from tax competition is more likely. Generally, this is the case when the value of the cut-off tax is high, i.e., the productive externality is relatively easy to reach. This happens when new ideas are adopted quickly (a is small), a large fraction of ideas is suitable for others to use (λ is high), the population is large (K is large), and the externality is substantial (β is high). While one would expect that a low externality threshold $\widehat{I}$ would also raise the cut-off tax and therefore make a bifurcation of steady states more likely this is not always the case. Because the steady state solution $n(\tau)$ in (18) depends on $\widehat{I}$ the cut-off tax may decline when the threshold parameter $\widehat{I}$ gets smaller (details of these effects are given in Appendix F).[17]

When the externality threshold is high and/or the low productivity Nash equilibrium prevents the high output regime from being reached via competitive tax reductions, there is a new role for cooperative policy. In this case and in contrast to the standard tax coordination recommendation, the optimal coordinated policy now requires that, *starting from the low joint optimum, taxes are reduced below the cut-off tax rate so that the high productivity regime is attained.* The cooperative tax reductions should continue until the high regime optimum at τ^{opt}_{High} is reached. Figure 2 illustrates.

3.3 Breaking Expectational Barriers

A further result can be obtained using Figures 1 and 2, and this brings forth the role of expectational dynamics.

Assume, as earlier, that the initial equilibrium is located at n_{Low} on offer curve F in Figure 1 where the common tax rate is τ^{opt}_{Low}. Suppose that, in an effort to guide the economy toward the higher productivity regime, the two countries undertake a joint tax reduction. As a result, the offer curve pivots up; let it reach the position F'.[18]

The shift from offer curve F to F' in Figure 1 yields a bifurcation which is different from the shift from F to F^{NE}_{High} discussed above. The bifurcation from F to F' expands the set of equilibria: while the low output steady state still exists on F' at n'_{Low}, two additional equilibria at n_U and n'_{High} now appear. Of these three, the high steady state at n'_{High} is clearly the most desirable one as welfare there is the highest. The question becomes: can the steady state at n'_{High} be actually reached, if initial expectations support the low equilibrium at n_{Low}? In view of what has been assumed about expectational dynamics this cannot be the case.

While entrepreneurs do respond positively and expectations are adjusted upwards as taxes decline, these expectational dynamics come to a halt once the steady state at n'_{Low} on F' is reached. This occurs because n'_{Low} is a stable equilibrium under adaptive learning. Accordingly, to the left of n'_{Low}, expectations and behavior are adjusted toward n'_{Low}; to the right of n'_{Low}, an analogous adjustment process brings realized temporary equilibria back to n'_{Low}. The presence of the unstable steady state at n_U guarantees that n'_{High} cannot be attained.

Figure 1, however, suggests a remedy for the expectational impasse that supports the low productivity steady state: taxes must be cut more decisively so that the hold of low expectations is broken. In Figure 1, this requires that offer curve F' pivots up

[17] For example, the value of $\widehat{I}$ is lower in Table 1 than in Table 2 and yet the parameter set of Table 2 yields a bifurcation jump in well-being whereas the augmented parameter set of Table 1 does not. See Section 4 below for a complete discussion.

[18] In Figure 2, this tax reduction corresponds to a leftward movement along curve W_{Low} to the region of W_{Low}, above τ_{cut}, where the high equilibrium (curve W_{High}) also exists.

sufficiently far so that both the low steady state near n'_{Low} and the unstable equilibrium at n_U cease to exist. But, according to our previous discussion and assuming that the low output Nash equilibrium n^{NE}_{Low} does not pose a barrier (i.e., $\tau^{NE}_{Low} < \tau_{cut}$ in Figure 2), *tax competition between the two countries can be a means of reaching this precise outcome* (compare offer curves F' and F^{NE}_{High} in Figure 1).[19]

While it is true that in a situation such as depicted in Figure 1 governments have an incentive to cooperate and by doing so they may be able to attain a high output state (near n^{opt}_{High} in Figure 2), our point is that, even if such policy cooperation were feasible and sufficiently effective, it may not yield very significant further welfare gains. Seemingly noncooperative policies can yield an outcome (n^{NE}_{High} on curve W_{High} in Figure 2) that is significantly better than the initial, perhaps cooperative, equilibrium. Regarded this way, *tax competition may at times be a reasonable substitute for tax cooperation* when such coordinated action is not possible.

4 Nash Equilibrium and Policy in the High Regime

Thus far, we have emphasized the role of tax competition in reaching the high productivity regime when external social gains potentially exist. When tax reductions are uncoordinated, one may worry about the relative inefficiency of the final Nash equilibrium (it will be worse in welfare terms than the potential joint optimum on curve W_{High} in Figure 2) and whether it is possible to aim for the global joint social optimum (on W_{High}) without simultaneously destroying the positive incentives and expectations that support the high output regime.

Fortunately, we may appeal to the same expectational inertia that, in the previous section, created the low equilibrium trap. In the case here, once the high productivity regime near the high employment Nash equilibrium (at n^{NE}_{High} on W_{High}) has been established, a *coordinated tax reform toward the global optimum can be undertaken without causing a plunge back to the low productivity state.* This is because expectational inertia maintains the high steady state near n^{NE}_{High} once this equilibrium has been realized. Only a very large increase in taxation that severely impacts expectations can re-establish the low productivity steady state. In Figure 2, this sort of a tax increase would have to raise the common tax above the rate τ^u_{cut} that denotes the upper limit of taxation at which both the high steady state and the unstable equilibrium near n_U cease to exist. In Figure 1, this corresponds to offer curve F^{NE}_{High} pivoting down to a position near curve F. A shift from curve F^{NE}_{High} to F' would not suffice because as long as the unstable steady state exists it serves as an expectational barrier that supports the high productivity regime.

There is also an issue about the direction of the optimal coordinated policy in the high regime. Above, and in Figure 2 as well, we have maintained the usual intuition whereby taxes at the Nash equilibrium are necessarily lower than what is socially optimal. Then, the optimal intervention at the Nash equilibrium is a coordinated tax increase. But,

[19] However, if $\tau^{NE}_{Low} \geq \tau_{cut}$ tax competition must be supplemented by further cooperative tax reductions.

as it turns out, this is not always the case. When productive externalities are strong, it is quite possible that the Nash equilibrium in taxes in the high productivity regime occurs to the *right* of the joint optimum (on curve W_{High} in Figure 2). In this case, the noncooperatively established Nash taxes are actually *higher* than what is socially optimal and *the optimal coordinated policy should aim toward reducing them further.* Table 3 gives a numerical example.[20]

TABLE 3: Nash Equilibrium and Social Optimum

Parameters: $\alpha = 0.9,\ \beta = 2.5,\ a = 0.2,\ \lambda = 0.02,$
$\widehat{I} = 0.75,\ K = 100,\ \sigma = 0.6,\ \mu = 1.$

	Low Joint Optimum	High Nash Equilibrium	High Joint Optimum
τ	0.35	0.32	0.17
W	1.70	15.62	17.81

This raises the question about the relative location of the high Nash equilibrium and the global joint optimum. In particular, what is the role of the various parameter assumptions in creating a paradoxical case such as in Table 3, and when does the usual intuition about the Nash equilibrium prevail (as in Table 2)? Some light can be shed on this issue with the help of Figure 3 and Tables 4A and 4B below.

FIGURE 3: Effort Regions in the High Regime

Whether the Nash equilibrium involves taxes that are lower or higher than what is jointly optimal depends on the sign of the unilateral welfare derivative at the joint optimum. If $\partial W^H(\tau_H, \tau_F)/\partial \tau_H < 0$ when $\tau_H = \tau_F = \tau_{Low}^{opt}$, country H will compete with country F by lowering its tax rate and the Nash equilibrium then occurs to the left of the joint optimum. In the opposite case, when $\partial W^H(\tau_H, \tau_F)/\partial \tau_H > 0$ at the joint optimum, each country will raise its domestic tax whereby the Nash equilibrium must lie to the right of the optimum.

The sign of the welfare derivative $\partial W^H(\tau_H, \tau_F)/\partial \tau_H$, in turn, crucially depends on the direction and size of the reactions in (steady state) entrepreneurial effort when taxes are changed. Of particular importance are the derivatives $\partial n_H(\tau_H, \tau_F)/\partial \tau_H$ and $(\partial n_H(.)/\partial \tau_H + \partial n_F(.)/\partial \tau_H)$ that indicate the changes in the domestic supply of effort and the total (domestic and exported) supply of effort in each country (see equation (76) in Appendix E). Expressions for these derivatives are obtained in Appendices A and B below, and Appendix B also determines the boundary values for the regions in which the signs of the derivatives $\partial n_H/\partial \tau_H$ and $\partial n_F/\partial \tau_H$ vary. These possibilities are indicated by the signs of the derivatives in Figure 3, where the boundary values are denoted by

[20] For the parameter values of Table 3, $\tau_{cut} = 0.326 > \tau_{High}^{NE} = 0.32$, so that tax competition that starts from the low joint optimum does cause a shift to the high productivity regime. The low Nash equilibrium does not pose a barrier to the jump because $\tau_{Low}^{NE} = 0.15 < \tau_{cut} = 0.326$.

$n_{\widehat{I}}$, n_4, n_3 and n_1.[21] In Appendix B, the various regions are discussed as Cases (i) - (v), which are also shown in Figure 3.

If there are no productive externalities at all, $\partial n_H/\partial\tau_H < 0$ and $\partial n_F/\partial\tau_H > 0$ as intuition would suggest. In Figure 3, this no externality region occurs to the left of $n_{\widehat{I}}$ which is the level of individual effort that, when aggregated across all entrepreneurs, yields a total supply of effort precisely equal to the externality threshold. Thus, to the right of $n_{\widehat{I}}$ the positive externality is present. In this region, when the entrepreneurs work sufficiently hard, the impact of the externality is eventually mitigated by the individual decreasing returns (recall the discussion of the production function (1)). There is a limit value n_1, so that when $n > n_1$ we again have $\partial n_H/\partial\tau_H < 0$ and $\partial n_F/\partial\tau_H > 0$. Below n_1, where the impact of the externality is stronger, the derivatives $\partial n_H/\partial\tau_H$ and $\partial n_F/\partial\tau_H$ can take opposite signs as shown in Figure 3. As an extreme example, in the region $n \in (n_{\widehat{I}}, n_4)$ *both* effort derivatives are positive. Table 4A gives the boundary values of effort obtained using the parameters in Tables 2 and 3.

TABLE 4A: Effort Derivative Regions

	Table 2 parameters	Table 3 parameters
n_1	9.99	30
n_3	1.15	2.43
n_4	0.83	0.70
$n_{\widehat{I}}$	0.10	0.22

TABLE 4B: Effort Value Range

	Table 2 parameters	Table 3 parameters
$n_{\max}$	29.98	3.33
$n_{\min}$	12.47	2.24
n_{High}^{NE}	14.62	2.26
n_{High}^{opt}	14.30	2.79

It is because the signs of the n_H and n_F derivatives at the joint optimum can vary that the Nash equilibrium in taxes in the high regime can be located on either side of the joint optimum. The boundary values, $n_{\widehat{I}}, ..., n_1$, depend on the technology parameters $(\alpha; a, \lambda, \beta, K)$ and the preference parameter σ, and these parameters also determine the range of the actual values of effort that can be realized at symmetric steady states, including the joint optimum. The symmetric n-solution $n(\tau)$ can be computed from the offer curve equation (14), from which we can obtain the maximum value of effort, $n_{\max}$, as the value $n(0)$, while the the smallest level of effort, $n_{\min}$, equals $n(\tau_{cut})$.[22] Table 4B gives the range of n-values for the parameters in Tables 2 and 3. In Figure 3, these ranges are shown as the two shaded bars, labeled Table 2 and Table 3 respectively.

Recall that the parameter values in Table 2 yield the high Nash equilibrium in which taxes are lower than jointly optimal, whereas the parameters in Table 3 give the opposite conclusion. Using Figure 3, we can clearly detect that the two examples are very different. The parameters in Table 2 yield a range of feasible values of effort that falls in

[21] In fact, Figure 3 and the Appendices use the more convenient derivatives $\widehat{n}_H \equiv (\partial n_H/\partial\tau_H)/n_H$ and $\widehat{n}_F \equiv (\partial n_F/\partial\tau_H)/n_F$.

[22] Note that $n_{\min} = n(\tau_{cut})$ as long as both the joint optimum and the Nash equilibrium in the high regime occur to the left of τ_{cut} in Figure 2.

its entirety within the area $n > n_1$ where a reduction in τ_H expands domestic activity at the expense of effort directed to the foreign market. It is in this example that the Nash taxes are lower than optimal. For the parameters in Table 3, however, the range of steady state values for n falls to one of the regions (n_4, n_3) or (n_3, n_2) where the effect of the externality is stronger. Here, entrepreneurial effort does not respond to taxation as usually expected and because of this there is little incentive to reduce taxes. The Nash equilibrium tax rate in the high regime consequently remains close to the cut-off tax τ_{cut} $(= 0.326)$ and while the modest tax reduction to τ^{NE}_{High} $(= 0.32)$ yields a bifurcation jump in welfare, a tax rate that is much lower is globally jointly optimal.

Though changes in parameter values have been minimized in Tables 2 and 3, there are systematic differences between them. First, while the externality parameters a, λ, and β are the same in both tables, the number of entrepreneurs is larger in Table 2. The larger population raises the value of the externality function $\Psi(.)$ and reduces $n_{\widehat{I}}, ..., n_1$, thus pushing the n-value range to the right in Figure 3. This works toward reducing the Nash equilibrium tax. An analogous effect is obtained by choosing a lower value of α $(= 0.5)$ in Table 2 than in Table 3 ($\alpha = 0.9$). By this change, the impact of diminishing returns is made larger in Table 2 and this further reduces n_1. The preference parameter σ is higher in Table 3 than in Table 2 and this magnifies the previous changes by reducing incentive to work in Table 3 (when σ is high, less utility is attained from any given level of consumption). Finally, the weight of public consumption in preferences, μ, is higher in Table 2; the added importance of public consumption raises the jointly optimal tax rate and mitigates the impact of the production externality that tends to lower it.

In sum, the numerical examples established by Tables 2 and 3 show that, depending on the location of the local Nash equilibrium with externality, the optimal coordinated policy in the high production regime may involve both tax increases and tax reductions.

5 Conclusions

In this paper, we have analyzed entrepreneur mobility and tax competition in a simple two-country overlapping generations model. A special feature of the model is the multiplicity of equilibria that reflects the possibility of increasing social returns in production. Learning dynamics play an important role in identifying the steady states that are stable under adaptive adjustment of expectations. This sort of dynamics can influence the outcome of the tax competition game by determining production regime that unilateral tax reduction can attain.

We have shown that there are circumstances in which competitive tax setting can positively enhance (and not reduce) the gains from factor mobility. In particular, tax competition can be the source of large bifurcational gains in welfare. In the present model, such gains are realized when unilateral tax reductions cause a sufficiently large shift in the supply of entrepreneurial effort and this change significantly alters individual expectations. Then, it may be possible for the economy to reach a new high effort, high output, steady state in which external benefits from the high level of activity are realized.

Because expectations have an effect on the current supply of effort, the large gains that potentially exist may not be reaped if individual expectations remain persistently low (this happens when the existence of an unstable equilibrium creates a trap that expectational dynamics cannot overcome). In such circumstances, tax competition may be helpful because it can significantly perturb the status quo, thus encouraging all individuals to work much harder. Most significantly, once the existence of a new high output production regime has been learned by all, this high productivity steady state can still be sustained even if some further cooperative tax increases are undertaken. In other words, the seemingly radical policy choice of free tax competition combined with cooperative tax increases at a later stage may sometimes yield better results than a gradual, cooperative, approach that never shocks expectations out of their present rut and fails to reach the economy's highest potential.

Of course, bifurcational gains from tax competition are conditional on the existence of multiple equilibria that are stable under adaptive learning. In the present paper, in an effort to make the analysis as transparent as possible, we have used a simple threshold externality and specific functional forms to create such multiplicity. Elsewhere, we have shown that bifurcational jumps in economic growth can occur when capital goods are complementary to each other (Honkapohja and Turunen-Red (2002)). Since technological complementarities and external influences on productivity (through sharing of ideas) appear increasingly important in the most modern high-technology sectors, we believe research on the effects of economic policy should not ignore the possibility of large gains in these settings.

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Appendices:

(A) Solving the Model at a Steady State for the entrepreneurial effort variables: The first-order conditions (8) yield the following equations:

$$n_H: \quad \alpha(1-\tau_H)n_H^{\alpha-1}\Psi(N_H) = c_H^\sigma, \tag{19}$$

$$n_F: \quad \alpha(1-\tau_F)n_F^{\alpha-1}\Psi(N_F) = c_H^\sigma, \tag{20}$$

$$n_F^*: \quad \alpha(1-\tau_F)n_F^{*(\alpha-1)}\Psi(N_F) = c_F^{*\,\sigma}, \tag{21}$$

$$n_H^*: \quad \alpha(1-\tau_H)n_H^{*(\alpha-1)}\Psi(N_H) = c_F^{*\,\sigma}. \tag{22}$$

In (19)-(22),

$$c_H = (1-\tau_H)n_H^\alpha\Psi(N_H) + (1-\tau_F)n_F^\alpha\Psi(N_F), \tag{23}$$

$$c_F^* = (1-\tau_H)(n_H^*)^\alpha\Psi(N_H) + (1-\tau_F)(n_F^*)^\alpha\Psi(N_F), \tag{24}$$

and, in (21)-(22) and (24), n_H^* and n_F^* denote the effort that Foreign born entrepreneurs invest in countries H and F, respectively.

Equations (19)-(22) give

$$n_F = n_H\left[\frac{\Psi(N_F)}{\Psi(N_H)}\frac{(1-\tau_F)}{(1-\tau_H)}\right]^{\frac{1}{1-\alpha}} \equiv n_H T^{\frac{1}{1-\alpha}}, \tag{25}$$

$$n_H^* = n_F^* T^{\frac{1}{\alpha-1}}. \tag{26}$$

Further, using (23) and (25) in (19), we obtain

$$n_H = \frac{\alpha^{\frac{1}{z}}(1-\tau_H)^{\frac{1}{1-\alpha}}\Psi_H^{\frac{1}{1-\alpha}}}{\left[(1-\tau_H)^{\frac{1}{1-\alpha}}\Psi_H^{\frac{1}{1-\alpha}} + (1-\tau_F)^{\frac{1}{1-\alpha}}\Psi_F^{\frac{1}{1-\alpha}}\right]^{\frac{\sigma}{z}}}, \tag{27}$$

$$z \equiv 1-\alpha(1-\sigma),$$

and symmetrically,

$$n_F^* = \frac{\alpha^{\frac{1}{z}}(1-\tau_F)^{\frac{1}{1-\alpha}}\Psi_F^{\frac{1}{1-\alpha}}}{\left[(1-\tau_H)^{\frac{1}{1-\alpha}}\Psi_H^{\frac{1}{1-\alpha}} + (1-\tau_F)^{\frac{1}{1-\alpha}}\Psi_F^{\frac{1}{1-\alpha}}\right]^{\frac{\sigma}{z}}}. \tag{28}$$

Then, by (27) and (28), $n_H = n_F^* T^{\frac{1}{\alpha-1}}$ which implies, using (26), that

$$\begin{aligned} n_H &= n_H^*, && (29) \\ n_F &= n_F^*. && (30) \end{aligned}$$

Therefore, we can express the model solution in terms of n_H and n_F.

Using (27) and (28),

$$n_H = \frac{\alpha^{\frac{1}{z}} x}{(x+y)^{\frac{\sigma}{z}}}, \tag{31}$$

$$n_F = \frac{\alpha^{\frac{1}{z}} y}{(x+y)^{\frac{\sigma}{z}}}, \tag{32}$$

where

$$x \equiv (1-\tau_H)^{\frac{1}{1-\alpha}} \Psi_H^{\frac{1}{1-\alpha}}, \;\; y \equiv (1-\tau_F)^{\frac{1}{1-\alpha}} \Psi_F^{\frac{1}{1-\alpha}}, \tag{33}$$

$$\Psi_j(n_j) = \max\left[\widehat{I}, \frac{\lambda K(2n_j)}{1+a\lambda K(2n_j)}\right]^\beta, \quad j = H, F. \tag{34}$$

(B) Derivatives of Entrepreneurial Effort with respect to taxes: Taking logarithms and differentiating (31) and (32) with respect to τ_H yields:

$$A_x \widehat{n}_H = B_x - C_y \widehat{n}_F, \;\; A_y \widehat{n}_F = D_x - C_x \widehat{n}_H, \tag{35}$$

where

$$\widehat{n}_H \equiv \frac{dn_H/d\tau_H}{n_H}, \;\; \widehat{n}_F \equiv \frac{dn_F/d\tau_H}{n_F}, \tag{36}$$

$$A_i \equiv \left[1 - \frac{\beta}{1-\alpha}\left[1 - \frac{\sigma}{z}\frac{i}{x+y}\right](1 - a\Psi_H^{\frac{1}{\beta}})\right], \quad i = x, y, \tag{37}$$

$$B_x \equiv -\frac{1}{(1-\alpha)(1-\tau_H)}\left[1 - \frac{\sigma}{z}\frac{x}{x+y}\right], \tag{38}$$

$$C_i \equiv \frac{\sigma}{z}\frac{i}{x+y}\frac{\beta}{(1-\alpha)}(1 - a\Psi_F^{\frac{1}{\beta}}), \;\; i = x, y, \tag{39}$$

$$D_x \equiv \frac{\sigma}{z}\frac{x}{x+y}\frac{1}{(1-\alpha)(1-\tau_H)}. \tag{40}$$

At a symmetric equilibrium, $A_x = A_y$ and $C_x = C_y$ and $x/(x+y) = y/(x+y) = 1/2$ in (37)-(40). Thus, given symmetry, (35) yields

$$\widehat{n}_H(A_x^2 - C_x^2) = A_x B_x - C_x D_x, \tag{41}$$

where

$$\begin{aligned} A_x^2 - C_x^2 &= (A_x - C_x)(A_x + C_x) \\ &= \left[1 - \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}})\right] \left[1 - \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}})\left(1 - \frac{\sigma}{z}\right)\right], \end{aligned} \tag{42}$$

$$A_xB_x - C_xD_x = -\frac{1}{(1-\alpha)(1-\tau_H)}\left[\left(1 - \frac{\sigma}{2z}\right) - \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}})\left(1 - \frac{\sigma}{z}\right)\right]. \tag{43}$$

Expression (42) is positive if and only if

$$\frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) < 1 \quad \text{or} \quad \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) > \frac{1}{1-\frac{\sigma}{z}}, \tag{44}$$

and negative otherwise, while (43) is positive if and only if

$$\frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) < \frac{1-\frac{\sigma}{2z}}{1-\frac{\sigma}{z}} \quad \left(< \frac{1}{1-\frac{\sigma}{z}}\right) \tag{45}$$

and negative otherwise. Thus, we obtain that $\widehat{n}_H < 0$ if and only if

$$\frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) < 1 \quad \text{or} \quad \frac{1-\frac{\sigma}{2z}}{1-\frac{\sigma}{z}} < \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) < \frac{1}{1-\frac{\sigma}{z}}. \tag{46}$$

Next, we apply (35) and (46) to completely describe the signs of $\widehat{n}_H$ and $\widehat{n}_F$.

Case (i): Suppose $\widehat{n}_H < 0$ because $\frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) < 1$. Then, by (37), $A_x > 0$ on the left-hand side of (35), while the right-hand side is positive and so $\widehat{n}_F > 0$.

Case (i) occurs when there are no externalities, i.e., $\beta = 0$. Then, (35) yields

$$\widehat{n}_H = \frac{B_x}{A_x} = -\frac{1}{(1-\alpha)(1-\tau_H)}\left(1 - \frac{\sigma}{2z}\right) < 0, \tag{47}$$

$$\widehat{n}_F = \frac{D_x}{A_x} = \frac{\sigma}{2z}\frac{1}{(1-\alpha)(1-\tau_F)} > 0. \tag{48}$$

Second, Case (i) applies when $\beta > 0$ but the equilibrium solution for entrepreneurial effort is too low for the positive externality appear, or using (34)

$$\frac{\lambda K(2n_H)}{1 + a\lambda K(2n_H)} < \widehat{I} \Leftrightarrow n_H < n_{\widehat{I}} \equiv \frac{1}{2\lambda K}\left[\frac{\widehat{I}}{1 - a\widehat{I}}\right]. \tag{49}$$

Third, Case (i) occurs when the solution for n_H is sufficiently large so that the local impact of the externality is small enough for the inequality $\beta(1 - a\Psi_H^{\frac{1}{\beta}}) < 1 - \alpha$ to be satisfied. This requires

$$n_H > n_1 \equiv \frac{1}{2a\lambda K}\left[\frac{\beta}{1-\alpha} - 1\right]. \tag{50}$$

Case (ii): Suppose $\widehat{n}_H > 0$ because $1 < \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) < \frac{1}{1-\frac{\sigma}{2z}}$. Then , $A_x > 0$ as in Case (i) and from (35),

$$\widehat{n}_F A_x = \frac{\sigma}{2z}\frac{1}{(1-\alpha)(1-\tau_H)}\left[1 + \frac{a(c-ab)}{(1-a)(1-ab)}\right],\ a \equiv \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}), \tag{51}$$

where the expressions b and c are obviously defined. We can write

$$1 + \frac{a(c-ab)}{(1-a)(1-ab)} = \frac{1 - a(1+b-c)}{(1-a)(1-ab)}, \tag{52}$$

where $(1-a) < 0$ and $(1-ab) > 0$ because $(1-ab) > 0$ (i.e., $a < \frac{1}{1-\frac{\sigma}{z}}$). Given $a < \frac{1}{1-\frac{\sigma}{z}}$, the numerator $(1 - a(1+b-c))$ is positive. The right-hand side of (51) is therefore negative, whereby $\widehat{n}_F < 0$. Thus in Case (ii), $\widehat{n}_H > 0$ and $\widehat{n}_F < 0$.

Case (ii) is observed when the equilibrium effort satisfies the inequality

$$\frac{1}{2a\lambda K}\left[\frac{\beta}{R(1-\alpha)} - 1\right] < n_H < n_1, \quad R \equiv \frac{1}{1-\frac{\sigma}{2z}}. \tag{53}$$

Case (iii): Suppose $\widehat{n}_H > 0$ because $\frac{1}{1-\frac{\sigma}{2z}} < \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) < \frac{1-\frac{\sigma}{2z}}{1-\frac{\sigma}{z}}$. In this region, $A_x < 0$ and expression (52) is positive, whereby $\widehat{n}_F < 0$. In Case (iii) as well, $\widehat{n}_H > 0$ and $\widehat{n}_F < 0$. This region corresponds to the n-values

$$n_3 \equiv \frac{1}{2a\lambda K}\left[\frac{\beta}{S(1-\alpha)} - 1\right] < n_H < \frac{1}{2a\lambda K}\left[\frac{\beta}{R(1-\alpha)} - 1\right], \quad S \equiv \frac{1-\frac{\sigma}{2z}}{1-\frac{\sigma}{z}} \tag{54}$$

Case (iv): Suppose $\widehat{n}_H < 0$ because $\frac{1-\frac{\sigma}{2z}}{1-\frac{\sigma}{z}} < \frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) < \frac{1}{1-\frac{\sigma}{z}}$. Then, $A_x < 0$ and because the right-hand side of the expression for $\widehat{n}_F$ in (35) is positive, we obtain $\widehat{n}_F < 0$. In Case (iv), $\widehat{n}_H < 0$ and $\widehat{n}_F < 0$. The corresponding inequality for n_H is

$$n_4 \equiv \frac{1}{2a\lambda K}\left[\frac{\beta}{T(1-\alpha)} - 1\right] < n_H < n_3, \quad T \equiv \frac{1}{1-\frac{\sigma}{z}}.$$

Case (v): Suppose $\widehat{n}_H > 0$ because $\frac{\beta}{1-\alpha}(1 - a\Psi_H^{\frac{1}{\beta}}) > \frac{1}{1-\frac{\sigma}{z}}$. Then, $A_x < 0$ and (52) is negative. In Case (v), $\widehat{n}_H > 0$ and $\widehat{n}_F > 0$. This case occurs if $n_H < n_4$.

By calculations analogous to above and by appealing to symmetry,

$$\widehat{n}_H = \frac{dn_F/d\tau_F}{n_F}, \ \frac{dn_H/d\tau_F}{n_H} = \widehat{n}_F,$$

and $dn_F^*/d\tau_F = dn_H/d\tau_H$ and $dn_H^*/d\tau_F = dn_F/d\tau_H$. Accordingly, the n-derivatives are completely characterized by the expressions for $\widehat{n}_H$ and $\widehat{n}_F$.

Finally, we determine the sign of $\left(\frac{\partial n_H}{\partial \tau_H}+\frac{\partial n_F}{\partial \tau_H}\right)$ representing the response in individual entrepreneurial effort to a domestic tax increase. At a symmetric equilibrium, we have

$$\frac{\partial n_H}{\partial \tau_H}+\frac{\partial n_F}{\partial \tau_H}=n_H\left[\widehat{n}_H+\widehat{n}_F\right], \tag{55}$$

and so the sign of (55) is determined by the sign of $(\widehat{n}_H+\widehat{n}_F)$. From (35), we obtain

$$\widehat{n}_H+\widehat{n}_F=\frac{B_x+D_x}{A_x+C_x}=\frac{-\frac{1}{(1-\alpha)(1-\tau)}(1-\frac{\sigma}{z})}{1-\frac{\beta}{1-\alpha}(1-a\Psi^{\frac{1}{\beta}})(1-\frac{\sigma}{z})}. \tag{56}$$

The numerator of (56) is negative and the denominator is positive if and only if

$$\frac{\beta}{1-\alpha}(1-a\Psi^{\frac{1}{\beta}})<\frac{1}{1-\frac{\sigma}{z}}. \tag{57}$$

Thus, $\widehat{n}_H+\widehat{n}_F<0$ if and only if Cases (i)-(iv) hold.

(C) The offer curves in Figure 1: Offer curves are characterized by the equation

$$n_t = 2^{-\sigma}\alpha(1-\tau)^{1-\sigma}(n_{t+1}^{\alpha}\Psi_{t+1})^{1-\sigma}, \tag{58}$$

$$\Psi_{t+1} = \max\left[\widehat{I},\frac{\lambda N_{t+1}}{1+a\lambda N_{t+1}}\right]^{\beta}. \tag{59}$$

Since $0<\alpha(1-\sigma)<1$, n_t is an increasing and concave function of n_{t+1} if Ψ_{t+1} is a constant. This yields the first concave segment along the offer curves in Figure 1.

For values of n_{t+1} at which Ψ_{t+1} is not a constant (to the right from the kink on offer curves in Figure 1) equation (58) gives

$$\frac{\partial n_t}{\partial n_{t+1}}=L(1-\sigma)y^{1-\sigma}\left[\frac{\partial y_{t+1}}{\partial n_{t+1}}\frac{1}{y_{t+1}}\right]=L(1-\sigma)\left[\frac{y_{t+1}^{1-\sigma}}{n_{t+1}}\right]\epsilon_n^y>0, \tag{60}$$

$$\frac{\partial^2 n_t}{\partial n_{t+1}^2}=\frac{L(1-\sigma)}{n_{t+1}}\left[\frac{y_{t+1}^{1-\sigma}}{n_{t+1}}\epsilon_n^y((1-\sigma)\epsilon_n^y-1)+y_{t+1}^{1-\sigma}\frac{\partial\epsilon_n^y}{\partial n}\right], \tag{61}$$

where $L=2^{-\sigma}\alpha(1-\tau)^{1-\sigma}$ and

$$\epsilon_n^y\equiv\frac{(dy/dn)n}{y}=\alpha+\beta(1-a\Psi^{\frac{1}{\beta}})=\alpha+\frac{\beta}{1+a\lambda N} \tag{62}$$

is the *elasticity of individual output with respect to effort.* The sign of the derivative (61) determines the curvature of the offer curve. Because

$$\frac{\partial\epsilon_n^y}{\partial n}=-\frac{2a\lambda}{(1+2a\lambda n)^2}<0 \tag{63}$$

in (61), the offer curve (58) is concave if

$$(1-\sigma)\epsilon_n^y - 1 < 0 \Leftrightarrow n > n_\epsilon \equiv \frac{1}{2a\lambda K}\left[\frac{(1-\sigma)(\alpha+\beta)}{1-(1-\sigma)\alpha} - 1\right]. \tag{64}$$

Observing that

$$\frac{(1-\sigma)(\alpha+\beta)}{1-(1-\sigma)\alpha} - 1 < \frac{\beta}{1-\alpha} - 1, \tag{65}$$

we obtain that condition (64) is satisfied when $n > n_1$, i.e., in Case i) of Appendix B. If there are no productive externalities, we obtain the first concave segment of the offer curves in Figure 1. When externalities are operative ($n_{t+1} > n_{\hat{I}}$), condition (64) determines a range of high values of n_{t+1} for which the offer curve is concave. Concavity holds even for values of n_{t+1} that are somewhat smaller than n_ϵ because $\partial\epsilon_n^y/\partial n < 0$ in (61). Thus, the second concave segment of the offer curves exists when n_{t+1} is sufficiently large.

The offer curve is convex if $\epsilon_n^y > 1/(1-\sigma)$ and such that (61) is positive. Using (64), it is evident that such convex segments, if they occur, can only appear when n_{t+1} is between the low value $n_{\hat{I}}$ at which the externality appears and the high value n_ϵ at which the offer curve already is concave. Thus, the offer curves in Figure 1 conform to the general description obtained from equation (58).

(D) Welfare as a function of the tax rate in Figure 2: In this appendix, we evaluate $\partial W^H/\partial\tau_H + \partial W^H/\partial\tau_F$ at a symmetric equilibrium, determining the slope of the curves $W(\tau)$ in Figure 2. Using (13),

$$\frac{\partial W^H}{\partial\tau_H} = U'\left[\frac{\partial c_H}{\partial\tau_H}\right] - \left[\frac{\partial n_H}{\partial\tau_H} + \frac{\partial n_F}{\partial\tau_H}\right] + \mu U'\left[\tau_H\frac{\partial Y_H^T}{\partial\tau_H} + Y_H^T\right], \tag{66}$$

$$\frac{\partial W^H}{\partial\tau_F} = U'\left[\frac{\partial c_H}{\partial\tau_F}\right] - \left[\frac{\partial n_H}{\partial\tau_F} + \frac{\partial n_F}{\partial\tau_F}\right] + \mu U'\left[\tau_H\frac{\partial Y_H^T}{\partial\tau_F}\right], \tag{67}$$

where $Y_H^T = f(n_H, N_H) + f(n_H^*, N_H)$ is the total (per capita) output in H.

Furthermore, since $c_H = (1-\tau_H)f(n_{Ht}, N_{Ht}) + (1-\tau_F)f(n_{Ft}, N_{Ft})$ at a steady state,

$$\frac{\partial c_H}{\partial\tau_H} = -f(n_H, N_H) + (1-\tau)(f_1' + 2f_N')\left[\frac{\partial n_H}{\partial\tau_H} + \frac{\partial n_F}{\partial\tau_H}\right], \tag{68}$$

$$\frac{\partial c_H}{\partial\tau_F} = -f(n_F, N_F) + (1-\tau)(f_1' + 2f_N')\left[\frac{\partial n_H}{\partial\tau_F} + \frac{\partial n_F}{\partial\tau_F}\right], \tag{69}$$

and

$$\frac{\partial Y_H^T}{\partial\tau_H} = 2(f_1' + 2f_N')\frac{\partial n_H}{\partial\tau_H}, \qquad \frac{\partial Y_H^T}{\partial\tau_F} = 2(f_1' + 2f_N')\frac{\partial n_H}{\partial\tau_F} \tag{70}$$

The derivatives of n_H and n_F with respect to the tax variables are discussed in Appendix B above.

Next, we substitute (68)-(70) into (66)-(67) and apply (11) to obtain

$$\begin{aligned}\frac{\partial W^H}{\partial \tau_H} &= -U'(c_H)f(n_H, N_H) + \mu U'(G_H)Y_H^T \\ &\quad +2\mu\tau U'(G_H)(f_1' + 2f_N')\frac{\partial n_H}{\partial \tau_H} + 2U'(c_H)(1-\tau)f_N'\left[\frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H}\right],\end{aligned} \tag{71}$$

$$\begin{aligned}\frac{\partial W^H}{\partial \tau_F} &= -U'(c_H)f(n_F, N_F) + 2\mu\tau U'(G_H)(f_1' + 2f_N')\frac{\partial n_H}{\partial \tau_F} \\ &\quad +2U'(c_H)(1-\tau)f_N'\left[\frac{\partial n_H}{\partial \tau_F} + \frac{\partial n_F}{\partial \tau_F}\right].\end{aligned} \tag{72}$$

Therefore, by symmetry

$$\begin{aligned}\frac{\partial W^H}{\partial \tau_H} + \frac{\partial W^H}{\partial \tau_F} &= \mu U'(G_H)Y_H^T - U'(c_H)\left[f(n_H, N_H) + f(n_F, N_F)\right] \\ &\quad +2\mu\tau U'(G_H)(f_1' + 2f_N')\left[\frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_H}{\partial \tau_F}\right] \\ &\quad +4U'(c_H)(1-\tau)f_N'\left[\frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H}\right].\end{aligned} \tag{73}$$

The first term on the right-hand side of (73) is positive and the other terms are negative as long as condition (57) of Appendix B is satisfied, which corresponds to Cases (i)-(iv).

The sum of the first two terms on the right-hand side of (73) equals

$$\left[\mu U'(G_H) - U'(c_H)\right] Y_H^T = [\mu\tau^{-\sigma} - (1-\tau)^{-\sigma}](Y_H^T)^{1-\sigma}. \tag{74}$$

When τ approaches zero, the term (74) in (73) grows large whereby (73) eventually must be positive. Thus, when τ is sufficiently small, welfare is increasing in the common value of τ. However, as τ increases, the other (negative) terms in (73) will eventually dominate. Then, welfare in decreasing in τ.

When the productive externality effect is very large so that condition (57) is violated (this is Case (v) of Appendix B), the previously negative third and fourth terms on the right-hand side of (73) become positive. This means that, given very strong externality, welfare can be an increasing function of τ for a wider range of τ-values (in Figure 2, $W(\tau)$ curves shift to the right).

(E) Unilateral incentives to lower the tax rate at a local optimum: A locally jointly optimal (symmetric) tax rate, τ^{opt}, satisfies the first order condition

$$\left[\frac{\partial W^j}{\partial \tau_H} + \frac{\partial W^j}{\partial \tau_F}\right]|_{\tau^{opt}} = 0, \qquad j = H, F, \tag{75}$$

and the derivative expressions are defined in (73). Clearly, $\partial W^H/\partial \tau_H < 0$ if and only if $\partial W^H/\partial \tau_F > 0$.

Using (72) and taking into account (75), we obtain

$$\frac{\partial W^H}{\partial \tau_F} \mid_{\tau^{opt}} = Y_H \left[\frac{U'(c_H)}{2} - \mu U'(G_H) \right] - 2\mu\tau U'(G_H)(f_1' + 2f_N') \frac{\partial n_H}{\partial \tau_H} \tag{76}$$
$$-2U'(c_H)(1-\tau) f_N' \left[\frac{\partial n_H}{\partial \tau_H} + \frac{\partial n_F}{\partial \tau_H} \right].$$

The signs of the n-derivatives in (76) can vary. Three possibilities arise.

First, in Cases (i) and (iv) of Appendix B, the second and third terms of (76) are positive. The first term positive as well if

$$MRS_{cG} \equiv \frac{U'(c_H)}{\mu U'(G_H)} = \frac{1}{\mu} \left[\frac{\tau^{opt}}{1-\tau^{opt}} \right]^{\sigma} > 2. \tag{77}$$

This condition is satisfied when the utility weight of public services μ tends to zero.. Thus, for sufficiently low values of μ, (76) is positive in Cases (i) and (iv) of Appendix B. For higher μ the incentive to lower the domestic tax is weakened but it can still exist.

Second, in Cases (ii) and (iii) of Appendix B, the second term on the right-hand side of (76) is negative but the third term is positive. Then $\partial W^H / \partial \tau_H$ is less negative than in the previous case, implying that Nash equilibrium occurs at a higher level of taxation.

Third, in Case (v) of Appendix B, both the second and third terms of 76) are negative. In this case, when the externality is very strong, it is likely that $\partial W^H / \partial \tau_H > 0$ unless the weight of public consumption in preferences is very low. In this case, therefore, the Nash equilibrium is likely to involve taxes higher than jointly optimal.

(F) Externality parameters and the cut-off tax, τ^{cut}: The cut-off tax is obtained from (17) and (18). Equation (18) defines a downward sloping curve in (τ, n)-space, and the slope of this curve equals

$$\frac{\partial n(\tau)}{\partial \tau} = -2^{-\frac{\sigma}{z}} \alpha^{\frac{1}{z}} \widehat{I}^{\frac{\beta(1-\sigma)}{z}} \frac{(1-\sigma)}{z} (1-\tau)^{\frac{1-\sigma}{z}-1} \quad (<0). \tag{78}$$

Since

$$\frac{\partial \Psi(n)}{\partial n} = \beta \left(\frac{2\lambda K n}{1 + 2a\lambda K n} \right)^{\beta-1} \frac{2\lambda K}{(1+2a\lambda K n)^2} \quad (>0), \tag{79}$$

a reduction in a and and increase in λK both increase the slope (79) thus reducing the cut-off value of n. Using (18) this means that the cut-off tax increases.

When β increases, the function curve $n(\tau)$ and the slope (79) both increase. Together, these changes imply that the cut-off tax increases. When $\widehat{I}$ decreases, the cut-off value of n clearly declines. However, the reduction in $\widehat{I}$ also causes a reduction in the slope (78) and it is possible that the cut-off tax also declines.

Figure 1: Offer Curves and Steady States

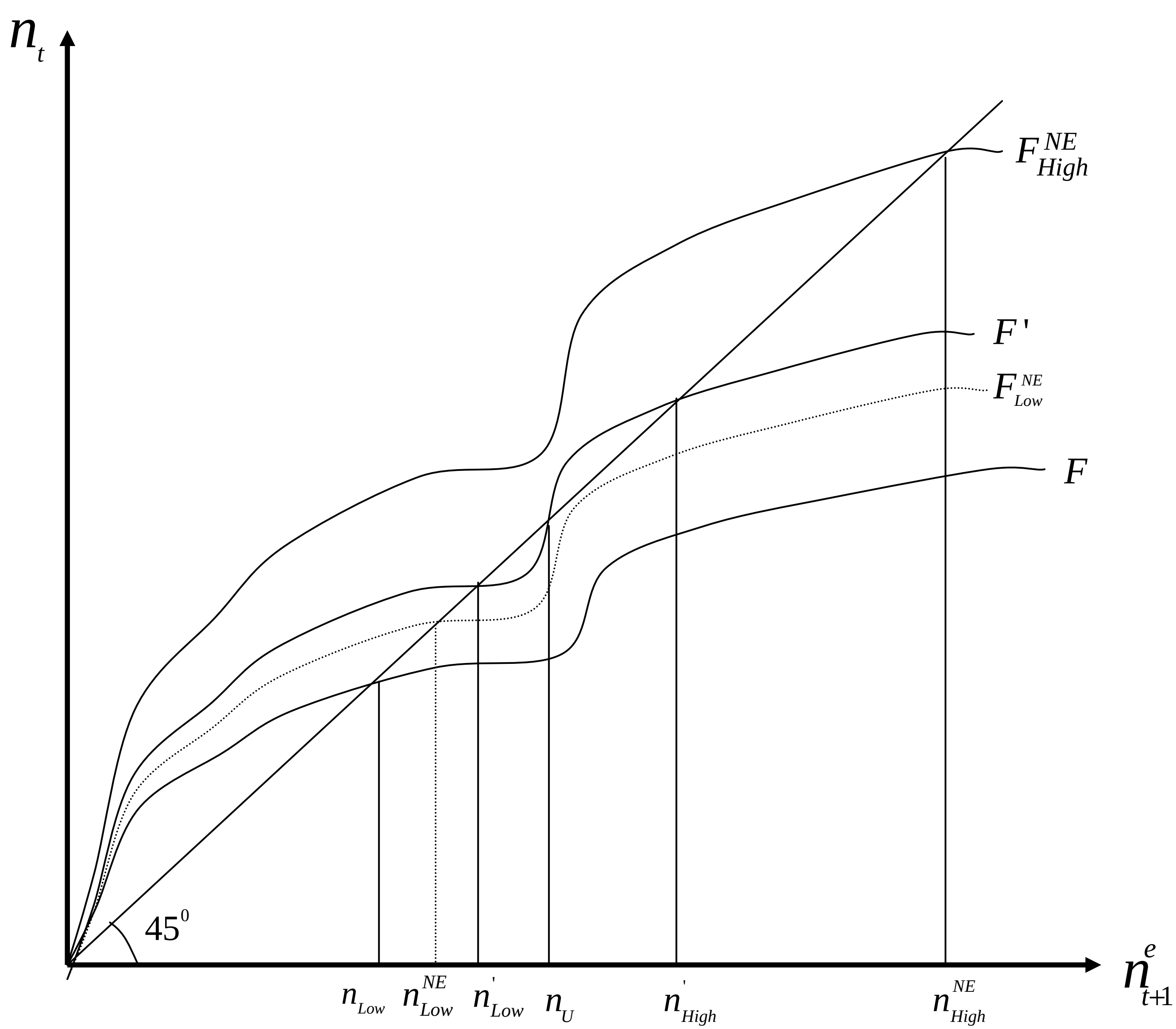

Figure 2: Taxes and Welfare

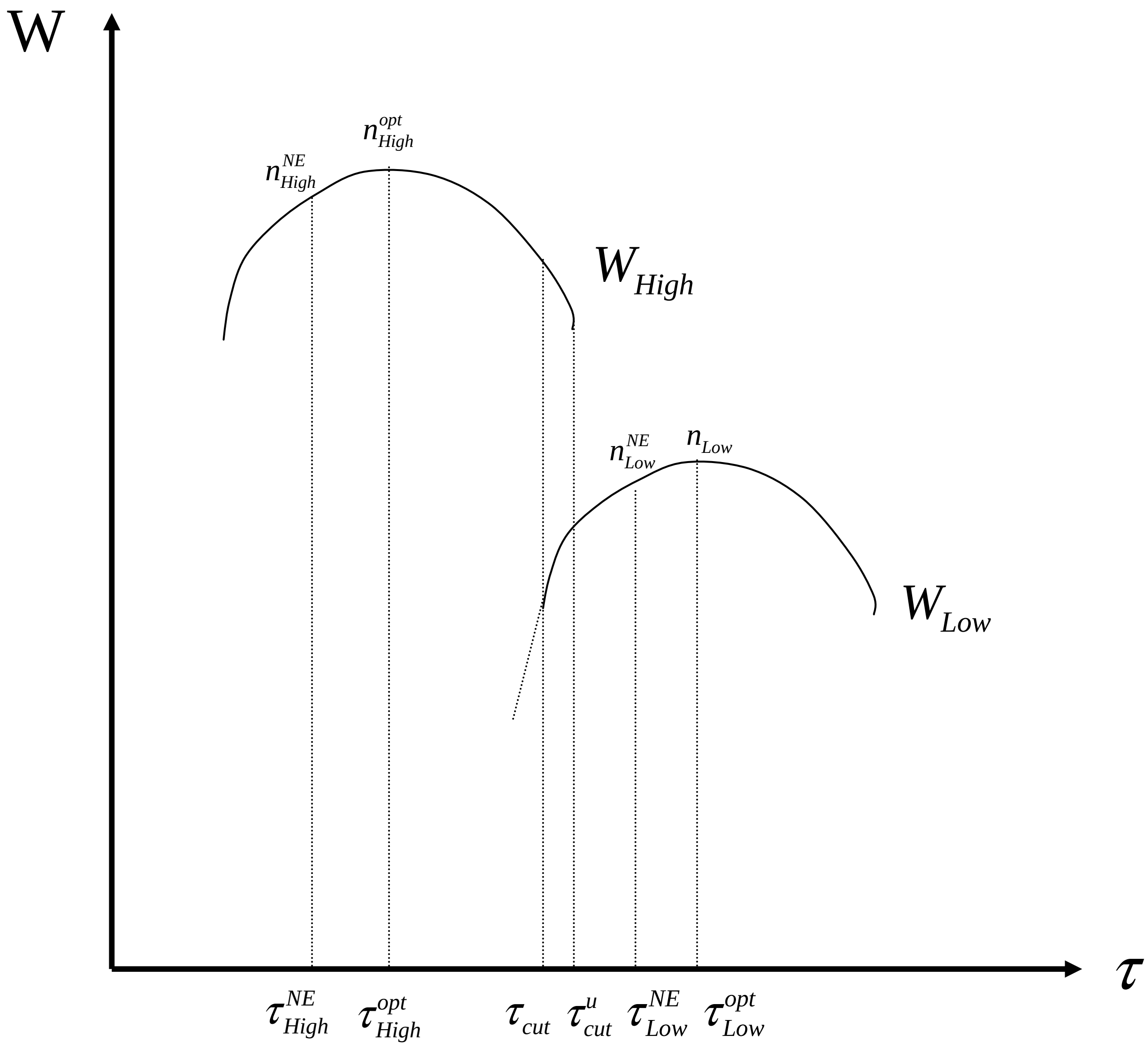

Figure 3: Effort Regions in the High Regime