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Abstract

This paper investigates the impacts of free trade on the structure of urban systems, skill distribution, and income disparities. The paper proposes a model that integrates international trade theory and the theory of urban system. This is done in a two sector, spatial general equilibrium model of a North-South trade. Each country is populated with a continuum of unskilled workers with heterogeneous potential ability. Through differential training costs, workers with different potential ability can achieve the same productivity. Workers can acquire a skill by investing in training. Thus, skill distribution in both countries is determined endogenously in the model through self-selection. The economy produces a final good with the use of a high-tech intermediate input and unskilled workers. Horizontally differentiated skilled workers produce the high-tech intermediate input. Cities are formed in this model as a result of investment in setup cost, i.e., public infrastructures. I characterize two different types of spatial equilibria: a closed-economy equilibrium, in which each country consists of a system of cities without trade, and a free-trade equilibrium, in which we allow for trade between cities and countries.

Key words: Potential ability, training, cities, income disparity
JEL classification: R13; R51; F16

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1. INTRODUCTION

The role of national government in influencing trade has been declining with the emergence and the expansion of the European Union and with NAFTA, GATT, and some trade agreements between some European countries and some developing countries in North Africa. With this trend, eventually the role of a city in a network of urban system will become more important and the comparative advantage of a city, not a national trade policy, will shape the future of trade patterns. When taken together, all of these facts will lead us to conclude that a comprehensive theory of system of cities is not only an interesting theoretical exercise, but also an essential part of understanding international trade in a broader context.

Several studies have indicated that the income disparity in the U.S. and other developed nations has been rising during the past two decades.[1] As a result, a dramatic decrease in the real wage of low-skilled labor as well as an increase in the wage of high-skilled labor has materialized. Specifically, Juhn, Murphy, and Topel (1991) found that for the U.S., the wages of the lowest 20% of the American workforce in the 1990's had fallen to 25% below the 1973 real wage. Also Juhn, Murphy and Pierce (1993) found that between 1963 and 1989, the average weekly real wage for the least skilled workers decreased by 5%, where the same for skilled workers increased by 40%.[2] Furthermore, Machin (1996) showed that income inequality has been rising in the UK within skilled labor as well as between skilled and unskilled labor during the 1978-1992 period. He suggested that within-group disparity, in hourly income, increased by 23% over the same period.[3] This rise in inequality has been the result of a relative wage growth at the top of the wage distribution and a fall at the bottom of the wage distribution.

[1] For general trends supporting this see Murphy and Welch (1993).
[2] See Richardson (1995) for a survey of the impact of trade on U.S. wage inequalities.

On the other hand, Cawley, Heckman, and Vytacil (2001) showed that potential ability is correlated with the wage. This was shown especially in the case of white-collar workers. Furthermore, they showed that a significant portion of the rise in within-group inequality is unexplained by age and education. Hoxby and Terry (1999) decomposed the growing divergence into three sources. The first is the increasing diversity of people attending college. The second is an increasing return to aptitude. The third is the increasing correlation between the average aptitude of a college's student and its expenditure on education.

As indicated above, the combination of the growing number of free trade agreements, trade liberalization and increased globalization, together with the above trends in income disparities, raises some important questions concerning their impacts on each country's labor market. In the international trade literature, there is a lively debate on whether free trade between a developed and a less-developed country has a detrimental effect on the welfare of the unskilled workers in the developed nation. Leamer (1993) and Wood (1995) suggested that the declining welfare of unskilled workers in the developed nation is caused primarily by the expansion of trade with the LDC. This is because of the link between the labor market of the developed country and that of the LDC. This link resulted in a decline in the wage of the North's unskilled labor and a rise in the wage of the South's unskilled workers, thus, leading to a change in the distribution of skill in both markets as well as widening income disparities. On the other hand, Lawrence and Slaughter (1993) among others, argue that trade is not a major factor in determining income disparity. In spite of this disagreement on whether or not free trade is the major factor behind the growing income disparity, it is one of the factors that affected the labor market.[4]

[4] There are some models that associate the growing income disparity to skill biased technical change, Acemoglu (2002).

On the other hand, the urban literature presented some initial work that analyzes the impacts of income disparity on the skill distribution, social welfare, and on the structure of urban system, Abdel-Rahman (2002).[5] This was done in a one-sector general equilibrium model of a system of specialized cities. The main finding is that the structure of urban system is affected by the skill distribution and by income disparity.

From these two bodies of literature, one can conclude that there is a need for a spatial, general equilibrium model of a North-South trade that can analyze the impact of free trade on the spatial distribution of skill within a system of cities. This will enable us to examine the impact of free trade not only on the labor market, but also on the structure of the system of cities in both the North and the South.[6] The objective of this paper is to propose a model in this direction. Thus, the paper addresses the following questions: *What are the factors that determine the distribution of skill within a system of cities? What are the determinants of income disparities? Does free trade affect the structure of the urban system? What is the impact of free trade on interregional and intraregional income disparities?*

In addressing the above questions, the paper proposes a model that integrates international trade theory and the theory of urban system. The framework is a two sector, spatial general equilibrium model of a North-South trade. The economy produces a final good with the use of a high-tech intermediate input and unskilled labor. The intermediate input is produced with the use of horizontally differentiated skilled workers. Each country is populated with a continuum of unskilled workers with heterogeneous potential ability. Through differential training costs, the workers with different potential ability can achieve the same productivity. In

[5] For a recent survey of the theory of system of cities see Abdel-Rahman (2004).
[6] Fischer and Serra (1996) presented an international trade model that analyzes the opposition of a developed country to free trade but with exogenous skill distribution.

the context of the model, a worker can acquire a skill by investing in training. Thus, skill distribution in both countries is determined endogenously in the model through self-selection. Cities are formed in this model by competitive developers investing in fixed setup cost, i.e., public infrastructures.[7] I characterize two different types of spatial equilibria: a closed-economy equilibrium, in which each country consists of a system of cities without trade, and a free trade equilibrium. Then, I investigate the impact of free trade on the system of cities, skill distribution within the system of each country, and income disparity.

It is assumed that the output and labor markets in this economy are perfectly competitive. In modeling city formation, we typically consider two types of forces: forces that lead to a concentration of population and forces that lead to population dispersion. When these forces are balanced at the margin, the equilibrium city size is determined. In the context of this model, the concentration force is a result of fixed setup costs. On the other hand, the de-concentration force is a result of the increase in transportation costs due to the physical expansion of the city Mills (1967). In the context of the model, we will consider a core–periphery system of cities where the core is populated with skilled workers while the periphery is populated with unskilled workers.

The main findings of the paper are as follows: First, I showed that free trade, in the context of a core-periphery system, will increase interregional income disparity as well as intraregional income disparity. Second, free trade will result in change in the structure of the system of cities. On the other hand, in the case of a core-periphery system of cities, free trade will increase the number of core cities in the North and decrease the number of periphery cities. Third, I showed that free trade might result in vanishing core cities in a core-periphery system in

[7] This city formation mechanize has been adopted to simplify the analysis so that we will be able to focus on the impacts of free trade on the urban systems.

the South. Finally, I showed that free trade would result in larger income disparities within the skilled group as well as between skilled and unskilled groups in the North.

The organization of this paper is as follows: Section 2 presents the assumptions and the specification of the model. Section 3 develops the equilibrium and establishes the existence of the equilibrium city system, skill distributions, and efficiency under autarky. Section 4 investigates the determinants of income disparities and their impacts on urban system. Section 5 establishes the existence of the equilibrium system of cities under free trade. Finally, Section 6 concludes.

2. THE MODEL

2.1 Assumptions and specification

We consider a world with two countries: the highly developed North and a less developed South, indexed by $i \in \{N, S\}$. Each country consists of a system of a system of specialized cities, indexed by $j \in \{x, z\}$ spreading over a flat, featureless plane.[8] The total population of a given country is given by M_i consumers/workers (to be described below) where each is endowed with one unit of labor. One final good, X, is produced with the use of unskilled workers and a high-tech intermediate input, Z, under constant returns to scale (CRS) technology. Good X can be used for consumption, commuting, and for the setup costs required for city formation. Each household/worker in a given country is perfectly mobile within the country, and resides in the city that maximizes his utility level. Furthermore, all consumers are identical in every aspect except for their potential ability and each has a utility that is a function of the only consumption good, X, in the economy. Workers in both countries are identical in terms of their productivity of good X. However, they are heterogeneous in their potential ability if employed in the high-tech

[8] We define a specialized system if each city produces x or z.

intermediate input industry, *Z*. Workers of type δ are uniformly distributed on a unit interval such that $M_i(\delta) = M_i \forall \delta \in [0,1]$.[9] The mass of workers can be written as, $M_i = \int_0^1 M_i(\delta) d\delta$.

Furthermore, suppose that the production function of *Z* for any skilled worker in a given city is *Z* = *L*, where *L* is amount of time devoted to production by skilled workers. Thus, the productivity of any trained worker of type δ is 1 unit of good *Z* given that he devoted all of his time to production. Labor can acquire training in a given country by investing TR_i units of his time. Suppose that the training cost for a worker of type δ in country i is given by a linear function of the potential ability, $TR_i = \tau_i - \alpha_i \delta$, where α_i and τ_i are positive constants to be specified later. Thus, the training of a worker with potential ability $\delta = 0$. is τ_i, while the training time of a worker with potential ability $\delta = 1$ is $\tau_i - \alpha_i$. Therefore, training cost consists of a fixed training time given by $\tau_i - \alpha_i$ and a variable training that depends on the worker ability. This can be interpreted as firms in industry *Z* have job requirements that best match the potential ability of workers of type $\delta = 1$. On the other hand, a worker with potential ability $\delta = 0$ has the poorest match with the skill requirements by industry *Z* and would have to spent τ_i units of his time to acquire training. Furthermore, we will impose the following assumption:

Assumption 1.

$$\alpha_{ij} = \begin{cases} \alpha_i & if [1 - \underline{\delta_{ij}}] = [1 - \delta_{Ui}] \\ 0 & if [1 - \underline{\delta_{ij}}] \neq [1 - \delta_{Ui}] \end{cases}$$

where $\underline{\delta_{ij}}$ is the worker with the lowest potential ability in city type j, while $(1 - \delta_{Ui})$ is the measure of skilled workers in the country, to be determined later in section 3. This assumption

[9] The assumption of uniform distribution is imposed for simplicity and does not affect the results of the model.

implies that training time will be shorter in a city populated by all types of skilled workers. In other words, it implies that diversity in skilled workers in a given city results in shorter time required for training by skilled workers. Thus, skilled worker, in this framework, acquire knowledge through interaction and exchange of information with all other types of skilled workers in the city. It is not our intention to support one of the above two assumptions, but rather to explore the impact of free trade on the resulting system of cities under each assumption.

The productivity of a worker with potential ability $\delta \in [0,1]$, in country $i \in \{N, S\}$ in city *j*, if he acquires training, in terms of good *Z* is given by

$$Z_i(\delta) = 1 - \tau_i + \alpha_i \delta \qquad (1)$$

where the price of good Z is normalized to be one. Assume that $\tau_i > 1$, and define $T_i \equiv \tau_i - 1$. Thus, (1) indicates that the high-tech intermediate input can be produced with a continuum of CRS technology. Furthermore, without loss of generality, it is assumed that $T_N < T_S$ and $\alpha_N > \alpha_S$. This can be viewed also as a difference in the training technology. Thus, the potential ability in the North is higher than in the South and the training cost is lower. In addition, the assumption that $\alpha_N > \alpha_S$ implies that each worker with potential ability, δ, in the North is more productive in the high-tech intermediate input than the South. This can be further interpreted with the Ricardian view that trade will be a result of difference in technology between countries. With this view, the North has access to more productive technology than the South in the high-tech intermediate input.

Both countries produce homogeneous final good, *X*, according to a CRS, Cobb Douglas production function. The production function of a given firm in city x and country *i* is given by

$$X_i = L_i^{\beta} Z_i^{1-\beta} \qquad i \in \{N, S\} \quad (2)$$

where $\beta \in (0,1)$, Xi is the output country i, and Z_j is the quantity of a high-tech intermediate input, and L_i is the quantity of unskilled workers. Furthermore $(1-\delta_{Ui})$ is the measure of skilled workers in the country i, to be determined later in section 3. Thus it is implied that the diversity of skilled workers in a given city results in higher productivity in the production of the final good X. Thus, unskilled worker, in this framework, acquire knowledge through interaction and exchange of information with diversified group of skilled workers in the city. This is because all types of skilled workers are different in terms of potential ability. The behavior of a given firm in this industry is characterized by $\left\{\max_{L_i,Z_i} \pi_i = P_i X_i - W_i L_i - Z_i \middle| (2)\right\}$. Solving for the first order conditions of this problem, we have

$$\beta P_i X_i = W_i L_i \qquad (3)$$

$$(1-\beta) P_i X_i = Z_i \qquad (4)$$

From (3) and (4) we have

$$\frac{\beta}{(1-\beta)} Z_i = W_i L_i \qquad (5)$$

The distribution of skill in a given country will be determined through a self-selection process. The marginal worker in any representative city will be the worker who is indifferent between acquiring skill or not. This worker will determine the measure of untrained worker in a given city as well as in the economy. Thus, the marginal worker, δ_{Ui}, is the one for which the wage offered to him by industry Z is equal to his opportunity cost, $W_i^* + TR_i$, where W_i^* is the equilibrium wage of the untrained worker, to be determined later.[10] Thus, the marginal worker can be determined as

[10] Note that if Assumption 1 if satisfied, then all cities in a given country will be identical in terms of size and skill

$$\delta_{Ui} = \frac{W_i + T_i}{\alpha_i} \qquad (6)$$

Consequently, the measure of trained labor in the intermediate good industry, *Z*, is $(1-\delta_{Ui})$. It is assumed that wages will be determined competitively for skilled and unskilled workers.

2.2 Household and city formation

Each household in a given city can reside only at one location and have only a single job that requires commuting to the central business district (CBD) where all firms are located.[11] For simplicity, we postulate that each household consumes one unit of land, i.e., demand for land is perfectly inelastic.[12] In addition, all households in the economy have an identical utility function, which is linear in the only final good, $U = x$. The budget constraint facing a household residing at distance *r* from the CBD in a given city and county is given as

$$P_i x(r_i) + P_i t_i r_i + R(r_i) = W_i(\delta) \qquad (7)$$

where t_i represents the amount of good *x* used to commute one unit distance; *R(r)* represents the unit land rent at distance *r* from a CBD of a given *country i*, and W_i denotes the wage income of a household, as defined below. Given the assumption of free mobility of worker between cities in a given country, the wage of a worker of a given type must be equalized between cities. Note that the only type of commuting cost, in the context of the model, is the monetary cost in terms of good *x*. Recall that each household consumes exactly one unit of land. Thus the total population of a representative city can thus be derived as:

$$N_i = \int_0^{f_i} 2\pi r_i dr = \pi f_i^2 \qquad (8)$$

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distribution. This will be discussed later in section 3.

[11] Thus, it is assumed that production can only take place in the CBD.

[12] Virtually all authors modeling system of cities impose this assumption for calculation simplicity.

where f represents the urban fringe distance for a given city in a given country. Equilibrium requires that all workers in a given city achieve the same utility level. Hence, from (7) and given the utility function, equilibrium require that $R(r_i) + P_i t_i r_i = R(f_i) + P_i t_i f_i$, must hold at each $r \leq f$. We can normalize the opportunity cost of land to be zero, so that $R(f_i) = 0$. Thus, in equilibrium, the land rent schedule becomes: $R(r_i) = P_i t_i (f_i - r_i)$. Utilizing (8) and integrating this land rent schedule, we can derive the aggregate land rent, *ALR*, in each city as:

$$ALR_i = \int_0^{f_{ij}} R(r_{ij}) 2\pi r_i dr = \mu_i P_i N_i^{3/2} \qquad (9)$$

where $\mu_i = t_i / 3\pi^{1/2}$ in a city which produces good *X*. The indirect utility for a household at distance *r* in a given city and country is given as

$$V_i(\delta) = P_i^{-1}\left[W_i(\delta) - 3\mu_i P_i N_i^{1/2}\right] \qquad (10)$$

This equation will be crucial in solving the model. This will become apparent later, as we examine the model further.

We assume that developers form cities by acquiring the land required for the development of the city from absentee landlords at the agricultural rent, which has been normalized to be zero. Then the developers sublet the land to households at the market price. Each developer provides the infrastructure required for city formation, F_i, where F_i is the quantity of infrastructure in terms of good X. Thus, the profit from city development is given by

$$\pi_{ij} = \mu_i P_i N_{ij}^{3/2} - P_i F_i \qquad (11)$$

Given free entry of developers into the city formation market in each country, developers will continue to form cities until profits from city development are driven to zero. Thus, the equilibrium city size in a given country is[13]

$$N_i^* = (F_i / \mu_i)^{2/3} \qquad (12)$$

From equation (12), it can be seen that the city size is increasing in fixed set up costs and in commuting costs. Furthermore, each country will consist of a system of identically sized cities. Thus the equilibrium number of cities is given by $m_i = M_i / N_i^*$.

Given the determination of the city size by the developer, the competitive market determines everything within the city. In other words, the land, the output, the intermediate input, and the labor markets are all competitive. Each household of a given type in a city must achieve a common utility level in equilibrium. In order to solve for the equilibrium system of cities, first we have to derive the indirect utility for a representative household of a given type as a function of city size. Note that the size of a city is independent of the type of workers that will reside in it. This is because the city size does not affect the wage rate. Substituting (12) into (10), we can derive the equilibrium utility of unskilled and skilled workers as

$$V_{Ui} = P_i^{-1}\left[W_{Ui} - P_i 3\mu_i^{2/3}(F_i)^{1/3}\right] \qquad (13)$$

$$V_i(\delta) = P_i^{-1}\left[W_i(\delta) - P_i 3\mu_i^{2/3}(F_i)^{1/3}\right] \qquad (14)$$

From the above indirect utility functions and the structure of the model, one can conclude that the model will result in a system of identically sized cities.[14]

[13] This equilibrium city formation mechanism implies that ALR will be used to finance the set-up cost, which is known as the Henry George theorem, Flatters, Henderson, and Mieszkowski (1974).

[14] It is not our intention in this model to explain different sizes of cities. This is why we used this city formation mechanism. For a resent survey of models of system of cities see Abdel-Rahman (2004).

3. SYSTEM OF CITIES UNDER AUTARKY

As it can be seen from equations (1), (13), and (14) and Assumption 1, the model will result in a system of specialized cities, a core-periphery system. One type of city is populated by skilled workers and produces the high-tech intermediate input, the core, while the other type of city is populated by unskilled workers and produces the final good, the periphery.[15] Now let us characterize the equilibrium system of cities.

Under autarky, each country must produce the high-tech intermediate good as well as the final good. Thus, an autarky equilibrium is defined by $\left\{N_i^*, m_{ij}^*, Z_{iz}^*, X_{ix}^*, V_{ij}^*(\delta) > 0 \forall\, \delta, \delta_{Ui}^*; \delta \in [0,1],\ j \in \{x,z\},\ i \in \{N,S\}\right\}$, *such that* $\left\{P_i^*, R_i^*(r), W_i^*, W_{ij}^*(\delta);\ r \le f_{ij}, \delta \in [0,1],\ j \in \{x,z\},\ i \in \{N,S\}\right\}$ which will result in the following conditions: profit maximization for firms in both the final good and the intermediate good sectors; zero profit for firms in both sectors; zero profit for developers; equality of supply and demand in the labor and the intermediate good markets in each city; a worker will acquire training if it will result in higher net wage; equal utility for each type of worker within a given country; and workers will reside in the city that provide them with the highest equilibrium utility level.

In this case all cities will be identical in size. One type of city will produce the high-tech intermediate input while the other type of city will produce the final good. Furthermore, suppose that equilibrium exists. Then, the aggregate supply of unskilled labor in country $i \in \{N,S\}$ is given by

[15] Henderson and Thisse (2001) in a non-spatial model examined the distribution of households differentiated by income between communities where community developers provide public good and behave strategically. In that model, the income distribution is given exogenously and public goods are differentiated by quality.

$$L_i = M_i \delta_{Ui} = \left[M_i \left(W_i + T_i \right) \right] \alpha_i^{-1} \qquad (16)$$

where the equilibrium N_i is given by equation (12). In addition, the aggregate supply of the intermediate input in a given city, in country $i \in \{N, S\}$ is given from equation (1) as

$$Z_i = M_i \left[\int_{\delta_{ui}}^{1} \left(1 - \tau_i + \alpha\delta \right) d\delta \right] = M_i \left[\int_{\delta_{Ui}}^{1} \alpha_i \delta d\delta - \left(1 - \delta_{Ui} \right) T_i \right] \qquad (17)$$

where δ_{Ui} is the measure of untrained labor in the country. By integrating this equation, we have

$$Z_i = M_i \left[\frac{\alpha_i}{2} \left(1 - \delta_{Ui}^2 \right) - T_i \left(1 - \delta_{Ui} \right) \right] \qquad (18)$$

Substituting (6) into the above equation, we have

$$Z_i = \frac{M_i}{2\alpha_i} \left[\left(\alpha_i - T_i \right)^2 - W_i^2 \right] \qquad (19)$$

Equating the aggregate supply and demand by substituting (16) and (19) into (5), we have

$$W_i T_i = \frac{\beta}{2\left(1 - \beta\right)} \left\{ \left(\alpha_i - T_i \right)^2 - \left(\frac{2 - \beta}{\beta} \right) W_i^2 \right\} \qquad (20)$$

This is a quadratic equation that has two roots. The relevant one determines the equilibrium wage as[16]

$$W_i^* = \frac{-T_i \left(1 - \beta \right) + \sqrt{T_i^2 \left(1 - \beta \right)^2 + \left(T_i - \alpha_i \right)^2 \beta \left(2 - \beta \right)}}{\left(2 - \beta \right)} \qquad (21)$$

From equation (21), it can be seen that the W^* is positive as long as $\alpha_i \neq T_i$. Thus, we impose the following regularity assumption:

Assumption 2: $\alpha_i > T_i$, $i \in \{S, N\}$

[16] Katz and Autor (2000) indicated that within-group inequality accounts for a substantial fraction of overall earned

This assumption implies that, in each city or country, there exists at least some type of worker that will have a positive wage in industry *Z*, net of training cost. Note that if Assumption 2 is not satisfied in a given country, *i*, no worker will acquire training. From the above specification of the model, we now obtain the following:

Theorem 1. *If Assumption 1 and 2 are satisfied, then there exists a set of parameters* $\{\alpha_N, \alpha_S, T_N, T_S\}$ *such that a unique equilibrium system of cities under autarky exists, where each country consists of a core –periphery system of cities.*[17]

Proof. Given Assumption 1and equations (13) and (14), the highest utility will be achieved if each city has all types of worker. This will be the case since the skilled worker will be more productive, while the unskilled worker will be indifferent between location with the skilled worker or in a city of there own. Then we show that for equilibrium to exist we must have $\alpha_i - T_i > W_i^*$. It can be seen that if $\alpha_i - T_i \leq W_i^*$, then no worker will acquire training. Thus, the high-tech intermediate input will not be produced and production of the final good is not feasible. Hence, utility will be zero, which contradicts the initial definition of equilibrium. Thus, for autarky equilibrium to exist, we must have $\alpha_i - T_i > W_i^*$. Second, by substituting (21) into $\alpha_i - T_i > W_i^*$ and rearranging terms, we have $\alpha_i > T_i$ ■

In this case the system of cities will consists of a core, city type *z*, in which cities are producing the high-tech intermediate input and populated with all types of skilled workers and the periphery, city type *x*, in which cities are producing the final good with unskilled worker. However, all cities will be of identical size, since the reason for the formation of any city in the

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income inequality.

[17] The equilibrium wage in the case of two types of cities is the same as in the case of diversified city. Thus, we will skip the derivation. Furthermore, the proof of this Theorem is the same as Theorem 1.

model is the fixed set-up cost which is the same for all cities.[18] Trade among cities in this system will occur due to specialization. The number of cities of each type in each country will be given by $m_{ix} = M_i \delta_{iU} / N_i^*$ and $m_{iz} = M_i (1 - \delta_{iU}) / N_i^*$.

Straightforward differentiation of equation (21) will lead to the following result:

Result 1: *Given Assumption 2, the equilibrium wage for the unskilled workers, W_i^*, is increasing in α and β* and *decreasing in T.*

The intuition behind this result is that an increase in α will increase the supply of trained workers. This is because it will increase the incentive for workers to acquire training, and consequently, will decrease the supply of untrained workers. This will result in an increase in the wage of unskilled workers. On the other hand, an increase in *T* will lower the incentive to acquire training, which will increase the supply of unskilled labor, thus, resulting in a decrease in the wage of unskilled workers. Finally, an increase in β will increase the demand for unskilled workers, *L*, and thus resulting in an increase in the wage of the unskilled labor. Recall the assumption that $\alpha_N > \alpha_S$ and $T_S > T_N$. From this, we can conclude that the unskilled worker's equilibrium wage in the North will be higher than in the South.

Next, we examine the impact of the parameters on the equilibrium amount of untrained workers. Totally differentiating (16) with respect to *in* β, T, and α, with the use of Result 1, will lead to the following:

Result 2: *The equilibrium number of periphery cities will increase*

(i) as β and T increase.

(ii) as α decrease.

[18] It is not our in this model to explain the factors underlining the formation of different size cities but rather to examine the distribution of skills in the urban system.

First, note that an increase in the equilibrium amount of unskilled workers, in a give country, will result in a larger number of cities type x in a core-periphery system. Given this, the intuition behind this result is that a higher β will result in a higher demand for unskilled labor and thus, higher wages for the unskilled workers and a larger amount of untrained workers. Both α and T affect δ_{Ui} directly and indirectly through W^* as can be seen from (17). However, the direct effect dominates the indirect effect in both cases. Next, a higher cost of training, T, will decrease the incentive for labor to train and thus increase the mass of untrained workers. On the other hand, a higher α leads to higher wages for the trained labor and consequently increase the incentive for labor to train. Thus resulting in a larger amount of unskilled labor. Thus, since the amount of skilled workers in the economy is given by $M_i(1-\delta_{Ui})$ and given the assumption that $\alpha_N > \alpha_S$ and $T_S > T_N$, we can conclude that *the North will have larger number of core cities than the South.*

Finally, the equilibrium price of good X in a given country can be derived by substituting (3) and (4) into (2) as

$$P_i^* = B\left(W_i^*\right)^{\beta} \qquad (22)$$

where $B = \beta^{-\beta}(1-\beta)^{-(1-\beta)}$. Totally differentiating this equation with the use of (21), we can conclude the following:

Result 3: *The price of the final good, X, under autarky, is increasing in* α *and decreasing in T.* The reason for this result is that P^* is increasing in W^*, which is increasing in α and decreasing in T, from Result 1. Given the assumptions imposed on the parameters T and α, Result 3 implies that the price of the final good is higher in the North, $P_N > P_S$. This result will be used in the following section to derive the pattern of trade under trade equilibrium.

4. Income Inequalities and Social Welfare

In general, there are several ways to measure income disparities. In this paper, I focus on earned wage inequality.[19] Thus, I define a measure of income disparity that will measure income disparity *between* the skilled and unskilled workers as well as a measure of income disparity *within* the skilled workers in a given city and country. Given a core-periphery system, then the measure of income disparity between skill and unskilled workers represent an interregional income disparity. In other words, it is an income disparity between the core and the periphery.

The measure that we adopt is defined as the difference between the highest wage of the most skilled worker and the lowest wage of the least skilled worker, which is given by $ID^{W}{}_{i} = \alpha_i(1-\delta^{*}_{Ui})$. However, since in equilibrium the skill distribution is determined endogenously within the model by the equality of the wages of skilled and unskilled workers. This measure will be given by

$$ID_i = \alpha_i - T_i - W_i^{*} \qquad (23)$$

Observe that this measure defines the disparity within the skilled workers and also as a measure of disparity between the skilled and unskilled workers in a given country.[20] Totally differentiating this equation with the use of (21) and Result 1, we can conclude the following:

Result 4: *If the country consists of a system of diversified cities, then income disparity within the skilled group as well as between skilled and unskilled workers, in a given country and a given city, is decreasing in T and β and increasing in α.*

The intuition behind this result is that an increase in α or a decrease in T will affect income disparity in opposite directions. First, it will decrease income disparity indirectly,

[19] The net income inequality will be the same, in this model, since per-capita expenditure on rent and commuting costs are the same in all cities. The reason for that is that all cities are of equal size.
[20] This is because the income of the lowest skilled worker is the same as the income of the unskilled worker.

through the equilibrium wage of the unskilled workers. Second, it will increase income disparity through the direct effect on *ID*. However, the direct effect dominates the indirect effect. Intuitively, an increase in α and a decrease in T will increase the net income of the most productive skilled worker, which increases income disparity. The impact of β on the income inequality is only expressed through the equilibrium wage. This is also intuitive since β will increase the wage of unskilled labor, which results in a lower income disparity. *Thus, given the assumption that* $\alpha_N > \alpha_S$ *and* $T_S > T_N$*, income disparity in the North is higher than in the South under autarky.*

Result 5: *In a core-periphery system of cities, income disparity within the core as well as between the core and the periphery, in a given country, is decreasing in T and* β *and increasing in* α *.*

Thus, an increase in α or a decrease in *T and* β will result in interregional as well as intra regional diversion.

Define the social welfare function by uniformly aggregating all workers' equilibrium utilities in all cities in a given country. It is assumed that individual utility is linear only in the consumption good, *x*. Thus, by integrating the sum of equation (13) and (14) we have[21]

$$\omega_i = P_i^{*-1}\left\{M_i\left[\delta_{Ui}^* W_i^* + \int_{\delta_{Ui}}^{1} \alpha_i \delta_i d\delta_i - T_i(1-\delta_{Ui})\right] - M_i P_i 3\mu^{2/3}(F/2)^{1/3}\right\} \quad (24)$$

By integrating equation (24) we can derive the social welfare as

$$\omega_i = BM_i W_i^{*(-\beta)}\left[W_i^*\left(\frac{W_i^* + T_i}{\alpha_i}\right) + \left(\frac{(\alpha_i - T_i)^2 - W_i^{*2}}{2\alpha_i}\right)\right] - M_i 3\mu^{2/3}(F/2)^{1/3} \quad (25)$$

[21] Observe we were able to derive that because all cities in the economy are of identical sizes. Thus, the sum of commuting cost and land rent is the same for all households in the economy.

The first term between brackets is the welfare of the unskilled worker, while the second term is the welfare of the skilled worker. It can be seen that the welfare of the unskilled worker is increasing in W^*, while the welfare of the skilled worker is decreasing in W^*. Totally differentiating equation (26) we have the following result:

Result 6: *The social welfare is increasing in* α *and decreasing in T, t, and F.*

It can be seen that both parameters α *and* T affect social welfare directly and indirectly through the wage. But, since it can be shown that the derivative of ω with respect to W evaluated at δ_{Ui}^* is zero, the direct effect is the relevant one on ω. Observe that an increase in α will increase productivity and thus, intuitively it will increase social welfare. Also an increase in T will decrease resources devoted to non-consumption activities and consequently will decrease social welfare. Thus, Result 6 follows directly from differentiating ω with respect to α and T. On the other hand, the impact of the spatial factors t and F negatively affect the social welfare since both parameters result in resources being devoted to non-consumption activities, such as commuting and city formation cost.

Now we can characterize the relationship between income disparity and equilibrium utility of unskilled workers. From Results 5 and 6 we can conclude the following**:**

Result 7: A*n increase*

(i) in α *increases income disparity and increases the social welfare.*

(ii) in T reduces income disparity and reduces the social welfare.

(iii) in t and F reduce social welfare and do not affect income disparity.

From Result 7, and given the assumptions on the parameters, it can be seen that under autarchy, the North will have both higher income disparity and higher social welfare when

compared with the South. Furthermore, if $F_S < F_N \quad and \quad t_S > t_N$ then from equation (12) the cities in the North can be smaller or lager than the cities in the South.

5. Trade Equilibrium

Now consider an integrated North-South world economy in which we have free trade and no labor mobility between countries. In the absence of friction, or transportation cost, trade will result in price parity for the final good *X* such that $P_N = P_S = P$. Given the parameters of the model, the price of the final good is higher in the North as can be seen from Result 3. Thus, the pattern of trade will result in the North importing the final good in return for exporting the high-tech intermediate input to the South. As a result, the unskilled workers' wages will be equalized between both countries. Thus, wage disparity between unskilled worker in the North and South will vanish.

Given $\left\{P_i^{**}, R_{ij}^{**}(r), W_i^{**}, W_i^{**}(\delta); \delta \in [0,1], j \in \{d, x, z\}, i \in \{N, S\}\right\}$, then we define a trade equilibrium by $\left\{N_i^{**}, m_{ij}^{**}, Z_{iz}^{**}, X_{ix}^{**}, V_i^{**}(\delta), \delta_{Ui}^{**}; \delta \in [0,1], j \in \{d, x, z\}, i \in \{N, S\}\right\}$ which results in the following conditions: profit maximization for firms in both the final good and intermediate good sectors; zero profit for firms in both sectors; zero profit for city developers; workers/households of all types maximize utility; equal utility for unskilled workers; a worker will acquire training if it will result in higher net wage; and equality of supply and demand in the labor and intermediate good markets in both countries. From the above specification of the model, we now obtain the following:

Theorem 2. *If Assumption 1 and 2are satisfied, then there exists a set of parameters* $\{\alpha_N, \alpha_S, T_N, T_S\}$ *such that a unique free-trade equilibrium exists, in which each country consists of core-periphery system of cities.*

Proof. Given Assumption 1 and equations (13) and (14), the highest utility will be achieved if each city has all types of worker. This is since the skilled worker will be more productive while the unskilled worker will be indifferent between location with the skilled worker or in a city of there own. Now the material balance condition is given by equating the aggregate supply and the aggregate demand in both countries as

$$\tfrac{(1-\beta)}{\beta}\sum W_i L_i = \sum_i Z_i \qquad (26)$$

Substituting (8), (10), and (12) into (26) we have

$$\frac{(1-\beta)}{\beta}\sum_i W_i M_i\left(\frac{W_i + T_i}{\alpha_i}\right) = \sum_i \frac{M_i}{2\alpha_i}\left[(\alpha_i - T_i)^2 - W_i^2\right] \qquad (27)$$

Without a loss of generality, we can assume that both countries are of the same size, $M_S = M_N$ and that $\alpha_S = \gamma\alpha_N = \gamma\alpha$ *and* $T_N = \theta T_N = \theta T$ where $\theta, \gamma \le 1$. From these conditions, we have

$$\frac{(1-\beta)}{\beta}W_N\left(\frac{W_N + \theta T}{\alpha}\right) + \frac{\left[W_i^2 - (\alpha - \theta T)^2\right]}{2\alpha_N} = \frac{\left[(\gamma\alpha - T)^2 - W_S^2\right]}{2\gamma\alpha} - \frac{(1-\beta)}{\beta}W_S\left(\frac{W_S + T}{\gamma\alpha}\right) \qquad (28)$$

Equation (28) is a quadratic with two roots. The relevant one determines the equilibrium wage as

$$W^{**} = \frac{-(1-\beta)(1+\theta\gamma)T + \sqrt{(1-\beta)^2(1+\theta\gamma)^2T^2 + 2(2-\beta)\beta(1+\gamma)\left[\alpha^2\gamma(1+\gamma) - 2T\alpha\gamma(1+\theta) + T^2(1+\theta\gamma)^2\right]}}{(2-\beta)(1+\gamma)} \qquad (29)$$

Thus, there exists a unique equilibrium wage for the unskilled workers ■

It can be seen that the wage rate will be equalized in equilibrium for the unskilled workers, Samuelson (1945). However, the wage will not be equalized for the skilled workers of a given type between the countries. The reason for this is that workers of a given type in both countries have different potential abilities. Thus, when they acquire training, their productivity

will be different. Thus, under a fee-trade equilibrium each country will consists of a system of cities each of which will have the same skill distribution as the national skill distribution. Next, we will examine the impact of free trade on a sorting equilibrium.

Now observe that if both countries are the same in every respect, then the wage of the unskilled workers before and after trade remains unchanged. But if $\gamma > 0 \quad and/or \quad \theta > 0$, then the wage of the unskilled worker in the North after trade will be lower than before trade. In other words, a positive value in either of the above parameters is sufficient to represent a difference between the two countries. Thus, a relatively small value of one of these parameters indicates a relatively large difference between the North and South in terms of training technology and potential ability. In addition, smaller values of θ or larger values of γ will result in smaller difference between wages of the unskilled workers in the North before and after trade. As a result of that, we can conclude the following:

Result 8: *In a free trade equilibrium, there exists a set of parameters such that:*

(i) $\delta_N^{**} \in (0,1)$ *and* $\delta_S^{**} \in (0,1)$. *The North and the South will produce both the high-tech intermediate input and the final good in a core-periphery system.*

(ii) $\delta_N^{**} \in (0,1)$ *and* $\delta_S^{**} = 1$. *The North will produce the high-tech intermediate input as well as the final good in a core-periphery system, while the South will completely specialize in the production of the final good in a system of one type city.*

This result follows from equation (17) and from the fact that wage rate equalization of the untrained workers under free trade equilibrium will decrease the equilibrium wage rate in the North while it increases the wage rate in the South for these workers. The result indicates that free trade will increase the mass of skilled workers in the North, which has been documented by Machin (1996). Thus, free trade will induce acquisition of human capital in cities in the North.

However, the opposite will occur in cities in the South. Furthermore, free trade will result in change in the structure of the system of cities. In the case of identical diversified cities in each country, free trade will result in an increase in the parentage of unskilled worker in the South. *In other words, skilled workers in the South will be better off working in the final good industry and earning the unskilled wage than working in the production of the high-tech intermediate input. On the other hand, in the case of a core-periphery system of cities, free trade will increase the number of core cities in the North and decrease the number of periphery cities. Furthermore, it* is interesting to note that *the South can completely specialize in the production of the final good if* $W^{**} \geq \gamma\alpha - T$, where W^{**} is given by (29). In this case the South will consists of only periphery cities, while the North will consists of a core –periphery system. Thus, as a result of free trade, cities in the South will be populated with unskilled workers and will specialize in the production of the final good.

Turning to the impact of free trade on income disparity, given both measures of income, disparity within a group and between groups, we can conclude the following:

Result 9: *Free trade will result in*

(i) a higher within the core and within the skilled group income disparity in the North, while a lower income disparity will occur in the South.

(ii) a higher interregional and between skilled and unskilled groups in the North, while a lower income disparity will occur in the South.

This result follows from the wage rate equalization of the unskilled workers in both countries and from the fact that the wages of the skilled workers will not be affected by free trade. The reason that the skilled workers' wages will not be affected as a result of free trade is the choice of numeraire and the fact that these workers have differing potential ability profiles.

This implies that workers are different among countries, since each country has a different potential ability profile, and consequently, a different productivity profile. *Thus, free trade will increase interregional income disparity as well as intraregional income disparity. Furthermore, it will increase the number of core-cities in the North.* As fare as I know, no empirical test has been done on the regional level of free trade impacts on the structure of urban system. However, the decline of national income disparity in the South has been supported for some developing countries in East Asia, Wood (1995). On the other hand, the widening income disparity between skilled and unskilled labor in the North has been supported by Wood (1995), Katz and Murphy (1992), and Bound and Johnson (1992) among others. This rise in income disparity within the skilled group in the North has been documented by Machin (1996) and Katz and Autor (2000). Furthermore, the widening income disparity within the skilled group in the North is due to the increase in the diversity of skilled workers, which has been supported by Hoxby and Terry (1999).

6. Conclusion

This paper presented a spatial, general equilibrium model of North-South trade in which both countries produce a final good and a high-tech intermediate input. The model integrates international trade theory and urban economics to examine the impacts of free trade on the urban system. Each country is populated by a continuum of unskilled workers with differing potential abilities. Developers invest in public infrastructure to form cities in the North and South. Workers in the North and South can acquire skill by investing in training or education. Thus, the skill distribution in the North and South is determined endogenously in the model through self-selection. The model analyzed the impact of a bilateral, free trade agreement on the income inequalities and skill distribution in each country. Furthermore, the model examined the impact

of free trade on the structure of the urban system in the North and South.

It has been shown, in the context of a core-periphery system, that free trade will increase interregional income disparity as well as intraregional income disparity. Furthermore, free trade will result in change in the structure of the system of cities. In the case of a system of identical diversified cities, free trade will result in an increase in the parentage of skilled worker in the North. On the other hand, in the case of a core-periphery system of cities, free trade will increase the number of core cities in the North and decrease the number of periphery cities. In addition, free trade may lead to complete specialization in cities in the South but will not lead to complete specialization in cities in the North. It has been shown that free trade will result in larger income disparities within the skilled group as well as between skilled and unskilled workers in the North.

In this model we adopted a simple city formation mechanize. As a result, all cities in a given country were identical in size, which simplified the structure of the urban system. Thus, the first natural extension of the model is to consider city formation mechanize that would result in different size cities. This will enable us to examine the impact of free trade on skill distribution and the structure of urban system as well as the sizes of cities in each country. The second potential candidate for an extension is micro foundation model of productivity due to interaction between different types of worker.

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