Budget deficit and monetary growth targets: macroeconomic consequences in an optimizing model

M. Badrul Haque
University of New Orleans

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M. Badrul Haque*

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* Assistant Professor of Economics, Department of Economics, University of New Orleans.

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Budget Deficit and Monetary Growth Targets: Macroeconomic consequences in an optimizing model

Abstract

In this paper, I analyze financial programs of the types adopted in the U.K., U.S.A., and in many developing countries under the auspices of the International Monetary Fund. I show that such a program has the effect of stabilizing the relevant macroeconomic system, as to be expected following a result established by Tobin and Buiter (1976). Additionally, I show that when budget deficit and monetary targets are pursued the government must choose at least two policy variables endogenously. I find support for these results in the macroeconomic performances of the U.K. economy in the 1980s. I also examine the effects of changes in certain policies on the steady state values and on the values along the transition path to such a state.
Budget Deficit and Monetary Growth Targets: 
Macroeconomic consequences in an optimizing model

1. Introduction

The purpose of this paper is to assess theoretical implications of a neoclassical optimizing monetary growth model and relate these implications to actual macroeconomic performances following a simultaneous adoption of targets for government budget deficit levels and monetary growth rates. Although such financial programs are now common in both developing and developed countries, this paper is motivated by the policy package introduced in the U.K. in 1980. The government announced a comprehensive strategy to reduce then high rates of inflation and budget deficits by adopting a Medium Term Financial Strategy (MTFS) consisting of a monetary and a budget deficit target. Because of the sustained commitment to reducing government budget deficits and controlling the growth rates of money, examination of macroeconomic performances of the U.K. economy since the adoption of the MTFS provides valuable insights into the likely economic effects of implementing such a policy.

The U.S. and several developing countries introduced a somewhat similar policy but without the same level of sustained commitment. The U.S. Congress became worried in 1984 that the actual and projected high budgetary deficits as well as the debt-service ratio would jeopardize the economic recovery that began in the previous year. The Gramm-Rudman-Hollings law was enacted in 1985 with a view to achieving successively lower projected budget deficit levels and a balanced budget now by 1995. The targeting of the monetary aggregates was effectively abandoned by then, but because of the separation of powers between the Treasury and the Federal Reserve the money supply remains an exogenous
policy variable to the fiscal authorities. In this sense, current U.S. policies are comparable to a financial program in other countries. Among developing countries, over 90 countries have adopted such programs between 1950 and 1989, usually requiring a reduced ratio of government budget deficit to gross domestic product and a ceiling on the money supply. On the average, each developing country implemented these programs for 8 years while receiving balance of payments support from the International Monetary Fund (IMF). Therefore, these programs were not always implemented by the country authorities over a continuous period.

Although policy implications of maintaining a financial program are important in a broad range of countries, existing theoretical literature has not dealt with the simultaneous adoption of targets for budget deficits and money growth. There are two predominant views in the existing literature on the effect of budget deficits. According to the Keynesians, budget deficits are expansionary for output, employment, and prices. On the other hand, neoclassical economists assert that in a closed economy high budget deficits will be associated with higher interest rates, private consumption, lower savings and investment activities, and a slower subsequent growth in the economy’s output. In a small open economy with flexible exchange rates and capital mobility, domestic interest rate will not rise as the demand for savings will be satisfied by incipient capital inflows. Any discrepancy between national and foreign interest rates will be reflected in the foreign currency premia or discount in the forward market where foreign currency traders use the interest parity condition to make forward exchange quotations. Yellen (1989)

1The ceiling is usually specified on domestic credit expansion, which is on the asset side of a monetary survey.
argues that a combination of these outcomes is likely. On tight monetary policy, Friedman (1968) asserted that controlling the growth rate of money will ultimately contribute to a reduced inflation path. This view was challenged by Sargent and Wallace (1981) in situations where real interest rates on government bonds are positive. A number of authors, including Drazen (1985), Liviatan (1984), and Obstfeld and Rogoff (1983), confirm, in the context of a neoclassical model with output treated fixed, the Sargent and Wallace result that attempts to lower inflation with reduced money growth could imply high current and future inflation. Haque (1985) contradicts this result within a neoclassical model that allows movements in the capital stock to induce effects on money demand.

The literatures on budget deficits and tight money are not appropriate in discussing financial packages such as those pursued in the U.K., U.S.A., and developing countries. Because such a package involves a budget deficit and money supply targets, these restrictions must be satisfied simultaneously with the government budget constraint and private behavioral equations. A neoclassical growth model is used in this paper to derive the theoretical implications of a financial package similar to the U.K. MTFS. The principal policy changes in the U.K. in the 1980s has been embodied in the MTFS. Therefore, in the following analysis I neglect the exchange rate policy and the performance of the external sector. Macroeconomic performances in the U.K. during 1980s provide some evidence for the main results of this paper regarding dynamic stability and endogeneity of two policy variables in a typical financial program. However, because of the following changes made mid-stream in the selection of endogenous policies and the limited period for which data is available, a comparison of the transitional effects of a policy change under
different combinations of endogenous policies is not meaningful. The theoretical result indicates a reduction in the fiscal deficit will not affect the transitional real interest rate (and, hence, capital accumulation) if bond issue and lump sum taxes are the two endogenous policies. In the case of the U.K., North Sea oil revenues initially helped in satisfying the constraints and such revenues are not dissimilar to lump sum taxes. Subsequently, discretionary government expenditures were made the second endogenous policy which according to the theory will affect the transitional real interest rate.

Tables 1 and 2 present selected economic indicators in the U.K. between 1974 and 1987. Table 1 shows that the authorities successfully reduced the budget deficit to gross national product ratio by 2 percentage points following the implementation of the MTFS. In the early 1980s, this reduction was achieved because of improved revenue performances owing to higher royalties and taxes from the North Sea oil production. Higher royalties and oil-related taxes permitted the authorities to raise the expenditure ratios without raising the budget deficit ratios. Beginning 1984, as the oil sector weakened, discretionary expenditures were reduced to achieve lower budget deficit ratios. With respect to money growth, the authorities targeted a variety of national monetary aggregates. These definitions differ from those found in the IMF's International Financial Statistics, the data source for this paper. Nevertheless, IMF definitions used in Table 1 provide a useful benchmark for comparing yearly changes. Narrow and broad money grew more rapidly in the 1980s compared to in the 1970s. Both concepts of money grew less rapidly in the early 1980s before they started to rise again. These outcomes reflect several factors. First, the difficulties the authorities experienced in targeting monetary aggregates. Second, distortions in the published data series owing to
Table 1. United Kingdom: Consolidated central government budget and monetary survey, 1974-87

|------|----------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|-----------------
| Total revenues and grants         | 31.5 | 35.2 | 31.5 | 31.8 | 33.0 | 31.0 | 35.9 | 36.5 | 39.2 | 37.7 | 37.8 | 37.6 | 37.4 | 37.1 | 31.2 | 37.4
| Tax revenues                      | 30.2 | 30.7 | 30.2 | 30.5 | 28.9 | 28.6 | 31.0 | 31.7 | 34.1 | 32.8 | 32.8 | 33.1 | 33.2 | 33.6 | 32.9 | 32.8
| Other revenues and grants         | 1.3  | 4.5  | 4.3  | 4.3  | 4.1  | 4.4  | 4.9  | 4.8  | 5.1  | 4.9  | 4.8  | 4.6  | 4.6  | 4.7  | 4.2  | 4.6
| Total expenditure and net lending|      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      
| Of which: interest payments       | (3.6) | (3.3) | (3.4) | (3.6) | (3.8) | (4.1) | (4.1) | (3.8) | (4.1) | (4.1) | (4.0) | (4.0) | (3.4) | (3.4) | (4.0) | (3.4)
| Published overall deficit         | -4.4 | -7.3 | -5.7 | -3.4 | -5.2 | -5.7 | -4.8 | -4.8 | -3.3 | -3.2 | -3.1 | -1.9 | -0.8 | -5.3 | -3.3 |        
| Adjusted overall deficit          | -6.3 | -9.3 | -7.5 | -5.1 | -6.9 | -7.8 | -7.3 | -6.9 | -5.5 | -5.5 | -4.9 | -5.1 | -3.4 | -2.2 | -7.2 | -5.2
| Monetary survey                   |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      
| (Changes in percent of current gross national product) |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      
| Foreign assets, net               | -2.9 | -0.3 | 2.3  | 4.3  | -0.4 | -0.2 | 0.6  | -0.1 | -1.7 | -1.8 | -2.0 | 0.9  | 0.3  | 1.5  | 0.5  | -0.3  
| Domestic credit                   | 8.7  | 3.8  | 5.8  | 2.5  | 3.9  | 3.6  | 4.9  | 7.3  | 6.4  | 5.5  | 7.8  | 5.8  | 9.4  | 9.4  | 4.7  | 7.1
| Central government, net           | 3.1  | 2.7  | 1.9  | -0.4 | 0.6  | -1.2 | 0.1  | -2.0 | 0.7  | -0.3 | 0.7  | 1.6  | -0.1 | -1.6 | -1.1 | -0.3
| Official entities                 | 0.1  | 1.1  | 0.5  | 0.2  | 0.4  | 0.4  | 0.6  | 1.7  | -0.1 | -0.5 | -0.6 | 0.3  | -0.7 | -0.6 | -0.2 | 0.5  | 0.0
| Private sector                    | 5.4  | 1.0  | 3.0  | 2.5  | 2.7  | 3.9  | 4.2  | 4.1  | 4.9  | 4.1  | 1.2  | 3.8  | 5.0  | 6.4  | 2.8  | 4.3
| Other financial institutions      | 0.0  | 0.9  | 0.5  | 0.2  | 0.6  | 0.6  | 0.2  | 3.4  | 1.3  | 2.2  | 4.0  | 1.0  | 5.2  | 3.1  | 0.5  | 2.6
| Broad money                       | 5.0  | 2.5  | 3.7  | 2.9  | 4.3  | 3.6  | 5.0  | 8.1  | 1.9  | 4.5  | 4.3  | 8.9  | 9.3  | 4.1  | 5.9
| Narrow money                      | 1.7  | 2.6  | 1.6  | 2.3  | 2.3  | 1.3  | 0.5  | 2.1  | 1.5  | 1.5  | 2.7  | 3.6  | 4.1  | 2.1  | 2.3  |        
| Quasi-money                       | 3.3  | -0.1 | 1.1  | 0.1  | 1.6  | 2.3  | 3.0  | 3.0  | 2.4  | 1.6  | 5.3  | 5.2  | 2.0  | 3.6  |        |        
| Other items, net                  | 0.8  | 1.0  | 0.8  | 0.6  | 0.7  | 0.4  | 0.6  | 1.3  | 0.9  | 1.1  | 1.2  | 2.4  | 0.9  | 1.6  | 0.7  | 1.3
| (In percent of domestic credit)   |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      
| Credit to private sector          | 56.3 | 50.2 | 50.4 | 53.3 | 51.7 | 59.6 | 62.8 | 61.8 | 63.8 | 65.2 | 63.5 | 63.8 | 62.1 | 65.7 | 56.3 | 61.6
| Credit to government sector       | 38.0 | 42.8 | 42.5 | 39.7 | 37.5 | 32.0 | 29.3 | 23.5 | 29.7 | 16.4 | 13.1 | 13.5 | 10.0 | 5.6  | 38.0 | 16.5
| Credit to other financial         | 5.7  | 7.0  | 7.1  | 7.1  | 7.8  | 8.5  | 7.9  | 14.6 | 15.4 | 18.5 | 23.4 | 22.8 | 27.9 | 28.6 | 5.7  | 19.9
| institutions                      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      
| Memorandum items:                 |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      
| Gross national product            | 85.5 | 107.0 | 128.0 | 158.8 | 168.9 | 188.0 | 239.5 | 259.9 | 278.5 | 304.7 | 325.9 | 356.6 | 382.5 | 411.2 |      
| (In percent)                      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      
| Growth rate of broad money        | 12.9 | 7.1  | 11.6 | 9.5  | 14.6 | 12.5 | 18.5 | 27.8 | 11.4 | 12.8 | 12.1 | 11.3 | 22.5 | 21.0 | 11.4 | 17.2
| Growth rate of narrow money        | 10.8 | 13.6 | 11.4 | 20.8 | 16.3 | 9.1  | 4.0  | 17.7 | 11.3 | 11.1 | 15.4 | 18.1 | 22.0 | 22.8 | 14.5 | 15.3
| Changes in crude petroleum         | 4   | 283.3 | 878.3 | 215.6 | 42.7 | 44.2 | -13.9 | 11.9 | 14.7 | 33.3 | 9.9  | 1.7  | -0.5 | -2.8 | 292.8 | 6.8


1 Arithmetic mean of the yearly figures.
2 Removed from other revenue category the following: capital revenue consisting of sales of fixed capital assets and sales of stocks, non-tax revenue from nonfinancial public enterprises and public financial institutions, and non-tax revenue in the form of royalties, etc. The amount so removed is considered a special financing of the thus adjusted overall deficit.
3 Royalties fixed at the previous year's level of £1.2 bn.
4 Published data contain numerous discrepancies between total assets and liabilities plus net worth.
5 Government sector consists of central government plus official entities.
6 1974 recorded first oil production and it was 0.6 percent of the level achieved in 1980.
Table 2. United Kingdom: Selected Other Economic Indicators, 1974-87

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<td>99.5</td>
<td>102.6</td>
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<tr>
<td>Growth rate of real GDP (in percent)</td>
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<td>3.8</td>
<td>1.0</td>
<td>3.9</td>
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1. Components may not sum to total due to rounding errors.
2. Arithmetic mean of the yearly figures.
3. Data unavailable at the time of writing this paper.
reclassifications of financial institutions several times during the period. Third, sizable statistical discrepancies in the published data. Examining the asset side of the monetary survey provides more insight into factors that affected monetary growth. In the 1980s, domestic credit expanded more rapidly, contributed primarily by lending to the private sector and closely followed by lending to other institutions. Credit expansion to the government sector was nearly zero and, as a result, the share of the domestic credit taken by the government sector fell sharply while those of the private and others rose. The rise in the private sector share and a corresponding fall in the government sector were partially contributed by the selling-off of the state-owned enterprises to the private sector.

Against the background of a successful budget deficit reduction program and attempted targets for money aggregates, performances of selected other economic variables are highlighted in Table 2. It shows that the consumer price increase slowed sharply and that the nominal bank lending rate fell somewhat. Employment level fell severely in the first two years and, although it began rising starting 1983, employment in 1986 (latest year for which data is readily available) remained 3 percent below its 1979 level. The ratio of gross national savings to GNP fell, and the gross domestic investment ratio fell more sharply. Clearly, the domestic economy could not absorb all the national savings. Indeed, in the first seven years, residents used some of their savings to invest abroad. In 1987, a reversal in net investment abroad took place on a small scale. Overall, the mean savings ratio fell in the 1980s relative to the previous period and while the mean investment ratio also fell it remained virtually flat in each year. Whereas foreign savings, acquired by maintaining external current account deficits, provided almost 5 percent of total investment
in the 1970s, even the low national savings in the 1980s exceeded investment needs by over 6 percent. This happened when foreign interest rates fell more rapidly than U.K. interest rates. Following a recession in 1980 and 1981, GDP grew between 1 and 4 percent so that the mean growth in the 1980s was 1.8 percent compared with 1.5 percent in the previous period.

In the balance of this paper, I discuss in section 2 the theoretical implications of the model developed in the Appendix A. The model allows for changes in the capital stock and, hence, in the level of output. But, in Appendix B, I consider how the results might change if output is considered fixed. Finally, in section 3, I make some concluding remarks.

2. An optimizing model: Theoretical implications

I develop a Sidrauski-Brock optimizing monetary growth model in Appendix A and establish the following results for a financial program. First, the imposition of a budget deficit target to a model with a government budget constraint requires now two endogenous policy variables instead of one. Second, the steady state solutions of the model are unique. Third, the economy always evolves along its unique stable path to this steady state. Given these results, I analyze in and out of steady state the implications of a change in the deficit ratio, in nominal money, and in nominal money growth.

A Financial Program

A financial program consists of a government budget constraint, a budget deficit target, and an independent path for the money supply. For simplicity, consider a closed economy where (a) the government does not undertake investment and (b) the sources of the government's revenues are costless issuance of base money, interest-bearing securities issued as perpetuity bonds, and the
collection of lump sum and marginal taxes on private incomes. Because the
government budget deficit is equal to the excess of government spending plus net
interest payments over tax revenue, the government budget constraint is simply
the requirement that any deficit must be financed either by issuing money or
bonds. At each instant, this requirement implies that
\[ g + (1-t)eb - \tau - tf(k) = \dot{m} + \lambda \dot{b} + (m+\lambda b)\pi. \] (1)
The left hand side of this equation is the real budget deficit while the right
side is an indication that this deficit may be financed by either issuing real
money \((M/P)\) or real bonds \((\lambda B/P)\). The variables are defined as follows:
g = real government spending, \(t = \) marginal tax rate, \(e = \) nominal coupon payment
on a bond, \(b = B/P = \) real bonds, \(B = \) number of bonds, \(P = \) price of goods,
\(\tau = \) lump sum tax, \(f(k) = \) production as a function of capital to labor ratio \((k),
m = M/P = \) real money balance, \(\lambda = \) price of a real bond \((b),\) and \(\pi = \dot{P}/P = \) fully
anticipated rate of inflation. In this specification of the government budget,
government policy decisions relate to five variables. These are real government
spending \((g),\) real taxes \((t\) and \(\tau),\) monetary policy \((M),\) and bond issues \((B).\)

Literature addressing government budget constraint has now established that
at most four of these preceding policy variables may be chosen independently and
the fifth then determined to balance the government budget, given the structure
of the economy and its agents' preferences. The second specification in a
financial program involving an additional constraint imposed through a
restriction on the size of the budget deficit will place a further limitation on
the government's ability to choose freely its policy variables. It does not
matter analytically whether this restriction on deficit is specified in levels
or in ratio to gross domestic product. Consider
\[ g + (1-t)eb - \tau - tf(k) = Rf(k), \quad R > 0. \] (2)
Equation (2) indicates that the budget deficit ratio is required to be some arbitrary ratio $R$. This type of formulation is more common in the IMF-supported financial programs introduced in the developing countries. In the U.K., the MTFS has included a deficit target such as (2) with $R$ declining over time. The preceding equation may be used in analyzing the feasibility of a financial program.

Finally, an independent monetary policy requires specifying a path for the money supply. If nominal money grows at a constant rate $\theta$ and at the initial date the outstanding nominal money stock ($M_0$) is predetermined, then real money balances grow according to

$$\dot{m} = (\theta - \pi)m.$$  

In the U.K., $\theta$ has been specified for inside money as well as, since 1984, outside money (M0) that is close to the definition of $M$ in this paper. The MTFS specified a declining $\theta$ over time.

Equations (A18) and (A20) in the appendix together imply that higher fiscal deficits $Rf(k^*)$ are associated with higher real stock of bonds, higher lump sum taxes, unchanged goods prices and hence a higher path for the number of bonds. Since the capital stock remains unchanged, higher deficit ratios do not alter the steady state real interest rate. A previously unanticipated once and for all change in the nominal money stock $M$ is neutral in the steady state. In particular, real balances remain intact which implies that price level $P^*$ change by an equivalent rate. With the real stock of bonds unchanged too, the higher price level implies a higher path for the number of bonds. Similarly, changing $\theta$ does not affect the steady state values of real consumption, capital stock, or

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2Henceforth, an A preceding the Roman numbers for an equation indicates the equation is in the Appendix A.
output. In this sense, money is superneutral. The effect on the inflation rates is one for one:

\[ \frac{d\tau^*}{d\theta} = 1. \]

Despite the preceding results, the deficit ratio may influence real interest rates and the effect of a change in money growth on inflation may also be different in the transition path to the steady state. Such dynamic paths are analyzed below for changes in these two policies. The differential equations system (A23) is used to discuss comparative dynamics. First, consider the dynamic consequences of a change in the deficit ratio. The effect on the transition path is assessed by calculating the eigenvectors corresponding to the stable eigenvalues. This is not necessary in this particular case. From the preceding discussion of comparative statics, changes in \( R \) only alters the number of bonds, real bonds, and lump sum taxes. In the differential equation system (A23), \( R \) and \( b^* \) appear only in the fifth row so changes in \( R \) only influence the fifth elements of the two eigenvectors. Since \( k^*, c^* \), and \( m^* \) are unaffected, and the first three elements of the eigenvectors are independent of \( R \) and \( b^* \), from equations (A24) and (A25) inflation and real interest rates along the transition path are independent of \( R \). However, if government spending rather than lump sum taxes is endogenous, then changes in capital accumulations and bondholdings are induced directly. The intrinsic dynamics of the system remains stable, but changes in real interest and inflation rates follow. An alternative formulation to this paper would incorporate government spending in the welfare function (A1) in a nonlinear fashion in which \( g \) would alter agent's marginal rates of substitution between consumption and real balances. In and out of steady state analysis of such a general version will provide a richer insights on the real consequences of high deficit ratios. In the interest of brevity, and as a first approach, the simple version of (A1) is considered in this paper.
Consider now the consequences of a reduced \( \theta \). A rigorous assessment is possible by solving for the unique negative eigenvalue of the top left hand side 3x3 block of the differential equations system (A23), but this is not necessary. It was shown earlier that in the steady state the inflation rate will fall whenever money growth is reduced. Given also the regular saddlepath condition, at the very worst, attempts to reduce the path of inflation through monetary contraction will initially raise it. For an additive separable utility function, it is shown below that inflation rates will rise before gradually falling to the desired rate.

By definition, \( \text{U}_{cm}^* = 0 \) implies \( E = 0 \). Then, immediately from (A23), three roots are \(( -J_2^*, -\theta, \phi + \theta )\): only one of these is negative. The remaining two roots are obtained from the top left hand side 2x2 submatrix:

\[
2\lambda_1 = (\phi + \delta t)/(1-t) + \sqrt{(\phi + \delta t)/(1-t) + rDF} > 0 \quad (4a)
\]

\[
2\lambda_2 = (\phi + \delta t)/(1-t) - \sqrt{(\phi + \delta t)/(1-t) + 4DF} < 0 \quad (4b)
\]

The two initial conditions on equations system (A23) remain relevant when \( E = 0 \), and therefore the dynamic system continues to be a regular saddlepath. The evolution of dynamic variables depends on the eigenvectors corresponding to the stable eigenvalues: \(-\theta \) and \( \lambda_2 \). The eigenvector of \(-\theta \) is \((0,0,0,0,n)\), where \( n \) is an arbitrary constant. This is convenient because the path of the inflation rate is influenced by changes in the path of \((\hat{k}, \hat{c}, \hat{m})\) as equations (A24) and (A25) suggest. From the steady state analysis, neither consumption nor capital stock is influenced by changes in the growth rate of nominal money supply \( \theta \). In other words, with \( D \) and \( f^n \) unaffected, the unique stable root \( \lambda_2 \) and the unstable root \( \lambda_1 \) remain the same.
The paths of interest and inflation rates in deviation from their steady state values are, up to a factor of constant proportionality,\(^3\)

\[
\begin{align*}
\hat{r} &= \epsilon F \ e^{\lambda_2 s} \\
\hat{\pi} &= \epsilon \lambda_2 \ (J_1 \lambda_1 - F)/(\lambda_2 + J_2 m^*) \ e^{\lambda_2 s}
\end{align*}
\]

where \(\epsilon = k_o - k^*\).

Changes in the nominal growth rate of money do not affect \(\lambda_1, \lambda_2,\) and \(F\). The constant \(\epsilon\) is also unaffected since its value is determined from the initial condition provided by the predetermined capital stock \(k_o\) which cannot alter instantaneously in response to an unanticipated policy change. With these

\(^3\)The solutions to the differential equations are obtained by calculating the eigenvectors corresponding to the stable eigenvalues. The eigenvector corresponding to \(\lambda_2\) is derived as follows. Denote the transpose of the non-trivial eigenvector corresponding to \(\lambda_2\) as \(V_T = (V_1, V_2, V_3, V_4, V_5)\). Then from equations system (A23), the first three elements may be written as

\[V_2 = \lambda_1 \ V_1 \quad V_3 = V_1 m^* (F - J_1 \lambda_1)/(J_2 m^* + \lambda_2)\]

where \(V_1\) is arbitrary. Hence, by taking \(V_1 = 1\), the paths for \((k, c, m)\) in deviations are

\[
\begin{align*}
\hat{k} &= \epsilon \ e^{\lambda_2 s} \\
\hat{c} &= \epsilon \lambda_1 \ e^{\lambda_2 s} \\
\hat{m} &= \epsilon V_3 \ e^{\lambda_2 s}.
\end{align*}
\]

The constant \(\epsilon\) is derived from the initial condition on the capital stock, namely, \(\epsilon = k_o - k^*\). Substitute these solutions in (A24) and (A25) to obtain equations (5). See Haque (1985) for more details.
results in mind, the dynamic implication of changes in $\theta$ on the path of inflation is obtained by differentiating equation (5b)$^4$:  

$$-(\lambda_2 + m^*J_2)^2 e^{-\lambda_2 s} \frac{dx}{d\theta}$$

$$= -\epsilon \lambda_2 \left[ (\lambda_2 + m^*J_2) \lambda_1 dJ_1/d\theta - (\lambda_1 J_1 - F) d/d\theta(m^* J_2) \right]$$

$$= \epsilon \lambda_2 \left[ (\lambda_2 + m^*J_2) \lambda_1 \frac{U^*_c}{U^*_c} + (\lambda_1 J_1 - F) (m^* U_{mmm}/U^*_c \frac{dm^*}{d\theta} + 1) \right].$$

Equation (6) provides the new path for inflation in the transition period to the steady state. Once the steady state is reached, the relevant equation for examining inflation path is (A10): when a reduction in money growth will have a reduced steady state inflation. Consider the case where the initial endowment of the capital stock is less than the steady state level. If the utility function is quadratic, so that $U_{mmm} = 0$, the deviation of inflation from its steady state rate is positive, indicating that inflation will rise in the transition path following a cut in money growth. These results, together with the fact that the dynamic system (A23) is a regular saddlepoint convergent, suggest that after an initial increase in the inflation rates such rates will

$^4$From the definition of $J_1$ and $J_2$ when $U_{cm}^* \equiv 0$, 

$$J_1 \equiv - \frac{U^*_c U^* / U^*_c}{U_{mm}/U^*_c} > 0 \quad J_2 \equiv \frac{U^*_c U^* / U^*_c}{U_{mm}/U^*_c}.$$ 

Hence, 

$$dJ_1/d\theta = - \frac{U^*_c U^* / U_{mm}/U^*_c \frac{dm^*}{d\theta}}{U^*_c/U^*_c} > 0$$ 

and 

$$d/d\theta(m^* J_2) = m^* \frac{U_{mm}/U^*_c \frac{dm^*}{d\theta}}{U^*_c} + 1.$$ 

Substituting for these derivatives, the second RHS expression of equation (6) is obtained.
begin to fall along the transition path to equal the lower rate prevailing in
the steady state.

The phase during which inflation rises following a cut in money growth
depends on the parameter values, initial endowment of capital stock relative to
its steady state value, and the steady state solutions. For purely illustrative
purpose, consider: \( \phi = 0.10, \theta = 0.08 \) (ex post), \( \delta = 0.05, t = 0.25, g = 1,\)
\( f(k) = k^{0.8}, \) and \( U(c,m) = c^{0.8} + m^{0.2}. \) These configurations imply, in common
units, \( k^* = 1024, c^* = 203.8, \) and \( m^* = 5.7; \lambda_1 = 0.26, \lambda_2 = -0.1134, \) and the
steady state elasticity of real money balance with respect to the growth rate of
money is a minus 0.56. In the example where \( \gamma = k_0/k^* < 1, \) the inflation effect
is given by the following relationship:

\[
\frac{d\pi}{d\theta} = (1-\gamma) 0.0685 e^{0.1134s} > 0.
\]

The preceding relationship implies that the larger is the \( \gamma \)-ratio, the longer it
takes for the inflation rates to reach the steady state values.\(^5\) Second, a
reduced \( \theta \) induces higher inflation rates in the transition path; if \( \gamma = 0.5, \) the
lower steady state rate is attained after 29 periods.

Thus far a variable capital stock, and hence a variable output, has been
assumed. The implications of a fixed output for a dual financial target is
considered in Appendix B.

\(^5\) That is, in developing countries such as Brazil and India, for example, where
the current capital stock is low relative to their steady state level a lower
inflation path is achieved much faster than in industrialized countries where
this ratio is high.
3. Concluding Remarks

In this paper, I have derived two main results that add to the government budget constraint literature. A well known result in this literature is that at least one government policy variable must be endogenously determined to balance the government budget. I show that when budget deficit and monetary targets are pursued, the government must choose at least two policy variables endogenously. A second well established result in the literature is that endogenous bond financing is least stable among alternative types of financings. I show that when a budget deficit target is pursued the dynamic stability of a macroeconomic system is unambiguous. This result is similar to that established by Tobin and Buiter (1976) in the IS-LM context. These results appear to be confirmed by the performances of the U.K. economy in the 1980s. Thus, financial programs of the types adopted in the U.K., U.S.A., and in developing countries under the auspices of the International Monetary Fund have the effect of stabilizing the relevant macroeconomic system but the programs limit further independent choices of government policies.
Appendix A: The Model

In this appendix, I specify an optimizing model to derive private behavioral equations (section 1), and then I analyze the uniqueness of the steady state (section 2) and dynamic stability (section 3).

1. The Private Economy

A financial program consisting of equations (1) to (3) in the text is satisfied by the simultaneous interactions with private behavioral equations as well as with the goods market-clearing condition. I develop a Sidrauski-Brock optimizing model over an infinite horizon and under perfect foresight to obtain conditions for private behavior. Although the issues addressed in this paper could be analyzed by extending the model as in Blanchard (1985), the model developed is of sufficient generality for the main point of this paper: exogenous money supply and a budget deficit target requires two endogenous policy variables and this has the effect of stabilizing the macroeconomic system.

Suppressing time subscripts, the representative agent's instantaneous welfare function is denoted as

\[ W = U(c, m) \]

where \( c \) and \( m \) denote real private consumption and real money balances. The representative agent maximizes (A1) over time subject to the stock and flow constraints which are described below. The wealth of each agent is composed of real money balances, equity, and the (per capita) market value of outstanding government bonds. Initially, it is assumed that the coupon payments on government bonds are fixed in nominal values. If \( B \) is the number of bonds and \( P \) is the price of goods, then denote \( b = B/P \) so that \( \lambda b \) is the real value of bonds
and \( \lambda \) is the price of \( b \). Assuming that Tobin's \( q \) is unity, the agent's real wealth is:

\[
a = m + k + \lambda b. \tag{A2}
\]

The time derivative of the preceding relation gives the following equation for asset accumulation:

\[
\dot{a} = \dot{m} + \dot{k} + \lambda \dot{b} + \lambda \dot{b}. \tag{A3}
\]

The representative agent is assumed to receive all production income as well as debt interest on public bond holdings. The production function is a linear homogeneous function of labor and the capital stock \( (k) \). Assume the labor supply is perfectly inelastic and is fully described by \( f(k) \). Taxation consists of a lump sum \( \tau \) and a marginal rate \( t \) applicable to all sources of income. In this formulation, depreciation expenditures are not tax deductible. The net of tax income is used to consume, make gross investment, or accumulate cash balances and bond holdings. Thus, the income constraint is

\[
(1-t)[f(k) + eb] - \tau = c + \dot{k} + \delta k + \dot{m}/P + \lambda \dot{b}/P
\]

\[
= c + \dot{k} + \delta k + \dot{m} + m\pi + \lambda (\delta + b\pi) \tag{A4}
\]

where \( e \) is the fixed nominal coupon value on a perpetuity, \( \delta \) is the rate of capital depreciation, \( \pi \equiv \dot{P}/P \) is the fully anticipated rate of inflation. Replacement investment is \( \delta k \) while net investment is \( k \). Substituting (A3) into (A4) yields the following flow constraint:

---

1The assumption that the agent receives all production income is consistent with a corporate sector financing its investment from borrowed funds. Brock and Turnovsky (1981) have shown that the dynamic structure of the economy would alter when corporate investments are financed by alternative methods. In the interests of simplicity, in what follows corporate investment are financed by borrowing from private agents.

2The assumption of inelastic labor supply is implicit in the specification of the welfare function (A4) when the latter is derived from a consumption-leisure choice framework where money facilitates reduced transaction costs.
\[(1-t)[f(k)+eb] - \tau = c + \delta k + (m+\lambda b)\tau + \dot{a} - \lambda b. \tag{A5}\]

Thus, the representative agent's decision problem is to choose a path for \((c,k,M,B,a)\), given an expected path for \((\lambda,P,t,\tau,g)\) and the initial values \((M_0,B_0,k_0)\) to maximize the present value of \((A4)\) subject to equations \((A2)\) and \((A5)\). In particular, applying the constant rate of time discount \(\phi\), identical agents are assumed to maximize the present value of welfare over an infinite horizon and under perfect foresight. Hence, denoting the Lagrangian multipliers of the stock and flow constraints as \(\beta\) and \(\alpha\), respectively, the Lagrangian is

\[
J^\infty [U(c,m) + \beta(a-m-k-\lambda b) + \alpha((1-t)[f(k)+eb] - \tau - c - \delta k - (m+\lambda b)\tau - \dot{a}) - \lambda b)] e^{-\phi s} ds \tag{A6}
\]

over all future time \(s\).

Assume \(U\) is concave so that

\[
U_{cc}U_{mm} - U_{cm}^2 \geq 0. \tag{A7}
\]

Then, sufficient conditions for the problem in \((A6)\) to have an internal optimum are the first order conditions, Euler equation \((A8.4)\) below, and the transversality condition

\[
\lim_{s \to \infty} \alpha a e^{-\phi s} = 0.
\]

This condition may be checked by using the finite steady state solutions.³

Letting \(r = \beta/\alpha\) and eliminating \(\alpha\), the equations for an internal optimum when \((A6)\) is solved are:

³Since \(a\) is the Lagrangian multiplier corresponding to the flow constraint expressed in terms of the asset accumulation \(\dot{a}\), it represents the additional value to welfare from increased asset accumulation. Thus \(a\) is the shadow price of asset accumulation. Hence, the transversality condition is the requirement that the present value of assets, evaluated at the shadow price, must equal zero. This condition is satisfied in dynamic systems which converge to a steady state.
Money market: \[ U_m = U_c (r + \pi) \] (A8.1)

Capital market: \[ (1-t)f'(k) = r + \delta \] (A8.2)

Bond market: \[ \lambda / \lambda + (1-t)e / \lambda = r + \pi \] (A8.3)

Euler equation: \[ - \frac{U_c}{U_c} = r - \phi \] (A8.4)

where \( f'(k) = \frac{d}{dk}(f(k)) \). The preceding four equations include a particular individual optimal value of \( c, m, \) and \( k \). They do not include such an optimal value for \( b \) because the instantaneous relationship in (A6) is linear in \( b \). So, while (A8.2) ties down a unique value of capital \( (k) \), there is no equivalent condition for bonds \( (b) \). Also, the level of government spending does not appear in the preceding conditions because the welfare function does not include such spending in a nonlinear fashion. Such a specification does not allow analyzing the issues involved when government spending is a substitute for private consumption. 4

Goods market-clearing is assumed. Since output is divided among private plus public consumption and gross private investment, this implies

\[ f(k) = c + g + \delta k + \delta k. \] (A9)

2. Steady State Analysis

The feasibility of the financial program is assured if the model represented by equations (1) to (3) in the paper and (A8) to (A9) in this

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4Equations system (A8) is used to discuss the term structure of interest. Because the real (implicit) return on (long) bonds is \( r \) and on real balance is \( -\pi \), the additional risk premium on bonds is \( r+\pi \). This is exactly equal to the nominal interest rate. Similarly, because the real return on capital (after tax and depreciation adjustments) is \( (r+\delta)/(1-t) \), the additional risk premium on equity is \( (r+\delta)/(1-t) + \pi \). This is the marginal product of capital plus the fully anticipated inflation rate. That is, the risk premium on equity exceeds that on bonds to account for capital depreciations and marginal tax rates. The additional risk premium on equity relative to bonds is \( (\delta+rt)/(1-t) \). When the tax rate is zero, this is equal to the rate of capital depreciations.
appendix has a unique forward-looking path as its solution. The necessary and sufficient conditions for this path being unique are that there exist unique steady state values, a convergent subspace in the equations system that describes the dynamic evolution of the economy, and an exact number of initial conditions to tie down a unique point in the convergent subspace.

Denote the steady state values by superscript *. I assume the real money stock is constant in the steady state so the perfectly anticipated inflation rate equals the rate of nominal money growth (see equation 3) -- i.e.,

\[ r^* = \theta. \] (A10)

From equation (A8.4), the real interest rate is equal to the rate of pure time discount:

\[ r^* = \phi \] (A11)
because I assume real consumption and real money balances are constant.

Equation (A8.2) then implies

\[ (1-t)f'(k^*) = \phi + \delta \] (A12)

and so this modified golden rule establishes a unique capital stock. Because the nominal interest rate \((\phi + \theta)\) and the coupon payment on bonds \((\epsilon)\) are constant, the bond price \(\lambda^*\) must be constant. Indeed, from equation (A8.3)

\[ \lambda^* = (1-t)e/(\phi + \theta). \] (A13)

Since real wealth must be constant, and \(\lambda^*\) is constant, it follows that

\[ \delta = 0. \] (A14)

The expression for the goods market clearing condition is:

\[ f(k^*) = c^* + g + \delta k^* \] (A15)
and it determines a unique private consumption level \( c^* \), given the level of real public spending and the rate of capital depreciation.\(^5\)

With a unique real consumption \( (c^*) \), demand for real money balances \( (m^*) \) is then solved from the money market condition (A8.1).

\[
U_m(c^*,m^*)/U_c(c^*,m^*) = \phi + \theta. \tag{A16}
\]

But, because this condition is nonlinear, \( m^* \) may have multiple solutions. The following analysis establishes that, provided money and consumption are normal or inferior goods and remain so far all feasible solutions, then the money market condition solves for a unique \( m^* \). The slope of the money market condition in the \((c,m)\)-plane is

\[
\frac{dm^*}{dc^*} = -J_1/J_2, \tag{A17}
\]

where \( J_1 \equiv (U_{cm}U_c - U_{cc}U_m)/U_c^2 \) and \( J_2 \equiv (U_{mm}U_c - U_{cm}U_m)/U_c^2 \).

Both \( J_1 \) and \( J_2 \) are evaluated at their steady state values. Restrictions \( J_1 > 0 \) ensure money and consumption are normal goods.\(^6\) Thus, the slope \( dm^*/dc^* \) is negative. Figures 1 and 2 show the graph for the money market condition, under the assumption of both normal or inferior goods. In two possible cases, given a unique \( c^* = c_0 \) the money market condition provides a unique \( m^* = m_0 \).

\(^5\)Condition (A15) also shows that private consumption in the steady state is positive provided that output net of capital depreciation exceeds public consumption of goods and services.

\(^6\)The normality conditions are derived as follows. First, maximize \( U(c,m) \) subject to the flow constraint \( pc + m = y \). Second, take total derivative of \( c \) and \( m \) with respect to \( y \). Conditions \( dc/dy > 0 \) and \( dm/dy > 0 \) imply that \( c \) and \( m \) are normal goods. See Fischer (1979).
Money and consumption are normal goods or both are inferior. Either money or consumption is an inferior good.

Thus, even though the money market condition is nonlinear, \( m^* \) is unique under the standard assumption that money and consumption goods are normal.

There remain expressions for the government budget constraint and the budget deficit targets:

\[
\begin{align*}
    m^* \theta &= g - \tau - tf(k^*) + \phi(1-t)eb^*/(\phi+\theta) \\
    Rf(k^*) &= g - \tau - tf(k^*) + (1-t)eb^*.
\end{align*}
\]

A knowledge of \( k^* \), \( m^* \), and \( \lambda^* \) and the constant parameters is sufficient to solve for a unique real bond stock \( b^* \) from the government budget constraint, simultaneously satisfying itself and the private sector's optimal behavioral equations. The real bond stock thus obtained must be unique precisely because the equation is linear in the unknown.\(^7\) However, unique values of the capital stock and the number of bonds will not generally satisfy the steady state version of the real budget deficit target, equation (A19). Hence, given a real budget deficit target, it is not generally feasible to set independent paths for

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\(^7\)From equation (A18), the real bond stock is positive if total government revenue, inclusive of inflation tax, exceeds government expenditure on goods and services. At an intuitive level, a positive real bond stock consistent with a steady state is possible if the government's tax revenue is sufficient to finance interest payments on a constant real stock of perpetuity bonds.
government spending, taxes, and the money supply. At least one must be determined endogenously, together with the number of bonds. If the marginal tax rate is specified as an endogenous policy variable, then the steady state values are not unique in general. This means that it is not possible to rule out multiple perfect foresight paths. Instead, either real government spending or lump sum taxes must be made endogenous to avoid this problem of multiple paths. In the formulation of this paper, these two have qualitatively similar steady state and dynamic stability properties because government spending does not enter the utility function in a nonlinear fashion.

To show how a real stock of bonds $b^*$ and lump sum tax $\tau^*$ or discretionary government spending $g^*$ are chosen to balance the government budget and satisfy the budget deficit target, substitute for $\tau$ or $g$ from the government budget constraint into (A18) to obtain

$$ Rf(k^*) = \theta(m^* + \lambda^* b^*). $$

With $k^*, m^*, \lambda^*$ already solved, the preceding equation determines $b^*$ given the steady state fiscal deficit $Rf(k^*)$. In particular,

$$ b^* = \frac{Rf(k^*)}{\theta} - m^*/\lambda^*. $$

By substituting this unique value of $b^*$ in the government budget constraint (A17), the following equation is obtained:

$$ g + \phi Rf(k^*)/\theta = (\phi + \theta)m^* + \tau + tf(k^*). $$

This equation then determines a unique $g^*$ or $\tau^*$ to satisfy the government budget constraint.

The establishment of unique steady state values in the preceding analyses confirms that a financial program similar to the MTFS in the U.K. is feasible.
with bond issues and lump sum taxation or government spending as two simultaneous endogenous policy decisions. The question of dynamic stability is addressed below.

3. Stability Analysis

The dynamic path is examined by linearizing the behavioral equations, the money supply rule, the government budget constraint, and the real budget deficit target in the neighborhood of the unique steady state equilibrium. The government budget constraint (1), the budget deficit target (2), and the money supply rule imply (3):

$$R_f(k) = m\theta + \lambda(b + b\pi).$$  (A21)

Denoting vector transpose by $T$, the following notation is adopted:

$$x^T = (k,c,m,\lambda,b) \quad \dot{x} = x - \dot{x}^* \quad \ddot{x} = d/ds(\dot{x}).$$

Linearizing the dynamic equations and neglecting the second and higher order terms in the Taylor expansion, the following expressions are obtained

$$\dot{k} = (\phi + \delta t)/(1-t) \quad \ddot{k} - \dddot{c}$$  (A22.1)

$$J_1 \dddot{c} + J_2 \dddot{m} = \dddot{r} + \dddot{\pi}$$  (A22.2)

$$F \dddot{k} = \dddot{r}$$  (A22.3)

$$\dddot{\lambda} = \lambda^* (\dddot{r} + \dddot{\pi}) + (\phi + \theta) \dddot{\lambda}$$  (A22.4)

$$U_{cc} \dddot{c} + U_{cm} \dddot{m} = - \dddot{r} U_c^*$$  (A22.5)

$$R_f' \dddot{k} = \theta m + \lambda^* (b + b^* \dddot{r} + \theta b) + \theta b^* \dddot{\lambda}$$  (A22.6)

$$\dddot{m} = - m^* \dddot{\pi}$$  (A22.7)

---

8This feasibility will not in general extend to the case of indexed coupon payments on government bonds. The compensation offered to economic agents for inflation implies that there is no inflation tax on bonds (i.e., $\theta b^* \dddot{m}$ does not appear in equation (A20)). This means that in the steady state the inflation tax on money ($\theta m^*$) must equal the real budget deficit which in turn must equal $R_f(k^*)$. With $k^*$, $m^*$, and $\theta$ following a constant path, $R$ cannot then be an exogenous variable.
where \( F \equiv (1-t)f'' \) and \( f'' = \frac{d}{dk}(f'(k)) \).

The preceding seven equations are reduced to five by eliminating \( \hat{r} \) and \( \hat{\pi} \). In matrix notation, variational linearizations of the private behavioral equations, the government budget constraint, the budget deficit and money supply targets under endogenous bond and lump sum tax finance are

\[
\begin{bmatrix}
\dot{k} \\
\dot{c} \\
\dot{m} \\
\lambda \\
\dot{b}
\end{bmatrix} =
\begin{bmatrix}
\frac{(\phi+\delta t)}{(1-t)} & -1 & 0 & 0 & 0 \\
-F(D+Em^*) & Em^*J_1 & Em^*J_2 & 0 & 0 \\
Fm^* & -m^*J_1 & -m^*J_2 & 0 & 0 \\
0 & \lambda^*J_1 & \lambda^*J_2 & (\phi + \theta) & 0 \\
Fb^* + Rf'/\lambda^* & -b^*J_1 & -b^*J_2 - \theta/\lambda^* & -\theta b^*/\lambda^* & -\theta
\end{bmatrix}
\begin{bmatrix}
k \\
c \\
m \\
\lambda \\
b
\end{bmatrix} \tag{A23}
\]

where \( D \equiv U_{cc}/U_{cc}^* \) and \( E \equiv U_{cm}/U_{cc}^* \). The solutions obtained for \((k, c, m)\) are then used to solve for \( \hat{r} \) and \( \hat{\pi} \) in the following equations.

\[
\hat{r} = Fk \tag{A24}
\]

\[
\hat{\pi} = J_1 \hat{c} + J_2 \hat{m} - \hat{r} \tag{A25}
\]

The triangular structure of the coefficient matrix in the differential equations system (A23) shows that two eigenvalues are \(- \theta\) (negative provided \( \theta > 0 \)) and \( \phi + \theta \) (positive). The remaining three eigenvalues are obtained from the top left hand side 3x3 submatrix, whose determinant is negative while the trace is positive. Hence, the 5x5 differential system has exactly two negative and three positive eigenvalues and is a regular saddlepath if, and only if, there are exactly two initial conditions. Real consumption (c), bond price (\( \lambda \)) and the price of goods (p) are endogenous and capable of instantaneous jumps. Nominal money and the number of bonds are instantaneously predetermined.
Although their real values may take on any value at the initial date, the jump in the price level cannot alter the money-to-bond ratio and this imposes one initial condition. The inherited capital stock $k_0$ (the outcome of previous investment decisions) is a second initial condition. Thus, under budget deficit and monetary targets, endogenous issuance of bonds and lump sum taxes or discretionary government spending, the economy is characterized by a regular saddlepath and it proceeds along its unique self-fulfilling and forward-looking convergent path to the steady state from the initial date on.
Appendix B: The case of fixed output

If the capital stock is fixed, then from the goods market condition \((A9)\) consumption is also fixed. The steady state solutions are again unique. The differential equations system \((A23)\) is replaced by the following 3x3 sub-block:

\[
\begin{bmatrix}
\dot{m} \\
\lambda \\
\dot{b}
\end{bmatrix} =
\begin{bmatrix}
-m^* J_2 & 0 & 0 \\
\lambda^* J_2 & (\phi + \theta) & 0 \\
-b^* J_2 - \theta/\lambda^* & -\theta b^* /\lambda^* & -\theta
\end{bmatrix}
\begin{bmatrix}
\dot{m} \\
\lambda \\
\dot{b}
\end{bmatrix} = \begin{bmatrix}
\hat{m} \\
\hat{\lambda} \\
\hat{b}
\end{bmatrix} \quad (B1)
\]

The preceding dynamic equations system has a stable eigenvalue if \(\theta\) is positive; this is assumed. The non-trivial eigenvector corresponding to \(-\theta\) is \((0, 0, n)\), where \(n\) is an arbitrary constant. Hence, the solution to \((B1)\) is

\[
\begin{align*}
\hat{m} &= 0 \\
\hat{\lambda} &= 0 \\
\hat{b} &= \epsilon_0 n \ e^{-\theta t}
\end{align*}
\]

where \(\epsilon_0 = b_0 - b^*\) (see Haque, 1985). That is, the stock of bonds \(b\) converges to its steady state value at the rate \(\theta\). All other variables instantaneously achieve their steady state values.

Consider what this implies for the inflation path. From \((A24)\) and \((A25)\), since \(\hat{c} = \hat{m} = \hat{k} = 0\), \(\hat{\pi} = 0\). Together with the money supply rule \((3)\), this implies \(\pi = \pi^* = \theta\) for all time, where \(\theta\) is the growth rate of nominal money. That is, tight money will always reduce inflation, and this result holds for the general utility function specified in this paper. Such a strong result is obtained precisely because of the specification of a budget deficit target, additional to a money supply target.

In a related literature, Liviatan (1984) has shown that when a money supply rule is pursued, and the utility function is logarithmic additive separable in consumption and real money balances, equations \((B1)\), \((A24)\), and \((A25)\) imply high current and future inflation when restrictive monetary policy is pursued.
Drazen (1985) has shown that in all other types of additive separable utility functions provided the interest elasticity of demand for money is inelastic, the Sargent/Wallace and Liviatan result for dynamic quantity theory demand schedule has to be qualified. In particular, tight money now implies eventually a high $\theta$ and inflation. This is exactly the result obtained in the original Sargent and Wallace example, although in their case the result held only for constant velocity quantity theory demand schedule.
REFERENCES


