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Monetary and Extended Monetary Growth Models: The Question of Uniqueness in the Steady State

Abstract

I show that in a monetary growth model Edgeworth-substitutability between consumption and real balances do not in general imply multiple steady state solutions as has been widely believed following Brock (1974). I then show that when the government budget constraint is explicit and the deficit is money-financed with fixed real coupons on outstanding bonds, it is not possible to rule out multiple steady states.

1. Introduction

I show that in a monetary growth model Edgeworth-substitutability between consumption and real balances do not in general imply multiple steady state solutions as has been widely believed following Brock (1974). I then show that when the government budget constraint is explicit and the deficit is money-financed with fixed real coupons on outstanding bonds, it is not possible to rule out multiple steady states.

Following Sidrauski (1967), a number of authors, including Brock (1974; 1975), Calvo (1979) and Begg and Haque (1984), introduce real money balances in a utility function together with the real consumption level. However, Brock (1974; 1975) has argued that if real consumption and real money balances are Edgeworth-substitutes then multiple steady states may exist.[1] Although for such an utility function Obstfeld (1984) states that "[g]iven existence, uniqueness of the steady state follows from the assumed normality of consumption" (fn. 7, page 226) the result by Brock has discouraged many authors including Obstfeld and Rogoff (1983) from considering non-additive, separable utility functions. A more recent example is Liviatan (1988). But the assumption of additive separability in consumption and real money balance removes an important argument for introducing real money stock and flow consumption in the utility function. An additive separable function has the feature that any increase in consumption arising from increased income will not induce a change in money demand. Yet, the usual argument for introducing real money balance in the utility function is

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[1]It is well-known that in perfect foresight and rational expectations models, for a unique self-fulfilling convergent path to a unique steady state to exist, necessary and sufficient conditions are (i) that there is a convergent subspace in the dynamic equation system which describes the evolution of the economy over time and (ii) there are a sufficient number of initial conditions to tie down a unique point on the convergent subspace. If steady state solutions are not unique, there will be no unique convergent path even if the other conditions are satisfied.

that it facilitates reducing transaction costs and thereby improving leisure and utility levels.

I demonstrate that unless consumption or real money demand, but not both, change from being normal to inferior commodity in the feasible range of solutions, the problem of nonuniqueness does not arise. I then point out a source of multiple equilibria in monetary macro-models in the case of endogenous money finance of government deficits and fixed real coupon values on outstanding government bonds.

2. Non-uniqueness in Monetary Models

In this section, I first describe the conditions that give rise to non-unique steady state solutions and then I establish reasons why such non-uniqueness is not likely to be a feature of a non-additive separable utility function in consumption level and money stock balance.

(a) Reasons for non-uniqueness

Brock (1974; 1975) uses the following money market condition to argue the non-uniqueness of the steady state:

$$U_m(c,m) = r\, U_c(c,m) \tag{1}$$

where r is the constant nominal interest rate, U_c is the marginal utility of consumption and U_m is the marginal utility of real money balances. Condition (1) is the familiar optimum condition: at an optimum the marginal rate of substitution between consumption and money is equal to the nominal interest rate.

Without any loss of generality, consider output is fixed. Then goods market clearing implies

$$y = c + g, \tag{2}$$

where c is real private consumption and g is the real government spending on goods and services. Thus, for a given g and y, the goods market condition determines a unique c. For a given c and r, equation (1) then determines the steady state real balances. However, it is typically asserted that because of the non-linearity in (1), if consumption and real balances are Edgeworth-substitutes (i.e., $U_{cm}<0$) then there are multiple money balances corresponding to a unique consumption level which simultaneously satisfies condition (1). This can only happen if the money market condition (1) has both positive <u>and</u> negative slopes in the (c,m)-plane as in Figures 1a and 1b.

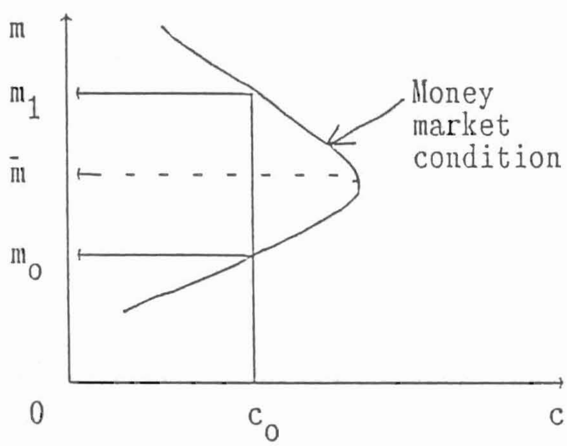

Figure 1a

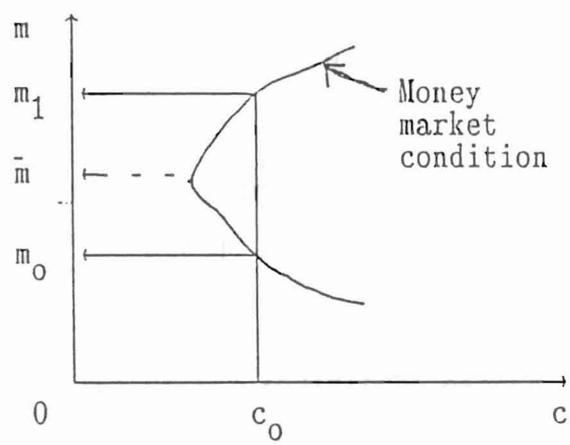

Figure 1b

(b) Uniqueness of the steady state re-examined

A natural question to ask is what is the intuition for a money market condition to have a shape such as in Figures 1a and 1b. It is generally assumed that both money and consumption are normal goods--that is, demand for both these commodities rises as income rises. This is true for all money-consumption combinations up to ($\bar{m}$, $\bar{c}$), where $\bar{c}$ is the maximum consumption level and $\bar{m}$ is the threshold money balance beyond which money becomes an inferior good. If beyond $\bar{m}$, money is an inferior good--that is, as income rises demand for real money balances falls--then income and financial market innovations are positively related beyond a threshold income level that necessitates less money holding. Suppose that threshold income corresponds to a consumption level $\bar{c}$, so that the money market condition has the backward-bending shape. Income levels beyond this threshold induce reduced demand for real money balance and consumption. The falling consumption level violates the goods market clearing condition. Therefore, the proposition that the money market condition has shapes such as in Figures 1a and 1b are not consistent with market clearing assumptions. I show below that standard assumptions imply a unique steady state solution irrespective of whether or not consumption and money demand are Edgeworth-substitutes.

As noted in Fischer (1979), conditions for real consumption and real money balances to be normal goods are

$$J_1 \equiv (U_{cm}U_c - U_{cc}U_m)/U_c^2 \; > 0 > (U_{mm}U_c - U_{cm}U_m)/U_c^2 \; \equiv J_2 \tag{3}$$

Suppose conditions (1) to (3) hold in the steady state. Furthermore, the steady state nominal interest rate r is constant (see, for example, Haque (1985)). Then the slope of the relationship (1) in the (c,m)-plane is

$$\frac{dm}{dc} = -J_1/J_2 \tag{4}$$

which is positive given the conditions in (3). Furthermore, if either consumption or real balances are inferior goods, dm/dc is negative. That is, whatever U_{cm} might take, the money market condition (1) is always either upward or downward sloping in the (c,m)-plane. Thus, given a unique value of c, condition (1) will imply a unique value for m. This is illustrated in Figures 2a and 2b for internal solutions of the monetary model. It follows that only if either money <u>or</u> consumption change from being normal to inferior good in the feasible region of solutions, then the question of multiplicity arises (see Figures 1 and 2).[2]

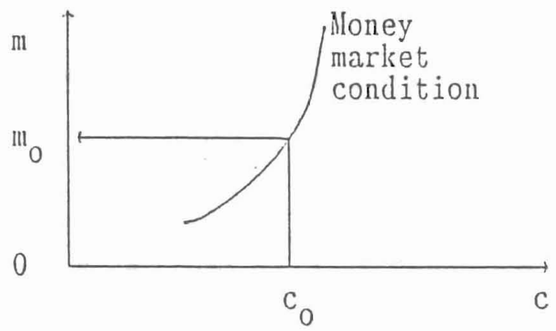

Figure 2a:
Money and consumption are normal goods or both are inferior.

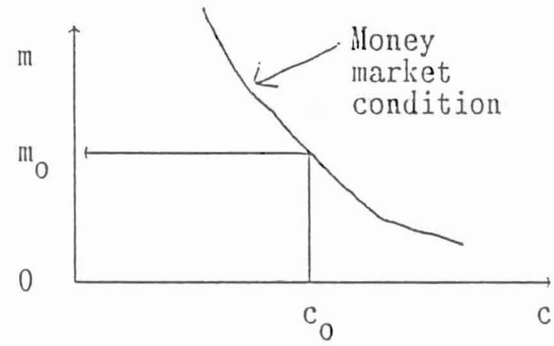

Figure 2b:
Either money <u>or</u> consumption is an inferior good.

3. Non-uniqueness in Extended Monetary Models

Haque (1983) has demonstrated an alternative source of non-uniqueness for steady state solutions when the government budget constraint is explicit and coupon values on government bonds are fixed in <u>real</u> terms. In this case, under residual money finance of fiscal deficit it is not possible to rule out multiple steady states. Furthermore, even if the perfect foresight steady state solutions are unique the rational expectations solutions might not be. At an intuitive level, this is because real balances and inflation are simultaneously determined to satisfy the government budget constraint and the money market condition; and these amount to two nonlinear relationships.

Specifically, suppose that

$r = \phi+\pi$

[2]It is trivial to demonstrate the preceding result of a unique steady state solution by considering a CES utility function.

where ϕ is the real interest rate and π is fully anticipated inflation rate so that we may re-write equation (1) as

$$U_c/U_m = \phi + \pi \tag{5}$$

Suppose further that the government maintains a positive deficit. Without any loss of generality, consider zero income tax revenue. In particular, government spending <u>plus</u> debt interest payments are financed by an inflation tax on money balances:

$$m\pi = g + \phi\, qb_o \tag{6}$$

where b_o is the number of bonds and q is its price such that $q = e/\phi$, e is the fixed <u>real</u> coupons. Nominal money is an endogenous policy decision while bond issues are exogenous. The money market condition (5) implies

$$\frac{dm}{d\pi} = 1/J_2 < 0.$$

The preceding slope clearly depends indirectly on π, through m in J_2. On the other hand, the government budget balance (6) is also a rectangular hyperbola in m and π:

$$\frac{dm}{d\pi} = -m/\pi < 0.$$

In view of the preceding two slopes, it is not possible to rule out at this level of generality the possibility of multiple steady state solutions for m and π. For illustrative purposes, consider a Cobb-Douglas type utility function:

$$U = c^{\alpha}\, m^{\beta} \qquad \alpha + \beta \leq 1 \qquad \alpha\, ,\, \beta \geq 0 \tag{7}$$

The money market condition implied by (7) is drawn as curve II in Figure 3. Curve 1 denotes the relationship implied by the government budget constraint.

In this example, the steady state solutions for m and π are unique under the assumption of perfect foresight.

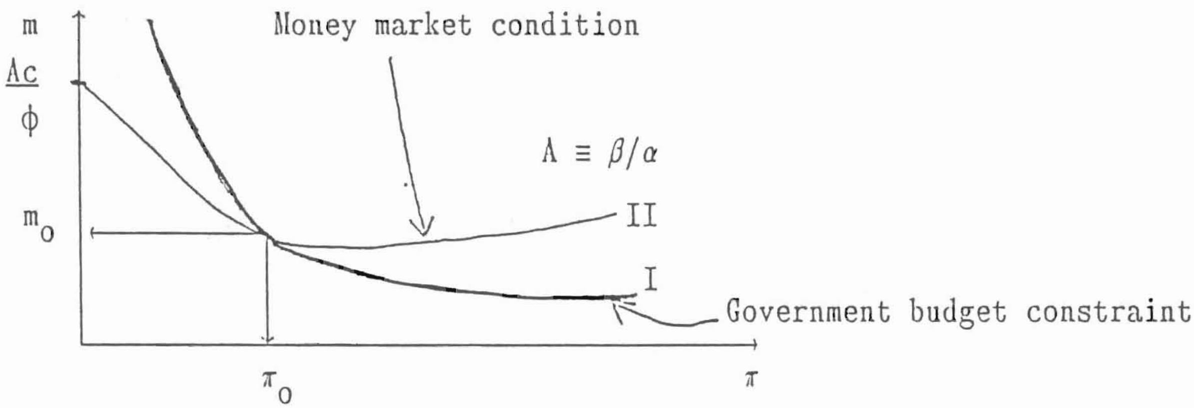

Figure 3

Nevertheless, the steady state under rational expectations may still be non-unique. This is precisely because when the assumption of perfect foresight is replaced with rational expectations, equations (5) and (6) are in expectations of products of m and π. In order to solve for the expected values of m and π, the expectations of products need to be substituted with products of expectations <u>plus</u> the covariances of m and π. The covariance being quadratic, and hence non-linear, it generally implies multiple solutions for expected values of m and π. Thus, even if the perfect foresight steady state solutions for m and π are unique, under rational expectations assumption there may be multiple steady states.[3]

[3] In an extensive analysis, Haque (1983) found non-uniqueness of steady states in one other case only: an endogenous marginal tax rate policy. All other feasible government policies have a unique steady state for fixed real as well as nominal coupon bonds under the assumption of both perfect foresight and rational expectations.

4. Conclusion

I have demonstrated the inaccuracy of the result that in monetary growth models multiple steady state solutions cannot be ruled out when consumption and real balances are Edgeworth-substitutes. Nevertheless, when the government budget constaint is explicit and the deficit is money financed with fixed real coupons on outstanding bonds, it is not possible to rule out multiple steady states. Even if the steady state is unique under perfect foresight this might not extend to rational expectations models.

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