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Relations between amplitude reflectances and phase shifts of the p and s polarizations when electromagnetic radiation strikes interfaces between transparent media

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Relations between amplitude reflectances and phase shifts of the \( p \) and \( s \) polarizations when electromagnetic radiation strikes interfaces between transparent media

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When an electromagnetic plane wave strikes the planar interface between two linear homogeneous and isotropic media, the complex amplitude Fresnel reflection coefficients \( r_p \) and \( r_s \) for the parallel \( p \) (or TM) and perpendicular \( s \) (or TE) polarizations are interrelated by

\[
1 \quad r_p = f(r_s) \quad \text{of Eq. (1)}
\]

where \( \phi \) is the angle of incidence. We assume the \( \exp(j\omega t) \) time dependence and \( p \) and \( s \) directions according to the Nebraska (Muller) conventions.

If the medium of incidence is transparent and the medium of refraction is absorbing, both \( r_p \) and \( r_s \) are in general complex, and the function \( r_p = f(r_s) \) of Eq. (1) can be studied graphically as a conformal mapping between the complex planes of \( r_s \) and \( r_p \). In this Letter we examine the special but important case when both media are transparent.

In the absence of absorption two situations are physically distinguishable:

1. Partial (internal or external) reflection, in which case \( r_s \) and \( r_p \) are real and \( |r_s| < 1, \nu = p, s \).

2. Total (internal) reflection, in which case \( r_p = r_s = 1, \nu = p, s \).

If we substitute \( r_p = r_s = \exp(j\delta_p) \) in Eq. (1) we get

\[
\delta_p = \delta_s + \arctan \left( \frac{\sin \delta_s}{\cos \delta_s - \cos 2\phi} \right) + \arctan \left( \frac{\sin \delta_s \cos 2\phi}{1 - \cos \delta_s \cos 2\phi} \right), \quad (3)
\]

as may be obtained by taking the argument of both sides of Eq. (2). Equation (3) provides a direct relation between the phase shifts \( \delta_p \) and \( \delta_s \), that the \( p \)- and \( s \)-polarized components of the incident wave experience upon total (internal) reflection. Here we also plot \( \delta_p \) vs \( \delta_s \) with the angle of incidence \( \phi \) as a parameter.

Figure 1 shows \( r_p \) vs \( r_s \) (denoted by RP and RS) at one angle.

References

5. H. R. Gordon, Appl. Opt. 18, 1161 (1979). This paper contains several typographical errors. These are: (1) \( \omega \) should be \( \mu \) in Eq. 7, (2) \( \Omega \) should be \( \lambda_0 \) in Eq. 8, (3) the last line of Eq. 9 should read: \( \times \exp[-K(\lambda_0)Z/f(\lambda_0, \lambda_F)] \), (4) the first sign in the equation defining \( f(\lambda_0, \lambda_F) \) should be minus not plus, (5) the \( a_0 \) scale in Fig. 3 is \( \times 10^3 \) not \( 10^4 \), and (6) in line 6 on page 1165 W m\(^{-1}\)cm\(^2\) should be mW cm\(^{-1}\)cm\(^2\).
13. This is defined to be the depth over which the number of downwelling quanta falls to \( 1/e \) of the number at the surface.
Fig. 1. Relation between Fresnel's reflection coefficients for the $p$ and $s$ polarizations (denoted by $R_P$ and $R_S$) at a fixed angle of incidence $\phi$ (taken here equal to 30°). The curve represents partial reflection at interfaces between transparent media. The significance of the points marked on the curve is discussed in the text.

Fig. 2. Relation between Fresnel's reflection coefficients for the $p$ and $s$ polarizations (denoted by $R_P$ and $R_S$) at 19 angles of incidence between 0° and 90° in equal steps of 5°. This figure applies to partial (internal and external) reflection at all possible interfaces between transparent media and throughout the electromagnetic spectrum.

Fig. 3. Relation between the total-reflection phase shifts $\delta_p$ and $\delta_s$ for the $p$ and $s$ polarizations (denoted by $\Delta P$ and $\Delta S$) at 19 angles of incidence between 0° and 90° in equal steps of 5°. This figure applies to all possible instances of total reflection and throughout the electromagnetic spectrum.

Fig. 4. Relation between the total-reflection phase shifts $\Delta = \delta_p - \delta_s$ and $\delta_s$ (denoted by $\Delta P$ and $\Delta S$) at 19 angles of incidence between 0° and 90° in equal steps of 5°. This figure applies to all possible instances of total reflection and throughout the electromagnetic spectrum.
of incidence $\phi$ (30°). The origin $O(r_s = r_p = 0)$ represents the limiting case of wave reflection from a vanishing interface when the two surrounding media become the same. The point $B(r_s = \cos 2\phi)$, $r_p = 0$) represents reflection at the Brewster angle. The point of minimum $r_p$, $M[r_s = \tan(\phi - 45°), r_p = -\tan(\phi - 45°)]$ corresponds to wave refraction at 45°.  

From Eq. (1) notice that while there is only one value of $r_p$ for each value of $r_s$, a given value of $r_p$ leads to two values of $r_p$. (An $r_s$ and $r_p$ at two points, or does not intersect it at all. At $M$ the two points of intersection coincide.) From Eq. (1) the two values of $r_p$ that correspond to the same value of $r_s$ are given by

$$r_p = 0.5 \cos 2\phi (1 - r_s) \pm \left[ (1 - r_s^2) + 0.25 \cos 2\phi (1 - r_s^2) \right]^{1/2}$$

Figure 2 shows a collective view of $r_p$ vs $r_s$, at 19 equispaced angles of incidence from 0° to 90° in steps of 5°. The following features can be deduced from Fig. 2:

1. If we exclude normal and grazing incidence ($\phi = 0, 90°$), as $r_s$ scans the full range from $-1$ to $+1$, $r_p$ scans (twice) the truncated interval, $-\tan(\phi - 45°) \leq r_p \leq 1$, at any given angle of incidence $\phi$.

2. The limiting cases of normal incidence, $\phi = 0$, and grazing incidence, $\phi = 90°$, are represented by the straight lines $r_p = -r_s$ and $r_p = r_s$, respectively. These lines define the boundaries of the domain of all physically possible pairs $(r_s, r_p)$, where $|r_p| \leq |r_s|$.

3. The curve of $r_p$ vs $r_s$ becomes the symmetrical parabola $r_p = r_s^2$ when the angle of incidence is $45°$.  

4. Two curves of $r_p$ vs $r_s$ associated with two angles of incidence equally above and below $45°$ (i.e., $\phi = 45° \pm \theta$) are mirror images of one another with respect to the axis $r_s = 0$.

5. The locus of the point of minimum $r_p$ ($M$ in Fig. 1), as the angle of incidence is varied, is the inverted parabola $r_p = -r_s^2$. This locus represents all possible instances of wave refraction at $45°$.  

6. When $r_s = -1$, we have $r_p = 1$ at all angles of incidence except grazing incidence, $0 \leq \phi < 90°$; when $\phi = 90°$, $r_p = -1$. Likewise, when $r_s = 1$, we have $r_p = 1$ at all angles of incidence except normal incidence, $0 < \phi \leq 90°$; when $\phi = 0$, $r_p = -1$.

Figure 3 gives $\delta_\rho$ vs $\delta_\phi$ (denoted by DELTA P and DELTA S) under conditions of total reflection as computed using Eq. (3) for 19 angles of incidence from 0° to 90° in equal steps of 5°. The following can be noted from Fig. 3:

1. The limiting cases of normal incidence, $\phi = 0$, and grazing incidence, $\phi = 90°$, are represented by the straight lines $\delta_\rho = \delta_\phi + 180°$ and $\delta_\rho = -\delta_\phi$, respectively. These lines bound the domain of all permissible pairs $(\delta_\phi, \delta_\rho)$, where $\delta_\rho \leq \delta_\rho + 180°$.

2. Incidence at $45°$ is represented by the straight line $\delta_\rho = 2\delta_\phi$.

3. Two curves of $\delta_\rho$ vs $\delta_\phi$ for two angles of incidence equally above and below $45°$ (i.e., $\phi = 45° \pm \theta$) are symmetrical with respect to the straight line $\delta_\rho = 2\delta_\phi$, in the sense that any $\delta_\rho = \delta_\rho$ constant straight line intersects the two curves at two points that are equidistant from the point of intersection of the same straight line with $\delta_\rho = 2\delta_\phi$.

For completeness, we show in Fig. 4 $\Delta = \delta_\rho - \delta_\phi$ (denoted by DELTA) vs $\delta_\phi$ (denoted by DELTA S) at the same angles of incidence as in Fig. 3. Both $\delta_\phi$ and $\Delta$ are limited between 0° and 180°. 

Mirror reflection with respect to the straight line $\Delta = \delta_\phi (\phi = 45°)$ relates any two $\Delta$ vs $\delta_\phi$ curves at $\phi = 45° \pm \theta$. This symmetry property is somewhat simpler than that stated above for the $\Delta$ vs $\delta_\phi$ curves.

Finally, we emphasize that all the results presented here are valid independent of the specific media that define the interface and are applicable throughout the electromagnetic spectrum.

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References
3. Equation (3) can be obtained by deleting the relative refractive index $n$ as the common variable between $\delta_\rho$ and $\delta_\phi$ expressed as functions of $n$. Such functions can be found, for example, in M. Born and E. Wolf, Principles of Optics (Pergamon, New York, 1975), p. 49.
5. Values of $r_p$ in the remaining interval, $-1 \leq r_p < -\tan^2(\phi - 45°)$, make $r_p$ complex. This interesting situation is possible only in the presence of absorption and is discussed in Ref. 1.
9. For total internal reflection at interfaces between transparent media at a given angle of incidence, the maximum attainable phase shift for the $s$ polarization is equal to double the angle of incidence, $\delta_s = 2\phi$, and occurs when the ratio of refractive indices of the two media tends to infinity. The associated phase shift for the $p$ polarization reaches a maximum of $\pi$. It follows that, in Fig. 3, $\delta_\rho$ and $\delta_\phi$ for total reflection at interfaces between transparent media are confined to the domain denoted by 1 below the horizontal straight line $\delta_\rho = 180°$, which is also the locus of $\delta_s = 2\phi$. Domain II, above this line, represents total reflection at an interface between a transparent medium and a medium with a negative real dielectric constant (e.g., a plasma). The phase shifts in this case are also interrelated by Eq. (3).
10. In Fig. 4, the diagonal straight line $\Delta = 180° - \delta_\phi = (180° - 2\phi)$ defines the boundary between two domains I and II with significance as indicated in the foregoing footnote.

Moiré strain analysis in cryogenic environments
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Contemporary demands such as superconducting magnets for energy storage operating at cryogenic environments necessitate the development of adequate strain measuring techniques under such conditions. While commercially available electrical strain gauges are employed at cryogenic temperatures, they are less than adequate. Moreover, the full-field capability of optical methods such as moiré warrants their development in cryogenic environments. It is the purpose of this Letter to describe the extension of moiré analysis of a component contained in liquid nitrogen (77 K). Moiré is demonstrated both with and without a tenfold fringe multiplication. The authors are unaware of any previously published application of moiré under cryogenic conditions.

Moiré is extremely well suited for strain (stress) analysis and metrology of physical components at room and elevated temperatures and under extreme conditions of loading. The method records the basis of continuum physics, i.e., dis-