Angular range for reflection of p-polarized light at the surface of an absorbing medium with reflectance below that at normal incidence

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The range of incidence angle, $0 < \varphi < \varphi_e$, over which $p$-polarized light is reflected at interfaces between transparent and absorbing media with reflectance below that at normal incidence is determined. Contours of constant $\varphi_e$ in the complex plane of the relative dielectric constant $\varepsilon$ are presented. A method for determining the real and imaginary parts of the complex refractive index, $\varepsilon^{1/2} = n + jk$, which is based on measuring $\varphi_e$ and the pseudo-Brewster angle $\varphi_{\text{PB}}$, is viable in the domain of fractional optical constants, $n, k < 1$. © 2002 Optical Society of America


1. INTRODUCTION

The reflection of collimated monochromatic $p$-polarized light at the planar interface between a transparent medium of incidence of real dielectric constant $\varepsilon_1$ and an absorbing medium of refraction of complex dielectric constant $\varepsilon_2$ is governed by the Fresnel reflection coefficient

$$r_p = [\cos \varphi - (\varepsilon - \sin^2 \varphi)^{1/2}]/[\cos \varphi + (\varepsilon - \sin^2 \varphi)^{1/2}],$$

where

$$\varepsilon = \varepsilon_2/\varepsilon_1,$$

and $\varphi$ is the angle of incidence; see Fig. 1. For a given complex $\varepsilon$, the absolute reflectance $r_p r_p^*$ initially decreases as $\varphi$ increases from 0, reaches a minimum at the pseudo-Brewster angle $\varphi_{\text{PB}}$, and then increases monotonically from minimum reflectance to 1 as $\varphi$ increases from $\varphi_{\text{PB}}$ to $90^\circ$. Explicit solutions for $\varphi_{\text{PB}}$ for a given complex $\varepsilon$ have been derived by several authors.2–4 Azzam and Ugbo5 also determined analytically the contours of constant $\varphi_{\text{PB}}$ in the complex $\varepsilon$ plane.

In this paper we are interested in the angular range $0 < \varphi < \varphi_e$, over which the reflectance for $p$-polarized light at oblique incidence is less than that at normal incidence. The upper limit $\varphi_e$, which lies between $\varphi_{\text{PB}}$ and $90^\circ$, is determined by equating the oblique and normal-incidence reflectances, i.e.,

$$r_p(\varphi)r_p^*(\varphi) = r_p(0)r_p^*(0).$$

For the special case of an interface between two transparent media ($\varepsilon$ real and $>0$), the minimum reflectance is zero, $\varphi_{\text{PB}}$ reverts to the usual Brewster angle $\varphi_{\text{B}} = \arctan \varepsilon^{1/2}$, and Eq. (3) has an explicit solution $\varphi = \varphi_e$ given by

$$\tan \varphi_e = (\varepsilon^2 + \varepsilon)^{1/2}.$$ (4)

Another interesting conclusion from Ref. 6 is that the difference $\varphi_e - \varphi_{\text{PB}}$ reaches a maximum of 13.9852° when $\varepsilon = 3.6135$, and $\varphi_{\text{B}} = 62.528^\circ$.

For the general case of an absorbing medium of refraction (complex $\varepsilon$), no analytical solution exists for Eqs. (1) and (3), and $\varphi_e$ must be determined numerically. Our approach in this paper is to determine all possible values of $\varphi_e - \varphi_{\text{PB}}$ that are consistent with a given $\varphi_{\text{PB}}$ (Section 2). The maximum difference ($\varphi_e - \varphi_{\text{PB}}$)max $= 20.447^\circ$ occurs in the limit when $\varepsilon$ is real negative, and $\varphi_{\text{PB}} = 44^\circ$. We also determine the constant-$\varphi_e$ contours in the complex planes of $\varepsilon$ and $\varepsilon^{1/2} (n + jk)$, the relative complex refractive index) in Section 3. In Section 4, we propose a technique for determining $n$ and $k$, which is based on measuring the two angles $\varphi_{\text{PB}}$ and $\varphi_e$.

2. ANGULAR RANGE $\varphi_e - \varphi_{\text{PB}}$ FOR SPECIFIED PSEUDO-BREWSTER ANGLE $\varphi_{\text{PB}}$

All possible values of complex $\varepsilon = |\varepsilon| \exp(j\theta)$, that are consistent with a given $\varphi_{\text{PB}}$ are determined by

$$|\varepsilon| = \varepsilon \cos(\xi/3),$$ (5)
where

\[ \eta = 2 \tan^2 \varphi_{PB} \left( 1 - \frac{2}{\pi} \sin^2 \varphi_{PB} \right)^{1/2}, \quad (6) \]

\[ \zeta = \arccos \left[ -\cos \theta \cdot \cos^2 \varphi_{PB} \left( 1 - \frac{2}{\pi} \sin^2 \varphi_{PB} \right)^{-3/2} \right], \quad (7) \]

by scanning \( \theta \) from 0 to 180°. Constant-\( \varphi_{PB} \) contours in the complex \( \varepsilon \) plane were presented in Ref. 5 based on Eqs. (5)–(7).

Figure 2 shows a family of reflectance-versus-angle \((r, r^*)\)-versus-\( \varphi \) curves for 13 values of complex \( \varepsilon = |\varepsilon| \exp(j \theta) \) that share the same pseudo-Brewster angle \( \varphi_{PB} = 50° \), as obtained by allowing \( \theta \) to assume values from 0 to 180° in steps of 15°. Both the normal-incidence reflectance and the minimum reflectance at \( \varphi_{PB} = 50° \) increase monotonically with \( \theta \). In the limit of \( \theta = 180° \) (i.e., \( \varepsilon \) is real negative), the reflectance is total (=1) at all angles.

Figure 3 shows the minimum reflectance, \( (r, r^*)_{min} \), as a function of \( \theta \) for constant values of \( \varphi_{PB} \) from 5° to 80° in steps of 5°. For small pseudo-Brewster angles (5° to 15°), an initial steep rise of the minimum reflectance with \( \theta \) is followed by a more gradual increase toward 1. For \( \varphi_{PB} > 30° \), the increase of minimum reflectance with \( \theta \) appears parabolic and is nearly independent of \( \varphi_{PB} \).

Figure 4 shows \( \varphi_e - \varphi_{PB} \) as a function of \( \theta \) for constant \( \varphi_{PB} \) from 5° to 80° in steps of 5°. The maximum difference \( (\varphi_e - \varphi_{PB})_{max} = 20.447° \) occurs when \( \theta = 180° \) and \( \varphi_{PB} = 44° \). For large values of \( \varphi_{PB} \), \( (\varphi_e - \varphi_{PB}) \) is nearly constant (e.g., at \( \varphi_{PB} = 80° \), \( \varphi_e - \varphi_{PB} \) increases from 8.270° to 8.351° as \( \theta \) increases from 0 to 180°).

### 3. CONSTANT-\( \varphi_e \) CONTOURS IN THE COMPLEX PLANES OF \( \varepsilon \) AND \( \varepsilon^{1/2} \)

Over the range of incidence angles \( 0 < \psi < \varphi_e \), the \( p \) reflectance at oblique incidence is less than that at normal incidence. It is of interest to consider the constant-\( \varphi_e \) contours in the complex \( \varepsilon \) plane. Figure 5 shows a family of such contours for \( \varphi_e \) from 45° to 80° in steps of 5° and \( \varphi_e \) from 80° to 85° in steps of 1°. These results are obtained by solving Eqs. (1) and (3) numerically. The curves resemble a family of semicircles centered at the origin. (However, each contour is not a semicircle.) Figure 6 shows the corresponding family of contours in the complex-refractive-index plane, \( \varepsilon^{1/2} = n + jk \). The de-
viation of each contour from a quadrant of a circle is more apparent at lower angles (e.g., at \( \varphi_e = 45^\circ \)).

4. TECHNIQUE FOR DETERMINING \( n \) AND \( k \) FROM THE MEASURED ANGLES \( \varphi_{pB} \) AND \( \varphi_e \)

Azzam described an analytical technique for determining the optical constants \( n \) and \( k \) of an absorbing medium from two pseudo-Brewster angles measured in two transparent incidence media.\(^7\) It is of interest to consider whether \( n \) and \( k \) can be determined from the two angles \( \varphi_{pB} \) and \( \varphi_e \) measured in the same medium of incidence. In general, angular measurements are attractive, because no absolute reflectance measurements are required. (For a review of numerous reflectance-based techniques, the reader may consult papers by Humphreys-Owen\(^2\) and Hunter\(^3\)).

Figure 7 shows two superimposed families of constant-\( \varphi_{pB} \) and constant-\( \varphi_e \) contours in the \( n-k \) plane in the domain of fractional optical constants \((n, k < 1)\).

Figure 8 is similar to Fig. 7, except that values of \( n, k > 1 \) are now considered. In Fig. 8 the families of...
constant-\(w_pB\) and constant-\(w_e\) contours are generated for \(\varphi_pB = 5\) to 70° in steps of 5°, and for \(\varphi_e = 60\) to 80° in steps of 5°, and \(\varphi_e = 83°\). Because of the smaller intersection angles, the present two-angle method would not provide an accurate method of determining \(n\) and \(k\).

**5. SUMMARY**

We have determined the range of incidence angles, \(0 < \varphi < \varphi_e\), over which the reflectance of \(p\)-polarized light at oblique incidence is less than that at normal incidence, for any transparent medium/absorbing medium interface. Constant-\(\varphi_e\) contours in the complex planes of the dielectric constant \(\varepsilon\) and refractive index \(\frac{\varepsilon}{\sqrt{2}} = n + jk\) are obtained. Finally, it is shown that fractional optical constants \(n\) and \(k\) can be determined if the pseudo-Brewster angle and the angle \(\varphi_e\) [which satisfies Eq. (3)] are measured.

**REFERENCES**

1. See, for example, R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, Amsterdam, 1987), Chap. 4.