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Extinction of the *p* and *s* polarizations of a wave on reflection at the same angle from a transparent film on an absorbing substrate: applications to parallel-mirror crossed polarizers and a novel integrated polarimeter

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The p- and s-polarized components of light can be suppressed on reflection at the same angle of incidence from an absorbing substrate coated by a transparent thin film if the wave is refracted in the film at 45° and the constraint $\mathrm{Re}[(\epsilon_2 - \alpha)/(1-\alpha)]^{1/2} = \alpha + |\epsilon_2 - \alpha|$ is satisfied, where 2α and ϵ_2 are the ratios of dielectric constants of the film and substrate, respectively, to that of the ambient. For high-reflectance metal substrates ($|\epsilon_2| \gg 1$), $\alpha \simeq 1$, the ratio of film to ambient refractive index approaches $\sqrt{2}$, and the unextinguished reflectances approach 1. The least film thicknesses required to suppress the p and s polarizations are in the ratio 2:1. The analysis is applied to Si and Al substrates in the near UV–visible–near-IR spectral range. It is found that the film refractive index and thickness should be controlled to within ±0.01 and ±5 Å, respectively, for an Al substrate at 550 nm. Significant applications are proposed that include parallel-mirror crossed polarizers, a novel polarimeter that integrates the polarization-analysis and photodetection functions, high-reflectance crossed thin-film reflection polarizers integrated on the same substrate, and division-of-wavefront polarizing beam splitters.

1. INTRODUCTION

In this paper we determine the conditions that are required for the extinction of the parallel (p) *and* perpendicular (s) polarized components of a plane wave of light (or any other electromagnetic radiation) on reflection at the *same* angle of incidence from an absorbing substrate coated by a transparent thin film. For given substrate and ambient media, suppression of both polarizations at a common angle of incidence occurs for a certain ratio of film to ambient refractive index that approaches $\sqrt{2}$ for high-reflectance metallic substrates. We find that the least film thickness required to suppress the p polarization is exactly double that required for suppression of the s polarization. The associated unextinguished reflectances are also determined and found to be high when the substrate reflectance is high.

The analysis is applied to Si and Al substrates over a range of wavelengths from the near UV to the near IR, and the effects of errors of film refractive index, thickness, and incidence angle are considered.

Several interesting and important applications are proposed. They include new devices (integrated crossed thin-film reflection polarizers, division-of-wavefront polarizing beam splitter and half-shade device, parallel-mirror crossed polarizers), a novel polarimeter that integrates the polarization analysis and photodetection functions, and optimum conditions for film-thickness metrology using reflection interferometry and ellipsometry.

2. CONDITIONS FOR EQUAL p AND s POLARIZING ANGLES

In Fig. 1, a plane wave of monochromatic light traveling in a transparent medium of refractive index N_0 is incident at an angle ϕ on an absorbing substrate of complex refractive index N_2 coated by a transparent thin film of refractive index N_1. The film has a uniform thickness $d = d_p$ over one area of the substrate and a different uniform thickness $d = d_s$ over the remaining area. The media of incidence, film, and substrate, denoted by 0, 1, and 2, respectively, are assumed to be linear, homogeneous, isotropic, and nonmagnetic and are separated by parallel-plane interfaces.

For given media in contact with the film, i.e., for given N_0 and N_2, we seek the conditions required to suppress the p-polarized component of the wave reflected from the film of thickness d_p and the s-polarized component of the wave reflected from the film of thickness d_s *at the same angle of incidence* $\phi = \phi_{p,s}$.

The complex-amplitude reflection coefficient of the film–substrate system is[1]

$$R_\nu = (r_{01\nu} + r_{12\nu}X)/(1 + r_{01\nu}r_{12\nu}X), \tag{1}$$

where $r_{01\nu}$ and $r_{12\nu}$ are Fresnel's reflection coefficients of the 01 and 12 interfaces for the ν (= p or s) polarization, and

$$X = \exp(-j2\pi\zeta), \tag{2}$$

$$\zeta = d/D_\phi, \tag{3}$$

$$D_\phi = \frac{\lambda}{2}(N_1{}^2 - N_0{}^2 \sin^2 \phi)^{-1/2}. \tag{4}$$

ζ is the normalized film thickness, D_ϕ is the film thickness period, and λ is the free-space wavelength of light. Extinction of the ν polarization on reflection occurs when $r_{01\nu} + r_{12\nu}X = 0$, or

$$X = -r_{01\nu}/r_{12\nu}, \tag{5}$$

as is evident from Eq. (1). If $N_1 > N_0 \sin\phi$, i.e., in the absence

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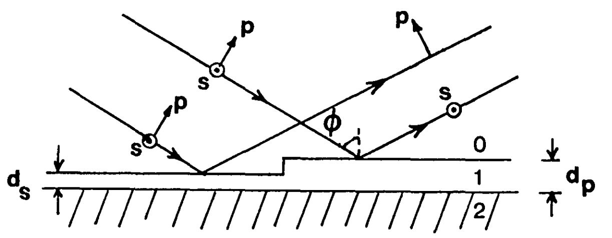

Fig. 1. Reflection of a plane wave by an absorbing substrate coated by a transparent thin film of uniform thicknesses d_p and d_s over adjacent areas of the substrate. Conditions are found for the extinction of the p- and s-polarized components at the same incidence angle $\phi = \phi_{p,s}$ from the d_p- and d_s-coated areas, respectively.

of *total* internal reflection at the 01 interface, D_ϕ and ζ are real; hence $|X| = 1$, as can be seen from Eqs. (2)–(4). Consequently, from Eq. (5), extinction of both polarizations at the same angle requires that

$$|r_{01s}| = |r_{12s}| \tag{6a}$$

and

$$|r_{01p}| = |r_{12p}| \tag{6b}$$

be satisfied *simultaneously.*

If Eq. (6a) is satisfied, Eq. (6b) will also be satisfied simultaneously *if and only if the angle of light refraction in the film is 45°.* Light refraction at 45° guarantees that[2]

$$r_{01p} = -r_{01s}{}^2 \tag{7}$$

and[3]

$$r_{12p} = r_{12s}{}^2. \tag{8}$$

The Fresnel reflection coefficients of the film–ambient (10) and film–substrate (12) interfaces for the s polarization at 45° incidence are given by[4]

$$r_{10s} = (S_1 - S_0)/(S_1 + S_0), \tag{9a}$$

$$r_{12s} = (S_1 - S_2)/(S_1 + S_2), \tag{9b}$$

where

$$S_0 = (N_0{}^2 - \tfrac{1}{2}N_1{}^2)^{1/2},$$
$$S_1 = N_1/\sqrt{2}, \tag{10}$$
$$S_2 = (N_2{}^2 - \tfrac{1}{2}N_1{}^2)^{1/2}.$$

S_2 is complex; S_1 is real, and so is S_0 when $N_0 > N_1/\sqrt{2}$.

Because $r_{01\nu} = -r_{10\nu}$, Eq. (6a) is satisfied if

$$r_{10s}{}^2 = r_{12s}r_{12s}{}^*. \tag{11}$$

Substitution of Eqs. (9) into Eq. (11) gives

$$S_2 + S_2{}^* = 2S_0(S_1{}^2 + S_2S_2{}^*)$$

or

$$\mathrm{Re}(S_2) = S_0(S_1{}^2 + |S_2|^2). \tag{12}$$

If we substitute Eqs. (10) into Eq. (12), and use the normalization

$$\epsilon_2 = N_2{}^2/N_0{}^2 = n_2{}^2, \tag{13a}$$

$$\alpha = N_1{}^2/2N_0{}^2 = n_1{}^2/2, \tag{13b}$$

we obtain

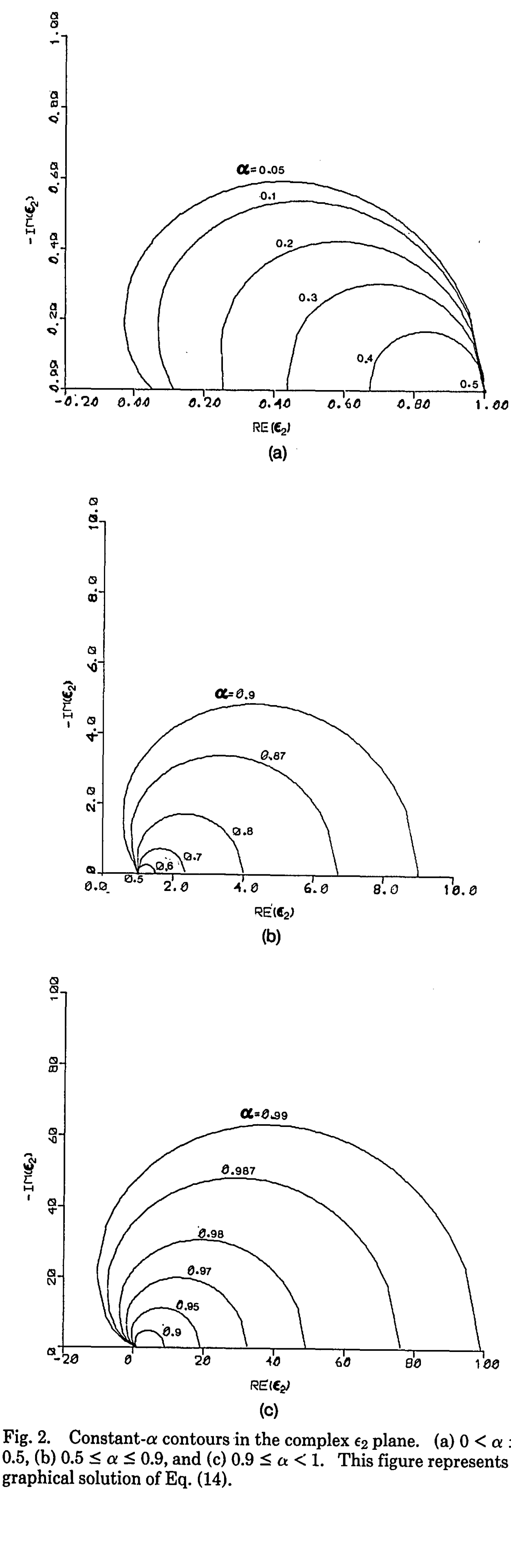

Fig. 2. Constant-α contours in the complex ϵ_2 plane. (a) $0 < \alpha \le 0.5$, (b) $0.5 \le \alpha \le 0.9$, and (c) $0.9 \le \alpha < 1$. This figure represents a graphical solution of Eq. (14).

$$\mathrm{Re}\left(\frac{\epsilon_2 - \alpha}{1 - \alpha}\right)^{1/2} = \alpha + |\epsilon_2 - \alpha|. \quad (14)$$

Equation (14) is an important result of this paper. It represents the constraint on the optical properties of the system of Fig. 1 such that suppression of both the p and s polarizations is possible at the same incidence angle. The normalized dielectric constants ϵ_2 (complex) and $2\alpha = \epsilon_1$ (real) are ratios of the dielectric constants of the substrate and film, respectively, to that of the ambient, as is indicated by Eqs. (13). Equation (14) is a transcendental equation that can be solved for α for any given complex ϵ_2. The normalized film refractive index n_1 is related to α by Eq. (13b):

$$n_1 = N_1/N_0 = (2\alpha)^{1/2}. \quad (15)$$

Constant-α contours in the complex ϵ_2 plane are plotted in Figs. 2(a), 2(b), and 2(c) for the following ranges of α: (a) $0 < \alpha \leq 0.5$, (b) $0.5 \leq \alpha \leq 0.9$, and (c) $0.9 \leq \alpha < 1$, respectively. $\alpha = 0.5$ is represented by the single point $\mathrm{Re}(\epsilon_2) = 1$, $\mathrm{Im}(\epsilon_2) = 0$ and marks the transition between internal reflection ($\alpha < 0.5$) and external reflection ($\alpha > 0.5$) at the 01 (ambient–film) interface. It is evident from Figs. 2(a) and 2(c) that contours for $\alpha < 0.05$ and $\alpha > 0.99$ would fill the rest of the plane. Thus a solution of Eq. (14) for α exists for *every* complex ϵ_2; hence a film refractive index can always be found to equalize the p and s polarizing angles for all ambient and substrate media.

Appendix A describes the procedure used to generate Fig. 2 from Eq. (14). In Section 5 solutions of Eq. (14) are given for specific material systems.

When ϵ_2 is real and $>\alpha$, Eq. (14) reduces to

$$\left(\frac{\epsilon_2 - \alpha}{1 - \alpha}\right)^{1/2} = \epsilon_2,$$

from which

$$\alpha = \epsilon_2/(\epsilon_2 + 1). \quad (16)$$

In terms of the normalized film and substrate refractive indices n_1 and n_2 [Eqs. (13)], Eq. (16) reads

$$n_1^2 = 2n_2^2/(n_2^2 + 1), \quad (17)$$

which is identical to the result we previously obtained[5] for this special case of a transparent film on a transparent substrate.

Once α is obtained by solving Eq. (14) for a given ϵ_2, the common polarizing angle of incidence for the p and s polarizations is determined by the condition of light refraction in the film at 45° and Snell's law:

$$\sin \phi_{p,s} = n_1/\sqrt{2} \quad (18a)$$

$$= \alpha^{1/2}. \quad (18b)$$

3. REQUIRED FILM THICKNESSES

The required normalized film thicknesses[6] for suppression of the p and s polarizations are obtained from Eqs. (2) and (5):

$$\zeta_p = (\delta_{12p} - \delta_{01p} \pm \pi)/2\pi, \quad (19a)$$

$$\zeta_s = (\delta_{125} - \delta_{01s} \pm \pi)/2\pi, \quad (19b)$$

where $\delta_{ij\nu} = \arg (r_{ij\nu})$ is the reflection phase shift at the ij interface for the ν polarization.

Under the condition of 45° refraction, Eqs. (7) and (8) are satisfied; hence

$$\delta_{01p} = 2\delta_{01s} \pm \pi, \qquad \delta_{12p} = 2\delta_{12s}. \quad (20)$$

Substitution of Eqs. (20) into Eq. (19a) and comparing the result with Eq. (19b) show that

$$\zeta_p = 2\zeta_s. \quad (21)$$

Multiplication of Eq. (21) by the film-thickness period D_ϕ evaluated at the common p and s polarizing angle of incidence $\phi_{p,s}$ gives

$$d_p = 2d_s. \quad (22)$$

Thus we reach the interesting conclusion that the film thickness required to suppress the p polarization is exactly double that required to suppress the s polarization at the common polarizing angle $\phi_{p,s}$, independent of the optical constants of the system.

4. UNEXTINGUISHED REFLECTANCES

The unextinguished (nonzero) complex-amplitude reflection coefficients for the s and p polarizations and their ratio are obtained from Eqs. (1), (5), (7), and (8) as

$$R_s = r_{01s}(r_{01s} + r_{12s})/(r_{01s}{}^3 + r_{12s}), \quad (23)$$

$$R_p = -r_{01s}(r_{01s} + r_{12s})/(1 + r_{01s}{}^3 r_{12s}), \quad (24)$$

$$R_p/R_s = -(r_{01s}{}^3 + r_{12s})/(1 + r_{01s}{}^3 r_{12s}). \quad (25)$$

We can write

$$r_{12s} = re^{j\delta}, \qquad r_{01s} = \mp r, \quad (26)$$

where the − and + correspond to external ($\alpha > 0.5$) and internal ($\alpha < 0.5$) reflection at the 01 interface, respectively.[7] Substitution of Eqs. (26) into Eqs. (23) and (24) gives

$$R_s = -r(1 - e^{j\delta})/(r^2 - e^{j\delta}), \quad (27)$$

$$R_p = -r^2(1 - e^{j\delta})/(1 - r^4 e^{j\delta}), \quad (28)$$

when $\alpha > 0.5$, and

$$R_s = r(1 + e^{j\delta})/(r^2 + e^{j\delta}), \quad (29)$$

$$R_p = -r^2(1 + e^{j\delta})/(1 + r^4 e^{j\delta}), \quad (30)$$

when $\alpha < 0.5$. Equations (27)–(30) express the unextinguished reflection coefficients in terms of the single Fresnel reflection coefficient r_{12s}. This film–substrate interface reflection coefficient for the s polarization at 45° incidence is given by

$$re^{j\delta} = \frac{1 - [(\epsilon_2/\alpha) - 1]^{1/2}}{1 + [(\epsilon_2/\alpha) - 1]^{1/2}}, \quad (31)$$

from Eqs. (9b), (10), and (13).

Of interest also are the intensity (power) reflectances:

$$\mathcal{R}_\nu = |R_\nu|^2, \qquad \nu = p, s. \quad (32)$$

For the special case of a transparent substrate, we have $\delta = \pi$ when $\alpha > 0.5$, and $\delta = 0$ when $\alpha < 0.5$. If $\delta = \pi$ is substituted into Eqs. (27) and (28) and $\delta = 0$ into Eqs. (29) and (30), it can be readily verified that the resulting reflection coefficients satisfy the relation

$$R_p = -R_s^2/(2 - R_s^2). \tag{33}$$

From Eqs. (32) and (33), the associated intensity reflectances are interrelated by

$$\mathcal{R}_p = [\mathcal{R}_s/(2 - \mathcal{R}_s)]^2. \tag{34}$$

Equations (33) and (34) reproduce exactly what we obtained before for this special use.[5]

5. EXAMPLES OF SEMICONDUCTOR AND METALLIC SUBSTRATES

The analysis of Sections 2–4 is applied to Si and Al as representative examples of semiconductor and metallic substrates. We take the optical constants of Si in the 206.6–826.3-nm (1.5–6-eV) spectral range of Aspnes and Studna[8] and those of Al in the 400–950-nm range from a recent review by Ordal *et al.*[9] The medium of incidence is assumed to be air or vacuum ($N_0 = 1$).

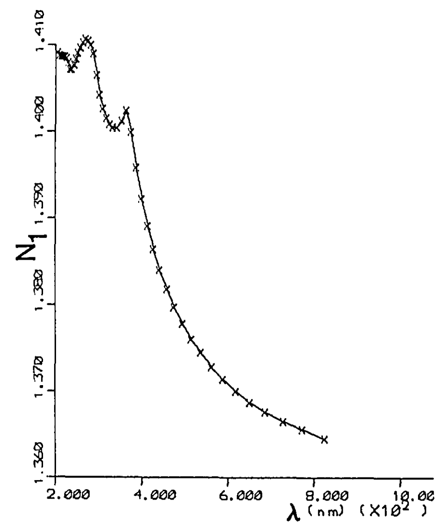

Fig. 3. Refractive index N_1 of a transparent layer on a Si substrate required for equal *p*- and *s*-polarizing angles versus wavelength λ (in nanometers). Light is incident from air or vacuum, and the optical constants of Si are obtained from Ref. 8.

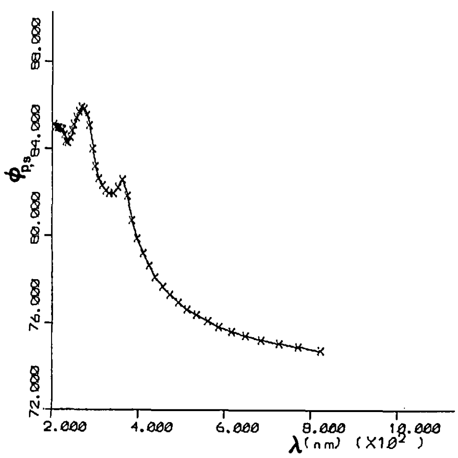

Fig. 4. The common *p*- and *s*-polarizing angle $\phi_{p,s}$ versus wavelength λ (in nanometers) for a Si substrate covered by a transparent thin film of refractive index given by Fig. 3.

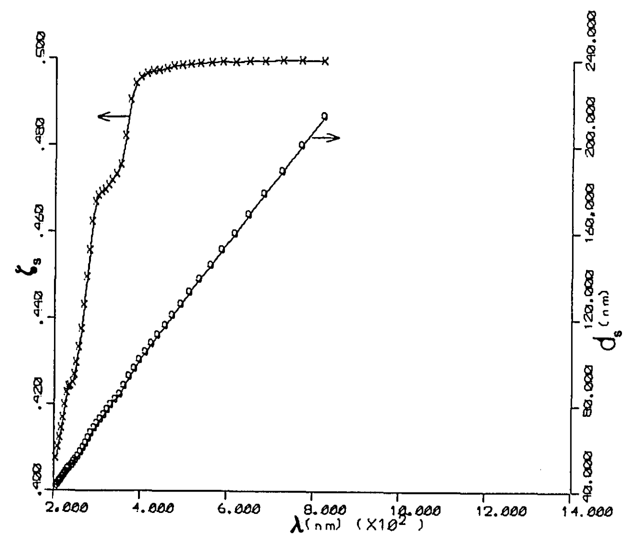

Fig. 5. The normalized (ζ_s) and actual (d_s, in nanometers) film thicknesses required to suppress the *s* polarization versus wavelength λ (in nanometers) for a Si substrate covered by a transparent thin film of refractive index given by Fig. 3. ζ_p and d_p that suppress the *p* polarization equal $2\zeta_s$ and $2d_s$, respectively.

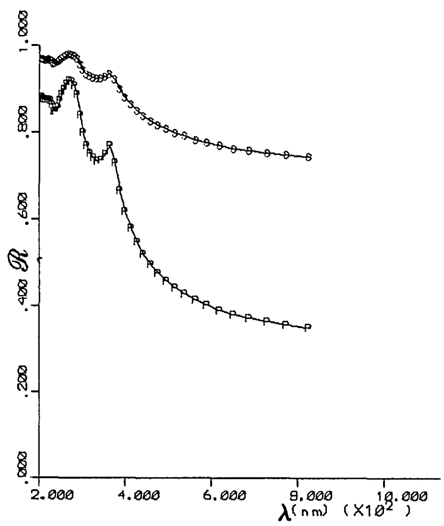

Fig. 6. The unextinguished reflectances ($\mathcal{R}$) versus wavelength λ (in nanometers) for the *s* polarization (upper curve) and *p* polarization (lower curve) of reflection polarizers that use a Si substrate coated by a thin film of refractive index given by Fig. 3.

Plotted in Figs. 3–6 versus wavelength λ (in nanometers) for the Si substrate are

(1) The film refractive index[10] N_1 for equal polarizing angles obtained by solving Eq. (14) (Fig. 3);

(2) The common polarizing angle $\phi_{p,s}$ from Eq. (18) (Fig. 4);

(3) The normalized (ζ_s) and actual (d_s in nanometers) film thicknesses required to suppress the *s* polarization from Eqs. (19b), (3), and(4) (Fig. 5); the normalized and actual film thicknesses required to suppress the *p* polarization are given by $\zeta_p = 2\zeta_s$ and $d_p = 2d_s$ [Eqs. (21) and (22)], hence are represented also by Fig. 5 if its ordinate scales are multiplied by 2;

(4) The unextinguished reflectances $\mathcal{R}_p$ and $\mathcal{R}_s$ from Eqs. (29)–(32) (Fig. 6).

Table 1. Characteristics of Thin-Film Reflection Polarizers on a Si Substrate with Equal p and s Polarizing Angles[a]

λ (nm) ($h\nu$)	N_2 (n_2–jk_2)	N_1	$\phi_{p,s}$ (deg.)	ζ_s	d_s (nm)	$\mathcal{R}_p$ (%)	$\mathcal{R}_s$ (%)
206.57 (6 eV)	1.010–j2.909	1.409071	85.112	0.40756	42.25	88.24	96.87
247.88 (5 eV)	1.570–j3.565	1.408445	84.823	0.42709	53.15	87.31	96.60
309.85 (4 eV)	5.010–j3.586	1.402582	82.646	0.46912	73.28	77.09	93.45
413.14 (3 eV)	5.222–j0.269	1.389117	79.190	0.49663	104.44	57.90	86.42
516.42 (2.4 eV)	4.215–j0.060	1.376035	76.656	0.49880	132.37	44.14	79.83
619.70 (2 eV)	3.906–j0.022	1.370030	75.640	0.49948	159.76	39.04	76.91
729.06 (1.7 eV)	3.752–j0.010	1.366512	75.076	0.49974	188.53	36.36	75.23
826.27 (1.5 eV)	3.673–j0.005	1.364545	74.770	0.49986	214.03	34.94	74.30

[a] The medium of incidence is assumed to be vacuum or air ($N_0 = 1$). λ is the wavelength of light. N_2 is the complex refractive index of Si from Ref. 8, and N_1 is the film refractive index for equal polarizing angles [from Eq. (14)]. $\phi_{p,s}$ is the common polarizing angle [Eq. (18)]. ζ_s and d_s are the normalized and actual least film thicknesses that suppress the s polarization, respectively. ($\zeta_p = 2\zeta_s$, $d_p = 2d_s$ for suppression of the p polarization.) $\mathcal{R}_p$ and $\mathcal{R}_s$ are the unextinguished p and s reflectances. The suppressed reflectances (not shown) are less than 10^{-6}.

Table 2. Characteristics of Thin-Film Reflection Polarizers on an Al Substrate with Equal p and s Polarizing Angles[a]

λ (nm)	N_2 (n_2–jk_2)	N_1	$\phi_{p,s}$ (deg.)	ζ_s	d_s (nm)	$\mathcal{R}_p$ (%)	$\mathcal{R}_s$ (%)
400	0.40–j3.92	1.413867	88.732	0.42350	84.72	99.17	99.79
450	0.49–j4.32	1.413844	88.690	0.43014	96.81	99.13	99.78
500	0.62–j4.80	1.413809	88.630	0.43686	109.25	99.05	99.76
550	0.76–j5.32	1.413798	88.611	0.44288	121.83	99.03	99.76
600	0.97–j6.00	1.413785	88.590	0.44933	134.84	99.01	99.75
650	1.24–j6.60	1.413733	88.506	0.45416	147.65	98.90	99.72
700	1.55–j7.00	1.413627	88.350	0.45723	160.10	98.66	99.66
750	1.80–j7.12	1.413491	88.168	0.45849	172.02	98.35	99.59
800	1.99–j7.05	1.413321	87.964	0.45866	183.58	97.97	99.49
850	2.08–j7.15	1.413298	87.938	0.45940	195.37	97.92	99.47
900	1.96–j7.70	1.413581	88.286	0.46158	207.80	98.56	99.64
950	1.75–j8.50	1.413855	88.710	0.46441	220.65	99.18	99.79

[a] The medium of incidence is assumed to be vacuum or air ($N_0 = 1$). λ is the wavelength of light. N_2 is the complex refractive index of Al from Ref. 9 and N_1 is the film refractive index for equal polarizing angles [from Eq. (14)]. $\phi_{p,s}$ is the common polarizing angle [Eq. (18)]. ζ_s and d_s are the normalized and actual least film thicknesses that suppress the s polarization, respectively. ($\zeta_p = 2\zeta_s$, $d_p = 2d_s$ for suppression of the p polarization.) $\mathcal{R}_p$ and $\mathcal{R}_s$ are the unextinguished p and s reflectances. The suppressed reflectances (now shown) are less than 10^{-6}.

Table 1 lists part of the numerical data used to generate Figs. 3–6 at several wavelengths.

The structure of the curves of Figs. 3–6 is due to the spectral structure of the complex dielectric function ϵ_2 (or n_2, k_2) of Si.[8] Figure 6 shows that high unextinguished s and p reflectances (>88%) are attainable in the near UV where the normal-incidence reflectance of Si is high. Figure 3 indicates that the associated film refractive index approaches $\sqrt{2} = 1.414$. This is expected from Eq. (14) and from Fig. 2(c), which shows that $\alpha \rightarrow 1$, hence $n_1 \rightarrow \sqrt{2}$, as Re(ϵ_2), Im(ϵ_2) become large. Figure 5 shows that ζ_s approaches 0.5 at longer wavelengths (in the near IR). $\zeta_s = 0.5$ holds exactly when the substrate is transparent.[5]

For brevity, we present the results for Al in tabular form only (Table 2). For such a high-reflectance metal substrate, polarization on reflection occurs near grazing incidence ($90° - \phi_{p,s} < 3°$), the film refractive index for equal p and s polarizing angles essentially equals $\sqrt{2}$, and the unextinguished reflectances are very high (>97%) at all wavelengths.

To preserve the integrity of the numerical solution of Eq. (14), N_1 is given in Tables 1 and 2 to six decimal places. The computed *extinguished* p and s reflectances are always $<10^{-6}$.

For the two cases considered, the unextinguished p reflectance is always less than the unextinguished s reflectance. In fact, $\mathcal{R}_p < \mathcal{R}_s$ holds for *all* cases, as can be proved from Eqs. (25) and (26). Furthermore, we have found that Eq. (34), which strictly applies for transparent substrates,[5] is satisfied with negligible error ($\ll$1%) when the substrate is absorbing, as the reader can verify using the $\mathcal{R}_p$ and $\mathcal{R}_s$ data of Tables 1 and 2.

To use transparent films of refractive indices $> \sqrt{2}$, a dense

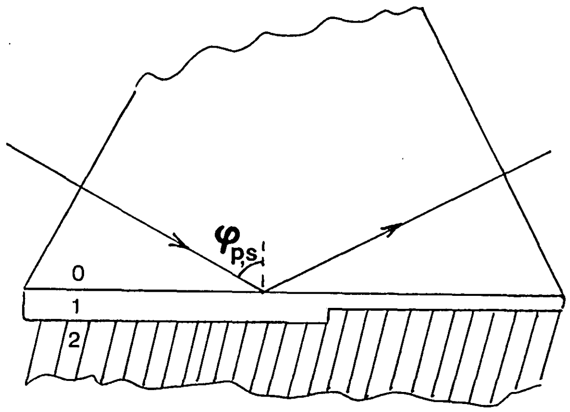

Fig. 7. A scheme similar to that of Fig. 1 but with a dense medium of incidence—a prism that now acts as a substrate. The polarizing, stepped-thickness, dielectric film is first deposited onto the prism face and is subsequently overcoated by an opaque metal layer. This internal-reflection arrangement permits the use of dielectric thin films of refractive index $>\sqrt{2}$.

medium of incidence is required. For example, light may be incident in glass ($N_0 = 1.5$) onto a polished face coated by a stepped-thickness dielectric film of refractive index $N_1 = N_0(2\alpha)^{1/2} \simeq 1.5\sqrt{2} = 2.12$. An opaque metal (Al) layer is subsequently deposited over the dielectric film, as shown in Fig. 7. In this design, the solid medium of incidence acts as the support or substrate.

6. ERROR ANALYSIS

It is important to consider the effects of small errors of refractive index, thickness, and angle of incidence on the suppressed p and s reflectances. For illustration, we assume the reflection of 550-nm light in air ($N_0 = 1$) from an Al substrate ($N_2 = 0.76$–$j5.32$) coated by a dielectric film of refractive index $N_1 = 1.4138 \simeq \sqrt{2}$. Film thicknesses required to suppress

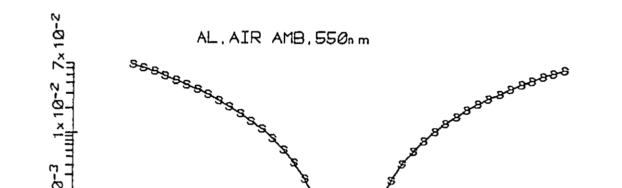

Fig. 8. Effect of errors of film refractive index N_1 on the suppressed p and s reflectances (lower and upper curves, respectively) of common-polarizing-angle thin-film reflection polarizers that use an Al substrate ($N_2 = 0.76$–$j5.32$) coated by a dielectric layer whose refractive index should be $N_1 = 1.4138 \simeq \sqrt{2}$. Light of wavelength $\lambda = 550$ nm and an air ambient are assumed.

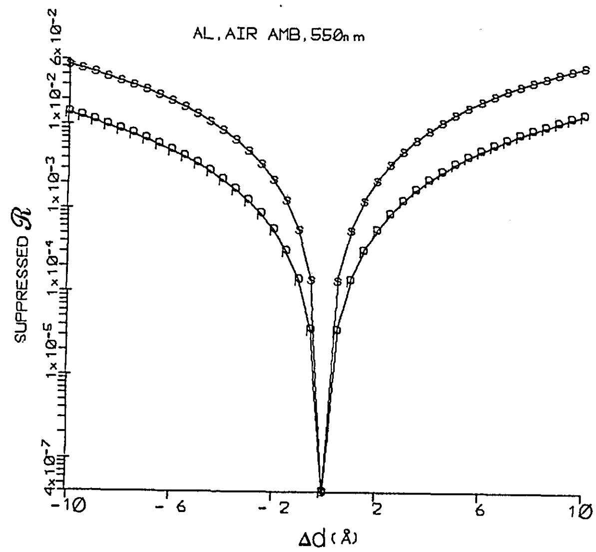

Fig. 9. Same as in Fig. 8 but for thickness shifts of up to ±10 Å around $d_p = 243.66$ nm and $d_s = 121.83$ nm.

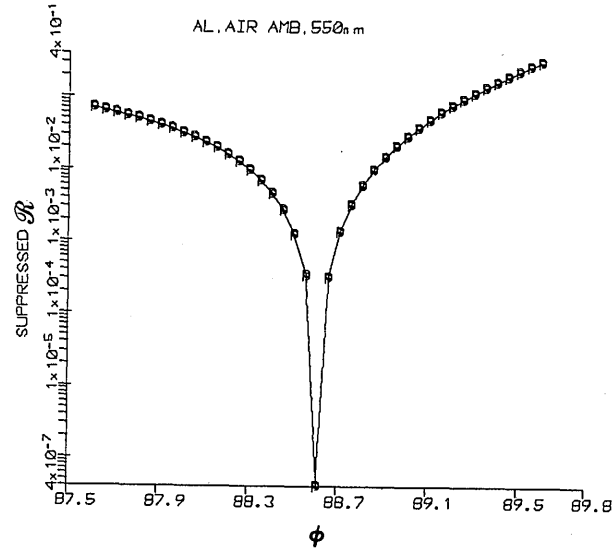

Fig. 10. Same as in Fig. 8 but for angular shifts of up to ±1° around the common polarizing angle $\phi_{p,s} = 88.61°$. The curves of $\mathcal{R}_p$ and $\mathcal{R}_s$ nearly coincide.

the p and s polarizations at the same angle, $\phi_{p,s} = 88.61°$, are $d_p = 243.66$ nm and $d_s = 121.83$ nm, respectively. The associated passed reflectances are $\mathcal{R}_s = 99.76$ and $\mathcal{R}_p = 99.03\%$. Suppressed reflectances are computed as each parameter is shifted from its proper value, one at a time.

Figure 8 shows the variation of the suppressed reflectances $\mathcal{R}_p$, $\mathcal{R}_s$ as N_1 is shifted by ±0.05. $\mathcal{R}_p$ is stationary with respect to small changes of N_1 around $N_1 = \sqrt{2}$. This can be traced back to the stationary property of the Fresnel coefficient r_{01p} at 45° refraction.[2] We conclude that it is advantageous to make p-suppressing thin-film reflection polarizers with a dielectric layer of refractive index $\simeq\sqrt{2}$ on a high-reflectance metal. By comparison, $\mathcal{R}_s$ increases rapidly with $|\Delta N_1|$. To keep $\mathcal{R}_s < 3 \times 10^{-3}$, N_1 should be controlled to within ±0.01.

Figure 9 displays $\mathcal{R}_p$, $\mathcal{R}_s$ as d shifts from d_p or d_s by ±10 Å. Both $\mathcal{R}_p$ and $\mathcal{R}_s$ are sensitive to thickness errors, with $\mathcal{R}_s$ being the more sensitive of the two reflectances. To maintain $\mathcal{R}_s < 0.014$, d_s should be controlled to within ±5 Å.

Figure 10 demonstrates that the suppressed reflectances $\mathcal{R}_p$, $\mathcal{R}_s$ are equally sensitive to angle shifts of $\pm 1°$ around $\phi_{p,s} = 88.61°$. A shift of $\pm 0.05°$ increases the suppressed reflectances from 4×10^{-7} to 3×10^{-4}.

We have also considered the effects of shifts of N_1, d, and ϕ on the passed reflectances. We find that the s and p unextinguished reflectances remain essentially unchanged with changes of film refractive index and thickness of ± 0.05 and ± 10 Å, respectively. However, these reflectances increase linearly with ϕ around $\phi_{p,s}$ with a modest slope ($\mathcal{R}_s$ increases from 99.76 to 99.93%, and $\mathcal{R}_p$ increases from 99.03 to 99.73%, as ϕ increases from 88.61° to 89.61°). Graphs for the passed reflectances are omitted, to save space.

While the required film refractive index (± 0.01) and thickness (± 5 Å) control is demanding for the specific example taken here, it is within the scope of today's technology. Furthermore, conditions can be sought for more error-tolerant designs.

7. APPLICATIONS

Many interesting and important applications are based on the new ideas presented in this paper. They may be grouped into three categories: A, novel devices; B, a novel polarimeter; and C, applications to thin-film metrology.

A. New Devices

In the scheme of Fig. 1 we accomplish (1) integrated crossed thin-film reflection polarizers (ICTFRP's) on the same substrate, (2) a division-of-wavefront polarizing beam splitter (DOWPBS), and (3) a thin-film reflective half-shade device[11] (TFRHSD). With high-reflectance metal substrates, we realize high unextinguished reflectances (e.g., >99% for Al at 550 nm), whereas only modest reflectances are possible with dielectric substrates.[5] Polarized optical wave fronts with juxtaposed fields of alternating orthogonal linear polarization states can be generated by reflecting a plane wave at the common polarizing angle from a metallic substrate that is covered by a dielectric layer of certain refractive index [that satisfies Eq. (14); $N_1 \simeq \sqrt{2}$]. The layer thickness alternates between the polarizing thicknesses d_s and $d_p = 2d_s$ in a pattern that corresponds to the desired binary polarization pattern of the reflected wave. Such a film can be prepared by well-established lithographic techniques. Films with refractive index $>\sqrt{2}$ can be used if the medium of incidence is dense, as shown by the internal-reflection arrangement of Fig. 7.

We also achieve the simplest pair of parallel-mirror crossed polarizers (Fig. 11). In this case, parallel planar metallic mirrors (which may consist of a dielectric, e.g., glass, substrate covered by an *opaque* metal, e.g., Al, film) are coated by transparent layers of the same material of the proper refractive index ($\simeq\sqrt{2}$ for a high-reflectance metal) but of different thicknesses, d_p and d_s, on the two mirrors. With monochromatic light incident at the common polarizing angle, this system functions as a pair of crossed polarizers. Such a system may replace the conventional crossed crystal or dichroic-sheet polarizers in a variety of instruments such as the polarizing microscope, photoelasticity and flow-birefringence apparatus. These crossed thin-film reflection polarizers are not area limited (basically) so that they may be made to large size to suit special needs.

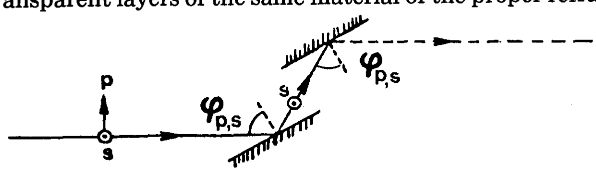

Fig. 11. Parallel-mirror crossed polarizers using the same dielectric-film–metal-substrate system. The p-polarizing thickness of the film on one mirror is double that of the s-polarizing thickness on the other mirror. Light strikes both mirrors at the common polarizing angle $\phi_{p,s}$.

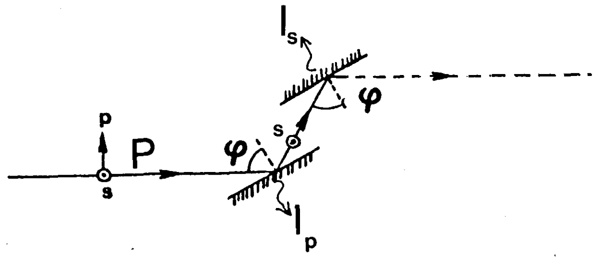

Fig. 12. The system of Fig. 11 can be used as a novel polarimeter that integrates the polarization-analysis and photodetection functions. The component polarization that is suppressed on reflection is absorbed by the substrate and generates a proportional signal I_ν ($\nu = p, s$). The degree of linear polarization of the incident flux, given by $P = (I_p - I_s)/(I_p + I_s)$, is measured.

B. Novel Integrated Polarimeter

The linearly polarized component of the incident radiation that is not reflected by a thin-film reflection polarizer is, of course, absorbed by the substrate. A proportional (e.g., photoelectric or photoacoustic) signal can be generated as a result of this absorption that may be detected. By integrating the polarization and detection functions, we achieve the simple and novel polarimeter shown in Fig. 12. It consists of the same pair of parallel-mirror, crossed, thin-film reflection polarizers shown in Fig. 11, with the further provision of recording signals I_p and I_s that result from the suppression (and absorption) of the p and s polarizations of the incident radiation on reflection from the first and second mirrors, respectively. For simplicity, let us assume the unextinguished s reflectance to be ≈100% (e.g., reflectance >99% is achieved with an Al substrate and >95% with Si in the UV). This polarimeter gives the degree of linear polarization of the input flux,

$$P = (I_p - I_s)/(I_p + I_s), \tag{35}$$

in terms of the two detected signals I_p and I_s. Because the unextinguished s reflectance is higher than the unextinguished p reflectance, it is preferable to make the p-suppressing mirror as the first-to-be-encountered polarizer, as suggested by Fig. 12.

Although this polarimeter does not measure all four Stokes parameters of light (i.e., it is *not* a complete polarimeter),[12] the degree of linear polarization is significant in atmospheric, astronomical, and linear-dichroism studies.[13,14] In ellipsometry, it amounts to measuring ψ only, which is sufficient for certain applications.[15]

C. Thin-Film Metrology

In the study of variable-thickness films by reflection interferometry, maximum fringe visibility is obtained when incident p- or s-polarized light shines at the corresponding polarizing angle.[16] For a given film, the ambient and/or substrate may be selected to equalize the p and s polarizing angles. This achieves optimum fringe visibility with either the p or s polarization and also with incident light that is unpolarized or arbitrarily polarized.

Reflection ellipsometry[1] to study film growth under the conditions described in this paper is also optimal in the sense that the maximum possible changes of the ellipsometric angle ψ occur, from 0 when $d = d_p$ to 90° when $d = d_s$. Thus ψ becomes a sensitive parameter to film growth, in contrast with the usual circumstance of relying mainly on the relative phase shift Δ.

8. SUMMARY

Conditions for the extinction of the p and s polarizations of a plane wave of light on reflection at the same angle from an absorbing substrate coated by a transparent thin film have been derived. This occurs when light refracts in the film at 45° and when a certain constraint on the optical constants of the system, Eq. (14), is satisfied. Figure 2 demonstrates the nature of this constraint. A film refractive index can always be determined that achieves equal polarizing angles [given by Eqs. (18)] for given substrate and ambient refractive indices. For high-reflectance substrates, the ratio of film to ambient refractive index approaches $\sqrt{2}$. The film thickness required to suppress the p-polarized component of light is exactly double that required for extinction of the s-polarized component at the common polarizing angle. Specific results are given is Section 5 for Si and Al as representative examples of absorbing semiconductor and metallic substrates. High reflectance of the bare substrate leads to high unextinguished p and s reflectances of the film–substrate reflection polarizers. Sensitivity to errors of film refractive index, film thickness, and angle of incidence is studied in Section 6. It is found that precise control of refractive index (±0.01), thickness (±5 Å), and angle of incidence (±0.25°) may be required.

Several important applications have been briefly discussed in Section 7. They include new devices, such as integrated crossed thin-film reflection polarizers, division-of-wavefront polarizing beam splitter, and a half-shade device. Parallel-mirror crossed polarizers are suggested for the first time to the author's knowledge. Such a system can also be employed as a novel polarimeter in which the polarization-analysis and photodetection functions are integrated. Finally, it is indicated that the conditions described in this paper are optimal in thin-film metrology based on reflection interferometry and ellipsometry.

APPENDIX A: SOLVING EQUATION (14) AND PLOTTING FIGURE 2

Equation (14) cannot be solved explicitly for α in terms of complex ϵ_2. For a given ϵ_2, α is determined numerically by iteration.

However, for a given α and $\mathrm{Re}(\epsilon_2)$, Eq. (14) can be solved explicitly for $\mathrm{Im}(\epsilon_2)$. This makes possible the direct generation of data required to plot the constant-α contours of Fig. 2, by assigning a sequence of values to $\mathrm{Re}(\epsilon_2)$, for a given α, and solving for the corresponding values of $\mathrm{Im}(\epsilon_2)$. The procedure is as follows. First we set

$$\epsilon_2 = \epsilon_{2r} - j\epsilon_{2i} \tag{A1}$$

and

$$\beta = \epsilon_{2r} - \alpha \tag{A2}$$

in Eq. (14) and get

$$\mathrm{Re}(\beta - j\epsilon_{2i})^{1/2} = (1-\alpha)^{1/2}[\alpha + |\beta - j\epsilon_{2i}|]. \tag{A3}$$

In terms of the angle

$$\theta = \arg(\beta - j\epsilon_{2i}), \tag{A4}$$

Eq. (A3) becomes

$$(\beta/\cos\theta)^{1/2}\cos(\theta/2) = (1-\alpha)^{1/2}[\alpha + (\beta/\cos\theta)]. \tag{A5}$$

Squaring both sides of Eq. (A5), the result can be put in the form of a quadratic equation:

$$A\cos^2\theta + B\cos\theta + C = 0, \tag{A6}$$

where

$$\begin{aligned} A &= \alpha^2(1-\alpha) - \tfrac{1}{2}\beta, \\ B &= 2\alpha\beta(1-\alpha) - \tfrac{1}{2}\beta, \\ C &= \beta^2(1-\alpha). \end{aligned} \tag{A7}$$

Equation (A6) is solved for $\cos\theta$:

$$\cos\theta = [-B \pm (B^2 - 4AC)^{1/2}]/2A. \tag{A8}$$

Finally, ϵ_{2i} is obtained from β and $\cos\theta$ as

$$\begin{aligned} \epsilon_{2i} &= \beta\tan\theta \\ &= \beta(1-\cos^2\theta)^{1/2}/\cos\theta. \end{aligned} \tag{A9}$$

Both roots in Eq. (A8) that lead to $\epsilon_{2i} > 0$ are acceptable for certain ranges of α and ϵ_{2r}, as is evident from Fig. 2.

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