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Explicit equations for the polarizing angles of a high-reflectance substrate coated by a transparent thin film

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Simple explicit equations are derived that determine the angles of incidence at which the parallel and perpendicular polarization components of light are extinguished on reflection from a transparent film coating a high-reflectance (metallic) substrate. The polarizing angles obtained from our approximate expressions are in excellent agreement with those determined by iterative numerical solution of the exact nonlinear equations that govern such angles. For the approximation to be valid, the intensity reflectance of the film–substrate interface, evaluated at the critical angle of the film–ambient interface, must exceed 0.5.

Polarization of monochromatic or quasi-monochromatic light by reflection from a dielectric or metallic substrate coated by a dielectric thin film is well known.[1] For given optical constants of the (transparent) film and (absorbing) substrate, the required angles of incidence for zero reflection of the parallel (p) and perpendicular (s) polarizations are determined by transcendental equations that can be solved only iteratively.[2]

In this communication, we derive simple *explicit* equations for the p- and s-polarizing angles of a film–substrate system. The equations are approximate but accurate when the substrate reflectance is sufficiently high. High-reflectance (metallic) substrates (e.g., Ag or Al) are needed to achieve high throughput for the unextinguished component and hence a more efficient polarizer.[2]

The basis for the approximation can be ascertained by considering Fig. 1. In this figure, we plot the Fresnel-intensity reflectances[3]

$$\mathcal{R}_{ij\nu} = |r_{ij\nu}|^2 \tag{1}$$

of the ambient–film ($ij = 01$) and film–substrate ($ij = 12$) interfaces for the parallel ($\nu = p$) and perpendicular ($\nu = s$) polarizations as functions of the grazing angle of incidence θ between the light beam and the reflecting surface. [$\theta = (\pi/2) - \phi$, where ϕ is the usual angle of incidence between the beam direction and the surface normal.] The data of Fig. 1 are for a representative case of an Al substrate of complex refractive index[4] $N_2 = 1.212 - j6.924$ coated by a transparent film (Al_2O_3) of refractive index $N_1 = 1.6$. Light of wavelength $\lambda = 6328$ Å (from a He–Ne laser) is assumed to be incident upon this film–substrate system from air or from vacuum ($N_0 = 1$).

Suppression of a given (p or s) polarization on reflection is possible by interference in the thin film at the angle of incidence that makes the ambient–film and film–substrate interface reflectances equal,[2,5] i.e., when

$$\mathcal{R}_{01\nu} = \mathcal{R}_{12\nu}, \qquad \nu = p \text{ or } s. \tag{2}$$

The points of intersection of the p and s pairs of reflectance curves, denoted by P and S, respectively, determine the corresponding polarizing angles θ_p and θ_s, as shown in Fig. 1.

Two observations regarding Fig. 1 make the derivation of accurate explicit equations for the polarizing angles possible:

(1) The grazing polarizing angles are small (of the order of a few degrees for a high-reflectance metal substrate[2,5]).

(2) $\mathcal{R}_{12p}$ and $\mathcal{R}_{12s}$ stay virtually constant with θ (they change imperceptibly, by <0.2%), as θ increases from 0 to 10°.

Correspondingly, two sensible approximations can be made. The first is to replace the exact Fresnel equations of $\mathcal{R}_{01\nu}$ ($\nu = p, s$) by quadratic Taylor-series expansions valid about $\theta = 0$. The second is to set

$$\mathcal{R}_{12\nu}(\theta_\nu) \cong \mathcal{R}_{12\nu}(0), \quad \nu = p, s. \tag{3}$$

The first three terms of the Taylor-series expansion of the ambient–film Fresnel (amplitude-reflection) coefficients, valid near grazing incidence, can be written as[6]

$$r_{01p} = -1 + 2E(E-1)^{-1/2}\theta - 2E^2(E-1)^{-1}\theta^2 + \ldots, \tag{4a}$$

$$r_{01s} = -1 + 2(E-1)^{-1/2}\theta - 2(E-1)^{-1}\theta^2 + \ldots, \tag{4b}$$

where

$$E = N_1^2/N_0^2 \tag{5}$$

and N_0, N_1 are the ambient and film refractive indices, respectively. By squaring Eqs. (4), the corresponding intensity reflectances are obtained:

$$\mathcal{R}_{01p} = 1 - 4E(E-1)^{-1/2}\theta + 8E^2(E-1)^{-1}\theta^2, \tag{6a}$$

$$\mathcal{R}_{01s} = 1 - 4(E-1)^{-1/2}\theta + 8(E-1)^{-1}\theta^2, \tag{6b}$$

where terms including θ^n ($n > 2$) have been dropped.

At the polarizing angles, $\mathcal{R}_{01p}$ and $\mathcal{R}_{01s}$, which appear on the left-hand sides of Eqs. (6a) and (6b), can be replaced by $\mathcal{R}_{12p}(0)$ and $\mathcal{R}_{12s}(0)$, respectively, according to Eqs. (2) and (3). Next, quadratic Eqs. (6) are solved for θ to give

$$\theta_{pa2} = \frac{(E-1)^{1/2}}{4E}\{1 - [2\mathcal{R}_{12p}(0) - 1]^{1/2}\}, \tag{7a}$$

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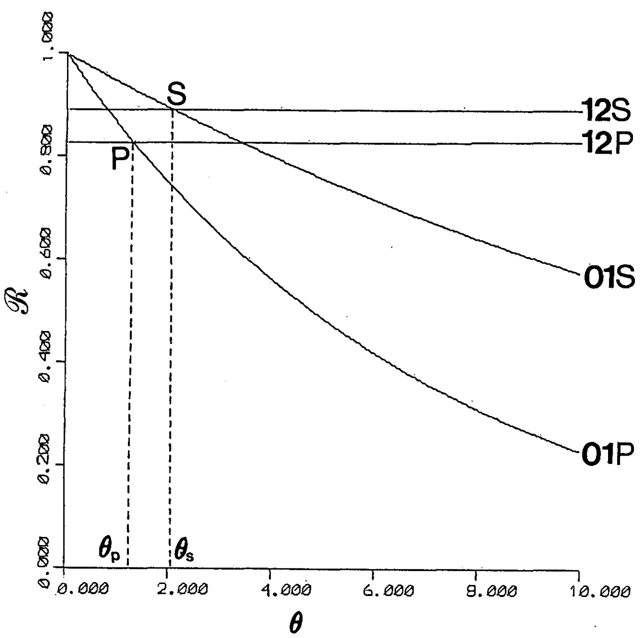

Fig. 1. Intensity reflectances $\mathcal{R}$ of the ambient–film (01) and film–substrate[12] interfaces for the p and s polarizations as functions of the grazing-incidence angle θ in degrees. Reflection of He–Ne laser light ($\lambda = 6328$ Å) in air ($N_0 = 1$) by an Al_2O_3 ($N_1 = 1.6$) film coating an Al ($N_2 = 1.212 - j6.924$) substrate is assumed. The points of intersection P and S determine the polarizing angles θ_p and θ_s, respectively.

$$\theta_{sa2} = \frac{(E-1)^{1/2}}{4}\{1 - [2\mathcal{R}_{12s}(0) - 1]^{1/2}\}. \tag{7b}$$

Equations (7) are the desired explicit equations for the p- and s-polarizing angles in radians. The subscript $a2$ refers to the quadratic approximation used.

Expressing Eqs. (7) in degrees and as functions of the film refractive index N_1 (and assuming an air or vacuum ambient $N_0 = 1$) yields

$$\theta_{pa2} = (45/\pi)\frac{(N_1^2 - 1)^{1/2}}{N_1^2}\{1 - [2\mathcal{R}_{12p}(0) - 1]^{1/2}\}, \tag{8a}$$

$$\theta_{sa2} = (45/\pi)(N_1^2 - 1)^{1/2}\{1 - [2\mathcal{R}_{12s}(0) - 1]^{1/2}\}. \tag{8b}$$

In Eqs. (7) and (8), $\mathcal{R}_{12\nu}(0)$, the Fresnel-intensity reflectances for the $\nu = p, s$ polarizations of the film–substrate, are functions of N_1 and are evaluated at external grazing incidence $\theta = 0$ (i.e., when light is refracted in the film at the critical angle of the film–ambient interface).

For the Al_2O_3–Al film–substrate system, with the optical constants at $\lambda = 6328$ Å as given above, we calculate $\mathcal{R}_{12p}(0) = 0.8261$ and $\mathcal{R}_{12s}(0) = 0.8908$. Substitution of these reflectances and $N_1 = 1.6$ into Eqs. (8) gives the following *approximate* polarizing angles:

$$\theta_{pa2} = 1.344°, \qquad \theta_{sa2} = 2.073°. \tag{9}$$

The *exact* polarizing angles, obtained by numerical iteration[7] (to bring $|r_{01\nu}/r_{12\nu}|$ to within $<10^{-6}$ of 1), are given by

$$\theta_{pe} = 1.335°, \qquad \theta_{se} = 2.070°. \tag{10}$$

The excellent agreement between the approximate and exact polarizing angles verifies the high accuracy of Eqs. (8).

From Eqs. (7) and (8) it is evident that the condition

$$\mathcal{R}_{12\nu}(0) > 0.5 \qquad \nu = p, s \tag{11}$$

must be satisfied for the approximate equations to yield meaningful real answers. We have verified that the accuracy of the approximation improves steadily as $\mathcal{R}_{12\nu}(0)$ increases above this lowest limit of 0.5. The polarizing angles from the approximate and exact equations agree to within a few percent for $\mathcal{R}_{12\nu}(0) > 0.7$. This condition is met readily when a high-reflectance metal substrate is used.

For further illustration, the exact and approximate p- and s-polarizing angles are calculated for a system that consists

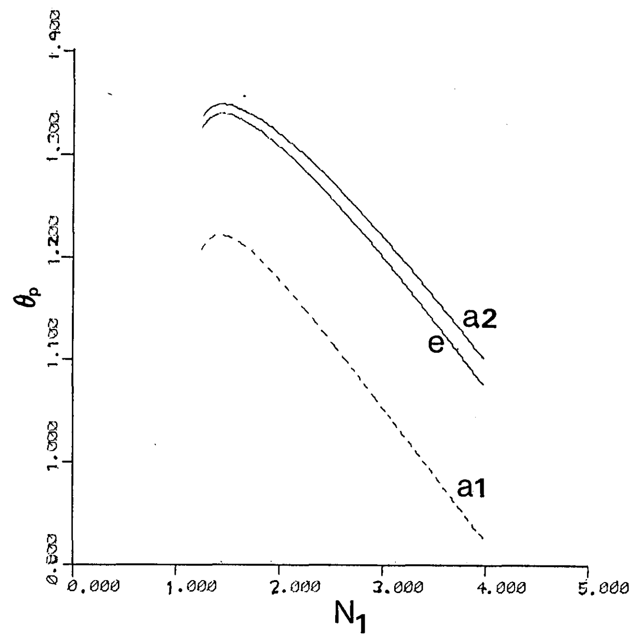

Fig. 2. Approximate ($a2$) and exact (e) p-polarizing angles θ_p (in degrees) as functions of the refractive index N_1 of a transparent film on an Al substrate ($N_2 = 1.212 - j6.924$). The dashed curve $a1$ represents the linear (instead of parabolic, $a2$) approximation to the polarizing angle.

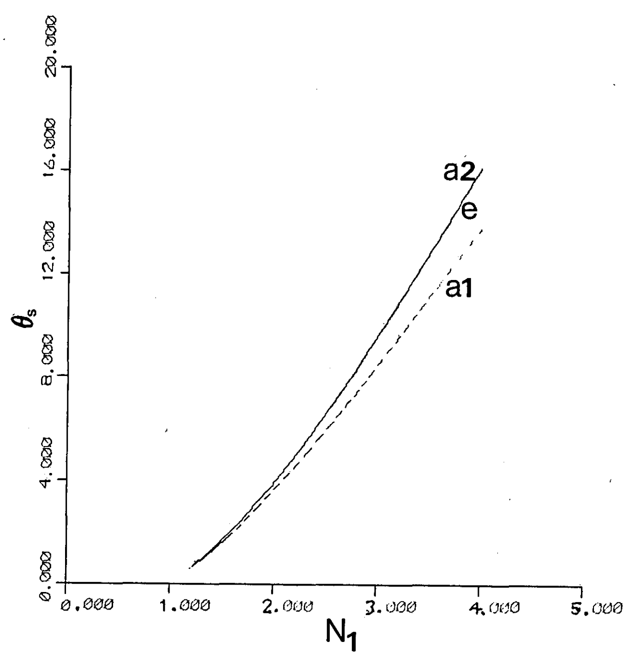

Fig. 3. Same as Fig. 2 but for the s polarization.

Table 1. **Comparison between the Approximate and Exact p ($\theta_{pa2}, \theta_{pe}$) and s ($\theta_{sa2}, \theta_{se}$) Grazing Polarizing Angles of Incidence of Transparent Films on Metal Substrates**[a]

Substrate (λ)	N_1	$\mathcal{R}_{12p}(0)$	θ_{pa2}	θ_{pe}	$\mathcal{R}_{12s}(0)$	θ_{sa2}	θ_{se}
Ag, $N_2 = 0.05 - j2.87$	1.5	0.9637	0.263	0.26	0.9801	0.322	0.32
(0.5 μm)	2.0	0.9626	0.237	0.24	0.9736	0.663	0.66
	3.0	0.9626	0.172	0.17	0.9695	1.256	1.26
Al, $N_2 = 0.64 - j5.497$	1.5	0.8561	1.111	1.11	0.9178	1.375	1.37
(0.492 μm)	2.0	0.8422	1.072	1.06	0.8814	3.141	3.14
	3.0	0.8118	0.947	0.94	0.8349	7.355	7.34
Rh, $N_2 = 1.62 - j4.633$	1.5	0.6093	3.789	3.51	0.7608	4.448	4.40
(0.564 μm)	2.0	0.5797	3.727	3.36	0.6706	10.320	10.05
	3.0	0.5219	3.558	2.9	0.5710	25.247	23.75

[a] N_1 is the film refractive index, N_2 is the substrate complex refractive index at the indicated wavelength λ. $\mathcal{R}_{12p}(0)$ and $\mathcal{R}_{12s}(0)$ are the film–substrate intensity reflectances evaluated at exact grazing incidence ($\theta = 0$). θ_{pa2} and θ_{sa2} are the approximate polarizing angles calculated from Eqs. (8) and truncated to three decimal places. θ_{pe} and θ_{se} are the exact polarizing angles as determined by Ruiz-Urbieta and Sparrow.[2] All angles are in degrees.

of a transparent film of refractive index that takes values from 1.2 to 4 on the same Al substrate ($N_2 = 1.212 - j6.924$ at λ = 6328 Å). The results are compared in Figs. 2 and 3. The maximum absolute differences are 0.026° for θ_p and 0.05° for θ_s. The curves of θ_{se} and θ_{sa2} appear coincident in Fig. 3; the maximum difference is <0.5% in this case.

For reference, we have superimposed dashed curves upon Figs. 2 and 3 that represent other approximate polarizing angles, namely,

$$\theta_{pa1} = (45/\pi)\frac{(N_1{}^2 - 1)^{1/2}}{N_1{}^2}[1 - \mathcal{R}_{12p}(0)], \qquad (12a)$$

$$\theta_{sa1} = (45/\pi)(N_1{}^2 - 1)^{1/2}[1 - \mathcal{R}_{12s}(0)]. \qquad (12b)$$

These are the first-order approximations to the polarizing angles obtained when the quadratic (θ^2) terms are dropped from Eqs. (6). They represent the situation in which the curves of $\mathcal{R}_{01p}$ and $\mathcal{R}_{01s}$ in Fig. 1 are replaced by their straight-line tangents at $\theta = 0$ (instead of parabolas for the quadratic approximation). For the Al_2O_3–Al system at 6328 Å, one obtains

$$\theta_{pa1} = 1.215°, \qquad \theta_{sa1} = 1.953°. \qquad (13)$$

The degree of match with the exact solution [Eqs. (10)] is an order of magnitude worse for the linear approximation [Eqs. (13)] than it is for the quadratic approximation [Eqs. (9)]. Only when the substrate reflectance is very high (>95%) can the linear approximation be useful.

The quadratic approximation is far superior and is the one to use. The simplification in going from Eqs. (8) to Eqs. (12) is too slight to justify paying further attention to the linear approximation.

We also compared the p- and s-polarizing angles calculated from our approximate Eqs. (8) with those (exact) angles obtained numerically by Ruiz-Urbieta and Sparrow.[2] Table 1 gives results for the two high-reflectance metals Ag and Al (normal-incidence reflectances $\mathcal{R}_0$ = 97.9% for Ag and 92.2% for Al at λ = 0.5 and 0.492 μm, respectively) and for film refractive indices of 1.5, 2.0, and 3.0. Again, our approximate explicit equations [Eqs. (8)] yield results that agree with the exact numerical data of Ruiz-Urbieta and Sparrow to within ~0.01° for these metals and films.

To test the range of validity of Eqs. (8), Table 1 compares our approximate polarizing angles with the exact ones of Ref. 2 for the low-reflectance metal Rh (R_0 = 77.1% at λ = 0.564 μm). Poorer agreement is obtained, as expected. The disagreement becomes more pronounced as the reflectances $\mathcal{R}_{12\nu}(0)$ approach 0.5, the lowest limit of validity of our approximate equations.

Finally, we should mention that Eqs. (8) continue to hold when the metal substrate is replaced by a high-reflectance multilayer substructure. In this case, $\mathcal{R}_{12\nu}(0)$ represents the reflectance of the film–substructure interface at external grazing incidence ($\theta = 0$) for the ν polarization.

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