Explicit determination of the complex refractive index of an absorbing medium from reflectance measurements at and near normal incidence

R. M. A. Azzam

University of New Orleans, razzam@uno.edu

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Explicit determination of the complex refractive index of an absorbing medium from reflectance measurements at and near normal incidence

R. M. A. Azzam

Department of Electrical Engineering, University of New Orleans, Lakefront, New Orleans, Louisiana 70148

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Measurement of reflectance at normal incidence \( R \) and its fractional change \( \Delta R/R \) caused by a change of the angle of incidence from 0 to a small angle \( \phi \) (\( \phi \leq 20^\circ \)) permits explicit determination of both the refractive index \( n \) and extinction coefficient \( k \) of an isotropic absorbing medium. The medium of incidence (ambient) is assumed to have a known refractive index (e.g., \( n = 1 \) for vacuum or air), and the incident light is either \( p \) or \( s \) linearly polarized.

A variety of reflectance methods\(^1\) is available to determine the complex refractive index \( N = n - jk \) of an absorbing medium. Few of those methods\(^5,6,9,10\) provide explicit solutions for \( n \) and \( k \) in terms of the measured reflectances. In this Letter we propose a new method that provides simple, direct, and explicit determination of \( n \) and \( k \) in terms of the intensity reflectance measured at normal incidence \( R \) and the fractional change of such reflectance \( \Delta R/R \) that results from a given change of angle of incidence from 0 to \( \phi \), where \( \phi \) is a small angle (\( \leq 20^\circ \)).

Figure 1 shows the normal-incidence reflection of light by the planar interface between a transparent ambient of known refractive index \( N_0 \) and an absorbing substrate (mirror) of complex refractive index \( N_1 \) to be determined. Both media are assumed to be homogeneous and isotropic. The mirror can be rotated about an axis \( z \) in its surface through the point of reflection by a known angle \( \phi \). The incident light is linearly polarized with its electric vector vibrating either parallel or perpendicular to the rotation axis. (These are the conventional \( s \) and \( p \) polarizations, respectively.) A complex reciprocal relative refractive index defined by

\[
N_r = N_0/N_1 = n_r + jk_r
\]  

is more readily determined first by using the proposed method. Of course, once \( N_r \) is found, \( N_1 \) is given by

\[
N_1 = N_0/N_r = n_1 - jk_1.
\]  

The signs of the imaginary parts in Eqs. (1) and (2) are consistent with the Nebraska (Muller) conventions.\(^1^0\)

The first equation to be used is\(^1^4\)

\[
\frac{\Delta R_s}{r_s} = \pm N_r \phi^2,
\]  

which gives the fractional change of Fresnel’s complex interface reflection coefficient for the \( s \) polarization (+ for \( s \) and − for \( p \)) that results from changing the angle of incidence from 0 to \( \phi \), where \( \phi \) is a given small angle. The measurable fractional change of intensity reflectance, \( \Delta R_s/R_s \), is related to \( \Delta r_s/r_s \) by

\[
\Delta R_s/R_s = 2 \text{Re}(\Delta r_s/r_s),
\]  

where \( \text{Re} \) means the real part of. Substitution of Eq. (3) into Eq. (4) and use of Eq. (1) give

\[
|\Delta R_s/R_s| = 2n_r \phi^2.
\]  

Equation (5) readily determines \( n_r \):

\[
n_r = |\Delta R_s/R_s|/2\phi^2.
\]  

(If \( |\Delta R_s/R_s| \), measured at various values of small \( \phi \), is plotted versus \( \phi^2 \), the slope of the resulting straight line gives a precise estimate of \( n_r \).) Subsequently, \( k_r \) is found from the normal-incidence reflectance

\[
R_r = |(1 - N_r)/(1 + N_r)|^2
\]  

or

\[
R_r = |(1 - n_r)^2 + k_r^2|/|(1 + n_r)^2 + k_r^2|.
\]  

Equation (8) gives

\[
k_r = \left[2n_r \left[\frac{1 + R_r}{1 - R_r} - (n_r^2 + 1)\right]\right]^{1/2}.
\]  

Equations (6) and (9) show explicitly how the complex reciprocal relative refractive index, \( N_r = n_r + jk_r \), is determined from the normal-incidence reflectance \( R \) and its fractional change \( \Delta R/R \) caused by changing incidence from normal to a small angle \( \phi \). The substrate complex refractive index \( N_1 \) is calculated from \( N_r \) and the known refractive index \( N_0 \) of the ambient (usually air, \( N_0 = 1 \)) by using Eq. (2).

The incident light is assumed to be either \( p \) or \( s \) polarized. The method in its present form would not work if the incident light were unpolarized. This is because \( \Delta R_p/R_p = -\Delta R_s/R_s \), so that

\[
\Delta R_s/R_s = 0
\]  

to second order in \( \phi \) (the subscript \( s \) denotes unpolarized).

\( \Delta R/R \) can be accurately determined by a lock-in technique if the mirror is periodically oscillated (e.g., sinusoidally, so that \( \phi = \phi \sin \omega t \), where \( \phi \) is a small-amplitude angular excursion) and the modulation of the reflected light intensity \( \Delta I/I \) is

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determined. With the intensity of the incident light constant, it is easy to show that
\[ \Delta R/R = \Delta I/I. \]  

The method can be considered as an interesting special case of angle-of-incidence derivative ellipsometry and reflectometry\textsuperscript{15,16} and, more closely, of a method previously described by Hunderi\textsuperscript{17}.

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REFERENCES


