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Determination of Mueller matrix elements in the presence of imperfections in optical components

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Determination of Mueller matrix elements in the presence of imperfections in optical components

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ABSTRACT

The Polarizer-Sample-Analyzer (PSA) arrangement with the optical components P and A rotating with a fixed speed ratio (3:1) was originally introduced to determine nine Mueller matrix elements from Fourier analysis of the output signal of a photodetector. The arrangement is modified to the P'PSAA' arrangement where P' and A' represent fixed polarizers that are added at both ends with the speed ratio of the rotating components (P and A) remaining the same as before. After determination of the partial Mueller matrix in the ideal case, azimuthal offsets and imperfection parameters are introduced in the straight-through configuration and the imperfection parameters are determined from the Fourier coefficients. Finally, the sample is reintroduced and the full Mueller matrix elements are calculated to show the deviation from the ideal case and their dependency on the offsets and imperfection parameters.

Keywords: Polarization, Ellipsometry, Jones calculus, Mueller calculus, polarizer, Fourier series

CHAPTER 1

INTRODUCTION

Polarization is a property that is common to all vector waves. It is defined by the behavior with time of one of the field vectors appropriate to that wave, observed at a fixed point in space. Light waves are electromagnetic in nature and are described by four field vectors: electric field strength $\vec{\mathrm{E}}$, electric displacement density $\vec{\mathrm{D}}$, magnetic field strength $\vec{\mathrm{H}}$, and magnetic flux density $\vec{\mathrm{B}}$. The electric field strength $\vec{\mathrm{E}}$ is used to define the state of polarization. This is because the force exerted on electrons by the electric field is much greater than that exerted by the magnetic field of the wave. The polarization of the three field vectors can be determined using the Maxwell's field equations, once the polarization of $\vec{\mathrm{E}}$ has been determined [1].

Ellipsometry is an optical technique for the characterization of interfaces and films and is based on the polarization transformation that occurs as a beam of polarized light is reflected from or transmitted through the interfaces or films [1].

We now present the four parameters that define an ellipse of polarization and we will describe the optical elements in the arrangement.

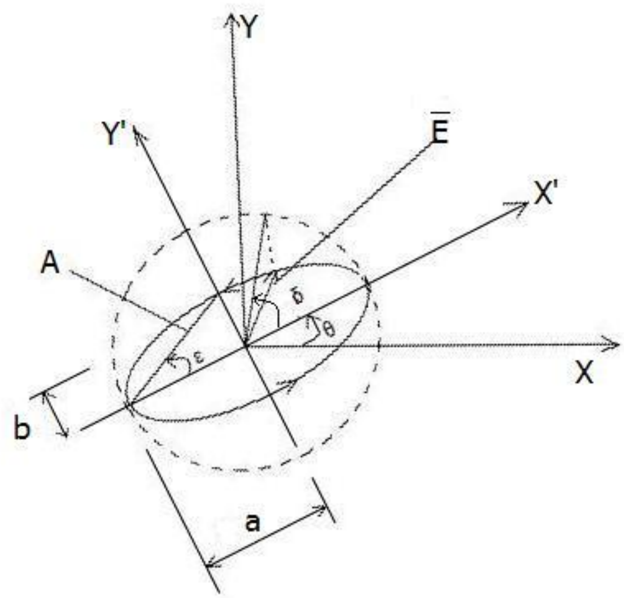

Fig 1.1. Four parameters defining the ellipse of polarization in its plane (1) the azimuth θ of the major axis from a fixed direction X, (2) the ellipticity e=$\pm$b/a=$\pm$tan ε, (3) the amplitude A=$(a^2+b^2)^{1/2}$ and (4) the absolute phase δ.

(1) The azimuth θ is the angle between the major axis the ellipse and the positive direction of the X axis and defines the orientation of the ellipse.

$$-\frac{\pi}{2} \leq \theta < \frac{\pi}{2} \tag{1.1}$$

(2) The ellipticity e is the ratio of the semi-minor axis of the ellipse b to the semi-major axis a,

$$e=\frac{b}{a} \tag{1.2}$$

(3) The amplitude of the elliptic vibration can be defined in terms of the lengths a and b as

$$A = (a^2 + b^2)^{1/2} \tag{1.3}$$

(4) The absolute phase vector δ determines the angle between the initial position of the electric vector at t=0 and the major axis of the ellipse. All possible values of δ are limited by

$$-\pi \leq \delta < \pi \tag{1.4}$$

The electric vectors of light linearly polarized in the X and Y directions are given by,

$$\vec{E}_x = A_x \cos(\omega t + \varphi_x)\hat{\imath} \tag{1.5}$$

$$\vec{E}_y = A_y \cos(\omega t + \varphi_y)\hat{\jmath} \tag{1.6}$$

For elliptically polarized light the electric vector is then given by

$$\vec{E} = \vec{E}_x + \vec{E}_y = A_x \cos(\omega t + \varphi_x)\hat{\imath} + A_y \cos(\omega t + \varphi_y)\hat{\jmath} \tag{1.7}$$

Or in matrix (column) form as,

$$\vec{E} = \begin{bmatrix} \vec{E}_x \\ \vec{E}_y \end{bmatrix} = \begin{bmatrix} A_x e^{i(\omega t + \varphi_x)} \\ A_y e^{j(\omega t + \varphi_y)} \end{bmatrix} = e^{j\omega t}\underline{\mathbf{J}} \tag{1.8}$$

Taking out the time dependence from Eq. (1.8), we have the Jones vector

$$\underline{\mathbf{J}} = \begin{bmatrix} A_x e^{i\varphi_x} \\ A_y e^{i\varphi_y} \end{bmatrix} \tag{1.9}$$

When light interacts with an optical system, the Jones vector transforms according to

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} \tag{1.10}$$

or

$$\underline{\mathbf{J}}' = \underline{\mathbf{T}}\underline{\mathbf{J}} \tag{1.11}$$

Here $\underline{\mathbf{T}}$ is defined as the Jones matrix of the optical system, the matrix elements t_{ij} are determined from the known properties of the optical element encountered by the light and the equation is simply the transformation matrix that converts $\underline{\mathbf{J}}$ into $\underline{\mathbf{J}}'$.

Similar to the Jones calculus, the Mueller calculus also represents the light by a vector; premultiplication of this vector by a matrix that represents an optical element yields a resultant vector that describes the beam after its interaction with the optical device. The vector representing the light is called a Stokes vector and is a 4 X 1 real vector instead of 2 X 1 complex Jones vector. The 4 elements of the Stokes vector represent intensities of the light. Therefore these elements are real. A common representation of the Stokes vector is given by

$$\underline{\mathbf{S}} = \begin{bmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} {A_x}^2 + {A_y}^2 \\ {A_x}^2 - {A_y}^2 \\ 2A_xA_y\cos\delta \\ 2A_xA_y\sin\delta \end{bmatrix} \tag{1.12}$$

where $\delta = \delta_y - \delta_x$ and $-\pi \leq \delta \leq \pi$.

A deterministic complex Jones matrix cannot be used to express the depolarizing incoherent interaction between the incident wave and the optical system. Thus we resort to the more powerful Mueller matrix. The output coherency vector can be written in terms of the input coherency vector as

$$\underline{\mathbf{J}}_0 = (\mathbf{T} \times \mathbf{T}^*)\underline{\mathbf{J}}_\mathrm{i} \tag{1.13}$$

The coherency vector **J** can be related to the Stokes vector **S** as **S** = **AJ**,[1]

$$\text{where} \quad \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & -\mathrm{j} & \mathrm{j} & 0 \end{bmatrix} \tag{1.14}$$

The relationship between the Stokes vector $\mathbf{S}_\mathrm{o}$ and $\mathbf{S}_\mathrm{i}$ of the outgoing and incident waves is given by,

$$\mathbf{S}_0 = [\mathbf{A}(\mathbf{T} \times \mathbf{T}^*)\mathbf{A}^{-1}]\mathbf{S}_\mathrm{i} \text{ or } \mathbf{S}_0 = \mathbf{M}\mathbf{S}_\mathrm{i} \tag{1.15}$$

where $\mathbf{M} = \mathbf{A}(\mathbf{T} \times \mathbf{T}^*)\mathbf{A}^{-1}$

We now introduce the main optical device that is used in the optical measurement arrangement and point out its characteristics and orientation.

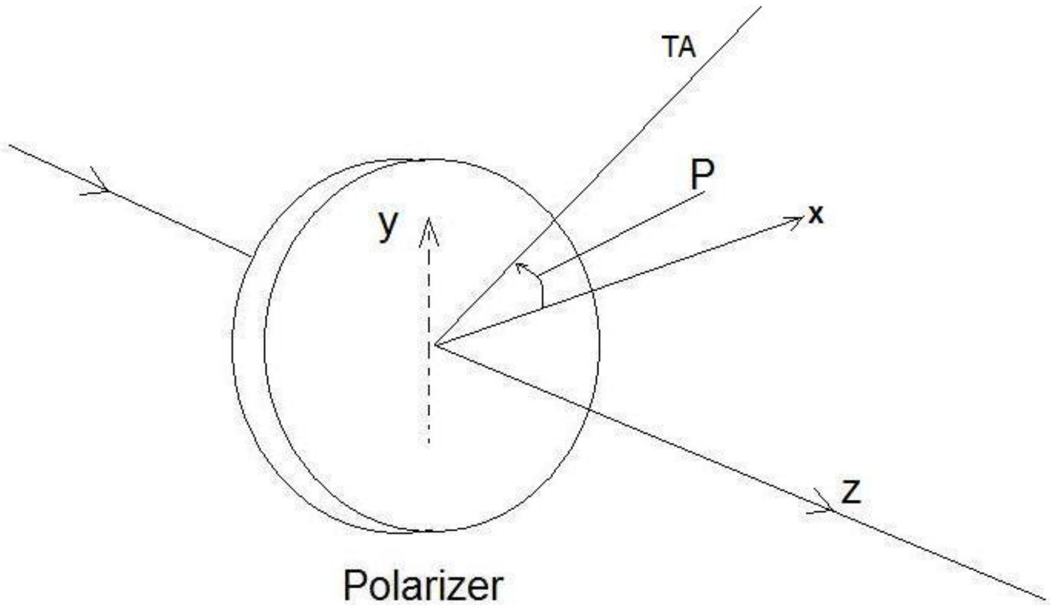

Fig 1.2. The polarizer azimuth P is measured from the positive X direction as a reference in an anticlockwise sense. The direction Z is the direction of propagation of light. TA is the transmission axis of the polarizer.

A linear polarizer transmits a beam of light whose electric field vector oscillates in a plane that contains the beam axis and transmission axis TA. The orientation of this plane in space can be varied by rotation of the polarizer about the beam axis. As shown Fig 1.2, the azimuth of the polarizer P is measured from the positive direction of X axis, where Z is the direction of propagation of light.

The Jones matrix of any optical device is a function of the azimuthal orientation of the input and output transverse coordinate axes, around the incident and outgoing wave vectors.

The matrix $\underline{\mathbf{T}}'$ for a linear polarizer or retarder with its optical axis rotated by an angle θ from the coordinates is found from $\underline{\mathrm{T}}$ by a matrix rotation such that

$$\underline{\mathbf{T}}'=R(-\theta)\mathbf{T}R(\theta) \tag{1.16}$$

$$\text{where } R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{1.17}$$

For a system of devices as shown below the resulting Jones vector is given by

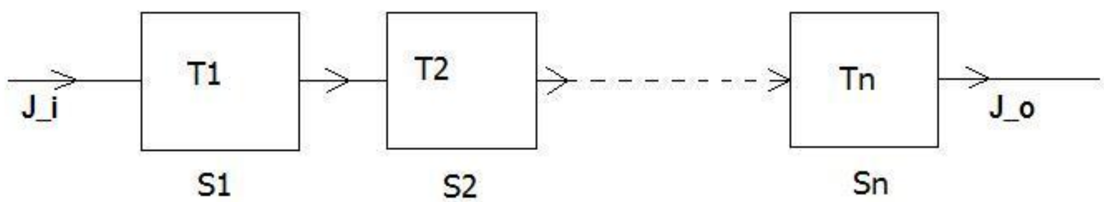

Fig 1.3. A combined effect of n optical systems S1, S2,…,Sn of Jones Matrix T1, T2,…, Tn.

$$\underline{\mathbf{J}}' = \mathrm{T_n}\underline{\mathbf{J}}'' \tag{1.18}$$

$$\underline{\mathbf{J}}'' = \mathrm{T_{n-1}}\underline{\mathbf{J}}''' \tag{1.19}$$

So the combination of the system of devices will give

$$\underline{\mathbf{J}}_\mathrm{o} = \mathrm{T_n}\,\mathrm{T_{n-1}}\ldots\mathrm{T_1}\underline{\mathbf{J}}_\mathrm{i} \tag{1.20}$$

where T_i is the Jones matrix for the i^{th} element and $\mathbf{J}_i$ is the Jones vector at the input. A similar analysis applies in the Stokes Mueller matrix calculus as will be used in subsequent chapters.

CHAPTER 2

OPTICAL SYSTEM AND MODIFICATION

2.1 Introduction

The original arrangement [2] is that of a typical ellipsometric arrangement with the sample placed between a polarizer and analyzer. The polarizer and analyzer are rotated with a fixed speed ratio of 3:1 respectively. The figure below shows this arrangement and the directions of rotation and azimuths of the optical elements. The 3:1 speed ratio is important in the fact that it gives the best results in terms of linearity of the obtained Mueller matrix elements in terms of the Fourier coefficients.

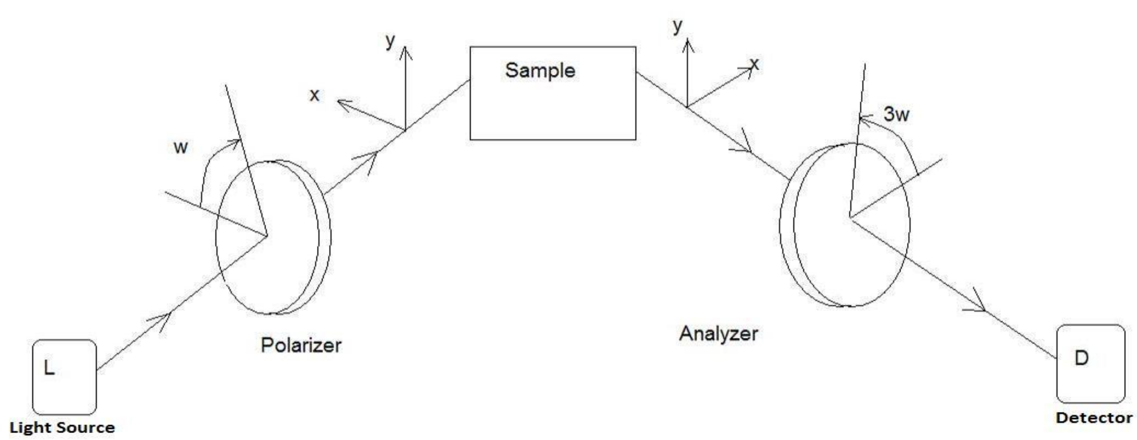

Fig 2.1. Original PSA arrangement with both the polarizer and the analyzer rotating at a speed of ωt and 3ωt respectively. The orthogonal directions to the direction of propagation are shown as x and y.

Nine elements of the sample Mueller matrix for the above arrangement are obtained as linear combination of the Fourier coefficients of the detected signal. The light source L is unpolarized, circularly polarized or partially circularly polarized. A depolarizer is needed in front of the detector to have a polarization insensitive photodetector as the polarization of light leaving the analyzer varies with time.

The photopolarimeter arrangement of Fig. 2.1 is modified by adding a fixed polarizer (FP) and fixed analyzer (FA) at the beginning and end as shown in Fig. 2.2. The direction x and y are parallel and perpendicular, respectively, to the scattering plane described by the directions of incident and transmitted light.

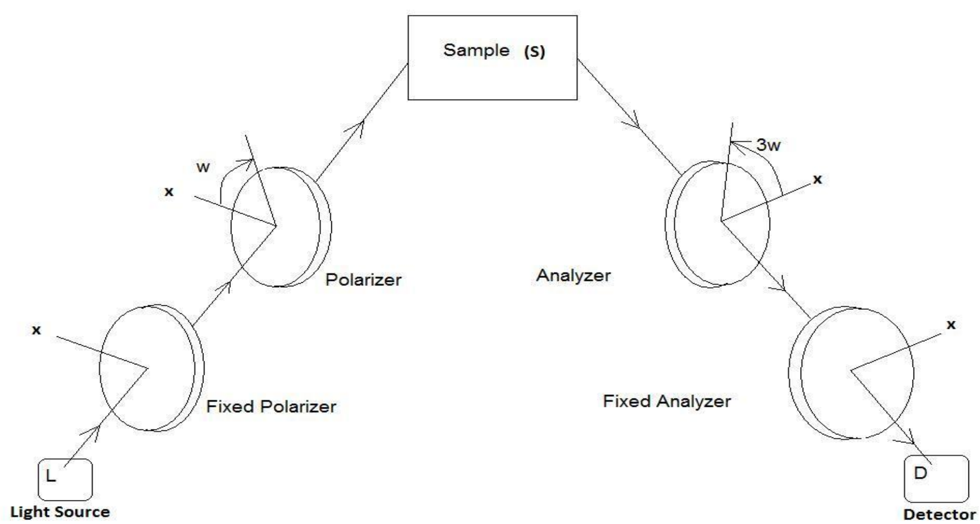

Fig 2.2. Fourier photopolarimeter with added fixed polarizer and analyzer. S is the optical system under measurement; L and D are the light source and detector respectively. x and y are parallel and perpendicular to the scattering plane and orthogonal to the direction of propagation.

2.2 Fourier analysis with ideal conditions

The azimuths of the rotating polarizer and analyzer P and A are measured from the positive direction of the x axis. Let $\underline{\mathbf{S}}_{\mathrm{FP}}$ be the Stokes vector of light leaving the polarizer, $\underline{\Gamma}_{\mathrm{FA}}$ the first row of the Mueller matrix of the fixed analyzer, $\underline{\mathbf{M}}_{\mathrm{A}}$ the Mueller matrix of the rotating analyzer, $\underline{\mathbf{M}}_{\mathrm{P}}$ the Mueller matrix of the rotating polarizer and $\underline{\mathbf{M}}_{\mathrm{S}}$ the Mueller matrix of the sample under measurement. Then the output signal of the detector D is given by the equation,

$$i = k\underline{\Gamma}_{\mathrm{FA}}\underline{\mathbf{M}}_{\mathrm{A}}\underline{\mathbf{M}}_{\mathrm{S}}\underline{\mathbf{M}}_{\mathrm{P}}\underline{\mathbf{S}}_{\mathrm{FP}}, \tag{2.1}$$

where k represents the detector sensitivity. Now substituting $\underline{\mathbf{S}}_{\mathrm{FP}} = \mathrm{I}[1\ \cos(2\mathrm{P}')\ \sin(2\mathrm{P}')\ 0]^{\mathrm{T}}$ (where I is the intensity of light leaving the polarizer and T indicates transpose), $\underline{\mathbf{M}}_{\mathrm{S}} = (\mathrm{m}_{\mathrm{ij}})$, i, j=1,2,3,4 , $\underline{\Gamma}_{\mathrm{FA}} = \tau[1\ \cos2\mathrm{A}'\ \sin2\mathrm{A}'\ 0]$, $\underline{\mathbf{M}}_{\mathrm{A}}$ and $\underline{\mathbf{M}}_{\mathrm{P}}$ are the Mueller matrices of the rotating analyzer and polarizer. Taking the fixed polarizer to have zero azimuth, we have the Stokes vector $\underline{\mathbf{S}}_{\mathrm{FP}} = [1\ 1\ 0\ 0]^{\mathrm{T}}$. Putting all these values in Eqn. (2.1), we get

$$\begin{aligned} i = XY[\, & \mathrm{m}_{11} + \mathrm{m}_{12}\cos(2P) + \mathrm{m}_{13}\sin(2P) + \mathrm{m}_{21}\cos(2A) + \mathrm{m}_{22}\cos(2A)\cos(2P) + \mathrm{m}_{23}\sin(2P)\cos(2A) + \\ & \mathrm{m}_{31}\sin(2A) + \mathrm{m}_{32}\sin(2A)\cos(2P) + \mathrm{m}_{33}\sin(2A)\sin(2P)] \end{aligned} \tag{2.2}$$

where

$$X = 1+\cos(2A')\cos(2A)+\sin(2A')\sin(2A), \qquad Y = 1+\cos(2P') \tag{2.3}$$

Since the 4th elements of $\underline{\mathbf{S}}_{\mathrm{FP}}$ and $\underline{\Gamma}_{\mathrm{FA}}$ are equal to zero the 4th row and 4th column of the Mueller matrix of the sample do not contribute to the detected signal. The polarizer and analyzer rotate in the same direction at angular speeds of ω and 3ω respectively. We also assume that the transmission axes of these elements are parallel to the x axis at t=0. Therefore we can write,

$$P = \omega t, \text{ and } A = 3\omega t \tag{2.4}$$

Substituting P and A from Eqn. (2.4) in Eqn. (2.2), and using the trigonometric identities,

sin x cos y = ½[sin(x+y)+sin(x-y)], sin x sin y = ½[cos(x-y)-cos(x+y)] and cos x cos y = ½[cos(x-y)+cos(x+y)], we get the Fourier series of the detected signal as,

$$i = a_0 + \sum_{n=1}^{8}(a_n \cos n\omega' t + b_n \sin n\omega' t), \tag{2.5}$$

where ω'= 2ω is the fundamental frequency of the current measured by the detector.

The Fourier amplitudes a_0, (a_n, b_n) from Eqn. (2.5) are given in Table 2.1 below, when the azimuth of the fixed analyzer A'=0.

$4a_0 = 4m_{11} + 2m_{12} + 2m_{21} + m_{22}$	$4b_1 = 3m_{13} + m_{23} + m_{32}$
$4a_1 = 4m_{11} + 5m_{12} + 2m_{21} + 3m_{22} + m_{33}$	$4b_2 = 2m_{13} - m_{23} + 2m_{31} + 2m_{32}$
$4a_2 = 2m_{11} + 4m_{12} + 2m_{21} + 3m_{22} + 2m_{33}$	$4b_3 = -m_{23} + 4m_{31} + 2m_{32}$
$2a_3 = 2m_{11} + m_{12} + 2m_{21} + m_{22}$	$8b_4 = 4m_{13} + 5m_{23} + 4m_{31} + 5m_{32}$
$8a_4 = 4m_{11} + 4m_{12} + 4m_{21} + 5m_{22} - 3m_{33}$	$4b_5 = m_{13} + m_{31} + 2m_{32}$
$4a_5 = m_{12} + m_{21} + 2m_{22}$	$2b_6 = m_{31}$

$4a_6 = 2m_{21} + m_{22}$	$4b_7 = m_{23} + m_{31} + m_{32}$
$4a_7 = m_{21} + m_{22} - m_{33}$	$8b_8 = 8m_{23} + m_{32}$
$8a_8 = m_{22} - m_{33}$	

Table 2.1. Fourier coefficients of the detected signal for the modified arrangement under ideal conditions

2.3 Mueller matrix

From the Fourier amplitudes $a_0, (a_n, b_n)$ of Table 2.1 the 3X3 Mueller matrix $\mathbf{M}_{3X3}$ can constructed as follows:

$$\mathbf{M}_{3X3} = \begin{bmatrix} a_3 + 6a_7 - 2a_5 + 2a_6 + 15a_8 & 4a_5 + 12a_7 - 8a_6 - 30a_8 & 3b_6 - b_7 - b_3 + 2b_5 \\ 4a_7 - 8a_8 & 4a_6 - 8a_7 + 16a_8 & 4b_7 - 2b_6 \\ 2b_6 & 2b_7 + 2b_3 - 5b_6 & 4a_8 - 4a_6 + 8a_7 \end{bmatrix} \tag{2.6}$$

It can thus be seen from the above that under ideal conditions the Mueller matrix elements can be expressed as linear combinations of the measured Fourier coefficients of the detected signal.

CHAPTER 3

OFFSETS AND IMPERFECTIONS

3.1 Introduction

The original PSA arrangement [2] gives us the partial 3x3 Mueller matrix. In order to determine the absolute Mueller matrix elements, we need to measure i_0. This can be done by removing the sample and setting the polarizer and analyzer azimuth 0 and π/4, respectively. This is the so called straight-through configuration as the optical arrangement falls in a straight line.

In the modified arrangement we also remove the sample and generate the straight-through configuration. Parameters that represent the deviations from the ideal case are introduced gradually and the imperfection parameters are determined in terms of the Fourier coefficients.

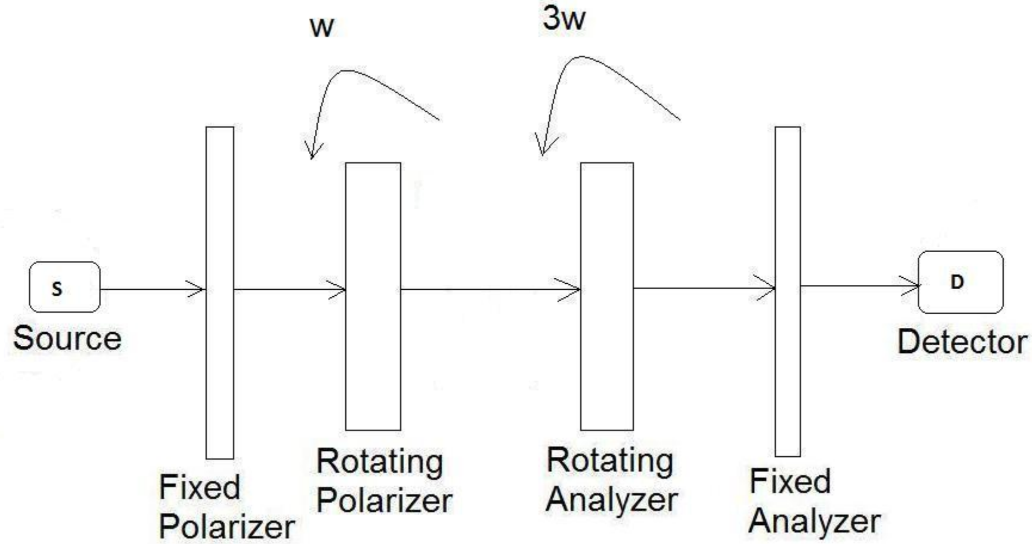

Fig 3.1. Straight through configuration: the sample is removed and all optical elements are in line.

3.2 Fourier analysis and determination of imperfection parameters from Fourier coefficients

We now analyze the case where the sample is removed which enables us to measure i_0 directly. In this setting the polarizer and the analyzer come in a straight-through position with the fixed polarizer azimuth set at P'= 0 and the fixed analyzer azimuth at A'. The Mueller matrix of the sample thus becomes the identity matrix. The result for such an arrangement is obtained as a Fourier series given by,

$$i_0 = \mathrm{a}_0 + \sum_{\mathrm{n}=1}^{8}(\mathrm{a}_\mathrm{n}\cos(\mathrm{n}\omega' \mathrm{t}) + \mathrm{b}_\mathrm{n}\sin(\mathrm{n}\omega' \mathrm{t})), \tag{3.1}$$

where ω'=2ω and the Fourier coefficients $\mathrm{a}_{0,}(\mathrm{a}_\mathrm{n}, \mathrm{b}_\mathrm{n})$ are given in Table 3.1.

$a_0 = 1 + \cos 2A'/4$	$b_1 = \sin 2A'/2$
$a_1 = 3/2 + \cos 2A'/2$	$b_2 = 3\sin 2A'/4$
$a_2 = 1 + 3\cos 2A'/4$	$b_3 = \sin 2A'$
$a_3 = 1/2 + \cos 2A'$	$b_4 = 3\sin 2A'/4$
$a_4 = 3\cos 2A'/4$	$b_5 = \sin 2A'/2$
$a_5 = \cos 2A'/2$	$b_6 = \sin 2A'/2$
$a_6 = \cos 2A'/4$	$b_7 = 0$
$a_7 = 0$	$b_8 = 0$
$a_8 = 0$	

Table 3.1. Fourier coefficients with straight-through configuration and simple case of fixed analyzer azimuth A'

We now consider the same straight-through configuration but introduce phase angles for the rotating polarizer and analyzer. So, let

$$P = \omega t + \beta \text{ and } A = 3\omega t + \alpha \tag{3.2}$$

while the fixed analyzer azimuth remains A'.

The final equation that we get by calculating $i = \underline{\Gamma}_{FA}\,\underline{\mathbf{M}}_A\,\underline{\mathbf{M}}_S\,\underline{\mathbf{M}}_P\,\underline{\mathbf{S}}_{FP}$ where $\underline{\mathbf{M}}_S$ is identity matrix, is given by,

$$i = [1+\cos(2P)+\cos(2A)\cos(2A')+\cos(2A)\cos(2P)\cos(2A')+\sin(2A)\sin(2A')+\sin(2A)\cos(2P)\sin(2A')]$$
$$[1+\cos(2A)\cos(2P)+\sin(2A)\sin(2P)] \tag{3.3}$$

Substitution of A and P from Eqn. (3.2) into (3.3) a Fourier series with coefficients

$(a_0, a_1 .. a_8, b_1 .. b_8)$ given in Table 3.2 is obtained.

$a_0 = 1+\frac{1}{4}[\cos(2A')]$

$a_1 = \frac{\cos(2A')}{2}[\cos 2\beta] + \frac{\sin(2A')}{2}[\sin 2\beta] + \frac{1}{2}[\cos 2\beta \cos 2(\alpha-\beta)+\sin 2\beta \sin 2(\alpha-\beta)] + \cos 2\beta$

$a_2 = \cos 2(\alpha-\beta) + \frac{\sin(2A')}{2}[\sin 2(\alpha-\beta)] + \frac{\cos(2A')}{2}[\cos 2(\alpha-\beta)] + \frac{\cos(2A')}{4}[\cos 2(\alpha+\beta)\cos 2(\alpha-\beta)+$
$\sin 2(\alpha+\beta)\sin 2(\alpha-\beta)] - \frac{\sin(2A')}{4}[\cos(\alpha-\beta)\sin 2(\alpha+\beta)-\cos 2(\alpha+\beta)\sin 2(\alpha-\beta)]$

$a_3 = \cos(2A')[\cos 2\alpha] + \sin(2A')[\sin 2\alpha] + \frac{1}{2}[\cos 2\beta \cos 2(\alpha-\beta)-\sin 2\beta \sin 2(\alpha-\beta)]$

$a_4 = \frac{\cos(2A')}{2}[\cos 2(\alpha+\beta)] + \frac{\sin(2A')}{2}[\sin 2(\alpha+\beta)] + \frac{\cos(2A')}{4}[\cos^2 2(\alpha-\beta)] + \frac{\sin(2A')}{4}[2\cos 2(\alpha-\beta)\sin 2(\alpha-\beta)]$

$a_5 = \frac{\cos(2A')}{2}[\cos 2\alpha \cos 2(\alpha-\beta)-\sin 2\alpha \sin 2(\alpha-\beta)] + \frac{\sin(2A')}{2}[\cos 2\alpha \sin 2(\alpha-\beta)+\sin 2\alpha \cos 2(\alpha-\beta)]$

$a_6 = \frac{\cos(2A')}{4}[\cos 2(\alpha-\beta)\cos 2(\alpha+\beta)-\sin 2(\alpha+\beta)\sin 2(\alpha-\beta)] + \frac{\sin(2A')}{4}[\cos 2(\alpha+\beta)\sin 2(\alpha-\beta) -$
$\sin 2(\alpha+\beta)\cos 2(\alpha-\beta)]$

$a_7 = 0$

$a_8 = 0$

$b_1 = -\sin 2\beta + \frac{1}{2}[\sin 2\beta \cos 2(\alpha-\beta)] + \frac{\cos(2A')}{2}[\cos 2\alpha \sin 2(\alpha-\beta)-\sin 2\alpha \cos 2(\alpha-\beta)] + \frac{\sin(2A')}{2}[\cos 2\alpha \cos 2(\alpha-\beta)$
$+\sin 2\alpha \sin 2(\alpha-\beta)]$

$b_2 = -\sin 2(\alpha-\beta) + \frac{\cos(2A')}{2}[-\sin 2(\alpha-\beta)] + \frac{\sin(2A')}{2}[\cos 2(\alpha-\beta)] + \frac{\cos(2A')}{4}[\cos 2(\alpha+\beta)\sin 2(\alpha-\beta)-$
$\sin 2(\alpha+\beta)\cos 2(\alpha-\beta)] + \frac{\sin(2A')}{4}[\cos 2(\alpha-\beta)\cos 2(\alpha+\beta)+\sin 2(\alpha-\beta)\sin 2(\alpha+\beta)]$

$b_3 = \sin(2A')\cos 2\alpha - \cos(2A')\sin 2\alpha - \frac{1}{2}[\cos 2\beta \sin 2(\alpha-\beta)+\sin 2\beta \cos 2(\alpha-\beta)]$

$b_4 = \frac{\sin(2A')}{2}[\cos 2(\alpha+\beta)] - \frac{\cos(2A')}{2}[\sin 2(\alpha+\beta)] - \frac{\cos(2A')}{4}[\sin 4(\alpha-\beta)] + \frac{\sin(2A')}{4}[\cos 4(\alpha-\beta)]$

$b_5 = \frac{\sin(2A')}{4}$ [2cos(4α-2β)]- $\frac{\cos(2A')}{4}$ [2sin(4α-2β)] $b_6 = \frac{\sin(2A')}{4}$ [cos(4α)]- $\frac{\cos(2A')}{4}$ [2sin4α] $b_7 = 0$ $b_8 = 0$

Table 3.2. Fourier coefficients with the imperfection parameters introduced namely (1) fixed analyzer offset azimuth A', (2) phase of rotating analyzer α and (3) phase of rotating polarizer β.

The above equations can be solved for the parameters A', α and β as follows:

$$4(a_0-1) = \cos(2A') \qquad (3.4)$$

The value of A' from Eqn. (3.4) is used in the following equation to determine α:

$$a_3+b_3 = [\cos 2\alpha - \sin 2\alpha][1/2 + \cos(2A')] + \sin(2A')[\sin 2\alpha + \cos 2\alpha] \qquad (3.5)$$

Finally the value of α recovered from Eqn. (3.5) is used in the following equation

$$a_6+b_6 = \frac{\cos(2A')}{4}[\cos 4\alpha \cos 4\beta] + \frac{\sin(2A')}{4}[\cos 4\alpha + \sin 4\alpha] \qquad (3.6)$$

to determine β.

Let us now analyze the arrangement taking into consideration the imperfections of the polarizers. We take out the rotating analyzer from the arrangement and consider the fixed elements as ideal elements. The Jones matrix of an ideal linear polarizer is

$$\underline{\mathbf{J}}_{\mathrm{FP}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{3.7}$$

and hence the first column for the Mueller matrix for the same linear polarizer will be

$$\underline{\mathbf{M}}_{\mathrm{FP}} = \frac{1}{2}[1\ 1\ 0\ 0]^{\mathrm{T}} \tag{3.8}$$

where the T represents the transpose of the matrix.

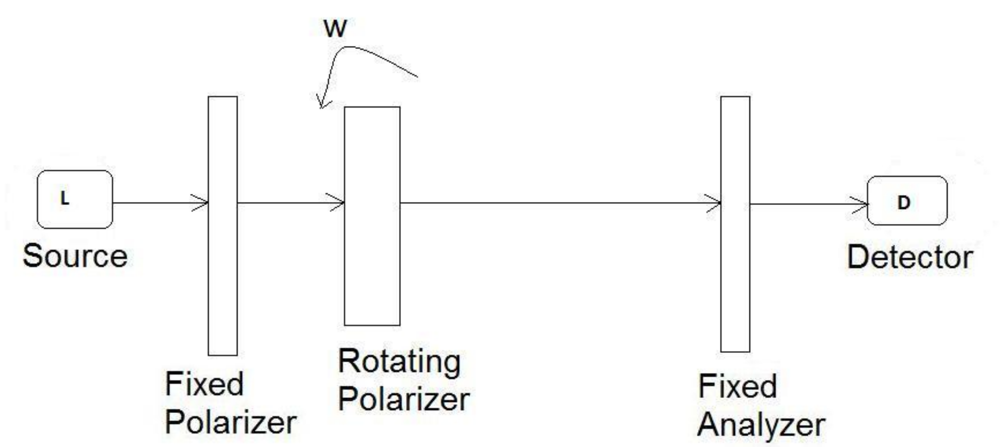

Fig 3.1. Arrangement without the rotating analyzer in the straight-through configuration.

In certain frequency bands a medium may exhibit both linear birefringence and linear dichroism concurrently. A linear dichoric retarder is one which has the optic axis of birefringence and dichroism coincident and parallel with the bounding planar faces. An imperfect polarizer can be modeled as one with such a property. For such a medium we replace the real index of refraction n by the complex index of refraction (n-jk) where k is the extinction coefficient of the medium.

The Jones matrix of the imperfect rotating polarizer can thus be written as

$$\underline{\mathbf{J}}_P = \begin{bmatrix} 1 & 0 \\ 0 & \rho e^{-j\delta} \end{bmatrix} \tag{3.9}$$

where

$$\delta = \frac{2\Pi d}{\lambda}(n_0 - n_e),\ \rho = \frac{\rho_1}{\rho_2} = \exp[-2\pi d(k_o - k_e)/\lambda]\ , \qquad \rho << 1 \tag{3.10}$$

where n_0 and n_e are the ordinary and extraordinary refractive indices of the medium and (k_e-k_o) is called the dichroism of the medium and may be positive or negative.

The Mueller matrix of the rotating polarizer can thus be written as

$$M_P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & \rho\cos\delta & \rho\sin\delta \\ 0 & 0 & -\rho\sin\delta & \rho\cos\delta \end{bmatrix} \tag{3.11}$$

This is for polarizer azimuth P = 0^0.

For P$\neq 0^0$, the Mueller matrix can be written as

$$M=\begin{bmatrix} \frac{1}{2} & a & b & 0 \\ a & x & m & -p \\ b & m & y & p \\ 0 & p & q & c \end{bmatrix} \qquad (3.12)$$

where,

$a=\frac{\cos 2P}{2}$, $b=\frac{\sin 2P}{2}$, $c=\rho\cos\delta$, $p=2b'd$, $q=-2a'd$ (3.13)

where a' and b' are the terms with rotating analyzer azimuth A instead of P

$m= 4ab(\frac{1}{2}-c)$, (3.14)

$x=\cos^2(2A)(\frac{1}{2}-c)+c$, (3.15)

$y=4ab(\frac{1}{2}-c)$ (3.16)

Now the output current is calculated as

$i=\Gamma_{FA} M S_{FP}$ and

Putting $P= \omega t + \beta$ we obtain,

$I= ½ + a + \cos(2A')(a+c) + b\sin(2A)' + \cos(2A')\cos^2(2P)(1/2 - c) + \sin(2A')4ab(1/2 - c)$ (3.15)

Putting this in Fourier series form we have,

$2a_0 = 1+\cos 2A'$	$b_1 = \frac{\sin 2A'}{2}\cos 2\beta - \frac{\sin 2\beta}{2}(1+\cos 2A')$
$a_1 = \frac{\cos 2\beta}{2}(1+\cos 2A') + \frac{\sin 2\beta}{2}(\sin 2A')$	$b_2 = \frac{\sin 2A'\cos 4\beta}{2}\left(\frac{1}{2} - \rho\cos\delta\right) - \frac{\cos 2A'\sin 4\beta}{2}\left(\frac{1}{2} - \rho\cos\delta\right)$
$a_2 = \frac{\cos 2A'\cos 4\beta}{2}\left(\frac{1}{2} - \rho\cos\delta\right) + \frac{\sin 2A'\sin 4\beta}{2}\left(\frac{1}{2} - \rho\cos\delta\right)$	

Table 3.3. Fourier coefficients in terms of imperfection parameters for the case of 'missing rotating analyzer'.

From Table 3.3 ρ and β can be calculated as:

$$2a_0 - 1 = \cos(2A') \tag{3.16}$$

$$b_1 = \frac{\sin(2A')}{2}\left(\frac{a_1}{a_0} - \frac{\sin(2A')\sin 2\beta}{2a_0}\right) - a_0\sin 2\beta \tag{3.17}$$

$$2b_2 = \left(\frac{1}{2} - \rho\cos\delta\right)\left(\sin(2A')\cos 4\beta - \cos(2A')\sin 4\beta\right) \tag{3.18}$$

The value of A' is calculated from Eqn. (3.18) and using that in Eqn. (3.19) the value of β can be obtained. Substitution of these values in Eqn. (3.20) gives the factor ρcosδ.

Now we take the more general case where we include the rotating analyzer in the straight-through configuration.

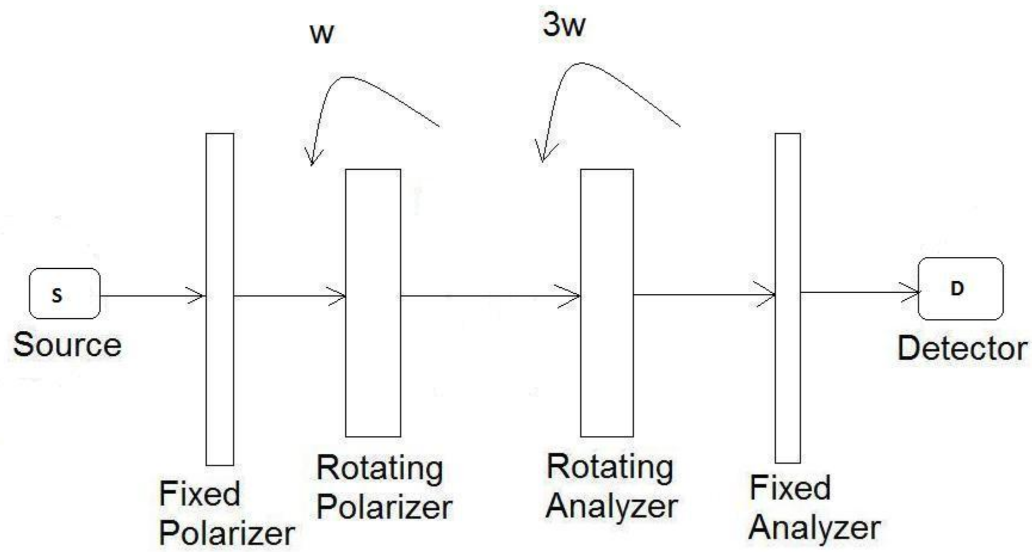

Fig 3.2. Straight-through arrangement with ideal fixed components and imperfections taken into account in the rotating elements.

The output current equation is given by,

$$i_D = \underline{\Gamma}_{FA}\,\underline{\mathbf{M}}_A\,\underline{\mathbf{I}}_S\,\underline{\mathbf{M}}_P\,\underline{\mathbf{S}}_{FP} \tag{3.21}$$

where I_s is the identity matrix. The rotating analyzer is assumed to be imperfect and the fixed analyzer offset is A'.

Putting the values of A=3ωt and P=ωt, we obtain the Fourier coefficients listed in Table 3.4.

$a_0 = \frac{1}{4} + c^2\cos(2A') + (\frac{1}{2} - c)\cos(2A')(c + \frac{1}{4}(\frac{1}{2} - c))$	$b_1 = \frac{\sin(2A')}{4} - (\frac{1}{2} - c)\frac{\sin(2A')}{4}$
$a_1 = \frac{\cos(2A')}{2}(\frac{1}{2} - c) + \frac{1}{4} + \frac{5}{32}(\frac{1}{2} - c)$	$b_2 = \frac{\sin(2A')}{8} - \frac{1}{2}(\frac{1}{2} - c)^2 \frac{\sin(2A')}{2} + \frac{\sin(2A')}{4}(\frac{1}{2} - c)$
$a_2 = \frac{\cos(2A')}{8} + \frac{1}{4} + \frac{1}{2}(\frac{1}{2} - c)c.\cos(2A') + \frac{1}{4}(\frac{1}{2} - c)^2\cos(2A') + \frac{d^2}{2}(\sin(2A') - \cos(2A'))$	
$a_3 = \frac{\cos(2A')}{4} + \frac{c}{2} + \frac{1}{4}(\frac{1}{2} - c)$	$b_3 = \frac{\sin(2A')}{4}$
$a_4 = \frac{\cos(2A')}{8} + (\frac{1}{2} - c)^2 \frac{\cos(2A')}{4} - \frac{d^2}{2}(\sin 2A' - \cos 2A')$	$b_4 = \frac{\sin(2A')}{8} + (\frac{1}{2} - c)^2 \frac{\sin(2A')}{2}$
$a_5 = \frac{3}{32}(\frac{1}{2} - c) - \frac{\cos(2A')}{8}(\frac{1}{2} - c)$	$b_5 = \frac{\sin(2A')}{8}(\frac{1}{2} - c)$
$a_6 = \frac{1}{2}(\frac{1}{2} - c)\cos(2A')(c + \frac{1}{2}(\frac{1}{2} - c))$	$b_6 = \frac{\sin(2A')}{4}(\frac{1}{2} - c)^2 + c.\frac{\sin(2A')}{8}(\frac{1}{2} - c)$
$a_7 = \frac{\cos(2A')}{8}(\frac{1}{2} - c)$	$b_7 = -\frac{\sin(2A')}{4}(\frac{1}{2} - c)$
$a_8 = 0$	$b_8 = 0$

Table 3.4. Dependency of the Fourier coefficients in terms of the parameters A', ρ and δ for the full straight-through configuration considering only the fixed polarizer azimuth to be 0°

From Table 3.4 ρ and A' are overdetermined as:

$$2\frac{b_5}{b_3} = (\frac{1}{2} - c) \tag{3.22}$$

$$4b_3 = \sin(2A') \tag{3.23}$$

The value of $(\frac{1}{2}-c)$ and sin2A' can now be used to get ρ and δ as

$$\cos(2A') = \frac{3b_5 - 16b_3a_5}{4b_5} \tag{3.24}$$

$$c = \rho\cos\delta = \frac{b_3 - 4b_5}{2b_3} \tag{3.25}$$

$$a_4 = \frac{3b_5 - 16b_3a_5}{16b_5}\left(\frac{1}{2} + 4\frac{{b_5}^2}{{b_3}^2}\right) - \frac{d^2}{2}\left(4b_3 + \frac{4b_3a_5}{b_5} - \frac{3}{4}\right) \tag{3.26}$$

The Eqns. (3.24) to (3.26) establish the relation between the imperfection parameters and Fourier coefficients of the detected signals. Thus we can determine each of the offset and imperfection parameters in terms of the measured Fourier coefficients. These parameters are then used to obtain the Mueller matrix elements as discussed in Chapter 4. If the experimental results to the Fourier series are available for the above discussed cases, the solution of the introduced unknowns can be determined accommodating all the equations using the non-linear least square method.

CHAPTER 4

DETERMINATION OF MUELLER MATRIX ELEMENTS

4.1 Introduction

After the determination of the offset and imperfection parameters from the straight-through configuration as described in Chapter 3 we now reintroduce the sample between the rotating polarizer and rotating analyzer. The sample's Mueller matrix elements are m_{ij} (i,j=0,..,3).

We derive the equation of the detector signal and from there Mueller matrix elements in terms of combinations of Fourier coefficients and the imperfection parameters introduced.

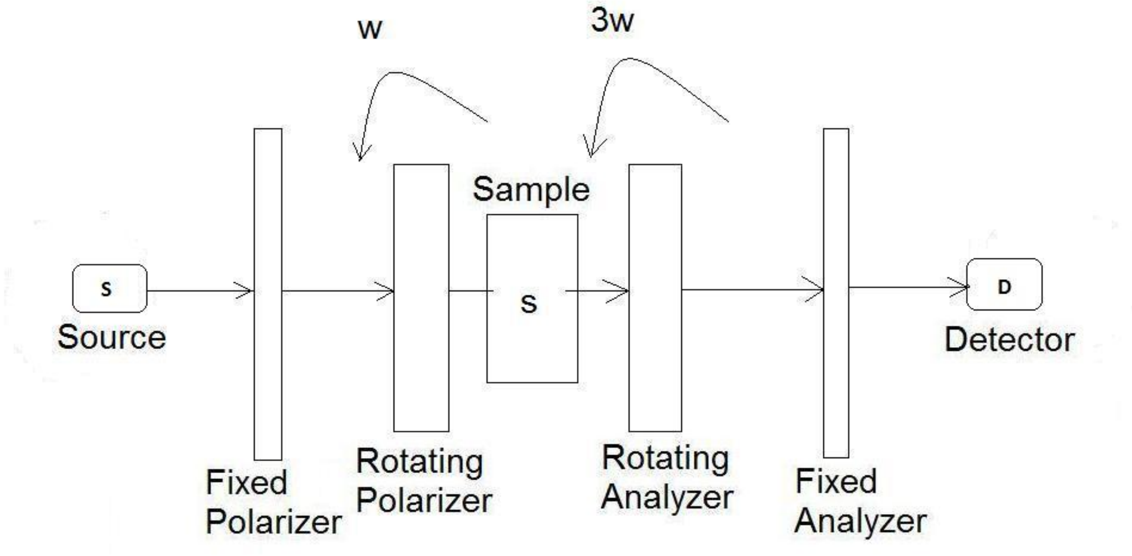

Fig 4.1. Full arrangement where the sample is introduced between the rotating components and all the imperfection parameters are considered.

4.2 Fourier coefficients in terms of imperfection parameters

The Fourier coefficients are shown below in terms of the Mueller matrix elements and the parameters A', c and d where,

$$c=\rho os\delta \tag{4.1}$$

$$d=\rho \sin\delta \tag{4.2}$$

a_0=m_{00}+m_{10}[$\frac{\cos 2A'}{16}$($\frac{1}{2}$-c)+$\frac{c}{2}$.cos2A']+m_{20} $\frac{\sin 2A'}{4}$($\frac{7}{8}$+$\frac{c}{4}$)+m_{01}[($\frac{1}{2}$-c)+$\frac{c}{2}$]+m_{11}[$\frac{3\cos 2A'}{128}$ ($\frac{1}{2} - c)^2$+ c.$\frac{\cos 2A'}{4}$($\frac{1}{2}$-c)+

c^2cos2A']+m_{21}[$\frac{\sin 2A'}{16}$($\frac{1}{2}$-c)-$\frac{\sin 2A'}{128}$ ($\frac{1}{2} - c)^2$+c.$\frac{\sin 2A'}{2}$($\frac{7}{8}$+$\frac{c}{4}$)]

a_1=m_{10}[$\frac{\cos 2A'}{16}$($\frac{1}{2}$-c)+c.$\frac{\cos 2A'}{2}$]-m_{20} $\frac{\sin 2A'}{16}$($\frac{1}{2}$-c)+$\frac{m_{01}}{4}$+m_{11}[$\frac{\cos 2A'}{16}$($\frac{1}{2}$-c)+c.$\frac{\cos 2A'}{2}$]+m_{21} $\frac{\sin 2A'}{4}$+m_{02} $\frac{\sin 2A'}{4}$($\frac{1}{2}$-c)+

m_{12} $\frac{\sin 2A'}{8}$($\frac{1}{2}$-c)+m_{22}[$\frac{\cos 2A'}{8}$($\frac{1}{2}$-c)+($\frac{1}{2}$-c)]+m_{32}.d.(sin2A'-cos2A')($\frac{1}{2}$-c)+m_{23} $\frac{\cos 2A'}{32}$.d.($\frac{1}{2}$-c)-m_{21} $\frac{\sin 2A'}{16}$($\frac{1}{2}$-c)

a_2=m_{00}[$\frac{1}{4}$+$\frac{\cos 2A'}{8}$]+$\frac{m_{10}}{8}$+m_{01} $\frac{\cos 2A'}{8}$+$\frac{m_{11}}{8}$+m_{02} $\frac{\sin 2A'}{8}$+$\frac{m_{22}}{8}$+m_{32}.$\frac{d}{4}$(sin2A'-cos2A')+m_{03}.d.$\frac{\sin 2A'}{4}$+m_{23}.$\frac{d}{4}$+

m_{33}.$\frac{d^2}{2}$(sin2A'-cos2A')

a_3=m_{00} $\frac{\cos 2A'}{4}$+$\frac{m_{10}}{4}$+m_{20} $\frac{\sin 2A'}{4}$+m_{01}[$\frac{3os 2A'}{32}$($\frac{1}{2}$-c)+c.$\frac{\cos 2A'}{2}$]+m_{11}[($\frac{1}{2}$-c).$\frac{3}{32}$+$\frac{c}{2}$]

a_4=m_{00} $\frac{\cos 2A'}{8}$+$\frac{m_{10}}{8}$+m_{01} $\frac{\cos 2A'}{8}$+$\frac{m_{11}}{8}$-m_{02} $\frac{\sin 2A'}{8}$+m_{12} $\frac{\sin 2A'}{8}$ ($\frac{1}{2} - c)^2$+m_{22}[$\frac{\cos 2A'}{8}$ ($\frac{1}{2} - c)^2$-$\frac{1}{8}$]-

m_{32}.$\frac{d}{4}$(sin2A'-cos2A')-m_{03} $\frac{\sin 2A'}{8}$.d-m_{23}.$\frac{d}{4}$-m_{33}.$\frac{d^2}{2}$(sin2A'-cos2A')

a_5=m_{10} $\frac{\cos 2A'}{32}$($\frac{1}{2}$-c)-m_{20} $\frac{\sin 2A'}{32}$($\frac{1}{2}$-c)+m_{11} $\frac{\cos 2A'}{32}$($\frac{1}{2}$-c)-m_{21} $\frac{\sin 2A'}{32}$($\frac{1}{2}$-c)-m_{02} $\frac{\sin 2A'}{4}$($\frac{1}{2}$-c)-m_{12} $\frac{\sin 2A'}{8}$($\frac{1}{2}$-c)+

$m_{22}[\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})\text{-}\frac{1}{4}(\frac{1}{2}\text{-c})]\text{+}\frac{m_{32}}{4}(\frac{1}{2}\text{-c})\text{-}m_{23}.\text{d}.\frac{\cos 2A'}{32}(\frac{1}{2}\text{-c})$

$a_6\text{=}m_{10}\,\frac{\cos 2A'}{16}(\frac{1}{2}\text{-c})\text{-}m_{20}\,\frac{\sin 2A'}{16}(\frac{1}{2}\text{-c})\text{+}\frac{m_{01}}{16}(\frac{1}{2}\text{-c})\text{+}m_{11}[\frac{\cos 2A'}{32}\,(\frac{1}{2}-c)^2\text{+c}.\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})\text{+c}.\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})]\text{+}$

$m_{21}[\frac{\sin 2A'}{16}(\frac{1}{2}\text{-c})\text{-}\frac{\sin 2A'}{32}\,(\frac{1}{2}-c)^2\text{-c}.\frac{\sin 2A'}{8}(\frac{1}{2}\text{-c})]$

$a_7\text{=}\frac{\cos 2A'}{32}(\frac{1}{2}\text{-c})[m_{10}\text{+}m_{11}]\text{-}\frac{\sin 2A'}{32}(\frac{1}{2}\text{-c})[m_{20}\text{+}m_{21}]$

$a_8\text{= -}m_{12}\,\frac{\sin 2A'}{8}\,(\frac{1}{2}-c)^2\text{-}m_{22}\,\frac{\cos 2A'}{8}\,(\frac{1}{2}-c)^2$

$a_9\text{=}m_{01}\,\frac{\cos 2A'}{32}(\frac{1}{2}\text{-c})\text{+}\frac{m_{11}}{16}(\frac{1}{2}\text{-c})$

$a_{10}\text{=0}$

$a_{11}\text{=0}$

$a_{12}\text{=}m_{11}\,\frac{\cos 2A'}{128}\,(\frac{1}{2}-c)^2\text{-}m_{21}\,\frac{\sin 2A'}{128}\,(\frac{1}{2}-c)^2$

$b_1\text{=}m_{02}[\frac{1}{4}\text{-}\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})]\text{+}m_{12}[\frac{\cos 2A'}{16}(\frac{1}{2}\text{-c})\text{+c}.\frac{\cos 2A'}{2}\text{-}\frac{1}{8}(\frac{1}{2}\text{-c})]\text{+}m_{22}[\frac{\sin 2A'}{4}\text{-}\frac{\sin 2A'}{16}(\frac{1}{2}\text{-c})]\text{+}m_{03}.\frac{d}{2}\text{+}m_{13}\,\frac{\cos 2A'}{16}(\frac{1}{2}\text{-c}).\text{d+}$

$m_{23}[\text{d}.\frac{\sin 2A'}{2}\text{-}\frac{\sin 2A'}{8}(\frac{1}{2}\text{-c}).\text{d}]$

$b_2\text{=}m_{00}\,\frac{\sin 2A'}{8}\text{-}m_{20}\,\frac{\sin 2A'}{16}(\frac{1}{2}\text{-c})\text{+}m_{30}.\frac{d}{4}(\sin 2A'\text{-}\cos 2A')\text{+}m_{01}\,\frac{\sin 2A'}{8}\text{+}\frac{m_{21}}{8}\text{+}m_{02}[\frac{1}{4}(\frac{1}{2}\text{-c})\text{-}\frac{\cos 2A'}{8}]\text{+}$

$m_{12}[\frac{\cos 2A'}{16}(\frac{1}{2}\text{-c}).\text{c+}\frac{\cos 2A'}{16}\,(\frac{1}{2}-c)^2\text{-}\frac{1}{8}]\text{+}m_{22}[\frac{\sin 2A'}{4}(\frac{1}{2}\text{-c})\text{-}\frac{\sin 2A'}{16}\,(\frac{1}{2}-c)^2]\text{-}m_{03}\text{d}.\frac{\cos 2A'}{4}\text{-}m_{13}.\frac{d}{4}\text{+}$

$m_{31}.\frac{d}{4}(\sin 2A'\text{-}\cos 2A')$

$b_3\text{=}m_{00}\,\frac{\sin 2A'}{4}\text{+}m_{30}(\sin 2A'\text{-}\cos 2A').\frac{d}{8}\text{+}m_{01}[\frac{\sin 2A'}{32}(\frac{1}{2}\text{-c})\text{+}\frac{\sin 2A'}{2}.\text{c}]\text{+}m_{21}[\frac{1}{32}(\frac{1}{2}\text{-c})\text{+}\frac{c}{2}]\text{+}\frac{m_{20}}{4}\text{+}$

$m_{31}(\sin 2A'\text{-}\cos 2A')[(\frac{1}{2}\text{-c}).\frac{d}{2}\text{+d}.\frac{c}{2}]$

$b_4\text{=}m_{00}\,\frac{\sin 2A'}{8}\text{+}\frac{m_{20}}{8}\text{+}m_{30}(\sin 2A'\text{-}\cos 2A').\frac{d}{4}\text{+}m_{01}\,\frac{\sin 2A'}{8}\text{+}\frac{m_{21}}{8}\text{+}m_{31}(\sin 2A'\text{-}\cos 2A').\frac{d}{4}\text{+}m_{02}\,\frac{\cos 2A'}{8}\text{+}$

$m_{12}[\frac{1}{8}\text{-}\frac{\cos 2A'}{32}\,(\frac{1}{2}-c)^2]\text{+}m_{22}\,\frac{\sin 2A'}{16}\,(\frac{1}{2}-c)^2\text{+}m_{03}\,\frac{\cos 2A'}{4}.\text{d+}m_{13}.\frac{d}{4}$

$b_5\text{=}m_{10}\,\frac{\sin 2A'}{8}(\frac{1}{2}\text{-c})\text{+}m_{20}\,\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})\text{+}m_{11}\,\frac{\sin 2A'}{8}(\frac{1}{2}\text{-c})\text{+}m_{21}\,\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})\text{+}m_{02}\,\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})\text{+}$

$m_{12}[\frac{1}{8}(\frac{1}{2}\text{-c})\text{-}\frac{\cos 2A'}{32}(\frac{1}{2}\text{-c})]\text{+}m_{22}\,\frac{\sin 2A'}{32}(\frac{1}{2}\text{-c})\text{-}m_{13}\,\frac{\cos 2A'}{32}..\text{d}.(\frac{1}{2}\text{-c})\text{+}m_{23}\,\frac{\sin 2A'}{16}.\text{d}.(\frac{1}{2}\text{-c})$

$$b_6=m_{10}\frac{\sin 2A'}{4}(\frac{1}{2}\text{-c})+m_{20}\frac{\cos 2A'}{4}(\frac{1}{2}\text{-c})+m_{11}[\frac{\sin 2A'}{64}(\frac{1}{2}-c)^2+\frac{\sin 2A'}{2}(\frac{1}{2}\text{-c}).c]+m_{21}[\frac{\cos 2A'}{16}(\frac{1}{2}-c)^2+\frac{\cos 2A'}{2}(\frac{1}{2}\text{-c}).c]$$

$$b_7=\frac{\sin 2A'}{8}(\frac{1}{2}\text{-c})[m_{10}+m_{11}]+\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})[m_{20}+m_{21}]+\frac{\cos 2A'}{32}(\frac{1}{2}\text{-c})\ [m_{12}+d.m_{13}]-\frac{\sin 2A'}{32}(\frac{1}{2}\text{-c})[m_{22}+d.m_{23}]$$

$$b_8=m_{12}\frac{\cos 2A'}{32}(\frac{1}{2}-c)^2\text{-}m_{22}\frac{\sin 2A'}{32}(\frac{1}{2}-c)^2$$

$$b_9=m_{01}\frac{\sin 2A'}{32}(\frac{1}{2}\text{-c})+\frac{m_{21}}{32}(\frac{1}{2}\text{-c})+m_{31}\frac{d}{16}(\frac{1}{2}\text{-c})(\sin 2A'\text{-}\cos 2A')$$

$$b_{10}=0$$

$$b_{11}=0$$

$$b_{12}=m_{11}\frac{\sin 2A'}{128}(\frac{1}{2}-c)^2+m_{21}\frac{\cos 2A'}{128}(\frac{1}{2}-c)^2$$

Table 4.1. Relation between Fourier coefficients and Mueller matrix elements in terms of parameters A', ρ and δ where the sample is reintroduced

From the Fourier coefficients are the Mueller-matrix elements are determined. Note that the coefficients of the frequency of 20ωt and 22ωt are equal to zero, i.e., a_9, a_{10}, b_9 and b_{10} are equal to zero.

4.3 Calculation of Mueller-matrix elements

The solutions of the Mueller matrix elements are determined as given below:

$$m_{21}=\frac{64(b_{12}-a_{12}\tan 2A')}{(\sin 2A'\tan 2A'+\cos 2A')(\frac{1}{2}-c)^2.\frac{1}{2}} \tag{4.3}$$

$$m_{11}=\frac{a_{12}+\frac{\sin 2A'(b_{12}-a_{12}\tan 2A')}{\sin 2A'\tan 2A'+\cos 2A'}}{\frac{\cos 2A'}{4}(\frac{1}{2}-c)^2.\frac{1}{2}} \tag{4.4}$$

$$m_{22}=\frac{a_8+2\tan 2A'.b_8}{\frac{1}{16}\left(\frac{1}{2}-c\right)^2.\frac{1}{2}(\sin 2A'\tan 2A'-\cos 2A')} \tag{4.5}$$

$$m_{12}=\frac{b_8+\frac{\sin 2A'(a_8+2\tan 2A'\,b_8)}{\sin 2A'\tan 2A'-\cos 2A'}}{\frac{\cos 2A'}{16}(\frac{1}{2}-c)^2.\frac{1}{2}} \tag{4.6}$$

$$m_{01}=\frac{a_9-\frac{a_{12}+(b_{12}-a_{12}\tan 2A')\tan 2A'}{\frac{1}{8}\left(\frac{1}{2}-c\right)(\sin 2A'\tan 2A'+\cos 2A')}}{\frac{\cos 2A'}{16}\left(\frac{1}{2}-c\right).\frac{1}{2}} \tag{4.7}$$

By substitution of Eqn. (4.3) and Eqn. (4.4) in

$$a_7=\frac{\cos 2A'}{16}(\frac{1}{2}\text{-c}).\frac{1}{2}[m_{10}+m_{11}] - \frac{\sin 2A'}{16}(\frac{1}{2}\text{-c}).\frac{1}{2}[m_{20}+m_{21}] \tag{4.8}$$

and the values of Eqn. (4.3) and Eqn. (4.4) again in

$$a_6=m_{10}\frac{\cos 2A'}{16}(\frac{1}{2}\text{-c})\text{-}m_{20}\frac{\sin 2A'}{16}(\frac{1}{2}\text{-c})+\frac{m_{01}}{16}(\frac{1}{2}\text{-c})+m_{11}[\frac{\cos 2A'}{32}(\frac{1}{2}-c)^2+c.\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})+c.\frac{\cos 2A'}{8}(\frac{1}{2}\text{-c})]+$$
$$m_{21}[\frac{\sin 2A'}{16}(\frac{1}{2}\text{-c})\text{-}\frac{\sin 2A'}{32}(\frac{1}{2}-c)^2\text{-c.}\frac{\sin 2A'}{8}(\frac{1}{2}\text{-c})] \tag{4.9}$$

we obtain the values of m_{10} and m_{20}.

Further the value of m_{00} can now be calculated from the equation:

$$a_3=m_{00}\frac{\cos 2A'}{4}+\frac{m_{10}}{4}+m_{20}\frac{\sin 2A'}{4}+m_{01}[\frac{3\cos 2A'}{32}(\frac{1}{2}\text{-c})+c.\frac{\cos 2A'}{2}]+m_{11}[\frac{3}{32}(\frac{1}{2}\text{-c})+\frac{c}{2}] \tag{4.10}$$

where the only unknown is m_{00}.

Further from

$$b_9= m_{01}\frac{\sin 2A'}{32}(\frac{1}{2}\text{-c})+\frac{m_{21}}{32}(\frac{1}{2}\text{-c})+m_{31}(\sin 2A'-\cos 2A').\frac{d}{16}.(\frac{1}{2}\text{-c}) \tag{4.11}$$

m_{31} is determined as the other Mueller-matrix elements are already known.

Also since the equation for b_3 in Table 4.1 has only m_{30} as the only remaining unknown, it can be easily determined as well.

Let us now write the rest of the equations in terms of the remaining unknown parameters.

$$b_1 = f(m_{02}, m_{03}, m_{13}, m_{23}) \quad (4.12)$$

$$b_2 = f(m_{02}, m_{03}, m_{13}) \quad (4.13)$$

$$b_4 = f(m_{02}, m_{03}, m_{13}) \quad (4.14)$$

$$b_5 = f(m_{02}, m_{13}, m_{23}) \quad (4.15)$$

$$a_1 = f(m_{02}, m_{32}, m_{23}) \quad (4.16)$$

$$a_2 = f(m_{02}, m_{32}, m_{03}, m_{23}, m_{33}) \quad (4.17)$$

$$a_4 = f(m_{02}, m_{32}, m_{03}, m_{23}, m_{33}) \quad (4.18)$$

$$a_5 = f(m_{02}, m_{32}, m_{23}) \quad (4.19)$$

In equations (4.12), (4.14), (4.15), (4.16) and (4.19) we have 5 unknowns of m_{02}, m_{32}, m_{23}, m_{03}, m_{13}. These can be evaluated using simple algebraic calculations or using determinant method. Using the above solutions and equation (4.17) or (4.18) we can determine m_{33} as well.

Thus resulting Mueller matrix parameters are thus determined in terms of the Fourier coefficients and parameters introduced.

V SUMMARY

Mueller matrix has important practical applications. One of them is to determine the change in properties of a sample when placed in a polarizer-sample-analyzer (PSA) arrangement. The general PSA arrangement is modified in this case by adding fixed polarizer and analyzer at both ends of the optical train. Simple analysis of this structure gives us the 3x3 block of Mueller matrix of the sample when every optical device is considered to be ideal.

Deviations from ideality are accounted for through the introduction of the azimuthal offsets and component imperfections. Operation in the straight-through configuration permits the measurement of these imperfections and offsets from Fourier analysis of the detected signal. This gives us an estimate of imperfection parameters and their effects in deviation from the ideal case.

Finally, when the sample is reintroduced and the full Mueller matrix elements are obtained in terms of the imperfection parameters and the Fourier coefficients of the detected signal.

VI FUTURE WORK

The work presented here is theoretical and remains to be applied to an actual laboratory instrument. The arrangement if realized in a laboratory can be useful in the determination of the Mueller matrix elements of the sample in the presence of component imperfections and thus will help us formulate an error analysis that links theoretical and experimental results.

The error analysis can also further be developed by using the systematic error-propagation method in which the calculation is done by evaluating the Jacobian matrix that relates errors δm_{ij} of the Mueller-matrix elements of the sample to imperfection parameters that propagate through the Fourier coefficients δa_o, δa_{2n}, and δb_{2n}. A similar work was done using rotating compensators showing the influence of each optical element on the acquired parameters [6].

The determination of Mueller matrix elements in practical laboratory experiments can be used to analyze the sample in between for various factors such as change in properties and can be helpful in various medicinal purposes such as defining the interconnection between geometry of the biological tissue and their polarization properties.

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VITA

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