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Creation, Verification, and Validation of a Panel Code for the Analysis of Ship Propellers in a Steady, Uniform Wake

Stephen Gregory Jennings
University of New Orleans

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Creation, Verification, and Validation of a Panel Code for the Analysis of Ship Propellers in a Steady, Uniform Wake

A Thesis

Submitted to the Graduate Faculty of the University of New Orleans in partial fulfillment of the requirements for the degree of

Master of Science in Naval Architecture and Marine Engineering

by

Stephen Gregory Jennings B.S., University of New Orleans, 2006

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First, I would like to thank my wife Katie for her unwaivering support during my long nights at the computer.

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</table>
\( \Delta \phi \) Vector of perturbation potential differences around the trailing edge
\( \Delta V \) Vector of velocity magnitude differences around the trailing edge
\( BP(r', e, k) \) Location of blade pressure face surface
\( BS(r', e, k) \) Location of blade suction face surface
\( c \) Unit vector in the \( Pn - cs \) coordinate system described in the \( P - xyz \) coordinate system
\( MC(r', k) \) Location of blade mid-chord line
\( n_Q \) Surface normal vector at point \( Q \)
\( NT(r', k, e) \) Location of blade nose-tail line
\( n \) Surface normal vector
\( o \) Unit vector of the \( Pn - op \) Cartesian coordinate system described in the \( P - xyz \) coordinate system
\( P(x, y, z) \) Arbitrary field point
\( p \) Unit vector of the \( Pn - op \) Cartesian coordinate system described in the \( P - xyz \) coordinate system
\( Q \) Vector locating a point on the surface \( S \)
\( q_0, q_1, q_2, \& q_3 \) Coefficients of panel shape function
\( q_{++, q_{+-}, q_{--}, \& q_{-+}} \) Panel corner locations
\( R_{ijk} \) Vector pointing from influenced panel \( i \) to influencing panel \( j \) on blade \( k \)
\( R = Q - P \)
\( s \) Unit vector in the \( Pn - cs \) coordinate system described in the \( P - xyz \) coordinate system
\( V_{\text{inf}} \) Free stream velocity
\( V_1 \) Incident velocity on the propeller blade surface
\( a \) Coefficient of \( \phi_c(g) \) or \( \phi_s(g) \)
\( b \) Coefficient of \( \phi_c(g) \) or \( \phi_s(g) \)
$B_{ij}$ Influencing potential on panel $i$ from source distribution on panel(s) $j$

c Coefficient of $\phi_c(g)$ or $\phi_s(g)$

$C(r')$ Blade chord as a function of $r'$

$C_P$ Pressure coefficient

$C_{ij}$ Influencing potential on panel $i$ from doublet distribution on panel(s) $j$

$E$ Constant used in (2.8)

e Non-dimensional, unit chord, such that 0 is the leading edge and 1 is the trailing edge

$e_{\text{max}}(r')$ Location where the maximum camber occurs along the nose-tail line, function of $r'$

$f$ Variable in (3.3)

g Distance along the blade surface

$g_{++}$ Distance from the center of the panel of interest to panel $m + 2$

$g_\pm$ Linear distance between adjacent panel centers within a given panel strip in the direction of increasing or decreasing panel index depending on the subscript

$i$ Panel index

$j$ Panel index

$K$ Number of blades

$k$ Blade number 1, 2, $\ldots$, $K$

$L$ Number of streamwise wake panels

$l$ Index of streamwise wake panels

$m$ panel index

$N$ Number of panels on each propeller blade and the associated portion of the hub

$N_C$ Number of blade panels in the chordwise direction (streamwise for sphere)
$N_S$  
Number of blade panels in the spanwise direction (circumferential for sphere)

$P - xr\theta$  
Propeller cylindrical coordinate system

$P - xyz$  
Propeller cartesian coordinate system

$P_I$  
Pressure at the propeller center

$Pn - cs$  
Panel coordinate system

$Pn - op$  
Panel Cartesian coordinate system

$Q_W$  
Point on wake surface

$R$  
Propeller radius

$r$  
Radial distance from center of sphere

$r$  
Radial distance from x-axis in $P - xr\theta$

$r'$  
non-dimensional propeller radius

$R_H$  
Propeller hub radius

$R_{sphere}$  
Sphere radius

$S$  
Combined blade, hub, and wake surfaces

$s_1$  
Panel geometry parameter used in Equations (3.14) & (3.15)

$s_2$  
Panel geometry parameter used in Equations (3.14) & (3.15)

$s_3$  
Panel geometry parameter used in Equations (3.14) & (3.15)

$S_B$  
Propeller blade surface

$S_H$  
Propeller hub surface

$S_j$  
Surface area of panel $j$

$S_W$  
Propeller wake surface

$SLAE_1$  
First system of linear algebraic equations

$SLAE_2$  
Second system of linear algebraic equations

$u$  
Spanwise panel index (circumferential for sphere)

$U_\infty$  
Velocity at infinity
<table>
<thead>
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<th>Variable</th>
<th>Description</th>
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<tr>
<td>$v$</td>
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<tr>
<td>$v_o$</td>
<td>Perturbation velocity in the panel $o$ direction</td>
</tr>
<tr>
<td>$v_p$</td>
<td>Perturbation velocity in the panel $p$ direction</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Perturbation velocity in the panel spanwise direction</td>
</tr>
<tr>
<td>$V_{\pm j}$</td>
<td>Velocity magnitude at the center of the suction(+) or pressure(-) side trailing edge panels of the $j$th spanwise strip</td>
</tr>
<tr>
<td>$W_{ij\text{TE}}$</td>
<td>Influencing potential on panel $i$ from doublet distribution on wake panel(s) $j$</td>
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<tr>
<td>$z$</td>
<td>Distance along the $z$-axis in $P-xyz$</td>
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</tbody>
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Abstract

This report describes the governing equation and boundary conditions for a marine propeller operating in a uniform flow field of inviscid and irrotational fluid. A method is presented by which the velocity and pressure on the blade surface of the propeller can be numerically simulated, using hyperboloidal, constant strength source and doublet panels. Accuracy of the numerical method is verified through comparison with analytically known results and the ability of the numerical simulation to predict the thrust and torque on a propeller in open water is assessed through comparison with published experimental results. The thrust and torque results for the propeller are near the experimental measurements but do not converge to a common value as the panel size decreases.

Keywords: Propeller, Marine, Hyperboloidal, Panel, Source, Doublet, Inviscid, Open-water, Thrust, Torque,
1 Introduction

The design of a ship’s propeller can be described as an optimization problem with the objective of maximizing efficiency, and constrained by the limitations of strength and often cavitation. Systematic series have long been used to size propellers in early phase ship design, however their applicability is limited to the range of sizes and shapes which constitute the series, precluding the use of systematic series in the design of new and novel propeller concepts. Various classes of numerical methods exist for solving the flow around a propeller, each with their own set of advantages and disadvantages. Lifting surface methods have been shown to predict thrust and torque with a reasonable degree of accuracy [8]. Lifting surface methods however, do not model the thickness of the blade and thus cannot be used to analyze the effect of blade section shape on thrust, torque, or cavitation. This inability to evaluate the effects of different blade section shapes leaves lifting surface methods unable to evaluate the effects of structural constraints, which manifest themselves as constraints on blade section thickness. Finite volume or other methods used to solve the Navier-Stokes equations have the ability to accurately predict the hydrodynamic quantities needed in the design of a marine propeller and can evaluate the effects of different blade section shapes [12]. However, resource limitations preclude their use in an optimization routine which will require frequent and numerous evaluations of different geometry and boundary conditions. Boundary element methods which solve the Laplace equation for the inviscid and irrotational flow around the true geometry of the propeller blade, have the ability to evaluate differences in blade section shape and the computational resources needed to execute the method can be met by the average personal computer. This report will discuss one such boundary element method, adapted from Hoshino [2], and evaluate its ability to predict the hydrodynamic quantities needed in a propeller design. It is the hope of the author that this method may be later used as a hydrodynamic analysis tool in the optimization of a marine propeller, where the hydrodynamic and structural properties of the blade are simultaneously evaluated.
2 The Propeller Flow Problem

2.1 Propeller Geometry and Coordinate System

We define the propeller Cartesian coordinate system \( P-xyz \) such that the \( x \)-axis is concentric with the shaft axis and positive downstream, the \( z \)-axis extends upward from the shaft centerline bisecting the nose-tail line of a blade section at the propeller hub, and the \( y \)-axis is such that we have a right handed coordinate system. It will be convenient to define a cylindrical propeller coordinate system \( P-xr\theta \) such that the \( x \)-axis is the same as that of \( P-xyz \) but where \( r \) is the radial distance from the \( x \)-axis and \( \theta \) increases moving clockwise from the \( z \)-axis, looking downstream. The cylindrical coordinate system can be transformed into the Cartesian system \( P-xyz \) using the following relations:

\[
\begin{align*}
x &= x \quad \text{(2.1)} \\
y &= -r \sin \theta \quad \text{(2.2)} \\
z &= r \cos \theta \quad \text{(2.3)}
\end{align*}
\]

The midchord line at any radius can be located using the following relations:

\[
\mathbf{MC}(r', k) = \begin{bmatrix} MC_x \\ MC_y \\ MC_z \end{bmatrix} = \begin{bmatrix} \chi(r') \\ -r \sin (\xi(r') + \theta_k(k)) \\ r \cos (\xi(r') + \theta_k(k)) \end{bmatrix}
\quad \text{(2.4)}
\]

where:

\[
r' = \frac{r - R_H}{R - R_H}
\]
\( R \) is the propeller radius
\( R_H \) is the propeller hub radius
\( \theta_k(k) \) is the angular location of the blade midchord at the hub of blade \( k \), with respect to the \( z \)-axis
\( k \) is the blade number \( 1, 2, \ldots, K \)
\( K \) is the number of blades
\( \chi(r') \) is the blade rake as a function of \( r' \)
\( \xi(r') \) is the blade skew as a function of \( r' \)
Figure 2.1: Propeller Cartesian and cylindrical coordinate systems (looking downstream)
The nose-tail line can be located using the following relationship:

\[
\mathbf{NT}(r', k, e) = \begin{bmatrix} NT_x \\ NT_y \\ NT_z \end{bmatrix} = \begin{bmatrix} \chi(r') + \left( eC(r') - \frac{C(r')}{2r} \right) \sin \gamma(r') \\ -r \sin \left( \xi(r') + \theta_k(k) + \left( \frac{eC(c')}{r} - \frac{C(c')}{2r} \right) \cos \gamma(r') \right) \\ r \cos \left( \xi(r') + \theta_k(k) + \left( \frac{eC(c')}{r} - \frac{C(c')}{2r} \right) \cos \gamma(r') \right) \end{bmatrix} \tag{2.5}
\]

where:

- \( C(r') \) is the blade chord length as a function of \( r' \)
- \( e \) is the non-dimensional, unit chord, such that 0 is the leading edge and 1 is the trailing edge
- \( \gamma(r') \) is the blade pitch angle as a function of \( r' \)

It should be noted that for a propeller the chord \( C(r') \) is the length between a leading edge point and a trailing edge point along a plane of constant radius, thus \( C \) is the length of an arc measured along a helix. Also, here the blade pitch angle \( \gamma \) is the geometric pitch angle measured along the nose-tail line of the blade section.

The blade section shape at any radius can be described by a half-thickness shape function \( \tau(\tau_{max}, e) \) and a camber shape function \( \epsilon(\epsilon_{max}, \epsilon_{max}, e) \), where:

- \( \tau_{max} \) is the maximum thickness of the section and is a function of \( r' \)
- \( \epsilon_{max} \) is the maximum camber of the section and is a function of \( r' \)
- \( \epsilon_{max} \) is the location where the maximum camber thickness occurs along the nose-tail line and is also a function of \( r' \)

These section shape functions are analogous to those used to describe foil section series, like the NACA series. Using them we can locate the blade surface with the following equations:

\[
\mathbf{BS}(r', e, k) = \begin{bmatrix} NT_x(r', e, k) - \tau \sin \alpha \sin \gamma - \epsilon \cos \gamma - \tau \cos \alpha \cos \gamma \\ -r \sin \left( \xi + \theta_k + \left( \frac{eC}{r} - \frac{C}{2r} \right) \cos \gamma - \frac{\tau \sin \alpha}{r} \cos \gamma + \frac{\epsilon \sin \gamma + \frac{\tau \cos \alpha}{r} \sin \gamma}{r} \right) \\ r \cos \left( \xi + \theta_k + \left( \frac{eC}{r} - \frac{C}{2r} \right) \cos \gamma - \frac{\tau \sin \alpha}{r} \cos \gamma + \frac{\epsilon \sin \gamma + \frac{\tau \cos \alpha}{r} \sin \gamma}{r} \right) \end{bmatrix} \tag{2.6}
\]

\[
\mathbf{BP}(r', e, k) = \begin{bmatrix} NT_x(r', e, k) - \tau \sin \alpha \sin \gamma - \epsilon \cos \gamma + \tau \cos \alpha \cos \gamma \\ -r \sin \left( \xi + \theta_k + \left( \frac{eC}{r} - \frac{C}{2r} \right) \cos \gamma + \frac{\tau \sin \alpha}{r} \cos \gamma + \frac{\epsilon \sin \gamma - \frac{\tau \cos \alpha}{r} \sin \gamma}{r} \right) \\ r \cos \left( \xi + \theta_k + \left( \frac{eC}{r} - \frac{C}{2r} \right) \cos \gamma + \frac{\tau \sin \alpha}{r} \cos \gamma + \frac{\epsilon \sin \gamma - \frac{\tau \cos \alpha}{r} \sin \gamma}{r} \right) \end{bmatrix} \tag{2.7}
\]

where \( \mathbf{BS} \) and \( \mathbf{BP} \) are vectors to the blade surface on the suction and pressure sides respectively. Equations (2.6) and (2.7) will be used later in discretizing the blade surface.


2.2 Velocity Potential and Boundary Conditions

The flow field in which the propeller operates can be described by the linear combination of a free-stream velocity potential and a perturbation velocity potential $\phi$ which both satisfy the Laplace Equation.

$$\nabla^2 \phi = 0$$

The perturbation potential also must become zero at an infinite distance away from the propeller.

Consider a boundary surface $S$ which consists of a propeller blade surface $S_B$, a hub surface $S_H$ and a wake surface $S_W$ with normal vector $n$ pointing into the fluid domain. Applying Green’s identity, the perturbation velocity potential at any field point $P(x, y, z)$ can be found from a distribution of sources and doublets on the surface $S$. The relationship for $\phi$ can be written as follows [2]:

$$\frac{4\pi E(P)}{R(P, Q)} = \int_S \phi(Q) \frac{\partial}{\partial n_Q} \left( \frac{1}{R(P, Q)} \right) dS - \int_S \frac{\partial \phi(Q)}{\partial n_Q} \cdot \frac{1}{R(P, Q)} dS \quad (2.8)$$

where

$$E = \begin{cases} 
0 & \text{for the point } P \text{ inside } S, \text{(not in the fluid domain)} \\
1/2 & \text{for the point } P \text{ on } S, \\
1 & \text{for the point } P \text{ outside } S, \text{(in the fluid domain)} 
\end{cases}$$

$Q$ is a vector locating a point on the surface $S$

$n_Q$ is the normal vector at point $Q$ pointing into the fluid domain

$R = Q - P$

$\frac{\partial \phi}{\partial n_Q}$ is the normal derivative at $Q$

The solution of equation (2.8) is made unique through the application of a kinematic boundary condition, imposed on $S_B$ and $S_H$ such that the normal velocity on those surfaces is zero. This can be expressed through equation (2.9).

$$\frac{\partial \phi(Q)}{\partial n_Q} = -V_I \cdot n_Q = -(V_{inf} + \Omega \times Q) \cdot n_Q \quad (2.9)$$

where $V_{inf}$ is the free stream velocity, $\Omega$ is the rotational speed of the propeller, and $V_I$ is the resultant incident velocity on the propeller blade surface. Imposing the kinematic boundary condition in (2.9), the second term on the right hand side of equation (2.8) can be satisfied using a constant source distribution on surfaces $S_B$ and $S_H$.

We assume that there is no flow through nor a pressure jump across the wake surface $S_W$, however a discontinuity of the potential $\Delta \phi$ is allowed. For the steady problem, $\Delta \phi$ is constant along any stream line in the wake and can be written as

$$\Delta \phi = \phi_+ - \phi_- \quad (2.10)$$
where $\phi_+$ is the potential on the suction side of the wake and $\phi_-$ the potential on the pressure side.

Applying the kinematic boundary condition via equation (2.9) and the wake boundary condition via equation (2.10) to equation (2.8) we get equation (2.11) as per Hoshino [2].

\[
2\pi E\phi(P) - \int_{S_B+S_H} \phi(Q) \frac{\partial}{\partial n_Q} \left( \frac{1}{R(P, Q)} \right) dS - \int_{S_W} \Delta \phi(Q_W) \frac{\partial}{\partial n_{Q_W}} \left( \frac{1}{R(P, Q)} \right) dS = \int_{S_B+S_H} (V_I \cdot n_Q) \frac{1}{R(P, Q)} dS \tag{2.11}
\]

Here $Q_W$ indicates a point on the wake surface. Integrals over $S_B$ and $S_H$ are Cauchy principal value integrals. Equation (2.11) can be solved uniquely for $\phi$, and the resulting potential distribution can be differentiated to obtain velocities. However, we are not yet assured that the Kutta condition is satisfied. Hess [1] states that the Kutta condition can be applied numerically in 3D by making the surface pressures (or velocity magnitude in the case of potential flow) on the suction and pressure side surfaces have a common limit as the trailing edge is approached. It is by these means that the Kutta condition will be satisfied in this work. Satisfaction of the Kutta condition will be discussed in more detail in the next chapter.
3 Discretization and Numerical Method

Figure 3.1 illustrates the process used to numerically solve the propeller flow problem satisfying the Kutta condition. The important steps of this process will be detailed in this chapter.

3.1 Discretization of the Blade, Hub, and Wake Surfaces

The blade, hub, and wake surfaces will be discretized into a number of quadrilateral panels. The panel shape and any point on the panel surface is described by Morino [10] via equations (3.1) and (3.2), creating a hyperboloidal quadrilateral panel.

\[ Q = q_0 + \zeta q_1 + \eta q_2 + \zeta \eta q_3 \]  
\[ \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} q_{++} \\ q_{+-} \\ q_{-+} \\ q_{--} \end{bmatrix} \]  

Here \( q_{++}, q_{+-}, q_{-+}, \) and \( q_{--} \) are corner points of a panel and \( \zeta \) and \( \eta \) are local parametric variables with the domain \( \zeta, \eta \in [-1, 1] \). Figure 3.2 illustrates a typical panel and figure 3.3 shows an example of a discretized blade, the hub surface between it and an adjacent blade, and the associated wake surface. It is important to note that these panels are not planar and that their bounding edges may be twisted, allowing all panel corners to lay on the blade surface, increasing the accuracy in which the blade is modeled.

Using equations (2.6) and (2.7) to find the panel corners we can discretize the propeller hub and blades through systematic variations of \( r' \) and \( e \). The blade will be divided into \( N_S \) panels spanwise and \( N_C \) panels chordwise on each side of the blade, giving \( 2N_SN_C \) panels per blade. We will use a cosine spacing in the spanwise direction and a uniform spacing in the chordwise direction such that

\[ r' = 1/2 - \frac{1}{2} \cos f \]
Figure 3.1: Flow chart of numerical solution method
Figure 3.2: Hyperboloidal quadrilateral panel
Figure 3.3: Panels on the blade, hub, and wake for the key blade
Figure 3.4: Panels on the blade, hub, and wake for the propeller
where

\[ f = \begin{cases} 
0 & \text{for } u = 1 \\
\frac{(2u - 1)\pi}{2(N_S + 1)} & \text{for } u = 2, 3, \ldots, N_S + 1 
\end{cases} \]

and

\[ e = \frac{v}{N_C} \] (3.4)

where \( v = 0, 1, \ldots, N_C \).

The hub surface has been discretized in the chordwise direction using the same uniform spacing that is used to discretize the root section of the blade. The corner points of the hub panels lay on helical arcs which connect the corners of the blade root panels on the suction side of one blade to the corners of blade root panels on the pressure side of the adjacent blade. These helical arcs are uniformly divided into \( N_S \) panels, creating \( N_S N_C \) panels on the hub section between each blade. An example of the hub panels can be seen in figure 3.3.

The blade wake has been approximated by strips of constant strength doublet panels emanating from the pairs of trailing edge panels on the blade surface. These panels continue downstream from the trailing edge with a constant pitch equal to the geometric pitch \( \gamma \) of the blade at that radius. Each wake panel represents a \( \frac{\pi}{5N_S} \) radian angular displacement around the helix of that wake strip, thus the panel size on a given radial wake strip is constant along the helix of the that wake strip. Each wake strip consists of \( L \) panels.

### 3.2 First System of Linear Algebraic Equations

Having divided the blade and hub surfaces \( S_B \) and \( S_H \) into \( K \times N \) panels we can write equation (2.11) as a system of \( N \) linear algebraic equations. Here \( N \) is the number of panels on the key blade and its surrounding hub section. Due to the symmetry of the steady propeller problem we need only solve for the perturbation potential on one blade and its surrounding hub section, accounting for the influence of the other propeller surfaces on that key blade. \( \phi_j \) and \( V_1 \cdot n_Q \) are assumed to be constant within each panel and equal to the value at the centroid of the panel. The first system of linear equations \( SLAE_1 \) then can be written as

\[
\sum_{j=1}^{N} (\delta_{ij} - C_{ij}) \phi_j = \sum_{j=1}^{N} B_{ij} (V_1 \cdot n_Q) \] (3.5)

for \( i = 1, 2, \ldots, N \)

The source and body doublet influence coefficients are written as \( B_{ij} \) and \( C_{ij} \) respectively. In equation (3.5), the \( i \) index refers to the row of the system matrix and the point being affected by the influence coefficient. The \( j \) index indicates the influencing panel and the column in the left hand side matrix. The source and body doublet influence coefficients, \( B_{ij} \) and \( C_{ij} \), include the influence from sister panels on all blades onto the collocation point on the key blade. Equations for the calculation of the influence coefficients are provided later in this chapter which mathematically describe the process by which the influence from each
blade is included. In equation (3.5) $\delta$ is the Kronecker delta.

$SLAE_1$ is merely solved as the initial condition in the iterative method used to satisfy the Kutta condition. The solution of $SLAE_1$ does not include the influence of the wake nor does it satisfy the Kutta condition.

After solving $SLAE_1$ we will calculate and store two vectors $\Delta \phi$ and $\Delta V$, both with dimension $N_S$, to be used in the iterative satisfaction of the Kutta condition.

$$
\Delta \phi = \begin{bmatrix}
\Delta \phi_1 \\
\Delta \phi_2 \\
\vdots \\
\Delta \phi_{N_S}
\end{bmatrix} = \begin{bmatrix}
\phi_{+1TE} - \phi_{-1TE} \\
\phi_{+2TE} - \phi_{-2TE} \\
\vdots \\
\phi_{+N_S TE} - \phi_{-N_S TE}
\end{bmatrix}
$$

$$
\Delta V = \begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
\vdots \\
\Delta V_{N_S}
\end{bmatrix} = \begin{bmatrix}
V_{+1TE} - V_{-1TE} \\
V_{+2TE} - V_{-2TE} \\
\vdots \\
V_{+N_S TE} - V_{-N_S TE}
\end{bmatrix}
$$

Where $\phi_{\pm jTE}$ and $V_{\pm jTE}$ are the perturbation potential and velocity magnitude at the center of the suction(+) or pressure(-) side trailing edge panels of the jth spanwise strip. The $TE$ subscript indicates that the value appears only when panel $j$ is a trailing edge panel.

### 3.3 Second System of Linear Algebraic Equations

In our second system of linear algebraic equations $SLAE_2$, we have included the influence of the wake $W_{ijTE}$ on the right hand side of the system and will specify values of $\Delta \phi_{jTE}$ attempting to satisfy the Kutta condition. Here again the $TE$ subscript indicates that the wake influence is only added when panel $j$ is a trailing edge panel. The manner in which values of $\Delta \phi_{jTE}$ are chosen is discussed later in this chapter.

$$
\sum_{j=1}^{N} (\delta - C_{ij}) \phi_j = \sum_{j=1}^{N} \left[ B_{ij} (V_I \cdot n_Q) + W_{ijTE} \Delta \phi_{jTE} \right] \quad (3.6)
$$

for $i = 1, 2, \ldots, N$

The wake influence coefficient $W_{ijTE}$ is the influence of an entire streamwise wake strip consisting of $L$ panels and the influence of sister wake strips from other blades, onto a collocation point on the key blade. The equations used to calculate the wake influence coefficient are given in the next section.
3.4 Influence Coefficients

The influence coefficients used in equations (3.5) and (3.6) and the manner in which effects from the other blades have been taken into account is show below. We see that the influence of multiple blades is conglomerated into a single coefficient. The influence of each wake panel in a streamwise strip is accounted for in the same manner.

\[ C_{ij} = \sum_{k=1}^{K} \left[ \frac{1}{2\pi} \int_{S_j} \frac{\partial}{\partial n_j} \left( \frac{1}{R_{ijk}} \right) dS_j \right] \]  
\[ W_{ij} = \sum_{k=1}^{K} \sum_{l=1}^{L} \left[ \frac{1}{2\pi} \int_{S_j} \frac{\partial}{\partial n_j} \left( \frac{1}{R_{ijk}} \right) dS_j \right] \]  
\[ B_{ij} = \sum_{k=1}^{K} \left[ -\frac{1}{2\pi} \int_{S_j} \left( \frac{1}{R_{ijk}} \right) dS_j \right] \]

Here, \( R_{ijk} \) is defined as:
\[ R_{ijk} = Q_{jk}(\zeta, \eta) - P_i \]

We also define:
\[ s_1(\zeta, \eta) = \frac{\partial Q(\zeta, \eta)}{\partial \zeta} \]
\[ s_2(\zeta, \eta) = \frac{\partial Q(\zeta, \eta)}{\partial \eta} \]
\[ s_3(\zeta, \eta) = \frac{s_1 \times s_2}{|s_1 \times s_2|} \]

Using the terms \( R, s_1, s_2, \) and \( s_3 \) we define the following as per Hsin [4].

\[ I_D(\zeta, \eta) = \frac{1}{2\pi} \tan^{-1} \left( \frac{(R \times s_1) \cdot (R \times s_2)}{|R||R \cdot (s_1 \times s_2)|} \right) \]  
\[ I_S(\zeta, \eta) = -\frac{1}{2\pi} \left\{ -\frac{(R \times s_1) \cdot s_3(0,0)}{|s_1|} \sinh^{-1} \left( \frac{R \cdot s_1}{|R \times s_1|} \right) \right. \]
\[ + \left. \frac{(R \times s_2) \cdot s_3(0,0)}{|s_2|} \sinh^{-1} \left( \frac{R \cdot s_2}{|R \times s_2|} \right) \right\} \]

and
\[ \epsilon(\zeta, \eta) = \frac{R \cdot (s_1 \times s_2)}{|R \cdot (s_1 \times s_2)|} \]

Using equations (3.14) - (3.16), we can evaluate the integrals in our influence coefficients.
such that,

\[
\Phi_D = \frac{1}{2\pi} \int_{S_j} \frac{\partial}{\partial n_j} \left( \frac{1}{R_{ijk}} \right) dS_j \\
= \begin{cases} 
0 & \text{If, } I_D(1, 1) + I_D(1, -1) + I_D(-1, 1) + I_D(-1, -1) = 1 \\
\varphi_D - \frac{1}{2\pi |\Phi_D|} \varphi & \text{If, } \epsilon(1, 1)\epsilon(-1, -1) \epsilon(-1, 1) < 0 \\
\varphi_D & \text{Else}
\end{cases}
\]

(3.17)

where

\[
\varphi_D = \epsilon(1, 1)I_D(1, 1) - \epsilon(1, -1)I_D(1, -1) - \epsilon(-1, 1)I_D(-1, 1) + \epsilon(-1, -1)I_D(-1, -1)
\]

(3.18)

and

\[
- \frac{1}{2\pi} \int_{S_j} \left( \frac{1}{R_{ijk}} \right) dS_j = I_S(1, 1) - I_S(1, -1) - I_S(-1, 1) + I_S(-1, -1) \\
+ (R(0, 0) \cdot s_3(0, 0)) \Phi_D
\]

(3.19)

3.5 Satisfaction of the Kutta Condition

The iterative process through which the Kutta condition is satisfied is illustrated in figure 3.1 and we will refer to that illustration in this description of the process. The Kutta condition must be satisfied at every spanwise set of trailing edge panels and to accomplish this we will choose a vector \( \Delta \phi \) such that every term of vector \( \Delta V \) is zero.

As illustrated in figure 3.1, once \( SLAE_1 \) is solved, the initial vectors \( \Delta \phi \) and \( \Delta V \) are obtained from the solution. We will identify this set of potential and velocity vectors by the superscript 0. Arbitrarily modifying \( \Delta \phi^0 \), we create a new vector \( \Delta \phi^1 \), the terms of which will be used to compute the wake influence in \( SLAE_2 \). Solving \( SLAE_2 \) with the wake influence scaled by \( \Delta \phi^1 \), we obtain another set of velocity differentials at the trailing edge \( \Delta V^1 \).

If every component of \( \Delta V^1 \) does not vanish, we must estimate a new potential difference for each pair of trailing edge panels, \( \Delta \phi_{newTE} \), which will make \( \Delta V_{jTE} = 0 \) for that pair. Using our two sets of vectors \( \Delta \phi^0, \Delta V^0, \Delta \phi^1, \) and \( \Delta V^1 \), we can linearly model the relationship between \( \Delta \phi_{jTE} \) and \( \Delta V_{jTE} \) at each spanwise set of trailing edge panels and solve for the potential difference \( \Delta \phi_{jTE} \) which will make \( \Delta V_{jTE} = 0 \). \( \Delta \phi_{jTE} \) is calculated and \( SLAE_2 \) is solved for each spanwise set of trailing edge panels individually. After \( SLAE_2 \) has been solved for every spanwise pair of trailing edge panels, the vector \( \Delta V \) is checked to see if every component is sufficiently close to zero.

If a term of \( \Delta V \) is not sufficiently close to zero, the set of vectors used to compute the relationship between \( \Delta \phi_{jTE} \) and \( \Delta V_{jTE} \) is updated, and the process is repeated.
3.6 Calculation of Velocity and Pressure on the Blade Surface

The velocities on the blade surfaces are obtained through numerical differentiation. This differentiation is done with respect to the local panel coordinate system, the perturbation velocity is combined with the free stream velocity, and the resultant converted to the propeller Cartesian coordinate system.

The perturbation potential field on the blade surface is modeled as two quadratic polynomials, $\phi_c(g)$ and $\phi_s(g)$, in the chordwise and spanwise directions respectively. Here $g$ is the distance along the blade surface in either the chordwise or spanwise direction. The coefficients of the quadratic function are obtained from the solution of equation (3.20) for $a$, $b$, and $c$.\[\begin{bmatrix} 0 & 0 & 1 \\ g^2_+ & g_+ & 1 \\ g^2_- & g_- & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \phi_m \\ \phi_{m+1} \\ \phi_{m-1} \end{bmatrix} \tag{3.20}\]

Here $m$ is either the spanwise panel index $u$ or the chordwise panel index $v$ depending on the direction for which the quadratic function is being fit. $\phi_m$ is the perturbation potential at the center of the panel where the velocity is being computed, and $\phi_{m\pm1}$ indicates an adjacent panel with either an increasing or decreasing panel index. In equation (3.20) $g_\pm$ is the linear distance between adjacent panel centers within a given panel strip in the direction of increasing or decreasing panel index depending on the subscript. Figure 3.5 illustrates this curve fitting for a simple case.

For the calculation of the velocity on a panel at the edge of the blade, where there is only one adjacent panel per strip, the differentiation is shifted. In an example where there is no panel index less than $m$, the system of equations describing the potential polynomial is similar to equation (3.20), however the panel of decreasing index has been replaced by a panel of index two greater than the panel of interest; as seen in equation (3.20). This effectively shifts the domain of the quadratic equation in the direction of increasing panel index.

\[\begin{bmatrix} 0 & 0 & 1 \\ g^2_{++} & g_{++} & 1 \\ g^2_{+} & g_+ & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \phi_m \\ \phi_{m+1} \\ \phi_{m+2} \end{bmatrix} \tag{3.21}\]

Here $g_{++}$ is the distance from the center of the panel of interest to panel $m + 2$, and $\phi_{m+2}$ is the perturbation potential at panel $m + 2$. The same shift can be done in the opposite direction for the case where there is no panel with index greater than the panel where the velocity is to be computed.

The derivatives of $\phi_c(g)$ and $\phi_s(g)$ with respect to $g$ are the perturbation velocities in the chordwise and spanwise directions, $v_c$ and $v_s$. $v_c$ and $v_s$ are assumed to act in the direction of $q_1$ and $q_2$ respectively. We create a panel coordinate system $Pn - cs$ with unit vectors $c$ and $s$ in the direction of $q_1$ and $q_2$ respectively. $c$ and $s$ are not always normal to each other and the velocities $v_c$ and $v_s$ will need to be projected onto a Cartesian panel coordinate system.
Figure 3.5: Sketch of method used in numerical differentiation of the perturbation potential

Figure 3.6: Local panel coordinate systems
system $Pn − op$ to be combined with the incident velocity $V_1$. Coordinate systems $Pn − cs$ and $Pn − op$ are illustrated in figure 3.6. Figure 3.6 shows that $o = c$. The projection onto the $Pn − op$ coordinate system is accomplished through equations (3.22) and (3.23).

\[
\begin{align*}
  v_o &= v_c \quad \text{(3.22)} \\
  v_p &= \frac{v_s - (s \cdot o)v_c}{s \cdot p} \quad \text{(3.23)}
\end{align*}
\]

where:

- $s$ is a unit vector in the direction of $q_2$
- $o$ is a unit vector of the $Pn − op$ Cartesian coordinate system described in the $P − xyz$ coordinate system
- $p$ is a unit vector of the $Pn − op$ Cartesian coordinate system described in the $P − xyz$ coordinate system

Thus, the total velocity at the the panel center, in the panel Cartesian coordinate system is given by equation (3.24). The component normal to the panel surface is zero due to the application of the kinematic boundary condition on the blade surface.

\[
V = \begin{bmatrix} V_1 \cdot o + v_o \\ V_1 \cdot p + v_p \\ 0 \end{bmatrix} \quad \text{(3.24)}
\]

The pressure on a panel is obtained using the Bernoulli equation such that

\[
P = P_t + \frac{1}{2} \rho \left( |V_1|^2 - |V|^2 \right) \quad \text{(3.25)}
\]

where $P_t$ is the reference pressure at the propeller center.

The pressure coefficient $C_P$ follows as:

\[
C_P = 1 - \frac{|V|}{|V_1|} \quad \text{(3.26)}
\]

### 3.7 Calculation of Propeller Forces

The thrust $F_x$ and the torque $M_x$ acting on the propeller are obtained by integrating the pressure and viscous forces on the blade surfaces.

\[
F_x = K \sum_{j=1}^{N} [P_j \Delta A_x + F_{\nu x}] \quad \text{(3.27)}
\]
\[ M_x = K \sum_{j=1}^{N} [(P_j \Delta A_z y - P_j \Delta A_y z) + (F_{vy} y - F_{vy} z)] \] (3.28)

Where

- \( P_j \) is the pressure at panel \( j \)
- \( \Delta A = -n \Delta A \), subscript indicates component direction
- \( \Delta A \) is the panel area
- \( F \) is the viscous force vector, subscript indicates component direction
- \( y, z \) panel center location in propeller Cartesian coordinate system \( P - xyz \)

The magnitude of the viscous force per panel is equal to

\[ \frac{1}{2} \rho C_F \Delta A V^2 \] (3.29)

where \( C_F \) is the ITTC friction coefficient [9] and \( V \) is the velocity magnitude on the panel. The viscous forces are assumed to act in the direction of the local velocity vector.

The advance ratio, thrust coefficient, torque coefficient and open-water efficiency are defined as

\[
\begin{align*}
J & = \frac{V_{ix}}{nD} \\
K_T & = \frac{F_x}{\rho n^2 D^4} \\
K_Q & = \frac{M_x}{\rho n^2 D^5} \\
\eta_O & = \frac{J}{2\pi} \frac{K_T}{K_Q}
\end{align*}
\] (3.30) (3.31) (3.32) (3.33)
4 Verification of the Method and Code

4.1 Sphere in Uniform Flow

Katz and Plotkin [5] describe the velocity potential field around a sphere in an otherwise uniform flow field as,

$$\Phi = U_\infty \cos \alpha \left( r + \frac{R_{\text{sphere}}^3}{2r^2} \right)$$  \hspace{1cm} (4.1)

the resulting velocity field as,

$$V_r = U_\infty \cos \alpha \left( 1 - \frac{R_{\text{sphere}}^3}{r^3} \right)$$  \hspace{1cm} (4.2)

$$V_\alpha = -U_\infty \sin \alpha \left( 1 + \frac{R_{\text{sphere}}^3}{2r^3} \right)$$  \hspace{1cm} (4.3)

and the coefficient of pressure $C_P$ on the sphere surface as

$$C_P = \left( 1 - \frac{9}{4} \sin^2 \alpha \right)$$  \hspace{1cm} (4.4)

$U_\infty$ is the free stream velocity, $R_{\text{sphere}}$ is the radius of the sphere, $r$ is the radial distance from the center of the sphere and $\alpha$ is the angle from the free-stream flow. In each of the equations above, the first term inside the parenthesis is the contribution from the free-stream and the second term represents the perturbation caused by the sphere.

To verify the accuracy of the method described in Chapter 3, a grid convergence study was performed using a sphere as the geometry, so that the results could be compared with equations (4.1) - (4.3). A sphere of unit radius was discretized using equations (4.5) - (4.7) to determine location of the panel corners in a Cartesian coordinate system with the origin at the center of the sphere.

$$x = -R_{\text{sphere}} \cos \left( \frac{v\pi}{N_C} \right)$$  \hspace{1cm} (4.5)
\[ y = \begin{cases} 
-0.001 \cos\left(\frac{\pi u}{N_S}\right) & \text{If } v = 0 \text{ or } N_C, \\
-R_{\text{sphere}} \sin\left(\frac{\pi v}{N_C}\right) \cos\left(\frac{\pi u}{N_S}\right) & \text{Else.} 
\end{cases} \tag{4.6} \]

\[ z = \begin{cases} 
-0.001 \sin\left(\frac{\pi u}{N_S}\right) & \text{If } v = 0 \text{ or } N_C, \\
-R_{\text{sphere}} \sin\left(\frac{\pi v}{N_C}\right) \sin\left(\frac{\pi u}{N_S}\right) & \text{Else.} 
\end{cases} \tag{4.7} \]

Where

- \( R_{\text{sphere}} \) = Radius of the sphere
- \( N_C \) = number of streamwise panels on half of the sphere
- \( N_S \) = number of circumferential panels on half of the sphere
- \( u = 0, 1, \ldots, N_S \)
- \( v = 0, 1, \ldots, N_C \)

The special case where \( v \) equals either zero or \( N_C \) has been implemented to insure that no panel corners are co-located, which causes one of the principal vectors used to compute the panel influence functions to become degenerate.

The other half of the sphere is paneled by mirroring the panel corners about the X-Y plane. The term \( \frac{\pi v}{N_C} \) in equations (4.5) - (4.7) is equal to \( \alpha \) in equations (4.1) - (4.3) thus panel centers are located at intervals of \( \pi/N_C \) streamwise around the sphere. Figure 4.1 illustrates the panel arrangement on a sphere where \( N_C = N_S = 9 \). No wake has been modeled for this problem as no lift is developed.

Figure 4.2 shows the difference between the perturbation potential from equation (4.1) and the numerical results at panels centers, plotted over half of the sphere. Results are given for cases where \( N_C = N_S = 3, 9, \& 27 \). From figure 4.2 one can see that the results converge rapidly to equation (4.1) as the number of panels is increased. Figures 4.3 and 4.4 show values of \( V_0 \) and \( C_P \) on a sphere of unit radius in a unit free stream flow. The maximum and minimum values that occur for the quantities shown are printed at the top and bottom of the color scale. The sphere in figures 4.3 and 4.4 has been paneled with parameters \( N_C \) and \( N_S = 27 \).
Figure 4.1: Panels on Surface of a Sphere, $N_C = N_S = 9$
Figure 4.2: Difference between the exact velocity potential and the numerical value
Figure 4.3: $V_\alpha$ on surface of sphere, $R_{sphere} = 1$, $U_\infty = 1$
Figure 4.4: $C_p$ on surface of sphere, $R_{sphere} = 1$, $U_\infty = 1$
4.2 Van de Vooren Airfoil in Uniform Flow

To verify that the implementation of the Kutta condition produces the correct results we will compare the numerical results obtained using the methods presented in this report with 2D results from the analytical solution of flow around a planar, Van de Vooren airfoil [5]. Results will be compared at several angles of attack, illustrating convergence to the analytical solution with respect to panel resolution, and the number of wake panels. Because we will be comparing 3D numerical results with 2D analytical results we will also illustrate the differences between a 2D solution and a 3D solution with a finite wing span, showing convergence to the 2D solution as the span increases.

First, we will examine the convergence of the numerical solution to the analytical results as the the number of panels on the foil increases. Figure 4.6 shows the velocity magnitude on the foil surface (angle of attack = zero) for the 2D analytical solution, plotted with results from coarse, medium, and fine 3D numerical simulations. The coarse, medium, and fine numerical solutions have 10x20, 20x40 and 40x80 panels on the upper and lower faces of the planar foil, respectively. The planar foil in the numerical solutions has a span/chord ratio of approx. 2. The number of streamwise panels used to model the wake for the coarse, medium, and fine discretizations was 40, 80 and 160 respectively. Figure 4.5 shows the body and wake panels for the medium case.

Upon examination of figure 4.6 we notice that the numerical results converge very quickly, such that the medium and fine results nearly overlap. The most noticeable errors are seen near the leading and trailing edges but these reduce as the number of chordwise panels
Figure 4.6: Velocity Magnitude on surface of foil for coarse, medium and fine simulations
increases and the geometric approximations improve in these regions of high curvature. However, we also note the failure of the 3D numerical results to converge to the 2D analytical solution due to the 3D effects at the ends of the planar foil body.

The end effects seen in Figure 4.6 are quite small, however as the angle of attack increases 3D end effects become more noticeable. Figure 4.7 shows 3D numerical and 2D analytical results for the same airfoil pictured in figure 4.5, but with an angle of attack of 10 degrees. Numerical results are shown for planar foils with aspect ratios (span/chord) of 2, 4, and 8. The panels on each of these planar foils were the same size as was used in the fine discretization with zero angle of attack and the results were again taken at the spanwise midpoint. Thus figure 4.7 shows us the rate at which the 3D numerical results converge to the 2D analytical results as the foil end effect is moved farther away from the point of interest. Figure 4.7 also shows that those end effects can be significant when comparing the numerical and analytical results.

We have seen that 3D end effects can significantly affect the results at the midspan of a lifting planar foil. Because those end effects are proportional to angle of attack of the foil, we
use a more modest angle of attack to study the effect that the length of the modeled wake has on the velocities at the blade surface. Figure 4.8 shows the velocity magnitude on the surface of the same foil at a 3 degree angle of attack. The panel size is again the same as was used in the fine discretization with zero angle of attack and the aspect ratio of the foil was kept at 8 to reduce the end effect errors. The number of streamwise panels used to model the wake has been varied while the aspect ratio of the panels has been kept constant, thus changing the length of the modeled wake. Numerical simulations were performed with 20, 40 and 80 wake panels. From figure 4.8 we can see that the majority of the wake influence comes from the panels near the trailing edge and that variations in the length of the wake have little effect on the results.

Figure 4.8: Velocity Magnitude on surface of foil for 20, 40, and 80 wake panels
5 Validation

In this chapter we will simulate the flow around a Wageningen B-Series propeller at several advance ratios and compare the thrust and torque predicted by the numerical simulation to the published experimental results for the B-Series [11] [7].

5.1 B-Series Propellers

The Wageningen B-Series propellers were chosen for this validation effort because the experimental results are well known to the marine community and because the data was readily available. The series represents a set of propellers with four to seven blades, with a range of expanded area ratios from 0.45 - 1.05 and a range of pitch-diameter ratios from 0.5 - 1.4. Experimentally measured thrust, and torque coefficients have been published for advance ratios from 0.0 - 1.6. Advance ratio, thrust coefficient and torque coefficient are of the forms noted in Chapter 3.

The geometry of the propellers simulated in this report has been recreated from tables of shape parameters published in [11] and [7].

5.2 Propeller Forces

The flow around a five bladed propeller with an expanded area ratio of 1.05 and a pitch-diameter ratio of 0.8, acting in an otherwise uniform flow, was simulated for comparison with the published experimental data. A range of advance ratios (0.3-0.6) were simulated to evaluate the numerical predictions at several propeller loading conditions. Simulations were performed for four discretizations of the blade, hub and wake surfaces. Figures 5.1 - 5.4 illustrate the panels for these coarse, medium, fine, and xfine simulations. Table 5.1 lists the number of panels on the blade in the chordwise and spanwise directions, as well as the number of streamwise panels used to model the wake. One can see from figures 5.1 - 5.4 that in each of the simulations the wake extends approximately one revolution downstream.

Figures 5.5 and 5.6 plot the thrust and torque coefficients from the numerical simulation along with the published B-Series data. In figure 5.5 we see that the numerically predicted thrust coefficient is greater than the experimentally predicted values for all advance ratios and discretizations. This could be expected from an inviscid, irrotational model, which fails to model phenomena such as separation, however the differences of this magnitude will likely
restrict the usage the numerical model in its present state. These differences may be due to
the inviscid and irrotational assumptions in the physical model, however results published
by Hoshino [2] for a similar physical model achieved better results.

We also notice that the slope of the numerically predicted $K_T$ curves is less than that
of the B-Series values; indicating that the numerical method predicts a more constant de-
velopment of thrust over variations in advance ratio than is seen in the experimental data.
This difference between the slopes of the numerical and experimental curves could be caused
by several of the assumptions made in the physical model, including: the application of the
kutta condition at highly loaded blade sections, or the lack of a model for the blade tip
vortex.

In figure 5.6, which plots the $K_Q$ curves, we again see that the slope of the numerically
predicted results is less than that of the experimental results. Torque is highly affected by
the induced velocities from the blade tip vortex and the incomplete modeling of that vortex
in the method proposed in this report could contribute to the insensitivity of the $K_Q$ curves
to changes in advance ratio.

Also, in figure 5.6 we see that the torque coefficient reduces as the grid resolution in-
creases; starting above the experimental results in the Coarse case but falling to below the
experimental results in the xFine case; not converging to the experimental results. It is
important to note that in the plots of both $K_T$ and $K_Q$ that the difference between the
numerical values for any two discretizations does not appear to decrease as the panels be-
come smaller. This inability to converge, even to results other that the experimental results,
could be due to the inadequacies in the physical model mentioned above or to an improper
numerical implementation of the continuum equations. In any case, the lack of observed
convergence as the panel size is decreased is a problem which must be corrected prior to
extensive use of the code.

Colored contour plots of the velocity magnitude and pressure coefficient for the coarse,
medium, fine and xfine cases, for advance ratios 0.3-0.6 are given in the Appendix.
Figure 5.1: Panels for coarse simulation, $N_S = N_C = 5$

Figure 5.2: Panels for medium simulation, $N_S = N_C = 7$
Figure 5.3: Panels for fine simulation, $N_S = N_C = 10$

Figure 5.4: Panels for xfine simulation, $N_S = N_C = 14$
Figure 5.5: $K_T$ curves for B5-105, $P/D = 0.8$
Figure 5.6: $K_Q$ curves for B5-105, $P/D = 0.8$
6 Conclusions

This report describes the governing equation, and boundary conditions for a marine propeller operating in a uniform flow field of inviscid and irrotational fluid and a method is presented by which the velocity and pressure on the blade surface of the propeller can be numerically simulated. The method has been tested on several geometries including a sphere, a planar Van de Vooren airfoil, and a Wageningen B-Series propeller.

The numerical results compared well with the analytical results for both the sphere and the Van der Vooren airfoil in potential flow, however significant differences were seen when compared to the experimentally measured thrust and torque coefficients for the B-Series propeller. Most troubling is that neither the numerical results for thrust of torque on the B-Series propeller we observed to converge to as the panel size on the computational surfaces was decreased.

Results published by Hoshino [2] show a more favorable comparison with experimental data, however no grid convergence tests with respect to thrust and torque values were provided in [2]. Other applications of boundary element or panel methods suggests that this type of simulation can provide accurate predictions of the steady thrust and torque on a marine propeller [3].

Further research into the modeling of wake roll-up [6] and the blade tip vortex could improve the comparison with experimental data.
Bibliography


A Appendix
Figure A.1: Velocity Magnitude for B5-105, Coarse, $P/D = 0.8$, $J = 0.3$ suction side (left) and pressure side (right)

Figure A.2: $C_p$ for B5-105, Coarse, $P/D = 0.8$, $J = 0.3$ suction side (left) and pressure side (right)
Figure A.3: Velocity Magnitude for B5-105, Coarse, $P/D = 0.8$, $J = 0.4$ suction side (left) and pressure side (right).

Figure A.4: $C_P$ for B5-105, Coarse, $P/D = 0.8$, $J = 0.4$ suction side (left) and pressure side (right).
Figure A.5: Velocity Magnitude for B5-105, Coarse, $P/D = 0.8$, $J = 0.5$ suction side (left) and pressure side (right).

Figure A.6: $C_P$ for B5-105, Coarse, $P/D = 0.8$, $J = 0.5$ suction side (left) and pressure side (right).
Figure A.7: Velocity Magnitude for B5-105, Coarse, $P/D = 0.8$, $J = 0.6$ suction side (left) and pressure side (right).

Figure A.8: $C_P$ for B5-105, Coarse, $P/D = 0.8$, $J = 0.6$ suction side (left) and pressure side (right).
Figure A.9: Velocity Magnitude for B5-105, Medium, $P/D = 0.8$, $J = 0.3$ suction side (left) and pressure side (right).

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Figure A.27: Velocity Magnitude for B5-105, xFine, $P/D = 0.8$, $J = 0.4$ suction side (left) and pressure side (right).

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Figure A.30: $C_P$ for B5-105, xFine, $P/D = 0.8$, $J = 0.5$ suction side (left) and pressure side (right).
Figure A.31: Velocity Magnitude for B5-105, xFine, $P/D = 0.8$, $J = 0.6$ suction side (left) and pressure side (right)

Figure A.32: $C_p$ for B5-105, xFine, $P/D = 0.8$, $J = 0.6$ suction side (left) and pressure side (right)
Vita

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