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The Effects of Graphing Calculator use on High-School Students' Reasoning in Integral Calculus

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I would like to dedicate this dissertation to my family, for your love, support, and encouragement.

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GLOSSARY

Algebraic Manipulation – Problem solving involving algebraic methods, use of formulas, and algorithms

Algorithmic Techniques – See algebraic manipulation

Conceptual Understanding – One of the strands of mathematical proficiency defined by the National Research Council (2001, p. 5) as "comprehension of mathematical concepts, operations, and relations."

Constructivism – an epistemology which suggests that all knowledge is constructed and that students are active participants in the learning process

Definite Integral – Calculus concept representing the area between two curves on the Cartesian plane

Disk Method – Calculus procedure for finding the volume of solids of revolution

Handheld Graphing Technology – Graphing calculators

Integral Calculus – Part of calculus involving integration techniques

Manipulative Techniques – See Algebraic Manipulation

Non-routine problem solving – The process of solving problems with which students are not typically familiar

Reasoning – "The process of drawing conclusions on the basis of evidence or stated assumptions" (NCTM, 2009, p. 2)

Riemann Sums – Calculus procedure for approximating a definite integral

Routine problem solving – The process of solving problems with which students are familiar

Solids of Revolution – Calculus concept referring to a three-dimensional solid formed when a graph is rotated about a line on the Cartesian plane

Abstract

This mixed-method study investigated the impact of graphing calculator use on high school calculus students' reasoning skills through calculus problems when applying to concepts of the definite integral and its applications. The study provides an investigation of the effects on reasoning when graphing calculators are used, since it is proposed that, through reasoning, conceptual understanding can be achieved. Three research questions were used to guide the study: (1) Does the use of the graphing calculator improve high school calculus students' reasoning ability in calculus problems applying the definite integral? (2) In what specific areas of reasoning does use of the graphing calculator seem to be most and least effective? and (3) To what extent can students who have used the graphing calculator demonstrate ability to solve problems using pencil and paper methods? The study included a quantitative, quasi-experimental component and a qualitative component. Results of the quantitative and qualitative analysis indicate that (1) graphing calculators had a positive impact upon students' reasoning skills (2) graphing calculators were most effective in the areas of initiating a strategy and monitoring progress (3) students' reasoning skills were most improved when graphing calculators were used together with the analytic approach during both instruction and testing and (4) students who used the graphing calculator performed equally as well in all elements of reasoning as those who used pencil and paper to solve problems.

Keywords: Calculus, Constructivism, Conceptual Understanding, Graphing Calculators, Handheld Graphing Technology, Problem Solving Approaches, Reasoning

CHAPTER 1:

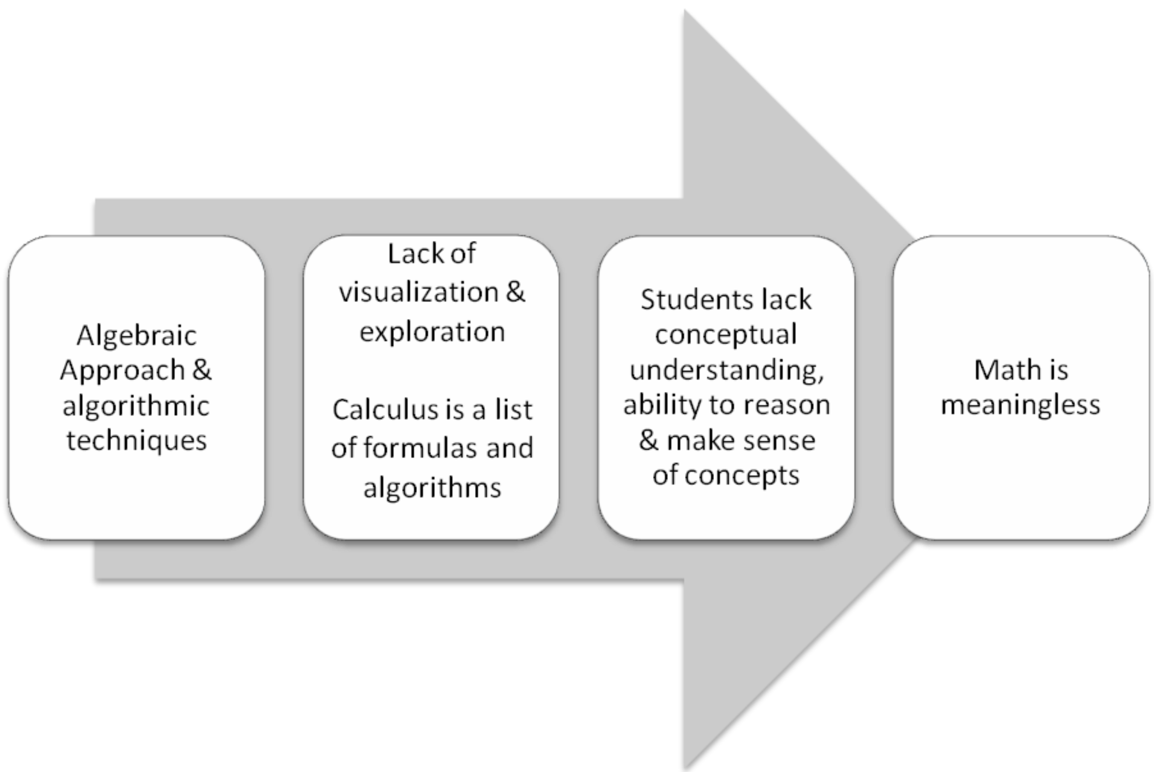
INTRODUCTION

The development of conceptual understanding is critical to students' success and ability to solve problems in calculus courses. Understanding concepts, such as the limit, derivative, and integral, requires a method of thinking different from other areas of mathematics. Many other areas of mathematics involve finite processes, whereas most calculus concepts, such as those listed above, involve infinite processes (NCTM, 1989). It has been recommended that curriculum and instruction in calculus focus on conceptual understanding rather than algebraic manipulation (NCTM, 1989, 2009). Despite this recommendation, it has been common for calculus courses to focus on algorithmic techniques. Several studies have documented the difficulties that students have faced in calculus as a result of the focus on algebraic manipulation and a lack of focus on conceptual understanding (Baker, Cooley, & Trigueros, 2000; Judson & Nishimore, 2005; White & Mitchelmore, 1996). For example, this lack of understanding has caused students to have difficulty in linking calculus concepts in order to solve problems, thus creating an obstacle for student success in this course (Baker et al.; Judson & Nishimore; White & Mitchelmore).

Problem and Purpose

Reasoning is one important component of students' mathematical learning and conceptual understanding in mathematics (NCTM, 2009). Development of conceptual understanding is important in mathematics and should be the goal of mathematics teaching rather than a focus on algebraic manipulation (NCTM). Calculus students, however, have often struggled to develop conceptual understanding with many of the topics that are central to calculus, and this lack of understanding appears to be a common problem (Baker et al., 2000, Judson & Nishimore, 2005, White & Mitchelmore, 1996). I have constructed Figure 1 below to illustrate the problem that students encounter in their calculus courses when the focus is placed on an algebraic approach and manipulative techniques.

Figure 1: Problem



There are several skills important to an understanding of math, of which reasoning is one (Garden et al., 2006). It has been suggested that student use of the graphing calculator can improve student success in areas such as concept development and problem solving while providing opportunities for visualization, access to multiple representations, and computation (Anderson et al., 1999; Brosnan & Ralley, 1995; Kimmins & Bouldin, 1996; NCTM, 1989, 2009; Porzio, 1997; Powell, 1998; Reznichenko, 2007; Waits & Demana, 1998); however, what was not known was whether the graphing calculator impacts students' ability to reason in solving calculus problems. NCTM postulates that use of technology may have an impact upon students' reasoning skills, but this has not been proven in previous research.

The purpose of this study was to investigate the impact of graphing calculator use on high-school calculus students' reasoning in solving problems on the definite integral and its applications. Garden et al. (2006) identified calculus as one of the traditional advanced mathematics content domains that deserves considerable emphasis, stating "calculus is a central tool in understanding the principles governing the physical world" (p.12) and "is the principal point of entry to most mathematically-based scientific careers" (p. 12). Since students' reasoning skills are part of their conceptual understanding in calculus, it is important to determine how these skills can be improved. The NCTM (2009) has identified several areas of mathematics, such as algebra, geometry, and statistics, that should include reasoning. As these areas form the foundation for calculus concepts, reasoning skills are an essential part of learning in calculus as well. Garden et al. (2006) also identifies calculus as one of the three main advanced mathematics content domains in which reasoning skills are useful. The NCTM has also suggested that technology be included to improve reasoning skills. Several types of technology, including graphing technologies such as graphing calculators, have been used in mathematics classrooms and in calculus classrooms, in particular. They have been used in order to improve student success in areas such as the development of problem-solving skills, and research has shown how the graphing calculator can be helpful in mathematics courses when it is used for visualization, computation, and access to multiple representations (Anderson et al., 1999; Brosnan & Ralley, 1995; Kimmins & Bouldin, 1996; NCTM, 1989, 2009; Porzio, 1997; Powell, 1998; Reznichenko, 2007; Waits & Demana, 1998). Although it has been suggested that the graphing calculator can impact one of the main aspects of understanding in calculus, that is, reasoning, this has not been studied in previous research. Recommendations for the inclusion of reasoning opportunities and the recent emphasis on reasoning skills reiterated the need for a study to investigate ways to improve these skills. This study addressed this need by investigating whether the use of graphing calculators could improve reasoning in calculus. This study provides information for the research community and for calculus teachers on whether the graphing calculator can be used to increase students' reasoning in calculus, so that conceptual understanding and success in calculus can be achieved.

Reasoning in Calculus

Currently, although reasoning is considered an important part of learning mathematics, these skills have not been given consistent attention in the mathematics curriculum (Fuson, Kalchman, & Bransford, 2005; NCTM, 2009). Furthermore, many students lack reasoning skills when they enter college (CUPM, 2004). As a result of the lack of reasoning abilities that exist, students often have not been able to see the meaning in mathematics, and ability to retain knowledge has been difficult (NCTM). Calculus students, in particular, have often struggled with a lack of understanding of concepts (Baker et al., 2000; Carson et al., 2003; White et al., 1996). Several studies have documented the areas where calculus students have struggled, such as the second derivative and the definite integral, and the apparent causes of these struggles, which include lack of understanding of functions and variables (Baker et al.; Judson et al.; White et al.). Little research, however, has focused on improving students' reasoning skills, particularly in calculus. While it has been suggested that appropriate use of the graphing calculator can help students in problem solving and reasoning through the solutions to problems, previous empirical research has not addressed the effects that graphing calculators have on reasoning in calculus.

Because of the potential benefits that reasoning may provide for students' understanding and success in mathematics, and calculus in particular, it is necessary to develop ways to improve reasoning skills in mathematics, but the strategies to do so are largely unexplored (CUPM, 2004). The NCTM suggests that technology be included throughout the mathematics curriculum in order to achieve this goal. It is suggested that technology can be used to provide multiple representations of calculus problems for students to see and that technology can provide more of these representations than students can often do by hand (NCTM, 2000, 2009). Because of the capabilities and opportunities afforded by technology, it can provide "a lens that leads to a deeper understanding of mathematical concepts" (NCTM, 2009, p. 14).

Conceptual Understanding in Calculus

The importance of conceptual understanding is emphasized widely (Council, 2001; CUPM, 2004; Fuson, Kalchman, & Bransford, 2005), and the importance of improving conceptual understanding in calculus courses has been well documented (CollegeBoard, 2009; Kissane & Kemp, 2008; Schwalbach & Dosemagen, 2000; Slavit, Lofaro, & Cooper, 2002), as this understanding can have a profound impact on students' ability to solve problems. The National Mathematics Advisory Panel (2008) has recommended that future research be focused on ways to improve conceptual understanding. One suggestion comes from the National Council of Teachers of Mathematics (NCTM, 2009). The NCTM recommends that refocusing the mathematics curriculum on reasoning and sense making will improve conceptual understanding (NCTM). Reasoning is defined by the NCTM (2009) as "the process of drawing conclusions on the basis of evidence or stated assumptions" (p. 2). Sense making is defined as "developing understanding of a situation, context, or concepts by connecting it with existing knowledge" (NCTM, 2009, p. 3). The processes of reasoning and sense making are said to be "intertwined," (NCTM, 2009, p. 3) as students can make sense of mathematical concepts as they engage in reasoning. The NCTM suggests that including opportunities for reasoning and sense making in the mathematics curriculum will help students to have a better understanding of mathematical procedures, when they are used, what they mean, and why they work. Likewise, mathematics should make sense and have meaning to students (NCTM). In addition, students are only able to make sense of mathematics if they have an understanding of the concepts involved. Concepts are said to be "the substance of mathematical knowledge" (NCTM, 1989, p. 223).

Graphing Calculators

Although many types of technology have been useful in mathematics classrooms, graphing calculators, in particular, have been beneficial in mathematics classes including calculus (Brosnan & Ralley, 1995; Porzio, 1997; Reznichenko, 2007). Graphing calculators can be used in a variety of ways in teaching and learning calculus (CUPM, 2004; NCTM, 1989, 2000). They can be used to increase the efficiency of calculations as well as to explore abstract concepts. Computing technology, such as the graphing calculator, "makes the fundamental concepts and applications of calculus accessible to all students" (NCTM, 1989, p. 182).

Teachers' Concern on Usage of the Graphing Calculator

My perspective toward the graphing calculator is based upon my experience as a high-school calculus teacher and my discussions with other teachers. I have used graphing calculators in calculus instruction for the past few years, and it has been a useful tool for my classes, yet many teachers question its use because they believe that it replaces students' thinking. Another concern among teachers that I have spoken with is how to help students in their ability to read and interpret graphs. On the ACT test, for example, students are tested on their ability to work with graphical representations on the math and science sections. Some of these teachers believe that students struggle with portions of the test due to lack of ability to read and interpret graphs. Furthermore, these teachers have commented that students often resort to recalling formulas and following procedures rather than relying on their reasoning skills to solve problems. These teachers have made a connection between reasoning skills and working with graphical representations.

Appropriate Use of the Graphing Calculator

Although overreliance on the calculator is a concern for some, the uses of the graphing calculator extend beyond mere ease of computation. They can be used to explore abstract concepts that are otherwise difficult to visualize, such as continuity, differentiability, end behavior, and concavity (NCTM, 1989). Features of the calculator such as the "zoom" feature and the ability to compute the definite integral make these explorations possible (NCTM, 2009). Numerous studies have documented the benefits associated with appropriate use of the graphing calculator on student learning in calculus and other mathematics (Anderson, Mueller, & Pedler, 1999; Beckmann, Thompson, & Senk, 1999; Brosnan & Ralley, 1995; Mesa, 2007; NCTM, 1989, 2000, 2009; Porzio, 1997; Waits & Demana, 1998). As with any technology, however, it is important to use this tool in a way that enhances opportunities for student learning and not to cause students to develop an overreliance on the calculator. With the proper curriculum design to employ this technology and appropriate use of the graphing calculator by the teacher and the students, this tool offers some advantages for student learning of mathematics, and calculus in particular, that is difficult or not possible without it (Doerr & Zangor, 2000).

My students and I have used the calculators not for mere computation or to replace their problem-solving skills but for exploration and visualization of functions and concepts. I have used the zoom feature, for example, to narrow in on part of a graph so that the students can understand the concept of local linearity when learning about the derivative and linear approximations. This is better understood by graph than it is algebraically, from my experience. Moreover, the graph can be manipulated through the graphing calculator. Drawing the graph for students on the board provides them with visualization, but zooming into the graph helps them to see more clearly and understand the meaning of the term local linearity. It is my perspective, though, that its effectiveness depends upon how it is used. Therefore, I was interested in determining whether, when graphing calculators are used strategically, for visualization and exploration, they have any effect on reasoning skills in integral calculus.

This study focuses on integral calculus, the part of calculus that deals with the definite integral and its applications. Graphing calculators can be used in integral calculus to demonstrate the area under a curve or between curves, to demonstrate the process used in computing a Riemann sum, a procedure used for approximating a definite integral, to approximate a definite integral, to compute area and volume of solids, and to teach the process used to find distance by demonstrating the motion of an object moving in a linear path. Although some of these concepts can be computed by hand and visualized through hand-drawn graphs, the graphing calculator provides a way to perform computations efficiently and provides graphical representations that can be manipulated.

Conceptual Framework

Constructivism

My conceptual framework for the study is the theory of constructivism, an epistemology which suggests that students are active participants in the learning process and construct their own knowledge through interactions with their environment, as explained by Terwel (1999). The foundation for the theory of constructivism was developed by Jean Piaget in 1937, and it was further expanded by von Glasersfeld (von Glasersfeld, 1996a).

There are several varieties of constructivism, but these varieties all share the belief that knowledge cannot simply be passed on from the teacher to the students or "internalized, stored, and reproduced at some later time" (Iran-Nejad, 1995, para. 7). Rather, knowledge is constructed by the student when he or she makes connections with previously existing knowledge. Constructivists believe, in fact, that all knowledge is constructed (Noddings, 1990). Through the processes of assimilation and accommodation, students construct their understanding by gaining new experiences and building upon what they already know (Ernst, 1996, Iran-Nejad). Their previous knowledge becomes the "building blocks" (Ernest, 1996, p. 336) for new constructions which they make. A critical part of this epistemology is that students are "active" in the learning process, not passive recipients of knowledge, and that students must participate in the learning process in order to achieve understanding (Harris & Graham, 1994). As stated by von Glasersfeld, "knowledge is not passively received but built up by the cognizing subject" (1996b, p. 18).

Types of Constructivism

There are several varieties of constructivism. Von Glasersfeld advocated a form called radical constructivism, which states as one of its principles, "the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality" (von Glasersfeld, 1996b, p. 18). Radical constructivism is "radical" in the sense that it denies the existence of an objective reality or truth and, because of this, radical constructivism cannot even claim that the theory of constructivism itself is true (Glasersfeld, 1990; Simon, 1995). There is also a form of constructivism that advocates a social element of learning, and this is called social constructivism. According to this theory, although students construct their own knowledge, this process is not solely an individual act. Rather, social constructivists believe in the importance of social interaction in learning and constructing knowledge (Green & Gredler, 2002). Students construct knowledge through interactions with their environment, and their environment includes the people around them (Ernest, 1996).

Constructivism in Mathematics Education Research

Constructivism is one of the most common theoretical perspectives used in research on mathematics education (Ernest, 1996). The constructivist perspective influences many areas of thought on mathematics teaching, including particular teaching strategies, the importance of conceptual understanding, reasoning, and the use of technology. In addition, constructivist theories about how students acquire knowledge have influenced the mathematics reform movement. The reform movement arose in response to the 1983 publication of Bell's *A Nation at Risk*, which called for standards-based education reform. In response to this, the NCTM published the *Curriculum and Evaluation Standards for School Mathematics* in 1989. The mathematics reform movement and these standards continue to be important and to have influence today (Byers, 2004). The reform movement generally supports student-centered learning rather than direct instruction. In student-centered learning, the student is involved in discovering information while the teacher serves more as a guide. "Discovery learning" (White-Clark, DiCarlo, & Gilchriest, 2008, p. 42) is one of the strategies associated with constructivist teaching. Direct instruction, on the other hand, can be thought of as being in opposition to constructivist teaching (Iran-Nejad, 1995). White-Clark et al. have suggested that constructivism is a more effective approach, yet their research has shown that, although the constructivist approach was favored, the direct instruction approach was more prominent in both high school and college classrooms.

Constructivism and Conceptual Understanding

Conceptual understanding is fundamental in the theory of constructivism. As suggested by Iran-Nejad (1995), “constructivism requires teachers to focus on depth of understanding” (para. 11). Memorization does not consitute knowing, and “is often not successful because knowledge is not a ready-made, transferable product but, rather, a product of the learner's thinking” (Iran-Nejad, 1995, para. 39). In addition, constructivists do not believe that “rote responses represent knowledge” (Noddings, 1990, p. 13). Simply reciting pieces of information, relying on memorization, and following procedures without understanding cannot be called knowledge. Rather, simple recitation of information is called a “knowledge claim” (Noddings, 1990, p. 13). As suggested by Noddings (1990), “students need building materials, tools, patterns, and sound work habits if they are to construct mathematical objects and relationships” (p. 15). The constructivist perspective on the use of symbolic manipulation versus conceptual understanding is seen in von Glasersfeld’s comment in response to the “formalist myth that all that matters in mathematics is the manipulation of symbols” (von Glasersfeld, 1996a, p. 312). Von Glasersfeld has stated that symbols are often treated as though there were no concepts that were behind them, and therefore students are unable to understand the meaning behind actions and procedures. However, von Glasersfeld, states “spoken words or marks on paper are symbols only if one attributes to them something they symbolize, that is, a meaning – and meaning is always conceptual” (von Glasersfeld, 1996a, p. 312). In the constructivist view, then, conceptual understanding in mathematics is of great importance. A focus on conceptual understanding will help students to construct meaning.

Constructivism and Reasoning

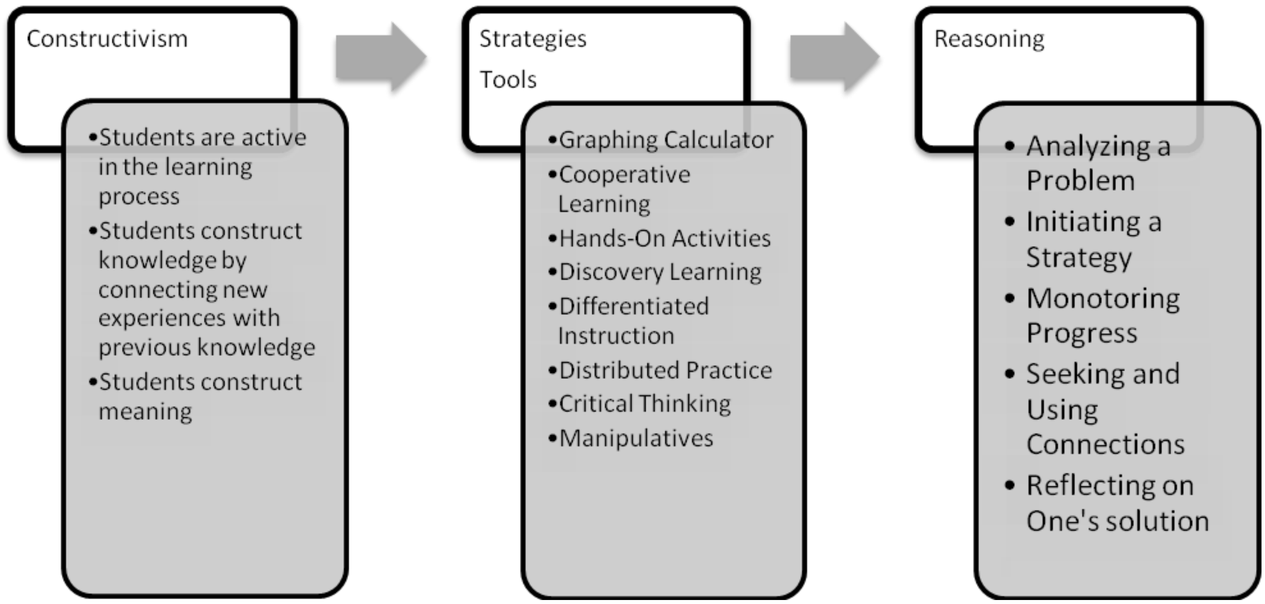
Reasoning is one important aspect of understanding (Garden et al., 2006). The constructivist view can be seen in the emphasis on reasoning skills, and the mathematics reform movement, which is influenced by constructivist theories, includes reasoning as one of its important principles for school mathematics. Reasoning, the process of drawing conclusions, is said to begin with explorations, conjectures, false starts, and partial explanations (NCTM, 2009). Constructivism encourages teachers to allow students to actively participate in learning through experimentation and exploration. When students' constructions lead to incorrect results, a constructivist would encourage the teacher to have the student explore whether the results could be accurate. The teacher would not simply provide the student with immediate criticism of the response. Reasoning also requires that students analyze a problem in order to develop strategies to solve the problem and, if necessary, to modify those strategies. In addition, it requires students to draw conclusions, question the reasonableness of a solution, make inferences, draw upon previous knowledge, and connect different concepts (NCTM). Teaching in a constructivist manner would involve asking students to analyze a problem, interpret results, classify terms or concepts, and to make predictions (Lunenberg, 1998). These activities are strongly connected to the reasoning process.

Constructivism and Technology

Use of technology is one of the strategies that is consistent with constructivist methods of teaching, and it is one way that students are able to explore concepts (White-Clark et al., 2008). Graphing calculators are tools that allow students to explore the behavior of functions, for example. The use of tools in mathematics classrooms, as explained by Hiebert et al. (1997), may have implications for students as they attempt to construct meaning. Constructing meaning with the use of tools in mathematics classrooms occurs when students work with tools for an extended period of time, explore ways to use the tools, and then see what happens as a result of using the tools. Students become familiar with the tools after working with them and constructing meaning for them. Then, the tools can be used to help solve problems. The meanings that different students may construct may be different for the same tools, though, and this is the idea behind constructivism. Students will develop different meanings for tools if they use them for simply calculating answers or if they use them for deciding why certain procedures work (Hiebert et al.). The graphing calculator is one of those tools that can be used in different ways. It can be used, on the one hand, to simply calculate an answer; however, it can also be used for exploration, an idea more consistent with constructivism. Although tools used in the mathematics classrooms extend beyond technologies such as the graphing calculator, the graphing calculator is one of the tools that may have the potential to allow students to construct meaning in mathematics. For example, students can use the calculator to "see the connections between a solution and what the word 'solution' means in terms of the graph" (White-Clark et al., 2008, p.43). Therefore, through the graphing calculator, students can connect abstract terms to what these terms actually mean (White-Clark et al.).

My study, which examines the effects of the graphing calculator on students' reasoning, is based on constructivist theories about how students learn. I emphasize the importance of conceptual understanding and reasoning in students' mathematical learning, and these aspects are supported by constructivist theories. In addition, I advocate a strategy based on constructivist theories – the use of a tool, the graphing calculator, in supporting students' mathematical learning by improving reasoning. I have constructed Figure 2 below to illustrate my approach to reasoning, with constructivism as the foundation.

Figure 2: Conceptual Framework



Methodology

This dissertation describes a mixed-method study that investigated the effects of graphing calculators on calculus students' reasoning skills when learning the definite integral and its applications. Four calculus classes, two honors and two Advanced Placement (AP), were used for the study. Two classes were from a private all-girls high school in a suburban area, and two classes were from a private co-educational high school in a rural area of the Southeast United States. The design of the quantitative part of this study was quasi-experimental because there was no random assignment to groups. The AP classes (control group) used traditional methods of graphing using pencil and paper. The honors classes (experimental group) used the graphing calculator, in addition to traditional methods of graphing, in learning applications of the definite integral and solving related problems. Both classes learned the same content. Two teachers were used for this study, the investigator and another teacher. Each teacher taught part of the control group and part of the experimental group. In addition, all classes learned the same lessons at the same level of difficulty. The difference between the two classes was only in the utilization of graphing calculators. All students completed a pretest to assess reasoning in order to identify individual differences in skill at the beginning of the study. Lessons created by the researcher on the definite integral and its applications took place in both classes over the course of a three week period. After three weeks, all students completed a reasoning assessment developed by the researcher and validated by a panel of two mathematicians. The assessment was completed by all students participating in the study. The data collected through this instrument were used to determine whether the graphing calculator has an effect on students' reasoning skills in calculus and, if so, in which specific areas of reasoning it was most and least effective. In addition, data were also analyzed to determine if students who used the graphing calculator could also solve problems using pencil and paper. After completion of this portion of the study, the AP students were instructed in the use of the graphing calculator as well and then interviews were conducted with one student from each calculus class. Observations and responses to the interview questions were scored by the researcher using a researcher-developed rubric in order to assess the reasoning processes used by students when solving problems on the definite integral and its applications when graphing calculators were used and when they were not used. The researcher also made observations during the interviews in order to identify the most effective uses of the graphing calculator and the most effective way to incorporate the graphing calculator into the calculus curriculum.

Research Questions

The following research questions guided the study:

1. Does the use of the graphing calculator improve high school calculus students' reasoning ability in calculus problems applying the definite integral?

2. In what specific areas of reasoning does use of the graphing calculator seem to be most and least effective?
3. To what extent can students who have used the graphing calculator demonstrate ability to solve problems using pencil and paper methods?

Research Hypothesis

H_0: The mean reasoning score of those who use pencil and paper graphing methods (control group) is greater than the mean reasoning score of those who use graphing calculators (experimental group) ($\mu_C > \mu_E$).

H_A: The mean reasoning score of those who use graphing calculators (experimental group) is greater than the mean reasoning score of those who use pencil and paper graphing methods (control group) ($\mu_C < \mu_E$).

Overview of the manuscript

In Chapter One, I have provided an overview of the topic of this dissertation, the purpose of this study, and the importance of the study to the research community. In Chapter Two, I provide a thorough review of the literature related to the main areas addressed in this dissertation, including conceptual development in calculus, reasoning in calculus and other mathematics courses, and the use of graphing calculators in calculus and other mathematics courses. The review of the literature on conceptual development in calculus and the use of graphing technology in calculus and other mathematics courses begins with reports and research conducted since 1986, the time that the calculus reform movement was established.

The review of the literature on reasoning also begins with reports and research conducted since 1989, when the NCTM established the curriculum and evaluation standards for mathematics. However, much of the specific information currently known about specific elements of reasoning is based on a recent report published by the NCTM in 2009.

Chapter Three provides a detailed description of the methodology used in conducting this study, including sampling and the selection of participants, research questions, instrumentation, procedures, data collection, data analysis procedures, ethical considerations, and limitations. Chapter Four provides a detailed analysis of the data, including that from posttest scores and from interviews, and report of the results and findings from the study. Chapter Five provides an in-depth interpretation of the results and discussion of the implications of the study.

CHAPTER 2:

REVIEW OF THE LITERATURE

This chapter provides a review of the literature relating to the main topics of this paper, which include reasoning, conceptual understanding in calculus, and graphing technology in calculus curriculum and instruction. The review begins with the literature on the inclusion of reasoning in the mathematics curriculum, with a look at the types of reasoning opportunities that students are provided with and how these skills can be improved. A review of the literature on conceptual understanding in calculus follows this, with a description of the common difficulties experienced by calculus students, the suggested causes of these difficulties, and the solutions that have been offered through commentaries and research studies. The review of the literature will conclude with an analysis of research that has focused on the inclusion of graphing technology, mainly handheld graphing technology, or graphing calculators, in the calculus curriculum. The focus will be on how graphing calculators could be used to improve reasoning in calculus courses, so that conceptual understanding can be achieved.

Reasoning in Mathematics

Reasoning is an important aspect of mathematical learning that has been recommended for inclusion in the mathematics curriculum. Reasoning is defined by several researchers. Reasoning is defined by the NCTM (2009) as “the process of drawing conclusions on the basis of evidence or stated assumptions” (p. 2). Kulpa (2009) defines reasoning as “knowledge processing” (p. 75), and Lithner (2000) defines it as “the line of thought, the way of thinking, adopted to produce assertions and reach conclusions” (p. 32). In addition, Garden et al. (2006)

describes reasoning as the processes of making conjectures, forming logical deductions from a set of assumptions, and justifying results. Although several different definitions for reasoning are provided by researchers, they all seem to involve the same general processes. In general, reasoning involves a higher level of thinking that leads one to form conclusions based on some initial set of assumptions. Adaptive reasoning, which is defined as the "capacity for logical thought, reflection, explanation, and justification," (p. 116) is included by the National Research Council (2001) as one of the five strands of mathematical proficiency. The Council suggests that students who cannot reason in mathematics are "cut off from whole realms of human endeavor" (2001, p. 16). Learning in mathematics, therefore, is not just about finding answers but about the ability to interpret them and reflect upon them. This is one area of reasoning.

In order to understand the process of reasoning, one must differentiate between routine and non-routine problems. Garden et al. (2006) explains the difference. Routine problems are those that students are familiar with and encounter often in a mathematics course. Non-routine problems, on the other hand, are those that students do not often encounter and are not familiar with solving. These types of problems require a level of thinking beyond that of routine problems, and as explained by Schoenfeld (1992), non-routine problem solving can occur after routine problem solving. Although reasoning skills are used to solve both types of problems, it is the non-routine problems for which reasoning skills are most useful. As explained by Garden et al., reasoning is used in solving both types of problems when students apply their existing knowledge to a new situation that they encounter; however, reasoning skills may be used in different ways for solving routine or non-routine problems.

Types of Reasoning

Reasoning takes more than one form. It can range from formal reasoning, often referred to as mathematical reasoning, or deducing and making conclusions on the basis of assumptions, to informal reasoning (NCTM, 2009). The process of reasoning, as explained by the NCTM, incorporates both types – formal and informal. Evans and Thompson (2004) classify formal reasoning as falling into two categories – deductive reasoning and statistical inference. They argue that formal reasoning is researched to a much larger extent than informal reasoning and that not enough emphasis is placed on the latter, since the majority of reasoning that occurs is informal. Kulpa (2009), who describes formal reasoning as “objective and rigorous,” (p. 76) also suggests that most reasoning used in solving problems is not usually purely formal. Another type of reasoning described by researchers is plausible reasoning. Plausible reasoning, as explained by Lithner (2000), “is founded on mathematical properties of the components involved in the reasoning” (p. 167) and “is meant to guide toward what probably is the truth, without necessarily having to be complete or correct” (p. 167). Lithner calls plausible reasoning a type of “mathematically well-founded reasoning” (Lithner, 2004, p. 407) The NCTM describes plausible reasoning as a type of reasoning used often in statistics and states that plausible reasoning is included in what is generally called mathematical reasoning. Two other types of reasoning described by Lithner (2004) are repeated algorithmic reasoning and reasoning based on established experiences. These are both examples of what Lithner (2004) calls superficial reasoning. In repeated algorithmic reasoning, the strategy that is followed is basically following algorithms or a set of procedures based purely on memory recall. Reasoning based on established experiences is based solely on an individual’s previous experiences but shares a commonality with plausible reasoning in that it guides toward the truth but may not be complete or correct.

These two types of reasoning, however, are not the types that are useful in non-routine problem solving.

Components of Reasoning

Different sources describe various components and steps involved in the reasoning process. Still, there seems to be close commonalities between the descriptions provided by all of these sources. Just as there are various definitions of reasoning that seem to share a common thread, so are there various components, which seem to all be related.

As explained by the NCTM (2009), there are five main areas which comprise reasoning, and these include analyzing a problem, initiating a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution. The first area, analyzing a problem, consists of looking for hidden structures, identifying patterns and relationships, making connections, making deductions, and determining whether a solution is appropriate. The second area, initiating a strategy, consists of selecting the appropriate concepts, representations, and procedures, making purposeful use of procedures, organizing solutions, and making logical deductions. The third area, monitoring one's progress, includes reviewing the strategy that one has chosen, making further analysis of a problem when necessary, and modifying selected strategies if necessary. The fourth area, seeking and using connections, includes connecting different mathematical domains, connecting different contexts, and connecting different representations. The last area, reflecting on one's solution, consists of interpreting the solution, assessing the reasonableness of solutions, justifying solutions, considering alternative ways of solving problems, generalizing solutions, and understanding the nature of a statistical conclusion (NCTM).

Other sources seem to provide some of the same components of reasoning, although they may describe them in more or less detail. In addition, the components described sometimes are given a different name but generally involve the same skills. Lithner (2000), for example, suggests that reasoning in non-routine problems includes four components. These include coming into contact with a problem, choosing a strategy, implementing the strategy, and reaching a conclusion. These steps share similarities with the steps involved in the reasoning process described by the NCTM.

Garden et al. (2006) also provide a list of components of the reasoning process. These are to analyze, generalize, synthesize and integrate, justify, and solve non-routine problems. Analysis involves using the information that is given in a problem in order to determine which facts are necessary to solve the problem as well as identifying and using the relationships that exist between the objects in a problem. Generalizing involves applying the results of a problem to more general situations. Synthesis and integration involves using different mathematical concepts and representations and connecting them in a way that will produce results. Justifying involves determining whether a statement is true or false and providing a justification for the result using mathematical properties. Solving non-routine problems involves using mathematics in often unfamiliar ways to solve problems that are both mathematical and in real-life contexts.

Reasoning in the Content Areas

The discussion on the inclusion of reasoning, although existent for many years, has in the past 20 years become a highly debated topic with the publication of standards by the NCTM highlighting the importance of reasoning in the mathematics curriculum. Furthermore, the importance of reasoning has been emphasized to a greater extent very recently with publication of the NCTM's *Focus in High School Mathematics: Reasoning and Sense Making* in 2009. In this document, they suggest that reasoning be included in the mathematics curriculum in specific content areas that include numbers and measurements, algebraic symbols, functions, geometry, and statistics. Reasoning with numbers and measurements include various aspects such as judging the appropriateness of solutions, choosing units, understanding the inherent error that exists in measurements, understanding number systems, and applying techniques of counting. Reasoning with functions includes understanding multiple representations of functions, developing mathematical models, and analyzing the effects of changing parameters in functions. Reasoning in geometry consists of making conjectures, making deductive arguments, choosing geometric approaches, and using geometry in other disciplines and in real-world situations. Reasoning with statistics and probability includes recognizing patterns and relationships through multiple representations of data, developing probability models, understanding the role of probability in statistical reasoning, and interpreting the results of statistical studies (NCTM, 2009). All of these areas form the basis for the concepts that are learned in calculus courses, and knowledge in all of these areas is necessary for problem solving in higher mathematics courses, such as calculus.

The NCTM (2009) suggests that recommendations for inclusion of reasoning skills in the high school curriculum apply as well to the undergraduate curriculum in college. The CUPM (2004) has provided a list of recommendations for the undergraduate mathematics curriculum. Among these recommendations it is stated that "every course should incorporate activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind" (CUPM, 2004, p. 7). In particular, this includes the ability to perform tasks such as stating the problem, modifying the problem when needed, stating assumptions, reasoning logically to reach conclusions, interpreting the results, using multiple approaches to problems, evaluating whether solutions are correct, posing questions, creating and testing conjectures, and communicating ideas. Problem solving, as defined by the CUPM, "does not mean following recipes – it requires the application of careful, organized, creative thought to the analysis of complex and often ill-defined questions in a diverse array of solutions" (p. 21). It requires activities and methods such as logic, proofs, counterexamples, visualization, intuition, and memory retrieval. For this reason, problem solving is deeply connected with the act of reasoning. The CUPM has further stated that many students in college have "underdeveloped reasoning skills" (p. 21). They emphasize the importance of improving these skills but suggest that much needs to be discovered about how this can be done and through which strategies this can occur.

Research on Reasoning

Most of the available literature on reasoning is not empirical in nature, but rather speculative. In order to find information on reasoning in calculus, I searched for relevant literature beginning in 1989, since it is at this time that the NCTM established the curriculum and evaluation standards. Consideration of reasoning is important in the discussion of calculus, and the calculus renewal movement has emphasized the importance of improving reasoning (CUPM, 2004). However, in general, previous empirical research has not addressed how reasoning in calculus can be improved.

Although it seems overall that reasoning is widely recommended for inclusion in the mathematics curriculum, reasoning is not usually given consistent attention in the mathematics curriculum (NCTM, 2009). As stated by Fuson et al. (2005), "mathematics instruction often overrides students' reasoning processes, replacing them with a set of rules and procedures that disconnects problem solving from meaning making" (p. 37). They suggest that students are sometimes taught one correct method for solving a problem and that this does not encourage the development of students' reasoning skills (Fuson et al.).

Kulpa (2009) identifies one type of informal reasoning, which is the diagram, but states that this tool is often thought of as unreliable. Kulpa states that there are two main problems with using diagrams for reasoning. These include a lack of generalizability and the diagram imprecision problem, which suggests that diagrams are not precise ways to solve problems. Kulpa argues, however, that diagrams can be used as reliable tools for solving problems and that more research needs to be done to examine how this can occur.

In 2000, Lithner studied three university students in an attempt to understand their reasoning through mathematical tasks. The tasks were related to calculus and included both routine and non-routine components. The researcher found that, in general, students seemed to choose reasoning based on established experiences to complete these tasks rather than plausible reasoning. However, the students experienced difficulty, and Lithner attributed this difficulty to their inability to use other reasoning approaches like plausible reasoning. This study seems to point to the idea that greater attention should be paid to helping students in solving mathematical tasks using the different types of reasoning. The students in this study seemed to only use strategies that they were familiar with; however, reasoning involves solving problems that one is familiar with as well as those that one is unfamiliar with. The students in this study did not appear to have the appropriate reasoning skills to do this. They seemed to only engage in what Lithner (2004) would refer to as superficial reasoning. Since this study only looked at three students, however, the results are not generalizable to a larger population.

In 2004, Lithner conducted a related study in which he investigated the types of reasoning required in calculus textbooks in Sweden. The researcher also looked at American calculus textbooks and found them to be similar. Lithner found that 90% of the textbook exercises required types of reasoning that involved students merely looking for similar examples or recalling information. The researcher concluded that the textbook did not have enough practice of global plausible reasoning skills, which he surmised could lead to a lack of conceptual understanding and poor problem solving strategies.

Overall, the research seems to point to the idea that reasoning is not given consistent attention in many mathematics classrooms and curricula, and that students may lack reasoning abilities when they enter college (CUPM, 2004). In addition, there is also a need for research on the strategies that may be used to improve reasoning skills.

Conceptual Understanding in Calculus

Conceptual understanding is a critical component to success in most mathematics courses. In *Adding It Up* (2001), the National Research Council outlines five strands of mathematical proficiency, and one of these strands is conceptual understanding, which the council defines as "comprehension of mathematical concepts, operations, and relations" (p. 5). Conceptual understanding in calculus, in particular, is important as well and can be defined as knowledge and understanding of the main concepts involved in calculus, including the limit, the derivative, and the integral, and how these concepts tie in to one another. This is in contrast to learning that focuses on manipulative techniques and algebraic methods of problem solving. In order to find information on this topic, I searched for literature that included commentaries from prominent mathematics teaching organizations, such as the NCTM, and research studies that have been conducted on understanding in calculus. I began my search with literature dated from 1986 to the present, since it is at this time that the Calculus reform movement was established.

An analysis of the literature on conceptual understanding in calculus is necessary in order to understand its importance to success in calculus courses.

Conceptual Understanding versus Algorithmic Techniques

The reform movement in calculus was established in order to develop ways to improve success rates in calculus courses. Conferences began taking place in 1986, which focused on conceptual understanding in calculus, use of technology in calculus, and viewing calculus as a "pump" rather than a "filter" (Cadena, Travis, & Norman, 2003, p. 1). Since this time, many calculus curricula have changed to reflect these aspects, and organizations focused on reform mathematics, such as the NCTM, have been dedicated to improving calculus instruction. The NCTM has emphasized that mathematics instruction can be improved through understanding, and it has defined four areas that contribute to students' understanding. These include reasoning, communicating, drawing connections, and solving real problems (Hiebert, et al., 1997).

The NCTM (1989) has suggested that calculus instruction, in particular, focus on conceptual understanding and not algebraic manipulation. The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics, or CUPM (2004), has recommended that undergraduate mathematics instruction focus on developing students' understanding of mathematical concepts rather than on procedures and computation. The CUPM has further stated that there is too often an overemphasis on problem solving by way of following specific procedures without true understanding of the subject. The National Research Council (2001) suggests as well that, although learning in mathematics is dependent upon understanding, memorization is often the focus, and Schoenfeld (1992) believes that treating mathematics as simply memorizing facts, formulas, and procedures "trivializes mathematics" (p. 3). Other researchers have concluded the same (Schwalbach & Dosemagen, 2000; Slavit et al., 2002).

Kissane and Kemp (2008) suggest that "although there is an emphasis on procedures in calculus in some classrooms, and an unavoidable emphasis on formal procedures in examinations, the concepts of calculus are the most important elements" (p. 2). Although much emphasis is placed upon conceptual understanding, however, this does not mean that the importance of procedural knowledge is diminished. In *Mathematical Understanding: An Introduction*, Fuson et al. (2005) outline three principles about "how people learn" (p. 37). The second principle states that developing an understanding of mathematics and obtaining mathematical proficiency requires both conceptual knowledge and procedural knowledge. Both are a necessary part of success in mathematics.

A variety of literature is available on the topic of conceptual understanding in mathematics, including calculus. This literature is both empirical and speculative, and the consensus among researchers and commentators alike seems to support the need for a focus on conceptual understanding. Calculus courses supported by the College Board Advanced Placement program also reflect the emphasis on conceptual understanding over algebraic manipulation. The course description for AP Calculus states learning goals, which include understanding connections among the different representations of functions, understanding the meaning of key concepts such as the derivative and integral, developing the ability to solve problems through derivatives and integrals, understanding the relationship between key concepts, developing the ability to communicate mathematics, modeling with functions, differential equations, and integrals, using technology to solve problems, experiment, interpret results, and draw conclusions, and determining the reasonableness of solutions (CollegeBoard, 2009). These learning goals reflect the importance of understanding.

Problems with Algorithmic Techniques

Educators and researchers in the field of education consistently attempt to develop ways to improve students' understanding and success in calculus courses. Calculus is a subject that has presented obstacles for many students, as the concepts involved are very different from most other mathematics courses that students have participated in. Judson and Nishimore (2005) describe the barrier faced by many students who take part in calculus. In their study of high school calculus students from both Japan and the United States, Judson and Nishimore found that the students thought calculus to be presented as more of a recipe book than a subject in which they could achieve a sound conceptual understanding. In both countries, students had a weak understanding of concepts such as the function and the fundamental theorem of calculus, and students lacked the ability to link calculus concepts in order to solve problems.

Many researchers have cited the difficulties faced by students because of their lack of conceptual understanding in calculus. Baker et al. (2000) found that calculus students at a large Midwestern university were able to retain certain concepts but not others. The main problem was in students' inability to see the first derivative as a function and thus had a misconception about the second derivative. They did not understand the relationship between the first and second derivatives. After this study, the researchers concluded that students have less of an understanding of calculus than originally believed. Since many other researchers had reached these same conclusions, Baker et al. surmised that this was a widespread problem and that a solution must be reached.

Educators and researchers have suggested that the problem with students' lack of understanding may be due to the emphasis that many calculus curriculums place upon manipulative techniques of problem solving. Less attention has been placed on conceptual understanding despite the recommendations that this approach would be helpful. White and Mitchelmore (1996) studied calculus students at a university in order to assess their ability to solve application problems and to observe the role of their conceptual understanding in solving these problems. They concluded that students lacked an understanding of concepts such as the variable and mainly saw variables as symbols to manipulate rather than knowing their meaning.

In order to solve the problem of this lack of conceptual understanding, organizations dedicated to improving mathematics instruction have offered some suggestions for improving the way that students learn calculus (CUPM, 2004; National Reseach Council, 2001), and they have stated that students will only be able to make sense of the mathematics if they understand the concepts that are involved (NCTM, 2009). As explained by Garden et al. (2006), understanding in mathematics involves operating in three cognitive domains – knowing, applying, and reasoning. Knowing involves mathematical facts, procedures, and concepts. Applying involves the use of knowledge to solve problems. The last cognitive domain, reasoning, involves solving non-routine problems and "the ability to use analytical skills, generalize, and apply mathematics to unfamiliar or complex contexts" (Garden et al., p. 17). Therefore, the process of reasoning is an integral part of understanding in mathematics.

Conclusion

The importance of conceptual understanding in mathematics, and particularly calculus, has been well-established by educators and researchers. The NCTM (2009) suggests that, in order to improve conceptual understanding, the mathematics curriculum be refocused on the development of reasoning and sense making skills. Without these skills, problem solving is difficult or even impossible. Problem solving, as described by Ellington (2003), involves two important components, which are the quantity of problems that are attempted and the ability to select the appropriate strategy. The NCTM states that the process standards, which include problem solving, reasoning and proof, connections, communication, and representation, are all based on the acts of reasoning and making sense of mathematics. Students' difficulties in mathematics, they claim, stem from their inability to see the meaning in mathematics and to make sense of it.

Handheld Graphing Technology in the Calculus Curriculum

The final section of this review of the literature focuses on the graphing calculator and its uses in mathematics courses, with particular attention placed on its uses in calculus classrooms. A review of the literature on the graphing calculator in calculus classrooms is necessary, as the purpose of this study will be to evaluate its use in specific ways, specifically its effect on reasoning skills in calculus. The literature review will focus on ways that the graphing calculator has been used in calculus classrooms and the benefits that it has provided to students learning calculus. The review of the literature on the uses of graphing calculators in calculus begins in 1986 since the calculus reform movement was established at this time.

Technology in the Mathematics Classroom

It is widely believed, with the changes that are taking place in society due to increasing use of technology, that the classroom reflect these changes and integrate technology as well into the way that students learn (Burrill et al., 2002). Different types of technology, such as computer software, graphing calculators, and the internet, have changed the way that students learn and offer the potential to increase learning in the classroom. Specifically in mathematics, it is suggested that technology can improve mathematical learning specifically in the areas of problem solving, concept and skill development, reasoning, and communication (Kimmins & Bouldin, 1996). Among the recommendations provided by the CUPM (2004), it is stated that "at every level of the curriculum, some courses should incorporate activities that will help all students progress in learning to use technology" (p. 13) as a tool for both problem solving and developing understanding of mathematical ideas. This review focuses mainly on handheld graphing technology, or graphing calculators.

Types of Graphing Calculators

Graphing calculators were first seen in 1985, when they were developed by Casio, and later were developed even further by Texas Instruments in 1995. With the invention of graphing calculators came a new way to deal with mathematics that provided access to mathematical problem solving that, before this time, could only be done on the computer. Some graphing calculators offer a CAS, or computer algebra system, that includes numerical solving, graphing, and symbol manipulating capabilities (Waits & Demana, 1998).

There are several varieties of graphing calculators, but all graphing calculators have certain functions and capabilities in addition to computation such as graphing, viewing tables, and running programs and applications. In addition, these calculators have a calculate function for computing roots, derivatives, extrema, and definite integrals. The TI-83 Plus and TI-84 Plus are two of the most basic graphing calculators that have all of these built-in functions. In addition, all graphing calculators have the ability to run programs input by the user or transferred to the calculator by connecting it with other calculators or by downloading from the Texas Instruments (TI) website. These programs add to the functionality of the calculator. Some more advanced graphing calculators, such as the TI-89 Titanium, have all of the functions of the TI-83 Plus and TI-84 Plus with the additional capabilities of graphing in three dimensions and symbolic manipulation. The most recent handheld graphing technology from Texas Instruments is the TI-Nspire. These graphing calculators have all of the capabilities of other graphing calculators in addition to the ability to view multiple representations on the same screen, to construct and animate geometric figures, and to receive documents that allow visualizations of solids of revolution.

Uses of Graphing Calculators

The CUPM (2004) suggests that technologies, and in particular handheld technologies, such as the graphing calculator, have impacted the way that mathematics is taught and can improve mathematics teaching. The National Research Council (2001) has also suggested that graphing calculators can be used to enhance conceptual understanding. The CUPM further states that, in calculus courses in particular, graphing utilities can improve the time spent on graphing functions and provide multiple approaches to functions, including graphical, numerical, and algebraic approaches, and they suggest that most calculus students do benefit from the availability of a graphing utility. One of these benefits is that of visualization. Visualization through the graphing calculator can help students to understand calculus concepts such as Riemann sums and Taylor polynomials (CUPM).

Since the invention of the graphing calculator, they have been increasingly integrated into mathematics classrooms and have become a required part of some mathematics programs. Programs such as the College Board Advanced Placement program, for example, require the use of the graphing calculator on some portions of the Calculus AP test taken at the end of the course. The College Board recommends that the graphing calculator be included in everyday teaching and learning of calculus and that instruction focus on specific ways to use the graphing calculator, including the ability to plot the graph of a function, find the roots of a function, calculate the derivative of a function, and calculate the integral of a function (NCTM, 2009). In addition, calculus teachers have increasingly used the graphing calculator in their own classrooms to increase understanding and help students make transitions from examples to proofs (Hurwitz, 1997, Touval, 1997).

Recommendations for Use

Those dedicated to calculus reform have emphasized that the use of technology to enhance learning in calculus be included throughout the curriculum. The Harvard Consortium Calculus, which was an approach to calculus based on the elements of reform, suggests an approach to calculus using "the rule of three," which includes the graphical, algebraic, and numerical approaches. It encourages students to look at problems from multiple perspectives, and it also encourages the use of technology for visualization in calculus (Knill, 2004). The NCTM, which is also dedicated to mathematics reform, offers some specific ways in which the graphing calculator can be used in mathematical learning and in calculus. The NCTM (2000) states that calculators and computers are "essential tools for teaching, learning, and doing mathematics" (p. 24). These tools offer benefits such as visualization of concepts, the ability to perform accurate computations, and analysis of data. The council suggests that, when tools such as these are available to students, they can spend time focusing on other activities, including problem solving and reasoning. In addition, graphing calculators can allow students to see multiple representations of functions in more ways than are either practical or possible by hand (NCTM). There are also specific ways in which the graphing calculator can be used in calculus instruction. According to the NCTM, calculus instruction should provide exploration of concepts from both a graphical perspective and a numerical perspective. This can be achieved through the use of both computer and calculator technology, allowing students to find maximum and minimum values of a graph and interpret the results, explore the concept of the limit by looking at areas under curves and infinite sequences and series, explore the concepts of limit, rate of change, area under a curve, and slope of the tangent line, and analyze the graphs of functions. The concepts of derivatives and integrals can be easily explored with the graphing calculator, in that it allows students the ability to calculate approximations of the area under a curve and to magnify a small part of a curve, making it look like a line segment, so the slope of the curve can be calculated. In addition to the major calculus concepts, such as limits, derivatives, and integrals, other important concepts essential to knowledge in calculus, such as continuity, asymptotes, end behavior, and concavity can also be explored through the graphing calculator (NCTM, 1989). Anderson et al.(1999) suggest that the ways in which graphing calculator can be specifically used in calculus include graphing functions, approximating derivatives and integrals, and in finding limits, roots, and extrema.

Research on Graphing Calculators

Graphing calculators have also been used to help students learn some of the underlying concepts that are important to understanding of calculus. Understanding the concept of the function, for example, is an important component to success in higher level mathematics courses such as calculus (Slavit et al., 2002). Mesa (2007) investigated the problem solving methods used by pairs of students, when working with functions and found that problem solving methods did not differ very much when graphing calculators were used; however, there were benefits associated with the use of the graphing calculator, in that students who used the graphing calculator were able to work with more functions in less time and thus had more time for exploration. Graphing calculators can be used in assessing students' understanding of functions as well. Beckmann et al. (1999) suggest that graphing calculators provide access to multiple types of representations and allow students to see how one can switch between algebraic, numerical, and graphical forms of functions. Students can use the multiple representations to solve problems, explain their reasoning, and to explain the meaning of the solution in the context of the problem. The CollegeBoard suggests that use of the graphing calculator can assist students in reasoning through their solution to a problem and in determining the reasonableness of their solution (CollegeBoard, n.d.).

On the other hand, it has also been suggested that students must be able to assess the reasonableness of solutions provided on the graphing calculator and to make sense of what is provided to them through the calculator (Rivera, 2007). Rivera conducted a qualitative study of precalculus students as they developed a graphical process for solving polynomial inequalities using the TI-89 graphing calculator. The researcher's goal was for students to make sense of solving polynomial inequalities without being given an algebraic procedure for doing so. Certain limitations of the graphing calculator, such as the capability to receive only explicit equations as opposed to implicit equations, required the students to rely on some algebraic procedures in addition to graphing; however, students used the calculator to explore the ideas of zeros, intercepts, and inequalities, and exploration through the graphing calculator led the class to draw conclusions about how to solve inequalities. Students used features of the graphing calculator such as the trace feature, the table, the math function, and the algebra function. In addition to using the calculator to solve problems, students referred to their experience using the calculator to graph functions and solve problems without the calculator. Rivera concluded that students used the graphing calculator as a "psychological tool" (p. 16) and that students were able to extend the possibilities for solving problems using this tool. The researcher did comment, however, that at times students used the tool for calculations that they could have done on paper.

The literature, in general, seems to support the use of graphing calculators in learning the concepts leading up to calculus as well as those taught in calculus. Porzio (1997) investigated the effects of three different approaches, including two technological approaches, to calculus among university students. These approaches consisted of the traditional approach, a graphing calculator approach, and an approach using Calculus and *Mathematica,* an electronic calculus course. Porzio found that the students who used the traditional approach had the most difficulty using graphical representations and understanding the connections between the numerical, graphical, and symbolic representations, whereas students who used the graphing calculator were "proficient" at using graphical representations but still had some difficulty understanding the connections between the representations. Thus, while students who used the graphing calculator still exhibited some difficulty, they still showed better understanding of the graphical representations than the students using the traditional method. Brosnan and Ralley (1995) also found positive effects associated with the use of the graphing calculator when they investigated a calculus teacher's perception of the impact of a reform-based calculus course on college students' mathematical learning. The graphing calculator was believed to benefit students' learning by providing new and different representations. The ability to provide multiple representations appears to be a common benefit of using the graphing calculator expressed in the literature.

All graphing calculators have the ability to provide multiple representations, and in particular, the new TI-Nspire graphing calculator has the ability to show multiple representations on the same screen. The TI-Nspire and other graphing calculators have other capabilities as well, that have been explored in research. One of these is data collection. In a mixed-methods study, O'Mahony, Baer, and Quynn (2008) investigated the effects of the TI-Nspire graphing calculators on high-school students' understanding in a hands-on discovery learning activity that connected math and science. Data collection in this study occurred in the control group through pencil and paper and in the experimental group through the use of the TI-Nspire graphing calculators. The researchers found that all participants showed an increase in knowledge but those that worked with the graphing calculator demonstrated understanding and the ability to draw inferences. In addition, lower-achieving students who worked with the graphing calculators seemed to show the most improvement.

The impact of graphing calculators on learning in mathematics has been the focus of several studies. Meta-analyses have been conducted to organize and synthesize the results of these various studies. Burrill et al. (2002) conducted a meta-analysis of 43 studies investigating the use of handheld graphing technology in mathematics instruction and suggest that there is “a lack of cumulative research in any one area related to handheld graphing technology” (p. 6). They found that after graphing, the three main ways in which graphing calculators are often used by students include as a computational device, a tool for visualization, and as a way to switch between different representations. The researchers reported that few studies addressed what students’ might do differently when there is access to the graphing calculator.

The meta-analysis conducted by Burrill et al. included several studies in which the graphing calculators used included a CAS. They found that students who used calculators with a CAS were better at applying calculus concepts. However, the calculators seemed to be more effective for lower-achieving students in achieving accuracy but not in conceptual development. Although the studies included in this meta-analysis varied in terms of the subject matter, some focused only on calculus instruction, and these ranged from qualitative, descriptive studies to quantitative, experimental studies. Results of these studies varied, with some showing no effect and some showing a positive effect associated with graphing calculator use. Results also showed that under-utilization of the graphing calculator was a concern and that graphing calculators were usually beneficial for lower-achieving students, a common finding in the literature. Ellington (2006) also conducted a meta-analysis of forty-two studies investigating the use of graphing calculators in mathematics instruction; however, Ellington's study differed from that of Burrill et al. in that it only included graphing calculators that did not include a CAS. In addition, Ellington only included studies that included a control group and treatment group, where the treatment consisted of use of the graphing calculator. The researcher's goal was to determine the effects of non-CAS graphing calculators on procedural skills, conceptual skills, overall achievement and attitude. The meta-analysis looked at studies of middle school, high school, and college and included courses ranging from algebra to calculus; however, 93% of the studies included in this meta-analysis included algebra and precalculus courses, and thus few focused on calculus. Ellington found that, when calculators are included in instruction but not testing, there is no benefit for students' ability to apply formulas and procedures, but the calculator is beneficial for student understanding. In addition, the researcher found no effect on overall mathematics achievement. Ellington found, on the other hand, that when calculators are included in both instruction and testing, there is a positive impact on students' performance in procedural skills, conceptual skills, and overall achievement. Results also showed a positive impact on students' attitudes toward mathematics. The researcher concluded overall that graphing calculators are beneficial in learning mathematics.

Reznichenko (2007) also conducted a review of the literature from 1990 to 2006 on the use of computers and calculators in algebra and calculus. The review revealed some studies that found positive results and some that showed no difference when calculators and computers are used in mathematics instruction. Those studies finding no difference in calculus involved only the use of computers and not graphing calculators. The researcher found that graphing calculators can be used in several ways to enhance the teaching and learning of calculus and to improve understanding of functions, modeling, and problem solving. In addition, the graphing calculator was found to be useful in symbolic manipulation, graphing functions, data analysis, collaborative learning, in problem-solving involving interpretation, and in visualizing mathematical theories.

Visualization of calculus concepts seems to be a common benefit of using a graphing calculator approach to calculus. Galindo (1995) investigated the differences between visualizers and non-visualizers in a traditional calculus course at a university, a course that used the graphing calculator, and a course that used the software Mathematica. Results of the study showed that non-visualizers actually performed better than visualizers in the sections using the traditional approach and the software program, but in the section using graphing calculators, there was no significant difference in achievement. This may indicate that the graphing calculator could provide advantages over the traditional approach for those who prefer to visualize calculus concepts.

Research has also addressed the potential of the graphing calculator to improve understanding in calculus. Lauten, Graham, and Ferrini-Mundy (1994) investigated the use of graphing calculators in college and high-school Advanced Placement calculus courses. They investigated students' understanding of functions and of the concept of limit through interviews with students participating in a calculus course where graphing calculators were used. The researchers found that, when the graphing calculator was used, students interviewed seemed to be more comfortable working with piecewise functions and seemed to display understanding of the concept of the limit, since the graphing calculator allowed them to trace along the curve. This was a qualitative study, however, that did not address the five elements of reasoning defined by the NCTM. In addition, this study was limited to the concepts of function and limit and did not address the definite integral.

The National Research Council (2001) suggests that “in a society saturated with advanced technology, people will be called on more and more to evaluate the relevance and validity of calculations done by calculators and more specific machines” (p. 16). As technology advances and students have access to calculators and computers, the ability to reason is becoming more important. Although the use of graphing calculators has been widely investigated in mathematics education research, little research has focused on their use in aiding reasoning skills. Magnani and Dossena (2005) suggest that magnification of diagrams should be studied in the context of reasoning. This is one of the tasks that can be approached through the graphing calculator. Magnani and Dossena recommend the use of optical diagrams in perceiving the “infinite and the infinitesimal” in calculus by zooming into these diagrams. They state that “the role of optical diagrams in a calculus teaching environment seems relevant” (p. 21). Scheuneman, Camara, Cascallar, Wendler, and Lawrence (2002) investigated the impact of calculator access, use, and type on performance on the SAT I: Reasoning Test in Mathematics. They used information from the SAT I tests administered in 1996 and 1997. They found that graphing calculators were used often and were second only to scientific calculators, although they reported that graphing calculator use did increase between the first and second test. They also found that those who used graphing calculators performed better than those who used other types of calculators or those who did not use calculators at all; however, they believed that the more able students were the ones with access to the calculators. The researchers concluded that students who are familiar with using the graphing calculator may use different strategies that might help them to perform better. In this study, use of calculators, and graphing calculators in particular, seemed to help students perform in the area of reasoning; however, this study did not focus on reasoning in calculus. It may, however, point to the idea that graphing calculators could be helpful in improving reasoning in calculus.

Appropriate and Effective Use of the Graphing Calculator

Although the graphing calculator appears to provide benefits for students in calculus and other mathematics courses, it is only able to provide advantages to the extent that is used appropriately, in ways that enhance learning and do not replace thinking. A study conducted by Alkhateeb and Wampler (2002) investigated the impact of graphing calculator use on students' understanding of the derivative at a point in a university setting and showed that the graphing calculator made a difference only for women. The researchers attributed the lack of a significant difference for the two groups as a whole on a curriculum that was not designed for use of the graphing calculator. Burrill et al. (2002) suggest that, although little research involving graphing calculator use focuses on equity, of those that do, some show differences in achievement based on factors such as gender, race, socio-economic status, and prior knowledge, and others show no significant difference. Burrill et al. do conclude, though, that continued access to graphing calculators has a positive effect on learning and that this effect is greatest for lower-achieving students.

A significant factor in the success of many educational technologies, including graphing calculators, is the teacher. Doerr & Zangor (2000) reported positive effects of the graphing calculator as a result of the knowledge and competence of the teacher. Stick (1997) recommends that both students and the teacher have to demonstrate an enthusiasm for using technologies like the graphing calculator in order to gain the most from activities. Burrill et al. (2002) state that teachers' attitudes toward mathematics play an important role in their use of the graphing calculator. The ways in which teachers use the calculator are dependent upon whether their approach to mathematics emphasizes conceptual understanding or computation. Burrill et al. (2002) state, “teachers who emphasize connections among representations and sense making in working with both the mathematics and the tool see the results in the performance of their students” (p. 7).

Appropriate use of the graphing calculator appears to be a common concern for mathematics educators and researchers (Kissane & Kemp, 2008; Lifshitz, 2004). It is important for students and teachers to learn ways of using the graphing calculator that will enhance learning and not cause an overdependence on the calculator. The CUPM (2004) recommends that instructors must decide how to use technology in a way that is appropriate for the lessons as well as for the students and that, at times, modifications should be made to achieve the most effective use of the technology. The CUPM further states that students should not use technology as a “crutch” (p. 24) to complete tasks that could be easily done by hand. They recommend that students should be trained in the use of technology so that the mathematics is the focus rather than how to use the technology. Kissane and Kemp describe multiple ways that the graphing calculator can be used to teach various calculus concepts. Through the use of the graphing calculator, new ways of learning these concepts can be explored; however, it is also necessary to use a curriculum and set of materials that properly integrate its use. Thus, a common thread found in the research on the use of the graphing calculator is that the extent to which the graphing calculator provides benefits in calculus, or any area of mathematics, may be dependent upon factors such as the curriculum, the role of the teacher, and the individual students (Galindo, 1995; Doerr & Zangor, 2000; Kissane & Kemp). The study conducted by Burrill et al. (2002) supports this idea, and the researchers suggest that, “the research supports what some might suspect: the curriculum, student teacher interaction, how the tool is used in the classroom, and students' existing mathematical knowledge and beliefs all appear to be significant factors in determining what mathematical knowledge and skills are learned by students who use handheld graphing technology and how they use this knowledge and these skills" (p. 9). Furthermore, although there can be challenges with the use of technology, the CUPM suggests that the benefits that it provides are worth the effort of dealing with these challenges.

Conclusion

The use of graphing calculators has been widely debated. Their use has been investigated in many areas of mathematics, and many studies have been conducted to investigate their effects in mathematics courses including calculus. Some of these studies have addressed aspects involved in the reasoning process, such as the ability to make inferences, but these studies have not focused on the development of overall reasoning skills in calculus or the specific elements of reasoning as defined by the NCTM. The National Mathematics Advisory Panel (2008) recommends that research be conducted to determine some of the particular uses of the graphing calculator, including their effects on problem solving, conceptual understanding, and computation skills. The majority of the literature available on their use in calculus has focused on the ability of the calculator to perform certain functions, such as calculating a derivative or integral, and on aspects of success in calculus aside from reasoning. The literature on the use of graphing calculators in calculus has largely excluded this aspect of students' understanding. Thus, there was a need for research on some of the particular uses of graphing calculators in calculus, such as their effect on the development of reasoning skills. This study investigated whether graphing calculators had an effect on students' reasoning skills in integral calculus, and in which areas of reasoning they were most and least effective. In addition, the qualitative portion of the study addressed appropriate use of the graphing calculator. It addressed how it was used by students, for either computation or multiple representations, and the most effective way of teaching the graphical approach, either concurrently with other approaches, or following instruction in the analytic approach.

CHAPTER 3:

METHODOLOGY

The purpose of this study was to investigate the impact of graphing calculator use on high-school calculus students' reasoning in solving problems on the definite integral and its applications. In order to investigate the effects of graphing calculator use on reasoning in calculus, a mixed-method study was conducted with both a quantitative quasi-experimental component and a qualitative component.

Research Questions

The following research questions guided the study:

1. Does the use of the graphing calculator improve high school calculus students' reasoning ability in calculus problems applying the definite integral?
2. In what specific areas of reasoning does use of the graphing calculator seem to be most and least effective?
3. Can students who have used the graphing calculator demonstrate ability to solve problems without the graphing calculator?

Research Hypothesis

H_0: The mean reasoning score of those who use pencil and paper graphing methods (control group) is greater than the mean reasoning score of those who use graphing calculators (experimental group) ($\mu_C > \mu_E$).

H_A: The mean reasoning score of those who use graphing calculators (experimental group) is greater than the mean reasoning score of those who use pencil and paper graphing methods (control group) ($\mu_C < \mu_E$).

Sampling Procedures and Participants

The sampling method used in this study was a blend of convenience sampling and purposeful sampling. Approximately 42 students were selected for participation in this study. The potential sample size was 44 students; however, two students were not able to participate since they were absent during a portion of the study. Participants were selected from two different schools: a private all-girls high school in a suburban area in the Southeast United States and a private co-educational school located in a rural area in the Southeastern United States. Participants were enrolled in either calculus honors or calculus AP. Participants in this study were both male and female students between the ages of 17 and 18 with some variation in race, although the majority of students were white. Participants were not excluded on the basis of race, gender, class, or any other distinguishing factor. See Table 1, which shows the demographics of students who participated in this study.

Table 1: Student Demographics

		Honors (Experimental Group)	**AP (Control Group)**
Race	White	86 %	96 %
	African American	0 %	4 %
	Hispanic	5 %	0 %
	Asian	9 %	0 %
Gender	Female	85%	64%
	Male	15%	36%

Four existing calculus classes were used for this study. Two calculus classes were from the all-girls school, and two calculus classes were from the co-educational school. Each school had both a calculus AP class and a calculus honors class. In school 1, the all-girls school, the honors class consisted of 12 students, and the AP class consisted of 10 students. In school 2, the co-educational school, the honors class consisted of 8 students, and the AP class consisted of 12 students. Participants were not further separated into groups. In each school, the calculus AP class served as part of the control group, and the calculus honors class served as part of the experimental group. The entire control group was composed of students from both schools enrolled in calculus AP, and the entire experimental group was composed of students from both schools enrolled in calculus honors. Two different instructors were used in this study, one of whom was the researcher. The researcher was the instructor for the study in school 1, and the other calculus teacher was the instructor for the study in school 2. Each teacher has been teaching for approximately five to six years and is accustomed to incorporating the graphing calculator into daily lessons. Since it has been shown that the teacher is a factor in the success of using technology in the classroom, each teacher taught both one class in the control group and one class in the experimental group. In addition, since the instructor in school 1 was also the researcher in this study, school administrators in this particular school conducted observations during administration of the pretest, the posttest, and the lessons.

The courses used in this study were similar in the topics that were covered but varied in the depth of this coverage. For this study, however, the depth of coverage remained the same. Placement of students in each of these courses was based on factors such as the students' precalculus grade point average, teacher recommendation, and students' preference. Concepts covered in these courses included the three main calculus concepts – limit, derivative, and integral. Topics covered in calculus honors included limits, continuity, the derivative and its applications, including analysis of curves and optimization, application of rates of change including velocity, speed, and acceleration, differential equations and slope fields, the definite integral and its applications to area, volume, and linear motion, and the Fundamental Theorem of Calculus. The calculus AP courses also covered these topics, with a more in-depth look at some topics, and placed special attention on preparation for the Advanced Placement Calculus AB test. In addition, three approaches to problem solving, the numerical, graphical, and algebraic approach, were addressed in these courses. Each course normally utilized the graphing calculator for both computation and exploration; however, for this study, only the experimental group used the graphing calculator. The two schools normally used a different edition of the same textbook, but for this study, the same edition of Larson *Calculus* (2002) was used for the study. One lesson was also taken out of Foerster's *Calculus Concepts and Applications* (2005). This study focused only on the definite integral and its applications, and topics included in this study were limited to area, volume, Riemann sums, and problems of linear motion, including velocity, distance, and displacement. In addition, the depth of coverage remained the same in both groups for each of these topics.

Before the onset of the study, the administrators of both schools were contacted and asked for permission to complete this study and were provided a full explanation of the details of the study. After permission was granted, each student and his or her parents were sent a letter containing the eight elements of informed consent. In this letter, they were informed of the purpose of the study and were asked if they were willing to participate. The letter also informed students that the results of the study would not in any way impact their grades and that credit would be given for participation in the study in the form of thoughtful responses to the questions asked. Students who agreed to participate were asked to provide a completed statement of informed consent. See Appendix A for the approved IRB application and Appendix B for participant solicitation materials. The two groups used in this study were distinguished by their use or lack of use of the graphing calculator. The control group was distinguished by non-use of the graphing calculator in learning the definite integral and its applications and consisted of students enrolled in calculus AP. The experimental group was distinguished by use of the graphing calculator in learning the definite integral and its applications and consisted of students enrolled in calculus honors. Since one may expect AP students to score higher than honors due to being placed at this level because of increased interest or skill, it was this group that did not use the graphing calculator, so that any impact on reasoning could be attributed to use of the graphing calculator rather than other distinguishing characteristics.

Variables

This study investigated the impact of the use of graphing calculators on students' reasoning skills in high school calculus courses. The independent variable was graphing calculator use, a nominal level variable, and the two levels of the independent variable were use of the graphing calculator and non-use of the graphing calculator. Use of the graphing calculator was defined as the incorporation of the graphing calculator in teaching the definite integral and its applications for both computation and exploration using features such as the table, the graphing feature, the trace feature, the calculate function, and the Riemann2 software for the graphing calculator. In addition, computer software such as TI Connect was used to display the functions of the graphing calculator for students over the LCD projector. The dependent variable was students' reasoning skills. Reasoning was defined as an interval level variable measuring the degree to which students were able to draw conclusions by successfully completing tasks in five areas: analysis of a problem (analyze), selecting an appropriate strategy (strategy), monitoring progress (monitor), using connections (seekconn), and reflecting on the solution to the problem (reflect).

Instrument Development

Lesson Plans

Lesson plans were created by the researcher and were used in all classes participating in the study. Practice problems and notes for the lesson plans were adapted from sections in the Larson 7th ed. *Calculus of a Single Variable* textbook. Lessons covered computation and applications of definite integrals, including Riemann sums, area, volume, distance, and displacement. See Appendix H for detailed lesson plans. An asterisk is located next to each item where there were differences between lessons for the control group and the experimental group, due to the use of the graphing calculator in the experimental group.

Pretest and Posttest

The instruments used for the pretest and posttest were researcher-developed calculus tests that were analogous to one another in order to establish test-retest reliability. The assessments were composed of five questions, with some parts taken or adapted from the TIMSS 2008 Advanced Mathematics test and some parts taken or adapted from the Larson 7th ed. *Calculus of a Single Variable* textbook, which was used by all students. Each question was subdivided into eight parts that measured students' success in the five main areas of reasoning. See Appendix C and Appendix D for the instruments used for the pretest and the posttest.

Calculus Questions

Each question included in the reasoning assessment could be solved using either traditional pencil and paper methods or the graphing calculator. One question involved applications of the definite integral, such as finding the distance traveled by an object moving in a linear path. This is a problem that students would have been expected to solve without the graphing calculator; however, the graphing calculator could be used in this problem to visualize the parabola that represents the motion of the moving object, to visualize the roots of the function, and to evaluate the definite integral. Another question involved applications of the definite integral, such as finding the area between two curves. Once again, this was a problem that students were usually expected to solve using pencil and paper, but the graphing calculator could have helped students to visualize the region between the two curves as it consisted of more than one region. Students could have also used the graphing calculator to find the points of intersection of the two functions and to evaluate the three definite integrals that are necessary. A third question involved evaluating the definite integral using Riemann sums. This question could have also been approached using algebraic methods, such as u substitution and Fundamental Theorem of Calculus, but the graphing calculator could have been useful in allowing students to calculate a Riemann sum and to view the rectangles that are used in calculating the sum through the Riemann1 program. Another question involved applications of the definite integral, such as finding the volume of solid of revolution using the disk method. This question could have been solved without the graphing calculator as well, but the graphing TI-89 calculator could have provided a visual of the solid as well as the cross-section and the TI-84 could have provided a visual of the function. In addition, the calculator could have been used to evaluate the definite integral. The last question on the assessment also involved applications of the definite integral, including volume of a solid of revolution. This question required that all students use traditional pencil and paper methods. No graphing calculators were permitted in solving this problem in order to determine whether the group using graphing calculators for the assessment was able to solve problems without the graphing calculator. Table 2 below provides a summary of the concepts tested on the pretest and posttest. The number of the question testing each concept is given in the table.

Table 2: Concepts Included on Assessments

Concept	Pretest Question Number	Posttest Question Number
Area	1	4
Volume	2	1
Distance	3	2
Riemann Sums	4	3
Volume	5	5

Reasoning Questions

Analyzing a problem

The first two parts of each question related to analyzing a problem. Students were asked to identify the mathematical concepts and relationships that are used in this problem and to draw some conclusions about the solution to the problem.

Initiating a strategy

The third part of each question related to initiating a strategy. Students were asked to name some of the approaches and strategies that they could use in the process of solving the problem.

Monitoring one's progress

The fourth part related to monitoring one's progress. In this part, students were asked to solve the problem and answer any questions that were posed. They were asked to write in ink throughout the test and not to erase any work, so that the thinking process was evident.

Seeking and using connections

The fifth part of each question involved seeking and using connections. In this part, students were asked to identify any previous ideas or concepts learned in math or any other subjects that they used to solve the problem.

Reflecting on one's solution

The last two parts of each question involved reflecting on one's solution. In these parts, students were asked to determine whether their solution was reasonable and justify the answer and to provide any other information that they could infer from the problem.

Table 3 below provides a summary of the elements of reasoning tested in each question on the tests.

Table 3: Elements of Reasoning Included on Assessments

Reasoning Element	Question
Analyzing a Problem	Part a
	Part b
Initiating a Strategy	Part c
Monitoring One's Progress	Part d
Seeking & Using Connections	Part e
Reflecting on One's Solution	Part f
	Part g

Scoring

In order to measure students' reasoning skills, a rubric was created for the grading of the assessments and was validated by a mathematics educator. Values for each part of each question ranged from 0 to 3, where 0 showed no evidence of reasoning skills and 3 showed great evidence of reasoning skills. The maximum score possible for each question was 21, and the maximum score possible on the test was 105. See Appendix E and Appendix F for sample solutions and Appendix G for the rubric used for grading the pretest and posttest.

Validity and Reliability

A panel of experts consisting of two mathematicians was asked to review both the pretest and the posttest in order to assess the validity and reliability of the two instruments. Validity was established by determining that the two reasoning assessments did, in fact, measure the five elements of reasoning. Inter-rater reliability was established through the use of two different graders. Test-retest reliability was assessed by determining that there was consistency between the two reasoning assessments.

Procedures

The series of lessons took place over the course of a three-week period, with time given to computation of the definite integral and to its applications. The control group used pencil and paper methods of graphing only. In the experimental group, graphing calculators were used in two ways. The students used the TI-84 plus graphing calculator for in-class work and outside of class homework assignments. The teacher of the course used the TI-84 plus graphing calculator and the TI-Nspire graphing calculator together with the TI-Connect computer software in order to display the graphing calculator screen for students. The length of the class period in the two schools differed; one school had 50 minute class periods and the other had 75 minute class periods. The school with the longer class periods, however, supplemented during the extra time with unrelated material. In addition, when additional time was available in either group, this time was filled with unrelated material rather than additional instruction.

The first two lessons were devoted to the conceptual understanding of the definite integral as the area under the curve and to numerical approximation of the definite integral using Riemann sums and trapezoid rule. This took place during approximately two class meetings, for an approximate time of 50 minutes for each lesson. Students learned how to find the area under a curve using rectangles and by counting squares. They also learned how to find a left, right, and midpoint Riemann sum as well as how to use the trapezoid rule to approximate the value of the definite integral. Students in the control group graphed functions by hand using pencil and paper.

Students in this class also used pencil and paper to compute the area under the curve using counting squares, Riemann sums, and trapezoid rule. In the experimental group, the TI-84 plus graphing calculator was used. Students in this class graphed functions on the graphing calculator using the grid, so that they could use the method of counting squares. The teacher also used the graphing calculator along with the TI-Connect program to demonstrate how to use the Rieman2 graphing calculator program for the students. The Rieman2 program graphed the function and drew rectangles and trapezoids to illustrate the meaning of the definite integral as the area between the curve and the x-axis. This program also calculated a left, right, and midpoint sum and computed the sum using the trapezoid rule. Students in the experimental group computed Riemann sums by hand but also used the Rieman2 program to approximate the value of a definite integral and the integral approximation on the graphing calculator to evaluate a definite integral.

The third lesson involved finding the area between two curves, an application of the definite integral. This lesson was covered in one class meeting, or approximately 50 minutes. Students in the control group graphed functions using pencil and paper and found the area by setting up an integral and using the Fundamental Theorem of Calculus. In order to find the limits of integration, students found the points of intersection algebraically. Students in the experimental group used the TI-84 plus graphing calculator to view the graphs of the functions and to find the points of intersection. They used the CALCULUS graphing calculator program or numerical approximation of the definite integral on the calculator to find the solution, although they still learned how to set up and evaluate the definite integral using the Fundamental Theorem of Calculus. CALCULUS is a program that has several functions, one of which graphs the two curves between the limits of integration, shades the region between the two curves, and calculates the area. In this class, the teacher used the TI-84 plus graphing calculator along with the TI-Connect program to demonstrate the process of finding area to the students.

The next two lessons involved finding the volume of solids of revolution and solids with known cross sections using the Disk method, another application of the definite integral. These topics were covered in two class meetings, which were approximately 50 minutes each. Students in the control group graphed functions using pencil and paper and used the Disk method to find the volume of the resulting solid when the function was rotated about an axis. Students in the experimental group learned how to compute volume by hand but also used the TI-84 plus graphing calculator to graph functions and the CALCULUS program to find the volume of a solid of revolution. In this class, the teacher also used the TI-Nspire graphing calculator along with the TI-Connect program to allow students to visualize a three dimensional solid in order to help students determine what a cross section of the solid would look like.

The final lesson involved applications of the definite integral, such as finding distance and displacement, given the velocity or position of an object moving in a linear path. One class meeting, or approximately 50 minutes, was devoted to completion of this lesson. Students in the control group graphed position and velocity functions using pencil and paper and set up integrals to find the distance and displacement using the Fundamental Theorem of Calculus. Students in the experimental group graphed position and velocity functions using the TI-84 plus graphing calculator and found distance and displacement using both the Fundamental Theorem of Calculus and the numerical approximation of the definite integral on the graphing calculator. In the experimental group, the teacher used the TI-84 plus graphing calculator in parametric mode along with the TI-Connect program to demonstrate linear motion to students. Using the graphing calculator, students were able to see the behavior of a velocity function as an object moved in a linear path both forward and backward.

After the series of lessons and upon completion of the posttest, students in the control group were given instruction in the use of the graphing calculator for applications of integral problems. This lesson took approximately forty minutes and covered using the graphing calculator for problems involving distance, area, Riemann sums, and volume. The purpose of the lesson was to provide the AP students with another approach to solving problems, in addition to the algebraic approach that they had been using. This lesson took place before the interviews were conducted, so that these students would have the option of using the graphing calculator throughout the interview, and so that it could be determined, through the interviews, whether it was more beneficial to learn both approaches simultaneously or with the graphical approach following the analytic approach.

The content and time frame for the series of lessons are summarized in Table 4.

Table 4: Lesson Content and Time Frame

Day	Length of Lesson	Lesson
1	50 minutes	Riemann Sums
2	50 minutes	Riemann Sums
3	50 minutes	Applications of Definite Integrals – Area
4	50 minutes	Applications of Definite Integrals – Volume
5	50 minutes	Applications of Definite Integrals – Volume
6	50 minutes	Applications of Definite Integrals – Distance

Data Collection

At the beginning of the study, all students completed the pretest. The control group did not use graphing calculators to complete the assessment. The experimental group used graphing calculators to complete the assessment, but was not allowed to use graphing calculators on the last question. Students in the experimental group completed the first four questions on the test, and then put away their graphing calculators. They were then administered the last question. Approximately three weeks after the administration of the pretest, and upon completion of the series of lessons on the definite integral and its applications, all students completed the reasoning assessment. The assessment was administered in a class period preceding a chapter test on these topics for as long as necessary for each student to complete the assessment. The reasoning assessment was administered by the teacher of the course. In the courses taught by the researcher, an administrator at the school was asked to observe during the administration of the tests. After completion of the reasoning assessments, the tests were brought to the same panel of experts for grading using the researcher-developed rubric. Each grader graded one school; therefore, each grader graded one class from the control group and one class from the experimental group. In order to facilitate inter-rater reliability, I met with each grader separately, and each grader graded one test question, consisting of seven reasoning questions, for the same student. Grading procedures were then discussed to ensure that grading was consistent. Inter-rater reliability was established by observing that the grades given by each grader were the same.

In order to conduct the qualitative portion of this research study, interviews were conducted with one student from each calculus class. These students were chosen based on their ranking in the class. The students with the highest average in calculus in each class were asked to participate in an interview. During this interview, one question was selected from the calculus reasoning posttest, and the students were asked to explain their reasoning through the solution to this problem aloud. All students were given the option of using the graphing calculator if they chose. An interview script was used to go through the interview process with the students. The goal of the interview was to describe the reasoning processes used by students solving definite integral problems when graphing calculators were used and when they were not used, depending on whether the students chose to use the calculator. In addition, the goal was to observe various uses of the graphing calculator, in order to make conclusions about appropriate use of the graphing calculator and appropriate instruction with the graphing calculator. Students were asked questions addressing math anxiety, use of graphing and graphing calculators for solving problems, and the student's reasoning process in solving problems on the definite integral. The students were asked to speak out loud as they worked a problem and to let me know what he or she was thinking through the process. Throughout the interview, I asked the student to describe what they were thinking. I also observed throughout the process whether the student used graphing methods and whether they used the graphing calculator to solve the problem. The interview session with each student was audiotaped and transcribed. See Appendix I for the full interview script and the rubric that was used for assessing responses to the interview questions.

Data Analysis

The purpose of the quantitative data analysis was to identify any significant differences that existed between the experimental group, who used graphing calculators, and the control group, who did not. Each analysis was conducted both for all participants and for each individual school.

Scores on the pretest were analyzed through an independent samples t-test, in order to determine any significant differences that existed between the two groups of students at the onset of the study.

To answer the first research question and to test the null hypothesis in order to reduce Type II Error, multivariate analysis of covariance (MANCOVA) was conducted. The goal of this analysis was to determine if there were significant differences between the two groups of students on a combination of the reasoning questions, using pretest scores as the covariate. The use of MANCOVA was appropriate, since there was a categorical independent variable and since the purpose of the analysis was to identify significant differences among groups on a combination of continuous interval-level dependent variables. Assumptions of the MANCOVA were checked using correlation and Box’s test. If significant results were found, in order to answer the second research question, a univariate analysis was then conducted on each area of reasoning, to determine in which specific areas of reasoning there were significant differences.

To answer the third research question, analysis of covariance (ANCOVA) was conducted on part 2 of the posttest, using part 2 of the pretest as the covariate. The ANCOVA was used in order to determine any effect that the use of graphing calculators during instruction had on the outcome variable, reasoning, when graphing calculators were not used in either group in testing. ANCOVA was appropriate since there was a categorical independent variable and a continuous interval-level dependent variable and since there may have been initial differences in reasoning skills of individual students in the two groups. The inclusion of the pretest scores, an interval-level continuous variable, as the covariate was chosen in order to control for variability that existed because of individual differences in interest and reasoning skills and to increase statistical power. As part of the ANCOVA, an F test was conducted to determine if the difference between the means of the two groups was significant, so that any effect of graphing calculators on reasoning could be inferred.

Qualitative analysis of data was conducted through the examination of interview responses. I sought to identify commonalities and differences between the reasoning processes of the participants who were allowed to use the graphing calculator and those who were not. A researcher-developed rubric was used for grading the responses to the interview questions, so that responses of individuals could be compared to identify differences in problem solving strategies and reasoning abilities. Analysis of the interview data was used in order to provide further information to answer the first and second research questions, and it was also used to make observations about the appropriate use of the calculator, concerning the number of ways in which it is used by the student and the order in which graphical and analytic approaches appear in instruction.

Ethical Considerations

In order to provide all students with the knowledge that they needed of both the graphing calculator and pencil and paper graphing methods, upon completion of the study, all students were taught both approaches to definite integrals and their applications. In addition, neither the results of the study nor individual responses to questions were used in calculating school grades. All students were given credit for participation in the study in the form of thoughtful responses to the questions asked.

Since the teacher of one of the two of the classes participating in the study was also the researcher, there may be some concern over the consistency of the teaching strategies used with each of the classes. In order to verify that the classes were taught in an unbiased and consistent fashion, with the single exception of the use of the graphing calculator, a school administrator was asked to observe classes at random and to provide a statement indicating her objective opinion regarding the actions of the teacher. Observations were conducted a total of four times throughout the study, with two observations taking place during administration of the pretest and posttest and two observations during other class periods. The observations during the periods in which the assessments were given were both announced, and the two observations during the regular class periods were unannounced. A statement from the administrator who conducted the observations is provided in Appendix J.

Limitations

Although the purpose of this study was to determine the impact of graphing calculator use on calculus students' reasoning skills, due to a small sample size, results of this study cannot be used to make generalizations to all populations. Results may only be generalized to those populations consisting of mostly white, female gifted students in advanced calculus courses.

CHAPTER 4:

RESULTS

This chapter presents the results of a mixed method study investigating the use of the graphing calculator in learning applications of the definite integral. The first part of the chapter presents the results of the quantitative portion of this study, which was conducted through the use of pretests and posttests. The second part presents the results of the qualitative part, which was conducted through interviews with four students. Results of the study were analyzed in order to test the null hypothesis that the mean reasoning score of those using pencil and paper graphing methods is greater than the mean reasoning score of those using graphing calculators and to answer the following research questions:

1. Does the use of the graphing calculator improve high school calculus students' reasoning ability in calculus problems applying the definite integral?
2. In what specific areas of reasoning does use of the graphing calculator seem to be most and least effective?
3. To what extent can students who have used the graphing calculator demonstrate ability to solve problems using pencil and paper methods?

Schools and Participants

The two schools participating in the study were a private, all-girls high school, which will be referred to as school 1, and a private, co-educational high school, which will be referred to as school 2. Participants in each school were asked not to identify themselves on the pretests and posttests, but rather to label their tests with a heading that specified their school, class, and number; thus, each student labeled the tests with school 1 or school 2, AP or honors, and a randomly chosen number. A total of 42 students took both the pretest and the posttest. Any tests on which the majority of questions were unanswered were excluded from the analysis, so that results would include data from only those students who provided thoughtful responses and so results would not be affected by scores considered to be outliers. Three of these tests, two from the experimental group and one from the control group, were excluded from the analysis, since over half of the test was not completed by the student. Therefore, a sample size of $n = 39$ was included in the quantitative analysis. Four students also participated in an interview. The top student in each calculus class in both schools was selected for an interview. The students interviewed included one student from the control group in school 1, Student C1, one student from the control group in school 2, Student C2, one student from the experimental group in school 1, Student E1, and one student from the experimental group in school 2, Student E2.

Quantitative Analysis

Posttest Components

The first part of the pretest was the main part used for the analysis, since it is on this part that the control group and the experimental group were differentiated. The experimental group, which was made up of all honors students, used a graphing calculator for this part, and the control group, which was made up of all AP students, did not. The first part consisted of four questions, which addressed applications of definite integrals, such as distance traveled, area, Riemann sums, and volume. The second part consisted of one question, for which none of the students could use the graphing calculator. Both parts were included in the analysis.

t – test

A t-test was computed to compare the two groups of students using both parts of the pretest. When all participants are included in the analysis, Levene's test was non-significant $(p = .195)$; therefore, equal variances were assumed. A Bonferroni adjustment was performed;

therefore, a significance level of $p = .01$ was used. Results of the t-test show no significant difference between the two groups of students on the first part $t(37) = -1.134, p = .264$ or the second part $t(37) = -2.614, p = .013$ of the pretest. In each school, when only the first part of the pretest is considered, there is also no significant difference between the control group and the experimental group in school 1 $t(20) = .165, p = .871$ or in school 2[1] $t(15) = -1.737, p = .103$. When only the second part of the pretest is considered, there is no significant difference in school 1 $t(20) = -.943, p = .357$ or in school 2 $t(15) = -2.846, p = .012$. Therefore, at the time that the pretest was given, we can assume that there was no significant difference between the two groups on the entire pretest or on each individual part of the pretest. However, in order to control for any differences that did exist between the groups, pretest scores were included as the covariate in running the multivariate analysis of covariance, subsequent univariate tests, and the analysis of covariance. Results from the independent samples t-tests are given in Table 5.

[1] Equal variances were not assumed; thus, results of the t-test were read from the second line of the table in SPSS.

Table 5: Independent Samples t-test

Results of Independent Samples t-test comparing groups on Posttest Part 1 and Part 2

	Experimental Group	Control Group	*T*	*Df*
		All Participants		
Pretest Part 1	40.78	36.90	-1.134	37
Pretest Part 2	9.39	6.76	-2.614	37
		School 1		
Pretest Part 1	44.08	44.80	.165	20
Pretest Part 2	9.83	8.50	-.943	20
		School 2		
Pretest Part 1	34.17	29.73	-1.737	14.978
Pretest Part 2	8.50	5.18	-2.846	15

Note: Using a Bonferroni correction, results were non-significant at the p < 0.01 level.

An independent samples t-test was also conducted to compare the two different schools participating in the study. When the entire pretest, including both parts, was included, Levene's test was non-significant ($p = .169$), and thus equal variances were assumed. Results show that students in school1 $(M = 53.73, SD = 11.65)$ scored significantly higher $t(37) = 4.70, p = .000$ on the pretest than students in school 2 $(M = 38.00, SD = 8.38)$. Therefore, at the time that the pretest was given, we can assume that there was a significant difference between students in the two schools participating in the study.

The First Research Question

The first research question asked, "Does the use of the graphing calculator improve high school calculus students' reasoning ability in calculus problems applying the definite integral?" In order to answer this question, multivariate analysis of covariance was conducted to determine if there were significant differences between the control group and the experimental group on posttest part 1 scores, using pretest part 1 scores as the covariate. Descriptive statistics for each group were all computed for part 1 of the posttest. Table 6 presents these descriptive statistics.

Table 6: Descriptive Statistics

Descriptive Statistics for Posttest Part 1 by Group

Group	*N*	*M*	*SD*	Minimum	Maximum
			All Participants		
Experimental	18	51.28	7.77	36	62
AP	21	44.81	10.57	17	57
			School 1		
Experimental	12	48.92	7.56	36	59
AP	10	50.10	7.03	37	57
			School 2		
Experimental	6	56.00	6.26	45	62
AP	11	40.00	11.20	17	56

MANCOVA

Multivariate analysis of covariance was conducted on the five different areas of reasoning tested on part 1 of the posttest. MANCOVA was conducted in each school and for all participants together in order to test the null hypothesis that the mean reasoning score of those using pencil and paper graphing methods is greater than the mean reasoning score of those using graphing calculators and to answer the first research question. Results of the MANCOVA are presented in Table 7.

Table 7: Summary of MANCOVA Results

MANCOVA Results for Posttest Part 1 Reasoning

Source	Pillai's Trace	*F*	*Df*	Error *df*
		All Students		
Posttest Part 1 Reasoning	.048*	2.536	37	32
		School 1		
Posttest Part 1 Reasoning	.753	.526	20	15
		School 2		
Posttest Part 1 Reasoning	.019*	4.635	15	10

Note: *Using a Bonferroni correction, results were significant at the p < .05 level.

School 1

Assumptions of homogeneity were checked using Box's test, which was non-significant $(p = .25)$. Multivariate normality could not be assessed, so univariate normality was checked using Kolmogorov-Smirnov. This test showed non-normality in the areas of analyzing a problem and initiating a strategy for the experimental group, and in the areas of analyzing a problem and initiating a strategy for the control group. However, this test is robust to violations of this assumption. Using Pillai's Trace and a Bonferroni adjustment, when all areas of reasoning were included in the analysis in school 1, we can conclude that the experimental treatment, graphing calculator usage, has a non-significant effect on students' posttest reasoning scores in school 1 $V = .15, F(5,15) = .526, p = .753$.

School 2

For school 2, Box's test, which was used to check assumptions of homogeneity, was also non-significant $(p = .128)$. Univariate normality was checked using Kolmogorov-Smirnov. This test showed non-normality in the areas of initiating a strategy and monitoring progress for the experimental group, and in the areas of initiating a strategy and seeking and using connections for the control group. However, this test is robust to violations of this assumption. Using Pillai's

Trace and a Bonferroni adjustment, when all areas of reasoning were included in the analysis in school 2, we can conclude that the experimental treatment, graphing calculator usage, had a significant effect $(p = .019)$ on students' posttest reasoning scores $V = .70, F(5,10) = 4.635, p = .019$.

All Participants

Assumptions of homogeneity were checked using Box's test, which was significant $(p = .01)$. Univariate normality was checked using Kolmogorov-Smirnov. This test showed non-normality in the areas of analyzing a problem, initiating a strategy, monitoring progress, and seeking and using connections for the experimental group, and in the area of initiating a strategy for the control group. However, this test is robust to violations of this assumption. When all areas of reasoning were included in the analysis, using Pillai's Trace and a Bonferroni adjustment, we can conclude that graphing calculator usage had a significant effect on students' posttest reasoning scores $V = .28, F(5,32) = 2.54, p = .048$.

Second Research Question

The second research question asked, "In what specific areas of reasoning does use of the graphing calculator seem to be most and least effective?" In order to answer this question, multivariate analysis of covariance was conducted to determine any significant differences between the two groups of students on the first part of the posttest, using the first part of the pretest as the covariate. This analysis was only conducted for all participants together and for school 2, since these groups had significant results on the MANCOVA. Results of the univariate tests are presented in Table 8.

Table 8: Summary of Univariate Results

Results of Univariate Tests following significant MANCOVA

Dependent Variable	*df*	*F*	Group	Means	95% Confidence Interval	
All Participants					**Lower**	**Upper**
Analyzing a Problem	37	.514	Experimental	11.588	10.124	13.051
			AP	10.977	9.524	12.230
Initiating a Strategy	37	6.312*	Experimental	11.714	11.187	12.241
			AP	10.817	10.329	11.304
Monitoring Progress	37	4.729*	Experimental	6.929	5.790	8.067
			AP	5.252	4.199	6.304
Seeking and Using Connections	37	.345	Experimental	8.601	7.697	9.505
			AP	8.961	8.125	9.797
Reflecting on One's Solution	37	2.610	Experimental	11.823	9.948	13.697
			AP	9.771	8.038	11.505
School 1 Participants Only – not included since multivariate tests were non-significant						
School 2 Participants Only						
Analyzing a Problem	1	2.290	Experimental	12.805	10.101	15.508
			AP	10.379	8.421	12.337
Initiating a Strategy	1	9.300*	Experimental	11.647	10.792	12.502
			AP	10.101	9.482	10.721
Monitoring Progress	1	3.080	Experimental	7.895	5.342	10.448
			AP	5.239	3.390	7.088
Seeking and Using Connections	1	.000	Experimental	8.049	5.889	10.209
			AP	8.064	6.500	9.628
Reflecting on One's Solution	1	5.539*	Experimental	13.274	9.127	17.420
			AP	7.487	4.484	10.490

Note: *Using a Bonferroni correction, results were significant at the $p < .05$ level.

Univariate Test Results

School 1

Univariate analysis was not conducted in school 1, since the results of the MANCOVA were not significant in this school.

School 2

When the effects of the pretest scores are taken out, students in the experimental group scored higher, on average, than students in the control group in the areas of analyzing a problem $(M_E = 12.81, M_C = 10.38)$, initiating a strategy $(M_E = 11.65, M_C = 10.10)$, monitoring progress $(M_E = 7.90, M_C = 5.24)$, and reflecting on one's solutions $(M_E = 11.27, M_C = 7.49)$, and students in the control group scored higher, on average, than students in the experimental group in the area of seeking and using connections $(M_E = 8.05, M_C = 8.06)$. These differences were significant in the areas of initiating a strategy $(p = .009)$ and reflecting on one's solution $(p = .034)$.

All Participants

When the effects of the pretest scores are taken out, students in the experimental group scored higher, on average, than students in the control group in the areas of analyzing a problem $(M_E = 11.59, M_C = 10.88)$, initiating a strategy $(M_E = 11.71, M_C = 10.82)$, monitoring one's progress $(M_E = 6.93, M_C = 5.25)$, and reflecting on one's solution $(M_E = 11.82, M_C = 9.77)$. These differences are significant in the areas of initiating a strategy $(p = .017)$ and monitoring progress $(p = .036)$. Students in the control group, scored higher, on average, than students in the experimental group in the area of seeking and using connections $(M_E = 8.60, M_C = 8.96)$.

The Third Research Question

The third research question asked, "To what extent can students who have used the graphing calculator demonstrate ability to solve problems using pencil and paper methods?" In order to answer this question, analysis of covariance was conducted on part 2 of the posttest, using pretest part 2 as the covariate. Results of the ANCOVA are presented in Table 9.

Table 9: Summary of ANCOVA Results

Results of ANCOVA for Posttest Part 2 by Group

Source	*Df*	*F*	H	*p*
		All Participants		
Posttest Part 2	1	.022	39	$p = .883$
		School 1		
Posttest Part 2	1	.350	22	$p = .561$
		School 2		
Posttest Part 2	1	.072	17	$p = .793$

Note: The data are non-significant at the p < .05 level.

ANCOVA

School 1

Analysis of Covariance was conducted to determine if there were significant differences between the control group and the experimental group in school 1 on the posttest part 2 scores, using pretest part 2 scores as the covariate. Levene's test was non-significant $(p = .39)$, and therefore we can assume that there was no significant difference between the variances of the two groups. When the effect of the pretest score is taken out, students in the control group $(M = 11.55, SE = .927)$ scored higher, on average, than students in the experimental group $(M = 10.80, SE = .844)$; however, the effect of the experimental treatment, graphing calculator usage, is non-significant $(p = .561)$.

School 2

Analysis of Covariance was conducted to determine if there were significant differences between the control group and the experimental group in school 2 on the posttest part 2 scores, using pretest part 2 scores as the covariate. Results of the ANCOVA showed that a student's pretest part 2 score was not a significant predictor of their posttest part 2 score $(p = .13)$. When the effect of the pretest score is taken out, students in the experimental group

$(M = 10.21, SE = 2.18)$ scored higher, on average, than students in the control group $(M = 9.43, SE = 1.51)$; however, the effect of the experimental treatment, graphing calculator usage, is non-significant $(p = .793)$.

All Participants

Analysis of Covariance was conducted to determine if there were significant differences between the control group and the experimental group on the posttest part 2 scores, using pretest part 2 scores as the covariate. Levene's test was non-significant $(p = .43)$, and therefore we can assume that there was no significant difference between the variances of the two groups. When the effect of the pretest score is taken out, students in the experimental group $(M = 10.62, SE = .910)$ scored higher, on average, than students in the control group $(M = 10.43, SE = .837)$; however, the effect of the experimental treatment, graphing calculator usage, is non-significant $(p = .883)$.

Qualitative Analysis

Interview Setting

The first two students interviewed were from school 1. These students were interviewed during their independent study period on a regular school day in a classroom in the participants' school. The other two students who participated in the interviews were from school 2. These students were interviewed during a regular class period on a regular school day in a classroom in the participants' school. Each interview lasted for approximately twenty to thirty minutes. The interviews were audio taped and then transcribed.

Interview Description

In each interview, students were asked about their math anxiety, their use of graphs and graphing calculators, and they were also asked to solve a problem that was on the posttest. They were also asked several questions relating to the reasoning process that they used in solving the problem. The students were asked to describe their thoughts throughout the process of solving the problem, so that I could understand their reasoning processes. Throughout the interview, I also observed the students' use of graphs and graphing calculators in reasoning through the problem. The goal was to identify which methods and strategies were most beneficial to the students as they reasoned through the problem. I noted the approach that was used by the students – analytic, numerical, or graphical, as well as the extent to which the graphing calculator was helpful, if at all, and the variety of uses. A full transcription of each interview is included in Appendix L.

Individual Interview Participants

Student C1 was a female student in calculus AP in school 1. Student E1 was a female student in calculus honors in school 1. Student E2 was a male student in calculus honors in school 2. Student C2 was a female student in calculus AP in school 2. The interviews took place in a math classroom in each participant's school on a regular school day during a regular class period. Students C1 and C2 had not used the graphing calculator for homework assignments when learning applications of the definite integral. The graphing calculator was also not used in instruction for these students throughout the entire study; however, these students were instructed in the use of the graphing calculator after the study was completed but before the interview took place. Students E1 and E2 were allowed to use the graphing calculator for homework assignments when learning applications of the definite integral. The graphing calculator was also used in instruction for these students throughout the entire study.

Interview Scores

Responses to the interview questions were graded according to a researcher-developed rubric. Scores were split into three areas to represent the similarity of the individual students' outside of the study, the students' problem solving approaches in the current study, and the students' reasoning abilities in the current study.

Analysis of Interviews

Preliminary Questions

Math Anxiety

The first question addressed math anxiety. Although math anxiety was not a focus of this study, this question was included to establish the atmosphere for the interview and to provide information for further study. Responses to this question are provided in Appendix M.

Sketches and Graphs

The students were then asked about their use of sketches and graphs when doing math homework or taking a math test. Students were asked, "In general, when you are doing math homework or taking a math test, how often do you use sketches or graphs to help you solve a problem?" Most of the students responded similarly to this question. Students C1 and C2 both responded that they do not use them very often and usually only when absolutely needed, although student C2 did state that she sometimes uses them when not asked to do so explicitly in the problem. Student E1 responded similarly that she almost never uses sketches and graphs. Only one student, student E2, stated that he often uses sketches and graphs to solve a problem, stating that he uses at least one sketch or graph on each set of problems. The responses were graded on a scale of 0 to 3, with 0 representing that a student never uses sketches and graphs and 3 representing that a student always uses sketches and graphs. Three out of four students were given a score of 1 to represent that they almost never use sketches and graphs for homework assignments and tests, and one student was given a score of 2 to represent that he often uses sketches and graphs on homework assignments and tests.

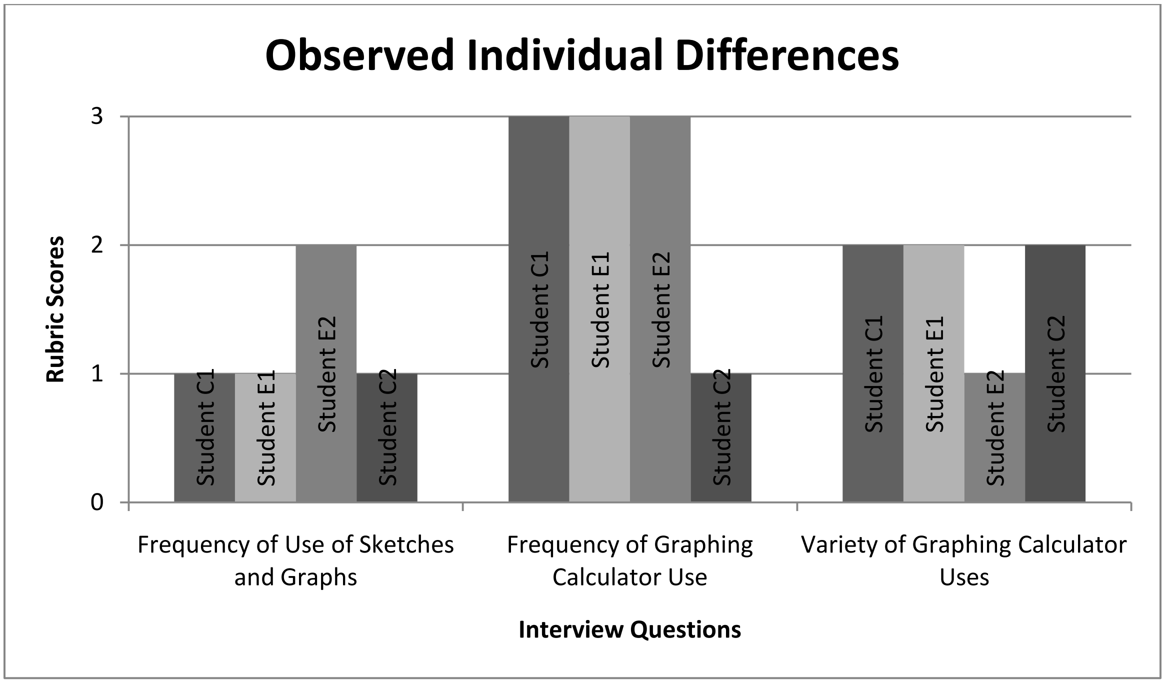
Graphing Calculator Usage

The next question addressed students' use of the graphing calculator when doing math homework or taking a math test. Students were asked, "In general, when you are doing math homework or taking a math test, how often do you use your graphing calculator to help you solve a problem?" They were first asked about the frequency of their graphing calculator use. The frequency of the graphing calculator use by the students was graded on a scale of 0 to 3, with 0 representing that the student never uses the calculator and 3 representing that the student always uses the calculator. Three out of four students responded similarly for this question as well, stating that they always use the graphing calculator, and therefore each of these students was given a score of 3 on this question. One student, however, stated that she uses the graphing calculator only sometimes on homework assignments and tests, and this student was given a score of 1 on this question. The students were then asked about the variety of ways in which they use their graphing calculator use. They were asked, "In what ways, if any, do you typically use the graphing calculator on homework assignments and tests?" The responses of three students were also similar to this question. Two students responded that they use the calculator for normal computation and to see the graphs of the functions, and one student responded that she uses the graphing calculator to view the graphs of functions if she does not already know what the graph looks like. Student E1 also provided an additional detail that she looks at the graphs of the equations to see if she can apply the graph to the problem. On the other hand, one student responded that use of the graphing calculator, for him, is usually for computation in order to save time and avoid doing mental math. The variety of uses of the graphing calculator by the students was on a scale of 0 to 3, with 0 representing that the student never uses the calculator and 3 representing that the student uses the calculator for multiple representations and features. Three students, students C1, E1, and C2 were each given a score of 2, since they use at least one feature of the calculator, such as graphing, and student E2 was given a score of 1, since he uses the calculator primarily for computation.

Conclusions

The students were each given a total score on the first four interview questions to represent how similar they are in terms of their normal math anxiety, use of sketches and graphs, and use of the graphing calculator unrelated to the current study. Students C1, E1, and E2 were all given a score of 6 out of a possible 9 points, and student C2 was given a score of 4 out of a possible 9 points. One can conclude that, although most students do not often use sketches and graphs when solving problems, the students often use their graphing calculators. The ways in which the graphing calculators are used varies; however, most of the students use it in at least two ways, and none of the students use all of the possible representations available on the calculator. Common uses of the graphing calculator include computation and visualization of graphs. In addition, sometimes the graphs are used to help solve the problem. Student E1, for example, when asked how she uses the graphing calculator, stated “usually for general calculation, but if I am given a function, I do use it just because I like to see the graphs to see if I really need to apply it to anything.” Figure 3 below presents the findings from the set of preliminary questions asked at the beginning of the interview.

Figure 3: Observed Individual Differences



Problem Solving Approaches and Representations

After a few preliminary problems, the students were handed one of the questions that they first saw on the posttest. This question assessed students understanding of volume using the disk method, one of the applications of definite integrals. The problem stated, "Approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations below about the $x-$ axis: $y = x\sqrt{4-x^2}$, $y = 0$." The students were asked to read the question to themselves and then to solve it, while explaining their thinking process and reasoning throughout solving the problem.

The processes used by all of the students were very different. Differences were evident in the problem solving approaches between the students who used the graphing calculator in the current study and those who did not, which is evident by the difference in interview scores for these students on the second part of the interview. The two students who were in the control group approached the problem from an almost entirely algebraic standpoint. One of the students in the control group solved the problem correctly, and one did not. The two students who used the graphing calculator in the study were much more likely to use a graphical approach and the graphing calculator in solving the problem. In addition, these students used the graphing calculator not only for computation and quick results but also to see the behavior of functions in order to solve parts of the problem. One of the students in the experimental group solved the problem correctly, and one did not.

Individual Students

Student C1 used an algebraic approach throughout the entire process of problem solving, including finding roots, setting up the integral, and using the Fundamental Theorem of Calculus. The student never picked up the graphing calculator to assist her in solving this problem, with the exception of the very end of the problem to perform a simple computation of adding two fractions. Throughout the process, she did not question her results; however, at the end of the problem, when she found an incorrect solution, she realized that the solution was incorrect, stating, “I’m thinking that I definitely got the answer wrong because it should not be zero.” After finding this answer, the student did not attempt another approach. Her mistake, however, was not in understanding but in an inaccurate application of the Fundamental Theorem of Calculus.

Student E1, upon looking at the problem, immediately picked up the graphing calculator in order to look at the graph of the function. She explained, upon looking at the graph, that she noticed that there was more than one interval that would be needed to find the volume and that one of these intervals was positive and one was negative. After looking at the graph, she then switched to an algebraic approach to find the zeros of the function. She also used the algebraic approach to set up the integral needed to find the volume; however, once this integral was set up, she moved back to the graphing calculator in order to find the volume using the CALCULUS program. She made observations throughout solving the problem, such as the fact that a square root would always be above 0, and she also referred to the concepts of "volume" and "revolution" throughout the problem solving process. The student then found the volume of each region separately. After finding the volume of the first region, she made a conclusion that two regions that are symmetric with respect to the origin will have the same volume, but she verified this assumption using the graphing calculator and found that her assumption was justified. She stated, "I'm gonna assume that the answer to the other problem is also the same answer, but I can never really assume in math, so I'm gonna do it (compute the integral using the graphing calculator) anyway." At the end of the problem, she found the correct answer and gave this answer in the proper units. Throughout solving the problem, student E1 moved freely between pencil and paper calculations and the graphing calculator, switching back and forth from the algebraic approach to the graphical approach. She used the graphing feature of the graphing calculator in order to determine how she would have to set up this problem algebraically. She also used the graphing calculator to view how many zeros she would have to find and then found these zeros algebraically. She also used the graph provided on the graphing calculator to view the symmetry of the function with respect to the origin, and used this observation to make the assumption that the volume of the two regions would be double the volume of one region.

Student E2 used a graphical approach to help solve the problem. He began the problem by graphing the given function on the graphing calculator. He then sketched the graph of the function that he saw on the calculator onto his paper. He also then drew what the solid would look like if he were to revolve the function about the x-axis. The student also discussed the shape of the resulting solid. He stated, "I'm gonna connect – this is a tip my teacher gave us – you connect the open ends to make it like a cone or whatever shape you're trying to find." He also used the graphing calculator to estimate the bounds of the function, and he estimated these correctly from the graph. The student, however, then switched to an algebraic approach in order to set up the integral needed to find the volume of the solid of revolution. In doing so, he did not use the formula for the area of a circle, and because of this, he used an incorrect formula for finding the volume of the solid of revolution. After setting up the integral, he evaluated it using the numerical integration function on the graphing calculator. He stated, "it plugs in and integrates for you, that way you don't have to do all the by hand stuff." He used the graphing calculator numerical integration function correctly, but since he had used the wrong formula, the answer given was incorrect. The student found the solution to be 0. The student had inadvertently used the formula to find the area of region rather than the volume of the solid formed when the region is rotated about an axis. The student also justified this answer afterwards, stating that 0 is reasonable since half of the function is above the axis and half is below. This justification would have been correct had the student been asked to find area. The student, then, confused the concepts of area and volume is solving this problem.

Student C2 also used both a graphical and an algebraic approach; however, the algebraic approach seemed to be the most useful in helping this student to solve the problem. The student began the problem by graphing the function by hand on paper and then sketching what the resulting solid would resemble when the function was rotated about the x-axis. She stated, "The graph's gonna be a square root, and it's gonna look like that, and then once you reflect it, it'll come out to look something like that, which would be a disk." In sketching the function on paper, however, the student only graphed the portion in the first quadrant; she did not graph the part of the graph located in the third quadrant. In addition, the sketch drawn by the student was not the correct shape for the function given, and therefore, the solid drawn was also the incorrect shape. After sketching the solid, the student set up the integral algebraically. She set it up correctly and also added that the cross sections would form circles. She then simplified the function in order to integrate. Afterwards, she then realized that she would need to find the bounds. She thought for a minute about how to do this and then remembered the correct procedure. She also found the bounds algebraically by setting the two functions equal to one another and solving. The student finally then used the fundamental theorem of calculus to find the answer, and she used the calculator for part of this step. This student only used the graphing calculator once, and the extent of its use was for the purpose of computation. This student was able to get the correct answer.

Scoring

The students' problem solving processes were graded in terms of the different types of approaches used to solve the problem, including the graphical approach, and on whether they used their graphing calculator. Scores for the first item were given on a scale of 0 to 3, with 0 representing no approach used to solve the problem and 3 representing the use of a graphical approach. Scores for the second item were also given on a scale of 0 to 3, with 0 representing no use of the graphing calculator and 3 representing use of the graphing calculator for multiple representations. Student C1 was given a score of 1 for both items, since she did not use a graphical approach and she used the graphing calculator for computation only. Student E1 was given a score of 3 on both items, since she used a graphical approach to help her solve the problem and since she used the graphing calculator for multiple representations or features, including graphing and the CALCULUS program. Student E2 was given a score of 3 on both items as well, since he used a graphical approach to solve the problem and used multiple functions on the graphing calculator. Student C2 was given a score of 3 on the first item since she used a graphical approach to solve the problem and a score of 1 on the second item since she used the graphing calculator for computation only.

Approach

After solving this problem, the students were asked to describe the approach that they used to solve the problem. Students were asked, "Which approach to problem solving do you believe was most helpful in solving this problem? Analytic or algebraic, numerical, or graphical? And how did you use this approach?" Student C1 responded correctly that she used the analytic approach in finding both the intervals and in applying the equation for volume. She was given a score of 1, on a scale from 0 to 3, where 0 represented that the student could not describe the approach and 3 represented that the graphical approach was used. Student E1 also responded correctly that she used the graphical approach and that, since she did not remember all of the steps, the graphing calculator was definitely helpful. This student was given a score of 3, since the graphical approach was used. Student E2 stated that he used the graphical approach as well and was also given a score of 3 on this item. This student stated that, although he may not always get the right answer, he prefers to visualize the problem before he solves it. He stated that he can usually get close if he looks at the graph. Student C2 stated that she used both graphical and numeric, since she has to know what the graph looks like in order to be able to solve the problem. This student was also given a score of 3 on this item. Three out of the four students, then, found that the graphical approach was helpful to them in solving this problem.

Graphing Calculator Usage

The students were then asked, "How helpful was the graphing calculator to you in solving this problem? Explain." Responses to this question were also graded according to the extent of use and variety of uses. Student C1 responded that the graphing calculator was only used in computation. She was given a score of 1 on both items, since the graphing calculator was only helpful at times and use was for computation only. Student E1 responded that the graphing calculator was very helpful in solving. She was given a score of 3 on both items, since she described it as very helpful in many ways and described use of the graphing calculator for looking at the graph, checking her assumptions, calculating the volume, and computation. Student E2 stated that the graphing calculator was extremely helpful, since he was able to use it both for graphing and for finding the integral of a function. He stated, “With the calculator, I can just punch in the equation.” He was therefore given a score of 3 on both items. Student C2 stated that the graphing calculator was not very helpful and that she only used it for computation, and thus this student was given a score of 1 on both items.

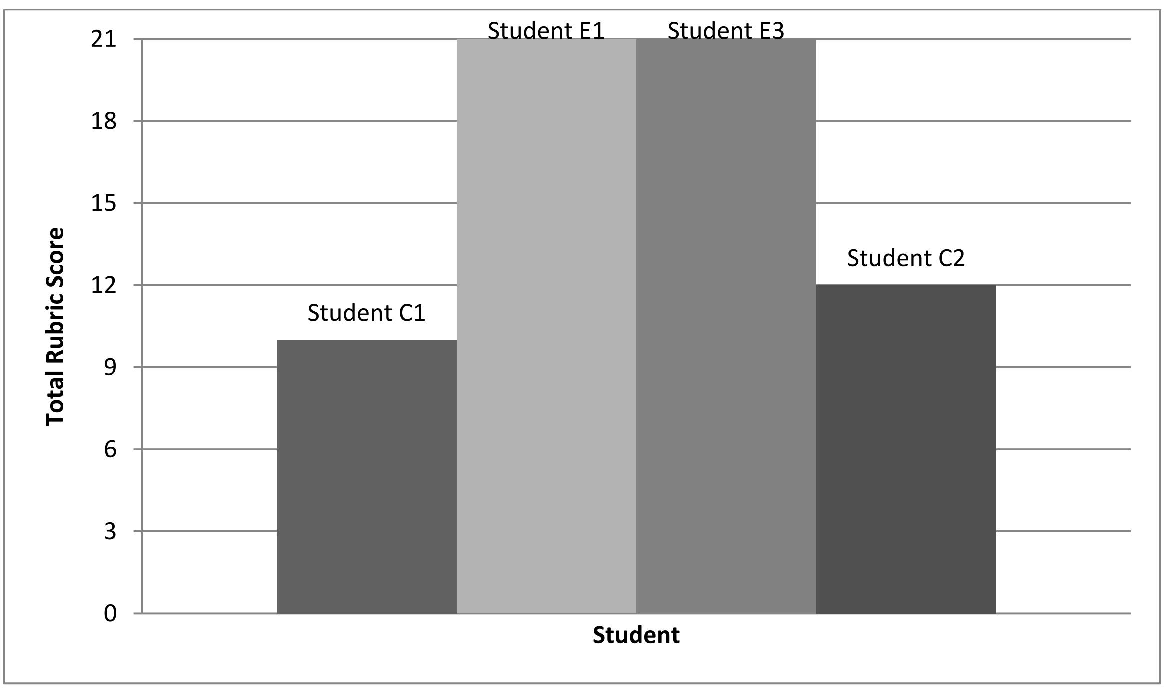
The next question was asked of all students but was directed mainly toward those that did not use the graphing calculator to solve the problem. Students were then asked, “How helpful do you believe the graphing calculator could have been in solving the problem other than the ways you used it?” Responses to this question were graded according to the extent of use and variety of uses of the graphing calculator. Scores for the first item were given on a scale of 0 to 3, with 0 representing that the graphing calculator was not helpful at all and 3 representing that the graphing calculator was extremely helpful. Scores for the second item were given on a scale of 0 to 3, with 0 representing that the graphing calculator would not be used at all and 3 representing that the graphing calculator would be used for multiple representations. Student C1 responded that it could have been extremely helpful if used to find the volume. She was given a score of 3 on the first item and 2 on the second item. Student E1 responded that she had used the graphing calculator to its full potential and that it could only have been more helpful if it had just given the answer, which it did. This student was given a score of 3 on both items. Student E2 also responded that he used the graphing calculator as much as possible, and was thus given a score of 3 on both items. Student C2 stated that the calculator could have been used to see the graph of a function if she did not already know what it looked like if it were a more difficult function. Since she only found the calculator to be helpful at times and only for one feature, this student was given a score of 1 on the first item and 2 on the second item.

Students who did not use the graphing calculator to fully solve the problem or believed that they could have used it differently were then asked, “In solving the problem, could you have used only the graphing calculator?” If the student then responded “yes,” he or she was asked to do so. The students were then asked to solve the problem using the graphing calculator. Student C1 began by going directly to the CALCULUS program without looking at the graph or table first. She encountered a problem when she attempted to find the volume using the program and the program stated that one function was not always greater than the other function. This problem arose because the student did not split the region into two intervals before finding the volume. When she encountered this problem, she did not attempt another approach. Student E1 was able to completely solve the problem on the graphing calculator using the graph of the function to find the limits of integration and number of intervals needed and the CALCULUS program to find the volume. Student E2 stated that he had used the calculator as much as he could. Student C2 used the graphing calculator to integrate the function, but she used the same bounds that she found algebraically and therefore did not solve the problem entirely on the calculator.

Conclusions

Scores on the seven questions addressing problem solving approaches and representations were combined to give each student another total score to represent the way that they approach a calculus problem. Higher scores represent use of the graphical approach, use of the graphing calculator beyond mere computation, and use of multiple representations. Student C1 received a score of 10, student E1 received a score of 21, student E2 received a score of 21, and student C2 received a score of 12 out of a possible 21 points. The two students who were part of the experimental group, in which the graphing calculator was used for instruction and assignments, scored higher than those students who did not use the calculator, indicating use of different approaches and representations, including the graphical approach. Figure 4 below presents the results of this part of the interview.

Figure 4: Problem Solving Approaches and Representations

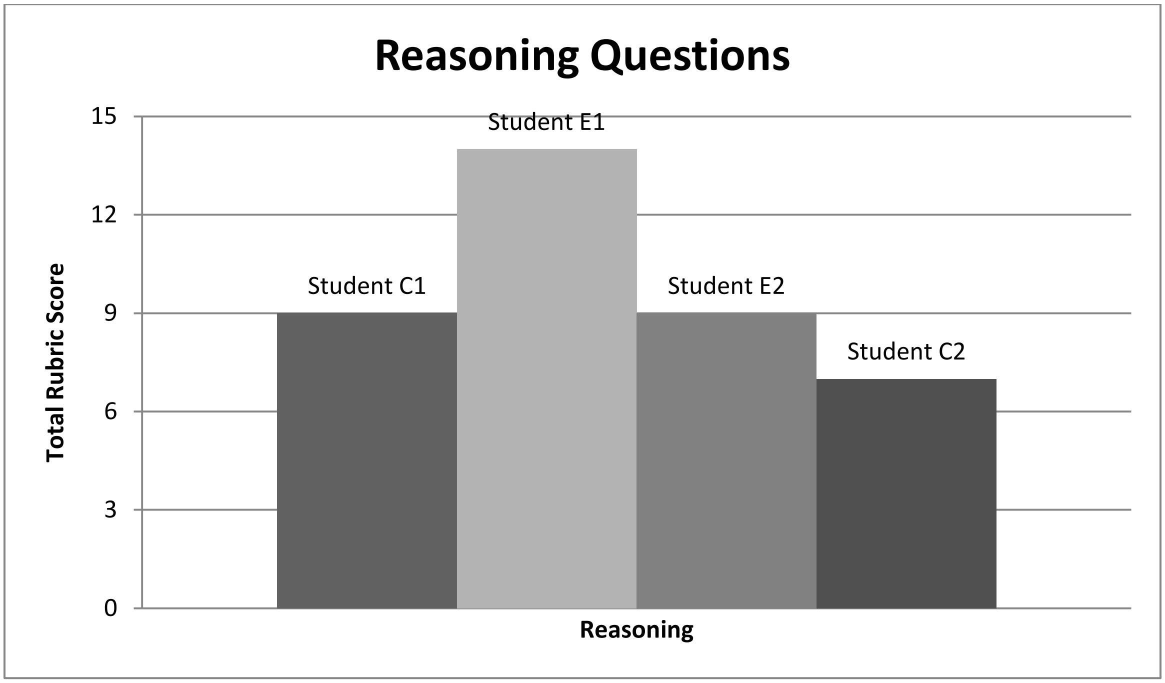


Reasoning

First Research Question

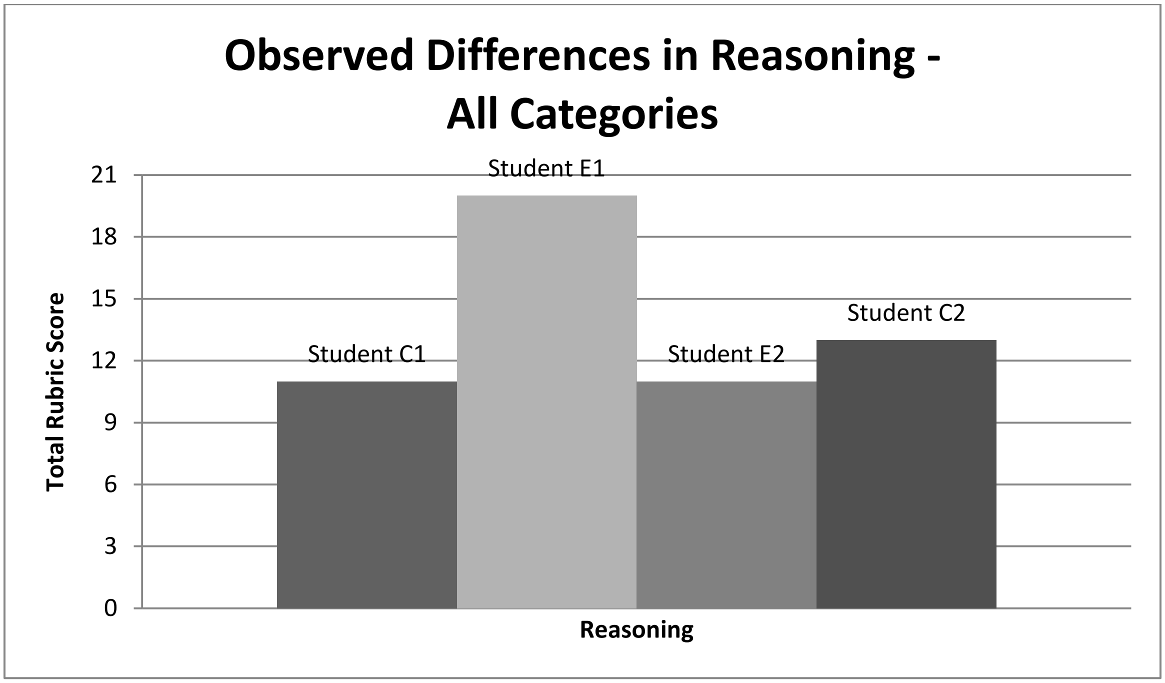
The first research question asked, “Does the use of the graphing calculator improve high school calculus students’ reasoning ability in calculus problems applying the definite integral?” In order to further examine this question after quantitative analysis, during each interview, the students were asked some questions relating to the reasoning process after solving the calculus problem. The last five interview questions related to the reasoning process. These questions specifically addressed the areas of analyzing a problem, initiating a strategy, and reflecting on one’s solution. Scores were combined on these last five questions to represent and compare the reasoning processes used by the students. Higher scores represent a greater demonstration of reasoning abilities. Student C1 received a score of 9, student E1 received a score of 14, student E2 received a score of 9, and student C2 received a score of 7 out of a possible 15. The scores for the students were more similar in this area than they were in the problem solving area; however, the student scoring the highest in this area was part of the experimental group and the student scoring the lowest was part of the control group. The student with the highest score is also the one that learned the analytic approach and graphing calculator approach concurrently during instruction and who used both approaches in solving the problem. Figure 5 below presents the total reasoning score earned by each student, based upon the reasoning questions asked in the interviews.

Figure 5: Reasoning Questions



The other two areas of reasoning, monitoring progress and seeking and using connections, were also observed throughout the process of solving the problem. I observed whether students were able to solve the problem entirely or in part, whether they monitored their progress in solving the problem, and whether they made connections to other concepts, subjects, or areas of math throughout solving the problem. When all five areas of reasoning are included, students C1 and E2 each earned 11 points, student E1 earned 20 points, and student C2 earned 13 points. Figure 6 presents the total reasoning score for each student from the interviews, based upon the interview questions and observations.

Figure 6: Observed Differences in Reasoning



Second Research Question

The second research question asked, “In what specific areas of reasoning does use of the graphing calculator seem to be most and least effective?” In order to further examine this question, students were asked questions during the interview relating to the concepts of analyzing a problem, initiating a strategy, and reflecting on one’s solution, and they were observed as they solved a problem in order to be assessed in the areas of monitoring progress and seeking and using connections.

The first question related to the stage of the reasoning process called “Analyzing a Problem.” Students were asked, “Can you think of any mathematical concepts and relationships between concepts that are used in this problem?” Scores were given on a scale of 0 to 3, with 0 representing that the student cannot describe any relationships and 3 representing that the students can describe at least two relationships, one of which includes a graphical approach. Student C1 was able to list two relationships between mathematical concepts, but one of these was incorrect. She correctly stated that the integral can be used to find the volume of a solid. She also stated that the volume equation for a circle can be found within the volume equation; however, this is incorrect, since it is the area equation for a circle that is found within the equation of the volume of a solid of revolution. This student was given a score of 1, since she was able to describe one correct relationship. Student E1 was able to describe two relationships. One of these is the relationship between derivatives and integrals, and one of these is between surface area and volume. Student E1 commented that she made one of these connections when she made an error on the graphing calculator. In attempting to find volume, she mistakenly found surface area and then noticed a relationship between the surface area and the volume of a figure. This student was given a score of 2, since she was able to describe two relationships but did not include a graphical approach. Student E2 was not able to describe any relationships between mathematical concepts, such as integration and graphing. He did, however, list some of the concepts that were used in solving the problem; therefore, this student was given a score of 1 on this item. Student C2 was able to list one relationship between mathematical concepts. She stated, “You use the formula pi r squared that’s the area of a circle, which is also used in geometry.” This student was given a score of 1 on this item as well.

The next question pertaining to the reasoning process also referred to the stage of the reasoning process called “Analyzing a Problem.” Students were asked, “Are there any conclusions that you would make at this point about the solution to the problem? If so, what would you say?” Scores for this question were given on a scale of 0 to 3, with 0 being no response given and 3 representing that the student could draw at least 2 correct conclusions. Student C1 was able to state one conclusion – that the solution would not be zero but would probably be a positive number, and therefore was given a score of 2. Student E1 was able to list two conclusions. She stated, “I automatically assumed it would be in units cubed. I also knew that the answer to the problem was going to have to be added together because I would get two separate answers.” The student knew that there were two regions because of the graph that she saw using the graphing calculator. This student was given a score of 3, since she was able to draw two conclusions. Student E2 attempted to answer this question, but he did not provide an appropriate response. Rather, he provided information about whether his answer was reasonable. This student was therefore given a score of 0 on this question. Student C2 provided one conclusion. She stated that the volume would be positive, and thus this student was given a score of 2 on this question.

The next question asked referred to the stage of the reasoning process called “Initiating a Strategy.” Students were asked, “What are some of the approaches you could use to solve this problem?” Scores for the this question were given on a scale of 1 to 3, with 1 representing that the students could not list any approaches and 3 representing that the student could list at least 2 approaches. Three students were able to list two approaches. Student C1 listed using the volume equation and the graphing calculator, student E1 listed solving for x and finding the integral, and student E2 listed graphing, integration, and algebra. All of these students were given a score 3. Only one student, student C2, was able to state one approach. She suggested graphing the function to determine what the shape is. This student was given a score of 2 on this question.

The next area of reasoning, “Monitoring Progress,” was observed as students solved the calculus problem given to them. The strategies used by the two students in the control group, who were part of the control group, were entirely algebraic. They relied upon their knowledge of the formula for volume and the Fundamental Theorem of Calculus to set up and evaluate a definite integral to find the volume. Student C1 did not, at any time, reference any method of graphing to solve the problem. Although her methods were almost completely correct, she made one error in computation and was not able to determine why this happened or attempt another approach. In addition, when asked to solve the problem on the graphing calculator, she knew the procedure for doing so, but was unable to follow through when the calculator screen displayed that one function was not always greater than the other function. A look at the graph of this function would have shown why this was true. Throughout solving the problem, she did not make assumptions or observations, but rather the student relied mainly on recalling formulas to solve the problem. Student C2 also used mainly algebraic methods but did use a graph at the beginning of the problem. She seemed to get confused for a minute when she realized that she needed bounds in order to solve the problem but then remembered the algebraic procedure for doing so. In addition, when she found the bounds, although they were correct, they did not match the graph that she had sketched on paper. Her sketch was limited only to the first quadrant, whereas the entire function was actually located in both the first and third quadrants. Although this student did rely mainly on her knowledge of the formula for volume and the fundamental theorem of calculus to solve the problem, she did make at least one assumption throughout the process of solving it. This is when she suggested that the shape of a cross-section would form a disk. I found that the reasoning process used by student E1, who was in the experimental group, differed greatly from this. Student E1 began by looking at the graph of the function before attempting to begin the problem. She stated that she always looks at the graph first so that she can picture what the shape of the three-dimensional solid will look like. By looking at the graph, she was able to determine that the volume would have to be determined by setting up two separate integrals, although at the end of the interview, she questioned whether her methods were correct. Throughout solving the problem, she used the graphing calculator to make assumptions and then also verified these assumptions using the graphing calculator. The graphing calculator seemed to very beneficial in the reasoning processes used by this student. Student E2, who was also in the experimental group, did use reasoning; however, his assumptions and conclusions were incorrect. He inadvertently found the area of the region rather than the volume and then justified the answer based on the information that he would used if he were in fact finding area. This student seemed to confuse the two concepts of area and volume, although he knew that the solid formed would be three dimensional. This showed a lack of true understanding by this student. The problem solving process used by the students was scored on a scale of 0 to 3, with 0 representing that the student used an incorrect approach in solving the problem, without monitoring progress, and 3 representing that the student followed the correct procedure throughout solving the problem. Student C1 could not find the correct answer to the problem. This student found an answer and realized that the solution was incorrect but did not attempt another approach; therefore, this student was given a score of 1 on this item. Student E1 found the correct answer to the problem and showed evidence of monitoring progress throughout; therefore, this student was given a score of 3. Student E2 found an incorrect answer to the problem by using an incorrect approach. This student did not show evidence of monitoring progress throughout; therefore, this student was given a score of 0. Student C2 found the correct answer to the problem, and was thus given a score of 3 on this item.

The next area of reasoning, "Seeking and Using Connections," was assessed by observing whether students made references to mathematical concepts used earlier in their calculus course, in other mathematics courses, or in any other subject areas. Observations were scored on a scale of 1 to 3, with 1 representing that the student made no connections to other concepts or courses and 3 representing that the student made at least two connections to other concepts or courses.

Student C1 made no connections and thus was given a score of 1 on this item. Student E1 referenced two concepts, derivatives and surface area, and therefore was given a score of 3. Student E2 referenced using what he learned in one other course, algebra, and therefore was given a score of 2. Student C2 referenced one other course, geometry, and one concept, fundamental theorem of calculus, and was therefore given a score of 3 on this item.

The next question referred to the stage of the reasoning process called "Reflecting on One's Solution." The students were then asked, "Do you think that the answer you got is reasonable? Why or why not?" Scores for this question were given on a scale of 0 to 3, with 0 representing that the student could not find an answer and 3 representing that the student is able to decide whether the answer is reasonable and to justify the answer. Three students were given a 3 on this question, since all were able to decide whether the answer was reasonable and to justify the answer. Student C1, for example, who got the wrong answer when solving the problem, realized that the answer was not reasonable. She stated, "No, I do not, because the volume should not be zero. There should be some volume." One student, student C2, stated that she believed her answer was reasonable but could not provide a reason, so this student was given a score of 2 on this question.

The last question referred to the stage of the reasoning process called "Reflecting on One's Solution." Students were asked, "Is there any other information you can gather from this problem, or any other comments that you would make after solving it?" Scores for this question were given on a scale of 0 to 3, with 0 representing that the student could not provide any other information and 3 representing that the student provides more than one additional inference, generalization, or comment. Students C1 and C2 were given a score of 0, since neither could provide any other information nor make any generalizations. The two students who were part of the experimental group that used the graphing calculators in this study were both able to come up with at least one conclusion. Student E1 stated that the shape would be an hourglass figure. She also stated that she wondered whether the volume should be computed by taking the integral from -2 to 2 or from -2 to 0 and 0 to 2. This student was given a 3, since she was able to make more than one comment or generalization. Student E2 suggested that if different bounds were given that he would get a different answer. This student was given a score of 2, since he was able to make one inference.

Summary

This chapter presented the results of the mixed-method study that investigated the impact of graphing calculator use on calculus students' reasoning in integral calculus. Results addressed the research hypothesis and the three research questions through quantitative analysis of posttest reasoning scores, using MANCOVA, univariate analyses following the MANCOVA, ANCOVA, and qualitative analysis of interview data.

During the interviews, graphing calculators were used as both a computational device and also as a means of making and checking assumptions, some of the activities that are part of the reasoning process. Two of the students interviewed used the graphing calculator for computation only, and two of the students used the graphing calculator in other ways. None of the students, however, used it in all of the ways possible, since none referred to the table feature. The use of the graphing feature on the graphing calculator was beneficial in the reasoning process as well by helping students to make conclusions about the solution to the problem before solving it.

Results indicated that, when all areas of reasoning are considered together, significant differences existed between the two groups of students, both for all participants overall and in school 2, but not in school 1. Overall, the experimental group scored significantly higher than the control group in the areas of initiating a strategy and monitoring progress. In school 2, the experimental group scored significantly higher than the control group in the areas of initiating a strategy and reflecting on one's solution. Results of the quantitative portion of the study lead one to reject the null hypothesis, which states that the mean reasoning score of those using pencil and paper graphing methods is greater than the mean reasoning score of those using graphing calculators. Results of the qualitative portion of the study showed that the students who had the graphing calculator throughout instruction had the highest scores in the area of problem solving, indicating use of multiple representations for problem solving. In addition, results of the qualitative portion of the study indicate that the student who used the graphing calculator together with the algebraic approach had the highest reasoning scores on the interview.

CHAPTER 5

DISCUSSION AND CONCLUSIONS

Purpose

Reasoning is viewed as an important part of understanding and learning in mathematics, and emphasis increasingly is being placed upon improving reasoning skills in mathematics. It is therefore important to investigate ways that reasoning skills can be improved, and technology is one of the ways suggested to improve these skills. The purpose of the current study was to investigate whether the use of handheld graphing technology, or graphing calculators, could improve reasoning skills in integral calculus.

First Research Question

The first research question asked, “Does the use of the graphing calculator improve high school calculus students’ reasoning ability in calculus problems applying the definite integral?” A multivariate analysis of covariance was conducted in each school and among all participants in order to answer this question and to test the null hypothesis, which states that the mean reasoning score of those using pencil and paper graphing methods is greater than the mean reasoning score of those using graphing calculators.

Results of the MANCOVA show that there was a significant difference between the control group and the experimental group when all aspects of reasoning are considered together. This difference is significant overall among all participants and specifically in school 2.The experimental group, which had access to graphing calculators during instruction and during the assessment, scored significantly higher than the control group, who did not have access to graphing calculators. This indicates that the graphing calculator was beneficial in improving reasoning skills overall among participants, but it was particularly useful in school 2. These results did not hold true for school 1, however; moreover, in school 1, it was the control group that scored higher on the posttest reasoning assessment, although this finding was not statistically significant. Therefore, results of the study lead one to reject the null hypothesis in favor of the alternative hypothesis.

During the course of the study, certain variables were controlled for, such as variability in the lessons and the textbook; therefore, this discrepancy in scores could be due to either individual differences in skill or to teaching styles. Examining the results of an independent samples t-test comparing students in the two different schools shows that there was a significant difference between students in school 1 and school 2 at the time that the pretest was given. Students in school 1 scored significantly higher on the pretest than students in school 2. Therefore, it may be helpful to consider the difference, if any, that the graphing calculator made in each individual school. In this case, graphing calculators seemed to improve reasoning skills with calculus students who began with lower reasoning skills. For those students who began with better reasoning skills, as compared to others taking the assessment, graphing calculators did not seem to be helpful or provide any significant effect in the area of reasoning. Thus, they may be more helpful in improving reasoning skills for students with lower reasoning abilities. This finding is consistent with the research literature. Burrill et al. (2002) also found that graphing calculators were more effective for lower-achieving students.

The qualitative portion of the study supports this conclusion. When considering only the reasoning questions, the highest score was earned by a student in the experimental group, and the lowest score was earned by a student in the control group. When reasoning questions and observations of reasoning skills are included, the scores are a little closer together; however, still, the highest reasoning score in the interviews was earned by a student in the experimental group, who had access to graphing calculators concurrently during instruction and also during testing. In addition, it was this student who also used both an algebraic approach and a graphical approach, primarily through the use of the graphing calculator, to solve the problem that was given. This student scored higher overall in reasoning than those who used either only an algebraic approach, only a graphical approach, or used both approaches but used pencil and paper for graphing.

Based on the results of the multivariate tests, one can conclude that, overall, students in the experimental group scored higher on reasoning questions than those in the control group. One can conclude, therefore, based on the results of the current study, that graphing calculators do improve high school calculus students' reasoning of the concept of the definite integral and its applications, although generalizations cannot be made to all populations due to a small sample size. Results from the qualitative portion of the study support the conclusion, while adding that a combination of the analytic and graphical approach during instruction and testing is the best approach to support reasoning skills.

Second Research Question

The second research question asked, "In what specific areas of reasoning does use of the graphing calculator seem to be most and least effective?" After the MANCOVA was conducted for each school and for all participants, if results of the MANCOVA were significant overall when all areas of reasoning were considered together, then univariate results were analyzed as well in order to determine the specific areas of reasoning for which the graphing calculator was most helpful.

When all students were included in the analysis, the areas of reasoning that were significantly improved when graphing calculators were used were that of initiating a strategy and monitoring progress. These two areas of reasoning require determining possible approaches to the problem and strategies that can be used throughout solving the problem and solving the problem while monitoring the steps that are used to ensure that they are appropriate and leading the student in the right direction. Students who used the graphing calculators were also more successful, on average, than those who did not in the areas of reasoning called analyzing a problem and reflecting on one's solution; however, these findings were not significant.

In school 2, the areas of reasoning that were significantly improved when graphing calculators were used were that of initiating a strategy and reflecting on one's solution. These areas of reasoning require determining possible approaches and strategies that can be used throughout solving the problem, reflecting on the reasonableness of one's solution, justifying the solution, and making inferences and generalizations after solving. The only area of reasoning that did not seem improved through the use of graphing calculators was that of seeking and using connections. In this area, those who did not use the graphing calculators scored slightly better, on average, by approximately one-hundredth of a point; therefore, this finding was not statistically significant.

The qualitative portion of the study also sought to answer this question. In general, there was no specific pattern that could be found, attributing higher scores to members of one group and lower scores to members of another group. The only pattern that could be found was that, in each category, a student in the experimental group, who approached the problem through the use of algebraic methods as well as the graphing calculator, received either the highest score or the maximum score obtainable for that category.

Although a distinct pattern could not be found in the scores, throughout the interview process, it was interesting to note comments made by some of the participants concerning the use of graphs. These comments, at times, related directly to some of the particular areas of reasoning. Student E1, for example, when asked about how she uses the graphing calculator, stated, “I used it in many ways. I used it to see the actual graph to make sure that all of my problems throughout working it were correct and my assumptions were correct as well.” The process of checking assumptions described by student E1 throughout solving relates directly to the area of monitoring progress. Therefore, graphing calculators were helpful to this student for this particular area of reasoning. In addition, this student not only described her use of the graphing calculator in this way but also demonstrated it when solving the problem. She stated, “(the graphing calculator) told me that the volume from 0 to 2 is 13.404, and because it was the same problem and it looked like it was the same on both sides (of the y-axis), from 0 to 2 and -2 to 0, I’m gonna assume that the answer to the other problem is also the same answer, but I can never really assume in math, so I’m gonna do it (perform the volume computation) anyway.” After making this assumption and checking her result on the calculator, she stated that, based on the graph that she saw, she knew that she would get her final answer by adding the two regions seen on the calculator together. Student E2 also made a comment that related to a reasoning skill. This student stated, “I like to look at the graph after you get your answer to see if it looks reasonable,” which is part of the area of reasoning called reflecting on one’s solution. The comments that were made that related directly to the reasoning process were both made by the students in the experimental group, who had access to graphing calculators throughout the study.

Therefore, based on the qualitative portion of this study, one can make observations about the potential impact of the graphing calculator based upon the interview responses, but no conclusions can be made, since the differences between students in each category were small and spread out between the two groups. Observations may lead one to conclude that graphing calculators were helpful to the students in the areas of analyzing a problem, monitoring progress, and reflecting on one's solution. In addition, results of the univariate tests following each significant MANCOVA lead one to conclude that graphing calculators are most effective in improving reasoning skills overall in the areas of initiating a strategy and monitoring progress.

Third Research Question

The third research question asked, "To what extent can students who have used the graphing calculator demonstrate ability to solve problems using pencil and paper methods?" In order to answer this question, analysis of covariance was conducted with part 2 of the posttest, since neither group could use the graphing calculator for this portion of the test, which consisted of one question on volume.

Results of the ANCOVA show that there were no significant differences between the two groups on this part of the posttest, both when all participants are considered together and when each school is considered separately. Overall and in school 2, students in the experimental group, scored higher, on average, than students in the control group; however, these findings were not significant. In school 1, students in the control group scored higher, on average, than students in the experimental group; however, this finding was not significant either.

Therefore, overall and in school 2, the experimental group scored significantly higher on part 1 of the posttest but not on part 2, on which the graphing calculator was not allowed. Based on these results, one may conclude that students who have used the graphing calculator can do equally as well as those who have not used the calculator when both groups use pencil and paper to solve problems. Based on these results, one can also conclude that graphing calculators are most effective when used in both instruction and testing. This finding is also supported by the research literature. Ellington (2006) also found a positive effect on achievement when graphing calculators were used in both instruction and testing.

Appropriate Instruction in and Use of Graphing Calculators

Appropriate use of the graphing calculator is an important factor in its success. Through the qualitative portion of this study, observations were made concerning the appropriate use of the graphing calculator. Appropriate use of the calculator refers, in part, to the ways in which it is used, that is, whether it is used purely for computation, for at least one representation, or for multiple representations. Throughout the interviews, the students made comments describing their use of the graphing calculator. Some used it for computation only. For example, student C1 stated, "I actually only used it to find out what two to the fifth was;" however, she also stated that the calculator could have been extremely helpful had she used the programs that were available. Another participant, student C2 stated, "I just used it to simplify the numbers. I could've done that by hand but (I used the calculator) just to make it quicker. And I already knew that the graph was a circle." Although this student only used the calculator for computation, she described other ways that it could be used. She stated, "If I didn't know what the graph already looked like, it could show me that, and if it was a more difficult problem, I'd have to plug it in to see the graph." Although the student sketched on paper what she believed the graph would resemble, her sketch was inaccurate, as it excluded the left quadrants; therefore, using the graphing calculator would have helped the student to see the entire graph. When asked about possible approaches, student C2 stated, "I would look at the graph first, honestly, because once they're asking for volume, they're gonna want to know the shape of it, and you're going to have to figure out what the 3D shape of the graph would be." Student E2 also commented about use of the graphing calculator, stating, “If I can look at it, I can typically figure it out and at least get close, you know.” He also stated, “I like to graph every problem, that way I can see what I’m trying to find before I try to find it.” Visualization, then, is important to this student, and he believes that visualizing the problem can help him to solve it. In addition, although this student stated in the interview that he usually only uses the graphing calculator for computation, I observed his use of the graphing calculator during the interview to be more than just for computation. Student E1 commented about using the graphing function of the calculator as well, stating, “I always try to keep that in mind whenever I’m doing these problems, is what kind of shape it makes.” For this study, visualization is a key aspect of the graphing calculator as well. The students used the graphing calculators in a variety of ways, with some using them only for computation and others using them to see the graphs of functions and compute integrals. None of the students, however, used all of the features on the graphing calculator; none referred to the table feature, for example. Therefore, the potential uses of the graphing calculator were not explored fully by any of the students interviewed. This finding is consistent with that of Burrill et al. (2002), who found in their meta-analysis of several graphing calculator studies that under-utilization of the graphing calculator was a concern.

Appropriate use can also refer to ensuring that students do not develop an overreliance on the calculator. During the interviews conducted for the qualitative portion of the study, observations were made concerning the ways in which the calculators were used and whether the students used only the graphing calculator to solve the problem, only the analytic approach, or a combination of the two approaches. Of the four students, two used a primarily analytic approach, one used only the graphing calculator, and one used a combination of the two approaches. Based on the interviews conducted with the four students, it appears that the student who performed the best in reasoning was the one who used a combination of the two approaches. This student used the graphing calculator as an additional tool to help solve the problem, and she referred back and forth between the multiple representations on the graphing calculator and her pencil and paper calculations. She used it to make conclusions about the solution as well. One could conclude, therefore, that an approach that combines the graphing calculator with analytic techniques is most effective in improving reasoning.

Another concern with appropriate use can be the timing with which instruction in the graphing calculator, or the graphical approach, is given. Two of the students interviewed were in the experimental group and were presented with the graphing calculator approach and the analytic approach at the same time. The other two students were in the control group; hence, they learned the analytic approach first, and then learned how to solve problems using the graphing calculator in a later lesson. Although the two students in the control group learned how to use the graphing calculator before the interviews were conducted, they did not use the calculators to help them solve the problem or in any of the areas of reasoning. The two students in the experimental group did use the calculators in a variety of ways to solve the problem and in reasoning. One of these students, particularly the one who used both approaches, was able to make more appropriate conclusions than the other student. Some educators suggest that manipulatives, which also include graphing calculators, should be presented one at a time, so that a student has an opportunity to master one before being presented with another. Some suggest that students should only be presented with graphing calculators after they master a technique by hand. This study, however, points to the idea that it is more helpful to learn the two approaches concurrently, and that it is best to use a combination of these approaches when solving a problem.

Other Considerations

A few concerns were mentioned by the mathematicians grading the assessments over responses to the reasoning questions and over the grading scale. One of the concerns was that some students did not seem to have enough of a mathematics vocabulary to be able to thoughtfully answer the questions. In some cases, responses were given in incomplete sentences, and in other cases, students may not have understood terminology. Thus, an implication of this is that some of the goals of mathematics instruction should be the mastery of mathematics terminology and to teach students how to effectively communicate ideas in mathematics. Another concern was expressed over the grading scale. The recommendation was that more emphasis should be placed on the part in which students actually solve the problem on paper, monitoring progress, and less emphasis should be placed on the other areas of reasoning. The recommendation arose out of a concern that there may not be consistency between this area of reasoning, monitoring progress, and all of the other areas of reasoning, which require reflection on the problem. In order to address this concern, a correlation was done between the area of reasoning called monitoring progress and the sum of all other areas on the second part of the posttest, on which all work had to be done using pencil and paper. The goal was to determine whether there was a relationship between scores in monitoring progress and scores in all other areas of reasoning. Results showed a moderate, positive correlation $r = .514, p = .001$ between these areas, and the correlation was significant at the $p < .01$ level; therefore, one can conclude that high scores in monitoring progress were significantly related to high scores in the other areas, and low scores in monitoring progress were significantly related to low scores in the other areas. This shows that students who were able to identify concepts and relationships, develop strategies, make conclusions, draw connections to other areas, and reflect upon their answer were also able to successfully solve the problem. In comparison, students who were not able to perform well in all of these areas requiring reflection were also not able to solve the problem. As a result of these findings, one of the goals of instruction should be to develop strategies to improve reasoning, so that students can also improve their problem solving. Table 10 below presents the results of the correlation.

Table 10: Correlation Results

Correlation Table for Monitoring Progress and All Other Reasoning Areas for n = 39 students

	Monitoring Progress	Total of all Other Elements of Reasoning
Monitoring Progress	---	.514*
Total of all Other Elements of Reasoning	---	---

Note: *Correlation is significant at the $p < .01$ level.

In reflecting upon the outcome of the study, I believe that the study supports the research literature and speculation about the use of graphing calculators, while adding to this to the potential for the graphing calculator to improve reasoning. However, it also points to particular ways of using them and incorporating them into instruction that may be more beneficial than others. Much of the research on graphing calculators supports their use but also often discusses appropriate use. There are often different sides to the argument concerning whether students should be allowed to use graphing calculators and in what order. It was interesting, therefore, to find that graphing calculators were in fact helpful in not only finding an answer but in improving reasoning skills when they were used in instruction and testing. In addition, it was interesting to find that students who learn the analytic and graphical approaches concurrently and who use both approaches in solving a problem appear to benefit the most in the area of reasoning. I would like to apply the results of the study to my own teaching and my own calculus classes, in particular.

As a result of the study, I will incorporate practice of reasoning skills so that students can solve problems. In addition, I will continue to teach multiple approaches simultaneously, and I will encourage students to use multiple approaches when solving the problem. As a result of this study, I believe that the calculator could be most useful if students use it for exploration, for multiple representations, and for computation. As a result of the study, I also found that some students provided more approaches and conclusions than I would have expected. Therefore, consistent with the theory of constructivism, allowing students to come up with their own ideas and ways of approaching a problem may be a valuable activity as well.

Conclusion

The current study addressed three primary research questions. These questions asked whether graphing calculators could improve high school students' reasoning in solving problems on the definite integral and its applications, in what specific areas of reasoning graphing calculators were most and least effective, and whether students who have used the graphing calculator can solve problems using pencil and paper.

Results of the study showed that graphing calculators were an effective way of improving reasoning in integral calculus both overall among participants and particularly in school 2. The particular areas that the graphing calculators seem to be most effective in are initiating a strategy, monitoring progress, and, in school 2, also in reflecting on one's solution. Therefore, for topics relating to applications of definite integrals, graphing calculators may help students to recognize the procedures that they can use to solve a problem and then to carry out those procedures, while recognizing the reasonableness and unreasonableness of those procedures throughout the process of solving the problem. In addition, graphing calculators may help students to assess the reasonableness of their answers, to justify their answers, and then to form conclusions, inferences, and generalizations based upon their solution to the problem. The only areas of reasoning for which there was no significant difference when graphing calculators were used were analyzing a problem and seeking and using connections. However, students who used graphing calculators performed equally as well as those who used pencil and paper; therefore, graphing calculators did not in any way hinder students' ability to solve problems or their reasoning abilities. These areas of reasoning require understanding relationships between concepts that are used in the problem, drawing conclusions about a solution before solving the problem, and identifying connections between concepts used in solving problems and concepts learned in other subject areas.

Those students who were part of the experimental group and had access to graphing calculators during instruction were also able to solve problems using pencil and paper, although they earned higher reasoning scores when using graphing calculators on the reasoning assessment. It does not seem, then, that students who used graphing calculators throughout the study became overly reliant on the graphing calculators to solve problems, since they were still able to successfully solve a problem when not allowed to use the graphing calculator. Consistent with the literature, though, reasoning scores were most improved, though, when graphing calculators were used in both instruction and testing.

During interviews conducted for the qualitative portion of the study, graphing calculators were used for computation, numerical approximation of definite integrals, visualization of graphs, and to make and check assumptions. Results from the qualitative portion of the study support the quantitative findings as well, while also providing additional information, mainly that appropriate use of graphing calculators to support reasoning skills includes use of multiple representations, use of multiple approaches, and instruction in the analytic and graphical approach concurrently.

Implications for Research and Practice

Improving reasoning skills in math courses, and particularly in calculus, is an important part of helping students to achieve a better understanding of mathematics. Helping students to understand mathematics involves helping them to see it as meaningful. It also involves approaching mathematics from the constructivist perspective by allowing students to construct their own understanding rather than providing them with a list of procedures and algorithms. Through the use of handheld graphing technology, such as the graphing calculator, in both instruction and testing, students may be able to explore ideas and concepts so that they can improve their reasoning skills and thereby construct understanding and see mathematics as meaningful. Results of this study indicate a positive impact of graphing calculators on reasoning skills used to solve problems in integral calculus, and reasoning skills seem most improved when graphing calculators are used in assessment as well as instruction. Calculus teachers, then, may consider including the graphing calculator in their instruction for topics relating to applications of the definite integral, so that reasoning skills may be improved. In particular, calculus teachers may consider this for those students whose reasoning skills are relatively low. In addition, as a result of the positive correlation between monitoring progress and all of the other elements of reasoning, calculus teachers may consider using strategies, such as use of handheld graphing technology, to improve reasoning so that students' problem solving skills can also be improved. Graphing calculators may be most beneficial, however, when used together with other analytic techniques and when instruction in both techniques is concurrent. Students who rely purely on their graphing calculator or purely on algebraic techniques may not be provided with the same benefits as if both approaches were used. In addition, when graphing calculators are isolated from instruction and incorporated after other techniques are mastered, students may be less likely to use the graphing calculator to help solve problems. Calculus teachers may, then, encourage use of graphing calculators, while showing students other analytic techniques, in order to improve reasoning in their calculus classes.

Recommendations for Future Research

When reflecting on this study, it appears that certain aspects could be modified or further research could be done to investigate this topic more thoroughly. The following recommendations are suggested to guide future research in this area:

1. In question 3 on the posttest and question 4 on the pretest, students are asked to "find" the area of a certain region and to determine whether it is an overestimate or an underestimate. In order to make these directions clearer, students should be asked to "approximate" the area of the region, so that this approximation can be classified as either an underestimate or an overestimate.
2. It was interesting to note that, although three of the students do not use sketches and graphs very often to assist them on homework assignments and tests, they almost always use the graphing calculator. This indicates that the students believe there is a difference between graphing by hand and graphing on the calculator. The students use the calculator for reasons beyond computation, that is, to see the graphs of the functions and, for one student, to possibly apply the information in the graph to the problems she is working. Responses to the interview questions indicate that, while providing sketches and graphs for students may not be preferable for students trying to learn calculus concepts, allowing students to use the graphing calculator is preferable. A further study may investigate the

reasons for this and the perceived differences between traditional graphing methods and the graphing calculator.

3. In an earlier study, Galindo (1995) found no significant differences in achievement in calculus between visualizers and non-visualizers when graphing calculators were used; however, when graphing calculators were used, this was the only situation in which non-visualizers did not outperform visualizers. This indicates that there may be some benefit for visualizers when graphing calculators are used. This study did not attempt to highlight differences between visualizers and non-visualizers in the area of reasoning when graphing calculators are used. Graphing calculators may aid in the reasoning processes for students who prefer to visualize calculus concepts, such as area and volume. This is an area in which further research could be done.
4. This study included only high school calculus students; however, reasoning is important in undergraduate mathematics as well. A future study could address the effects of graphing calculators on reasoning in college calculus courses.
5. A small sample was used for this study. A future study could include a larger sample, more calculus classes, and more calculus teachers.
6. The majority of participants in this study were white, female students. There may be cultural differences that would cause different effects on reasoning for different groups of students. A future study may investigate differences in reasoning between boys and girls or for different ethnicities when graphing calculators are used.
7. During the interviews, students were asked one question on math anxiety. Math anxiety was not a focus of this study; however, results of this question are in Appendix L. A

future study may investigate the relationship between math anxiety and reasoning skills, when graphing calculators are used and when they are not used.

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APPENDICES

Appendix A
IRB Application

APPLICATION FOR CONDUCTING RESEARCH

INVOLVING HUMAN SUBJECTS

A. FACE PAGE

PROTOCOL TITLE: The Effects of Graphing Calculator use on High-School Students' Reasoning in Integral Calculus		
ALTERNATE TITLE:		
TYPE OF REVIEW: ☒ NEW ☐ RENEWAL If renewal, are there substantive changes? Yes ☐ No ☐		
Principal Investigator: **Dr. Yvelyne Germain-McCarthy** **New investigator:** ☐	**Department:** **Curriculum & Instruction**	**University Affiliation:** ☒Professor ☐Associate Professor ☐Assistant Professor ☐Instructor ☐Other: (**Graduate students cannot serve as PI**)
Campus Address: **Bicentennial Education Bldg.** **2000 Lakeshore Drive** **University of New Orleans** **New Orleans, LA 70148**	**Phone:** **504-280-6533** **Preferred e-mail:** **ygermain@uno.edu**	
Co-Investigator: **Julie Spinato Hunter** **New investigator:** ☒	**Department:** **Curriculum & Instruction**	**University Affiliation:** ☐Faculty ☐Staff ☒Graduate Student ☐Undergraduate Student ☐Other:
Campus Address: **2200 Roosevelt Blvd.** **Kenner, LA 70062**	**Phone:** **504-812-9724** **Preferred e-mail:** **jspinato@uno.edu**	
Co-Investigator: **New investigator:** ☐	**Department:**	**University Affiliation:** ☐Faculty ☐Staff ☐Graduate Student ☐Undergraduate Student ☐Other:
Campus Address:	**Phone:** **Preferred e-mail:**	

Note: New investigators must submit a copy of their human subjects certification

(http://phrp.nihtraining.com/users/login.php)

B. Project Description

1. Provide an abstract of your project (do not exceed 250 words).

This mixed-method study will investigate the impact of graphing calculator use on high school calculus students' reasoning through calculus problems that relate to the concept of the definite integral and its applications. The study provides an investigation of the effects on reasoning when graphing calculators are used, since it is proposed that, through reasoning, conceptual understanding can be achieved. The study will include a quantitative, quasi-experimental component and a qualitative component. The participants will be students in four existing calculus classes at two private high schools in the Southeast United States. The two groups used for the study will be distinguished by use and non-use of the graphing calculator.

2. Provide a **brief** description of the **background, purpose, and design** of your research. Avoid using technical terms and jargon. Be sure to list all of the **means you will use to collect data** (e.g., instruments measures, tests, questionnaires, surveys, interview schedules, focus group questions, observations). Provide a short description of the tests, instruments, or measures and **attach copies of all instruments and questionnaires** for review. Descriptions should be at least 1 page and include citations.

The importance of conceptual understanding in calculus is well documented (CollegeBoard, 2009; Kissane & Kemp, 2008; Schwalbach & Dosemagen, 2000; Slavit, Lofaro, & Cooper, 2002), as is the importance of conceptual understanding in mathematics courses in general (Council, 2001; CUPM, 2004; Fuson, Kalchman, & Bransford, 2005). The development of conceptual understanding is critical to students' success and ability to solve problems in calculus courses. Understanding concepts, such as the limit, derivative, and integral, requires a method of thinking different from other areas of mathematics, which often involve finite processes, since most calculus concepts, such as these, involve infinite processes (NCTM, 1989). It has been recommended that curriculum and instruction in calculus focus on conceptual understanding rather than algebraic manipulation (NCTM, 1989, 2009). Despite this recommendation, it has been common for calculus courses to focus on manipulative techniques. Several studies have documented the difficulties that students have faced in calculus as a result of the focus on algebraic manipulation and a lack of focus on conceptual understanding (Baker, Cooley, & Trigueros, 2000; Judson & Nishimore, 2005; White & Mitchelmore, 1996). For example, this lack of understanding has caused students to have difficulty in linking calculus concepts in order to solve problems, thus creating an obstacle for student success in this course (Baker et al., 2000, Judson & Nishimore, 2005, White & Mitchelmore, 1996). The National Mathematics Advisory Panel (2008) has recommended that future research be focused on ways to improve conceptual understanding. One suggestion comes from the National Council of Teachers of Mathematics (NCTM, 2009). The NCTM recommends that refocusing the mathematics curriculum on reasoning and sense making will improve conceptual understanding.

Reasoning is an important part of understanding, that is comprised of analyzing a problem, initiating a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution (NCTM, 2009). Although an important part of understanding, reasoning has not been given consistent attention in the mathematics curriculum (Fuson, Kalchman, & Bransford, 2005; NCTM, 2009), and many students lack reasoning skills when they enter college (CUPM, 2004). As a result of the lack of reasoning abilities that exist, students often have not been able to see the meaning in mathematics, and ability to retain knowledge has been difficult (NCTM, 2009). The NCTM (2009) has suggested that technology may be used to improve reasoning, but this has been largely unexplored.

The purpose of this study is to investigate the impact of graphing calculator use on high-school calculus students' reasoning in solving problems on the definite integral and its applications. In order to investigate the effects of graphing calculator use on reasoning in calculus, a mixed-method study will be conducted with both a quantitative quasi-experimental component and a qualitative component.

The following research questions will guide the study:

4. Does the use of the graphing calculator improve high school calculus students' reasoning of the concept of the definite integral and its applications?
5. In what specific areas of reasoning does use of the graphing calculator seem to be most and least effective?
6. Can students who have used the graphing calculator demonstrate ability to solve problems without the graphing calculator?

The participants in this study will be high school students enrolled in either calculus honors or calculus AP at two high schools: a private all-girls high school in a suburban area in the Southeast United States and a private co-educational school located in a rural area in the Southeastern United States. Participants in this study will be both male and female students between the ages of 17 and 18 of varying races. Participants will not be excluded on the basis of race, gender, class, or any other distinguishing factor. Four existing calculus classes will be used for this study. Two calculus classes will be from the all-girls school, and two calculus classes will be from the co-educational school. Each school has both a calculus AP class and a calculus honors class. In the all-girls school, the honors class consists of 13 students, and the AP class consists of 10 students. In the co-educational school, the honors class consists of 8 students, and the AP class consists of 13 students. Participants will not be further separated into groups. In each school, the calculus AP class will serve as part the control group, and the calculus honors class will serve as part of the experimental group. The entire control group will be composed of students from both schools enrolled in calculus AP, and the entire experimental group will be composed of students from both school enrolled in calculus honors. Two different instructors will be used in this study, one of whom is the researcher. Each teacher will teach both one class in the control group and one class in the experimental group. All students will be taught the same lessons at the same level of difficulty. The two groups will be distinguished by use and non-use of the graphing calculator. Before the onset of the study, the administrators of the high school will be contacted and asked for permission to complete this study. They will be provided with a full explanation of the details of the study. After permission is granted, each student and her parents will be sent a letter containing the eight elements of informed consent. In this letter, they will be informed of the purpose of the study and asking if they would be willing to participate. The letter will also inform students that the results of the study will not in any way impact their grades and that credit will be given for participation in the study in the form of thoughtful responses to the questions asked. Students who agree to participate will be asked to provide a completed statement of informed consent. All participants will complete a researcher-developed pretest to assess reasoning skills in solving problems on the definite integral and applications. The pretest consists of five calculus questions with seven parts measuring the specific elements of reasoning. Students will then participate in the series of lessons, with the experimental group using the graphing calculator for graphing and the control group using pencil and paper graphing methods. Participants will then complete a researcher-developed post test that is analogous to the pretest. A validity check of the assessments was performed by a panel of two mathematicians. The same panel of mathematicians will grade both the pretests and post tests in order to establish inter-rater reliability. A rubric developed by the researcher and checked for validity by a mathematics educator will be used for grading the pretests and post tests. Analysis of covariance will be used in the analysis of data using pretest scores as the covariate. Afterwards, one student from each class will be selected for an interview. The researcher will use an interview script to guide the interview process. In each interview, the student will be asked to complete one of the questions on the post test. The researcher will assess the reasoning process used by the student in solving the problem. One of the main goals of the interview will be to describe the reasoning processes used by the students, the extent to which

graphical representations are part of that reasoning process, and the extent to which the students in the experimental group use the graphing calculator in reasoning through their solution to the problem.

C. Data Collection

1. Total **number of participants** that you plan to include/enroll in your study: 44
2. Age range of participants you plan to include / enroll in your study. 17 to 18
3. Will you recruit participants from any of the following groups? (check all that apply)

   ☒ **Minors (persons under the age of 18)**
   ☐ **Cognitively or psychologically impaired individuals**
   ☐ **Prisoners or parolees**
   ☐ **Specific medical population:**
   ☐ **Elderly**
   ☐ **Pregnant women**
   ☐ **Minority populations**
   ☐ **UNO students/employees**

If you checked any of the boxes in #3, describe how you will provide the **special protections** to which these participants may be entitled under federal regulations. (See a description of special considerations at: http://www.hhs.gov/ohrp/irb/irb_chapter6.htm. special protections are described at: http://ohrp.osophs.dhhs.gov/humansubjects/guidance/45cfr46.htm).

Since this study involves standard educational tests and an interview to assess performance on standard educational tests, this study will not involve any greater than minimal risks to the participants. For those participants under the age of 18, assent from the child and consent from the parents will be obtained prior to the study.

4. Will the recruitment of participants and/or data collection involve any of the following?:

   ☒ **Audiotapes, videotapes, photographs**
   ☐ **Electronic communications (e.g., e-mail, internet)**
   ☐ **Archival data that is not publicly available.**
   ☐ **Focus group**

If you checked any of the boxes in #4, describe how the media will be used (e.g., coded and then destroyed, kept for possible publication or broadcast, etc.).

This study will involve the use of audiotapes during interviews with participants. Audiotapes will be locked in a cabinet in Dr. Germain's office during data analysis and will be destroyed afterwards.

5. Does the proposed research require that you **deceive** participants in any way? ☐ Yes ☒ No

If yes, describe the type of **deception** you will use, why deception is necessary, and provide a copy of the debriefing script.
N/A

6. Describe how you will **recruit** participants and inform them about their role in the study. Please attach copies of advertisements, flyers, website postings, recruitment letters, oral or written scripts, or other materials used for this purpose.

I will recruit participants and inform them of their role in this study through a pre-prepared written script that will be read for the students. The written script is attached. The letters of informed consent and assent will also be read aloud for students. These letters are attached.

7. Project Start Date: __12-06-10_______________ Project End Date: __02-04-11________________
* Projects lasting more than 12 months must receive continuation approval before the end of the project.

D. Funding Source

1. Have you received any source of **funding** for the proposed research (federal, state, private, corporate, or religious organization support)? ☐ Yes ☒ No
2. Is this project currently **consideration** for funding (e.g., under review)? ☐ Yes ☒ No

If your response is "yes" to either 1 or 2, please indicate any source(s) of **funding** for the proposed research (e.g., NIH, NSF, departmental funds, private foundations or corporations).

3. Do you or the funding source(s) have any potential for financial or professional benefit from the outcome of this study? ☐ Yes ☒ No

If yes, please explain.

E. Risks to Participants

1. Consider to both the **actual and potential** risks to the participants that could reasonably be expected to occur during the course of the study. Check all that apply.
   - ☐ Disclosure of the participants' responses may place the participant at risk of **criminal or civil liability.**
   - ☐ Disclosure of the participants' responses may be damaging to their **financial standing, employability, or reputation.**
   - ☐ Participants may encounter **physical risk.**
   - ☐ Participants may be subjected to **stress** beyond that ordinarily encountered in daily life.
   - ☐ Participants may be asked to disclose information that they might consider **personal or sensitive.**
   - ☐ Participants may be asked to reveal **personal information that cannot be anonymous and/or there may be a limit to the confidentiality that can be guaranteed due to particular circumstances or procedures used in the study** (e.g., focus group or surveys submitted via email).
   - ☐ Participants may be presented with materials that they may consider **offensive, threatening, or degrading** or they may encounter other forms of **psychological or social risk**.
   - ☐ An individual's participation will be reported to an instructor so that the individual can receive **research or extra credit**.
   - ☐ As a result of this research, a **permanent record** will be created that will contain information (identifiers) that could reveal a participant's identity

If you checked any of these risks, discuss the risk below. Describe the steps you will take to **minimize risk** to the participant.

F. Informed Consent

1. Describe the procedures you will use to **obtain and document informed consent and/or assent.**

Students under the age of 18 will be given a letter to be sent home to their parents informing the parents of the purpose of the study and the students' role in the study. Parents who allow their child to participate will be asked to complete a written letter of informed consent. Students under the age of 18 will also be asked to complete a written letter of assent. Students ages 18 and over will be given a letter informing them of the purpose of the study and their role in the study. They will also be asked to complete a written letter of informed consent.

2. **Attach copies of the forms that you will use.** The UNO Human Subjects website has additional information on sample forms and letters for obtaining informed consent. (*in the case of secondary data, please attach original informed consent or describe below why it has not been included.)* Fully justify any request for a waiver of written consent or parental consent for minors. **All consent forms must be on current UNO letterhead and contain the 8 elements of consent: (http://www.humansubjects.uno.edu/8%20elements%20of%20consent.doc)**

G. Data Use

1. How will these data be use? Check all that apply.

☒	Dissertation	☐	Publication/journal article
☐	Thesis	☐	Results released to participants/parents
☐	Undergraduate honors thesis	☐	Results released to employer/school
☐	Conference/presentations	☐	Results released to agency/organization
☐	Other:		

2. Describe the steps you will take to ensure the confidentiality of the participants and data. Indicate how you will safeguard data that includes identifying or potentially identifying information (e.g., coding). Indicate when identifiers will be separated or removed from the data. Also, indicate where and how you will store the data and how long you plan to retain it. Describe how you will dispose of it (e.g., erasing tapes; shredding data). Be sure to include all types of data collected (e.g., audiotape, videotape, and questionnaire/survey).

The individual participants and their schools will remain anonymous in this study. Students will be asked not to put a name on their tests. In the reporting of results, names of students will not be used. Students' names will also not be used in the audiotapes. The tests and tapes will contain no identifying information. In the reporting of results, students will be referred to as Student A, B, C, and D. After analysis, the data (tests and audiotapes of interviews) will be stored in a locked file cabinet in Dr. Germain's office for approximately 2 months following the study. Afterwards, all data will be shredded.

Signature Page

Protocol Title:
The Effects of Graphing Calculator use on High-School Students' Reasoning in Integral Calculus

H. Principal Investigator's Assurance

I certify that the information provided in this application is complete and correct.

I understand that as Principal Investigator, I have ultimate responsibility for the conduct of the study, the ethical performance of the project, the protection of the rights and welfare of human subjects, and strict adherence to any stipulations imposed by the IRB.

I agree to comply with all UNO policies and procedures, as well as with all applicable federal, state, and local laws regarding the protection of human subjects in research, including, but not limited to, the following:

- performing the project by qualified personnel according to the approved protocol,
- implementing **no** changes in the approved protocol or consent form without **prior** UNO IRB approval (except in an emergency, if necessary to safeguard the well-being of human subjects),
- obtaining the legally effective informed consent from human subjects or their legally responsible representative, and using only the currently approved, stamped consent form with human subjects,
- **promptly reporting significant or untoward adverse effects to the UNO IRB in writing within 5 working days of occurrence.**

If I will be unavailable to direct this research personally, as when on sabbatical leave or vacation, I will arrange for a co-investigator to assume direct responsibility in my absence. Either this person is named as a co-investigator in this application, or I will advise UNO IRB by letter, in advance of such arrangements.

I also agree and understand that informed consent/assent records of the participants must be kept for at least three (3) years after the completion of the research.

Principal Investigator Name: (Print)	
Signature:	Date:
Co-Principal Investigator Name: (Print)	
Signature:	Date:
Co-Principal Investigator Name: (Print)	
Signature:	Date:

University Committee for the Protection of Human Subjects in Research University of New Orleans

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Campus Correspondence

Principal Investigator: Yvelyne Germain-MacCarthy

Co-Investigator: Julie Spinato Hunter

Date: November 15, 2010

Protocol Title: "The Effects of Graphing Calculator Use on High-School Students' Reasoning in Integral Calculus"

IRB#: 04Nov10

The IRB has deemed that the research and procedures described in this protocol application are exempt from federal regulations under 45 CFR 46.101 category 1B. This minimal-risk study will be conducted in established or commonly accepted educational settings, involving normal educational practices and will entail research on the effectiveness of or the comparison among instructional techniques, curricula, or classroom management methods.

Exempt protocols do not have an expiration date; however, if there are any changes made to this protocol that may cause it to be no longer exempt from CFR 46, the IRB requires another standard application from the investigator(s) which should provide the same information that is in this application with changes that may have changed the exempt status.

If an adverse, unforeseen event occurs (e.g., physical, social, or emotional harm), you are required to inform the IRB as soon as possible after the event.

Best wishes on your project.

Sincerely,

Robert D. Laird, Ph.D., Chair

Committee for the Protection of Human Subjects in Research

Appendix B
Participant Solicitation Materials

PARENTAL LETTER OF CONSENT FOR STUDENTS UNDER 18

Dear Parent:

I am a graduate student under the direction of Professor Germain-McCarthy in the Department of Curriculum and Instruction at the University of New Orleans. I am conducting a research study to investigate the impact of graphing calculator use on calculus students' reasoning skills.

I am requesting your child's participation, which will involve participating in a series of lessons on the definite integral and its applications, taking a calculus reasoning pretest and posttest during two class periods, and possible selection for an interview in which reasoning processes in solving calculus problems will be discerned. The topics to be included in this study are topics that students will be learning regardless of the study. This study will involve an experimental component, in which graphing calculator use will be the experimental treatment. At the end of the study, all students will learn both methods of graphing – using pencil and paper and using the graphing calculator, so the study will not affect your child's learning in any way. Your child's participation in this study is voluntary. If you choose not to have your child participate or to withdraw your child from the study at any time, there will be no penalty (it will not affect your child's grade). Likewise, if your child chooses not to participate or to withdraw from the study at any time, there will be no penalty. The results of the research study may be published, but your child's name will not be used.

Although there may be no direct benefit to your child, the possible benefit of your child's participation is the ability to determine whether the graphing calculator is useful in helping your child to learn certain calculus concepts.

The risks associated with participating are minimal and include minor test anxiety. These risks are not greater than those ordinarily encountered in daily life.

All assessments and audiotapes of interviews will be kept strictly confidential. They will be stored in a locked location in Dr. Germain's office during data analysis and will be shredded afterwards.

If you have any questions concerning the research study or your child's participation in this study, please call me [or Dr. Germain-McCarthy] at (504) 812-9724.

If you have any questions about you or your child's rights as a subject/participant in this research, or if you feel you or your child have been placed at risk, you can contact Dr. Ann O'Hanlon at the University of New Orleans at 504-280-6501.

Sincerely,

Julie Spinato Hunter

By signing below, you are giving consent for your child _______________ to participate in the above study.

______________________ ______________________ _____

Signature Printed Name Date

WRITTEN CHILD ASSENT FOR STUDENTS UNDER 18

I have been informed that my parent(s) have given permission for me to participate in a study concerning the impact of graphing calculator use on reasoning in calculus.

I will be asked to participate in a series of lessons on the definite integral and its applications, to take two tests (a pretest and a posttest) to assess reasoning skills during a regular class period, and possibly to participate in an interview in which reasoning processes in solving calculus problems will be discerned.

The risks or harm to me include minor test anxiety.

My participation in this project is voluntary and I have been told that I may stop my participation in this study at any time. If I choose not to participate, it will not affect my grade in any way.

______________________________ ______________________________ ____________

Signature Printed Name Date

LETTER OF CONSENT FOR STUDENTS 18 AND OVER

Dear Calculus student:

I am a graduate student under the direction of Professor Germain-McCarthy in the Department of Curriculum and Instruction at the University of New Orleans. I am conducting a research study to investigate the impact of graphing calculator use on calculus students' reasoning skills.

I am requesting your participation, which will involve participating in a series of lessons on the definite integral and its applications, taking a calculus reasoning pretest and a posttest during two class periods, and possible selection for an interview in which reasoning processes in solving calculus problems will be discerned. The topics to be included in this study are topics that students will be learning regardless of the study. This study will involve an experimental component, in which graphing calculator use will be the experimental treatment. At the end of the study, all students will learn both methods of graphing – using pencil and paper and using the graphing calculator, so the study will not affect your child's learning in any way. Your participation in this study is voluntary. If you choose not to participate or to withdraw from the study at any time, there will be no penalty, (it will not affect your grade). The results of the research study may be published, but your name will not be used.

Although there may be no direct benefit to you, the possible benefit of your participation is the ability to determine whether the graphing calculator is useful in helping students to learn calculus concepts.

The risks associated with participating are minimal and include minor test anxiety. These risks are not greater than those ordinarily encountered in daily life.

All assessments and audiotapes of interviews will be kept strictly confidential. They will be stored in a locked location in Dr. Germain's office during data analysis and will be shredded afterwards.

If you have any questions concerning the research study, please call me [or Dr. Germain-McCarthy] at (504) 812-9724.

Sincerely,

Mrs. Julie Spinato Hunter

By signing below you are giving consent to participate in the above study.

______________________ _________________________ __________

Signature Printed Name Date

If you have any questions about your rights as a subject/participant in this research, or if you feel you have been placed at risk, please contact Dr. Ann O'Hanlon at the University of New Orleans (504) 280-6501.

Script for Recruitment of Participants

I am a doctoral student at the University of New Orleans, and I am conducting a research study to investigate the impact of graphing calculator use on high-school calculus students' reasoning in solving problems on the definite integral and its applications. The definite integral is one of the important concepts that are covered in this course. I am interesting in finding out, if graphing calculators are used in learning about the definite integral and applications of the definite integral, whether they will have any impact on reasoning skills. This study could provide information about the potential uses of the graphing calculator in calculus and its effectiveness in learning calculus concepts.

This study involves participation in a series of lessons on the definite integral and its applications, taking two tests (a pretest and a posttest), and possible selection for an interview so that I can assess the process that you use to reason through solving calculus problems.

Honors Class: You will use the graphing calculator in solving definite integral problems

AP Class: You will not use the graphing calculator in solving definite integral problems

I am asking for your (or your parents') consent for you to participate in this study. If you are under 18 years old, I am asking for your parents to complete a letter of informed consent for you to participate in this study. I am also asking for you to complete a letter of assent to participate in this study. If you are 18 years old, I am asking for you to complete a letter of informed consent to participate in this study. I will provide you with these letters. If you are able to participate, please sign (or have your parents sign) this letter of informed consent, and if you are under 18, please also sign the letter of assent.

These are concepts and problems that the class will learn anyway. If you choose not to participate in this study (to complete the pretest and posttest and to provide an interview), you will not lose any points in this class. In addition, you may choose to drop out of the study at any time. However, as these concepts are a regular part of learning in this course, you will still be responsible for the concepts that are taught.

Your responses to the pretest and posttest questions will be graded by a panel of mathematicians. Otherwise, the results of your test as well as your individual responses to the test questions and interview questions will be kept confidential. In addition, neither your name nor the name of your school will be used in this study.

If you are able to participate, please sign (or have your parents sign) and return the letter of informed consent (and assent if necessary) by Friday, December 3, 2010.

Thanks.

Appendix C
Pretest

Calculus Reasoning Pretest

The Definite Integral & Applications

Score _______________ / 105

This test measures the five components of reasoning, as defined by the NCTM, which include analyzing a problem, initiating a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution. Each part of the question measures exactly one of these characteristics.

Directions: Please answer the following questions to the best of your ability. Provide thoughtful responses to each question. Use ink to answer all questions, and do not erase any steps. Rather, cross out any incorrect procedures with an X.

Part I

1. Approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations below about the x-axis.

$$y = x\sqrt{9 - x^2},\ y = 0$$

(Adapted from Larson Calculus, p429, #24)

a. ________ Identify some mathematical concepts and relationships that are used in this problem.

b. ________ What conclusions can you make about the solution to this problem?

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

2. A car starts braking as it approaches a road junction. After braking for t seconds, the velocity $v(t)$ of the car is given by $v(t) = -4t + 10$. How far does the car travel from the time the brakes are applied until it stops?

(Adapted from TIMSS 2008 Advanced, Calculus Applying)

a. ________ Identify some mathematical concepts and relationships that are used in this problem.

b. ________ What conclusions can you make about the solution to this problem?

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

3. Graph the region bounded by the graphs of the following functions and find the area of the region. $f(x) = x^3 - 6x, g(x) = x^4 + 6x^2$

(Adapted from Larson Calculus, p418 #36)

a. _________ Identify some mathematical concepts and relationships that are used in this problem.

b. _________ What conclusions can you make about the solution to this problem?

c. _________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

4. Consider the region bounded by the graphs of $f(x)=\frac{2x}{1-x}, x=2, x=6,$ and $y=0$. Find the area of this region to the nearest hundredth. Then decide whether this is an overestimate or an underestimate.

(Adapted from Larson Calculus, Graphical Reasoning, p263 #73)

a. _________ Identify some mathematical concepts and relationships that are used in this problem.

b. _________ What conclusions can you make about the solution to this problem?

c. _________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

At the end of this question, if you are using a graphing calculator, please put up your graphing calculator, turn in Part I of this test, and your teacher will hand you Part II.

Part II – YOU MAY NOT USE A CALCULATOR FOR THIS QUESTION.

5. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{4}x\sqrt{3-x}$ and the x-axis about the x-axis, where x and y are measured in meters. Find the tank's volume.

(Adapted from Larson Calculus, p429, #51)

a. ________ Identify some mathematical concepts and relationships that are used in this problem.

b. ________ What conclusions can you make about the solution to this problem?

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

Appendix D
Posttest

Calculus Reasoning Posttest **The Definite Integral & Applications**

Score _______________ / 105

This test measures the five components of reasoning, as defined by the NCTM, which include analyzing a problem, initiating a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution. Each part of the question measures exactly one of these characteristics.

Directions: Please answer the following questions to the best of your ability. Provide thoughtful responses to each question. Use ink to answer all questions, and do not erase any steps. Rather, cross out any incorrect procedures with an X.

Part I

1. A car starts braking as it approaches a road junction. After braking for t seconds, the velocity $v(t)$ of the car is given by $v(t) = -2t + 20$. How far does the car travel from the time the brakes are applied until it stops?

(Adapted from TIMSS 2008 Advanced, Calculus Applying)

a. ________ Identify some mathematical concepts and relationships that are used in this problem.

b. ________ What conclusions can you make about the solution to this problem?

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

2. Graph the region bounded by the graphs of the following functions and find the area of the region. $f(x) = x^4 - 4x^2, g(x) = x^3 - 4x$

(Larson Calculus, p418 #36)

a. _________ Identify some mathematical concepts and relationships that are used in this problem.

b. _________ What conclusions can you make about the solution to this problem?

c. _________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

3. Consider the region bounded by the graphs of $f(x)=\dfrac{8x}{x+1}, x=0, x=4,$ and $y=0$. Find the area of this region to the nearest hundredth. Then decide whether this is an overestimate or an underestimate.

(Adapted from Larson Calculus, Graphical Reasoning, p263 #73)

a. _________ Identify some mathematical concepts and relationships that are used in this problem.

b. _________ What conclusions can you make about the solution to this problem?

c. _________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

4. Approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations below about the x-axis.

$$y = x\sqrt{4 - x^2},\, y = 0$$

(Larson Calculus, p429, #24)

a. ________ Identify some mathematical concepts and relationships that are used in this problem.

b. ________ What conclusions can you make about the solution to this problem?

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

At the end of this question, if you are using a graphing calculator, please put up your graphing calculator, turn in Part I of this test, and your teacher will hand you Part II.

Part II – YOU MAY NOT USE A CALCULATOR FOR THIS QUESTION.

5. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y=\frac{1}{8}x^2\sqrt{2-x}$ and the x-axis about the x-axis, where x and y are measured in meters. Find the tank's volume.

(Larson Calculus, p.429, #51)

a. ________ Identify some mathematical concepts and relationships that are used in this problem.

b. ________ What conclusions can you make about the solution to this problem?

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

g. ________ What other information can you infer from this problem?

Appendix E
Pretest Sample Solutions

Calculus Reasoning Pretest Sample Solutions **Score _______________ / 105**
The Definite Integral & Applications

This test measures the five components of reasoning, as defined by the NCTM, which include analyzing a problem, initiating a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution. Each part of the question measures exactly one of these characteristics.

Directions: Please answer the following questions to the best of your ability. Provide thoughtful responses to each question. Use ink to answer all questions, and do not erase any steps. Rather, cross out any incorrect procedures with an X.

Directions for graders: The following assessment is intended to measure reasoning in calculus. Sample solutions are provided. Other solutions and responses are welcome. A rubric is attached at the back of this assessment for grading purposes. Please rate each student's response to each question from 0 to 3, with 0 meaning no evidence of reasoning skills and 3 meaning great evidence of reasoning skills and place the score for each question in the blank space next to question. The total score possible for each student is 105.

1. Approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations below about the x-axis.

$$y = x\sqrt{9-x^2}, y = 0$$

(Larson Calculus, p429, #24)

Analyzing a Problem

a. ________ Identify some mathematical concepts and relationships that are used in this problem?

- Definite Integral and Volume – The definite integral can be used to find the volume of a solid of revolution.
- Solid of Revolution and Disk Method – The Disk method is one strategy that can be used to find the volume of a solid of revolution.
- Area of Cross Section and Volume – The area of a cross section is used in finding the volume of a solid of revolution.

b. ________ What conclusions can you make about the solution to this problem?

- The answer will be a positive number.
- The answer will include the number π .
- The graph of the function will not be continuous for all real numbers.
- The domain of the function will be a closed interval.

Initiating a Strategy

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could find the domain of the function algebraically in order to find the limits of integration.
- I could use the graphing calculator to find the domain of the function in order to find the limits of integration.
- I could use the Disk method to set up an integral and the Fundamental Theorem of Calculus to evaluate it.
- I could use the Shell method to set up the definite integral.
- I could use the Disk method to set up the definite integral and evaluate it using a Riemann sum.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

$$V = \int_{-3}^{3} \pi y^2 dx = \int_{-3}^{3} \pi \quad x\sqrt{9-x^2} \quad ^{2} dx =$$

$$\int_{-3}^{3} \pi x^2 (9 - x^2) dx = \pi \int_{-3}^{3} (9x^2 - x^4) dx =$$

$$\pi \left(3x^3 - \frac{1}{5} x^5 \right) \Bigg|_{-3}^{3} = \frac{324}{5}\pi = 203.6$$

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Area of a Circle from Geometry
- Simplifying expressions from Algebra
- Domain from Algebra
- Using the graphing calculator to find the domain from pre calculus

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

Yes, The solution does seem reasonable because:

- It includes the number π.
- It is a positive number
- The limits of integration are the endpoints of the domain.

g. ________ What other information can you infer from this problem?

- The volume of a solid of revolution can always be found using the Disk method.
- Functions involving a root will have a closed interval as the domain.
- The volume of a solid of revolution will always be a positive number.

2. A car starts braking as it approaches a road junction. After braking for t seconds, the velocity $v(t)$ of the car is given by $v(t) = -4t + 10$. How far does the car travel from the time the brakes are applied until it stops?

(Adapted from TIMSS 2008 Advanced, Calculus Applying)

Analyzing a Problem

a. ________ Identify some mathematical concepts and relationships that are used in this problem?

- Velocity and Distance – The integral of the absolute value of the velocity function can be used to find the distance.
- Integral and Distance – The integral of the absolute value of the velocity function can be used to find the distance.

b. ________ What conclusions can you make about the solution to this problem?

- I will be able to find the distance traveled by the car.
- My solution to the problem should be a positive number.
- The units for my answer will be given in meters.
- There will be at most one root of the velocity function.
- The root can be identified by viewing on the calculator where the function crosses the x-axis.

Initiating a Strategy

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could set the velocity function equal to 0 to find the point at which the car comes to a stop.
- I could graph the line and locate the root by viewing where the function crosses the x-axis.
- I could use the Fundamental Theorem of Calculus to evaluate the integral from 0 to the time when the car stops.
- I could use a Riemann sum to approximate the value of the definite integral from 0 to the time when the car stops.
- I could find the root graphically by viewing on the graphing calculator where the function crosses the x-axis.
- I could use another method for evaluating a definite integral such as a Riemann sum.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

$$v(t) = -4t + 10 = 0$$

$$t = 2.5$$

$$\int_0^{2.5} \left|-4t + 10\right| dt = (-2t^2 + 10t)\Big|_0^{2.5}$$

$$= 12.5 \text{ meters}$$

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Solving equations from Algebra
- Formulas learned in physics such as $d = rt$
- Use of the graphing calculator to see relationships from pre calculus

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

Yes, my solution is reasonable because:

- It is a positive number
- The graph is a line
- The graph has one zero
- The graph is above the x-axis

g. ________ What other information can you infer from this problem?

- We can find distance traveled by an object moving in a linear path if we are given the velocity function.
- Definite integrals can always be used to find the distance traveled by an object moving in a straight line.
- After time $t = 2.5$, the car was traveling backwards.

3. Graph the region bounded by the graphs of the following functions and find the area of the region. $f(x) = x^3 - 6x, g(x) = x^4 + 6x^2$

(Larson Calculus, p418 #36)

Analyzing a Problem

a. ________ Identify some mathematical concepts and relationships that are used in this problem?

- Area and Definite Integral – The definite integral of a function is used to find the area between the curve and the x-axis.
- Points of Intersection and Limits of Integration – The points of intersection of two functions are used for the limits of integration to find the area between the two functions.
- Definite Integral and Fundamental Theorem of Calculus – The fundamental theorem of calculus is one way to evaluate the definite integral of a function.

b. ________ What conclusions can you make about the solution to this problem?

- The solution to this problem will be a positive number.
- There may be more than 2 points of intersection.
- The graph of the region bounded by the two curves may be composed of more than one region.

Initiating a Strategy

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could find the points of intersection algebraically by setting the two functions equal to one another.
- I could use the graphing calculator to view the points of intersection of the two functions.
- I could use an algebraic approach to set up the definite integral and evaluate it using the Fundamental Theorem of Calculus.
- I could use the numerical approach and evaluate the definite integral using a Riemann sum.
- I could use the calculate function on the graphing calculator to approximate the value of the definite integral.
- I could use the Riemann1 graphing calculator program to approximate the value of the definite integral using rectangles or trapezoids.
- I could find the area between the top curve and the x-axis and the area between the bottom curve and the x-axis and then subtract the two values.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

$$f(x) = x^3 - 6x, g(x) = -x^4 + 6x^2$$
$$x^3 - 6x = -x^4 + 6x^2$$
$$x^4 + x^3 - 6x^2 - 6x = 0$$
$$x(x^3 + x^2 - 6x - 6) = 0$$
$$x(x^2 - 6)(x+1) = 0$$
$$x = 0, -1, -\sqrt{6}, \sqrt{6}$$
$$\int_{-\sqrt{6}}^{-1} (-x^4 + 6x^2 - x^3 + 6x)dx + \int_{-1}^{0} (x^3 - 6x + x^4 - 6x^2)dx +$$
$$\int_{0}^{\sqrt{6}} (-x^4 + 6x^2 - x^3 + 6x)dx$$
$$\approx 20.42$$

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Solving equations and factoring from Algebra
- Finding points of intersection from Pre Calculus
- Using the graphing calculator to find points of intersection from pre calculus.

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

My solution is reasonable because:

- It is a positive number
- It includes more than two points of intersection
- It uses more than one definite integral

g. ________ What other information can you infer from this problem?

- The area between two curves will always be a positive number.
- We will get the same answer if we subtract the two curves and then find the area underneath or if we find the area underneath each curve and then subtract the two values.

4. Consider the region bounded by the graphs of $f(x) = -\dfrac{2x}{1-x}, x = 2, x = 6,$ and $y = 0$. Find the area of this region to the nearest hundredth. Then decide whether this is an overestimate or an underestimate.

(Adapted from Larson Calculus, Graphical Reasoning, p263 #73)

Analyzing a Problem

a. _________ Identify some mathematical concepts and relationships that are used in this problem?

- Area under the Curve and Definite Integral – The definite integral is used to find the area between a curve and the x-axis.
- Riemann Sum and Area under the Curve – A Riemann sum can be used to find the area under the curve.
- Discontinuity and Asymptote – The graph of a function with a non-removable discontinuity

b. _________ What conclusions can you make about the solution to this problem?

- The graph of this region will be above the x-axis.
- If a Riemann sum is used, the answer will be more accurate if a large number of rectangles are used.
- If a Riemann sum is used, this answer will be either an overestimate or an underestimate.

Initiating a Strategy

c. _________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could use the formula for a left, right, or midpoint Riemann sum.
- I could use the Riemann2 graphing calculator program to find the left, right, and midpoint sums.
- I could set up the definite integral algebraically and use substitution method along with Fundamental Theorem of Calculus to evaluate it.
- I could take the definite integral of the function using the substitution method along with the Fundamental Theorem of Calculus.
- I could find the definite integral by counting squares.
- I could use the graphing calculator to view the points of intersection.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

Upper sum or Left sum, n = 4

$$1(4+3+8/3+5/2)=\frac{73}{6}\approx 12.17$$

This is an overestimate.

Lower sum or Right sum, n = 4

$$1(3+8/3+5/2+2.4)=\frac{317}{30}\approx 10.57$$

This is an underestimate.

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Area from Geometry
- Finding points of intersection from pre calculus.
- Using the graphing calculator to view the points of intersection from pre calculus

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

Yes, the answer does seem to reasonable because:

- The answer is a positive number
- The answer is rounded to the nearest hundedth
- The graph is above the x-axis.

g. ________ What other information can you infer from this problem?

- The value of the definite integral can always be approximated using a Riemann sum.
- The midpoint Riemann sum will always be a closer estimate the upper or lower sums.
- The graph of a function with a non-removable discontinuity has an asymptote.

5. YOU MAY NOT USE A CALCULATOR FOR THIS QUESTION. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{4}x\sqrt{3-x}$ and the x-axis about the x-axis, where x and y are measured in meters. Find the tank's volume.

(Adapted from Larson Calculus, p429, #51)

Analyzing a Problem

a. ________ Identify some mathematical concepts and relationships that are used in this problem?

- Volume and Definite Integral – The definite integral is used in calculating volume.
- Solid of Revolution and Disk Method – The disk method can be used to find the volume of a sol of revolution.
- Zeros and Limits of Integration – The zeros of the function are used for the limit of integration in finding the volume.

b. ________ What conclusions can you make about the solution to this problem?

- The answer will be positive.
- The cross sections will be circles.
- The answer will include the number π.

Initiating a Strategy

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could find the zeros of the function algebraically and use these as the limits of integration.
- I could use the disk method to set up the definite integral.
- I could use the fundamental theorem of calculus to evaluate the definite integral.
- I could use a Riemann sum to calculate the definite integral.
- I could graph the function using pencil and paper and then draw the solid of revolution.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

$$\frac{1}{4}x\sqrt{3-x}=0$$

$$x=0,3$$

$$\int_0^3 \pi\left(\frac{1}{4}x\sqrt{3-x}\right)^2 dx=\left(\frac{\pi}{16}x^3-\frac{\pi}{64}x^4\right)\Bigg|_0^3$$

$$=\frac{27\pi}{64}\approx 1.33$$

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Finding zeros from pre calculus
- Graphing functions from algebra

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

- Yes, the solution is reasonable because it is a positive number.

g. ________ What other information can you infer from this problem?

- Solids of revolution will always have circular cross sections.
- The Disk method can always be used to find the volume of a solid of revolution.

Appendix F
Posttest Sample Solutions

Calculus Reasoning Posttest Sample Solutions **Score _______________ / 105**
The Definite Integral & Applications

This test measures the five components of reasoning, as defined by the NCTM, which include analyzing a problem, initiating a strategy, monitoring one's progress, seeking and using connections, and reflecting on one's solution. Each part of the question measures exactly one of these characteristics.

Directions for Student: Please answer the following questions to the best of your ability. Provide thoughtful responses to each question. Use ink to answer all questions, and do not erase any steps. Rather, cross out any incorrect procedures with an X.

Directions for graders: The following assessment is intended to measure reasoning in calculus. Sample solutions are provided. Other solutions and responses are welcome. A rubric is attached at the back of this assessment for grading purposes. Please rate each student's response to each question from 0 to 3, with 0 meaning no evidence of reasoning skills and 3 meaning great evidence of reasoning skills and place the score for each question in the blank space next to question. The total score possible for each student is 105.

1. [Position, Velocity, Distance, & Displacement] A car starts braking as it approaches a road junction. After braking for t seconds, the velocity $v(t)$ of the car is given by $s(t) = -2t + 20$. How far does the car travel from the time the brakes are applied until it stops?

(TIMSS 2008 Advanced, Calculus Applying)

Analyzing a Problem

a. ________ Identify some mathematical concepts and relationships that are used in this problem?

- Velocity and Distance – The integral of the absolute value of the velocity function can be used to find the distance.
- Integral and Distance – The integral of the absolute value of the velocity function can be used to find the distance.

b. ________ What conclusions can you make about the solution to this problem?

- I will be able to find the distance traveled by the car.
- My solution to the problem should be a positive number.
- The units for my answer will be given in meters.
- There will be at most one root of the velocity function.
- The root can be identified by viewing on the calculator where the function crosses the x-axis.

Initiating a Strategy

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could set the velocity function equal to 0 to find the point at which the car comes to a stop.
- I could graph the line and locate the root by viewing where the function crosses the x-axis.
- I could use the Fundamental Theorem of Calculus to evaluate the integral from 0 to the time when the car stops.
- I could use a Riemann sum to approximate the value of the definite integral from 0 to the time when the car stops.
- I could find the root graphically by viewing on the graphing calculator where the function crosses the x-axis.
- I could use another method for evaluating a definite integral such as a Riemann sum.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

$v(t) = -2t + 20 = 0$

$t = 10$

$\int_0^{10} \left|-2t + 20\right| dt = (-t^2 + 20t)\Big|_0^{10}$

$= -100 + 200 - 0 = 100$

Solution = 100 meters

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Solving equations from Algebra
- Formulas learned in physics such as $d = rt$
- Use of the graphing calculator to see relationships from pre calculus

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

Yes, my solution is reasonable because:

- It is a positive number
- The graph is a line
- The graph has one zero
- The graph is above the x-axis

g. ________ What other information can you infer from this problem?

- We can find distance traveled by an object moving in a linear path if we are given the equation of the velocity function.
- Definite integrals can always be used to find the distance traveled by an object moving in a straight line.
- After time $t = 10$, the car was traveling backwards.

2. [Area] Graph the region bounded by the graphs of the following functions and find the area of the region. $f(x) = x^4 - 4x^2, g(x) = x^3 - 4x$

(Larson Calculus, p418 #36)

Analyzing a Problem

a. ________ Identify some mathematical concepts and relationships that are used in this problem?

- Area and Definite Integral – The definite integral of a function is used to find the area between the curve and the x-axis.
- Points of Intersection and Limits of Integration – The points of intersection of two functions are used for the limits of integration to find the area between the two functions.
- Definite Integral and Fundamental Theorem of Calculus – The fundamental theorem of calculus is one way to evaluate the definite integral of a function.

b. ________ What conclusions can you make about the solution to this problem?

- The solution to this problem will be a positive number.
- There may be more than 2 points of intersection.
- The graph of the region bounded by the two curves may be composed of more than one region.

Initiating a Strategy

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could find the points of intersection algebraically by setting the two functions equal to one another.
- I could use the graphing calculator to view the points of intersection of the two functions.
- I could use an algebraic approach to set up the definite integral and evaluate it using the Fundamental Theorem of Calculus.
- I could use the numerical approach and evaluate the definite integral using a Riemann sum.
- I could use the calculate function on the graphing calculator to approximate the value of the definite integral.
- I could use the Riemann1 graphing calculator program to approximate the value of the definite integral using rectangles or trapezoids.
- I could find the area between the top curve and the x-axis and the area between the bottom curve and the x-axis and then subtract the two values.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

$$f(x) = x^4 - 4x^2, g(x) = x^3 - 4x$$
$$x^4 - 4x^2 = x^3 - 4x$$
$$x^4 - x^3 - 4x^2 + 4x = 0$$
$$x(x^3 - x^2 - 4x + 4) = 0$$
$$x(x-1)(x+2)(x-2) = 0$$
$$x = 0, 1, -2, 2$$
$$\int_{-2}^{0}(x^3 - 4x - x^4 + 4x^2)dx + \int_{0}^{1}(x^4 - 4x^2 - x^3 + 4x)dx +$$
$$\int_{1}^{2}(x^3 - 4x - x^4 + 4x^2)dx$$
$$= \frac{293}{30} \approx 9.77$$

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Solving equations and factoring from Algebra
- Finding points of intersection from Pre Calculus
- Using the graphing calculator to find points of intersection from pre calculus.

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

My solution is reasonable because:

- It is a positive number
- It includes more than two points of intersection
- It uses more than one definite integral

g. ________ What other information can you infer from this problem?

- The area between two curves will always be a positive number.
- We will get the same answer if we subtract the two curves and then find the area underneath or if we find the area underneath each curve and then subtract the two values.

3. [Riemann Sums] Consider the region bounded by the graphs of $f(x) = \dfrac{8x}{x+1}, x = 4,$ and $y = 0$. Find the area of this region to the nearest hundredth. Then decide whether this is an overestimate or an underestimate.

(Adapted from Larson Calculus, Graphical Reasoning, p263 #73)

Analyzing a Problem

a. _________ Identify some mathematical concepts and relationships that are used in this problem?

- Area under the Curve and Definite Integral – The definite integral is used to find the area between a curve and the x-axis.
- Riemann Sum and Area under the Curve – A Riemann sum can be used to find the area under the curve.
- Discontinuity and Asymptote – The graph of a function with a non-removable discontinuity will have at least one vertical asymptote.

b. _________ What conclusions can you make about the solution to this problem?

- The graph of this region will be above the x-axis.
- If a Riemann sum is used, the answer will be more accurate if a large number of rectangles are used.
- If a Riemann sum is used, this answer will be either an overestimate or an underestimate.
- The solution to this problem will approximate the area under the curve.

Initiating a Strategy

c. _________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could use the formula for a left, right, or midpoint Riemann sum.
- I could use the Riemann2 graphing calculator program to find the left, right, and midpoint sums.
- I could set up the definite integral algebraically and use substitution method along with Fundamental Theorem of Calculus to evaluate it.
- I could take the definite integral of the function using the substitution method along with the Fundamental Theorem of Calculus.
- I could find the definite integral by counting squares.
- I could use the graphing calculator to view the points of intersection.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

Lower sum = Left sum

$$1(0+4+16/3+6)=\frac{46}{3}\approx 15.33$$

This is an underestimate.

Upper sum = Right sum

$$1(4+16/3+6+6.4)=\frac{326}{15}\approx 21.73$$

This is an overestimate.

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Area from Geometry
- Finding points of intersection from pre calculus.
- Using the graphing calculator to view the points of intersection from pre calculus.

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

Yes, the answer does seem to reasonable because:

- The answer is a positive number
- The answer is rounded to the nearest hundedth
- The graph is above the x-axis.

g. ________ What other information can you infer from this problem?

- The value of the definite integral can always be approximated using a Riemann sum.
- The midpoint Riemann sum will always be a closer estimate the upper or lower sums.
- The graph of a function with a non-removable discontinuity has an asymptote.

4. [Volume] Approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations below about the x-axis.

$$y = x\sqrt{4 - x^2}, y = 0$$

(Larson Calculus, p429, #24)

Analyzing a Problem

a. ________ Identify some mathematical concepts and relationships that are used in this problem?

- Definite Integral and Volume – The definite integral can be used to find the volume of a solid of revolution.
- Solid of Revolution and Disk Method – The Disk method is one strategy that can be used to find the volume of a solid of revolution.
- Area of Cross Section and Volume – The area of a cross section is used in finding the volume of a solid of revolution.

b. ________ What conclusions can you make about the solution to this problem?

- The answer will be a positive number.
- The answer will include the number π.
- The graph of the function will not be continuous for all real numbers.
- The domain of the function will be a closed interval.

Initiating a Strategy

c. ________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could find the domain of the function algebraically in order to find the limits of integration.
- I could use the graphing calculator to find the domain of the function in order to find the limits of integration.
- I could use the Disk method to set up an integral and the Fundamental Theorem of Calculus to evaluate it.
- I could have use the Shell method to set up the definite integral.
- I could use the Disk method to set up the definite integral and evaluate it using a Riemann sum.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

$$V = \int_{-2}^{2} \pi y^2 dx = \int_{-2}^{2} \pi \quad x\sqrt{4-x^2} \quad {}^{2} \, dx =$$

$$\int_{-2}^{2} \pi x^2 (4 - x^2) dx = \pi \int_{-2}^{2} (4x^2 - x^4) dx =$$

$$\pi \left(\frac{4}{3} x^3 - \frac{1}{5} x^5 \right) \Bigg|_{-2}^{2} = \frac{128}{15} \pi = 26.8$$

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Area of a Circle from Geometry
- Simplifying expressions from Algebra
- Domain from Algebra
- Using the graphing calculator to find the domain from pre calculus

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

Yes, The solution does seem reasonable because:

- It includes the number π.
- It is a positive number
- The limits of integration are the endpoints of the domain.

g. ________ What other information can you infer from this problem?

- The volume of a solid of revolution can always be found using the Disk method.
- Functions involving a root will have a closed interval as the domain.
- The volume of a solid of revolution will always be a positive number.

5. [Volume] YOU MAY NOT USE A CALCULATOR FOR THIS QUESTION. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2-x}$ and the x-axis about the x-axis, where x and y are measured in meters. Find the tank's volume.

(Larson Calculus, p.429, #51)

Analyzing a Problem

a. _________ Identify some mathematical concepts and relationships that are used in this problem?

- Volume and Definite Integral – The definite integral is used in calculating volume.
- Solid of Revolution and Disk Method – The disk method can be used to find the volume of a sol of revolution.
- Zeros and Limits of Integration – The zeros of the function are used for the limit of integration in finding the volume.

b. _________ What conclusions can you make about the solution to this problem?

- The answer will be positive.
- The cross sections will be circles.
- The answer will include the number π.

Initiating a Strategy

c. _________ Name some of the possible approaches or strategies that you might use to solve this problem. Be explicit.

- I could find the zeros of the function algebraically and use these as the limits of integration.
- I could use the disk method to set up the definite integral.
- I could use the fundamental theorem of calculus to evaluate the definite integral.
- I could use a Riemann sum to calculate the definite integral.
- I could graph the function using pencil and paper and then draw the solid of revolution.
- I could use the graphing calculator to evaluate the definite integral.

Monitoring one's Progress

d. ________ Solve the problem. Show your work. Do not erase any steps. Box your answer.

$$\frac{1}{8}x^2\sqrt{2-x}=0$$

$$x=0,2$$

$$\int_0^2 \pi\left(\frac{1}{8}x^2\sqrt{2-x}\right)^2 dx=\left(\frac{\pi}{160}x^5-\frac{\pi}{384}x^6\right)\Bigg|_0^2$$

$$=\frac{\pi}{30}$$

Seeking and Using Connections

e. ________ In solving this problem, did you use any previous ideas or concepts learned in math or any other subjects? If so, which ones?

- Finding zeros from pre calculus
- Graphing functions from algebra

Reflecting on One's Solution

f. ________ Does your solution seem to be reasonable and appropriate? Justify your answer.

- Yes, the solution is reasonable because it is a positive number.

g. ________ What other information can you infer from this problem?

- Solids of revolution will always have circular cross sections.
- The Disk method can always be used to find the volume of a solid of revolution.

Appendix G
Pretest and Posttest Grading Rubric

Pretest Question 2 **Posttest Question 1**

	3	2	1	0
A: Analyzing a Problem	The student describes at least three relationships between mathematical concepts or terms.	The student is able to describe two relationships between mathematical concepts.	The student is able to list some mathematical concepts.	No response given
Example:	Student describes relationships between "Quadratic Function & Parabola," "Integral & Distance," and "Velocity & Distance."	Student describes relationships between "Integral & Distance" and "Velocity & Distance."	Student describes the relationship between "Velocity & Distance."	No response given
B: Analyzing a Problem	Student is able to draw at least 2 correct conclusions about the answer to the problem.	Student is able to draw at least 1 correct conclusion about the answer to the problem.	Student draws an incorrect conclusion about the answer to the problem.	No response given
Example:	Student states "the answer to this problem will be a positive number and will be given in meters."	Student states "the answer to this problem will be a positive number."	Student states "the units for my solution to the problem will be given in meters per second."	No response given
C: Initiating a Strategy	The student lists at least 1 reasonable approach.	The student lists one unreasonable approach.	No response given	N/A
Example:	Student states "I could graph the parabola and locate the roots by viewing where the function crosses the x-axis.	Student states "I could use the Fundamental Theorem to evaluate the definite integral."	No response given	N/A
D: Monitoring one's Progress	The correct procedure is followed through OR the student begins with an incorrect procedure but realizes mistake and follows through using a new approach.	The student begins with an incorrect procedure, realizes mistake, and attempts another approach OR begins the correct procedure and solves a certain portion but does not follow through.	The student begins with an incorrect procedure and realizes mistake but does not attempt another approach.	The student uses an incorrect approach in solving the problem and does not show evidence of monitoring her progress or leaves blank.
Example:	Student begins by setting the wrong function equal to 0 but then realizes the mistake and finishes the problem correctly.	Student begins by setting the wrong function equal to 0, realizes the mistake, and attempts another approach but does not finish correctly.	Student begins by setting the wrong function equal to 0, realizes the mistake, but does not attempt another approach.	Student solves the problem incorrectly and does not realize the mistake.

E: Seeking and Using Connections	Student connects this problem to previous knowledge in math before calculus or in other subjects in at least 2 ways.	Student is able to connect this problem to previous knowledge in math before or during calculus in 1 way.	No response given	N/A
Example:	Student writes, "I remember doing a problem similar to this in my physics class last semester, and I used the graphing calculator to see relationships, which we learned in pre calculus."	Student writes, "I am using solving equations from algebra."	No response given	N/A
F: Reflecting on One's Solution	Student finds correct solution, finds whether it is reasonable, and justifies the answer. The justification includes the fact that distance must always be positive.	Student finds an incorrect or partial solution, decides whether it is reasonable, and attempts to justify it.	Student finds the correct solution but cannot justify or find whether it is reasonable.	Student finds an incorrect solution and does not decide whether it is reasonable OR does not find answer.
Example of Student Response	Student finds the correct distance and states that it is reasonable because it is positive.	Student finds the correct distance and states that it is reasonable.	Student finds the correct distance and does not answer this question.	Student finds an incorrect distance and does not answer this question.
G: Reflecting on One's Solution	Student is able to make two correct inferences.	Student is able to make one correct inference.	Student attempts to make an inference, but it is incorrect.	No response given
Example of Student Response	Student states, "We can always find the distance traveled by an object moving in a linear path if given the position function, and the answer will always be a positive number."	Student states, "We can always find the distance traveled by an object moving in a linear path if given the position function."	Student states, "When asked for the distance traveled by an object moving in a linear path, we will always get a negative answer."	No response given

Pretest Question 3 **Posttest Question 2**

	3	2	1	0
A: Analyzing a Problem	The student describes at least three relationships between mathematical concepts or terms.	The student is able to describe two relationships between mathematical concepts.	The student is able to list some mathematical concepts.	No response given
For example:	Student lists, “Definite Integral & Area,” “Points of Intersection & Limits of Integration,” and “Definite Integral & FTC.”	Student lists, “Points of Intersection & Limits of Integration” and “Definite Integral & FTC.”	Student writes, “Definite Integral & Area”	No response given
B: Analyzing a Problem	Student is able to draw at least 2 correct conclusions about the solution.	Student is able to draw at least 1 correct conclusion about the solution.	Student draws an incorrect conclusion about the solution.	No response given
For example:	Student writes, “The solution will be a positive number and the graph will be composed of more than one region.”	Student writes, “The solution to this problem will be a positive number.”	Student writes, “The solution to this problem will be a negative number.”	No response given
C: Initiating a Strategy	The student lists at least 1 reasonable approach.	The student lists one unreasonable approach.	No response given	N/A
For example:	Student writes, “I could evaluate the definite integral using the FTC or using a Riemann sum.”	Student writes, “I could set up the definite integral and evaluate it using the FTC.”	No response given	N/A
D: Monitoring one’s Progress	The correct procedure is followed through OR the student begins with an incorrect procedure but realizes mistake and follows through using a new approach.	The student begins with an incorrect procedure, realizes mistake, and attempts another approach OR begins the correct procedure and solves a certain portion but does not follow through.	The student begins with an incorrect procedure and realizes mistake but does not attempt another approach.	The student uses an incorrect approach in solving the problem and does not show evidence of monitoring her progress or leaves blank.
For example:	Student begins by setting up one definite integral on the wrong interval, but realizes mistake and sets up and evaluates three definite integrals to find the solution.	Student begins by setting up one definite integral on the wrong interval, but then realizes mistake and sets up three definite integrals but does not evaluate.	Student begins by setting up one definite integral on the wrong interval and crosses out the incorrect work or does not finish the problem.	Student sets up and evaluates only one definite integral or does not attempt problem.

E: Seeking and Using Connections	Student connects this problem to previous knowledge in math before calculus or in other subjects in at least 2 ways.	Student is able to connect this problem to previous knowledge in math before or during calculus in 1 way.	No response given	N/A
For example:	Student writes, "Solving equations from Algebra and using the graphing calculator to find points of intersection from pre calculus."	Student writes, "Solving equations from Algebra."	No response given	N/A
F: Reflecting on One's Solution	Student finds correct solution, finds whether it is reasonable, and justifies the answer.	Student finds an incorrect or partial solution, decides whether it is reasonable, and attempts to justify it.	Student finds the correct solution but cannot justify or find whether it is reasonable.	Student finds an incorrect solution and does not decide whether it is reasonable OR does not find answer.
For example:	Student finds the area of the region, states that it is reasonable, and provides a reason such as that the answer is positive.	Student finds an area and correctly determines whether it is reasonable but does not provide a reason.	Student finds the correct area but does not provide a response to this question.	Student's solution is incorrect and she does answer this question.
G: Reflecting on One's Solution	Student is able to make two correct inferences.	Student is able to make one correct inference.	Student attempts to make an inference, but it is incorrect.	No response given
For example:	Student states that area between two curves will always be positive and that it can always be calculated in more than one way (subtracting curves and then integrating or integrating and then subtracting).	Student states that area between two curves will always be a positive number.	Student states that area between two curves will always be a negative number.	No response given

Pretest Question 4 **Posttest Question 3**

	3	2	1	0
A: Analyzing a Problem	The student describes at least three relationships between mathematical concepts or terms.	The student is able to describe two relationships between mathematical concepts.	The student is able to list some mathematical concepts.	No response given
For example:	Student describes relationships between "Discontinuity & Asymptote," "Definite Integral & Area under the Curve," and "Riemann sum & area under a curve."	Student describes relationships between "Area under the curve and definite integral" and "Riemann sum and area under the curve."	Student describes the relationship between "Area under the curve and Definite Integral."	No response given
B: Analyzing a Problem	Student is able to draw at least 2 correct conclusions about the solution.	Student is able to draw at least 1 correct conclusion about the solution.	Student draws an incorrect conclusion about the solution.	No response given
For example:	Student writes, "The graph of this region will be above the x-axis and the solution to this problem will approximate the area under the curve."	Student writes, "The graph of this region will be above the x-axis."	Student writes, "The graph of this region will be below the x-axis."	No response given
C: Initiating a Strategy	The student lists at least 1 reasonable approach.	The student lists one unreasonable approach.	No response given	N/A
For example:	Student writes, "I could use the formula for a Riemann sum. I could also use the Riemann1 graphing calculator program to approximate the definite integral."	Student writes, "I could use the formula for a Riemann sum."	No response given	N/A
D: Monitoring one's Progress	The correct procedure is followed through OR the student begins with an incorrect procedure but realizes mistake and follows through using a new approach.	The student begins with an incorrect procedure, realizes mistake, and attempts another approach OR begins the correct procedure and solves a certain portion but does not follow through.	The student begins with an incorrect procedure and realizes mistake but does not attempt another approach.	The student uses an incorrect approach in solving the problem and does not show evidence of monitoring her progress or leaves blank.
For example:	The wrong formula for a Riemann sum is used at first, but the student realizes her mistake and corrects the formula.	The wrong formula is used, the student realizes her mistake and attempts another formula but her process is still incorrect.	The wrong formula is used and the student realizes her mistake but does not attempt a different formula.	The wrong formula is used and the student does not realize her mistake.

E: Seeking and Using Connections	Student connects this problem to previous knowledge in math before calculus or in other subjects in at least 2 ways.	Student is able to connect this problem to previous knowledge in math before or during calculus in at least 1 way.	No response given	N/A
For example:	Student writes, "I used Finding points of intersection algebraically and using the graphing calculator to view points of intersection from pre calculus."	Student writes, "I used finding points of intersection from pre calculus."	No response given	N/A
F: Reflecting on One's Solution	Student finds correct solution, finds whether it is reasonable, and justifies the answer.	Student finds an incorrect or partial solution, decides whether it is reasonable, and attempts to justify it.	Student finds the correct solution but cannot justify or find whether it is reasonable.	Student finds an incorrect solution and does not decide whether it is reasonable OR does not find answer.
For example:	Student finds an appropriate estimate for the definite integral and states that it is reasonable since it is a positive number.	Student finds an incorrect approximation for the definite integral and states that it is reasonable.	Student finds an appropriate estimate for the definite integral.	Student finds an incorrect Riemann sum and does not answer this question.
G: Reflecting on One's Solution	Student is able to make two correct inferences.	Student is able to make one correct inference.	Student attempts to make an inference, but it is incorrect.	No response given
For example:	Student writes, "The definite integral can be approximated using a Riemann sum and the graph of a function with a non-removable discontinuity has an asymptote."	Student writes, "The value of the definite integral can always be approximated using a Riemann sum."	Student writes, "The value of the definite integral can be found exactly using a Riemann sum."	No response given

Pretest Question 1 **Posttest Question 4**

	3	2	1	0
A: Analyzing a Problem	The student describes at least three relationships between mathematical concepts or terms.	The student is able to describe two relationships between mathematical concepts.	The student is able to list some mathematical concepts.	No response given
For example:	Student describes relationships between "Definite Integral & Volume," "Solid of Revolution & Disk Method," and "Area of a Cross Section & Volume."	Student describes relationships between "Area of a Cross Section and Volume" and "Definite Integral & Volume."	Student describes the relationship between "Solid of Revolution & Disk Method."	No response given
B: Analyzing a Problem	Student is able to draw at least 2 correct conclusions about the answer to the problem.	Student is able to draw at least 1 correct conclusion about the answer to the problem.	Student draws an incorrect conclusion about the answer to the problem.	No response given
For example:	Student writes, "The answer will be positive and will include π ."	Student writes, "The answer will be a positive number."	Student writes, "The answer will be a negative number."	No response given
C: Initiating a Strategy	The student lists at least 1 reasonable approach.	The student lists one unreasonable approach.	No response given	N/A
For example:	Student writes, "I could use the Disk method to set up an integral and the Fundamental Theorem or a Riemann sum to evaluate it."	Student writes, "I could use the Disk method to set up an integral and the Fundamental Theorem to evaluate the definite integral."	No response given	N/A
D: Monitoring one's Progress	The correct procedure is followed through OR the student begins with an incorrect procedure but realizes mistake and follows through using a new approach.	The student begins with an incorrect procedure, realizes mistake, and attempts another approach OR begins the correct procedure and solves a certain portion but does not follow through.	The student begins with an incorrect procedure and realizes mistake but does not attempt another approach.	The student uses an incorrect approach in solving the problem and does not show evidence of monitoring her progress or leaves blank.
For example:	The student begins by using the wrong formula to find volume, but then realizes the mistake and follows through using the disk method correctly.	The student begins by setting up the wrong formula to find the volume, realizes the mistake and attempts to use the formula correctly but cannot.	The student begins by setting up the wrong formula to find the volume and realizes the mistake but does not attempt to correct the formula.	The student uses the wrong formula throughout solving the problem.

E: Seeking and Using Connections	Student is able to connect this problem to previous knowledge in math before calculus or in other subjects in at least two ways.	Student is able to connect this problem to previous knowledge in math before or during calculus in at least one way.	No response given	N/A
For example:	Student writes, "I used Area of a Circle from Geometry and Using the graphing calculator to find domain from pre calculus."	Student writes, "I used Area of a Circle from Geometry."	No response given	N/A
F: Reflecting on One's Solution	Student finds correct solution, finds whether it is reasonable, and justifies the answer.	Student finds an incorrect or partial solution, decides whether it is reasonable, and attempts to justify it.	Student finds the correct solution but cannot justify or find whether it is reasonable.	Student finds an incorrect solution and does not decide whether it is reasonable OR does not find answer.
For example:	Student finds the correct volume and states that it is reasonable since the answer is positive and includes the number π.	Student finds a solution for the volume and states whether it is reasonable.	Student finds the correct volume and does not answer this question.	Student finds an incorrect solution and does not answer this question.
G: Reflecting on One's Solution	Student is able to make two correct inferences.	Student is able to make one correct inference.	Student attempts to make an inference, but it is incorrect.	No response given
For example:	Student writes, "The volume of a solid of revolution can always be found using the Disk method, and the solution will always be positive.	The volume of a solid of revolution can always be found using the Disk method.	The solution to the problem will always be a negative number.	No response given

Pretest Question 5 **Posttest Question 5**

	3	2	1	0
A: Analyzing a Problem	The student describes at least three relationships between mathematical concepts or terms.	The student is able to describe two relationships between mathematical concepts.	The student is able to list some mathematical concepts.	No response given
For example:	Student describes relationships between "Volume & Definite Integral," "Solid of Revolution & Disk Method," and "Volume and FTC."	Student describes relationships between "Volume and Fundamental Theorem of Calculus" & "Definite Integral and Fundamental Theorem of Calculus."	Student describes the relationship between "Volume and Definite Integral."	No response given
B: Analyzing a Problem	Student is able to draw at least 2 correct conclusions about the answer to the problem.	Student is able to draw at least 1 correct conclusion about the answer to the problem.	Student draws an incorrect conclusion about the answer to the problem.	No response given
For example:	Student writes, "The answer will be positive and will include the number π."	Student writes, "The answer will be positive."	Student writes, "The answer will be negative."	No response given
C: Initiating a Strategy	The student lists at least 1 reasonable approach.	The student lists one unreasonable approach.	No response given	N/A
For example:	Student writes, "I could use a Riemann sum to calculate the definite integral. I could also use the FTC to evaluate the definite integral."	Student writes, "I could use the FTC to evaluate the definite integral."	No response given	N/A
D: Monitoring one's Progress	The correct procedure is followed through OR the student begins with an incorrect procedure but realizes mistake and follows through using a new approach.	The student begins with an incorrect procedure, realizes mistake, and attempts another approach OR begins the correct procedure and solves a certain portion but does not follow through.	The student begins with an incorrect procedure and realizes mistake but does not attempt another approach.	The student uses an incorrect approach in solving the problem and does not show evidence of monitoring her progress or leaves blank.
For example:	Student begins by using the wrong area formula in setting up the integral, realizes her mistake, and finishes the problem correctly.	Student begins by using the wrong area formula in setting up the integral, realizes her mistake and attempts another approach but is not successful.	Student begins by using the wrong area formula in setting up the definite integral and realizes her mistake but does not attempt to fix the problem.	Student uses the wrong area formula to evaluate the definite integral.

E: Seeking and Using Connections	Student is able to connect this problem to previous knowledge in math before calculus or in other subjects in at least two ways.	Student is able to connect this problem to previous knowledge in math before or during calculus in at least one way.	No response given	N/A
For example:	Student writes, “Finding zeros from pre calculus and graphing functions from algebra.”	Student writes, “Graphing functions from algebra.”	No response given	N/A
F: Reflecting on One’s Solution	Student finds correct solution, finds whether it is reasonable, and justifies the answer.	Student finds an incorrect or partial solution, decides whether it is reasonable, and attempts to justify it.	Student finds the correct solution but cannot justify or find whether it is reasonable.	Student finds an incorrect solution and does not decide whether it is reasonable OR does not find answer.
For example:	Student’s solution is correct. She decides that it is reasonable and justifies her answer.	Student’s solution is incorrect. She decides whether it is reasonable but does not justify the answer.	Student’s solution is correct but she does not answer this question.	Student’s solution is incorrect and she does not answer this question.
G: Reflecting on One’s Solution	Student is able to make two correct inferences.	Student is able to make one correct inference.	Student attempts to make an inference, but it is incorrect.	No response given
For example:	Student writes, “The Disk method can always be used to find the volume of a solid of revolution, and solids of revolution will always have circular cross-sections.	Student writes, “The Disk method can always be used to find the volume of a solid of revolution.	Student writes, “Solids of revolution will always have cross sections that are semi-circles.”	No response given

Appendix H
Lesson Plans

Course: Calculus (Control Group) **Day:** 1

Unit: Applications of Integration **Lesson:** Riemann Sums

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, “What is the meaning of the definite integral and what are some of the ways that the definite integral can be computed?”

Goals/ Objectives:

- Use sigma notation to write and evaluate a sum
- Understand the concept of area
- Understand the definition of a Riemann sum
- Approximate the area of a plane region

Standards: 2, 3, 6 – 10

Materials: Textbook, Graphing Paper, Basic or Scientific Calculator

Prerequisites: Sigma Notation, Basic Area Formulas from Geometry, Fundamental Theorem of Calculus

Introduction: Students will practice using Sigma Notation.

Evaluate the following using Sigma Notation:

1. $\sum_{i=0}^{2} i$
2. $\sum_{i=1}^{3} (2-i)$
3. $\sum_{i=1}^{n} (i^2+1)$

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Neither the teacher nor the students will use the graphing calculator. All graphing will be done by hand. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

The Definite Integral

In the last section, you learned about <u>antidifferentiation</u>, when we find the original function given the derivative. You also learned how to evaluate the definite integral of a function using the <u>Fundamental Theorem of Calculus</u>.

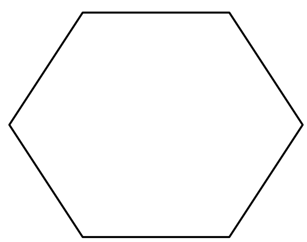
What is the notation that we use for the definite integral of a function f from a to b? ($\int_a^b f(x)dx$)

In the next section, you will learn the meaning of the definite integral and how we can evaluate the definite integral using other methods.

The <u>definite integral</u> of a function represents the area between the curve and the x-axis. They can be calculated various ways, including the Fundamental Theorem of Calculus, by counting squares, and by Riemann Sums.

Historical Perspective
In Euclidean geometry, how did we find the area of a rectangle? (using the definition $A = bh$)
We can also come up with definitions for the area of many other polygons. The ancient Greeks came up with a method for determining areas of other regions that are not polygons that was called the <u>exhaustion method</u>. Archimedes is well known for his use of this method.

How could we find the area of the figure below? (divide into triangles)



We can use a method similar to the exhaustion method for determining the area under a curve. We can split the region into a series of rectangles or trapezoids and sum the areas of these polygons to estimate the area under the curve. An example is shown below:

Example 1: Find the area of the region bounded by the curve $f(x) = -x^2 + 1$, the x-axis, and the vertical lines $x = 1$ and $x = 5$. What are two ways that we can solve this problem? (Fundamental Theorem of Calculus and the method of counting squares)

Example 2: How can we draw five rectangles to find two different approximations of the area of the region lying between the graph of $f(x) = -x^2 + 1$ and the x-axis between $x = 1$ and $x = 5$? (Draw rectangles outside of the curve and draw rectangles underneath the curve)

In the previous example, we found what is called an Upper Sum and a Lower Sum. What do these terms mean? (An Upper Sum is found by drawing circumscribed rectangles and summing the areas of these rectangles. A Lower Sum is found by drawing inscribed rectangles and summing the areas of these rectangles.)

These are the formulas for the Upper Sum and Lower Sum:

$$\text{Lower Sum} = s(n) = \sum_{i=1}^{n} f(m_i)\Delta x$$

$$\text{Upper Sum} = S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$$

where $f(m_i) =$ Minimum value of $f(x)$ on the ith subinterval
$f(M_i) =$ Maximum value of $f(x)$ on the ith subinterval

When using upper and lower sums, we chose partitions of equal width. Could we find the area if we divided the region into subintervals of unequal width? How? (Yes, we could draw rectangles of different widths)

Georg Friedrich Bernhard Riemann developed a more general form of the Riemann sum that the upper sum and lower sums that we saw before.

Definition of a Riemann Sum

Let f be defined on the closed interval [a, b], and let Δ be a partition of [a, b] given by

$$a = x_0 < x_1 < x_2 < ... < x_{n-1} < x_n = b$$

where Δx_i is the width of the ith subinterval. If c_i is any point in the ith subinterval, then the sum

$$\sum_{i=1}^{n} f(c_i)\Delta x_i, \; x_{i-1} \le c_i \le x_i$$

is called a Riemann sum of f for the partition Δ.

What other types of Riemann sums could we find? (A left sum and a right sum)
Other Riemann sums include the left Riemann sum and the Right Riemann sum. What do you think these two terms mean? (To find a left sum, we would use left endpoint of each subinterval, and to find a right sum, we would use the right endpoint of each subinterval.)

Try the example below:

Example 3: What are two types of Riemann sums that we could use to find the area of the region bounded by the graph of $f(x) = \frac{1}{2}x^2$ and the x-axis between $x = 0$ and $x = 3$. Find the area of this region.

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. No graphing calculators are allowed on this assignment.

P 261 #23-33 odd, 39-43 odd, 47-61 odd, 73a-d

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

Prove the following formula using mathematical induction: $\sum_{i=1}^{n} 2i = n(n+1)$

Course: Calculus (Experimental Group) **Day:** 1

Unit: Applications of Integration **Lesson:** Definite Integrals

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, "What is the meaning of the definite integral and what are some of the ways that the definite integral can be computed?"

Goals/ Objectives:

- Use sigma notation to write and evaluate a sum
- Understand the concept of area
- Understand the definition of a Riemann sum
- Approximate the area of a plane region

Standards: 2, 3, 6 – 10

Materials: Textbook, Graphing Paper, *Graphing Calculator

Prerequisites: Sigma Notation, Basic Area Formulas from Geometry, Fundamental Theorem of Calculus

Introduction: Students will practice using Sigma Notation.

Evaluate the following using Sigma Notation:

1. $\sum_{i=0}^{2} i$
2. $\sum_{i=1}^{3} (2-i)$
3. $\sum_{i=1}^{n} (i^2+1)$

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. *Both the teacher and the students will use the graphing calculator for graphing functions. They will also use the numerical approximation of the definite integral and the Rieman2 graphing calculator program. The teacher will use the TI-Connect software to display the graphing calculator screen for the students. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

The Definite Integral

In the last section, you learned about antidifferentiation, when we find the original function given the derivative. You also learned how to evaluate the definite integral of a function using the Fundamental Theorem of Calculus.

What is the notation that we use for the definite integral of a function f from a to b? ($\int_a^b f(x)dx$)

In the next section, you will learn the meaning of the definite integral and how we can evaluate the definite integral using other methods.

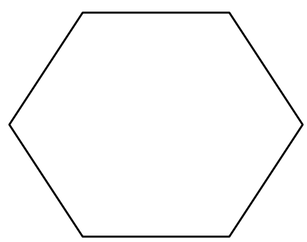
The definite integral of a function represents the area between the curve and the x-axis. They can be calculated various ways, including the Fundamental Theorem of Calculus, by counting squares, and by Riemann Sums.

Historical Perspective

In Euclidean geometry, we did we find the area of a rectangle? (using the definition $A = bh$)

We can also come up with definitions for the area of many other polygons. The ancient Greeks came up with a method for determining areas of other regions that are not polygons that was called the exhaustion method. Archimedes is well known for his use of this method.

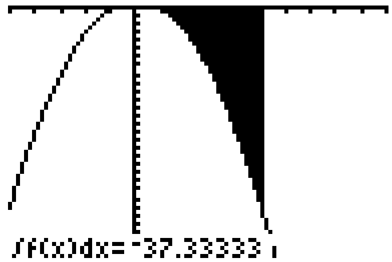
How could we find the area of the figure below? (divide into triangles)



We can use a method similar to the exhaustion method for determining the area under a curve. We can split the region into a series of rectangles or trapezoids and sum the areas of these polygons to estimate the area under the curve. An example is shown below:

Example 1: Find the area of the region bounded by the curve $f(x) = -x^2 + 1$, the x-axis, and the vertical lines $x = 1$ and $x = 5$. What are two ways that we can solve this problem? (Fundamental Theorem of Calculus and the method of counting squares)

*We can also use both the numerical approximation on the graphing calculator to evaluate the definite integral and to use the method of counting squares.



Example 2: How can we draw five rectangles to find two different approximations of the area of the region lying between the graph of $f(x) = -x^2 + 1$ and the x-axis between $x = 1$ and $x = 5$? (Draw rectangles outside of the curve and draw rectangles underneath the curve)

In the previous example, we found what is called an Upper Sum and a Lower Sum. What do these terms mean? (An Upper Sum is found by drawing circumscribed rectangles and summing the areas of these rectangles. A Lower Sum is found by drawing inscribed rectangles and summing the areas of these rectangles.)

These are the formulas for the Upper Sum and Lower Sum:

$$\text{Lower Sum} = s(n) = \sum_{i=1}^{n} f(m_i)\Delta x$$

$$\text{Upper Sum} = S(n) = \sum_{i=1}^{n} f(M_i)\Delta x$$

where $f(m_i) =$ Minimum value of $f(x)$ on the ith subinterval
$f(M_i) =$ Maximum value of $f(x)$ on the ith subinterval

When using upper and lower sums, we chose partitions of equal width. Could we find the area if we divided the region into subintervals of unequal width? (Yes, we could draw rectangles of different widths)

Georg Friedrich Bernhard Riemann developed a more general form of the Riemann sum that the upper sum and lower sums that we saw before.

Definition of a Riemann Sum

Let f be defined on the closed interval [a, b], and let Δ be a partition of [a, b] given by

$$a = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$$

where Δx_i is the width of the ith subinterval. If c_i is any point in the ith subinterval, then the sum

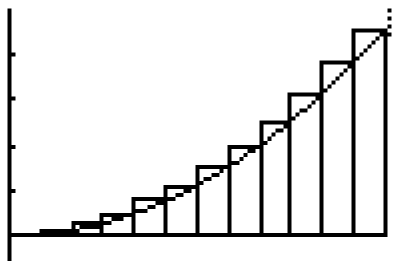
$$\sum_{i=1}^{n} f(c_i)\Delta x_i,\ x_{i-1} \le c_i \le x_i$$

is called a Riemann sum of f for the partition Δ.

What other types of Riemann sums could we find? (A left sum and a right sum)
Other Riemann sums include the left Riemann sum and the Right Riemann sum. What do you think these two terms mean? (To find a left sum, we would use left endpoint of each subinterval, and to find a right sum, we would use the right endpoint of each subinterval.)

Try the example below:

Example 3: What are two types of Riemann sums that we could use to find the area of the region bounded by the graph of $f(x)=\frac{1}{2}x^2$ and the x-axis between $x=0$ and $x=3$.
*Now using the graphing calculator, find the upper sum and the lower sum. [Teacher will demonstrate the Rieman2 graphing calculator program for students to find the upper sum, and then students will use the programs on their own to find the lower sum]



Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook.
*Graphing calculators are allowed on this assignment.

P 261 #23-33 odd, 39-43 odd, 47-61 odd, 73a-d

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

Prove the following formula using mathematical induction: $\sum_{i=1}^{n} 2i = n(n+1)$

Course: Calculus (Control Group) **Day:** 2

Unit: Applications of Integration **Lesson:** Riemann Sums

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, "What are some of the ways that the definite integral can be approximated, and how can we find the most accurate estimate of the definite integral?"

Goals/ Objectives:

- Find the area of a plane region using limits
- Approximate a definite integral using the Trapezoidal Rule.

Standards: 2, 6 – 10

Materials: Textbook, Graphing Paper, Basic or Scientific Calculator

Prerequisites: Upper and Lower Riemann Sums

Introduction:

Find the left and right Riemann sum for the region bounded by the graph of $f(x) = \frac{1}{2}x^3$ and the x-axis between x = 0 and x = 4.

How do the two sums compare to one another? (The lower sum is less than the upper sum)

What happens if we increase the number of rectangles used? (The difference between the two sums gets smaller)

Thus, as n, the number of rectangles gets larger, the sums approach the same value.

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Neither the teacher nor the students will use the graphing calculator. All graphing will be done by hand. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

As the number of rectangles increases or as n approaches _________ (infinity), the Upper Sum and Lower Sum approach the same number. We can say then, that as n approaches infinity, the limits of the Upper Sum and the Lower Sum approach one another.

Limit of the Lower and Upper Sums

$$\lim_{n\to\infty} s(n) = \lim_{n\to\infty} \sum_{i=1}^{n} f(m_i)\Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} f(M_i)\Delta x = \lim_{n\to\infty} S(n)$$

where $\Delta x = (b-a)/n$ and $f(m_i)$ and $f(M_i)$ are the minimum and maximum values of f on the interval.

Definition of the Area of a Region in the Plane

Let f be continuous and nonnegative on the interval [a,b]. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x,\ x_{i-1} \le c_i \le x_i$$

where $\Delta x = (b-a)/n$.

Try the next examples. For each problem, use the limit definition to find the definite integral and describe and use another method for checking your work.

Example 1: Find the area of the region bounded by the graph $f(x) = x^3$, the x-axis, and the vertical lines $x = 0$ and $x = 1$.

Example 2: Find the area of the region bounded by the graph of $f(x) = 9 - x^2$, the x-axis, and the vertical lines $x = 0$ and $x = 2$.

We have been looking at the area of regions bounded by a curve and the x-axis. We will now look at a region that is bounded by a curve and the y-axis.

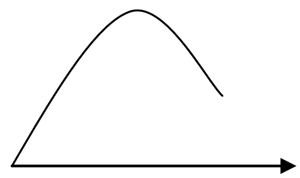
Example 3: Find the area of the region bounded by the graph of $f(y) = y^3$ and the y-axis for $0 \le y \le 1$.

What other shapes could we use to find the area under a curve besides rectangles? (Trapezoids)

What are some of the ways that we now know how to evaluate the definite integral or approximate the definite integral? (Fundamental Theorem of Calculus, counting squares, and Riemann sums). The trapezoidal rule is another way to approximate the value of the definite integral.

Let f be continuous on [a, b]. Then $\displaystyle\int_a^b f(x)dx \approx \frac{b-a}{n}\left[\frac{1}{2} f(x_0) + f(x_1) + ... + f(x_{n-1}) + f(x_n)\right]$

How we find the area underneath the curve below using trapezoids?



Try the following examples:

Example 4: Use the Trapezoidal Rule to approximate $\int_0^3 -x^2+4 \; dx$. Compare the results for different values of n. How can we achieve a better estimate of the definite integral?

The Midpoint Rule is another type of Riemann sum defined by: $\sum_{i=1}^{n} f\left(\frac{x_i + x_{i-1}}{2}\right)\Delta x$

In Midpoint Rule, function values are taken at the midpoint of each subinterval.

Example 5: Use both midpoint rule and Trapezoidal Rule to approximate $\int_0^4 \sqrt{4+x^2}$ for different values of n. Compare the results.

Example 3: Approximate the following definite integral using an upper sum, lower sum, midpoint rule, and Trapezoidal rule with $n=6$. Then, arrange these approximations, along with the actual value of the definite integral, from least to greatest. $\int_0^6 (x^2+1)dx$

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. Graphing calculators are not allowed on this assignment.

P 263 #63, 65
P 305 #1-9 odd, 43

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

Suppose that a function f is continuous on the interval [-4, 4] and that $\int_0^4 f(x)dx = 2$. Find $\int_{-4}^4 f(x)dx$ if f is even and if f is odd.

Course: Calculus (Experimental Group) **Day:** 2

Unit: Applications of Integration **Lesson:** Riemann Sums

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, "What are some of the ways that the definite integral can be approximated, and how can we find the most accurate estimate of the definite integral?"

Goals/ Objectives:

- Find the area of a plane region using limits
- Approximate a definite integral using the Trapezoidal Rule.

Standards: 2, 6 – 10

Materials: Textbook, Graphing Paper, *Graphing Calculator

Prerequisites: Upper and Lower Riemann Sums

Introduction:

Find the left and right Riemann sum for the region bounded by the graph of $f(x) = \frac{1}{2}x^3$ and the x-axis between x = 0 and x = 4.

How do the two sums compare to one another? (The lower sum is less than the upper sum)

What happens if we increase the number of rectangles used? (The difference between the two sums gets smaller)

Thus, as n, the number of rectangles gets larger, the sums approach the same value.

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. *Both the teacher and the students will use the graphing calculator program Rieman2. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

As the number of rectangles increases or as n approaches _________ (infinity), the Upper Sum and Lower Sum approach the same number. We can say then, that as n approaches infinity, the limits of the Upper Sum and the Lower Sum approach one another.

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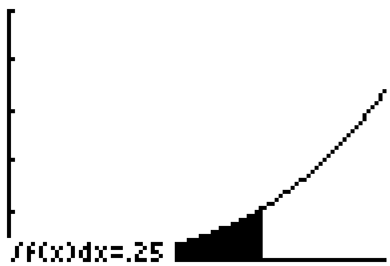
Let f be continuous and nonnegative on the interval [a,b]. The area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is

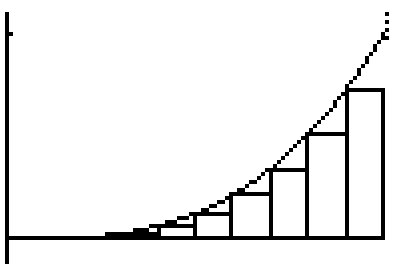
$$\text{Area} = \lim_{n\to\infty}\sum_{i=1}^{n} f(c_i)\Delta x,\ x_{i-1} \le c_i \le x_i$$

where $\Delta x = (b-a)/n.$

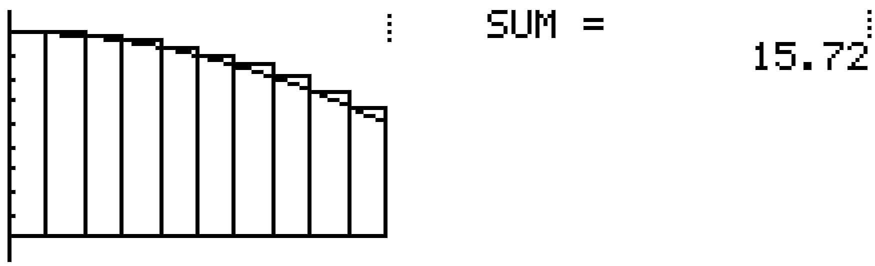
*Try the next examples. For each problem, describe and use a few different methods on the graphing calculator for finding the area of the region. Then find the definite integral using the limit definition.

Example 1: Find the area of the region bounded by the graph $f(x) = x^3$, the x-axis, and the vertical lines $x = 0$ and $x = 1$.





Example 2: Find the area of the region bounded by the graph of $f(x) = 9 - x^2$, the x-axis, and the vertical lines $x = 0$ and $x = 2$.



We have been looking at the area of regions bounded by a curve and the x-axis. We will now look at a region that is bounded by a curve and the y-axis.

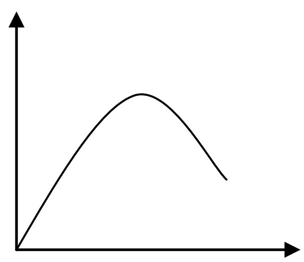
Example 3: Find the area of the region bounded by the graph of $f(y) = y^2$ and the y-axis for $0 \le y \le 1$.

What other shapes could we use to find the area under a curve besides rectangles? (Trapezoids)

What are some of the ways that we now know how to evaluate the definite integral or approximate the definite integral? (Fundamental Theorem of Calculus, counting squares, and Riemann sums). The trapezoidal rule is another way to approximate the value of the definite integral.

Let f be continuous on [a, b]. Then $\int_a^b f(x)dx \approx \frac{b-a}{n}\left[\frac{1}{2}f(x_0) + f(x_1) + ... + f(x_{n-1}) + f(x_n)\right]$

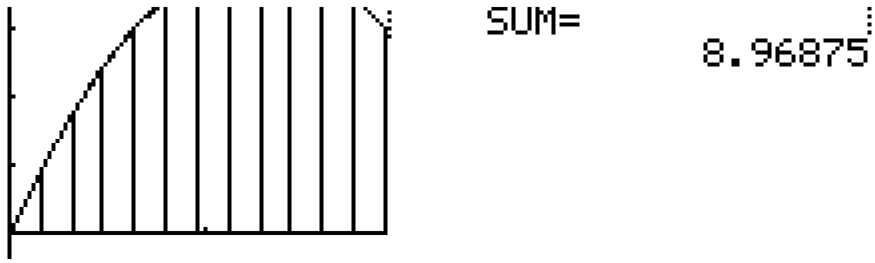
How we find the area underneath the curve below using trapezoids?



Try the following examples:

*For each example, use your calculator to solve the problem in addition to pencil and paper methods.

Example 4: Use the Trapezoidal Rule to approximate $\int_0^3 -x^2 + 4 \ dx$. Compare the results for different values of n. How can we achieve a better estimate of the definite integral?



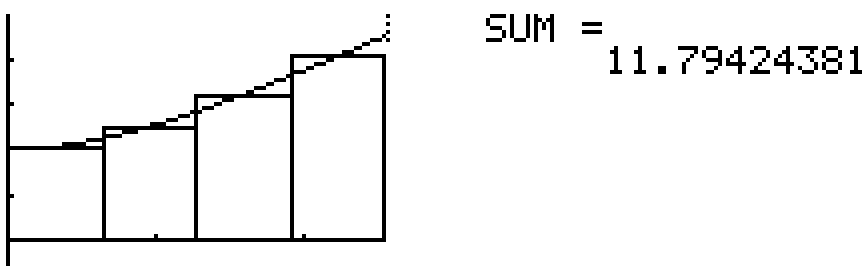
The Midpoint Rule is another type of Riemann sum defined by: $\sum_{i=1}^{n} f\left(\frac{x_i + x_{i-1}}{2}\right)\Delta x$

In Midpoint Rule, function values are taken at the midpoint of each subinterval.

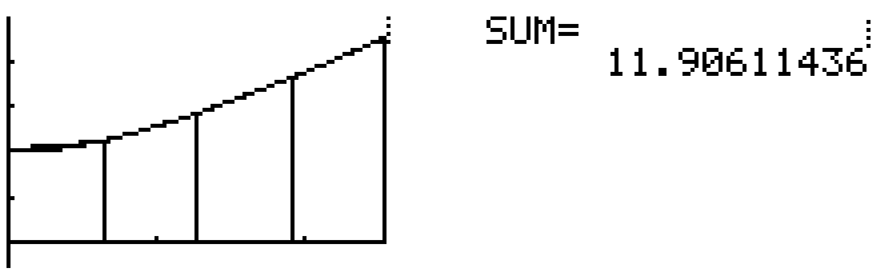
Example 5: Use both midpoint rule and Trapezoidal Rule to approximate $\int_0^4 \sqrt{4+x^2}$ for different values of n. Compare the results.

N = 4

Midpoint Rule

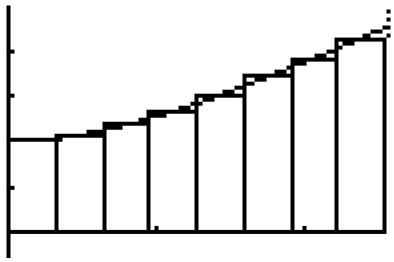


Trapezoid Rule

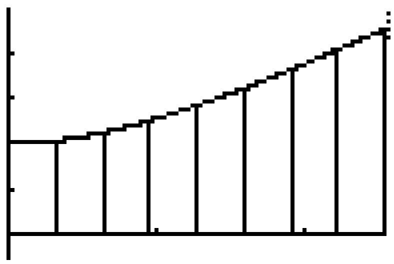


N = 8

Midpoint Rule



Trapezoid Rule



Example 6: Approximate the following definite integral using an upper sum, lower sum, midpoint rule, and Trapezoidal rule with $n = 6$. Then, arrange these approximations, along with the actual value of the definite integral, from least to greatest. $\int_0^6 (x^2 + 1)dx$

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. *Graphing calculators are allowed on this assignment.

P 263 #63, 65
P 305 #1-9 odd, 43

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

Suppose that a function f is continuous on the interval [-4, 4] and that $\int_0^4 f(x)dx = 2$. Find $\int_{-4}^4 f(x)dx$ if f is even and if f is odd.

Course: Calculus (Control Group) **Day:** 3

Unit: Applications of Integration **Lesson:** Area

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, “What are some of the applications of the definite integral, and what are some of the ways that we can find the area between two curves?”

Goals/ Objectives:

- Find the area of a region between two curves using integration
- Find the area of a region between intersecting curves using integration

Standards: 2, 6 – 10

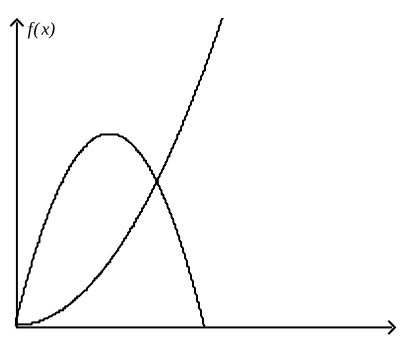
Materials: Textbook, Graphing Paper, Basic or Scientific Calculator

Prerequisites: Fundamental Theorem of Calculus

Introduction: Review Homework Questions

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Neither the teacher nor the students will use the graphing calculator. All graphing will be done by hand. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

We have now seen how to compute the area underneath a curve. How might we compute the area of a region between two curves such as the one below? (We could find the area underneath each curve and then subtract the two answers)



To find the area between two curves, we can either find the area under each and then subtract the values or we can subtract the two curves and then find the area underneath the new curve.

Consider the functions $f(x) = x^2 + 1$ and $g(x) = -x^2 + 4$.

In order to find the area of the region between two graphs, we have to know where the two functions intersect. How can we find the points where two functions intersect? (Set the functions equal to one another and solve)

The teacher should demonstrate for students how to find the points of intersection.

Does $\int_a^b [f(x) - g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$?

(The teacher should demonstrate graphing the functions $f(x)$ and $g(x)$ on the set of axes as well as a new function $f(x) - g(x)$. Then verify that we arrive at the same solution when find the definite integral of these two regions.)

To find the area between two curves f and g on the interval [a, b], we can use the formula

$$A = \int_a^b [f(x) - g(x)]dx$$

When computing the area between two curves, we always subtract one function from the other. How can we know which function should be subtracted from the other? (The bottom function should be subtracted from the top function) What would happen if we subtracted the top function from the bottom function? (We would get a negative answer)

Therefore, area between two curves should always be a _________ number? (Positive)

*Graph these two functions. Which function is greater on the interval between the two points of intersection? ($y = -x^2 + 4$) Is this function greater on the entire interval $(-\infty, \infty)$?

Example 1: Find the area of the region bounded by the graphs of $f(x) = x + 2$ and $g(x) = x^2$.

We can also find the area of a region between two graphs given two points other than the points of intersection.

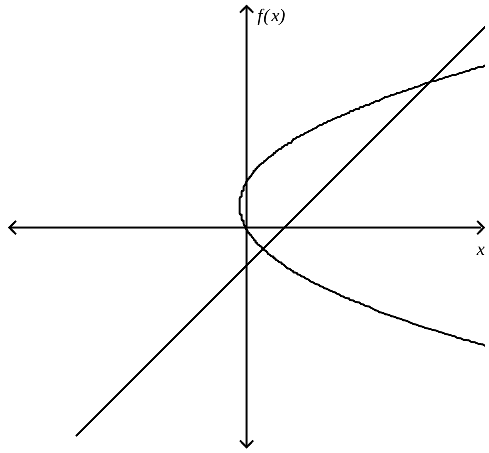
Example 2: Find the area of the region bounded by the graphs $y = 1 - x$ and $y = -x^2$ between x = -1 and x = 2.

Now consider the graphs of the trigonometric functions $y = \sin x$ and $y = \cos x$. At how many points do these functions intersect? (An infinite number of points)

Based on your answer, can we say that there may be more than one region bounded by two graphs? (yes)

Example 3: Find the area of the region bounded by the graphs $y = x^3 - 4x^2 + 1$ and $y = -x^2 + x - 2$.

Look at the region below. Can we use the same method to find the area of this region? (No) Why or why not? (The graph does not show a function of x; however, it is a function of y.) How might we find the area of this region? (Integrate with respect to y)



Example 4: Find the area of the region bounded by the graphs of $x = y^2 - y$ and $x = 2 + 2y$.

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. Graphing calculators are not allowed on this assignment.

P 418 #1-35 every other odd, 41-45 odd, 53-65 odd

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

For the following problem, find k such that the line $y = k$ divides the region bounded by the graphs below into two regions with equal areas.

$y = x^2 + 4, y = 0$

Course: Calculus (Experimental Group) **Day:** 3

Unit: Applications of Integration **Lesson:** Area

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, "What are some of the applications of the definite integral, and what are some of the ways that we can find the area between two curves?"

Goals/ Objectives:

- Find the area of a region between two curves using integration
- Find the area of a region between intersecting curves using integration

Standards: 2, 6 – 10

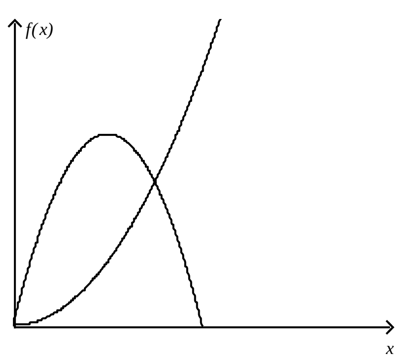
Materials: Textbook, Graphing Paper, Graphing Calculator

Prerequisites: Fundamental Theorem of Calculus

Introduction: Review Homework Questions

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Both the teacher and the students will use both pencil and paper methods of graphing as well as the graphing calculator for graphing, numerical approximation of the definite integral, and the CALCULUS program. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

We have now seen how to compute the area underneath a curve. How might we compute the area of a region between two curves such as the one below? (We could find the area underneath each curve and then subtract the two answers)



To find the area between two curves, we can either find the area under each and then subtract the values or we can subtract the two curves and then find the area underneath the new curve.

Consider the functions $f(x) = x^2 + 1$ and $g(x) = -x^2 + 4$.

In order to find the area of the region between two graphs, we have to know where the two functions intersect. How can we find the points where two functions intersect? (Set the functions equal to one another and solve *or use the graphing calculator to find the points of intersection)

*The teacher should demonstrate for students how to use the graphing calculator to both view and find points of intersection.

Does $\int_a^b [f(x) - g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx$?

*(The teacher should demonstrate using the graphing calculator to graph the functions $f(x)$ and $g(x)$ on the same screen as well as a new function $f(x) - g(x)$. Then use the numerical approximation of the definite integral on the graphing calculator to verify that we arrive at the same solution when find the definite integral of these two regions.)

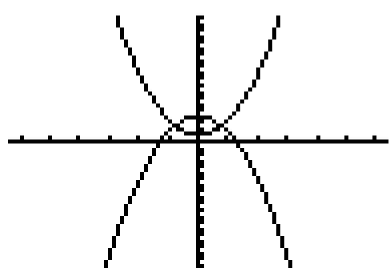
Hence, to find the area between two curves f and g on the interval [a, b], we can use the formula

$$A = \int_a^b [f(x) - g(x)]dx$$

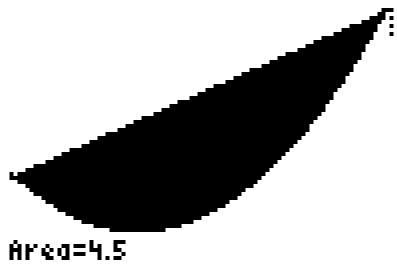
When computing the area between two curves, we always subtract one function from the other. How can we know which function should be subtracted from the other? (The bottom function should be subtracted from the top function) What would happen if we subtracted the top function from the bottom function? (We would get a negative answer)

Therefore, area between two curves should always be a _________ number? (Positive)

*Use your graphing calculator to view these two functions. Which function is greater on the interval between the two points of intersection? ($y = -x^2 + 4$) Is this function greater on the entire interval $(-\infty, \infty)$?



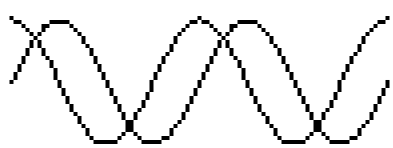
Example 1: Find the area of the region bounded by the graphs of $f(x) = x + 2$ and $g(x) = x^2$. *Use both pencil and paper graphing methods as well as the graphing calculator program CALCULUS to find the area of the region between the two curves.



We can also find the area of a region between two graphs given two points other than the points of intersection.

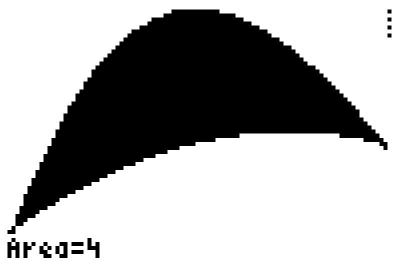
Example 2: Find the area of the region bounded by the graphs $y = 1 - x$ and $y = -x^2$ between x = -1 and x = 2. *Use pencil and paper graphing methods as well as your graphing calculator.

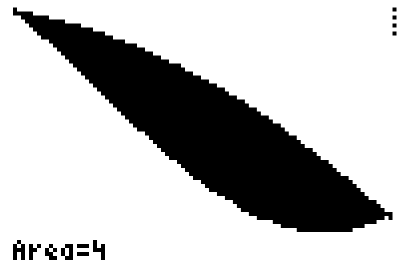
Now consider the graphs of the trigonometric functions $y = \sin x$ and $y = \cos x$. *View these functions on your graphing calculator. At how many points do these functions intersect? (An infinite number of points)



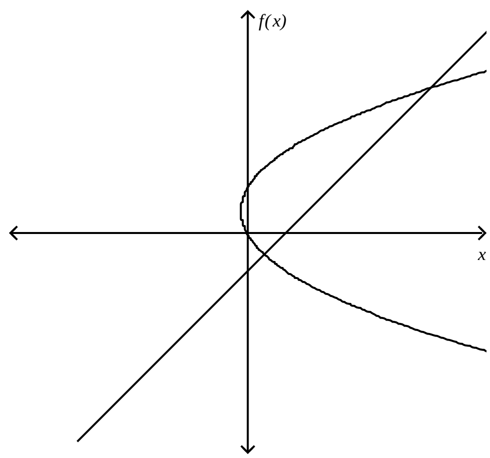
Based on your answer, can we say that there may be more than one region bounded by two graphs? (yes)

Example 3: Find the area of the region bounded by the graphs $y = x^3 - 4x^2 + 1$ and $y = -x^2 + x - 2$. *(The students may use their graphing calculators to view the functions and the program CALCULUS to compute the area between the two graphs)





Look at the region below. Can we use the same method to find the area of this region? (No) Why or why not? (The graph does not show a function of x; however, it is a function of y.) How might we find the area of this region? (Integrate with respect to y)



Example 4: Find the area of the region bounded by the graphs of $x = y^2 - y$ and $x = 2 + 2y$.

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. Graphing calculators are allowed on this assignment.

P 418 #1-35 every other odd, 41-45 odd, 53-65 odd

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

For the following problem, find k such that the line $y = k$ divides the region bounded by the graphs below into two regions with equal areas.

$y = x^2 + 4, y = 0$

Course: Calculus (Control Group) **Day:** 4

Unit: Applications of Integration **Lesson:** Volume

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, "What are some of the applications of the definite integral, and what are some of the ways that we can find the volume of a solid of revolution?"

Goals/ Objectives:

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.

Standards: 2, 3, 6 – 10

Materials: Textbook, Graphing Paper, Basic or Scientific Calculator

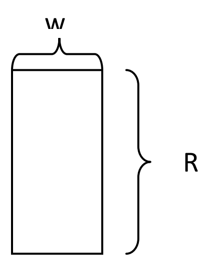
Prerequisites: Basic area formulas from Geometry, Fundamental Theorem of Calculus

Introduction: Review Homework Questions

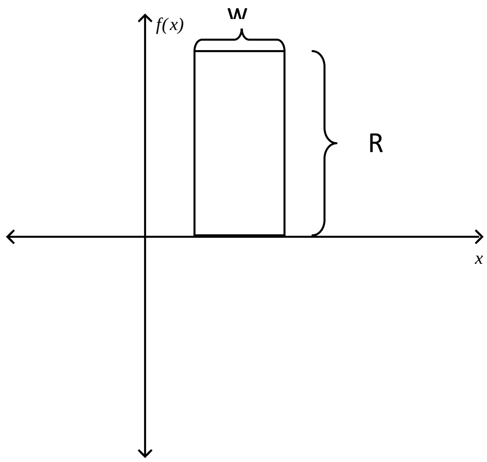
Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Neither the teacher nor the students will use the graphing calculator. All graphing will be done by hand. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

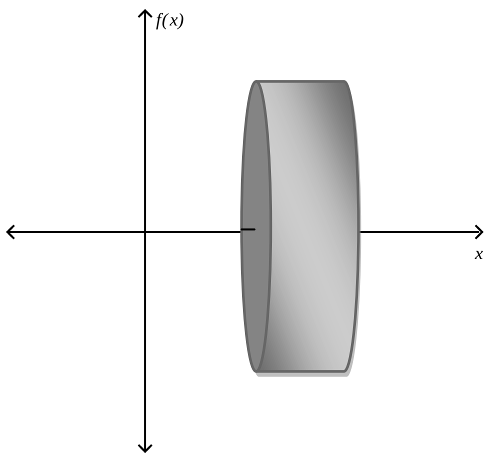
In this section, we will be looking at another application of the definite integral. What are some of the applications of the definite integral that we've seen so far? (Area underneath a curve and Area between two curves)

Look at the rectangle below. In geometry, how did we find the area of this rectangle? (length times width)



Now imagine that this rectangle were situated on the x-y axes and that we rotated this rectangle about the x-axis forming a three dimensional solid. (The teacher should draw a picture on the board to help students visualize this concept). What would this solid look like? (A cylinder) What shape would the cross sections be? (circles)





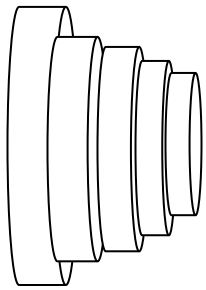
The volume of this disk would be V = _______________. ($V = \pi R^2 w$)

Example 1: What is the volume of the solid formed when the line $y = 3$ is rotated about the x-axis from $x = 2$ to $x = 6$?

Imagine that we used several disks to find the area of a solid of revolution. A solid of revolution is formed by rotating a region about a line. In the above example, we rotated a rectangle about the line $y = 0$.

How might we find the volume of a solid formed by rotating the graph of $y = e^x$ about the x-axis between $x = 0$ and $x = 2$? (Use several representative disks)

We can find the volume of solids of revolution through integration. Integration is an accumulation process. How is that demonstrated in the figure below? (To find the volume of this solid, we can sum the volume of the representative disks)



How does this relate to the process that we used to approximate the definite integral when finding Riemann sums? (When finding Riemann sums, we summed the areas of several rectangles)

How are the following concepts related? $\sum_{x=1}^{n} x$ and $\int_{1}^{n} x$

(The first is for a discrete function and the second is for a continuous function)

The Disk Method can be used to find the volume of a solid of revolution.

Horizontal Axis of Revolution

$$V = \pi \int_a^b \left[R(x) \right]^2 dx$$

How would this formula change for a vertical axis of revolution? (The radius is a function of y and we would integrate with respect to y)

Example 1: Find the volume of a solid formed when the portion of the graph of $f(x) = -x^2 + 4x$ in Quadrant I is rotated about the x-axis. (The teacher should draw a visual for students on the board of the function and what the solid of revolution would look like).

Example 2: Find the volume of the solid of revolution formed by revolving the region bounded by $y = 6 - x^2$ and $g(x) = 2$ about the line y = 1.

Example 3: Imagine now that we rotated the region bounded by the graphs in the example above by the x-axis instead. What would be the resulting shape? (A donut or washer) Find the volume of this solid. (The teacher should draw a visual for students on the board of the function and what the solid of revolution would look like).

Example 4: Find the volume of the solid of revolution formed by rotating the region bounded by $y = 2x^2$ and $y = 2\sqrt{x}$ about the x-axis and about the line y = -1. (The teacher should draw a visual for students on the board of the function and what the solid of revolution would look like).

Now create two positive functions on your own. (a) First, find the volume of the solid generated when each region is rotated about the x-axis. (b) Then, find the volume of the solid generated when the region between the two curves is rotated about the x-axis. What is the relationship between your answers to part a and part b?

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. Graphing calculators are not allowed on this assignment.

P 428 #1-5 odd, 15-17 odd, 23-29 odd, 43, 45

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

What is the volume of the solid generated by rotating the region bounded by the graphs of $y = e^x,\ y = 0,\ x = 0,$ and $x = 2$ about the line y = 10?

Course: Calculus (Experimental Group) **Day:** 4

Unit: Applications of Integration **Lesson:** Volume

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, "What are some of the applications of the definite integral, and what are some of the ways that we can find the volume of a solid of revolution?"

Goals/ Objectives:

- Find the volume of a solid of revolution using the disk method.
- Find the volume of a solid of revolution using the washer method.

Standards: 2, 3, 6 – 10

Materials: Textbook, Graphing Paper, TI-84 Plus Graphing Calculator, TI-Nspire Graphing Calculator

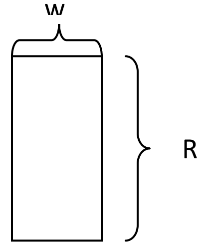
Prerequisites: Basic area formulas from Geometry, Fundamental Theorem of Calculus

Introduction: Review Homework Questions

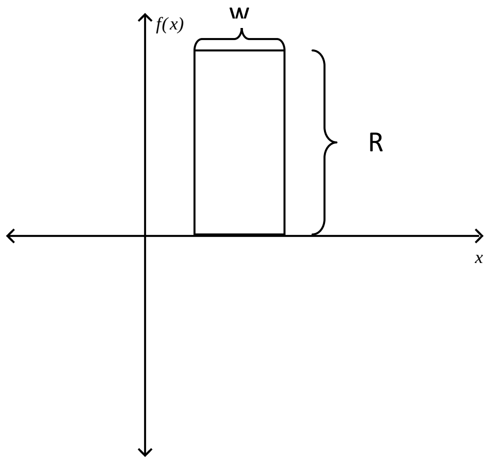
Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Both the teacher and the students will use both pencil and paper methods of graphing as well as the graphing calculator for graphing functions and the CALCULUS program. The teacher will also use the TI-Nspire graphing calculator to provide visualizations of solids of revolution. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

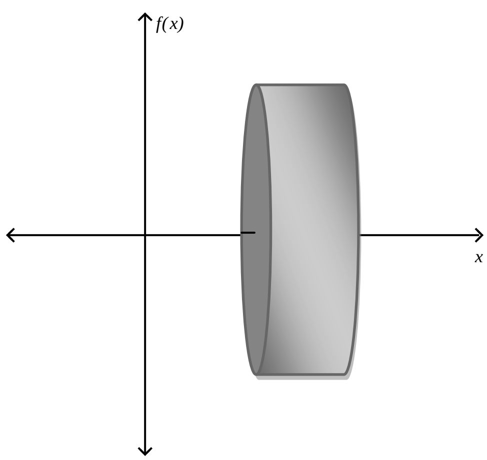
In this section, we will be looking at another application of the definite integral. What are some of the applications of the definite integral that we've seen so far? (Area underneath a curve and Area between two curves)

Look at the rectangle below. In geometry, how did we find the area of this rectangle? (length times width)



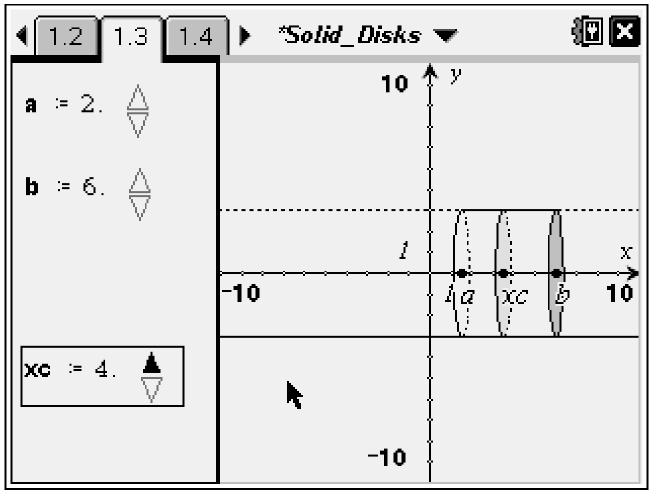
Now imagine that this rectangle were situated on the x-y axes and that we rotated this rectangle about the x-axis forming a three dimensional solid. *(The teacher should use the TI-Nspire document *Solid_Disks* to help students visualize this concept). What would this solid look like? (A cylinder) What shape would the cross sections be? (circles)





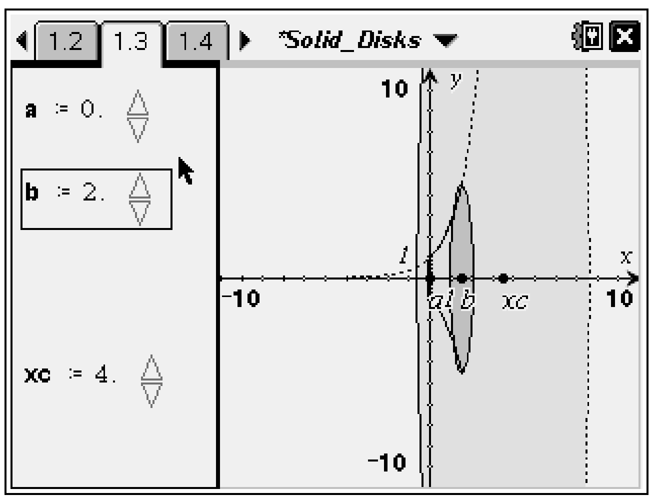
The volume of this disk would be V = _______________. ($V = \pi R^2 w$)

Example 1: What is the volume of the solid formed when the line $y = 3$ is rotated about the x-axis from $x = 2$ to $x = 6$? *(The teacher should use the TI-Nspire document *Solids_Disks* to show students the solid of revolution)

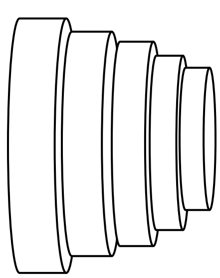


Imagine that we used several disks to find the area of a solid of revolution. A solid of revolution is formed by rotating a region about a line. In the above example, we rotated a rectangle above the line $y = 0$.

How might we find the volume of a solid formed by rotating the graph of $y = e^x$ about the x-axis between $x = 0$ and $x = 2$? (Use several representative disks)



We can find the volume of solids of revolution through integration. Integration is an accumulation process. How is that demonstrated in the figure below? (To find the volume of this solid, we can sum the volume of the representative disks)



How does this relate to the process that we used to approximate the definite integral when finding Riemann sums? (When finding Riemann sums, we summed the areas of several rectangles)

How are the following concepts related? $\sum_{x=1}^{n} x$ and $\int_{1}^{n} x$

(The first is for a discrete function and the second is for a continuous function)

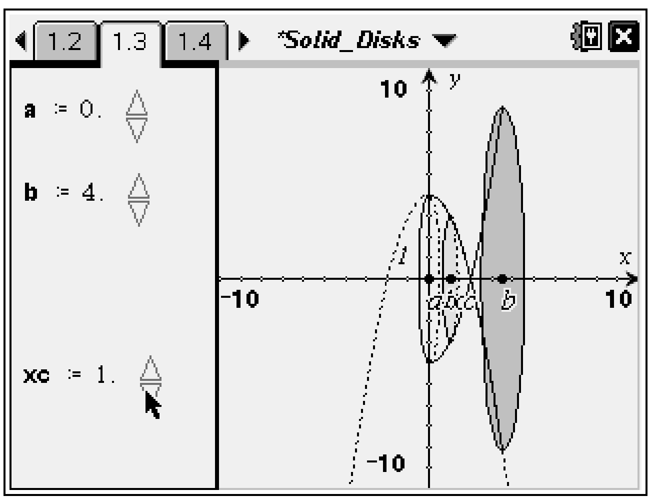
The Disk Method can be used to find the volume of a solid of revolution.

Horizontal Axis of Revolution

$$V = \pi \int_a^b R(x)^2 \, dx$$

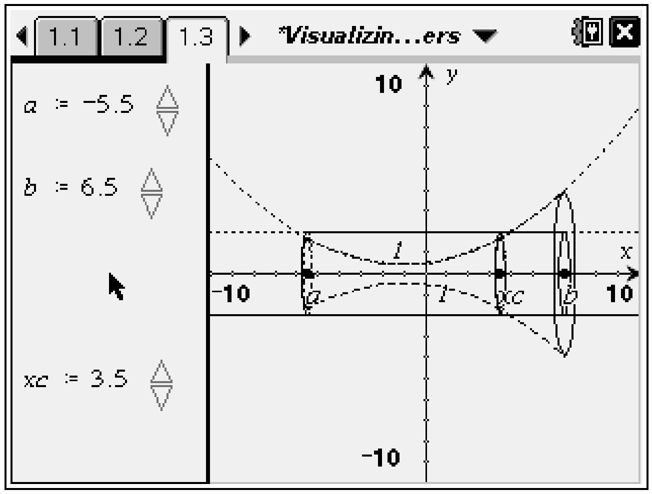
How would this formula change for a vertical axis of revolution? (The radius is a function of y and we would integrate with respect to y)

Example 1: Find the volume of a solid formed when the portion of the graph of $f(x) = -x^2 + 4x$ in Quadrant I is rotated about the x-axis. *(The teacher should use the TI-Nspire document *Solid_Disks* to show students the solid of revolution)

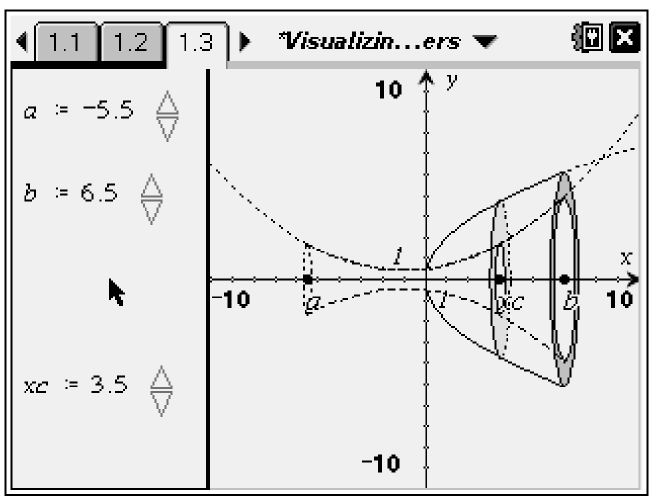


Example 2: Find the volume of the solid of revolution formed by revolving the region bounded by $f(x) = 6 - x^2$ and $g(x) = 2$ about the line y = 2.

Example 3: Imagine now that we rotated the region bounded by the graphs in the example above by the x-axis instead. What would be the resulting shape? (A donut or washer) Find the volume of this solid. *(The teacher should use the TI-Nspire document *Visualizing_Solids_of_Revolution* to show students the solid of revolution)



Example 4: Find the volume of the solid of revolution formed by rotating the region bounded by $y = 2x^2$ and $y = 2\sqrt{x}$ about the x-axis and about the line y = -1. *(The teacher should use the TI-Nspire document *Visualizing_Solids_of_Revolution* to show students the solid of revolution)



Now create two positive functions on your own. (a) First, find the volume of the solid generated when each region is rotated about the x-axis. (b) Then, find the volume of the solid generated when the region between the two curves is rotated about the x-axis. What is the relationship between your answers to part a and part b? *Use the TI-Nspire document *Visualizing_Solids_of_Revolution* to view the solids of revolution.

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. Graphing calculators are allowed on this assignment.

P 428 #1-5 odd, 15-17 odd, 23-29 odd, 43, 45

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

What is the volume of the solid generated by rotating the region bounded by the graphs of $y = e^x,\ y = 0,\ x = 0,$ and $x = 2$ about the line y = 10?

Course: Calculus (Control Group) **Day:** 5

Unit: Applications of Integration **Lesson:** Volume

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, "What are some of the applications of the definite integral, and what are some of the different methods of computing volume using definite integrals?"

Goals/ Objectives:

- Find the volume of a solid with a vertical axis of revolution.
- Find the volume of a solid with known cross sections.

Standards: 2, 6 – 10

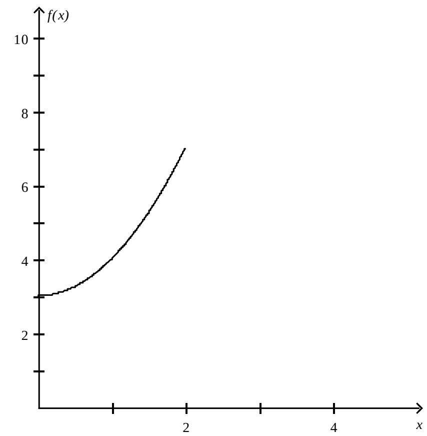
Materials: Textbook, Graphing Paper, Basic or Scientific Calculator, Donut or similarly shaped object

Prerequisites: Basic area formulas from Geometry, Fundamental Theorem of Calculus

Introduction: Find the volume of the solid that is formed when the graph of $y = x^2 + 3,\ 0 \le x \le 2$ is rotated about the x-axis.

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Neither the teacher nor the students will use the graphing calculator. All graphing will be done by hand. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

We found the volume of the solid that is formed when the graph below of $y = x^2 + 3,\ 0 \le x \le 2$ is rotated about the x-axis. What was the resulting volume? In order to find the volume, we made cross sections perpendicular to which axis? (The x-axis)



In the previous lesson, how did we make the cross sections to find the volume of a solid of revolution? (perpendicular to the x-axis). In this lesson, we will continue finding the volume of solids with cross sections made perpendicular to the y-axis and with known cross sections.

Imagine now that we rotated the region above by the y-axis. How could we find the volume of this solid? (Set up two integrals and integrate with respect to y)

We have been using the disk method to find the volume of solids of revolution.

What shape is the cross section when we use this method? (Circular)

What is the area of the cross sections that are formed? ($A = \pi r^2$)

Recall the following formula for the disk method:

$\text{Volume} = V = \int_a^b \pi \left[R(x) \right]^2 dx$ When the axis of revolution is horizontal

Can you find the formula for the area of a circle in this formula for volume? ($\pi \left[R(x) \right]^2$)

How could we modify this formula to find the volume of solids where the area of a cross section is not a circle? (Replace $\pi \left[R(x) \right]^2$ with the formula for the area of the cross section)

Suppose the area of the cross section was a square or a triangle. What general formula could we use to find volume?

$$\text{Volume} = V = \int_a^b A(x)\ dx$$

$$\text{Volume} = V = \int_c^d A(y)\ dy$$

Find the volume of a solid with a base bounded by $y = 1 - \frac{x}{2}$, $y = -1 + \frac{x}{2}$, and $x = 0$.

Closing Activity: Donut Lab: Cut a cross section of a donut (or similarly shaped object) and trace the top and bottom of the cross section on a piece of graph paper. Find the approximate area of the cross section. Then, use the disk method to find the volume if the region between the two functions was rotated about either the x or the y axis. Later, we will learn how to use the graphing calculator to find the volume.

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. Graphing calculators are not allowed on this assignment.

P.428 #7-13 odd, 19-21 odd, 31, 44, 53, 58, 61

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

Find the volume of the solid whose base is bounded by the unit circle, with cross sections that are semicircles.

Course: Calculus (Experimental Group) **Day:** 5

Unit: Applications of Integration **Lesson:** Volume

Unit: Applications of Integration **Lesson:** Volume

Textbook: Larson Calculus of a Single Variable

Aim: Students will be able to answer the question, "What are some of the applications of the definite integral, and what are some of the different methods of computing volume using definite integrals?"

Goals/ Objectives:

- Find the volume of a solid with a vertical axis of revolution.
- Find the volume of a solid with known cross sections.

Standards: 2, 5 – 10

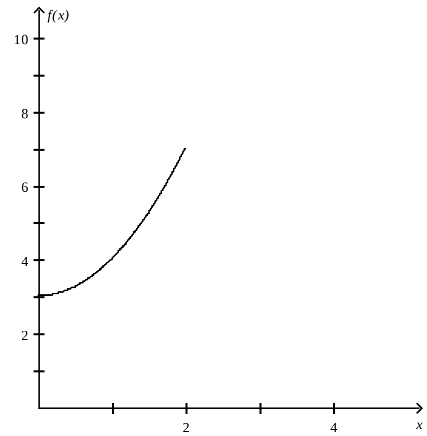
Materials: Textbook, Graphing Paper, TI-84 Plus Graphing Calculator, TI-Nspire Graphing Calculator, Donut or similarly shaped object

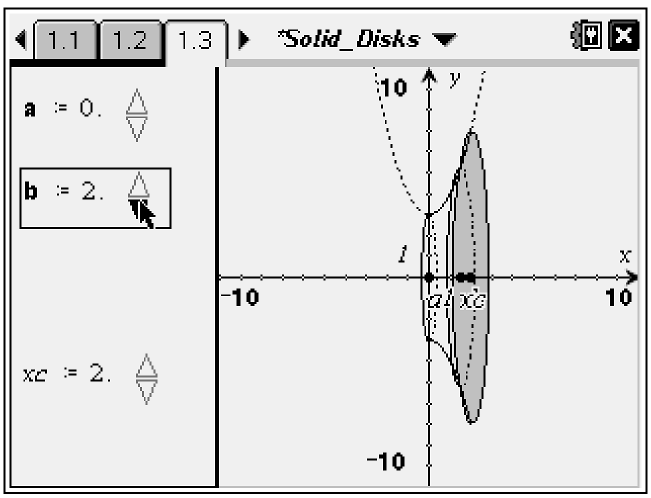
Prerequisites: Basic area formulas from Geometry, Fundamental Theorem of Calculus

Introduction: Find the volume of the solid that is formed when the graph of $y = x^2 + 3,\ 0 \le x \le 2$ is rotated about the x-axis.

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Both the teacher and the students will use both pencil and paper methods of graphing as well as the graphing calculator for graphing functions and the CALCULUS program. The teacher will also use the TI-Nspire graphing calculator to provide visualizations of solids of revolution. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

We found the volume of the solid that is formed when the graph below of $y = x^2 + 3,\ 0 \le x \le 2$ is rotated about the x-axis. What was the resulting volume? In order to find the volume, we made cross sections perpendicular to which axis? (The x-axis) *(The teacher should use the TI-Nspire document *Solid_Disks* to show students the solid of revolution).





In the previous lesson, how did we make the cross sections to find the volume of a solid of revolution? (perpendicular to the x-axis). In this lesson, we will continue finding the volume of solids with cross sections made perpendicular to the y-axis and with known cross sections.

Imagine now that we rotated the region above by the y-axis. How could we find the volume of this solid? (Set up two integrals and integrate with respect to y) *(The teacher should use the TI-Nspire document *Solid_Disks* to show students the two functions and the solid of revolution).

We have been using the disk method to find the volume of solids of revolution.

What shape is the cross section when we use this method? (Circular) *(The teacher should refer back to the visualization of the solid of revolution on the TI-Nspire).

What is the area of the cross sections that are formed? ($A = \pi r^2$)

Recall the following formula for the disk method:

$\text{Volume} = V = \int_a^b \pi \left[R(x) \right]^2 dx$ When the axis of revolution is horizontal

Can you find the formula for the area of a circle in this formula for volume? ($\pi \left[R(x) \right]^2$)

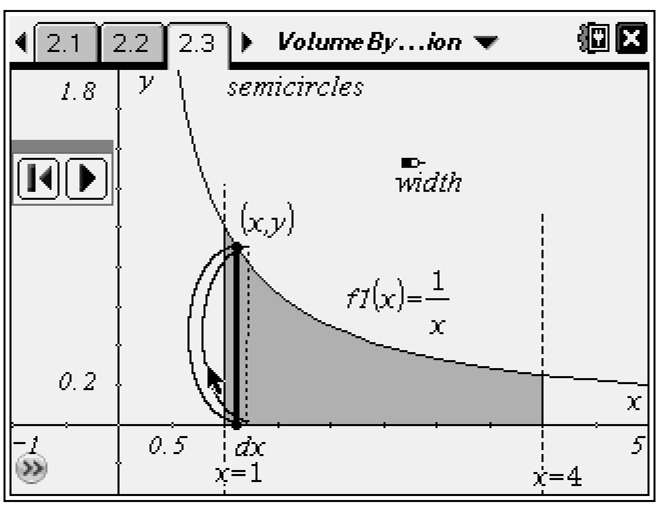
How could we modify this formula to find the volume of solids where the area of a cross section is not a circle? (Replace $\pi \left[R(x) \right]^2$ with the formula for the area of the cross section)

Suppose the area of the cross section was a square or a triangle. What general formula could we use to find volume?

$$\text{Volume} = V = \int_a^b A(x)\ dx$$

$$\text{Volume} = V = \int_c^d A(y)\ dy$$

Find the volume of a solid with a base bounded by $y = \frac{1}{x},\ x = 1,$ and $x = 4,$ where the cross sections are semi-circles . *(Use the TI-Nspire document *VolumeByCrossSection* to show students the solid).



Closing Activity: Donut Lab: Cut a cross section of a donut and trace the top and bottom of the cross section on a piece of graph paper. Enter the top sets of points and the bottom set of points separately into your graphing calculator and use regression to find a function that fits each set of points. Then, use the disk method to find the volume if the region between the two functions was rotated about either the x or the y axis.

Homework Assignment: Assignment from Larson Calculus of a Single Variable textbook. Graphing calculators are allowed on this assignment.

P.428 #7-13 odd, 19-21 odd, 31, 44, 53, 58, 61

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

Find the volume of the solid whose base is bounded by the unit circle, with cross sections that are semicircles.

Course: Calculus (Control Group) **Day:** 6

Unit: Applications of Integration **Lesson:** Distance and Displacement

Textbook: Foerster's Calculus Concepts and Applications

Aim: Students will be able to answer the question, "What are some of the applications of the definite integral, and how can we use the definite integral to find the distance and displacement of an object moving in a linear path?"

Goals/ Objectives:

- Given the velocity or acceleration function for an object in linear moving in a linear path, find the distance traveled in a given time and the displacement at a given time.

Standards: 2, 6 – 10

Materials: Graphing Paper, Basic or Scientific Calculator

Prerequisites: Velocity and Acceleration Functions by Derivatives, Relationship between functions and derivative functions, Relationship between Position, Velocity, and Acceleration Functions

Introduction: Suppose that a particle positioned along the x-axis is moving in a linear path, and the position of the particle is given by the function $x(t) = -3t^3 + 6t^2$ where $x(t)$ is in feet per second. Find the velocity and acceleration of the particle at time $t = 5$ seconds. Describe the motion of the particle in the first 5 seconds.

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Neither the teacher nor the students will use the graphing calculator. All graphing will be done by hand. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

In the problem above, we were given the position of an object moving in a linear path, and we were able to find the velocity and acceleration. We can also find the position or displacement of an object and the distance traveled by the object given the velocity or acceleration function.

If the original function given is that of the position of an object or displacement, the derivative represents the ________________. (Velocity) The derivative of this function then represents the ______________. (Acceleration)

Therefore, if we are given the velocity and need the position or displacement, we must __________ (Integrate).

Integral ↑ Position / Velocity / Acceleration ↓ Derivative

Displacement = $\int_a^b velocity \ dt$

If the velocity is positive, the displacement is ______________.

If the velocity is negative, the displacement is ______________.

What is the difference between displacement and distance? (Distance is always positive & Distance represents the entire amount traveled including the part traveled forwards and backwards)

If you stand at your desk and walk 5 feet forward and 5 feet backwards, what is your displacement from your desk? (0 feet) What is the total distance that you traveled? (10 feet) When were you traveling with positive velocity? (When walking forward) When were you traveling with negative velocity? (When walking backward)

Distance = $\int_a^b |velocity| dt$

Example 1: Suppose that an object is moving in a linear path with a velocity of $v(t) = t^2 - 6t + 8$ in feet per second in the time interval $t = 0$ to $t = 3$.

a) Find the time subintervals in which the distance between the object and its starting point is increasing and decreasing.
b) How far does the object travel in each of these subintervals?
c) What is the total distance traveled by the object in the interval [0,3] and the displacement of the object after 3 seconds?

Example 2: Given the acceleration function $a(t) = -3t^2 \ (ft/s)/s$ and that the velocity at time $t = 0$ was 27 ft/s, find the displacement and distance traveled by the object moving in a linear path.

Example 3: Suppose that a car traveling down the road is accelerating for 20 seconds. The acceleration in miles per hour per second is measured every 2 seconds and is given in the table below. Determine approximately how far the car traveled during this time.

Time (s)	Acceleration ((mi/h)/s)

0	0.7
2	1.1
4	1.5
6	1.9
8	2.2
10	1.7
12	1.3
14	0.8
16	0.3
18	-0.1
20	-0.4

Homework Assignment: Assignment from Foerster's *Calculus Concepts and Applications*. Graphing calculators are not allowed on this assignment.

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Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

Suppose that the acceleration of a moving object is given by the function $a(t) = 3t^2$ and $v(1) = 7$ and $x(0) = 10$. Find the displacement of the object at t = 3 seconds, and find the distance that the object travels in this time interval.

Course: Calculus (Experimental Group) **Day:** 6

Unit: Applications of Integration **Lesson:** Distance and Displacement

Textbook: Foerster's Calculus Concepts and Applications

Aim: Students will be able to answer the question, "What are some of the applications of the definite integral, and how can we use the definite integral to find the distance and displacement of an object moving in a linear path?"

Goals/ Objectives:

- Given the velocity or acceleration function for an object in linear moving in a linear path, find the distance traveled in a given time and the displacement at a given time.

Standards: 2, 6 – 10

Materials: Graphing Paper, TI-84 Plus Graphing Calculator

Prerequisites: Velocity and Acceleration Functions by Derivatives, Relationship between functions and derivative functions, Relationship between Position, Velocity, and Acceleration Functions

Introduction: Suppose that a particle positioned along the x-axis is moving in a linear path, and the position of the particle is given by the function $x(t) = -3t^3 + 6t^2$ where $x(t)$ is in feet per second. Find the velocity and acceleration of the particle at time $t = 5$ seconds. Describe the motion of the particle in the first 5 seconds. *Use parametric mode on your graphing calculator.

Procedures: The teacher will present the following lesson for the students. The teacher will work out all of the examples with the students, while asking for student input. Both the teacher and the students will use both pencil and paper methods of graphing as well as the graphing calculator for graphing functions in function and parametric mode and for regression. Frequently throughout the lesson, questions are posed for the students. Responses to these questions are provided in parentheses.

In the problem above, we were given the position of an object moving in a linear path, and we were able to find the velocity and acceleration. We can also find the position or displacement of an object and the distance traveled by the object given the velocity or acceleration function.

If the original function given is that of the position of an object or displacement, the derivative represents the _______________. (Velocity) The derivative of this function then represents the _____________. (Acceleration)

Therefore, if we are given the velocity and need the position or displacement, we must __________ (Integrate).

Integral ↑	Position Velocity Acceleration	↓ Derivative

Displacement = $\int_a^b velocity\ \ dt$

If the velocity is positive, the displacement is _____________.

If the velocity is negative, the displacement is _____________.

What is the difference between displacement and distance? (Distance is always positive & Distance represents the entire amount traveled including the part traveled forwards and backwards)

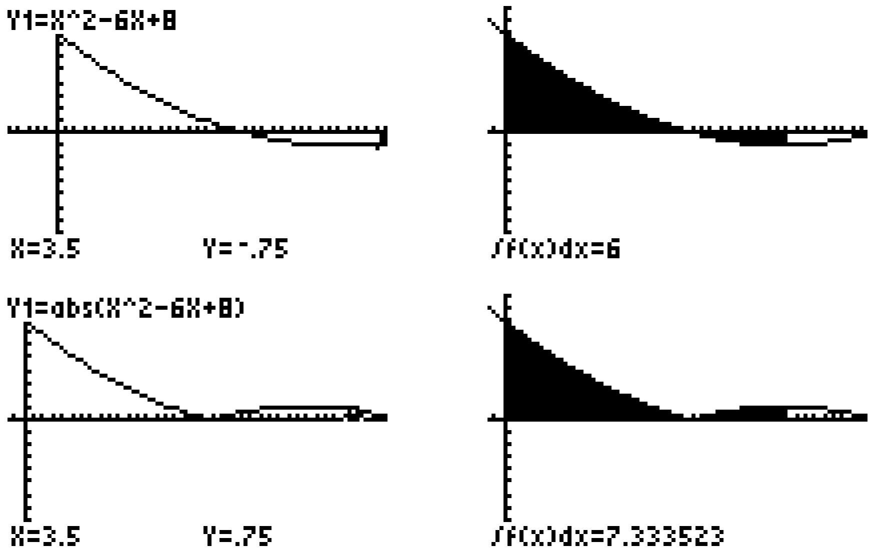
If you stand at your desk and walk 5 feet forward and 5 feet backwards, what is your displacement from your desk? (0 feet) What is the total distance that you traveled? (10 feet) When were you traveling with positive velocity? (When walking forward) When were you traveling with negative velocity? (When walking backward)

Distance = $\int_a^b |velocity|\, dt$

Example 1: Suppose that an object is moving in a linear path with a velocity of $v(t) = t^2 - 6t + 8$ in feet per second in the time interval $t = 0$ to $t = 3$. *(The teacher should demonstrate to students how to use the graphing calculator to view the function, to graph the absolute value function, and to view and compute the integral of the function and the absolute value function to find the distance and displacement).

d) Find the time subintervals in which the distance between the object and its starting point is increasing and decreasing.
e) How far does the object travel in each of these subintervals?

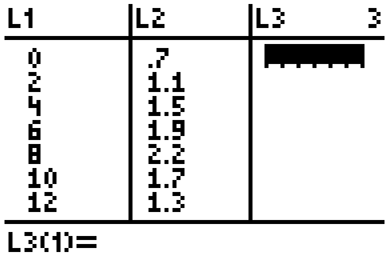
f) What is the total distance traveled by the object in the interval [0,3] and the displacement of the object after 3 seconds?

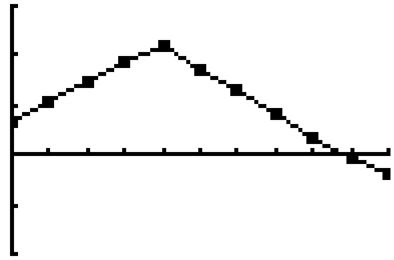


Example 2: Given the acceleration function $a(t) = -3t^2 \ (ft/s)/s$ and that the velocity at time $t = 0$ was 27 ft/s, find the displacement and distance traveled by the object moving in a linear path.

Example 3: Suppose that a car traveling down the road is accelerating for 20 seconds. The acceleration in miles per hour per second is measured every 2 seconds and is given in the table below. Determine approximately how far the car traveled during this time. *(The teacher should demonstrate to students how to use the regression capabilities of the graphing calculator to find an equation of the acceleration function and then use this equation to find the velocity, distance, and displacement).

Time (s)	Acceleration ((mi/h)/s)
0	0.7
2	1.1
4	1.5
6	1.9
8	2.2
10	1.7
12	1.3
14	0.8
16	0.3
18	-0.1
20	-0.4





Homework Assignment: Assignment from Foerster's *Calculus Concepts and Applications*. Graphing calculators are allowed on this assignment.

P.505 #1-13 odd

Accommodations for Individual Differences: The teacher will spend time after the lesson working with those students needing additional assistance, and the teacher will be available outside of class for these students as well. For those students who quickly master the concepts and need an additional challenge, the following question will be posed.

Suppose that the acceleration of a moving object is given by the function $a(t) = 3t^2$ and $v(1) = 7$ and $x(0) = 10$. Find the displacement of the object at t = 3 seconds, and find the distance that the object travels in this time interval.

Appendix I
Interview Script

Interview Script and Rubric

Hi. Thank you for doing this interview. As you know, I am working on my dissertation, and this is a big help to me. I'd like to ask you a few questions about problem solving and later I'll ask you to solve one of the problems that was on the posttest. At all times, it will be important that you talk out loud so that I can understand how you think through the problems.

1. How would you rate your level of math anxiety on a scale of 0 to 10, with 0 being a no math anxiety at all and 10 being a very high level of math anxiety? Why is it so high or so low?

	3	2	1	0
Math Anxiety	(0) No math anxiety, High Level of Comfort with Math, High Confidence when doing Math	(1 – 3) Very Little Math Anxiety, Comfortable with Math, Confidence in doing Math	(4 – 6) Moderate Math Anxiety, Comfort with Math is Dependent on Some Factor, Sometimes Confident in doing Math	(7 – 10) High Math Anxiety, Not Comfortable with Math, Lack of Confidence in Ability to do Math

2. In general, when you are doing math homework or taking a math test, how often do you use sketches or graphs to help you solve a problem?

	3	2	1	0
Use of Sketches and Graphs	Always uses sketches and graphs	Often uses sketches and graphs	Sometimes uses sketches and graphs	Never uses sketches and graphs

3. In general, when you are doing math homework or taking a math test, how often do you use your graphing calculator to help you solve a problem?

	3	2	1	0
Extent of Graphing Calculator Use	Always uses the graphing calculator	Often uses the graphing calculator	Sometimes uses the graphing calculator	Never uses the graphing calculator

4. In what ways, if any, do you typically use the graphing calculator on homework assignments and tests?

	3	2	1	0
Nature of Graphing Calculator Use	Use of multiple representations through graphing, tables, or data analysis	Use of at least one feature, such as graphing, tables, or calculate	Use of the computation feature only	Never uses the graphing calculator

Now I am going to give you one of the problems that you did on the posttest. Here is the question. (Hand student the question). This is the fourth question on the posttest. Please read the question. Is it clear to you what you have to do? As you are doing this problem, please talk out loud and tell me what you are thinking?

The question states, "Approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations below about the x-axis. Approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations below about the x-axis. $y = x\sqrt{4-x^2}, y = 0$."

Can you tell me what you are thinking? (The researcher will ask this question throughout the student solving the problem)

	3	2	1	0
Graphical Approach	Uses a graphical approach to solve the problem	Attempts to use a graphical approach but cannot interpret the graph to solve the problem	Does not use a graphical approach to solve the problem	Does not use any approach to solve the problem
Graphing Calculator (Experimental Group Only)	Uses the graphing calculator for multiple representations	Uses the graphing calculator for at least one representation or feature	Uses the graphing calculator for computation only	Does not use the graphing calculator

Observations about the use of graphs/graphing calculators:

____

____

____

Observations about the reasoning process used / use of graphs or graphing calculators in the reasoning process:

____
____
____
____

5. Which approach to problem solving do you believe was most helpful in solving this problem – analytic, numerical, or graphical? How did you use this approach?

	3	2	1	0
Approach to Problem Solving	Graphical Approach	Numerical Approach	Algebraic Approach	Cannot describe approach

6. How helpful was the graphing calculator to you in solving this problem? Explain.

	3	2	1	0
Extent of Use	extremely helpful, always helpful	fairly helpful, usually helpful	somewhat helpful, helpful at times	not helpful at all, never helpful
Nature of Use	student describes use of the calculator for viewing multiple representations	student describes use of the calculator for one feature	student describes use of the calculator for computation only	student states that graphing calculator was not used

7. (For those students who did not use the graphing calculator) How helpful do you believe the graphing calculator would have been in solving this problem? Explain.

	3	2	1	0
Extent of Use	extremely helpful, always helpful	fairly helpful, usually helpful	somewhat helpful, helpful at times	not helpful at all, never helpful
Nature of Use	student describes use of the	student describes use of the	student describes use of the	student states that graphing

	calculator for viewing multiple representations	calculator for one feature	calculator for computation only	calculator would not be used

Could you have solved the problem using a graphing calculator? If so, please do so.

8. Can you think of any mathematical concepts and relationships between concepts that are used in this problem?

	3	2	1	0
Analyzing a Problem	The student describes at least two relationships between mathematical concepts or terms. Includes a graphical approach.	The student is able to describe two relationships between mathematical concepts.	The student is able to describe a relationship between mathematical concepts.	The student cannot describe any concepts or relationships.

Observations about the use of graphs/graphing calculators:

____

____

____

9. Are there any conclusions that you can make at this point about the solution to the problem? If so, what would you say?

	3	2	1	0
Analyzing a Problem	Student is able to draw at least 2 correct conclusions about the answer to the problem.	Student is able to draw at least 1 correct conclusion about the answer to the problem.	Student draws an incorrect conclusion about the answer to the problem.	No response given

Observations about the use of graphs/graphing calculators:

____

____

____

10. What are some of the approaches that you could use to solve this problem?

	3	2	1	0
Initiating a Strategy	The student states at least 2 possible approaches.	The student is able to state one approach.	The student cannot think of any possible approaches.	N/A

Observations about the use of graphs/graphing calculators:

11. Do you think that the answer you got is reasonable? Why or why not?

	3	2	1	0
F: Reflecting on One's Solution	The student is able to decide whether the answer is reasonable and justify the answer.	The student is able to decide whether the answer is reasonable but cannot justify the answer.	The student cannot determine whether the answer is reasonable or justify the answer.	The student does not find an answer.

Observations about the use of graphs/graphing calculators:

12. Is there any other information you can gather from this problem or any other comments that you would make after solving it?

	3	2	1	0
Reflecting on One's Solution	Student provides other information, inferences, generalizations, or comments.	Student provides one other piece of information, inference, generalization, or comment.	Student attempts to provide another piece of information about the problem, but it is incorrect.	Student cannot provide any other information about the problem.

Observations about the use of graphs/graphing calculators:

Additional Observations:

	3	2	1	0
Monitoring Progress	The correct procedure is followed through OR the student begins with an incorrect procedure but realizes mistake and follows through using a new approach.	The student begins with an incorrect procedure, realizes mistake, and attempts another approach OR begins the correct procedure and solves a certain portion but does not follow through.	The student begins with an incorrect procedure and realizes mistake but does not attempt another approach.	The student uses an incorrect approach in solving the problem and does not show evidence of monitoring her progress or leaves blank.

	3	2	1	0
Seeking and Using Connections	Student connects this problem to previous knowledge in math before calculus or in other subjects in at least 2 ways.	Student is able to connect this problem to previous knowledge in math before or during calculus in at least 1 way.	No connections to other concepts or coursework given	N/A

Appendix J
School Administrator Observation Assessment

School Administrator Observation Assessment

I NAME OMITTED an administrator at a school participating in the study, "The Effects of Graphing Calculator Use on High School Students' Reasoning in Integral Calculus," conducted random observations during the administration of the pretests, post tests, and lessons taught by the researcher, Julie Spinato Hunter.

I found that the actions of the researcher in administering the pretest and post test were:

- [x] Fair and Unbiased
- [ ] Biased

I found that the researcher taught the two groups in a manner that was:

- [x] Fair, Unbiased, and Consistent, with the exception of graphing calculator use in the experimental group
- [ ] Biased or Inconsistent

X NAME OMITTED

School Administrator

Appendix K
Interview Transcriptions

Interview – Student C1

Researcher: Hi. Thank you for doing this interview. As you know, I'm working on my dissertation, and this is a big help for me. I'd like to ask you a few questions about problem solving a later I'll ask you to solve one of the problems that was on the posttest. At all times, it will be important that you talk out loud so that I can understand how you think through the problem. Are you ready?

Student C1: Yes

Researcher: First of all, how would you rate your level of math anxiety on a scale of 0 to 10, with 0 being no math anxiety at all and 10 being a very high level of math anxiety?

Student C1: A 5.

Researcher: Why is it so high or so low, do you feel?

Student C1: I stress out when I take tests for math. Regular classes are fine, it's just the tests.

Researcher: Okay, in general, when you are doing math homework or taking a math test, how often do you use sketches or graphs to help you solve the problem?

Student C1: I don't use them very often. Only when I absolutely need them.

Researcher: In general, when you are doing math homework or taking a math test, how often do you use your graphing calculator to help you solve a problem?

Student C1: I use it a lot, like.

Researcher: Would you describe it as never, sometimes, often, or always?

Student C1: Probably always.

Researcher: Thank you.

Researcher: Okay, in what ways, if any, do you typically use the graphing calculator on homework assignments and tests?

Student C1: For just normal computation and then to see the graphs of equations.

Researcher: Now I'm going to give you one of the problems that you did on the posttest. Here's the question. This is the fourth question on the posttest. Please read the question. Is it clear to you what you have to do?

Student C1: Yes

Researcher: Okay, as you are doing the problem, please talk out loud and let me know what you are thinking.

Student C1: Okay

Student C1: I am setting the two equations equal to each other to get the endpoints.

Researcher: Okay

Student C1: So I have 0, 2, and -2 for my intervals.

Researcher: Okay

Student C1: And then I'm going to set up my equation.

Student C1: I'm going to multiply pi times the integral from -2 to 2

Student C1: The integral of x times the square root of 4 minus x squared all squared from -2 to 0 and add it to the integral from 0 to 2 of x times the square root of 4 minus x squared, squared, oh, with dx behind both of them.

Researcher: Okay

Researcher: Can you tell what you're doing now or thinking now?

Student C1: Now I am simplifying the integrals themselves to get pi times the integral from 0 to -2 of x squared times 4 minus x squared dx plus the integral from 0 to 2 of the same equation, x squared times 4 minus x squared dx.

Researcher: Okay

Student C1: And then I'm going to simplify it even more. Pi times the integral from -2 to 0 of 4x squared minus x to the fourth dx plus the integral from 0 to 2 of 4x squared minus x to the fourth dx. And then integrate both of those. So pi times four thirds x cubed minus one fifth x to the fifth plus four thirds, oh, the first one from -2 to 0, and then plus four thirds x cubed minus one fifth x to the fifth from 0 to 2, and then I'm going to plug in the numbers from the intervals into the x's. So pi times four thirds, times 0, minus one fifth times 0, all minus four thirds times -2 cubed minus one fifth times -2 to the fifth plus four thirds times 2 cubed minus one fifth times 2 to the fifth. And then I'm going to simplify to get pi times negative 32 over 3 minus negative 32 over 5 plus 32 over 3 minus 32 over 5. And then I get 0…which is wrong.

Researcher: Okay, what are you thinking now?

Student C1: I'm thinking that I definitely got the answer wrong because it should not be 0.

Researcher: Okay. Well, is there anything else you want to do before you finish the problem?

Student C1: No. No.

Researcher: Which approach to problem solving do you believe was most helpful in solving this problem? Analytic, Numerical, or Graphical, and by Analytic, I also mean algebraic.

Student C1: Probably Analytic.

Researcher: And how did you use this approach?

Student C1: I started by setting them equal to each other…the two equations in order to get 0…the intervals…and then using the volume equation.

Researcher: Okay. How helpful was the graphing calculator to you in solving this problem?

Student C1: I actually only used it to find out what 2 to the fifth was.

Researcher: Okay. How helpful do you believe the graphing calculator would have been in solving this problem?

Student C1: I definitely could have used it to use the program for volume

Researcher: Okay

Student C1: About the x axis to get the right answer.

Researcher: Okay, would you say that it's not helpful at all, somewhat helpful, fairly helpful or usually helpful, or extremely helpful and always helpful.

Student C1: Probably extremely helpful.

Researcher: Okay. So could you have solved the problem using the graphing calculator?

Student C1: Yes.

Researcher: Okay. Can you do so?

Student C1: Do I speak out what I'm typing in?

Researcher: Yes

Student C1: Okay

Student C1: So I'm going to go to…okay I went to the program, then Calculus, the Calculus program. And I got to integral between curves and I'm gonna type in the first equation into f of x, x times the square root of 4 minus x squared. And then the second equation into g of x, and the lower bound I'm going to chose -2 and the upper bound 2. And I couldn't come up with anything because I think I did something wrong along the lines.

Researcher: Okay. Can you think of any mathematical concepts and relationships between concepts that are used in this problem?

Student C1: You can use the integral to find the volume of a solid when it's about either an axis or an equation.

Researcher: Okay. Are there any other concepts that you could describe? Concepts and relationships between concepts?

Student C1: The volume equation for a circle is found in, like within, the equation that we used to find the volume of the solid generated, when it's like revolved around something.

Researcher: Okay.

Researcher: Are there any conclusions that you can make at this point about the solution to the problem, and if so, what would you say?

Student C1: I would say that the solution is not 0, but it is probably a positive number. Yes.

Researcher: Okay.

Researcher: What are some of the approaches that you could use to solve this problem?

Student C1: You can use the volume equation with the pi times the integrals that with the equation squared, or you can use the calculator to find it.

Researcher: Okay. Thank you. Do you think that the answer you got is reasonable? Why or why not?

Student C1: No, I do not, because the volume should not be 0. There should be some volume.

Researcher: Is there any other information that you can gather from this problem, or any other comments that you would make after solving it?

Student C1: No, not really. I don't think so.

Researcher: Okay. And that's it. Thank you very much.

Student C1: Your welcome.

Interview – Student E1

Researcher: Hi, and thank you for doing this interview. As you know, I'm working on my dissertation, and this is a big help to me. I'd like to ask you a few questions about problem solving, and then later I'll ask you to solve one of the problems that was on the posttest. At all times, it will be important that you talk out loud so I can understand how you think through the problems. Are you ready?

Student E1: Yes ma'am.

Researcher: How would rate your level of math anxiety on a scale from 0 to 10, with 0 being no math anxiety at all, and 10 being a very high level of math anxiety?

Student E1: On this problem?

Researcher: Just in general, in math.

Student E1: I'd say it's about a 4 on a good day, 7 if it's been a long day.

Researcher: Okay

Student E1: Typically it's a five though. Typically.

Researcher: Typically a five, okay. Why is it so high or so low, do you feel?

Student E1: Math usually comes pretty easily to me, but when I'm learning new material, it can be a little tricky to get a hold of it.

Researcher: Okay

Researcher: In general, when you are doing math homework or taking a math test, how often do you use sketches or graphs to help you solve a problem?

Student E1: Almost never.

Researcher: Almost never, okay.

Student E1: Very rarely do I really need to draw a picture, except in physics.

Researcher: Okay. In general, when you are doing math homework or taking a math test, how often do you use your graphing calculator to help you solve a problem?

Student E1: Every problem.

Researcher: Every problem, okay.

Student E1: Every single one of them.

Researcher: Okay. In what ways, if any, do you typically use the graphing calculator on homework assignments and tests?

Student E1: What ways?

Researcher: Yes.

Student E1: Usually for general calculation, but if I am given a function, I do use it just because I like to see the graphs to see if I really need to apply it to anything.

Researcher: Okay. Now I'm going to give you one of the problems that you did on the posttest. Here's the question. Please read the question.

Student E1: Out loud?

Researcher: You can read it to yourself.

Student E1: Okay.

Student E1: Alright.

Researcher: Is it clear to you what you have to do?

Student E1: Yes ma'am.

Researcher: As you are doing the problem, please talk out loud and tell me what you're thinking.

Student E1: Okay. Well, I see that it's a y = problem, so I'm going to turn on my graphing calculator and plug it in. So, it's x square root of 4 minus x squared. And looking at the graph real quick. And I'm seeing that I'm going to have, when I set equal to 0 to find the interval that I need to find the volume, it's going to have a negative and a positive number. So, now I'm going to set the function equal to 0. And I know that because I have a x on the outside that x must equal 0. So, that's one of my first solutions. And then I set the square root of four minus x squared equal to 0, and I square both sides to get rid of the root, so I have four minus x squared equals 0. And, I'm just going to go ahead and say 4 minus both sides, multiply both sides by negative 1, will just give me x squared equals 4 and the square root of 4 is plus or minus 2.

Researcher: Okay.

Student E1: So, I know my interval is gonna be somewhere between, is gonna be -2 to 0 and 0 to positive 2. So I go ahead and I set up the problem, as from -2 to 0, x square root of 4 minus x squared, dx, equals and I'm going to leave that blank for now and then move on and set up my next problem, which is, from 0 to 2, x square root of 4 minus x squared, dx equals. I'm gonna leave that blank and then go ahead and plug that into my calculator. Put in my calculus problem, function program. It's the integral between curves, I'm gonna go ahead and say x square root of 4 minus x squared. And then for the second equation, put in 0. And for the lower bound, I'm first gonna put in 0 to 2 because I know that x square root of 4 minus x squared will always be above 0, so I'm gonna say 0 to 2. And find the volume of the circular revolution, horizontal line, y = 0, cause they ask me to revolve it around the x axis.

Researcher: What are you thinking right now?

Student E1: Well I'm thinking that this calculator takes a long time, to solve a problem, but it's told me that the volume from 0 to 2 is 13.404, and because it was the same problem and it looked like it was the same on both sides, from 0 to 2 and -2 to 0, I'm gonna assume that the answer to the other problem is also the same answer, but I can never really assume in math, so I'm gonna do it anyway.

Researcher: Okay.

Student E1: For the first function, I'm gonna plug in 0 and for the second function, I'm gonna actually plug in the function given, because I know it's below the 0 axis at that point, from -2 to 0. Volume of a circular revolution, horizontal line, y = 0. Now I'm wondering why it takes so long…Because it knows the answer or because it's animation. Who makes these programs? And again, the answer is 13.404. But I can't just leave it at that. I need to combine the answers to give me the actual volume. So, I close out the program and I'm gonna take those answers, take one answer and multiply it by two, and that's gonna give me 26.81 units cubed.

Researcher: Okay.

Student E1: And that'll be my answer.

Researcher: Okay, thank you.

Researcher: Which approach to problem solving do you believe was most helpful in solving this problem? Analytic or algebraic, numerical, or graphical? And how did you use this approach?

Student E1: What were my options?

Researcher: Analytic/Algebraic, Numerical, or Graphical.

Student E1: I think, now graphical would apply to my entire calculator or just looking at the graph?

Researcher: Either one.

Student E1: Then I'd say graphical, because, to be completely honest I don't remember all the steps to find the answer, so using the calculator definitely helped.

Researcher: Okay. How helpful was the graphing calculator to you in solving this problem? Explain.

Student E1: Very. It was very helpful to me because sometimes I forget the steps, so it helped in solving.

Researcher: Okay. Would you say that you used it for computation only, for one feature only, or for in multiple ways for multiple representations, or not at all?

Student E1: I used it in many ways. I used it to see the actual graph to make sure that all of my problems throughout working it were correct and my assumptions were correct as well. Then I used it in the program, for calculus, and I used it to combine the numbers just to make sure I didn't do silly math wrong.

Researcher: Thank you. How helpful do you believe the graphing calculator could have been in solving the problem other than the ways you used it?

Student E1: I think I used it to it's full potential. The only other way that it could have been helpful were if it just spat the answer out, which it pretty much did.

Researcher: Okay. In solving the problem, could you have used only the graphing calculator?

Student E1: Yes, I do believe that I could have.

Researcher: Could you show me?

Student E1: Well, to follow the steps that I did, I would simply look at the problem and I would assume that, I could figure out the intervals in my head from assuming that x equals 0, and then, um, I would just automatically see 4 square root of x squared and know that my answer was gonna be plus or minus 2, and then I would automatically know that those were my intervals. And then I would plug the equation into my graphing calculator and see what the answer, what the graph looks like, and then eventually, from there went on to my program and did it the exact way that I did.

Researcher: Thank you. Can you think of any mathematical concepts and relationships between concepts that are used in this problem?

Student E1: I'm sorry?

Researcher: Can you think of any mathematical concepts and relationships between concepts that are used in this problem?

Student E1: Oh, well I definitely see the relationships between integrals and derivatives, that the rate of one is the function of the other. And I see that the volume, though it only limits itself from -2 to 2, has a volume of 26.8. And I see the relationship between volume, and before I did this problem, I actually made the mistake on my posttest, of doing accidentally finding the surface area, and I saw the relationship between surface area and volume. It‘s a big gap, but I did see the relationship there.

Researcher: Okay. Are there any conclusions that you would make at this point about the solution to the problem? If so, what would you say?

Student E1: Well, I automatically assumed it would be in units cubed. I also knew that the answer to the problem was going to have to be added together because I would get two separate answers. And I think that those are all the assumptions that I could really make at that point.

Researcher: Okay. What are some of the approaches you could use to solve this problem?

Student E1: Solving for x of course. Knowing what the volume actually means, understanding that the x axis is y equals 0, knowing how to find the derivative and the integral, or in this case, just finding the integral. And I think that's about it.

Researcher: Okay. Do you think that the answer you got is reasonable? Why or why not?

Student E1: Yeah, I'd say it's reasonable, because if I know that the volume from -2 to 0 is 13, and I know that the volume from 0 to 2 is 13, then simply adding them together would give me my answer of 26.8.

Researcher: Okay. Is there any other information you can gather from this problem, or any other comments that you would make after solving it?

Student E1: Really the only comment that I would make is that the shape that it came out with was an hourglass figure, and I always try to keep that in mind whenever I'm doing these problems, is what kind of shape it makes, I don't know why, just does. And the only other comment would be that I'm a little unsure of just how accurate my intervals were, not that 2 was wrong, but more that, was the answer supposed to be from -2 to 2 or -2 to 0 and 0 to 2. So, that would be my only comment on that.

Researcher: Okay. Is that all?

Student E1: Yes ma'am.

Researcher: Thank you very much.

Interview – Student E2

Researcher: Well thank you very much for doing this interview. I'm working on my dissertation, and this is really a big help to me. I'm first going to ask you a few questions about problem solving and then later I'm going to ask you to solve one of the problems that was on the posttest. At all times, it's important that you talk out loud so I can understand how you think through the problems. So here's the first question. Are you ready?

(Student nods head)

Researcher: How would you rate your level of math anxiety on a scale from 0 to 10, with 0 being no math anxiety at all and 10 being a very high level of math anxiety?

Student E2: Probably about a 6.

Researcher: A 6. Ok. And why do you think it's so high or so low?

Student E2: Grades are really important to me. Not so much that it's math – it's just that math is hard. So, I'm trying to keep my grades up and I probably struggle the most with math out of all my classes.

Researcher: Ok. Thank you.

Researcher: In general, when you are doing math homework or taking a math test, how often do you use sketches or graphs to help you solve a problem?

Student E2: In calculus, just about every set of problems that I do. At least one of them, I have to use a graph or a sketch.

Researcher: Okay. In general, when you are doing math homework or taking a math test, how often do you use your graphing calculator to help you solve a problem?

Student E2: Pretty much every problem just so I don't have to slow down and do the mental math and stuff.

Researcher: Okay.

Student E2: Not always the big functions, but the simple stuff – multiplication and stuff.

Researcher: Right. Okay, well, that's kind of my next question. In what ways, if any, do you typically use the graphing calculator?

Student E2: Probably, probably 80, 85% of the time it's just like multiplication, fractions, just so you don't have to do that it your head.

Researcher: Okay, and now I'm going to give you one of the problems that you did on the posttest. And here's the question. Here's an extra sheet of paper if you need it. This is the fourth question that you did on the posttest. You can read the question to yourself.

(Student reading the question silently)

Researcher: Is it clear to you what you have to do?

Student E2: Yes.

Researcher: Okay. As you're doing the problem, please talk out loud and tell me what you're thinking.

Student E2: Alright, to start it, I'm just gonna graph. Can I use the calculator?

Researcher: Sure.

Student E2: I'm just gonna graph it and sketch the graph.

(Student entering the function into the calculator and drawing the graph on paper)

Student E2: I'm gonna revolve it to try and make a solid.

Researcher: Okay.

Student E2: I'm gonna connect – this is a tip my teacher gave us – you connect the open ends to make it like a cone or whatever shape you're trying to find.

Researcher: Okay.

Student E2: Then gonna try to determine my bounds, which from this it looks like -2 and 2. I'm gonna set up my integration – with the equation.

Student E2: And then use a calculator to integrate to save some time.

Researcher: Okay.

Student E2: We do them by hand sometimes and sometimes she lets us do this. It just depends on exactly what she's trying to get us to, what she's trying to test us on.

(Student working with calculator to integrate the function)

Researcher: And how are you using the calculator to integrate?

Student E2: I'm using the math 9 function and

Researcher: Okay.

Student E2: Type in your bounds and then the equation, and

Researcher: Okay

Student E2: Pretty much, it just, it plugs in and integrates for you, that way you don't have to do all the by hand stuff.

Researcher: Okay.

Student E2: And it gives me 0 because half of the graph is above the x-axis and half is below it, so when you find the volume, half is positive, half is negative, so it cancels out and gives you 0.

Researcher: Okay.

Researcher: Alright. Which approach to problem solving do you believe was most helpful in solving this problem? Do you think that it was the analytic, or numerical, or graphical approach?

Student E2: Probably graphical, I find, I think everybody in the class kind of had their own preference, but for me it was always the graphs and if I can look at it, I can typically figure it out and at least get close, you know. (Train in background) Might not always get the right answer but I'm better at looking at something so I like to graph every problem, that way I can see what I'm trying to find before I try to find it.

Researcher: Alright. And how helpful was the graphing calculator to you in solving this problem? Was it extremely helpful, fairly helpful, somewhat helpful, or not helpful at all?

Student E2: I would probably say extremely helpful, because in the time it would have took me to graph that by hand just thinking about it, it would have probably took me, I'm not that great, but it would have probably took me 4 or 5 minutes to be like alright this goes here, and with the calculator I can just punch in the equation.

Researcher: Okay.

Student E2: And see instantly.

Researcher: Thanks. And would you say that you used the calculator for computation only or in what ways did you specifically use the calculator?

Student E2: For this problem, I pretty much took the easy way out with it because I graphed the equation with it and then, instead of doing an integral by hand and then plugging in, the bounds, I just typed it in the calculator, skipping that step by hand to make it quicker and easier.

Researcher: Okay. How helpful do you believe the graphing calculator would have been in solving this problem other than the ways that you used it?

Researcher: Could it have been, extremely helpful, fairly helpful?

Student E2: I think I pretty much used it just about as much as I could.

Researcher: Okay.

(student laughing)

Researcher: Can you think of any mathematical concepts and relationships between concepts that are used in this problem?

Student E2: I mean obviously integration and graphing, and in this particular problem there wasn't much simple algebra, but if I had done it by hand instead of using the calculator, it would've been some simple algebra like adding like terms and things like that.

Researcher: Okay. And are there any conclusions that you can make at this point about the solution to the problem? If so, what would you say?

Student E2: Looking at the graph, it kind of, I like to look at the graph after you get your answer to see if it looks reasonable, and being the graph pretty much looks like it's exactly equal from the top of the axis to the bottom, it kind of makes 0 make a lot of sense.

Researcher: Okay. And what are some of the approaches that you could use to solve this problem?

Researcher: Any different approaches?

Student E2: Graphing and integration, I guess, and simple algebra if I had done it by hand.

Researcher: Do you think that the answer you got is reasonable? Why or why not?

Student E2: Yeah, I think it's reasonable because like I said, looking at the graph, it just, if it wouldn't be 0, it would probably be a really, really small like decimal number.

Researcher: Okay.

Student E2: So 0 like seems good.

Researcher: Alright, and one last question. Is there any other information that you can gather from this problem or any other comments that you would make after solving it?

Student E2: If they had given different bounds, you could've came out with a different answer, but being, being no bounds were given, you're pretty much finding the volume of the entire graph.

Researcher: Okay, and is that it?

Student E2: Yep.

Researcher: Alright, thank you very much.

Student E2: Alright. Hope it was helpful.

Interview – Student C2

Researcher: Well, first of all, thank you very much for doing this interview.

Student C2: No problem. (student laughing)

Researcher: I'm working on my dissertation, and this is really a big help to me. I'd first like to ask you a few questions about problem solving and then later I'll ask you to solve one of the problems that was on the posttest. At all times, it's going to be important that you talk out loud so I can understand your thinking process.

Student C2: Okay.

Researcher: Alright. Are you ready?

Student C2: Yes. (Laughing)

Researcher: How would you rate your level of math anxiety on a scale from 0 to 10, with 0 being no math anxiety at all and 10 being a very high level of math anxiety?

Student C2: I'd rate it probably like at a 5. It depends on whatever the topic is or what we're learning. But most of the time, I can understand like what's going on.

Researcher: Okay. So, why would you say it's so high or so low?

Student C2: I don't know. I just stress about my grades. Like I like to do well in the class, so that would probably be the only reason why – not being how difficult or easy the subject is.

Researcher: Okay. In general, when you are doing math homework or taking a math test, how often do you use sketches or graphs to help you solve a problem?

Student C2: Not often. I do whenever the problem requires you to draw the graph. Sometimes I'll use the graph without them telling but not often.

Researcher: Okay. In general, when you are doing math homework or taking a math test, how often do you use your graphing calculator to help you solve a problem?

Student C2: Probably half the time. Normally I'll do things by hand more than the graphing calculator but when it's harder things I'll use the calculator more.

Researcher: Okay. In what ways, if any, do you typically use the graphing calculator, when you do use it for homework assignments and tests?

Student C2: Probably for the graphs, like if I don't know what a function looks like, I'll type it in and see what the graph looks like if I need to use it.

Researcher: Okay.

Student C2: That's it.

Researcher: Alright. Now I'm going to give you one of the problems that was on the posttest.

Student C2: Uh huh.

Researcher: And here's the question. This is the fourth question that was on the posttest. You can read it to yourself.

Student C2: Okay.

(student reading question silently)

Researcher: Is it clear to you what you have to do?

Student C2: Yes ma'am.

Researcher: Okay. As you're doing the problem, just talk out loud and tell me what you're thinking.

Student C2: Okay, well first I'm gonna draw the graph to see what it looks like.

(Student C2rawing graph on paper)

Student C2: And it's gonna come over here, it's gonna be the, a disk, right?

Researcher: Okay.

Student C2: The graph's gonna be a square root, and it's gonna look like that, and then once you reflect it, it'll come out to look something like that, which would be a disk.

Researcher: Okay.

Student C2: And to solve it you're going to use pi r squared and you take the integral of that because once you would cut the disk into shapes it would form circles. And r would be the equation they give you, which is x square root of 4 minus x squared.

Researcher: Okay.

Student C2: And you integrate that. You'd have to simplify it first.

(student simplifying the expression inside the integral)

Student C2: So before you would integrate, you would get 4x squared minus x to the fourth, and you would do the bounds. (Student looking at problem seeming confused)

Researcher: What are you thinking?

Student C2: I was wondering whatever you have to plug in, like after you integrate the equation, you would have to plug in the x bounds that they would give you but.

Researcher: Okay.

Student C2: So integrating that would give - (student integrating the function and speaking quietly to herself)

Researcher: What are you thinking now?

Student C2: I'm trying to figure out what to plug in for the bounds.

Student C2: Oh, I'm going to set the two equations that they give you equal.

(student finds the bounds algebraically)

Student C2: You're bounds would be from -2 to 2.

Student C2: And then you use the fundamental theorem of calculus to plug in.

(student working problem on paper)

Researcher: And what is it that you're doing now?

Student C2: And gonna plug in my calculator to simplify it and then leave the pi and add that in later.

Researcher: Okay.

(student simplifying final answer using the calculator and pencil and paper)

Student C2: And I have 128 pi, well 128 over 15 pi would be my answer.

Okay. Thank you. Which approach to problem solving do you believe was most helpful in solving this problem? Would you say that it's analytic, numerical, or graphical?

Researcher: And by analytic, I mean algebraic, and then numerical or graphical.

Student C2: I'd say graphical and numeric, because you have to know what the graph looks like to be able to solve it before you can plug in the numbers and solve the equation.

Researcher: Okay. How helpful was the graphing calculator to you in solving this problem?

Student C2: It really wasn't much – I mean I just used it to simplify the numbers. I could've done that by hand but just to make it quicker. And I already knew that the graph was a circle, so.

Researcher: And how helpful do you believe the graphing calculator would have been in solving the problem or could have been?

Student C2: If I didn't know what the graph already looked like, it could show me that, and if it was a more difficult problem, I'd have to plug it in to see the graph.

Researcher: Okay. And could you have solved the problem using a graphing calculator?

Student C2: Yes.

Researcher: Okay, can you do so?

Student C2: Yes. You would plug in the (plugging function into the calculator) and the integral. My calculator is different from this.

Researcher: Oh okay.

Student C2: I'm trying to find the little.

Researcher: And what are you doing right now?

Student C2: I'm plugging in my equation once I've simplified before I took the integral, and then I plug in the bounds, if I can figure out how to type it in. Mine is different.

Researcher: Okay.

Student C2: Okay. And I got the same answer without the pi.

Researcher: Okay. Thank you. Can you think of any mathematical concepts and relationships between concepts that are used in this problem?

Student C2:Well you use the formula pi r squared that's the area of a circle, which is also used in geometry. So.

Researcher: Okay. Are there any conclusions that you can make at this point about the solution to the problem? If so, what would you say?

Student C2: Well the answer is positive so I would assume that the volume is positive.

Researcher: Alright, and what are some of the approaches that you could use to solve the problem?

Student C2: Well I would look at the graph first, honestly, because once they're asking for volume, they're gonna want to know the shape of it, and you're going to have to figure out what the 3D shape of the graph would be.

Researcher: Okay. Do you think that the answer you got is reasonable? Why or why not?

Student C2: I think it's somewhat reasonable. I mean I'm not sure if I did it entirely correct, but based off of the information, I think it's reasonable.

Researcher: Okay. And I have one last question. Is there any other information that you can gather from the problem or any comments that you would make after solving it?

Student C2: By information, what do you mean?

Researcher: Anything that you can infer from this problem or just any general comments, and just anything else that you wanted to add.

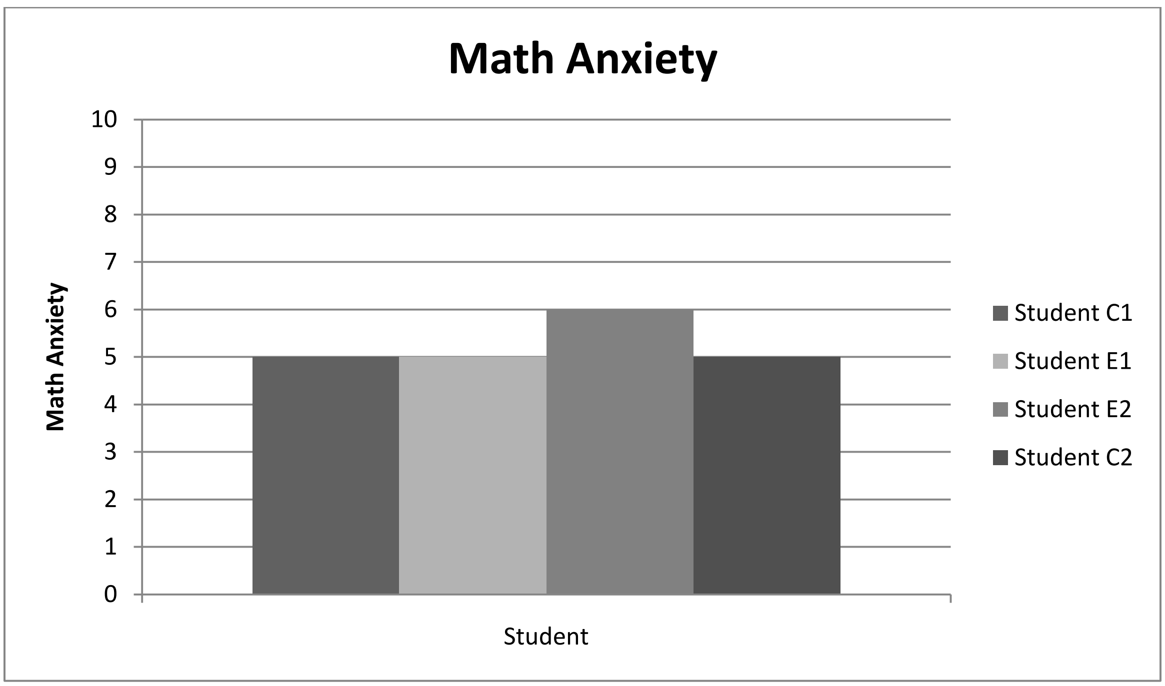
Student C2: No, I think once I look at it again, it tells you everything you need to solve. It gives you the information and you should be able to solve it if you know the concept and how to approach it.

Researcher: Okay, thank you very much. That’s it.

Student C2: Thank you.

Appendix L
Math Anxiety

Students were asked, “How would rate your level of math anxiety on a scale from 0 to 10, with 0 being no math anxiety at all, and 10 being a very high level of math anxiety?” They responded similarly, with students C1, E1, and C2 each responding that their levels of math anxiety are a 5, and student E2 responding that his level of math anxiety is a 6, on a scale of 0 to 10. Three out of four of these students, students C1, E1, and C2, believe that their confidence in math is dependent upon some factor. For student C1, this factor is taking a math test, for student E1, this factor is the idea of learning new material, and for student E2, math anxiety depends upon the particular topic. All of the students interviewed have a moderate amount of math anxiety.



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The author was born in New Orleans, Louisiana. She graduated *Magna Cum Laude* with a Bachelor of Science in Mathematics from Loyola University in 2005. She then received a Master of Education from the University of New Orleans. She then went on to pursue her doctoral degree at the University of New Orleans in 2011. She currently holds a position as a calculus teacher at a private high school.