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A Nonlinear Ship Wave Solution Method

Li Chen
lchen8@uno.edu

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Li Chen

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Abstract

Wave resistance on the hull is a major part of the total resistance. In ship design, we want to predict ship wave resistance and the wave pattern for a proper hull form at different velocities. This thesis is an attempt to build up the numerical model for a nonlinear Boundary Element Method similar to RAPID method of Hoyte C. Raven using Wigley hull as the test model of the program.

The RAPID method is known to have a higher accuracy than linearised codes because it closely models the full nonlinear free surface. These computations are carried out via an iterative procedure with convergence criteria using the residual error in the boundary conditions. This method is known to be superior to the convergence criteria of other nonlinear methods which only check that changes in the free surface become small. Comparisons between the linear and nonlinear codes show the nonlinear method to give better wave resistance results.

Key words: RAPID method, Boundary Element Method, nonlinear, potential flow, source

Chapter 1 Introduction

During Fall semester 2012, I participated in a graduate level course NAME 6160 Numerical Methods in Hydrodynamic offered by Dr. Lothar Birk, in Naval Architecture and Marine Engineering. That is the first time I get inside the world of Computational Fluid Dynamic(CFD). In our problem, we are using Boundary Element Method (BEM) in solving linearised free surface potential flow problem. The advantage of using BEM is change a three dimensional problem into a two dimensional problem. This method will save a lot of memory we are using in computer in solving the problem. And it is pretty amazing when we see the result of the ship wave propagation.

In practical, the resistance of a ship in still water is mainly consider aspect in ship design. Wave resistance takes over from 10 to 60 % of the total resistance of a ship in still water in the practical cases. While at relatively low speeds the wave resistance is actually zero, as the speed growing up it increases very quickly. A good CFD program can reduce 2% to 3% of the total resistance, that means we can save a lot of the energy during the life time of a ship. As we want to calculate for a more accurate result, I decided to further my study in Hoyte C. Raven's RAPID method, which is nonlinear Boundary Element Method and also a commercial CFD program in Maritime Research Institute Netherlands (MARIN).

We are using Fortran as the programming language in solving the problem. As Fortran is very stable and still very powerful in solving numerical problems. The concept of nonlinear method is based on linear method, except we are updating the parameter we are calculating in each iteration. In the following, we will discuss the theory and the concept in detail. Chapter 2 is the theoretical background of the problem. We eliminate some insignificant phenomena. Then use the mathematical theory we derived on class to transform our problem into a specific numerical problem. Detail derive of the free surface boundary condition will be available in this chapter

In Chapter 3 we describe how to build up the numerical model using the mathematical background we describe in the Chapter2.

We will discuss in Chapter 4, the nonlinear method developed from the RAPID method of Rave in detail. In this chapter we will discuss about the process of the program and also for the nonlinear problem the most important step the converge criteria.

Chapter 2 Theoretical Background

2.1 Limitation of the problem

We set the coordinate system fixed to the ship hull in still water. The ship is supposed to move straight forward with constant speed. Compare to the real situation, we have neglect some of the aspects. From Raven (1996).

Because different phenomena are governed by multiple physical laws and their theoretical prediction requires quite different mathematical models. For precise prediction methods, we split different phenomena into different mathematical problems. In our discussion here, we will neglect some of the phenomena and it presents as followed:

First of all, the propulsion effects will be disregarded. As the modeling of the propulsion effects are quite different from that we dealt with here, and the situation we are modeling here is consider the ship being towed. The neglect of propulsion may affect the prediction of stern waves, trim and sinkage.

Furthermore, we neglect the wave breaking effects. The physical description of wave breaking in present is still not completed, and no direct or indirect treatment of wave breaking is available for the problem considered. Fortunately only some of the case will have wave breaking in practical, though almost always present, it has relatively minor effect on the problem.

For similar reasons the spray effects will be neglected. Again, for most cases the neglect of spray will also have minor effect on the problem we discuss here.

The trim and sinkage are let out of account. As we are not going to consider the dynamic effect on the ship hull.

Viscous effects will provide little problem for slender hull which we discuss here, but we will have to consider viscous effects for transom stern ship hull.

Finally, surface tension effects will be disregarded, being insignificant for full-scale ships.

After we limit the problem into a certain area, we will use the basic equation as followed.

2.2 Basic Equation

As the continuity equation (conservation of mass) and Navier - Stokes Equations (momentum equations) are the basic fluid mechanic equation we are using here. We will briefly describe these equations.

Starting with the continuity equations we have

$$\frac{\partial}{\partial t}\iiint_V \rho \mathrm{dV} + \iint_S \rho \underline{v}^T \mathrm{d}\underline{S} = 0 \tag{2-1}$$

$$\frac{D}{Dt}\iiint_V \rho \mathrm{d}V = 0 \tag{2-2}$$

$$\frac{\partial \rho}{\partial t} + \underline{\nabla}^T (\rho \underline{v}) = 0 \tag{2-3}$$

$$\frac{D\rho}{Dt} + \rho \underline{\nabla}^T \underline{v} = 0 \tag{2-4}$$

In the equations, $\underline{\mathrm{v}}$ is the flow velocity vector. And equations above are the continuity equation in different forms. For equation (2-1) is the integral, conservation form, for finite control volume V fixed in space. Equation (2-2) is the integral, non-conservation form, for finite control volume moving with the fluid. Equation (2-3) is the differential, conservation form, for infinitesimal fluid element fixed in space. Equation (2-4) is the differential, non-conservation form, for infinitesimal fluid element moving with the fluid. The detail derive of the equation will be available in Birk(2012).

Assuming the fluid is incompressible, with constant density $\rho = const$, we get the continuity equation (2-3) for incompressible flow

$$\underline{\nabla}^T \cdot \underline{v} = 0 \tag{2-5}$$

For the Navier-Stokes equation, let us consider the flow temperature in the fluid to be constant. And the dynamic viscosity of water μ is largely depended on the temperature, therefore we can consider μ to be a constant. Then the Navier-Stokes equation becomes

$$\rho\left(\frac{\partial v}{\partial t} + (\underline{v}^T \cdot \underline{\nabla})\underline{v}\right) = \rho \underline{\mathrm{f}} - \underline{\nabla}\mathrm{P} + \mu \Delta \underline{v} + \frac{1}{3}\mu \underline{\nabla}(\underline{\nabla}^T \cdot \underline{v}) \tag{2-6}$$

Now we substitute the continuity equation (2-5) into equation (2-6) yields

$$\frac{\partial \underline{v}}{\partial t} + (\underline{v}^T \underline{\nabla})\underline{v} = \underline{\mathrm{f}} - \frac{1}{\rho}\underline{\nabla}\mathrm{P} + \nu \Delta \underline{v} \tag{2-7}$$

in the equation ν is the kinematic viscosity and kinematic viscosity $\nu = \frac{\mu}{\rho}$. The last term of equation (2-6) vanish as we substitute the incompressible continuity equation.

If we assume the fluid to be inviscid, as we are considering slender ship hull, with viscosity μ $= 0$. Then equation (2-7) becomes

$$\frac{\partial \underline{v}}{\partial t} + (\underline{v}^T \underline{\nabla})\underline{v} = \underline{\text{f}} - \frac{1}{\rho}\underline{\nabla}\text{P} \quad (2\text{-}8)$$

Equation (2-8) is known as Euler equations. We can obtain another useful form of the Euler equations by rearranging the convective acceleration term $(\underline{v}^T \underline{\nabla})\underline{v}$, from Birk(2012).

$$(\underline{v}^T \underline{\nabla})\underline{v} = \underline{\nabla}\left(\frac{1}{2}\underline{v}^2\right) - \underline{v} \times (\underline{\nabla} \times \underline{v}) \quad (2\text{-}9)$$

Substituting the rearranged convective acceleration term (2-9) into the Euler equations (2-8) we get

$$\frac{\partial \underline{v}}{\partial t} + \underline{\nabla}\left(\frac{1}{2}\underline{v}^2\right) - \underline{v} \times (\underline{\nabla} \times \underline{v}) = \underline{\text{f}} - \frac{1}{\rho}\underline{\nabla}\text{P} \quad (2\text{-}10)$$

We will assume the flow in the fluid to be irrotational, then for irrotational flow the curl of the velocity becomes $\underline{\nabla} \times \underline{v} = 0$. Then the Euler equation becomes

$$\frac{\partial \underline{v}}{\partial t} + \underline{\nabla}\left(\frac{1}{2}\underline{v}^2\right) = \underline{\text{f}} - \frac{1}{\rho}\underline{\nabla}\text{P} \quad (2\text{-}11)$$

After we compute the velocity field in the fluid, we will use equation (2-11) to compute for the pressure distribution.

2.3 The Laplace Equation

In physics it was discovered that certain force fields have a special property. The work done to move a body from point P_0 to point P_1 is independent from the path taken, see Fig 2.1. These force field are called conservative and the work done may be computed by computing the difference in potential between point P_0 and point P_1. The potential is a scalar function representing the conservative vector field. A simple example: [E = -*gz*] is the potential of the gravity field. In the equation, *g* is the gravitational acceleration and *z* is the distance measured from the earth center. The work *W* per unit mass needed to move a body from $P_0 = (x_0, y_0, z_0)$ to $P_1 = (x_1, y_1, z_1)$ is $W = E_1 - E_2 = -g(z_1 - z_0)$, from Birk (2012).

The sufficient condition for a force field $\underline{\text{f}}$ to be conservative, for example, if we want to have a potential, we can have $\underline{\nabla} \times \underline{\text{f}} = \underline{0}$ only if *rot* $\underline{\text{f}} = \underline{0}$.

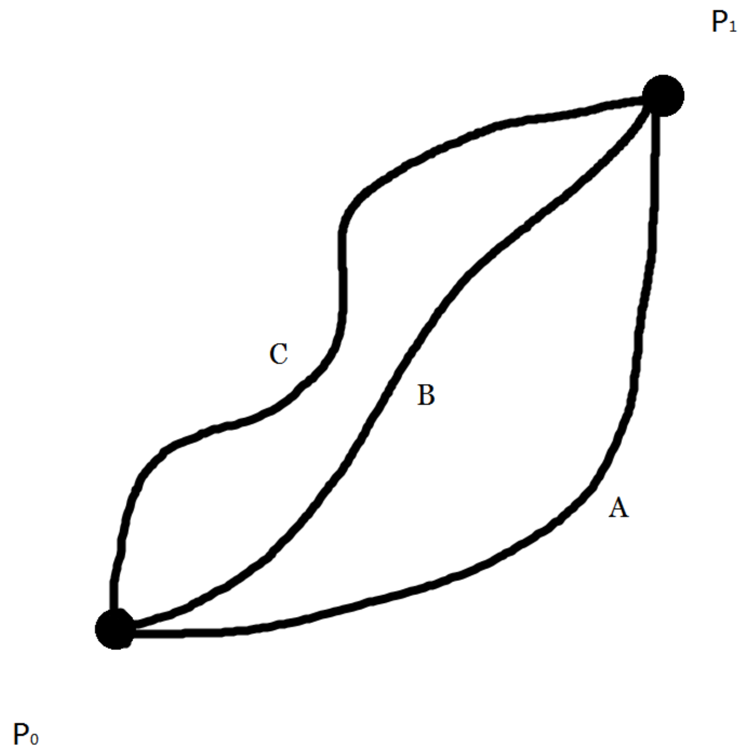

Fig 2.1: Potential force field

To be precise $rot\ \underline{f} = \underline{0}$ has to be enforced only in simply connected regions. A simply connected region is a region where all closed curves are reducible. For example, the region can be contracted to a point without leaving the region, see Fig 2.2.

We apply the concept of a potential to the velocity field of an irrotational flow. For a flow with $rot\ \underline{v} = \underline{0}$ in simply connected regions a velocity potential Φ exists with $\underline{v} = \underline{\nabla}\Phi$. In the equation Φ is a scalar function and its gradient represent the velocity field.

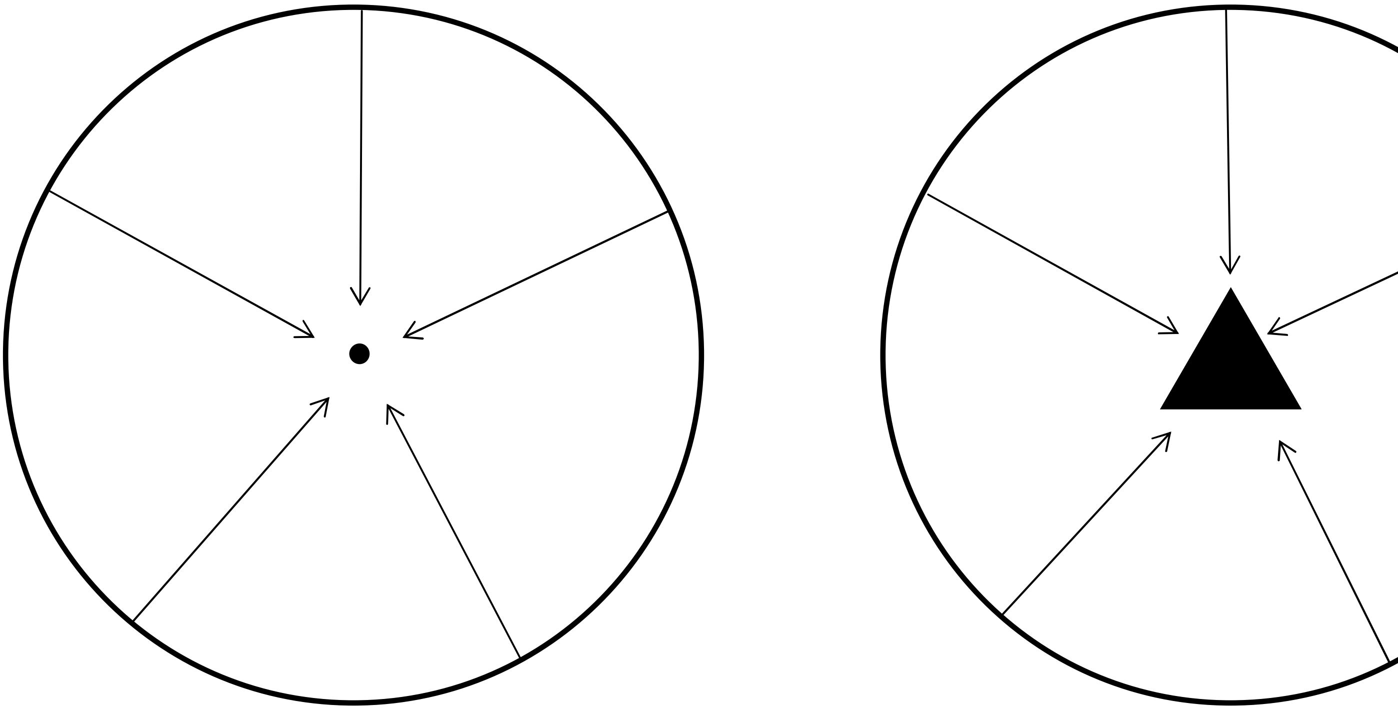

Fig 2.2: Different connected region

If we assume the flow is irrotational flow $rot\ \underline{v} = \underline{0}$, as we select the region around the object but not include the object, we have

$$\underline{v} = \underline{\nabla}\,\Phi \tag{2-12}$$

Now we can recall the continuity equation (2-5) for incompressible flow we described earlier

$$\underline{\nabla}^T(\underline{v}) = 0 \tag{2-13}$$

We substitute continuity equation(2-13) into potential equation (2-12), then the equation becomes

$$\underline{\nabla}^T(\underline{\nabla}\Phi) = 0 \tag{2-14}$$

$$\underline{\nabla}^2\Phi = 0 \tag{2-15}$$

$$\Delta\Phi = 0 \tag{2-16}$$

Now we have the Laplace equation (2-16) and the Laplace equation is Cartesian coordinates becomes

$$\Delta\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{2-17}$$

The Laplace equation is linear and partial differential equation of second order, and it is a special case for continuity equation in inviscid, incompressible and irrotational flow.

2.4 The Boundary Condition

In real case we are considering about using potential theory in different part of the domain. For instance, it can consist of a submerged body moving through a fluid with constant speed, for which we want to know the pressure distribution around the body. Other problem may include the interaction of a floating body with the free surface as it moving forward with constant speed and we want to know the resistance of the body during the movement.

We will use potential flow theory to solve the problems described above, which are in essence, two variations of one single general problem. Recall that potential flow implies incompressible, inviscid and irrotational flow. We will use the continuity equation (2-13) expressed in terms of the velocity potential, or Laplace equation (2-16) to solve the problem.

2.4.1 Ship Hull Boundary Condition

For the fully submerged body we required that no fluid flows through the body surface. In the coordinate system fixed to the ship hull, we have

$$\frac{\partial \phi}{\partial n} = 0 \tag{2-18}$$

This equation is using on the ship hull body surface S_B, □ is the total flow potential and $\underline{n}$ is the normal vector pointing into the ship hull, from Birk (2012).

The total flow potential □ is combined by the base potential Φ and a perturbation potential φ', which we will discuss in section 2.6.

Also the disturbance created by the motion should decay far from the ship hull body. If r is the distance between the field point we are measuring the motion to the ship hull body, r will tend to be infinity.

$$\lim_{r \to \infty} (\underline{\nabla} \phi - \underline{v}) = 0 \tag{2-19}$$

where $r=(x, y, z)$ and $\underline{v}$ is the relative velocity between the undisturbed fluid in V and the ship hull body.

2.4.2 Free Surface Boundary Condition

Free surface represents a stream surface, however the surface is unknown shape. We do not yet know the wave system shape. Let us describe free surface as an implicit function.

$$F(x, y, z) = 0 \tag{2-20}$$

We select:

$$F(x, y, z) = z - \eta(x, y) = 0 \tag{2-21}$$

In equation (2-21), η is the wave elevation. As mentioned before, we eliminate the breaking waves, so choice of $z - \eta(x, y) = 0$. The gradient of equation (2-21) is

$$\underline{\nabla} F = \begin{pmatrix} -\frac{\partial F}{\partial x} \\ -\frac{\partial F}{\partial y} \\ 1 \end{pmatrix} = \begin{pmatrix} -\frac{\partial \eta}{\partial x} \\ -\frac{\partial \eta}{\partial y} \\ 1 \end{pmatrix} \tag{2-22}$$

On the free surface, the flow velocity must be tangential to the free surface, from Raven(1996). There is no flow through free surface, then the normal velocity on S_F vanishes.

$$\underline{n}^T \underline{\nabla} \phi = 0 \tag{2-23}$$

As the normal vector of the free surface is vertical to the free surface. The normal vector of an implicit free surface is as followed

$$\underline{n} = \frac{\underline{\nabla} F}{|\underline{\nabla} F|} \tag{2-24}$$

We have the define of normal vector, let us substitute equation (2-24) into (2-23), we have

$$\left(\frac{\underline{\nabla} F}{|\underline{\nabla} F|} \right)^T \bullet \underline{\nabla} \phi = 0 \tag{2-25}$$

Substituting equation (2-22) into (2-25), then the equation becomes

$$-\eta_x \phi_x - \eta_y \phi_y + \phi_z = 0 \tag{2-26}$$

Then we have the kinematic boundary condition (2-26) on $z = \eta(x, y)$.

Dynamic free surface boundary condition: It is defined by the Bernoulli's law that the pressure related the velocities and the wave elevation must be constant at the free surface, from Raven (1996).

The Bernoulli equation is as followed

$$P_\infty + \frac{1}{2} \rho u_b^2 = P_\infty + \rho g z + \frac{1}{2} \rho |\underline{\nabla} \phi^2| \tag{2-27}$$

As the water density in the equation is considered to be constant, equation (2-27) yields

$$\frac{1}{2}\left(u_b{}^2-\phi_x^2-\phi_y^2-\phi_z^2\right)-gz=0 \tag{2-28}$$

Equation (2-28) is the Dynamic free surface boundary condition on $z = \eta(x, y)$.

Radiation condition: some care is to be taken for the behavior at infinity. Dimply requiring decay of the disturbance with distance from the body is not appropriate. The desired solution is that which includes only the waves generated by the ship, which roughly speaking are found downstream of the bow.

2.5 General solution of exterior flow problem

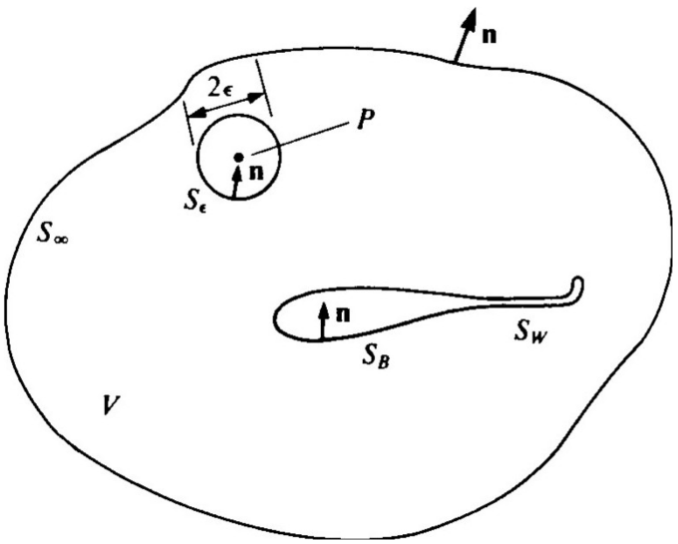

Fig 2.3: Nomenclature used to define the potential flow problem

From Katz(2001), Laplace's equation for the velocity potential must be solved for an arbitrary body with boundary S_B enclosed in a volume V, with the outer boundary of the region we consider S_∞ , in Fig. 2.3. The boundary conditions in equation (2-18) and (2-19) apply to S_B and S_∞. The normal vector we consider here is pointing out of the volume *V.* Now, the vector appearing in the divergence theorem is replaced be the vector $\phi_1\underline{\nabla}\phi_2-\phi_2\underline{\nabla}\phi_1$, where □$_1$ and □$_2$ are two scalar functions of position. This results in

$$\iint_S \left(\phi_1\underline{\nabla}\phi_2-\phi_2\underline{\nabla}\phi_1\right)^T \square\, \underline{n}\,\mathrm{d}S=\iiint_V \left(\phi_1\underline{\nabla}^2\phi_2-\phi_2\underline{\nabla}^2\phi_1\right)\mathrm{d}V \tag{2-29}$$

Equation (2-29) is one of Green's theorem. Here the surface integral is taken over all the boundaries S, including boundary of the infinite small sphere S_ε.

$$S = S_B + S_\infty + S_\varepsilon \tag{2-30}$$

Also we set

$$\phi_1 = \frac{1}{r} \quad and \quad \phi_2 = \phi \tag{2-31}$$

where ϕ is the potential of the flow we are interested in *V*, and *r* is the distance from a point *P(x, y, z)*, as shown in Fig 2.3. We shall see later, ϕ_1 is unbounded (1/r→∞) as *P* is approached as $r \rightarrow 0$ and is the potential of a source. In the case where the point is outside of *V* both ϕ_1 and ϕ_2 satisfy Laplace's equation and equation (2-29) becomes

$$\iint_S \left(\frac{1}{r} \underline{\nabla} \phi - \phi \underline{\nabla} \frac{1}{r} \right)^T \cdot \underline{n} \mathrm{d}S = 0 \tag{2-32}$$

We are more interested in the case when the point *P* is inside the volume *V*. The point *P* must now be excluded from the region of integration and it is surrounded by a small sphere of radius ε, then we create a simple connected region. So the potential ϕ satisfies Laplace's equation, when point *P* is outside of the sphere and in the remaining volume *V*. And equation (2-32) becomes

$$\iint_{S_B + S_\infty + S_\epsilon} \left(\frac{1}{r} \underline{\nabla} \phi - \phi \underline{\nabla} \frac{1}{r} \right)^T \cdot \underline{n} \mathrm{d}S = 0 \tag{2-33}$$

Let us consider about the integral over sphere S_ε

$$\iint_{S_\epsilon} \left[\frac{1}{r} \underline{\nabla} \phi - \phi \underline{\nabla} \left(\frac{1}{r} \right) \right]^T \underline{n} \mathrm{d}S = \iint_{S_\epsilon} \left[\frac{1}{r} \underline{\nabla} \phi \right]^T \underline{n} \mathrm{d}S - \iint_{S_\epsilon} \left[\phi \underline{\nabla} \left(\frac{1}{r} \right) \right]^T \underline{n} \mathrm{d}S \tag{2-34}$$

On the sphere surrounding point *P*, as ε→0 the first term in the first integral vanishes, and the second term becomes, more detail derive see Birk(2012).

$$\lim_{\epsilon \to 0} \iint_{S_\epsilon} \left[\phi \underline{\nabla} \left(\frac{1}{r} \right) \right]^T \underline{n} \mathrm{d}S = 4\pi \phi(P) \tag{2-35}$$

After we substitute equation (2-35) back to equation (2-34), the integral of the sphere surrounding point *P* yields

$$\iint_{S_\epsilon} \left[\frac{1}{r} \underline{\nabla} \phi - \phi \underline{\nabla} \left(\frac{1}{r} \right) \right]^T \underline{n} \mathrm{d}S = -4\pi \phi(P) \tag{2-36}$$

And equation (2-33) becomes

$$\phi(P) = \frac{1}{4\pi} \iint_{S_B + S_\infty} \left(\frac{1}{r} \underline{\nabla} \phi - \phi \underline{\nabla} \frac{1}{r} \right)^T \cdot \underline{n} \mathrm{d}S \tag{2-37}$$

Equation (2-37) gives the value of $\phi(P)$ at any point in the flow, within the volume *V*, in terms of the values of ϕ and $\partial\phi/\partial n$ on the boundaries *S*.

Let us consider about the case that point *P* lies on the boundary S_B. In order to exclude point *P* from *V*, the integration is carried out only around the surrounding hemisphere with radius *ε* and equation (2-37) becomes

$$\phi(P) = \frac{1}{2\pi} \iint_{S_B} \left(\frac{1}{r} \underline{\nabla} \phi - \phi \underline{\nabla} \frac{1}{r} \right)^T \cdot \underline{n} \mathrm{d}S \tag{2-38}$$

In our mathematical model the boundary condition on S_B makes sure that S_B is actually a steam surface with no flow across it. However there may be also a flow inside the body S_B. The interior of the body V_{inside} becomes the fluid domain of interest. The interior fluid flow is described by the potential ϕ_{inside}. We keep point P outside of V_{inside} and thus can reuse the first case of exterior flow.

$$\iint_{S_B} \left(\frac{1}{4\pi r} \underline{\nabla} \phi_{inside} - \phi_{inside} \underline{\nabla} \frac{1}{4\pi r} \right)^T \cdot \underline{n}_{inside} \mathrm{d}S = 0 \tag{2-39}$$

Note that the boundary consists of body surface S_B only. We replace the normal vector $\underline{n}_{inside}$ with the normal vector of the volume V.

$$\underline{n}_{inside} = -\underline{n} \tag{2-40}$$

Then equation (2-39) becomes

$$-\iint_{S_B} \left(\frac{1}{4\pi r} \underline{\nabla} \phi_{inside} - \phi_{inside} \underline{\nabla} \frac{1}{4\pi r} \right)^T \cdot \underline{n} \mathrm{d}S = 0 \tag{2-41}$$

Now, we add equation (2-37) and (2-41), as we are going to consider both point outside of region V and inside the region V.

$$\begin{aligned} \phi(P) + 0 = \frac{1}{4\pi} \iint_{S_B + S_\infty} \left(\frac{1}{r} \underline{\nabla} \phi - \phi \underline{\nabla} \frac{1}{r} \right)^T \cdot \underline{n} \mathrm{d}S \\ -\iint_{S_B} \left(\frac{1}{4\pi r} \underline{\nabla} \phi_{inside} - \phi_{inside} \underline{\nabla} \frac{1}{4\pi r} \right)^T \cdot \underline{n} \mathrm{d}S \end{aligned} \tag{2-42}$$

We adjust for the integral on the body S_B, then we have

$$\begin{aligned} \phi(P) = \iint_{S_B} \left(\frac{1}{4\pi r} (\underline{\nabla} \phi - \underline{\nabla} \phi_{inside}) - (\phi - \phi_{inside}) \underline{\nabla} \frac{1}{4\pi r} \right)^T \cdot \underline{n} \mathrm{d}S \\ + \iint_{S_\infty} \left(\frac{1}{4\pi r} \underline{\nabla} \phi - \phi \underline{\nabla} \frac{1}{4\pi r} \right)^T \cdot \underline{n} \mathrm{d}S \end{aligned} \tag{2-43}$$

We summaries the integral over S_∞ as the so-called far field potential ϕ_∞.

$$\phi_\infty(P) = \frac{1}{4\pi} \iint_{S_\infty} \left(\frac{1}{r} \underline{\nabla} \phi - \phi \underline{\nabla} \frac{1}{r} \right)^T \cdot \underline{n} \mathrm{d}S \tag{2-44}$$

For our exterior flow problem with a body moving at a constant speed $\underline{v}_\infty$ the far field potential would be

$$\phi_\infty(P) = -(u_\infty x + v_\infty y + w_\infty z) = -\underline{P}^T \cdot \underline{v}_\infty \tag{2-45}$$

The minus sign is a question definition. An observer on the body moving with speed $\underline{v}_\infty = (u_\infty, v_\infty, w_\infty)^T$ would see the water moving towards him. Substituting this result in (2-42) yields

$$\phi(P) = \iint_{S_B} \left[\frac{1}{4\pi r} (\underline{\nabla}\phi - \underline{\nabla}\phi_{inside}) \right]^T \square \underline{n} \mathrm{d}S$$
$$- \iint_{S_B} \left[(\phi - \phi_{inside}) \underline{\nabla} \left(\frac{1}{4\pi r} \right) \right]^T \square \underline{n} \mathrm{d}S + \phi_\infty(P) \tag{2-46}$$

We merge the normal vector with the nabla operator into the more common normal derivative.

$$(\underline{\nabla}\phi)^T \underline{n} = \underline{n}^T \underline{\nabla}\phi = \frac{\partial\phi}{\partial n} \tag{2-47}$$

$$\left[\underline{\nabla} \left(\frac{1}{4\pi r} \right) \right]^T \square \underline{n} = \frac{\partial}{\partial n} \left(\frac{1}{4\pi r} \right) \tag{2-48}$$

Let us substitute equation (2-47) and (2-48) into (2-46), then we have

$$\phi(P) = \iint_{S_B} \left[\frac{1}{4\pi r} \left(\frac{\partial\phi}{\partial n} - \frac{\partial\phi_{inside}}{\partial n} \right) \right] \mathrm{d}S$$
$$- \iint_{S_B} \left[(\phi - \phi_{inside}) \frac{\partial}{\partial n} \left(\frac{1}{4\pi r} \right) \right] \mathrm{d}S + \phi_\infty(P) \tag{2-49}$$

From equation (2-49) we can define

$$-\sigma = \frac{\partial\phi}{\partial n} - \frac{\partial\phi_{inside}}{\partial n} \tag{2-50}$$

$$-\mu = \phi - \phi_{inside} \tag{2-51}$$

In equation (2-50) is the unknown source strength and in equation (2-51) is the doublet strength.

We finally have the general solution of the exterior flow problem

$$\phi(P) = \iint_{S_B} \sigma \left(-\frac{1}{4\pi r} \right) \mathrm{d}S$$
$$- \iint_{S_B} \left[\mu \frac{\partial}{\partial n} \left(\frac{-1}{4\pi r} \right) \right] \mathrm{d}S + \phi_\infty(P) \tag{2-52}$$

The potential □=□*(P)* at a point *P* in the fluid domain can be represented by a combination of a source distribution $\left(-\frac{\sigma}{4\pi r} \right)$ and a doublet distribution $\mu \frac{\partial}{\partial n} \left(\frac{-1}{4\pi r} \right)$ over the body surface.

The difference between external and internal potential is the doublet strength. And the difference between the normal derivatives of external and internal potential yields the source strength.

We have a single integral equation for both source and doublet strength, thus we do not have a unique solution yet. A choice must be made how to combine or not to combine sources and doublets.

For out naval architecture problems it is more realistic to select □=□i on SB thus eliminating the doublet distribution.

$$\phi(p)=\iint_{S_b} \underbrace{\sigma\left(\frac{-1}{4\pi r}\right)}_{Rankine\ sources} \mathrm{d}S+\phi_{\infty}(p) \tag{2-53}$$

2.6 Derivation of Free Surface Boundary Condition

In this section, we are going to discuss the derivation of Kinematic Free Surface Boundary Condition(KFSBC), Dynamic Free Surface Boundary Condition(DFSBC) and the Combined Free Surface Boundary Condition (CFSBC).

We assume that the movement of the ship hull create a perturbation of the flow passing around and having a flat water surface.

$$\phi=\Phi+\varphi' \tag{2-54}$$

In equation (2-54), □ is the free surface flow potential, Φ is the base potential and φ' is the perturbation potential.

$$\eta=H+\eta' \tag{2-55}$$

In equation (2-55), η is the wave elevation of the free surface, H is the assumed wave surface and η' is a perturbation.

2.6.1 Kinematic Free Surface Boundary Condition

As we added perturbation in the flow potential and wave elevation, let us substitute equation (2-54) and (2-55) into kinematic free surface boundary condition (2-26)

$$-(\Phi_x+\varphi_x')(H_x+\eta_x')-(\Phi_y+\varphi_y')(H_y+\eta_y')+(\Phi_z+\varphi_z')=0 \tag{2-56}$$

After the multiplication of the first two terms, we have

$$\begin{aligned}&-(\Phi_x H_x+\Phi_x\eta_x'+H_x\varphi_x'+\varphi_x'\eta_x')\\&\qquad-(\Phi_y H_y+\Phi_y\eta_y'+H_y\varphi_y'+\varphi_y'\eta_y')+(\Phi_z+\varphi_z')=0\end{aligned} \tag{2-57}$$

In our assumption, the perturbation η' and φ' is small, so we neglect the product of $\eta'\varphi'$. The equation then becomes

$$\Phi_x H_x + \Phi_x \eta'_x + H_x \varphi'_x \\ + \Phi_y H_y + \Phi_y \eta'_y + H_y \varphi'_y - \Phi_z - \varphi'_z = 0 \quad (2\text{-}58)$$

We combine the common terms

$$\Phi_x (H_x + \eta'_x) + \Phi_y (H_y + \eta'_y) \\ + H_x \varphi'_x + H_y \varphi'_y - \Phi_z - \varphi'_z = 0 \quad (2\text{-}59)$$

Furthermore, the first two terms contain the partial derivative of equation (2-55) yields

$$\Phi_x \eta_x + \Phi_y \eta_y + H_x \varphi'_x + H_y \varphi'_y - \Phi_z - \varphi'_z = 0 \quad (2\text{-}60)$$

Equation (2-60) is the linearised kinematic free surface boundary condition.

2.6.2 Dynamic Free Surface Boundary Condition

We substitute equation (2-54) into dynamic free surface boundary condition (2-28) yields

$$\frac{1}{2}[u_b^2 - (\Phi_x + \varphi'_x)^2 - (\Phi_y + \varphi'_y)^2 - (\Phi_z + \varphi'_z)^2] - g\eta = 0 \quad (2\text{-}61)$$

After the calculation of the quadratic equation inside the bracket, equation (2-61) becomes

$$\frac{1}{2}[u_b^2 - (\Phi_x^2 + 2\Phi_x \varphi'_x + \varphi'^2_x) - (\Phi_y^2 + 2\Phi_y \varphi'_y + \varphi'^2_y) \\ - (\Phi_z^2 + 2\Phi_z \varphi'_z + \varphi'^2_z)] - g\eta = 0 \quad (2\text{-}62)$$

Neglect the small terms, we have

$$\eta = \frac{1}{2g}(u_b^2 - \Phi_x^2 - \Phi_y^2 - \Phi_z^2 - 2\Phi_x \varphi'_x - 2\Phi_y \varphi'_y - 2\Phi_z \varphi'_z) \quad (2\text{-}63)$$

Equation (2-63) is the linearised dynamic free surface boundary condition.

2.6.3 Combined Free Surface Boundary Condition

We must be careful when we are doing the substitution of the wave elevation η from the dynamic condition into the kinematic condition, since η is a function of *(x,y)* alone, while the right hand side of the dynamic condition in principle is a function *F(x,y,z)*. Consequently

$$\phi_x \eta_x + \phi_y \eta_y = (\phi_x F_x + \phi_x \eta_x F_z) + (\phi_y F_y + \phi_y \eta_y F_z) = \phi_x F_x + \phi_y F_y + \phi_z F_z \quad (2\text{-}64)$$

in which we have used the kinematic condition. However, in our implementation the partial derivatives of F are defined as differences between values in field points which are on the free surface, so the F_z contribution is inherently taken into account.
The resulting combined condition then reads:

$$\Phi_x \frac{\partial\left[\frac{1}{2g}(u_b^2-\Phi_x^2-\Phi_y^2-\Phi_z^2-2\Phi_x\varphi'_x-2\Phi_y\varphi'_y-2\Phi_z\varphi'_z)\right]}{\partial x}$$
$$+\Phi_y \frac{\partial\left[\frac{1}{2g}(u_b^2-\Phi_x^2-\Phi_y^2-\Phi_z^2-2\Phi_x\varphi'_x-2\Phi_y\varphi'_y-2\Phi_z\varphi'_z)\right]}{\partial y} \tag{2-65}$$
$$+\varphi'_x H_x+\varphi'_y H_y-\Phi_z-\varphi'_z=0$$

Let us do the partial derivative in each term inside the bracket, then we have

$$\Phi_x\left[\frac{1}{2g}(-\frac{\partial\Phi_x^2}{\partial x}-\frac{\partial\Phi_y^2}{\partial x}-\frac{\partial\Phi_z^2}{\partial x}-\frac{\partial 2\Phi_x\varphi'_x}{\partial x}-\frac{\partial 2\Phi_y\varphi'_y}{\partial x}-\frac{\partial 2\Phi_z\varphi'_z}{\partial x})\right]$$
$$+\Phi_y\left[\frac{1}{2g}(-\frac{\partial\Phi_x^2}{\partial y}-\frac{\partial\Phi_y^2}{\partial y}-\frac{\partial\Phi_z^2}{\partial y}-\frac{\partial 2\Phi_x\varphi'_x}{\partial y}-\frac{\partial 2\Phi_y\varphi'_y}{\partial y}-\frac{\partial 2\Phi_z\varphi'_z}{\partial y})\right] \tag{2-66}$$
$$+\varphi'_x H_x+\varphi'_y H_y-\Phi_z-\varphi'_z=0$$

Extract the minus sign outside the bracket, we have

$$-\frac{1}{2g}\Phi_x\frac{\partial}{\partial x}(\Phi_x^2+\Phi_y^2+\Phi_z^2+2\Phi_x\varphi'_x+2\Phi_y\varphi'_y+2\Phi_z\varphi'_z)$$
$$-\frac{1}{2g}\Phi_y\frac{\partial}{\partial y}(\Phi_x^2+\Phi_y^2+\Phi_z^2+2\Phi_x\varphi'_x+2\Phi_y\varphi'_y+2\Phi_z\varphi'_z) \tag{2-67}$$
$$+\varphi'_x H_x+\varphi'_y H_y-\Phi_z-\varphi'_z=0$$

After we combine the common terms, equation (2-67) becomes

$$-\frac{1}{2g}(\Phi_x\frac{\partial}{\partial x}+\Phi_y\frac{\partial}{\partial y})(\Phi_x^2+\Phi_y^2+\Phi_z^2+2\Phi_x\varphi'_x+2\Phi_y\varphi'_y+2\Phi_z\varphi'_z) \tag{2-68}$$
$$+\varphi'_x H_x+\varphi'_y H_y-\Phi_z-\varphi'_z=0$$

Again let us do the partial derivative for each terms inside the bracket, then we have

$$-\frac{1}{2g}[(\Phi_x 2\Phi_x\Phi_{xx}+\Phi_x 2\Phi_y\Phi_{yx}+\Phi_x 2\Phi_z\Phi_{zx}+\Phi_x 2\Phi_{xx}\varphi'_x+\Phi_x 2\Phi_x\varphi'_{xx}$$
$$+\Phi_x 2\Phi_{yx}\varphi'_y+\Phi_x 2\Phi_y\varphi'_{yx}+\Phi_x 2\Phi_{zx}\varphi'_z+\Phi_x 2\Phi_z\varphi'_{zx}) \tag{2-69}$$
$$+(\Phi_y 2\Phi_x\Phi_{xy}+\Phi_y 2\Phi_y\Phi_{yy}+\Phi_y 2\Phi_z\Phi_{zy}+\Phi_y 2\Phi_{xy}\varphi'_x+\Phi_y 2\Phi_x\varphi'_{xy}$$
$$+\Phi_y 2\Phi_{yy}\varphi'_y+\Phi_y 2\Phi_y\varphi'_{yy}+\Phi_y 2\Phi_{zy}\varphi'_z+\Phi_y 2\Phi_z\varphi'_{zy})]$$
$$+\varphi'_x H_x+\varphi'_y H_y-\Phi_z-\varphi'_z=0$$

Extract 2 out of the bracket

$$
\begin{aligned}
&-\frac{1}{g}[(\Phi_x^2\Phi_{xx}+\Phi_x\Phi_y\Phi_{yx}+\Phi_x\Phi_z\Phi_{zx}+\Phi_x\Phi_{xx}\varphi'_x+\Phi_x^2\varphi'_{xx}\\
&+\Phi_x\Phi_{yx}\varphi'_y+\Phi_x\Phi_y\varphi'_{yx}+\Phi_x\Phi_{zx}\varphi'_z+\Phi_x\Phi_z\varphi'_{zx})\\
&+(\Phi_x\Phi_y\Phi_{xy}+\Phi_y^2\Phi_{yy}+\Phi_y\Phi_z\Phi_{zy}+\Phi_y\Phi_{xy}\varphi'_x+\Phi_x\Phi_y\varphi'_{xy}\\
&+\Phi_y\Phi_{yy}\varphi'_y+\Phi_y^2\varphi'_{yy}+\Phi_y\Phi_{zy}\varphi'_z+\Phi_y\Phi_z\varphi'_{zy})]\\
&+\varphi'_xH_x+\varphi'_yH_y-\Phi_z-\varphi'_z=0
\end{aligned}
\qquad (2\text{-}70)
$$

The last step, we time gravity acceleration to both side of equation (2-70), finally we have the combined free surface condition

$$
\begin{aligned}
&(\Phi_x^2\Phi_{xx}+\Phi_x\Phi_y\Phi_{yx}+\Phi_x\Phi_z\Phi_{zx}+\Phi_x\Phi_{xx}\varphi'_x+\Phi_x^2\varphi'_{xx}\\
&+\Phi_x\Phi_{yx}\varphi'_y+\Phi_x\Phi_y\varphi'_{yx}+\Phi_x\Phi_{zx}\varphi'_z+\Phi_x\Phi_z\varphi'_{zx}\\
&+\Phi_x\Phi_y\Phi_{xy}+\Phi_y^2\Phi_{yy}+\Phi_y\Phi_z\Phi_{zy}+\Phi_y\Phi_{xy}\varphi'_x+\Phi_x\Phi_y\varphi'_{xy}\\
&+\Phi_y\Phi_{yy}\varphi'_y+\Phi_y^2\varphi'_{yy}+\Phi_y\Phi_{zy}\varphi'_z+\Phi_y\Phi_z\varphi'_{zy})\\
&-g\varphi'_xH_x-g\varphi'_yH_y+g\Phi_z+g\varphi'_z=0
\end{aligned}
\qquad (2\text{-}71)
$$

Equation (2-71) is a very important equation in our problem, it build up the relation between the velocity field and the wave height elevation.

Chapter 3 The nonlinear ship wave solution

3.1 Basic Integral Equation

In this section, we will transform the boundary equation into integral form. In order to solve from the potential problem, we will use equation (2-53) to compute the potential in the region. The equations will have to satisfy all the boundary equations we derived in the last section. The following is the build up process of the integral equation of the boundary conditions.

We use the Newmann-Kelvin condition as the initial condition of the problem. The condition define the initial velocity distribution and the initial wave height.

$$\Phi = \begin{pmatrix} -u_b x \\ 0 \\ 0 \end{pmatrix} \tag{3-1}$$

$$H=0 \tag{3-2}$$

The ship velocity is u_b, as the ship is traveling towards the positive x-direction, the flow potential in x-direction become $-u_b x$. The initial wave height distribution is zero.

3.1.1 The Integral equation on ship hull

The ship hull boundary condition is similar to the case we discuss in section 2.5. It is consider to a deeply submerged body and no flow can go through the body. Recall equation (2-18).

$$\frac{\partial \phi}{\partial n} = 0 \tag{3-3}$$

The normal vector in equation (3-3) is defined to point into the body. Then we can rewrite the equation

$$\underline{n}^T \cdot \underline{\nabla} \phi = 0 \tag{3-4}$$

We are looking for the perturbation potential on the ship hull. Then equation (3-4) becomes

$$\underline{n}^T \underline{\nabla} \varphi' = \underline{n}^T \begin{pmatrix} u_b \\ 0 \\ 0 \end{pmatrix} = n_1 u_b \tag{3-5}$$

Then the source strength distribution □ which provides us with the perturbation potential φ'.

$$\varphi'(p) = \iint_S \sigma(q) \left(\frac{-1}{4\pi r(P,q)} \right) \mathrm{d}S_q \tag{3-6}$$

Note that, point q is the field in the region we are considering and point P is all the points on the ship hull.

Substituting equation (3-6) into equation (3-5), we have

$$\underline{n}_{(P)}{}^T \underline{\nabla} \iint_S \sigma(q) \left(\frac{-1}{4\pi r(P,q)} \right) \mathrm{d}S_q = n_1 u_b \tag{3-7}$$

The normal vector and the gradient are taken with respect to the point P. The integration is over all points q. Since we also have to satisfy a free surface condition we have to distribute sources on the ship hull body surface as well as the free surface.
The integration domain then includes the ship hull body surface S_B and the free surface S_F.

For a point P on the surface S we have to consider point p is superimpose with point P for the region. As we have discussed in section 2.5, this special case will become the integral over the hull sphere Sε, as equation (2-38), then yields $-\frac{1}{2}\delta(\mathrm{P})$. Equation (3-7) becomes

$$-\frac{1}{2}\sigma(\mathrm{P}) + \underline{n}_{(P)}{}^T \underline{\nabla} \iint_S \sigma(q) \left(\frac{-1}{4\pi r(P,q)} \right) \mathrm{d}S_q = n_1 u_b \tag{3-8}$$

Equation (3-8) is the integral equation of the ship hull.

3.1.2 The Integral Equation on Free Surface

In comparison to the boundary value problem of a deeply submerged body difference come from the free surface and the resulting wave body interaction. Again we will follow an approach which attempts a solution using a surface distribution of Rankine sources, equation (3-6). This is by far the most common approach today. However, more complicated singularities have been used, for example, Havelock-source which already satisfy the Newmann-Kelvin linearization thus eliminating the need to distribute sources on the free surface. However these approaches are difficult to extend to nonlinear free surface conditions.

Let us recall the potential perturbation equation (2-54) here.

$$\phi = \Phi + \varphi' \tag{3-9}$$

We substitute equation (3-6) into equation (3-8). The total velocity potential at a point *P = (x, y, z)* then becomes

$$\phi(P) = \Phi(P) + \iint_S \sigma(q)\left(\frac{-1}{4\pi r(P,q)}\right)\mathrm{d}S_q \tag{3-10}$$

On the free surface we have to satisfy the combined free surface boundary condition, we recall equation (2-70)

$$\begin{aligned}
&(\Phi_x^2\Phi_{xx} + \Phi_x\Phi_y\Phi_{yx} + \Phi_x\Phi_z\Phi_{zx} + \Phi_x\Phi_{xx}\varphi'_x + \Phi_x^2\varphi'_{xx} \\
&+\Phi_x\Phi_{yx}\varphi'_y + \Phi_x\Phi_y\varphi'_{yx} + \Phi_x\Phi_{zx}\varphi'_z + \Phi_x\Phi_z\varphi'_{zx} \\
&+\Phi_x\Phi_y\Phi_{xy} + \Phi_y^2\Phi_{yy} + \Phi_y\Phi_z\Phi_{zy} + \Phi_y\Phi_{xy}\varphi'_x + \Phi_x\Phi_y\varphi'_{xy} \\
&+\Phi_y\Phi_{yy}\varphi'_y + \Phi_y^2\varphi'_{yy} + \Phi_y\Phi_{zy}\varphi'_z + \Phi_y\Phi_z\varphi'_{zy}) \\
&-g\varphi'_x H_x - g\varphi'_y H_y + g\Phi_z + g\varphi'_z = 0
\end{aligned} \tag{3-11}$$

Substituting equation (3-9) into(3-10) results in

$$
\begin{aligned}
& \Phi_x^2\Phi_{xx} + \Phi_x\Phi_y\Phi_{yx} + \Phi_x\Phi_z\Phi_{zx} \\
+ \quad & \Phi_x\Phi_{xx} \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_x^2 \iint_{s_F+S_b} \sigma(q)\frac{\partial^2}{\partial x^2}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \quad & \Phi_x\Phi_{yx} \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_y \iint_{s_F+S_b} \sigma(q)\frac{\partial^2}{\partial y\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \quad & \Phi_x\Phi_{zx} \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_z \iint_{s_F+S_b} \sigma(q)\frac{\partial^2}{\partial z\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \quad & \Phi_x\Phi_y\Phi_{xy} + \Phi_y^2\Phi_{yy} + \Phi_y\Phi_z\Phi_{zy} \\
+ \quad & \Phi_y\Phi_{xy} \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_y \iint_{s_F+S_b} \sigma(q)\frac{\partial^2}{\partial x\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \quad & \Phi_y\Phi_{yy} \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_y^2 \iint_{s_F+S_b} \sigma(q)\frac{\partial^2}{\partial y^2}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \quad & \Phi_y\Phi_{zy} \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_y\Phi_z \iint_{s_F+S_b} \sigma(q)\frac{\partial^2}{\partial z\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
- \quad & gH_x \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q - gH_y \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \quad & g\Phi_z + g \iint_{s_F+S_b} \sigma(q)\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q = 0
\end{aligned}
\tag{3-12}
$$

Put forward the common factor $\sigma(q)$ and move the parts that do not have $\sigma(q)$ to the right hand side of the equation, we have:

$$
\begin{aligned}
& \Phi_x\Phi_{xx} \iint_{S_F+S_b} \frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_x^2 \iint_{S_F+S_b} \frac{\partial^2}{\partial x^2}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \; & \Phi_x\Phi_{yx} \iint_{S_F+S_b} \frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_y \iint_{S_F+S_b} \frac{\partial^2}{\partial y\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \; & \Phi_x\Phi_{zx} \iint_{S_F+S_b} \frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_z \iint_{S_F+S_b} \frac{\partial^2}{\partial z\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \; & \Phi_y\Phi_{xy} \iint_{S_F+S_b} \frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_y \iint_{S_F+S_b} \frac{\partial^2}{\partial x\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \; & \Phi_y\Phi_{yy} \iint_{S_F+S_b} \frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_y^2 \iint_{S_F+S_b} \frac{\partial^2}{\partial y^2}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \; & \Phi_y\Phi_{zy} \iint_{S_F+S_b} \frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q + \Phi_y\Phi_z \iint_{S_F+S_b} \frac{\partial^2}{\partial z\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \qquad (3\text{-}13) \\
- \; & gH_x \iint_{S_F+S_b} \frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q - gH_y \iint_{S_F+S_b} \frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \\
+ \; & g \iint_{S_F+S_b} \frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(p,q)}\right)\mathrm{d}S_q \bullet \sigma(q) \\
= \; & -\left(\Phi_x^2\Phi_{xx} + \Phi_x\Phi_y\Phi_{yx} + \Phi_x\Phi_z\Phi_{zx} + \Phi_x\Phi_y\Phi_{xy} + \Phi_y^2\Phi_{yy} + \Phi_y\Phi_z\Phi_{zy} + g\Phi_z\right)
\end{aligned}
$$

Normally, we would again have to discuss the case P=q where the field point P approaches the source at q. However, it is common to take a different path for the free surface: we will desingularize the free surface condition. We remove the sources form the boundary z=0 and place them slight above it, this is so called raised panel method. The sources are not placed on S_F but on a raised, parallel surface S'_F. We have to satisfy the free surface boundary condition on *z=0*, thus *P* on S_F and consequently *P* is never equal to q. Of course now the integration has to be performed over the raised surface S'_F.

3.2 Discretization of the Integral equation

3.2.1 Discretization of the ship hull

The integral equation (3-8) are to be satisfied everywhere on the ship hull body surface S_B which consists of an infinite number of points *q*. In addition, the unknowns source strength □ is part of the integrand which makes a solution difficult.
Let us extract the unknown source strength □ from the integral equation.

In order to remove □ from the integral we have to know or assume something about how the quantity in distributed locally. Since it is impossible to guess for the complete surface we subdivide the body surface S_B into small parts, so called panels.

$$\iint_{S_B} \sigma(q) \frac{\delta}{\delta \underline{n}_P}\left(-\frac{1}{4\pi r}\right)\mathrm{d}S = \sum_{j=1}^{N} \iint_{S_j} \sigma(q) \frac{\delta}{\delta \underline{n}_P}\left(-\frac{1}{4\pi r}\right)\mathrm{d}S \tag{3-14}$$

$\underline{n}_p$ is the vector pointing from point P to panel center point q.

Note, this transformation itself is exact, as long as the collection of all surface panels S_j represent the true ship hull body surface.

The simplest approach to extract □ from the integral and by far the most widely used approach is to assume that □ is constant over a panel S_j and takes the value at its center q_j.

This will result is a so called zero order panel method. Other names are constant strength panel method. The integral equation now reads

$$-\frac{1}{2}\sigma(P) + \sum_{j=1}^{N} \sigma(\underline{q}_j) \iint_{S_j} \frac{\delta}{\delta \underline{n}_P}\left(-\frac{1}{4\pi r(P,q)}\right)\mathrm{d}S_{(\underline{q})} = \underline{n}_P^T \underline{v}_\infty \tag{3-15}$$

It is obvious that for N→∞ this will converge to the original integral equation. This equation still has to be satisfied at an infinite number of points P. Now we have to restrict the integral equation to a finite number of points.

Instead of satisfying (3-14) on infinite points in the space, we restrict ourselves to a finite number of points. In another words, we have finite number of panel centers P_i.

$$-\frac{1}{2}\sigma(P_i) + \sum_{j=1}^{N} \sigma(\underline{q}_j) \iint_{S_j} \underline{n}^T(P_i)\left(-\frac{1}{4\pi r(P_i,q)}\right)\mathrm{d}S_{(\underline{q})} = \underline{n}^T(P_i)\underline{v}_\infty \tag{3-16}$$

$$i = 1, 2, \cdots, N$$

We introduce some abbreviations:

$$\sigma(\underline{P}_i) = \sigma_i \tag{3-17}$$

$$\sigma(\underline{q}_j) = \sigma(\underline{P}_j) = \sigma_j \tag{3-18}$$

$$n(\underline{P}_i) = \underline{n}_i \tag{3-19}$$

$$\underline{n}^T(\underline{P}_i)\underline{v}_\infty = b_i \tag{3-20}$$

$$\iint_{S_j} \underline{n}_i \underline{\nabla}\left(-\frac{1}{4\pi r(P_i,q)}\right) \mathrm{d}S_{(\underline{q})} = D_{ij} \tag{3-21}$$

Then equation (3-16) becomes:

$$-\frac{1}{2}\sigma_i + \sum_{j=1}^{N} \sigma_j D_{ij} = b_i \qquad i = 1,2,\cdots,N \tag{3-22}$$

3.2.2 Discretization of the free surface

Equation (3-8) and (3-12) form a system of two coupled integral equations, to be satisfied at every point P on the body and the calm water surface.

To derive algebraic equations which we can solve on a computer we will select N_B points P_i($i = 1,2,...,N_B$) on the body surface and N_F points P_i ($i = N_B+1,\ N_B+2,\ ...,\ N_B+N_F$) on the free surface where we will check the boundary conditions. This yields

$$\begin{aligned}
& \Phi_x\Phi_{xx} \iint_{S_F+S_b} \frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q + \Phi_x^2 \iint_{S_F+S_b} \frac{\partial^2}{\partial x^2}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q \\
+ \;& \Phi_x\Phi_{yx} \iint_{S_F+S_b} \frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_y \iint_{S_F+S_b} \frac{\partial^2}{\partial y\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q \\
+ \;& \Phi_x\Phi_{zx} \iint_{S_F+S_b} \frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_z \iint_{S_F+S_b} \frac{\partial^2}{\partial z\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q \\
+ \;& \Phi_y\Phi_{xy} \iint_{S_F+S_b} \frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q + \Phi_x\Phi_y \iint_{S_F+S_b} \frac{\partial^2}{\partial x\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q \\
+ \;& \Phi_y\Phi_{yy} \iint_{S_F+S_b} \frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q + \Phi_y^2 \iint_{S_F+S_b} \frac{\partial^2}{\partial y^2}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q \\
+ \;& \Phi_y\Phi_{zy} \iint_{S_F+S_b} \frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q + \Phi_y\Phi_z \iint_{S_F+S_b} \frac{\partial^2}{\partial z\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q \\
- \;& gH_x \iint_{S_F+S_b} \frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q - gH_y \iint_{S_F+S_b} \frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q \\
+ \;& g \iint_{S_F+S_b} \frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q \bullet \sigma(q) \\
= \;& -\left(\Phi_x^2\Phi_{xx} + \Phi_x\Phi_y\Phi_{yx} + \Phi_x\Phi_z\Phi_{zx} + \Phi_x\Phi_y\Phi_{xy} + \Phi_y^2\Phi_{yy} + \Phi_y\Phi_z\Phi_{zy} + g\Phi_z\right)
\end{aligned} \tag{3-23}$$

Like for the submerged body we have to extract the unknown sources strength □ from the integrals. We will discretize the raised surface S'_F and the ship hull body surface S_B into quadrilaterals and then assume the general shape of the source strength distribution over the small panels For body and raised free surface we define as many panels as we have field points, body surface N_B panels and free surface N_F panels.

All integrals will be converted into sums of integrals over panels S_j. We assume the source strength to be constant on small panels S_j. We assume σ to be constant over an individual S_j with strength $\sigma(q_j)$. Note that q_j is the center of panel S_j.

Then equation (3-22) turns into

$$\left[\begin{array}{l}
\Phi_x\Phi_{xx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q+\Phi_x^2\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial x^2}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q \\
+\Phi_x\Phi_{yx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q+\Phi_x\Phi_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial y\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q \\
+\Phi_x\Phi_{zx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q+\Phi_x\Phi_z\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial z\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q \\
+\Phi_y\Phi_{xy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q+\Phi_x\Phi_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial x\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q \\
+\Phi_y\Phi_{yy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q+\Phi_y^2\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial y^2}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q \\
+\Phi_y\Phi_{zy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q+\Phi_y\Phi_z\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial z\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q \\
-gH_x\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q-gH_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q \\
+g\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)dS_q
\end{array}\right]\bullet\sum_{j=1}^{N_B+N_F}\sigma(q_j) \qquad (3\text{-}24)$$

$$=-\left(\Phi_x^2\Phi_{xx}+\Phi_x\Phi_y\Phi_{yx}+\Phi_x\Phi_z\Phi_{zx}+\Phi_x\Phi_y\Phi_{xy}+\Phi_y^2\Phi_{yy}+\Phi_y\Phi_z\Phi_{zy}+g\Phi_z\right)$$

As for the submerged body we have reduced the integrals to coefficients which only depend on geometric information we know.

Again we introduce some abbreviations:

$$
a_{i,j}=\left[\begin{aligned}
&\Phi_x\Phi_{xx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_x^2\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial x^2}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
&+\Phi_x\Phi_{yx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_x\Phi_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial y\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
&+\Phi_x\Phi_{zx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_x\Phi_z\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial z\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
&+\Phi_y\Phi_{xy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_x\Phi_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial x\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
&+\Phi_y\Phi_{yy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_y^2\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial y^2}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
&+\Phi_y\Phi_{zy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_y\Phi_z\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial z\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
&-gH_x\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q-gH_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
&+g\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q
\end{aligned}\right] \tag{3-25}
$$

$$
b_i=-\left(\Phi_x^2\Phi_{xx}+\Phi_x\Phi_y\Phi_{yx}+\Phi_x\Phi_z\Phi_{zx}+\Phi_x\Phi_y\Phi_{xy}+\Phi_y^2\Phi_{yy}+\Phi_y\Phi_z\Phi_{zy}+g\Phi_z\right) \tag{3-26}
$$

The equations (3-16) and (3-24) represent a system of N_B+N_F linear equation for the N_B+N_F unknown source strength values □. Then we replace the system equation with abbreviate equations (3-22), (3-25) and (3-26).

$$
\begin{bmatrix}
a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,N_B} & a_{1,N_B+1} & \cdots & a_{1,N_B+N_F}\\
a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{1,N_B} & a_{1,N_B+1} & \cdots & a_{2,N_B+N_F}\\
a_{3,1} & a_{3,2} & a_{3,3} & \cdots & a_{1,N_B} & a_{1,N_B+1} & \cdots & a_{3,N_B+N_F}\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\
a_{N_B,1} & a_{N_B,2} & a_{N_B,3} & \cdots & a_{N_B,N_B} & a_{N_B,N_B+1} & \cdots & a_{N_B,N_B+N_F}\\
\vdots & \vdots & \vdots & \ddots & & \ddots & & \vdots\\
a_{N_B+N_F-1,1} & a_{N_B+N_F-1,2} & a_{N_B+N_F-1,3} & \cdots & a_{N_B+N_F-1,N_B} & a_{N_B+N_F-1,N_B+1} & \ddots & a_{N_B+N_F-1,N_B+N_F}\\
a_{N_B+N_F,1} & a_{N_B+N_F,1} & a_{N_B+N_F,1} & \cdots & a_{N_B+N_F,N_B} & a_{N_B+N_F,N_B+1} & \cdots & a_{N_B+N_F,N_B+N_F}
\end{bmatrix}
\square
\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \vdots\\ \sigma_{N_B}\\ \vdots\\ \sigma_{N_B+N_F-1}\\ \sigma_{N_B+N_F}\end{pmatrix}
=
\begin{pmatrix}b_1\\ b_2\\ b_3\\ \vdots\\ b_{N_B}\\ \vdots\\ b_{N_B+N_F-1}\\ b_{N_B+N_F}\end{pmatrix} \tag{3-27}
$$

Now we want to solve for unknown source strength □ in equation (3-27).

Chapter 4 The Nonlinear Method

The process of the nonlinear method are now described in this chapter, and also the numerical method we are using in the method.

4.1 Process of the program

Hoyte Raven named his method as RAPID method, as the method is a RAised Panel Iterative Dawson method. After the building up of mathematical model in the previous chapters. We are using iterative method in solving the nonlinear problem.

The nonlinear method steps:

1. Initial free surface, Initial velocity distribution as we input the Froude number we calculate for the ship velocity.

2. Input panel file, including ship hull discretization, Δz for free surface panel distribution at a specified distance above the free surface and shifted forward Δx for the field points.

3.Move the free surface field points to the assumed wave height.

4. Apply linearised ship hull boundary condition and linearised combined free surface condition to both field points on the ship hull and the free surface. Calculated for the perturbations from the assumed velocity distribution.

5. Solving for unknown source strength □.

6. Computed for velocity distribution on the field points. Using the dynamic free surface condition calculate for a new estimate of wave elevation.

7. Recomputed for the velocity direction. Computed for the pressure coefficient, wetted surface, horizontal force and wave resistance coefficient.

8. Calculate the residual errors in the nonlinear free surface conditions. If these exceed the convergence criteria we defined, return to step 3.

In the program, we use the panel file as the input file for the wigleyfsmain.exe. The program is designed to calculate wave elevation of the free surface, source strength of the panels and the pressure coefficient on the ship hull in nonlinear method under different Froude numbers. Froude number is reading into the program from Frinput.txt file. Then we begin the calculation in the main program. First, the calculation for the coefficient matrix (a matrix), we call for subroutine panelinflunece for calculating the velocity and subroutine panelinflunence2 for calculating the second order derivatives of the potential. After finish calculating for the a matrix, we start the calculation for the right hand side matrix (b matrix) of the equation. Once we finish calculating for the two matrixes, we can start calculating for the sigma using subroutine SIMQIT2 which is a partial Gaussian elimination from slaemod.f90 provided by Dr. Lothar Birk. When done with the calculation for the sigma, we start computing for the pressure coefficient, wave elevation and wave resistance coefficient. Then output the data into two vtp file prepare for the paraview to read the data. Finally, we judge for convergence if not converge, repeat the full application.

In the program, we call for the subroutines as followed:

- Readgeometry
- Panelgeometry
- Panelinfluence
- Panelinfluence2
- SIMQIT2
- VtkXmlPolyDataCellScalar

The readgeometry subroutine is calling from the panlcgeom.f90, and it is using for reading the panel geometry, such as the points on the panel, the space location of the points, number of panels and the raised distance.

The panelgeometry subroutine is calling from the panlcgeom.f90, and it is using for reading the panel indices of the four corner points, the coordinates of all points, local coordinate system vectors, panel area and 3D panel center coordinates.

The panelinfluence subroutine is calling from the panlcfun1.f90, and it is using for calculating for the velocity induced by the panel and measure on the field point.

The panelinfluence2 subroutine is calling from the panlcfun2.f90, and it is using for calculating for the second order derivatives of the potential.

The SIMQIT2 subroutine is calling from slaemod.f90, and it is using for calculating the sigma, which is the unknown source strength of the panels.

The VtkXmlPolyDataCellScalar subroutine is calling from vtkxmlmod.f90, and it is using for gathering the data to generate a vtp file as an input of the paraview.

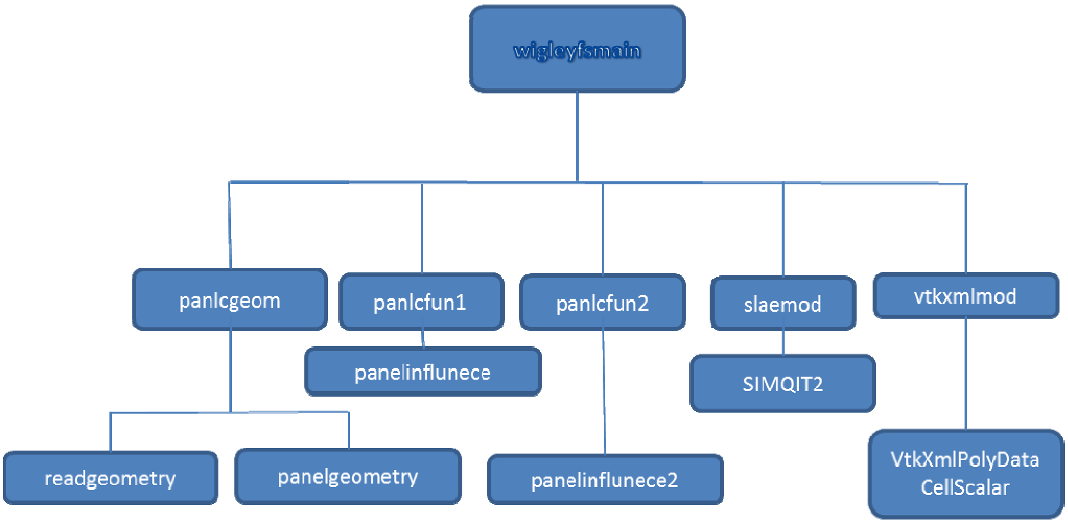

Fig 4.1: Call tree of the program

The call tree of the program we can see Fig 4.1.

4.2 Description of the method

4.2.1 The panel layout

The panel we are using is a double-body panel, we generate only half of the ship hull body and the free surface, and we mirroring the each field point of free surface and the ship hull body in the program. The basic panel is in Fig 4.2 the initial panel graph.

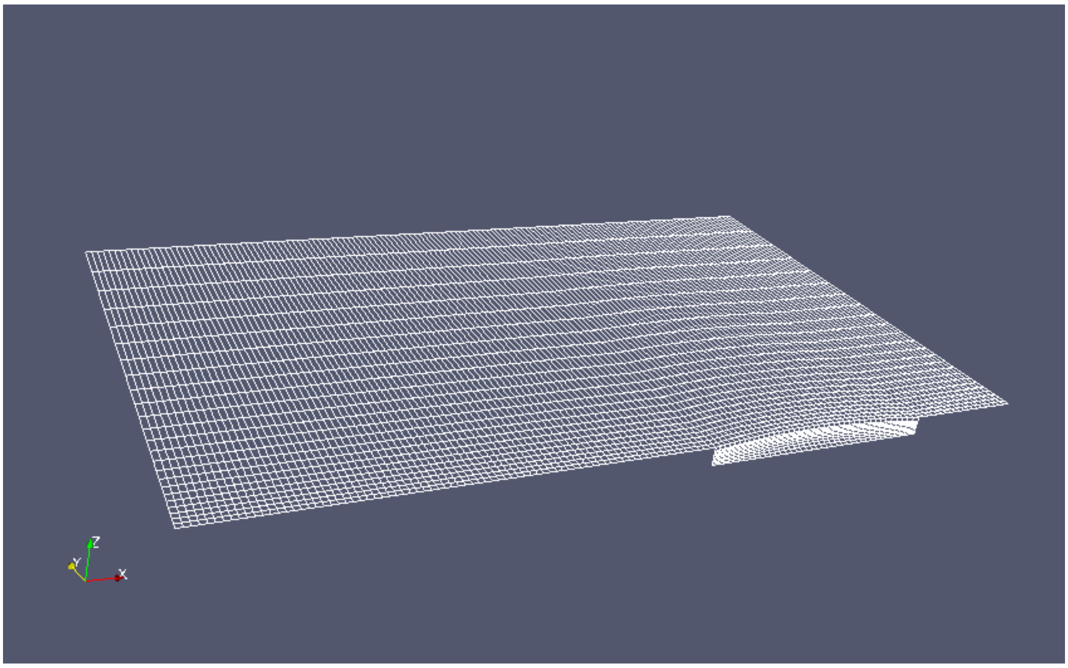

Fig 4.2: The initial panel layout

The ship hull body and the free surface are represented by flat quadrilateral panels with constant source strength. For the ship hull body, the source located on the hull body. At the meanwhile, for the free surface, the source located about 0.8 panel length under the free surface panel and also shifted forward in a small distance Δx, these small distance will help to reduce the possible of singularity in the system of algebraic equations.

The velocities and the second order derivative of potentials are exact solutions computed by Hess and Smith panel method.

4.2.2 The Free surface boundary condition

Here we are using the combined free surface boundary condition (3-23) we derived in last chapter. Recall the a-matrix calculation equation (3-25) as followed:

$$a_{i,j}=\left[\begin{array}{l}
\Phi_x\Phi_{xx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_x^2\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial x^2}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
+\Phi_x\Phi_{yx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_x\Phi_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial y\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
+\Phi_x\Phi_{zx}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_x\Phi_z\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial z\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
+\Phi_y\Phi_{xy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_x\Phi_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial x\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
+\Phi_y\Phi_{yy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_y^2\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial y^2}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
+\Phi_y\Phi_{zy}\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q+\Phi_y\Phi_z\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial^2}{\partial z\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
-gH_x\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial x}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q-gH_y\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial y}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q\\
+g\sum_{j=1}^{N_B+N_F}\iint_{S_F+S_b}\frac{\partial}{\partial z}\left(\frac{-1}{4\pi r(P_i,q)}\right)\mathrm{d}S_q
\end{array}\right] \quad (4\text{-}1)$$

After solving for a-matrix, we will compute for the right hand side of equation (3-23), so recalled b-matrix calculation equation (3-26).

$$b_i=-\left(\Phi_x^2\Phi_{xx}+\Phi_x\Phi_y\Phi_{yx}+\Phi_x\Phi_z\Phi_{zx}+\Phi_x\Phi_y\Phi_{xy}+\Phi_y^2\Phi_{yy}+\Phi_y\Phi_z\Phi_{zy}+g\Phi_z\right) \quad (4\text{-}2)$$

Now we have the solution for a-matrix and b-matrix, but there is still two unknown in a-matrix, H_x and H_y, we will use finite difference scheme to compute the value.

4.2.3 Finite Difference scheme

We are using the finite difference method for solving the partial derivative of wave height at x and y-direction.

For the points on the edges, we copy the wave elevation in the nearby points and extent one position. This will help to compute for the partial derivative on the free surface field points around the ship hull body.

For x-direction, as the flow is moving in this direction, we are using so call upwind difference scheme.

$$\frac{\partial f}{\partial x} \approx \frac{1.5 f_i - 2 f_{i-1} + 0.5 f_{i-2}}{\Delta x} \tag{4-3}$$

For y-direction, we are using central difference scheme.

$$\frac{\partial f}{\partial y} \approx \frac{f_{i+1} - f_{i-1}}{2\Delta y} \tag{4-4}$$

4.2.4 Initial solution

In our derivation of the free surface boundary condition, we have no specific assumptions on the base flow and the wave elevation. We can choose any initial condition we needed. Here we are using the Neumann-Kelvin condition as the input of the program that we start from a flat free surface and a uniform flow. It will be faster for converge, if we choose an more appropriate initial condition, but this is base on experience.

4.2.5 Update of wave surface and velocity field

After we calculate the result for unknown source strength □, we can compute for the velocity distribution. We use the dynamic free surface condition to solve for the wave elevation.

$$\eta = \frac{1}{2g}(u_b^2 - \phi_x^2 - \phi_y^2 - \phi_z^2) \tag{4-5}$$

As full application of the wave height update will frequently lead to divergence, we add a slight amount of underrelaxation in each iteration will help us largely decrease the possible of the divergence. The underrelaxation is 0.7 in the program. Then equation (4-5) becomes

$$\eta_{k+1} = 0.7\left[\frac{1}{2g}(u_b^2 - \phi_x^2 - \phi_y^2 - \phi_z^2) - \eta_k\right] \tag{4-6}$$

In the equation, k stand for the times of the iteration.

4.2.6 The base flow recalculation

The calculation of new base flow distributions is base on the last velocity field and a new estimated field point positions. After we shifted our field point to the new position, this will effect on the velocity directions and it is so call transfer effect. To cure this problem, we just simply do the calculation for the velocities again, then we can change the velocities to the correct direction.

4.2.7 Converge Criteria

The converge criteria of the program is judging for the residual error below

$$\epsilon_k = \phi_z - H_x\phi_x - H_y\phi_y \tag{4-7}$$

$$\epsilon_d = \frac{1}{2}\left(u_b{}^2 - \phi_x^2 - \phi_y^2 - \phi_z^2\right) - gH \tag{4-8}$$

In the equations above, ε_k is the residual error of kinematic free surface boundary condition and ε_d is the residual error of dynamic free surface boundary condition. It is not enough for judging for the change of wave elevation falls below a certain tolerance. The condition here are

$$\epsilon_k \le 0.04u_b \;\; and \;\; \epsilon_d \le 0.0025u_b{}^2 \tag{4-9}$$

Generally the change of wave elevation is less than 10^{-4} in the program.

Chapter 5 Results and Conclusions

In this chapter, we show the results of wave height profile in specific Froude numbers, wave pattern specific Froude numbers and wave resistance coefficient in different Froude numbers of the nonlinear method and the linear method compare with experiential data. More detail result will be available in Appendix.

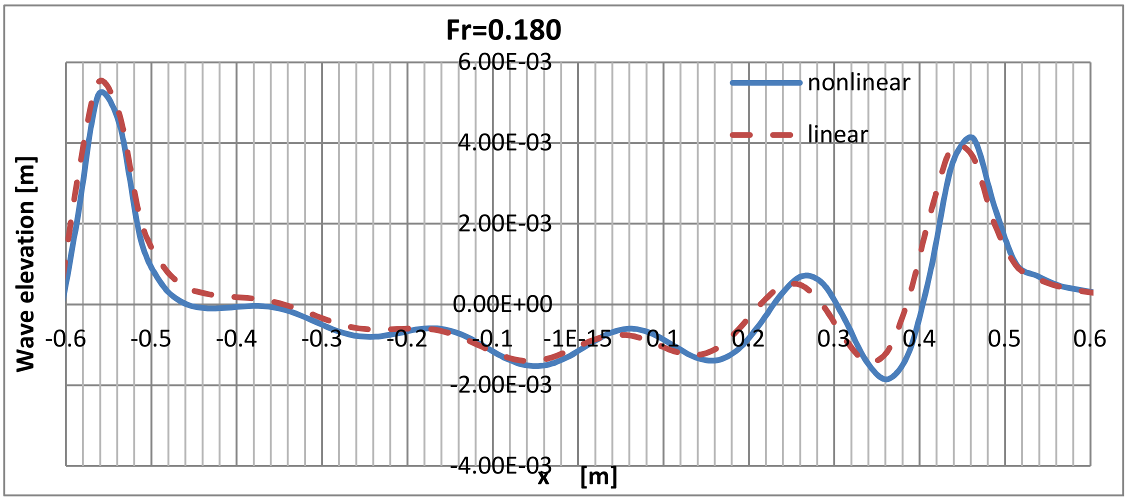

Fig 5.1: Wave height prediction Fr=0.180

In Fig 5.1, Wigley hull is in range of [-0.5, 0.5] and the ship is traveling in positive x-direction, y-axis is the predicted wave height, red line is the linear prediction and blue line is nonlinear prediction.

We can see from Fig 5.1, the wave height prediction around ship hull between linear and nonlinear case is very different. Compare between linear case and nonlinear case, we can see the result from linear calculation the bow wave height is lower than the nonlinear result and the stern wave is higher than the nonlinear case.

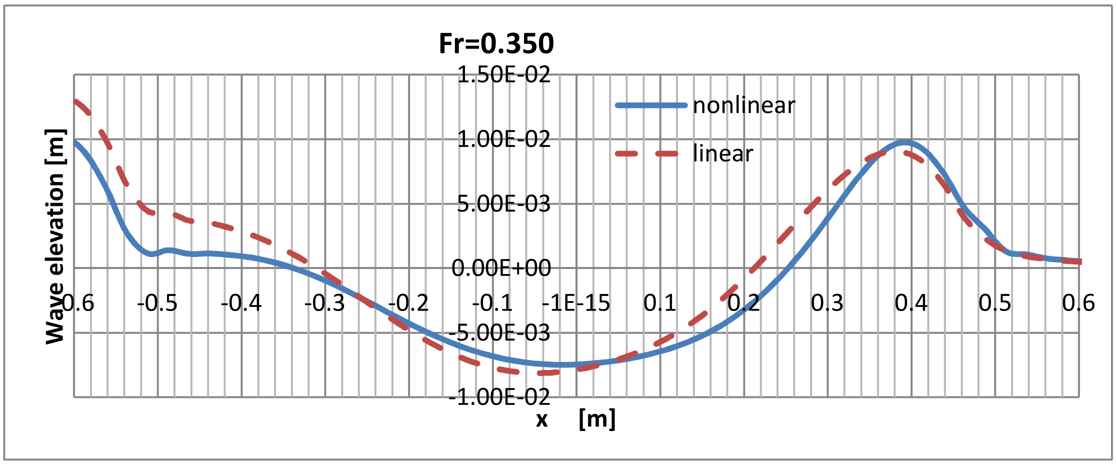

Fig 5.2: Wave height prediction Fr=0.350

For a higher Froude number Fr = 0.35, we can see more clear that the nonlinear method has a larger wave height in the bow and the crest is shifted forward a little bit, and the stern wave is lower than the linear method. The nonlinear method can better modify the flow separation in the stern and better wave height prediction at the bow wave generation.

After compare with the wave height profile between linear method and nonlinear method, let us look at the wave pattern in both methods.

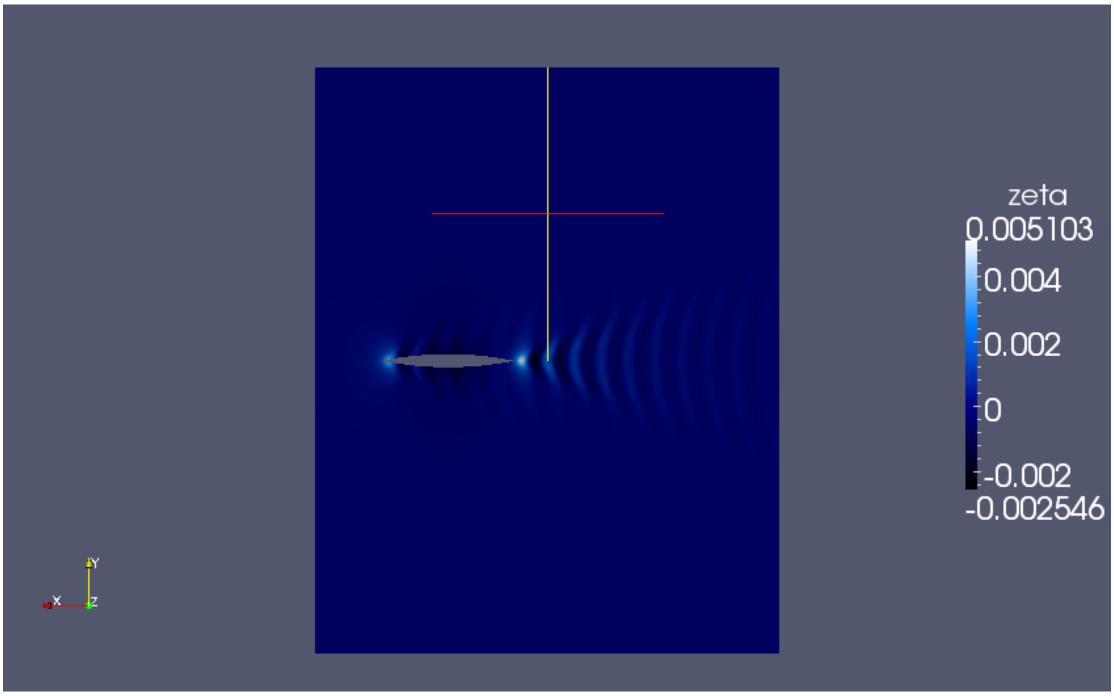

Fig 5.3: Linear method wave pattern Fr=0.180

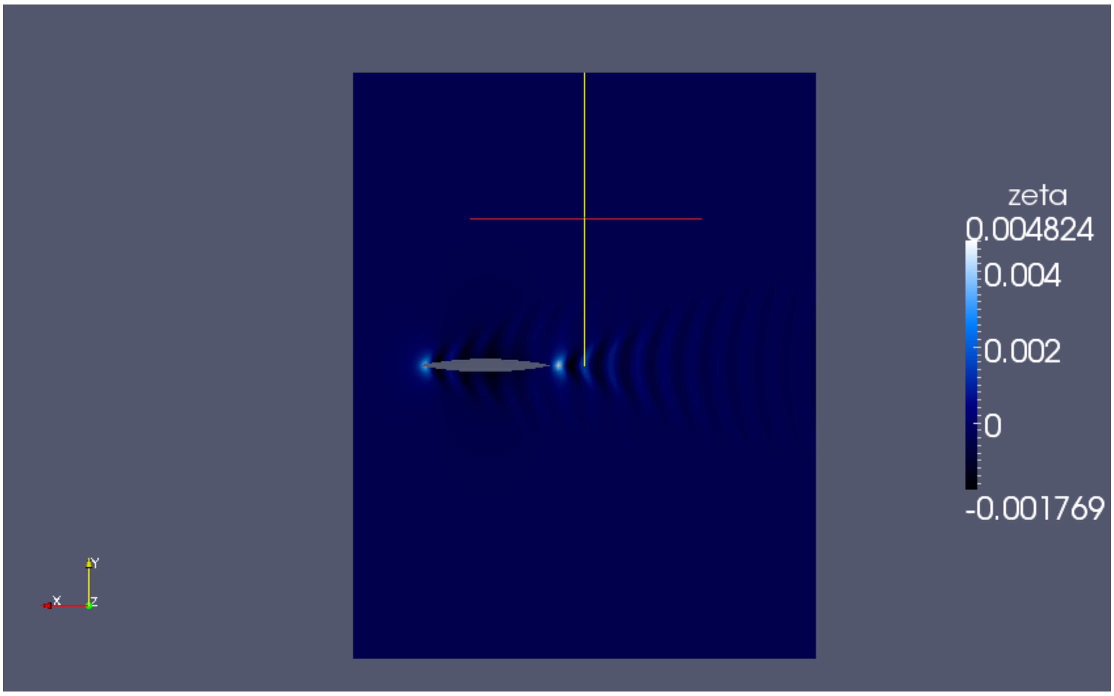

Fig 5.4: Nonlinear method wave pattern Fr=0.18

In the wave pattern figures, the x-axis is pointing to the left, the y-axis is pointing upwards and the z-axis is pointing outwards.

We can see from Fig 5.3 and Fig 5.4, the wave pattern around the ship hull is different. The nonlinear method has a deeper wave pattern around the ship hull than the linear method. The maximum wave height in linear method is larger than the nonlinear method, the minimum wave height in linear method is smaller than the nonlinear method. For the transome wave, both methods look similar in a low Froude number. Let us compare a higher Froude number.

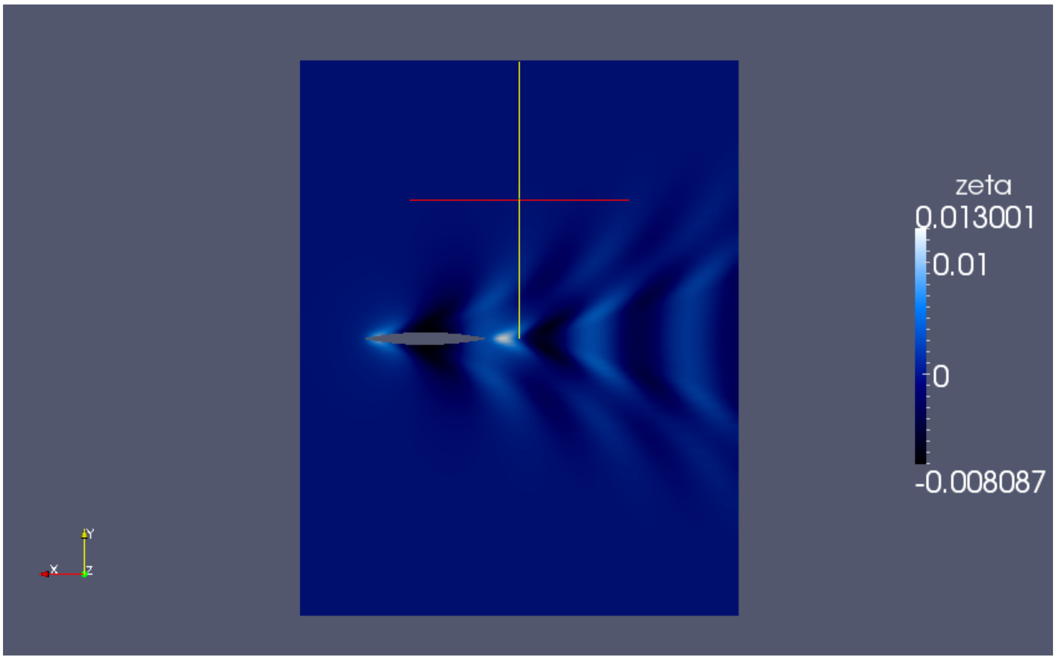

Fig 5.5: Linear method wave pattern Fr=0.350

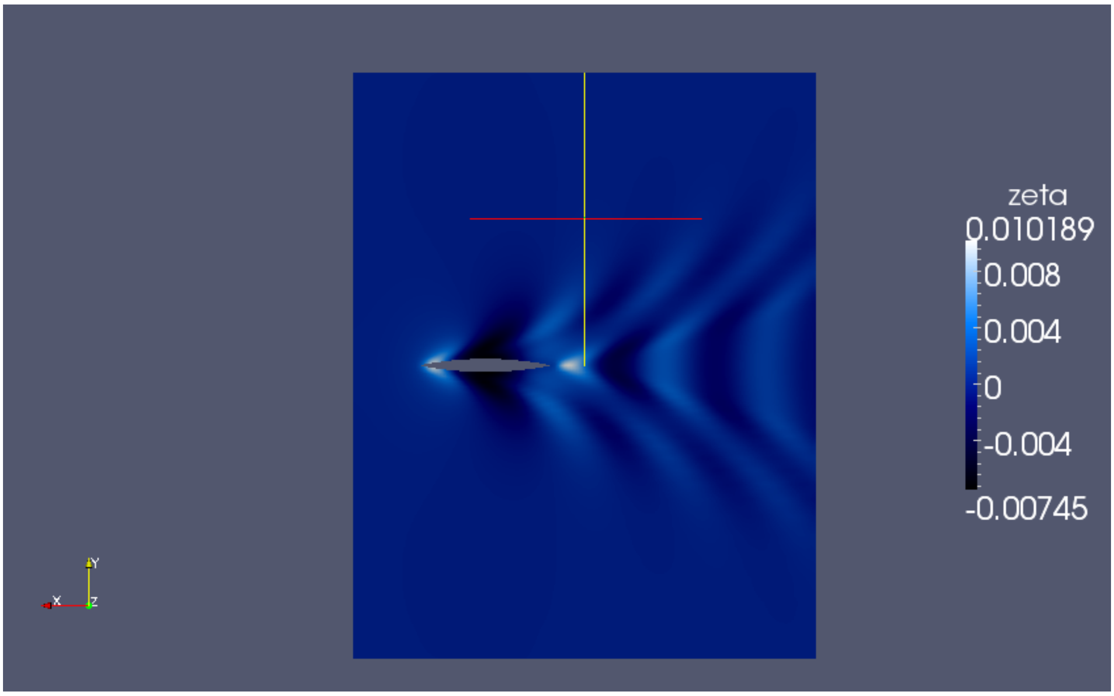

Fig 5.6: Nonlinear method wave pattern Fr=0.350

Fig 5.5 and Fig 5.6 are the wave pattern in Froude number Fr=0.350 in linear method and nonlinear method. The wave around the ship hull is different, in nonlinear method the wave goes wider and deeper at the stern. The same as Froude number Fr=0.0180, the maximum wave height in linear method is larger than the nonlinear method, the minimum wave height in linear method is smaller than the nonlinear method. For the transome wave looks very similar in both cases, we will look at the wave height profile at the transome for more detail analysis.

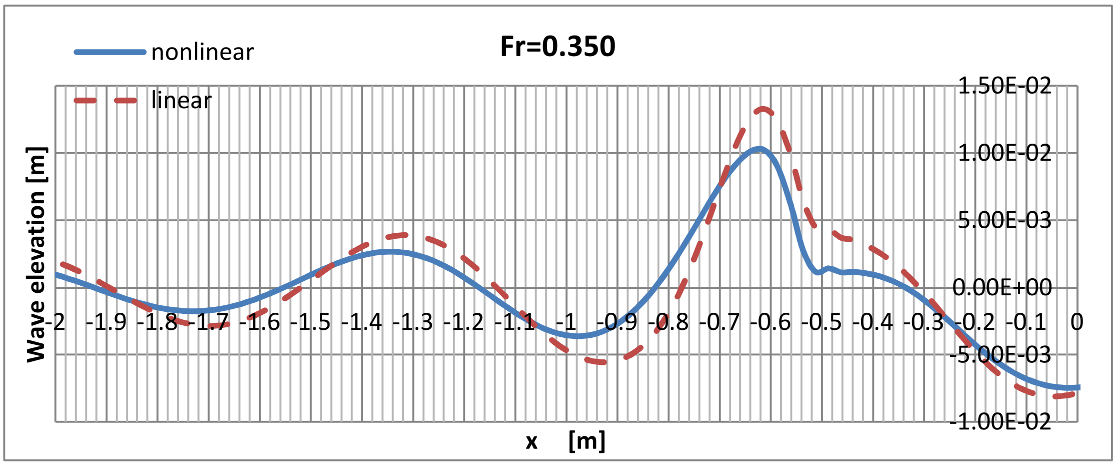

Fig 5.7: Transome wave profile Fr=0.350

In Fig 5.7, the transome wave is in range of [-0.5, -2.0], the ship is traveling in positive x-axis.

Form Fig 5.7, we can better look at the detail of the transome wave difference between linear method and nonlinear method. The transome wave is shifted backwards compare with the linear method result and the wave height is smaller than the linear method.

After comparison in wave height profile and wave patterns, let us compare the wave resistance results in different methods.

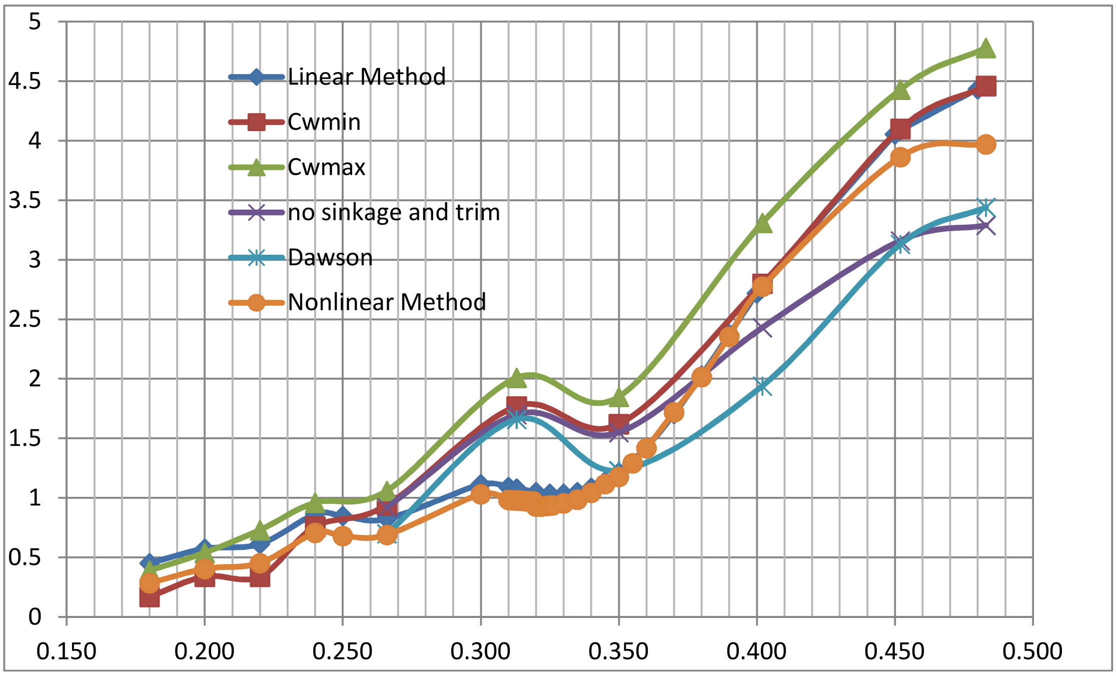

Fig 5.8: Wave resistance coefficient

From Fig 5.8, the nonlinear method has a better tendency to the experimental result compare with linear method. The result in low Froude number cases is in the range of experimental result and in nonlinear method can better modify the decrease tendency in Froude number range in [0.30, 0.35] than the linear method.

Chapter 6 Future Work

We can see from Fig 5.8, the result still need to be improve in high Froude number.

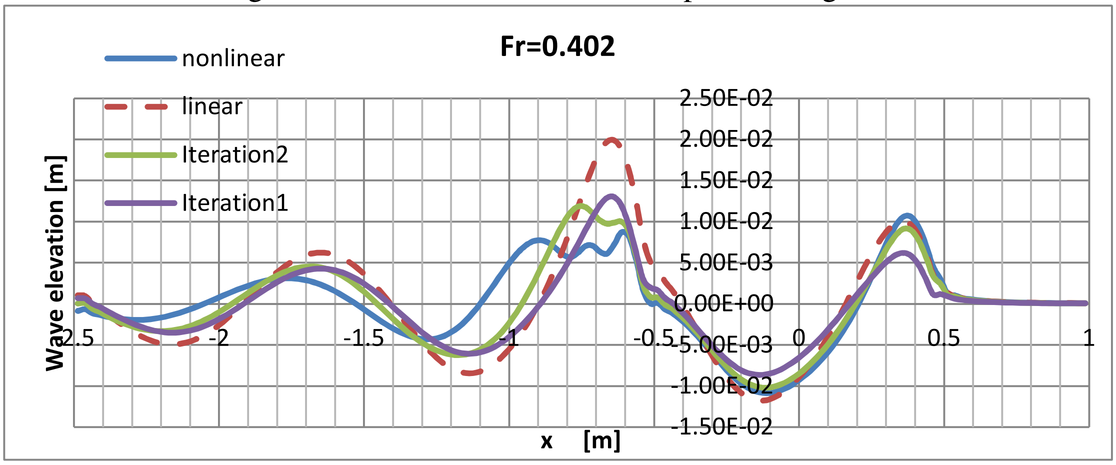

Fig 6.1: Wave height profile at Fr=0.402

In Fig 6.1, we can see that the calculation of the stern wave is not correct, as the maximum wave height is higher than the raised distance on the free surface panel, the wave elevation is affected by the panel. To better improve the calculation in high Froude number, we have to address the panel to a right distance above the estimated wave elevation.

The calculation for the actual wetted surface area will also need to be improved. As we are calculating the area when the center of the panel is lower than the wave elevation. Some part of the dry area is including in the calculation. So we need to increase the accuracy of the wetted surface calculation.

The improvement of finite difference scheme may also give a better result. As the finite difference scheme is improving nowadays, we can use a better finite difference scheme to decrease the residual error in the program.

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Appendix   Result Data

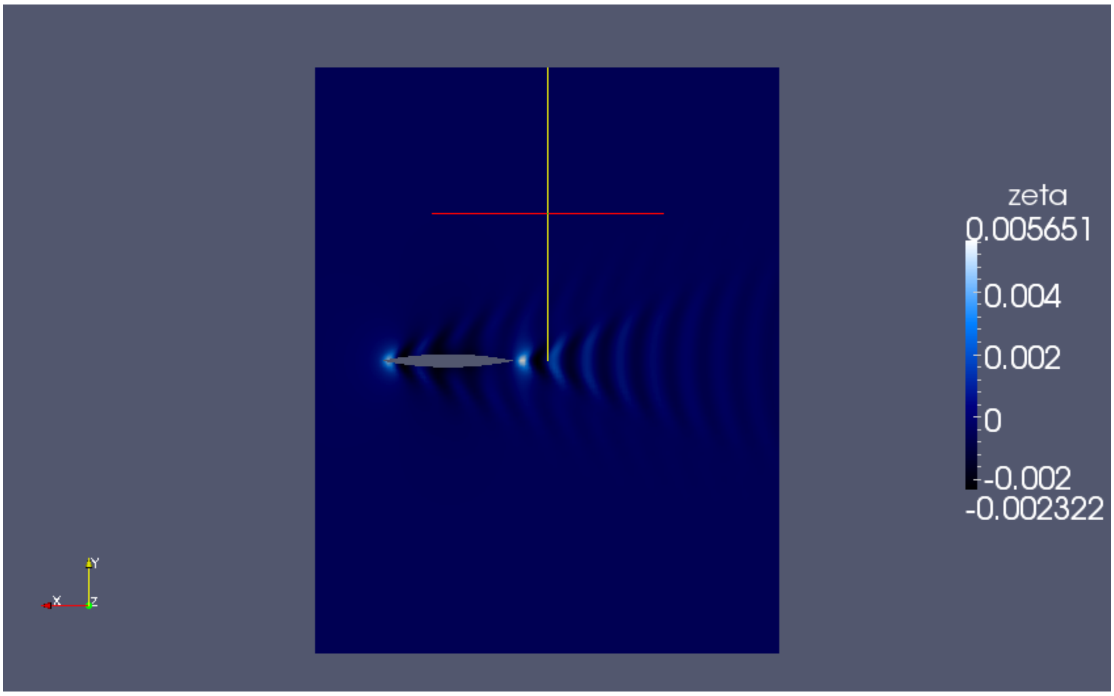

Fig1: Nonlinear wave patter Fr=0.200

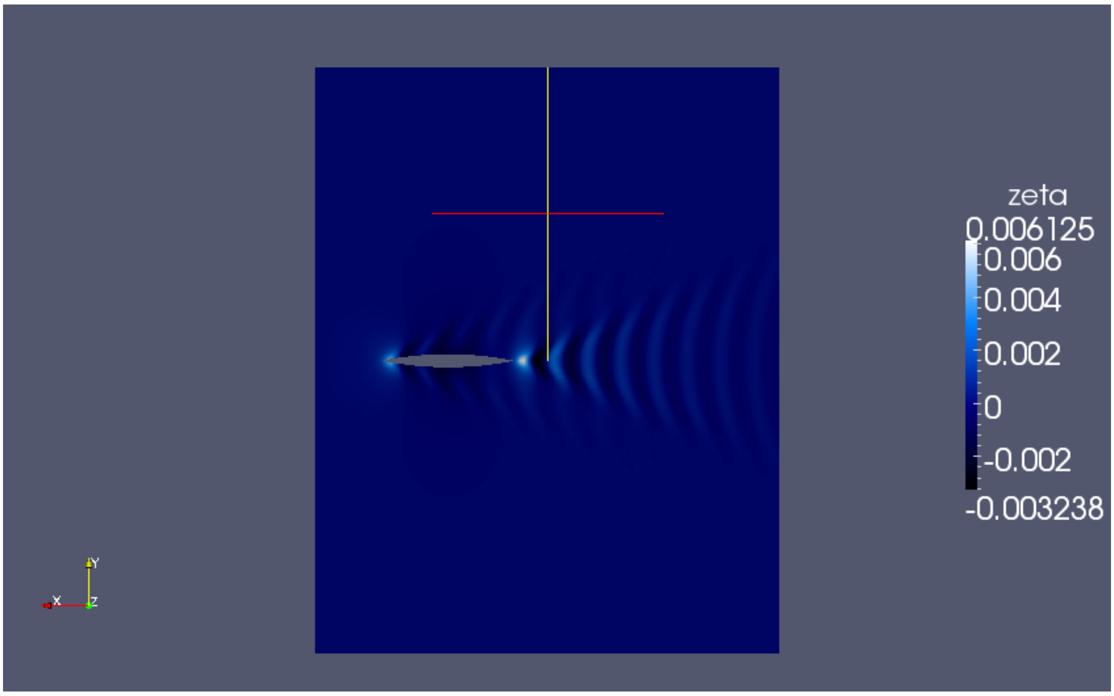

Fig2: Linear wave patter Fr=0.200

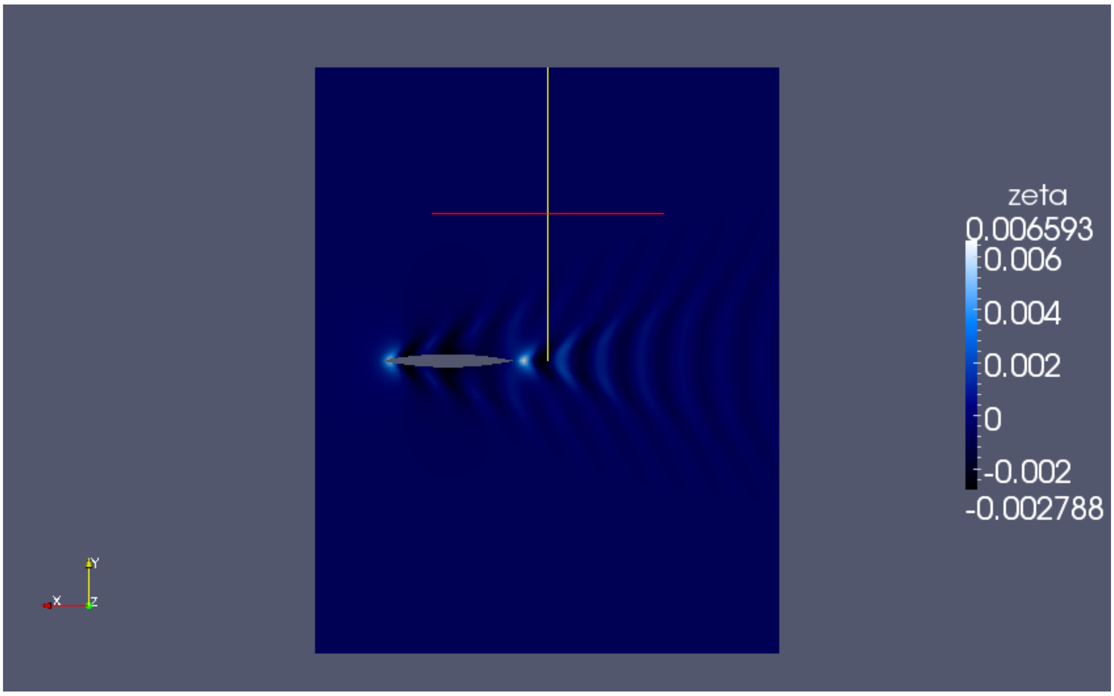

Fig3: Nonlinear wave patter Fr=0.220

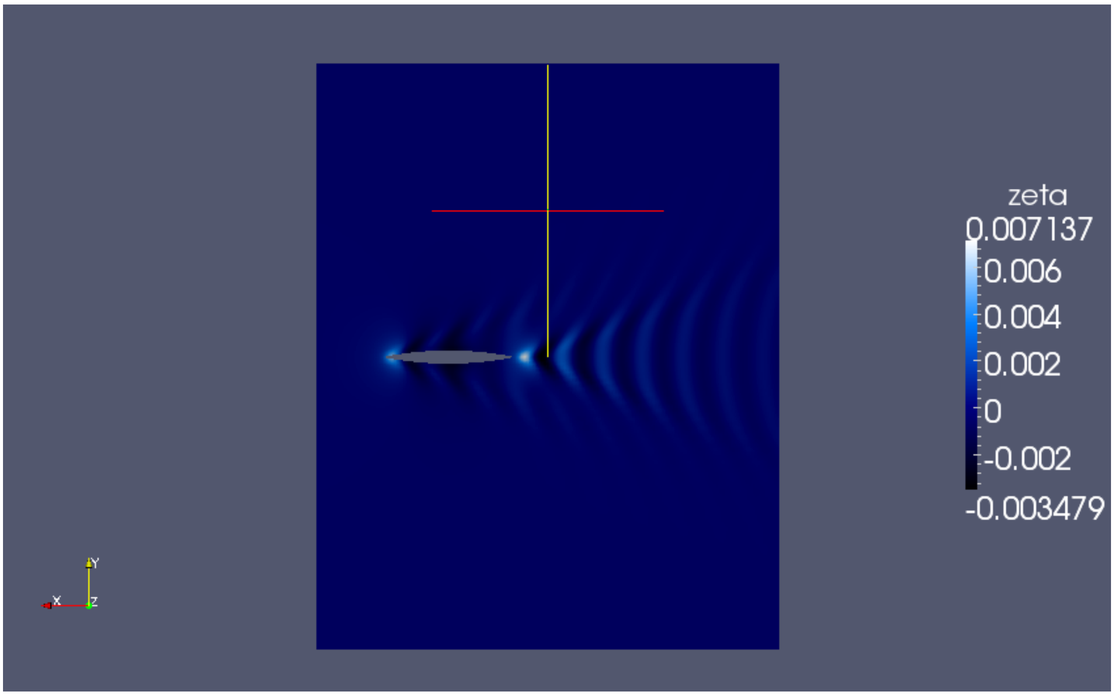

Fig4: Linear wave patter Fr=0.220

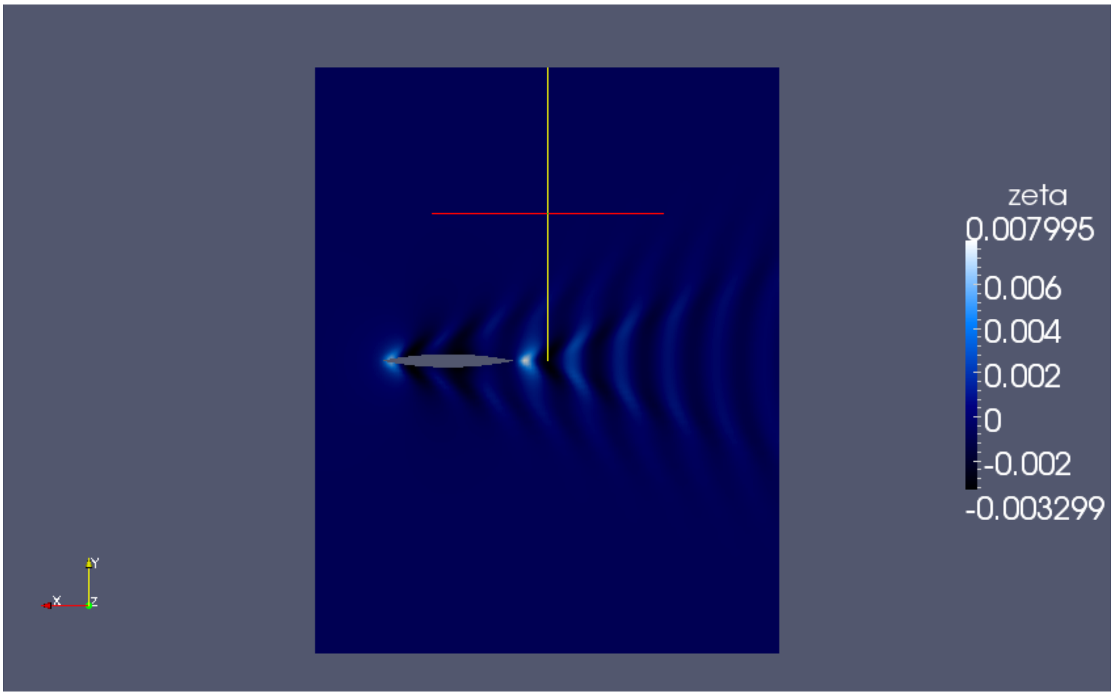

Fig5: Nonlinear wave patter Fr=0.240

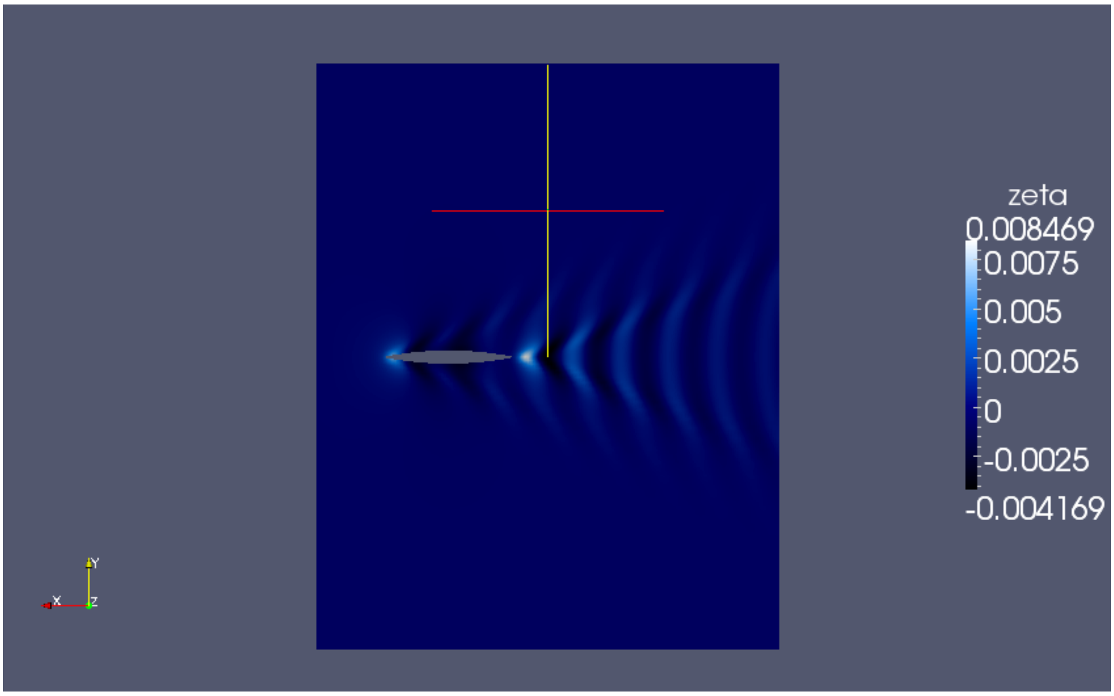

Fig6: Linear wave patter Fr=0.240

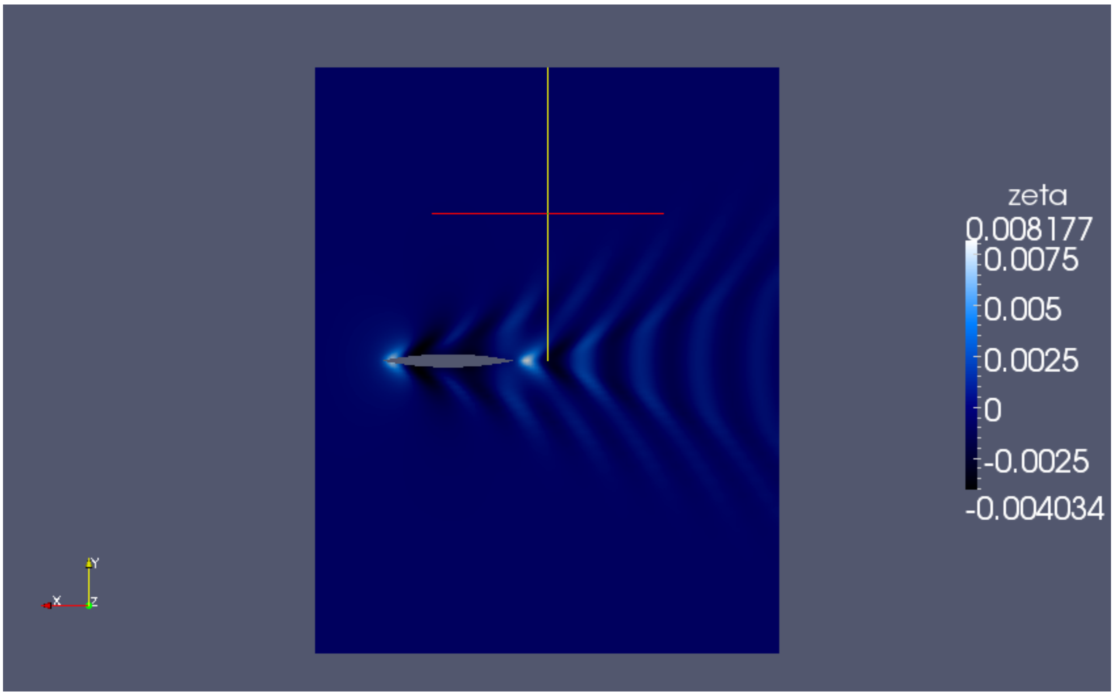

Fig7: Nonlinear wave patter Fr=0.266

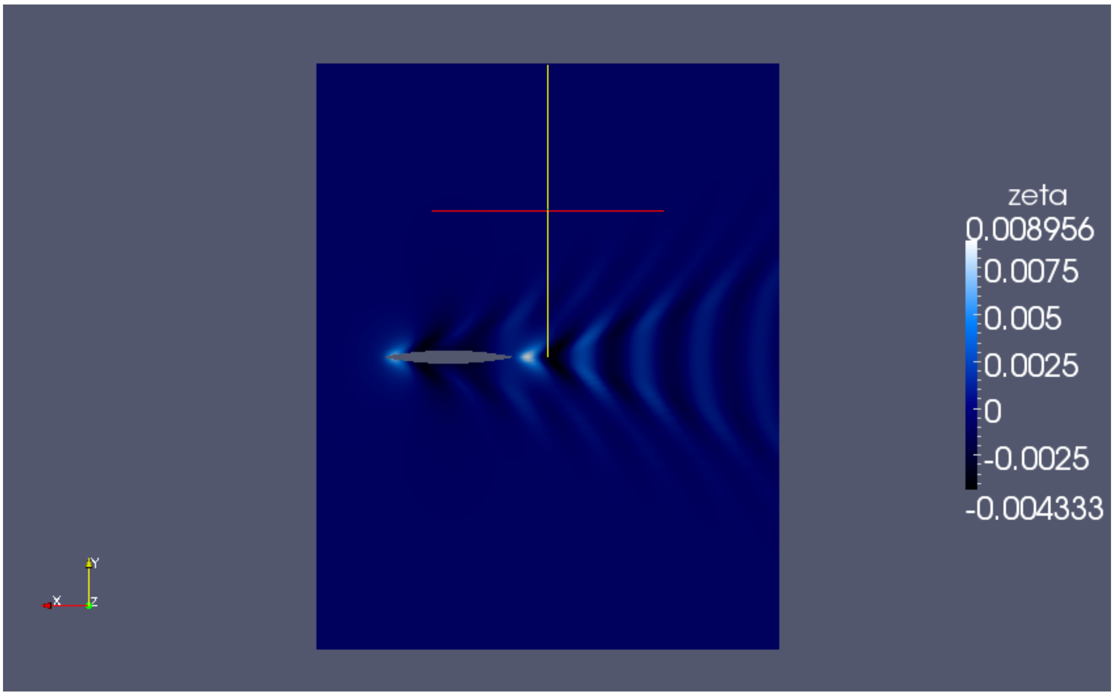

Fig8: Linear wave patter Fr=0.266

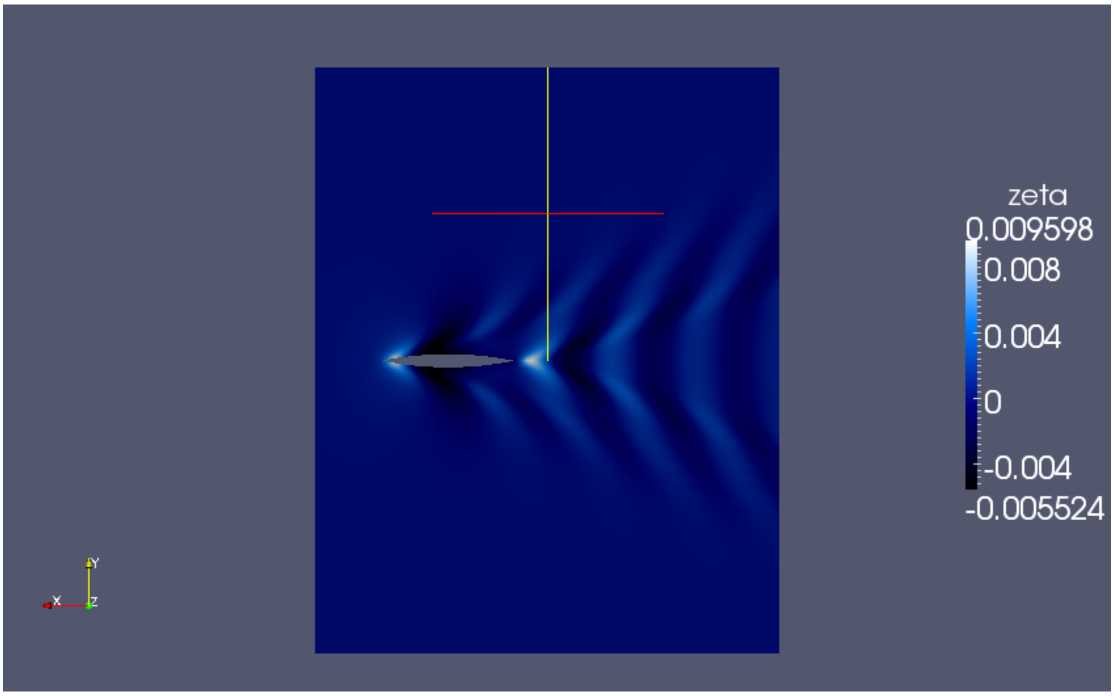

Fig9: Nonlinear wave patter Fr=0.313

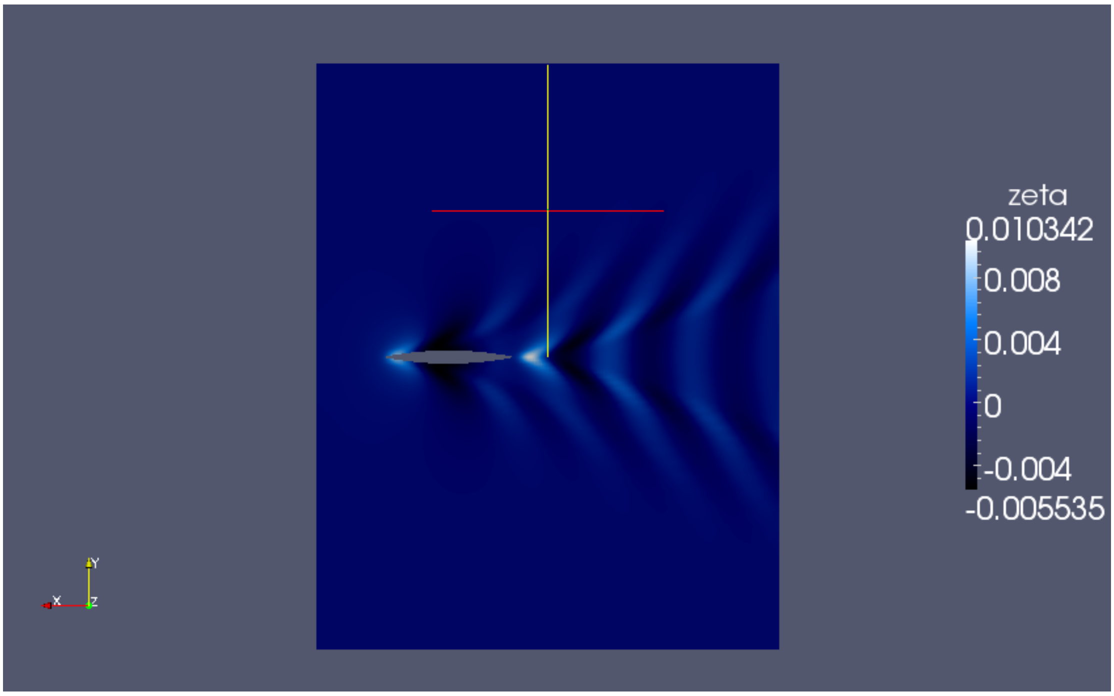

Fig10: Linear wave patter Fr=0.313

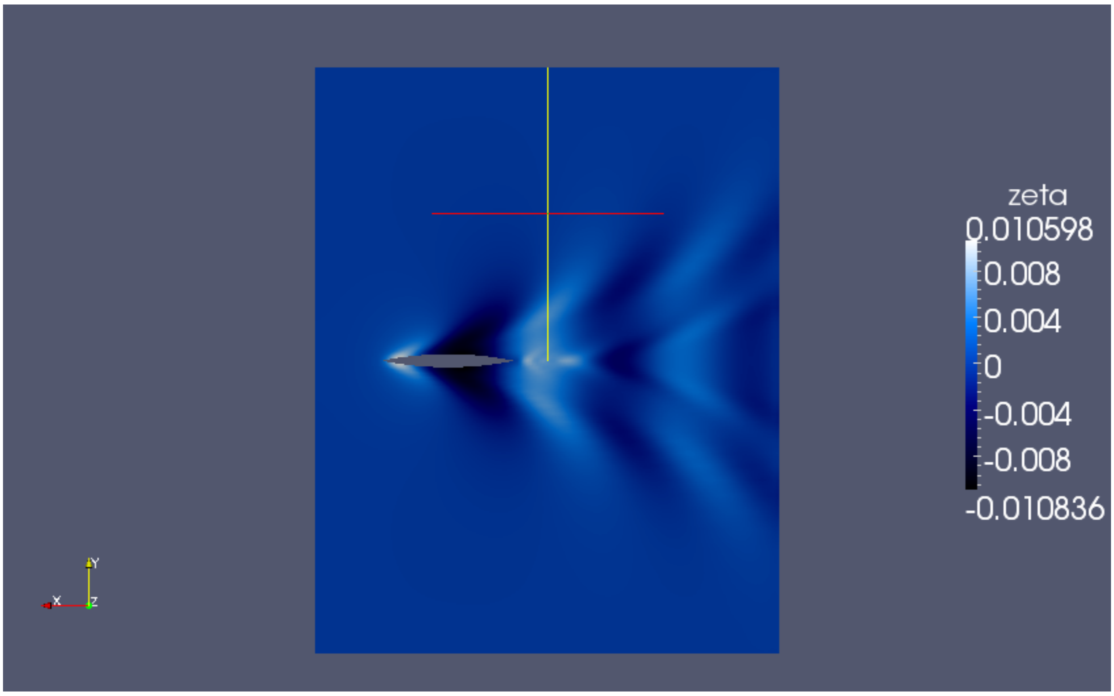

Fig11: Nonlinear wave patter Fr=0.402

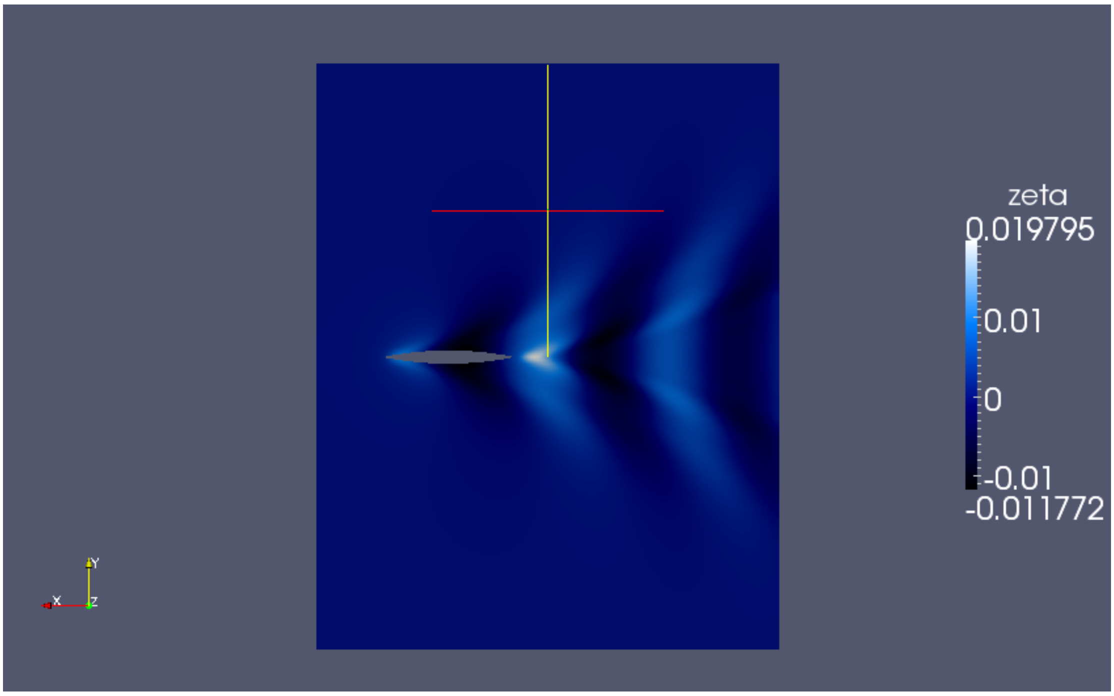

Fig12: Linear wave patter Fr=0.402

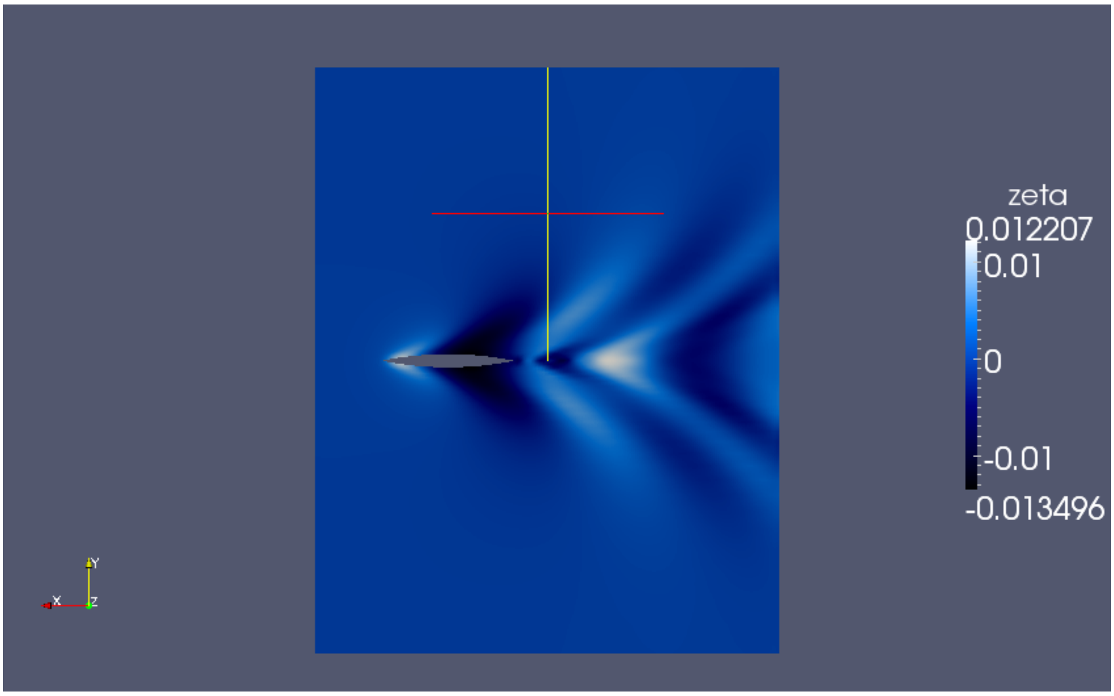

Fig13: Nonlinear wave patter Fr=0.452

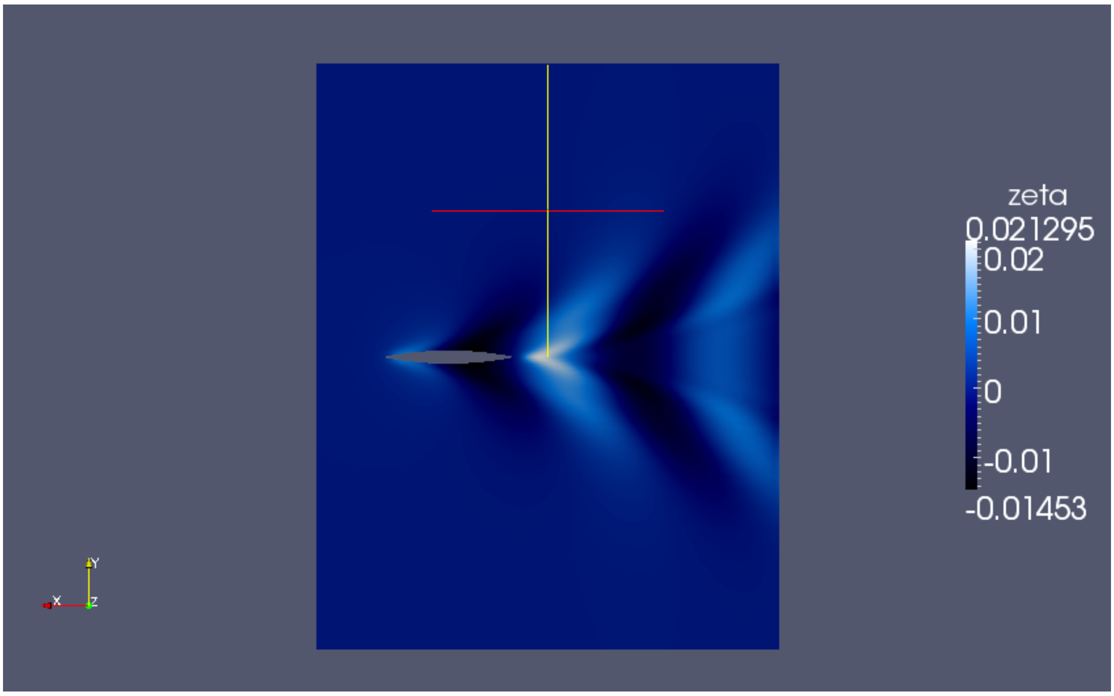

Fig14: Linear wave patter Fr=0.452

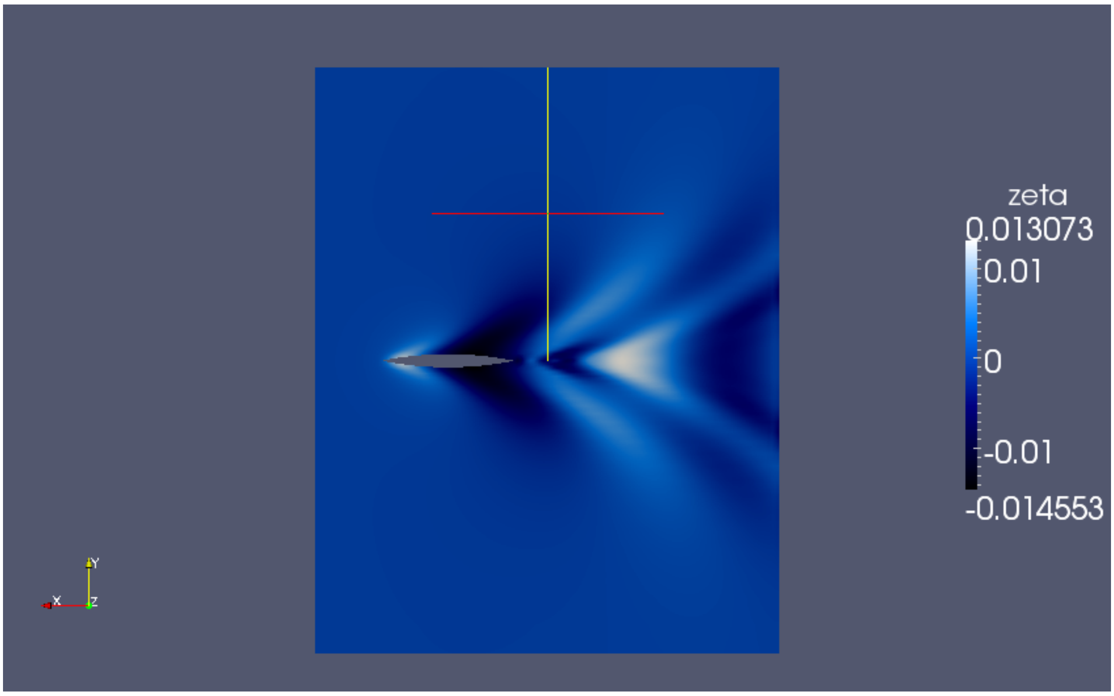

Fig15: Nonlinear wave patter Fr=0.483

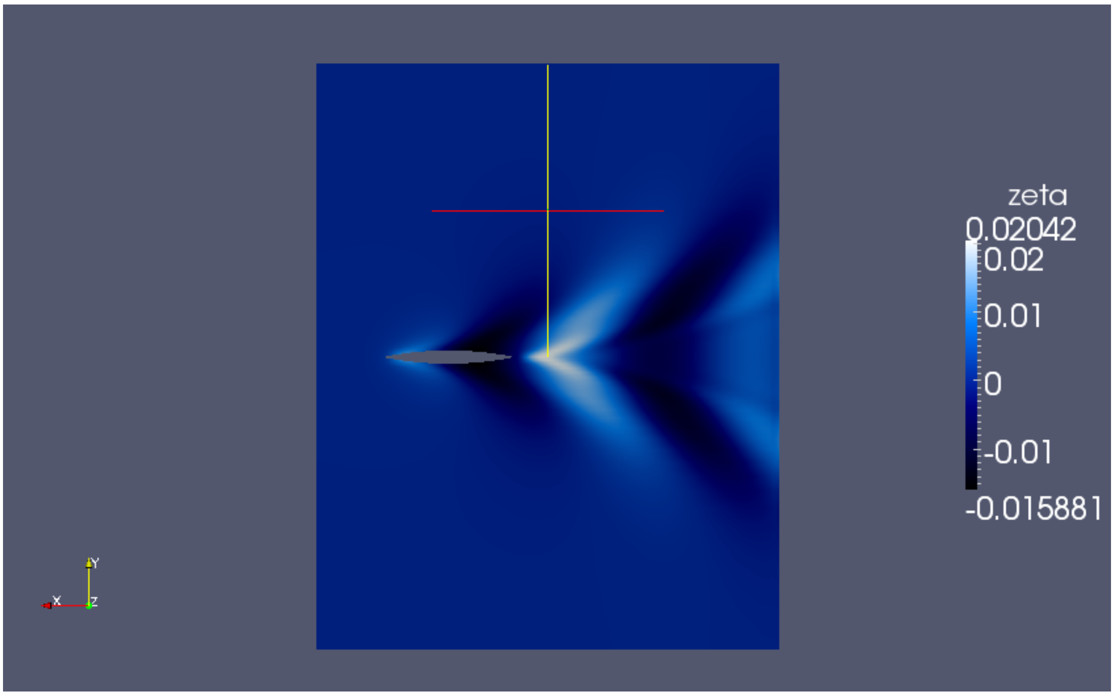

Fig16: Nonlinear wave patter Fr=0.483

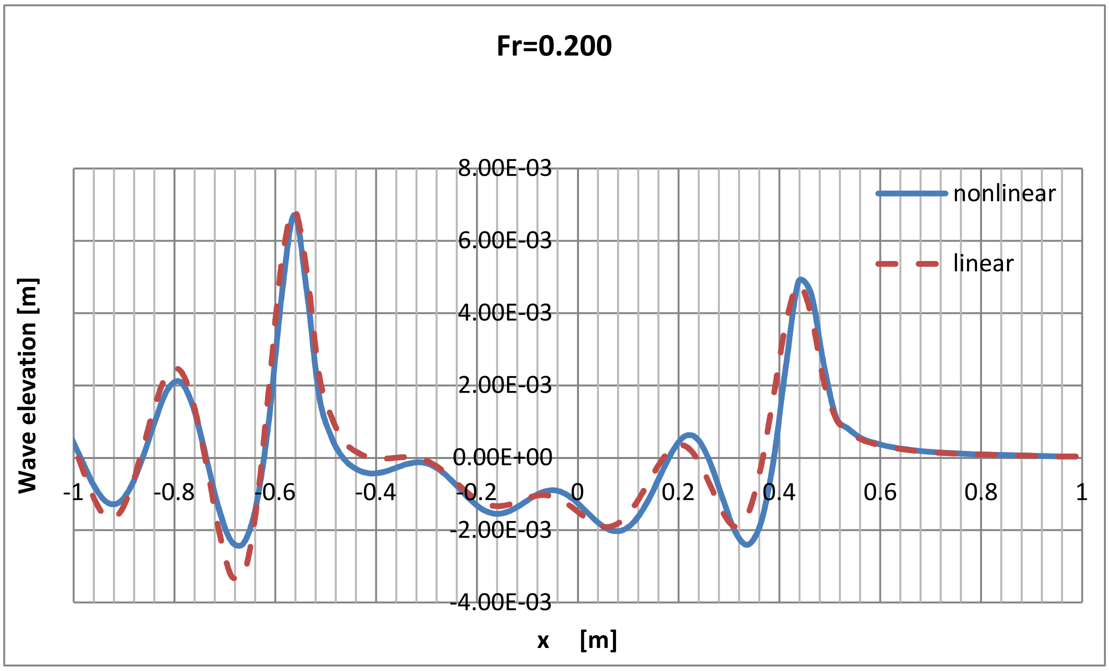

Fig17: Wave height profile at Fr = 0.200

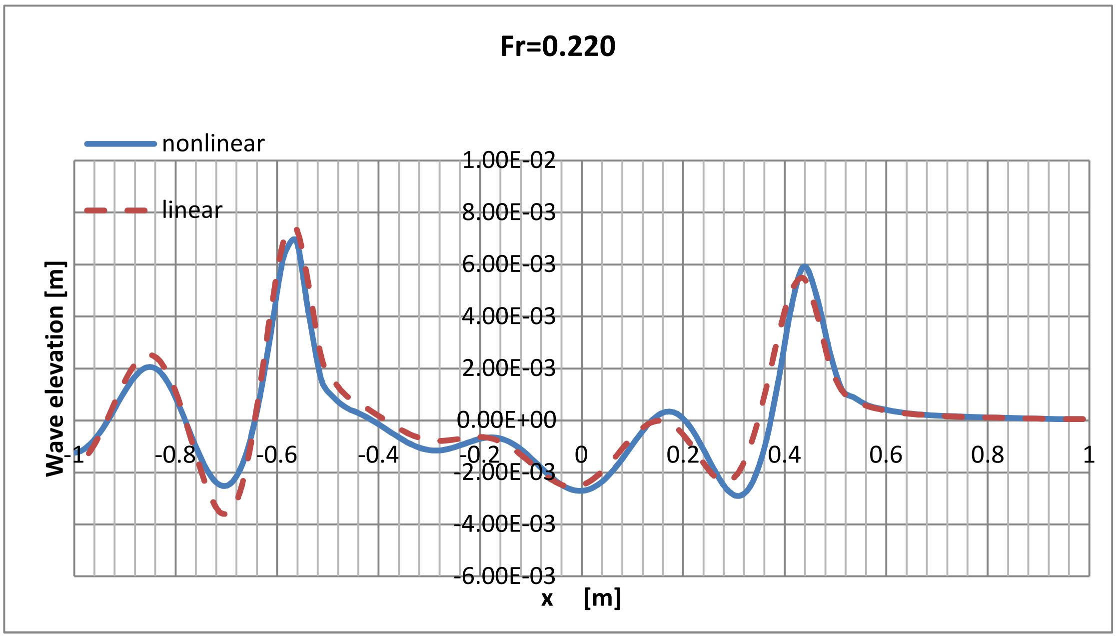

Fig18: Wave height profile at Fr = 0.220

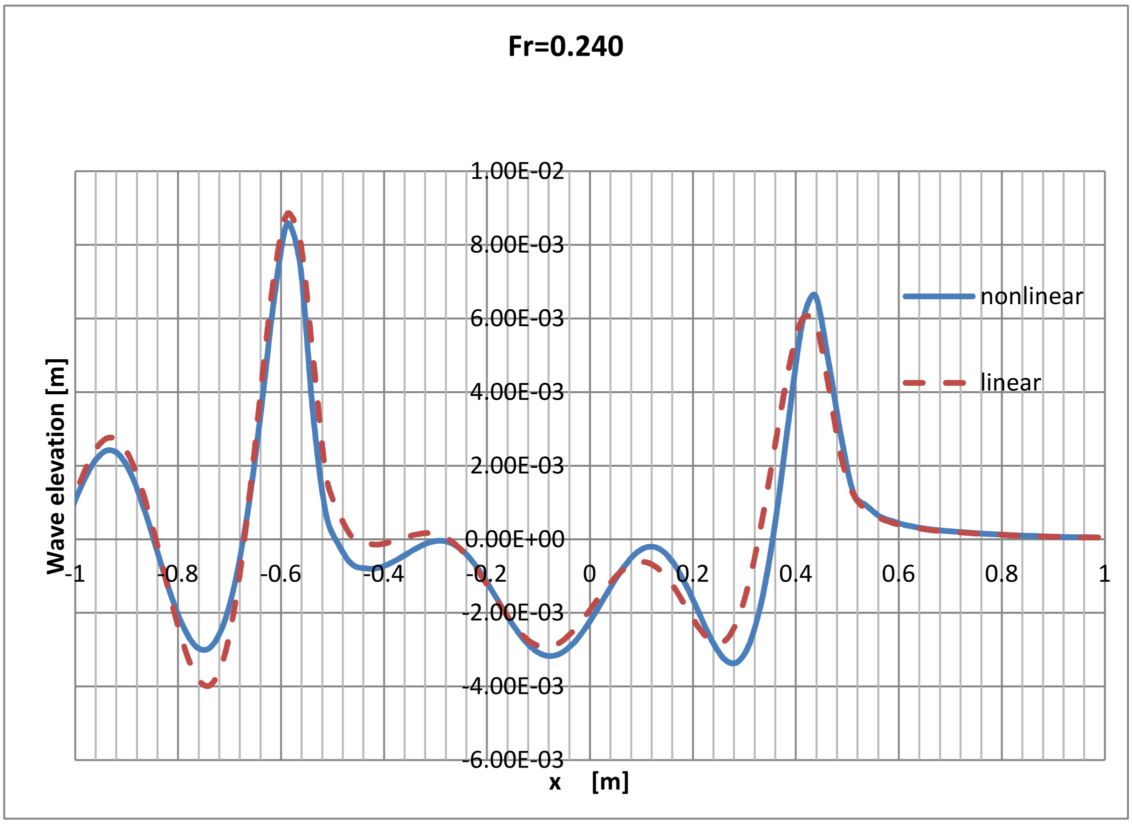

Fig19: Wave height profile at Fr = 0.240

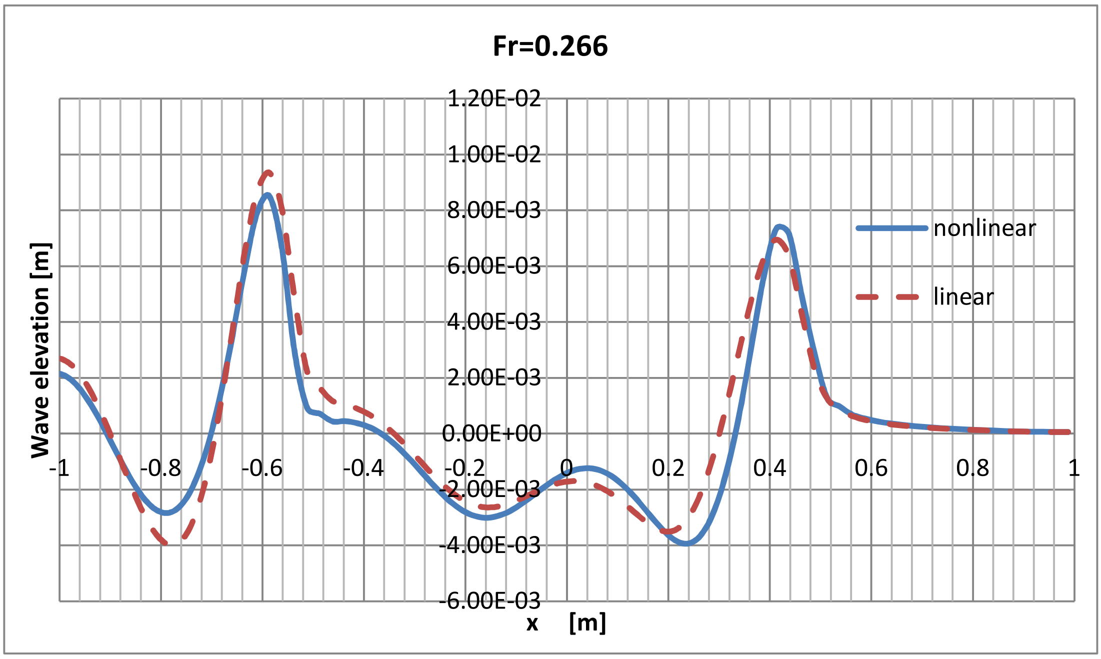

Fig20: Wave height profile at Fr = 0.266

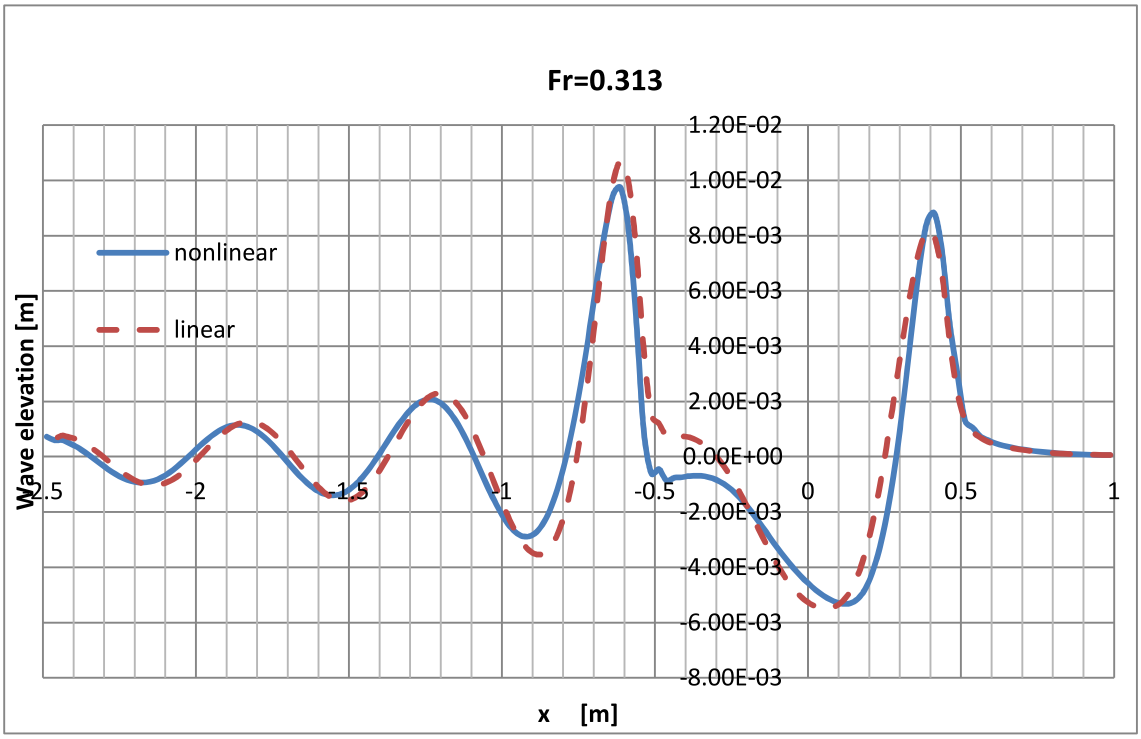

Fig21: Wave height profile at Fr = 0.313

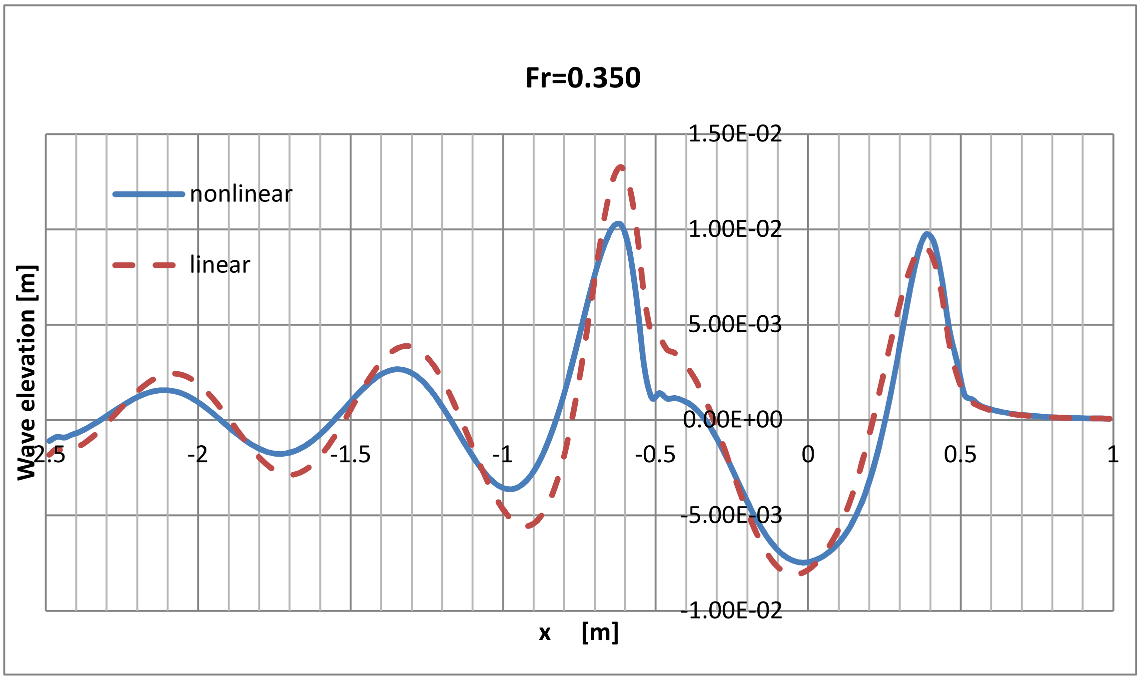

Fig22: Wave height profile at Fr = 0.350

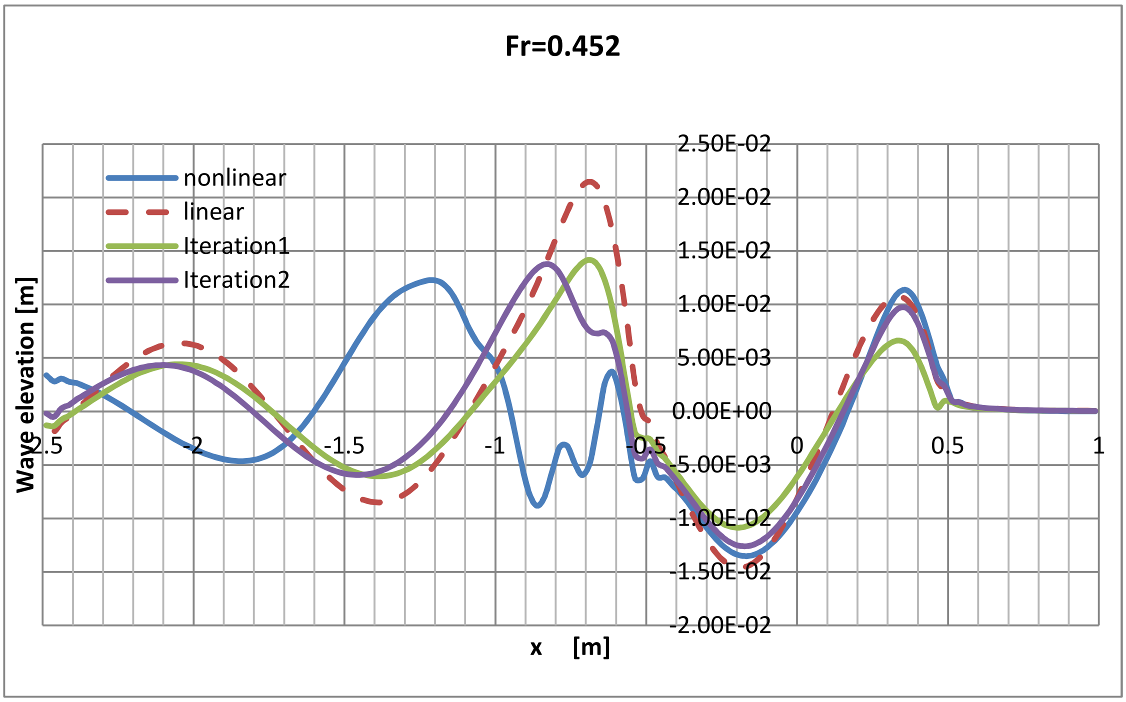

Fig22: Wave height profile at Fr = 0.452

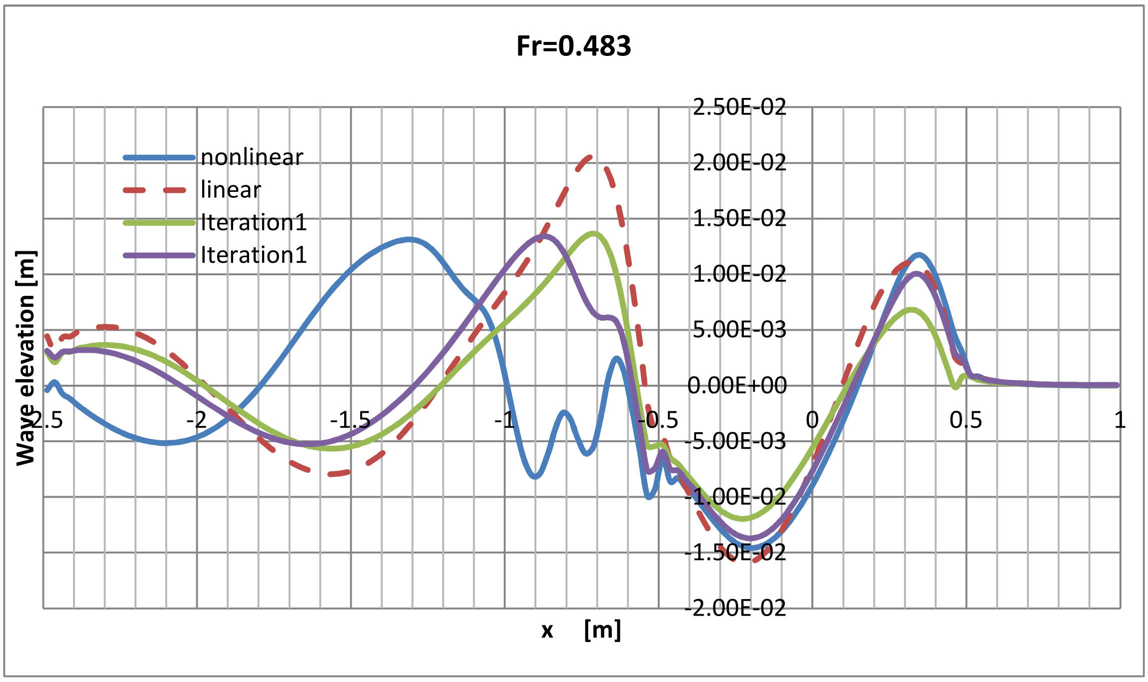

Fig23: Wave height profile at Fr = 0.483

Vita

The author was born in Guangzhou, China. In China, Guangzhou is one of the three major cities and has the third longest river - Pearl River. The shipbuilding industry is growing fast in Guangzhou. The author obtained his Bachelor of Engineering in Marine Engineering at the Wuhan University of Technology in June 2011. Wuhan University of Technology ranks 1st in this major in China. In order to widen his mind and improve his knowledge, the author decided to further his study on Naval Architecture & Marine Engineering and was admitted to a Master's program at the University of New Orleans. There, the author found his interest in Computational Fluid Dynamic (CFD) and decided to further his study in this field for his Master's thesis.