Spring 5-13-2016

Automated Sea State Classification from Parameterization of Survey Observations and Wave-Generated Displacement Data

Jason A. Teichman
University of New Orleans, New Orleans, jteichman71@gmail.com

Automated Sea State Classification from
Parameterization of Survey Observations
and Wave-Generated Displacement Data

A Thesis

Submitted to the Graduate Faculty of the
University of New Orleans
in partial fulfillment of the
requirements for the degree of

Master of Science
in
Applied Physics

by

Jason A. Teichman

B.S. Ferris State University, 2002

May, 2016

Approved for public release; distribution is unlimited.

ACKNOWLEDGEMENTS

There are a great number of people who have contributed to the realization of this thesis. My greatest thanks go out to Gus Michel, Lisa Pflug, Garry Owen, David Bates, and Wes Hillstrom. Your input and guidance kept me on-track and endlessly thinking. I would like to express my appreciation to Susan Sebastian, Clay Whittaker, and Theresa Anoskey for taking the time to walk me through many of the initial steps needed to gather my data. Thank you to the Naval Oceanographic Office, Mr. Randall Hill, and Dr. Michael Wild for graciously granting me the funding and time to work through this thesis, and to all of my colleagues who filled the void as I remained focused. Endless and ongoing thanks to my UNO thesis advisors Dr. Juliette Ioup and Dr. Stanley Chin-Bing for the years of education and patience. Lastly, warmest regards to my wife Samantha, who smiled as often as possible, especially when the times were hard.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABSTRACT

Sea state is a subjective quantity whose accuracy depends on an observer's ability to translate local wind waves into numerical scales. It provides an analytical tool for estimating the impact of the sea on data quality and operational safety. Tasks dependent on the characteristics of local sea surface conditions often require accurate and immediate assessment. An attempt to automate sea state classification using eleven years of ship motion and sea state observation data is made using parametric modeling of distribution-based confidence and tolerance intervals and a probabilistic model using sea state frequencies. Models utilizing distribution intervals are not able to exactly convert ship motion data into various sea states scales with significant accuracy. Model averages compared to sea state tolerances do provide improved statistical accuracy but the results are limited to trend assessment. The probabilistic model provides better prediction potential than interval-based models, but is spatially and temporally dependent.

Keywords: sea state; modeling; ship motion; Beaufort; World Meteorological Organization

CHAPTER 1 - INTRODUCTION

1.A. Sea State

The modern WMO (World Meteorological Organization) Sea State Scale in Table 1 (WOCE, 2002) describes the properties of locally driven, open-ocean wind waves. The code ranges from 0 (the calmest of conditions; the sea has a mirror-like appearance) to 9 (the worst conditions possible). The appearance of wind waves is predominantly generated by winds, the duration of the wind at speed, and the duration and size of the wind fetch; factors such as strong currents, precipitation, tides, and ice formations can also affect the developed sea state (White and Hanson, 2000). Swells are generally considered to be separate from wind waves, but the angle of the wind direction to the swell direction can significantly affect the agitation of local waves.

Table 1. WMO Sea State Scale

Code Figure	Descriptive terms	Wind-Wave Height (meters)
0	Calm (glassy)	0
1	Calm (rippled)	0 - 0.1
2	Smooth (wavelets)	0.1 - 0.5
3	Slight	0.5 - 1.25
4	Moderate	1.25 - 2.5
5	Rough	2.5 - 4
6	Very rough	4 - 6
7	High	6 - 9
8	Very high	9 - 14
9	Phenomenal	Over 14

Determination of sea state has traditionally been an in-situ process that requires subjective measurement, limited by the skill level of the observer and the observational conditions, and requires connecting wind forces with the state of the sea. In practice, the descriptive terms of the table are used as the primary guidance for classifying the seas; the wind-wave height becomes a secondary validation to the observer's sea state assessment.

Wind-wave conditions are often described and compared with the ubiquitous Beaufort Wind Scale which was developed and accepted into practice in the early 1800s. This scale is used in the maritime industry, including the organization responsible for the operation and maintenance of the survey ships that provided the data in this study. The Beaufort Wind Scale with corresponding WMO Sea State Codes is shown in Table 2 (Bowditch, 1995).

Table 2. Beaufort Wind Scale

Beaufort number or force	Wind speed (m/s)	World Meteorological Organization	Estimating wind speed	Sea State		
			Effects observed far from land	Term	Wave height (m)	Code
0	0.0-0.2	Calm	Sea like a mirror.	Calm (glassy)	0	0
1	0.3-1.5	Light air	Ripples with the appearance of scales are formed, but without foam crests.	Calm (rippled)	0-0.1	1
2	1.6-3.3	Light breeze	Small wavelets, still short but more pronounced; crests have glassy appearance and do not break.	Smooth (wavelets)	0.1-0.5	2
3	3.4-5.4	Gentle breeze	Large wavelets; crests begin to break; foam of glassy appearance; perhaps scattered whitecaps.	Slight	0.5-1.25	3
4	5.5-7.9	Moderate breeze	Small waves, becoming longer; fairly frequent whitecaps.	Moderate	1.25-2.5	4
5	8.0-10.7	Fresh breeze	Moderate waves, taking a more pronounced long form: many whitecaps are formed (chance of some spray).	Rough	2.5-4	5
6	10.8-13.8	Strong breeze	Large waves begin to form; the white foam crests are more extensive everywhere (probably some spray).	Very rough	4-6	6
7	13.9-17.1	Near gale	Sea heaps up and white foam from breaking waves begins to be blown in streaks along direction of the wind.			
8	17.2-20.7	Gale	Moderately high waves of greater length; edges of crests begin to break into spindrift; foam is blown in well-marked streaks along the direction of the wind.			
9	20.8-24.4	Strong gale	High waves; dense streaks of foam along the direction of the wind; crests of waves begin to topple, tumble and roll over; spray may affect visibility.			
10	24.5-28.4	Storm	Very high waves with long overhanging crests; the resulting foam, in great patches, is blown in dense white streaks along the direction of the wind; on the whole, the surface of the sea takes a white appearance; the tumbling of the sea becomes heavy and shock-like; visibility affected.	High	6-9	7
11	28.5-32.6	Violent storm	Exceptionally high waves (small and medium-sized ships might be for a time lost to view behind the waves); the sea is completely covered with long white patches of foam lying along the direction of the wind; everywhere the edges of the wave crests are blown into froth; visibility affected.	Very high	9-14	8
12	32.7 and over	Hurricane	The air is filled with foam and spray; sea completely white with driving spray; visibility very seriously affected.	Phenomenal	Over 14	9

The sea state codes used by the scientific teams aboard the survey vessels that provided the data in this study are a modified form of the WMO and Beaufort tables. This table is adjusted to represent specific mission requirements and is dependent primarily on wind speed and a description of the local sea conditions. Table 3 details the wind and sea surface conditions required to select operationally sound conditions for the surveys in this study (Velazquez-Aviles et al., 1999-2014). This sea state scale will be referred to as the Operational Scale. It is expected that this table has been followed to some degree of accuracy by all observers since 2004.

Table 3. Survey Operations Sea State Scale

OPERATIONAL SEA STATE DESCRIPTIONS				
SS#	**SEAS**	**DESCRIPTION**	**WIND (knots)**	**BEAUFORT**
0	Calm	Flat, glassy, calm. Smoke rises vertically.	<1	0
1	Smooth Sea	Ripples, no foam. Light air. Not felt on face.	1-4	1
2	Slight Sea	Small wavelets, no foam. Light to gentle breeze. Felt on face. Light flags wave.	4-10	2-3
3	Moderate Sea	Large wavelets. White caps begin to form. Gentle to moderate breeze. Light flags extended.	7-15	3-4
4	Rough Sea	Moderate waves, many white caps, some spray. Moderate to strong breeze. Wind whistles in rigging.	14-21	4-6
5	Very Rough Sea	Sea heaps up, with spindrift and foam streaks. Moderate to fresh gale. Walking resistance high.	21-41	6-8
6	High Sea	Waves begin to roll, dense streaks of foam, much spray. Strong gale. Loose gear and light canvas may part.	40-48	9
7	Very High Sea	Very high waves with overhanging crests. Sea appears white as foam scuds in very dense streaks. Visibility reduced.	48-55	10
8	Mountainous Sea	Very, very high-rolling breaking waves. Sea covered with foam. Very poor visibility. Storm.	55-65	11
9	Are you nuts?	Quit worrying about Sea State and go somewhere else!	>65	12

The Douglas Sea State Scale developed in the 1920s converts wave and swell heights into similar numerical codes, but given the subjective nature of these measurements, this scale is not included.

1.B. Limitations

1.B.1. *Observations*

Sea state measurements that are founded on ocean wave characteristics are limited by the ability of the observer to discern wave characteristics and categorize the observations into an appropriate scaling. The observer may have a limited skill set or may be impeded from making a sound observation by weather or visibility conditions that prevent an accurate assessment. This is particularly true at night, when determining a wind-wave sea state is almost impossible. In addition, mariners and scientists are given multiple scales to consider, and it is possible that scales used in one event are not the same scales used in another event. Although the confusion and subjectivity of observations have still produced many successful missions, there is a noticeable loss or corruption of data due to operations conducted in conditions that were misdiagnosed or rapidly changing.

The survey events in this study require the use of sea state as a predictor of data quality and operational safety. Some of the operations were conducted at night; sea states in these cases were highly subjective, and errors in observations are expected.

1.B.2. *Operations and Safety*

Reliable estimation of sea state is essential to decision support systems for effective oceanographic operations. Oceanographic operations that require calm conditions to perform adequately are dependent on the state of the sea. This study was developed from a need intrinsic to acoustic operations. In the survey events from which this study is derived, operational safety and noise are leading concerns and an accurate sea state assessment provides operations leaders with a more complete picture of the state of the sea. The subjective nature of the assessment has created a climate of contention in the community of operators. This study will attempt to alleviate the subjectivity and provide a more objectively quantifiable assessment base.

Every surface mission has sea-state limitations. In most cases, the limiting sea state is 4 or 5 (NAVOCEANO, 1999; NAVOCEANO Personnel, 2015). At sea state 4, concern for operational safety becomes significant. Sea states higher than 4 can severely impact operational safety. Higher sea states also render certain oceanographic missions ineffective. For example, sonobuoy radio frequency (RF) dropout created by a "washover" of the signal output equipment often occurs with seas that are above sea state 5 (NAVOCEANO, 1999).

Ambient noise levels increase with sea state, especially in the frequency range between 300 and 5000 Hz (NAVOCEANO, 1999). For each increase by 1 in the sea state code, ambient noise levels increase by approximately 6 dB (NAVOCEANO, 1999) and as much as 10 dB depending on the frequency and depth (Waite, 2005). In survey operations where good signal quality is critical, wind-wave interactions are a significant cause for data processing failures.

1.C. Prior Research of Sea State Automation

The connection of wave characteristics to the sea state has been studied throughout most of the second half of the 20^{th} century. Attempts to mitigate the problem of observational subjectivity have been a prominent subspace of this research. Although there have been many studies conducted by civilian and governmental organizations, only a few noteworthy publications are provided.

In the early 1950s, Diede and Thieme [5] conducted a general survey of the studies and instruments being developed for measurement of wind conditions and wave heights, lengths, and frequencies. They produced a summarization of the progress made in aircraft measurement devices and the use of optics to determine wave characteristics. They noted that at the time of the survey, only the U.S., England, Germany, and France were conducting studies in this area. Their conclusions made note of the optimism of the future for the research being conducted.

A preliminary study published by Clayton, Ivey, and Teegardin in 1954 [3] was conducted to develop a sea state meter using bare and dielectric-coated wires in conjunction with a slope-measuring unit created for the experiment. It was intended to produce height and slope data of ocean waves in an attempt to statistically determine sea state. Their experimental cycle rates were too low to produce the results they desired, but as of the time of the publication, they had determined a rate that showed promise.

Black and Adams [1] utilized vertically pointing aircraft photos to determine surface winds in an attempt to record Beaufort wind force values. This was done to assist in the training of personnel who utilized sea state measurements as a part of their working systems.

In the fall of 1989, scientists at the Naval Underwater Systems Center conducted experiments using the newly patented Submarine-Deployed Sea State Sensor (SUDSS) (Shonting et al., 1989). They were able to show the device had the ability to take a wide variety of sea surface measurements with varying degrees of accuracy. Other methods were eventually developed by the submarine community, and at present, the SUDSS is not being used operationally (NAVOCEANO Personnel, 2015).

Work published in 2000 by White and Hanson [29] at Johns Hopkins University utilized directional wave spectra and wind velocity data obtained from National Data Buoy Center (NDBC) weather buoys to create an effective wind speed that could then be translated into a Beaufort force and corresponding sea state codes. Their tests concluded that calculated sea states reasonably coincided with visually observed sea states.

As of the development of this thesis, no known attempts to connect sea state observations with vessel displacement measurements have been published.

1.D. Proposed Research

The intent of this study is to connect sea state observations with associated in-situ wave-riding characteristics of a naval survey vessel in an attempt to find a meaningful, deterministic model that can numerically assess the state of the sea with a significant degree of accuracy. It should be sufficient to utilize statistical methods to develop a parameterization of sea-state specific data that can be used to determine if information produced by displacement measurements fall within parameters of a given sea state. A distribution fitting method using tolerance and confidence intervals of combined displacement effects (as well as derivatives of these effects) will provide the limits that will be used in various modeling schemes. A variety of probability distribution types will be used. If a specific distribution provides a significantly more accurate assessment, the limits developed will drive the applied routines for this study. If there is a substantially uniform return for the distributions used, a more comprehensive use of the parameters may be examined.

Although wind contributes greatly to the state of local seas, its inherent variability in both speed and direction limit its use as an effective measurement for the purposes of this study. Average wind speeds are used to determine Beaufort and operational sea state codes in an attempt to determine if model results can additionally predict the measurements from these scales. It is expected that further development of this research will eventually include wind measurements in some form.

It should be noted that if a relationship between sea state and ship displacement exists and a deterministic method can be developed for predicting the state of the sea, the data must be managed in real time using shipboard acquisition and processing systems. Any model developed must contain algorithms that do not place unrealistic computational demands on the shipboard systems.

CHAPTER 2 - SURVEY DATA AND OBSERVATIONS

2.A. Data Collection

The data sets used for this study were collected from naval oceanographic surveys from 2004 to 2014. A total of 19 surveys were conducted in similar geographical locations, each survey consisting of between 7 and 36 individual survey events. Each event ranged from approximately 15 minutes to 3 hours. In earlier surveys, more than one event was conducted daily; later surveys are generally limited to one survey event per day. Collectively, there were a total of 410 separate survey events.

Table 4 details a parsing of the total number of recorded events by sea state. The unknown values represent events that had no sea state observation record.

Table 4. Pre-Selected Survey Event Count by Sea State

SS	n
0	9
1	86
2	136
3	149
4	19
5	4
6	2
Unknown	5
Total	410

The data were produced by crews and measurement systems aboard three naval survey vessels. Each ship was of the same construction class and sufficiently similar to be considered the same platform type for the purposes of measurement standardizations.

During each event, a shipboard inertial navigation system provided in-situ dynamic data, and an anemometer-based system measured both true and apparent wind speeds. The ship's Inertial Measurement Unit (IMU) utilized angular accelerometers to measure gravitation-based angular displacements (roll, pitch, and yaw), and linear accelerometers to measure non-gravitational accelerations that translate to linear motion displacements (heave, surge, and sway) (Eschbach et al., 1990; King, 1998). A calibrated gyroscopic element in the IMU is utilized to maintain an absolute plane of reference. The ship's axes were periodically updated to maintain a zeroed axis plane of reference (NAVOCEANO Personnel, 2015); the standard practice of ship axes calibration for the purpose of sensor performance and accuracy is expected and was confirmed by header data used to display each offset and its calibration. All dynamic and environmental data collected by the sensors were then chronologically recorded in an onboard system that parses and archives the data in the highest resolutions available.

A sea state observation based on the scaling in Table 3 was made at some time during the event (the exact time is never recorded). It was expected that the observer used the scale in Table 3, but observational tempo or limitations sometimes relegated the task of sea assessment to the ship's crew who exclusively used the Beaufort scaling in Table 2. In some cases the sea state changed during the event; in these instances, the higher sea states were used for this analysis. This was done to reflect the greater variability in the ship motion due to the increased sea conditions and represents a conservative estimate that leans in the direction of operational safety. Since surveys required operationally effective seas, only measurements of sea state 0 to sea state 6 were observed.

A shipboard anemometer was used to record the wind speed and direction. Processing software produced both apparent and true wind speeds and directions. For the purpose of this study, only true wind speeds are considered.

2.B. Extraction and Quality Control

The raw, time-series displacement and wind data from organizational archives were downloaded using LINUX/UNIX command-line functions and parsed by survey and Julian date.

To achieve the largest relevant sampling possible only events with periods greater than or equal to an hour were included; all other samples were rejected. Most samples were in excess of an hour but generally less than 2.5 hours. The only exception to this rejection criterion was a sea state 6 event with less than an hour of data. This event was not rejected in an attempt to provide sea state 6 data; there are only two sea state 6 events.

Events that were missing key information such as a sea state observation were rejected. Missing wind data was not considered a rejection criterion; wind is not used to provide limiting parameters in this study. Additionally, if the ship's course was not reasonably consistent, the event was rejected. Course changes can cause significant roll or pitch to occur which could contaminate the data for the purposes of this study.

A total of 23 survey events were rejected.

Since the purpose of this study is to utilize statistical arguments to justify the parameters needed to develop a sea state model, sample sizes in Table 4 are questionable. For example, given a standard deviation of $\sigma \approx 0.90521$ determined through preliminary testing, with a margin of error formula given by Meyer (1975),

$$ME = z^* \frac{\sigma}{\sqrt{n}}$$

where $z^* = 1.037,\ 1.282,\ 1.645, 1.960$ and 2.576 for confidence levels of 70%, 80%, 90%, 95%, and 99%, respectively, requires minimum sample sizes as detailed in Table 5 for margins of error ranging from ±1% to ±10%.

Table 5. Minimum Sample Sizes

ME\CONF	70%	80%	90%	95%	99%
±10%	88	135	222	315	544
±9%	109	166	274	389	671
±8%	138	210	346	492	850
±7%	180	275	453	642	1110
±6%	245	374	616	874	1510
±5%	352	539	887	1259	2175
±4%	551	842	1386	1967	3398
±3%	979	1496	2464	3498	6042
±2%	2203	3367	5543	7870	13593
±1%	8812	13467	22173	31478	54374

This criterion suggests that only parameters and results using sea state data from sea states 2 and 3 can be considered statistically significant. Specifically, the sample sizes in Table 5 allow a minimum margin of error of 9% for a 70% confidence level for sea state 2, and a minimum margin of error of 8% for a 70% confidence level or 10% for an 80% confidence. Due to sample sizes in this study, a large margin of error will exist at confidence levels higher than

70% for most of the data. Discussions relevant to all other sea states will be speculation based on trends.

2.C. Data Analysis

The following sections discuss the particular facets of the extracted data. Section 2.C.1. covers the extraction counts and partitions as well as a brief discussion of the sea state frequencies. An overview of the seasonal and diurnal observations is covered in section 2.C.2. A comparison of observed sea states to wind-generated Beaufort and Operational scales is accomplished in section 2.C.3. This analysis is conducted to provide comprehensive statistics of the selected data sets, and to provide conditions and validations for further analysis.

2.C.1. *Selected Data*

A total of 387 events were selected for inclusion in this study and assigned a number from 001 to 387. Table 6 is the selected event count by sea state. Table 7 details each survey and its associated Julian date range, number of individual selected events, and sampling resolution. Table 8 lists the number of survey events per ship platform.

Table 6. Selected Survey Event Count by Sea State

SS	n
0	9
1	81
2	129
3	146
4	16
5	4
6	2
Total	387

Table 7. Selected Survey Data

Survey #	JD Range	Event #	Res (sec)
1	144 – 159	21	0.52
2	170 – 191	32	0.52
3	037 – 061	21	0.52
4	073 – 095	25	0.52
5	169 – 192	23	1.04
6	202 – 227	20	1.04
7	231 – 251	23	0.52
8	264 – 284	17	0.52
9	240 – 254	23	0.50
10	264 – 284	12	0.50
11	058 – 083	30	0.20
12	135 – 158	33	0.20
13	173 – 196	18	0.20
14	180 – 199	24	0.10
15	217 – 233	22	0.10
16	292 – 304	12	0.10
17	326 – 336	7	0.10
18	282 – 310	17	0.10
19	320 – 332	7	0.10
Total		387	

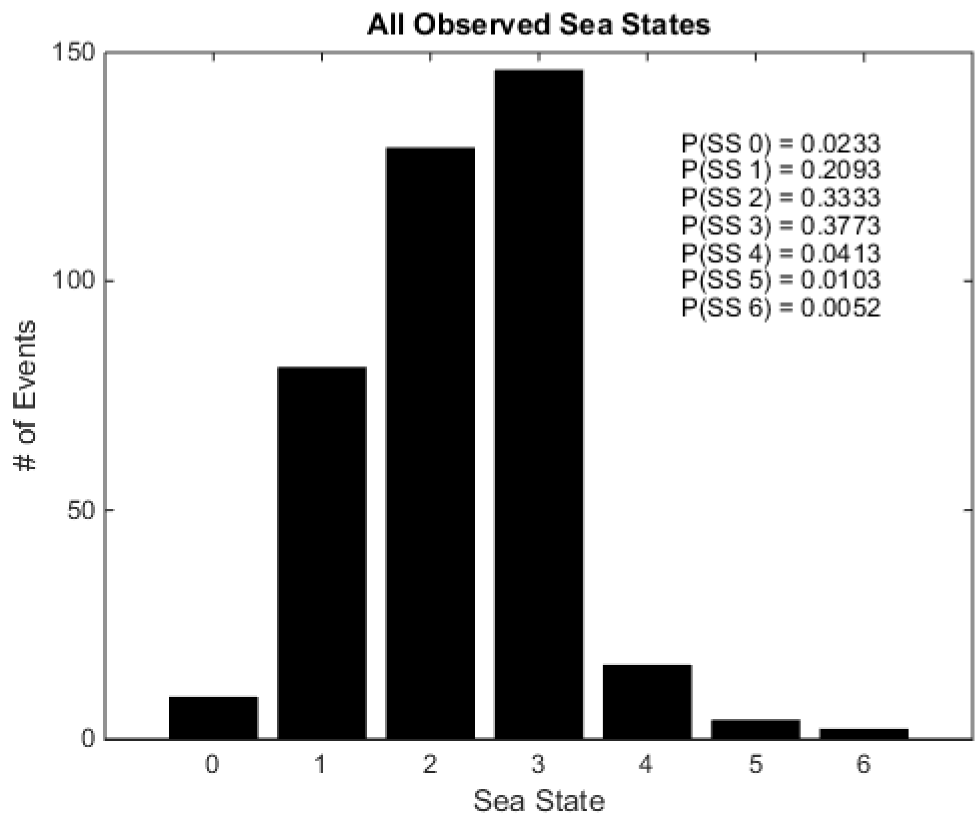
It should be noted that Surveys 5 and 6 do not have any associated wind data.

Table 8. Event Count by Ship Platform

Ship	# of Events
1	298
2	65
3	24

The boxplot in Figure 1 displays the event frequency and associated probability of the occurrence of each sea state. Given the wide wind variety available in the Operational scaling codes, there is a greater likelihood of sea states existing from 1 to 3. Operationally, sea state 3 conveys the greatest sea condition variability and the larger number of sea state 3 observations suggest validity to this claim.

Figure 1. Sea State Frequencies



A similar distribution of sea state frequencies is available in a technical report by Shonting, Hebda, McCarthy, and Chaves (1989).

2.C.2. *Seasonal and Visibility Information*

Surveys were conducted in both temperate and equatorial climates, under all possible conditions of visibility. The majority of surveys were done in the lower latitudes making the impact of seasonal data less significant. Although not specifically beneficial to this study, seasonal information provides an environmental backdrop for further research. Visibility conditions have a greater influence on the assessment of the state of the sea, and subsequently, survey events are parsed by light conditions to enhance environmental intelligence as it pertains to this study.

A meteorological season standard (NOAA, 2013) is used to partition survey events, such that for non-leap years, the Julian date ranges are: spring (60-151), summer (152-243), fall (244-334), and winter (1-59, 335-365), and for leap years: spring (61-152), summer (153-244), fall (245-335), and winter (1-60, 336-366). Given this scheme, there were a total of 85 spring surveys, 182 summer surveys, 97 fall surveys, and 23 winter surveys. Figures 2-5 are histograms that display the frequency of events in a season by sea state.

Figure 2. Spring Surveys

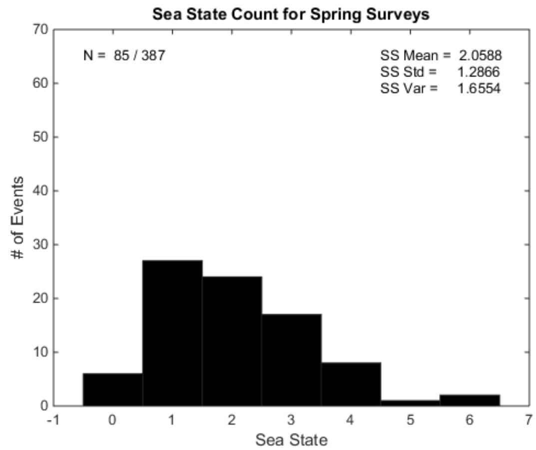


Figure 3. Summer Surveys

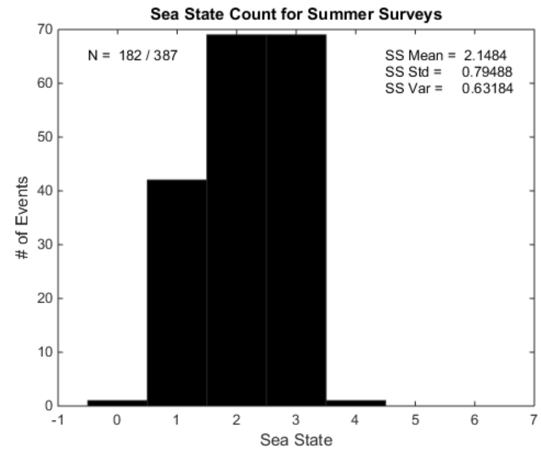


Figure 4. Fall Surveys

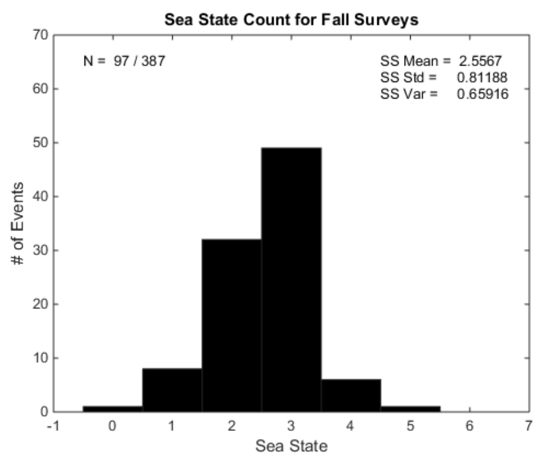
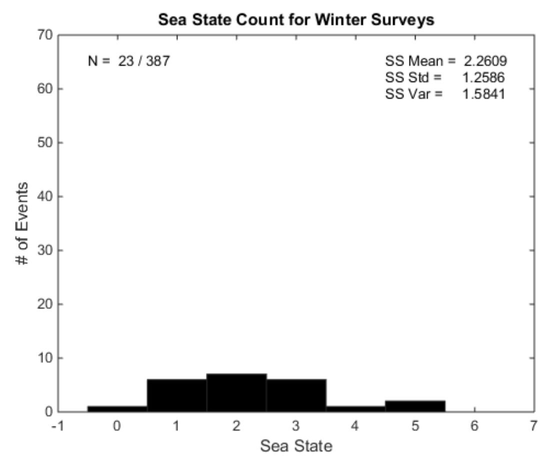


Figure 5. Winter Surveys



Visibility has a substantial impact on the assessment of the sea. Given the potential for inaccurate sea state assessments due to nighttime (poor light) observations, it is important to note the number of survey events conducted in good and poor light conditions. If it is expected that limited visibility begins due to nightfall, approximately 21:00 (military time) on average, which prevails until approximately 05:00, then observations that are made within this period will be considered limited by poor lighting. The light conditions and associated counts for each of the surveys are detailed in Table 9.

Table 9. Observational Visibility Counts

Survey #	Good Light	%	Poor Light	%
1	11	52.4	10	47.6
2	25	78.1	7	21.9
3	15	71.4	6	28.6
4	14	56.0	11	44.0
5	17	73.9	6	26.1
6	12	60.0	8	40.0
7	14	60.9	9	39.1
8	12	70.6	5	29.4
9	15	65.2	8	34.8
10	10	83.3	2	16.7
11	25	83.3	5	16.7
12	26	78.8	7	21.2
13	12	66.7	6	33.3
14	12	50.0	12	50.0
15	17	77.3	5	22.7
16	9	75.0	3	25.0
17	5	71.4	2	28.5
18	15	88.2	2	11.8
19	6	85.7	1	14.3
Total	272	70.3	115	29.7

2.C.3. *Wind Scale Comparisons*

The observed sea states in events that have associated wind data (Surveys 1-4, 7-19) were compared to Beaufort and Operational sea states using the mean of the true wind speed for the duration of the event. Although the wind speed average may not have coincided directly with the wind speed at the time of the observation, the mean was sufficient to show the trend of the wind in the event locale.

Average winds speeds for each event were converted to the Beaufort and Operational sea state codes on Tables 2 and 3, respectively. The sea states were then compared for each event and a count of matching sea states was determined. Figures 6 and 7 display the comparison between Beaufort and Operational sea states to observational sea states for events 001 to 099 and 142 to 387. These events have wind data that could be converted to a sea state scale.

Figure 6. Beaufort SS Conversion vs. Observed SS

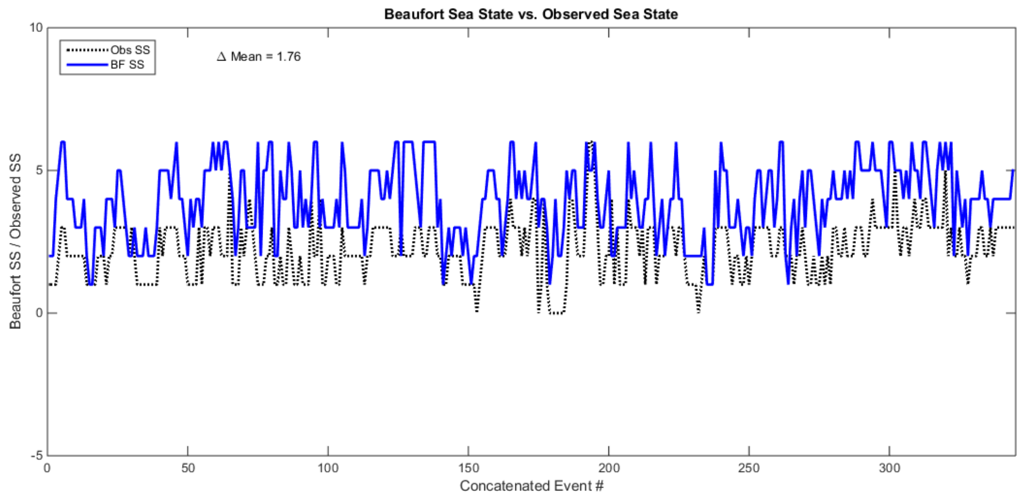
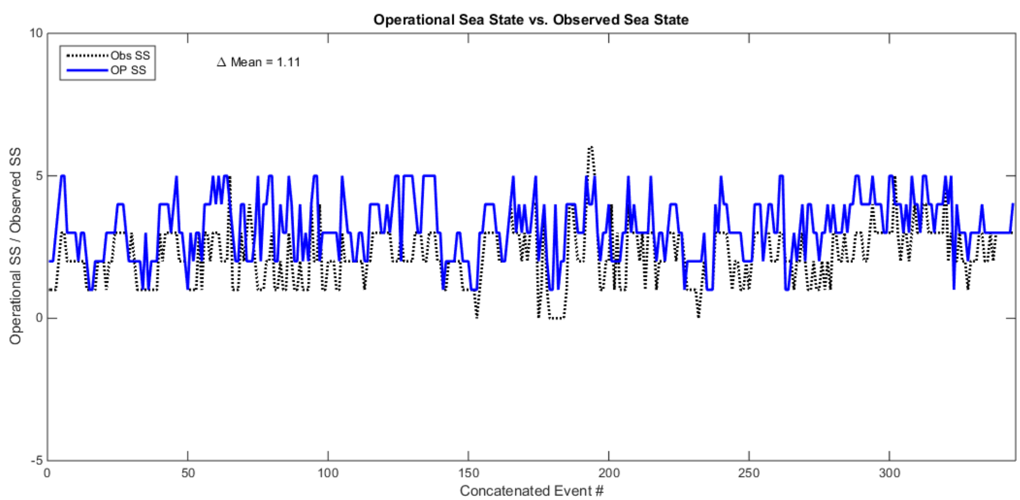


Figure 7. Operational SS vs. Observed SS



Comparison shows that 37 out of 344, or approximately 10.8 % of the Beaufort sea states match the observations made during each event. Operational sea states compared to the observed sea states show a closer comparison. In this case there were 80 of 344 matches, or approximately 23.3%. Both counts suggest that true wind speed converted to Beaufort or Operational scales may not accurately represent the state of the sea (assuming observations are accurate). It is possible that with further analysis, it may be determined that wind is accurate to define local seas.

The closer correlation of the observed sea state with the operational sea state scaling suggests that observers were following the Operational scale as a whole. It is known that occasional recording of Beaufort measurements taken from the bridge crews were entered into the logs due to reliance on the ship's crew for sea state, especially in limited visibility conditions (NAVOCEANO Personnel, 2015).

The mean errors greater than 1 for both comparisons suggest that observers generally underestimate the state of the sea in comparison to the wind-scaled codes (or conversely, the wind-scaled codes overestimate in comparison to the observer). Note that if the observed sea state was increased by 1 in each event, the observations would very closely match operational sea state codes.

It should be noted that it is common for wind speeds to precede or follow sea state measurements due to fetch activity or storm systems. In these cases, the wind may not represent the true state of the sea. This results in mismatched or misdiagnosed sea states when wind-based scales are used.

CHAPTER 3 – METHODS AND RESULTS

3.A. Considerations

To support the fundamental goal of developing a model or a set of models that will efficiently and accurately determine the sea state or potential of a sea state given real-time data supplied by a sea-going vessel's sensors, several components need to be considered.

In each survey event, the ship's course and speed remained constant. The course was maintained consistently with minor corrections from an automated control system; the variation was generally less than 10 degrees and carefully scrutinized in the extraction phase of this study. The speed of the ship was maintained at a constant value—either 4 or 8 knots. The relatively slow speed (generally just enough for steerage) should not significantly affect the displacement values.

Wind can have an obvious influence on the displacement motion of a ship—particularly the angular displacement of roll. Given the sea states of this study, the assumption can be reasonably made that winds under sea state 4 are generally not large enough (a wind average of 16.3 knots, or approx. 8.4 meters per second) to affect the displacement values significantly. Wind direction can greatly affect sea conditions if orthogonal to swells, but the sum effect will be considered present at the displacement of the vessel.

The development of any model in this study will require sufficiently significant independent variables. Displacement values and a power spectrum of the time-series data, realized in both arithmetic and root mean square terms, will provide the data to be fit to a variety of distributions. The distributions will determine limitations that can used to parameterize the individual sea states. Models can then be made to produce sea states from each distribution contribution. Ideally, a best distribution fit will be found and a final model can be developed using its associated limits. If a best fit is not determined, a combination of distributions may provide a broad result set. It may be possible to construct a set of values that will give the end-user a “picture” of the potential sea state(s) and provide a better-than-subjective representation.

3.B. Displacement

3.B.1. *Absolute Displacement (AD)*

This study uses three displacement values: angular displacements of roll and pitch measured in degrees, and motion displacement of heave, measured in meters. Roll represents angular motion (in degrees) to the left (port) or right (starboard) of the centerline vertical axis. Pitch represents angular motion (in degrees) of the front of the ship (bow) above or below the reference plane. Heave represents the motion displacement (in meters) of the entire ship above or below the reference plane. All values, given a datum of the motion reference plane, have the potential to be negative or positive. To represent magnitude, the absolute value of all displacement values is used exclusively. Each displacement measurement will be referred to as an *absolute displacement* (AD). Taking the magnitude of the displacement values also serves to assist in utilization of distributions designed for positive data.

Since there are two different units of measurement (degrees and meters), conversion of angular displacement to a linear displacement is required. A subtended length produced by a rotating lever arm radial to the calibrated center of the IMU was assumed, and the linear displacement (in meters) for roll and pitch were then determined using the formula,

$$l_{d,n} = \left(\frac{|\theta_{d,n}|}{360}\right)(2\pi r) = \frac{|\theta_{d,n}|\pi r}{180}$$

where $|\theta_{d,n}|$ is the absolute angular displacement of roll ($d = r$) or pitch ($d = p$) at index n, and r is the arm radius. A radial measurement of 1 meter was chosen arbitrarily and used exclusively throughout all analysis. Note that the choice of $r = 1$ meter does not result in true displacement amplitudes; scaled rather than true amplitudes will be used in the final model.

Assuming that principles of linearity apply, since all three displacement measurements are used to quantify the ship's motion, a summation of all three displacement values, in meters, with heave represented by h_n, such that

$$t_n = \frac{|\theta_{r,n}|\pi r}{180} + \frac{|\theta_{p,n}|\pi r}{180} + |h_n|$$

will be referred to as a *total absolute displacement* (TAD) and will provide the basis for model limit development.

3.B.2. *Mean Absolute Displacements*

Averages of the displacement values will provide the independent variable to be adjudicated with model limits. Both arithmetic mean and root mean square (RMS) of the displacement data through an entire event period are taken.

The arithmetic average of the TAD is

$$T_A = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{\pi r}{180}\left(|\theta_{r,i}| + |\theta_{p,i}|\right) + |h_i|\right]$$

where n is the number of displacement samples taken within a specific survey event, and will be referred to as the *arithmetic mean of the total absolute displacement* (AMTAD).

The root mean square of the displacements is produced in an attempt to "normalize" the displacement ranges being averaged so that no one displacement dominates the weighting. Significant changes in amplitude of one displacement type should not drastically change the total displacement. This is accomplished by producing the RMS of the each displacement's TAD as

$$T_R = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{|\theta_{r,i}|\pi r}{180}\right)^2} + \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{|\theta_{p,i}|\pi r}{180}\right)^2} + \sqrt{\frac{1}{n}\sum_{i=1}^{n}(|h_i|)^2}$$

and will be referred to as the *root mean square of the total absolute displacement* (RMSTAD). To verify the assumption that all three displacement measurements are significant and useful to this study, correlations between pre-scaled, arithmetic- and RMS-based MTAD values and sea state are determined. It is apparent from the values detailed in Table 10 that all displacements in arithmetic and RMS form are similar and subsequently significant as representations of the ship's motion.

Table 10. Pearson Correlation Coefficients of Pre-Scaled Displacement to Sea State

	Pre-Scaled AMTAD	Pre-Scaled RMSTAD
Roll	0.4360	0.4512
Pitch	0.3304	0.3931
Heave	0.4408	0.4454

3.B.3. *Scaling Factors*

Each displacement value has its own trend in amplitude, frequency, and phase. Observation of the raw data reveals that pitch and heave have similar levels of variability, while roll tends to be much higher. However, once the angular quantities are converted to linear, heave tends to have values that are one order of magnitude larger than roll or pitch. Since swells (measured by heave) generally have a very limited effect on the state of the sea (when not considered in conjunction with closely orthogonal wind directions) (White and Hanson, 2000), roll and pitch values must be normalized to reflect their individual and significant contribution to the characterization of the sea state. For the purposes of this study, no one value will hold a higher impact, and therefore each of the displacement values must be normalized (averages of roll, pitch, and heave will be set to unity to determine a *scaling factor* for each).

If the displacement data is parsed by sea state and $N_{d,ss}$ is the number of all event samples for a displacement type *d* in sea state *ss*, then the arithmetic averages can be represented by

$$A_{roll,ss} = \frac{1}{N_{roll,ss}} \sum_{j=1}^{N_{roll,ss}} \left(\frac{1}{n} \sum_{i=1}^{n} \frac{|\theta_{r,i}| \pi r}{180} \right)_j$$

$$A_{pitch,ss} = \frac{1}{N_{pitch,ss}} \sum_{j=1}^{N_{pitch,ss}} \left(\frac{1}{n} \sum_{i=1}^{n} \frac{|\theta_{p,i}| \pi r}{180} \right)_j$$

$$A_{heave,ss} = \frac{1}{N_{heave,ss}} \sum_{j=1}^{N_{heave,ss}} \left(\frac{1}{n} \sum_{i=1}^{n} |h_i| \right)_j$$

while the RMS averaging would be represented by

$$R_{roll,ss} = \sqrt{\frac{1}{N_{roll,ss}} \sum_{j=1}^{N_{roll,ss}} \left(\sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{|\theta_{r,i}| \pi r}{180} \right)^2} \right)_j}$$

$$R_{pitch,ss} = \sqrt{\frac{1}{N_{pitch,ss}} \sum_{j=1}^{N_{pitch,ss}} \left(\sqrt{\frac{1}{n} \sum_{i=1}^{n} \left(\frac{|\theta_{p,i}| \pi r}{180} \right)^2} \right)_j}$$

$$R_{heave,ss} = \sqrt{\frac{1}{N_{heave,ss}} \sum_{j=1}^{N_{heave,ss}} \left(\sqrt{\frac{1}{n}\sum_{i=1}^{n}(|h_i|)^2}\right)_j} .$$

If $n(ss)$ is the number of sea states, a mean, M_d, of the individual sea state averages for each displacement is produced for data with arithmetic means using

$$M_d = \frac{1}{n(ss)} \sum_{k=1}^{n(ss)} (A_d)_k.$$

For data with RMS averaging,

$$M_d = \sqrt{\frac{1}{n(ss)} \sum_{k=1}^{n(ss)} (R_d)^2{}_k}.$$

If the product of each M_d and a scaling variable S_n are set to unity, division of the expectations will yield scaling factors in Table 11.

Table 11. Scaling Factors

	M_d	S_n
Arithmetic		
Roll	0.01435	69.67944
Pitch	0.00908	110.11395
Heave	0.18515	5.40095
RMS		
Roll	0.02028	49.30800
Pitch	0.01201	83.23810
Heave	0.27904	3.58367

Applying the scaling factors to the AMTAD and RMSTAD formulas in section 3.B.2 yields the scaled sums of

$$T_{sA} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{\pi r}{180}\left(S_1|\theta_{r,i}| + S_2|\theta_{p,i}|\right) + S_3|h_i|\right]$$

and

$$T_{sR} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[\frac{\pi r}{180}\left(S_4|\theta_{r,i}| + S_5|\theta_{p,i}|\right) + S_6|h_i|\right]^2}$$

To confirm that the scaling factors are similarly useful for representing pre-scaled displacement values, Pearson correlation coefficients were calculated for all sea states and are given in Table 12. The results for sea state 3 are shown in Figures 8 and 9 where pre-scaled MTAD values are plotted versus scaled MTAD values. The high correlation coefficients suggest that the scaled values will accurately represent the displacements before scaling.

Table 12. Pearson Correlation Coefficients of Pre-Scaled vs. Scaled Data

	Arithmetic *r*	RMS *r*
Sea State 0	0.836	0.876
Sea State 1	0.896	0.924
Sea State 2	0.942	0.957
Sea State 3	0.935	0.943
Sea State 4	0.953	0.957
Sea State 5	0.995	0.994
Sea State 6	1.000	1.000

Figure 8. AMTAD Correlation

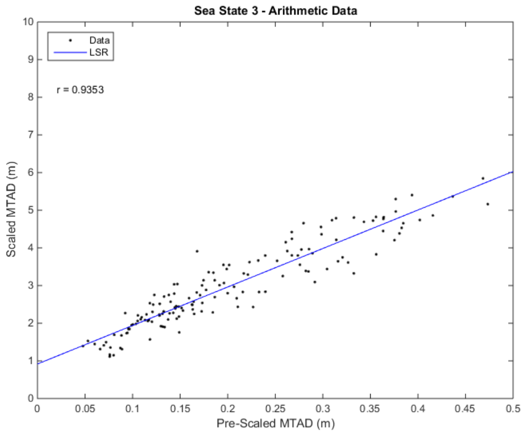
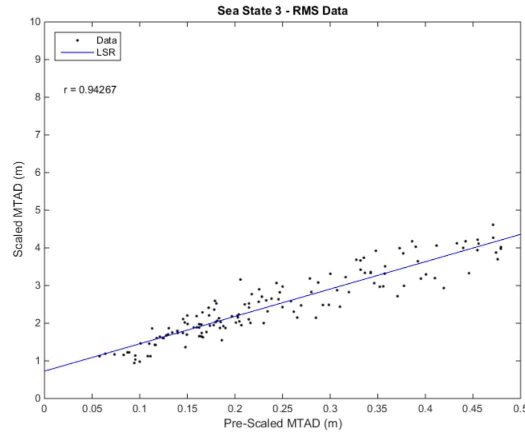


Figure 9. RMSTAD Correlation



3.B.4. *Power Spectral Density*

The absolute displacements are quantities that represent the motion of the ship, but given the variability of these values through the wind-wave interactions, spectral analysis of the waveforms is explored to determine if periodicities or distribution of energy with frequency can better represent the actual movements involved.

Given the scaled TAD time-series of an event, the spectrum of the data is produced using a Discrete Fourier Transform (Magrab et al., 2011), represented as

$$X(n\Delta f) = \Delta t \sum_{k=0}^{N-1} x_k \, e^{-i2\pi nk/N}$$

where $n = 0,1,2, \ldots, N-1$, and a subsequent Fast Fourier Transform (FFT) function defined in Matlab [17] as

$$X(k) = \sum_{j=1}^{N} x(j) \, \omega_N^{(j-1)(k-1)}$$

where $\omega_N = e^{(-2\pi i)/N}$ and $x(j)$ are defined as all the values in the time-series data sample.

The FFT size is determined by producing the next highest power of two from the length of the sample size of the event period; using this definition, the FFT size will always be greater than the length of the displacement data sample, and therefore the time-series data are padded with zeros.

To consider periodicities and character of frequency content, the power spectral density (PSD), or power spectrum, is used. If the discrete time-series data for a displacement is defined as

$$x_n = \frac{\pi r}{180}\left|\theta_{d,n}\right|$$

where $n = i\Delta t$ and i is the index of each displacement value for a time step of Δt and $1 \leq n \leq N$. If the Fourier transform of the data is

$$X(f) = \sum_{n=1}^{N} x_n\, e^{-i2\pi fn},$$

then the power spectral density is

$$P(f) = \frac{\Delta t}{N}\left|\sum_{n=1}^{N} x_n\, e^{-i2\pi fn}\right|^2.$$

This PSD estimator provides a reasonable representation of the frequency distribution of the time-series data. Concerns regarding spectral leakage and inconsistencies as N approaches infinity are not considered in the scope of this study.

Analysis of the PSD results shows that periodicity is greatly varied for each of the displacements. Examples of roll-based PSDs for two sea state 2 events in the same survey are detailed in Figures 10 and 11 where power amplitude is plotted versus frequency. Inspection of displacement values from other survey events confirms similar significant variation and lack of uniform periodicities.

Figure 10. Event #300 Roll Spectrum

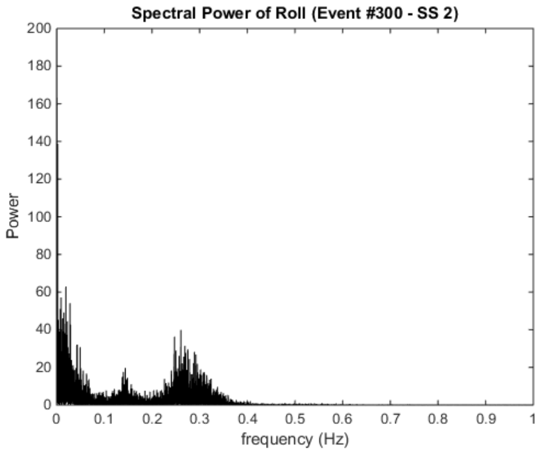
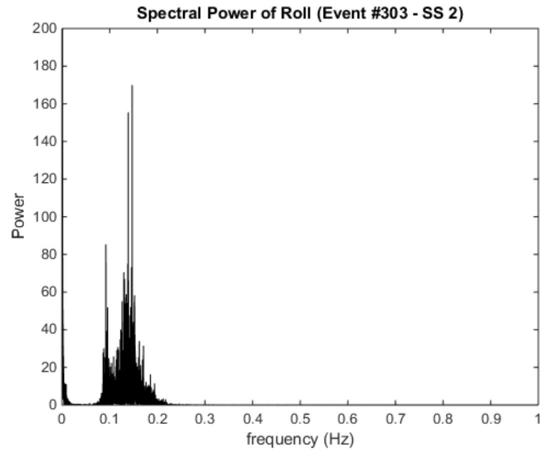


Figure 11. Event #303 Roll Spectrum



The PSD of the TAD provides a total power spectrum; this value can then be used to provide quantification for model development. By inspection of sample events, it is determined that the significant spectra are always less than 0.5 Hz. Prior studies with ocean wave spectrum components confirm this observation (Varkey, 1993; Nielsen, 2007). To avoid DC bias values and any phase drifting, spectra less than 0.05 Hz were rejected. This value is chosen from close inspection of several spectrum representations; there is a noticeable drop in significant periodicities at this frequency. Summation of $P(f)$ over the frequency interval $[0.05,0.5]$ provides a total power spectrum and is defined by

$$\sum_{f=0.05}^{f=0.5} \frac{\Delta t}{N} \left| \sum_{n=1}^{N} x_n \, e^{-i2\pi f n} \right|^2 .$$

Plots of power amplitude versus frequency in Figures 12 through 15 show the periodicities of each displacement for a given event. The regions in red are the rejected spectra outside of the chosen interval. Note that the TAD spectrum in Figure 15 is similar to the roll spectrum in Figure 12, indicating that roll dominates the cyclical component of the TAD.

Analysis of this and other events suggest that the PSD waveform of the TAD will reflect its most significant, individual component.

Figure 12. Power Spectrum of Roll

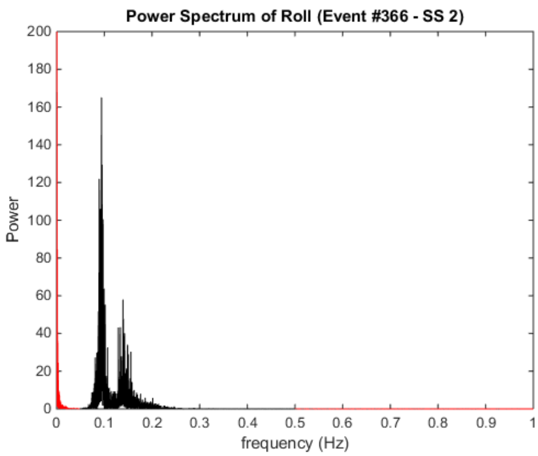


Figure 13. Power Spectrum of Pitch

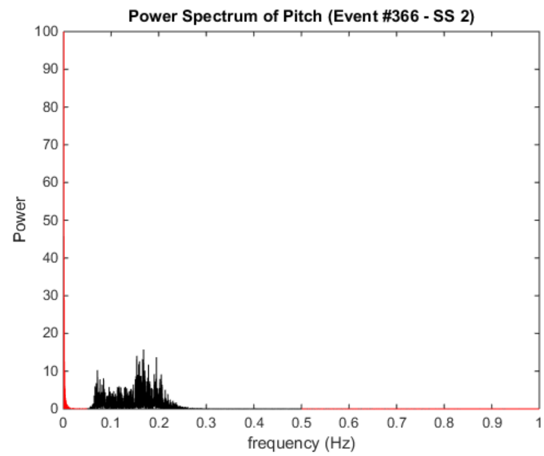


Figure 14. Power Spectrum of Heave

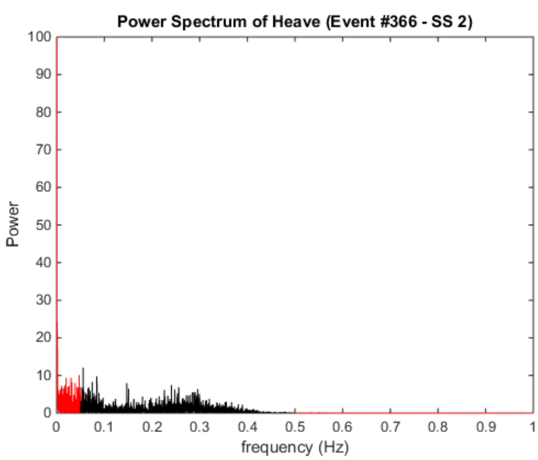
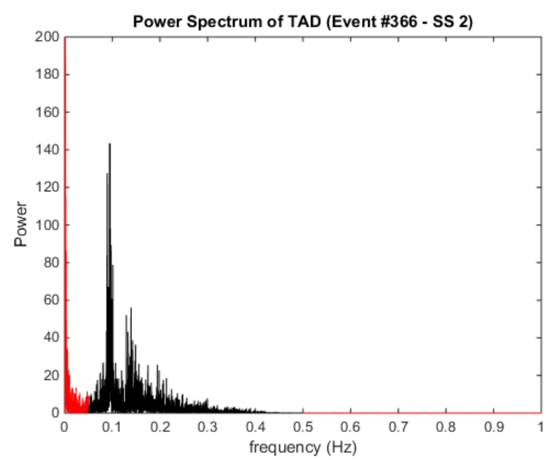


Figure 15. Power Spectrum of TAD



For arithmetic-mean-based scaled data, the total PSD for the relevant interval is referred to as the *Arithmetic Power Spectrum*, or ASPOW. The total PSD for the relevant interval of the RMS-based scaled data is referred to as the *Root Mean Square Power Spectrum*, or RSPOW.

3.B.5. *Outliers*

A 1.5-IQR test (Tukey, 1977) was applied to the scaled MTAD and power spectrum data. Although the 1.5 IQR test results in data symmetry, it provides a tool to scrub possible outliers resulting from mismatched sea state observations and wind-wave anomalies and extrema. Since the justification for outlier removal is not substantial in this study due to the natural aspect of the data, potential outliers are only removed for final comparisons; model sea state limits were developed using data containing the potential outliers.

If means $\bar{x}$ and quartiles Q_1 and Q_3 are determined, any data values x outside of the interval defined by

$$\bar{x} - 1.5(Q_3 - Q_1) \leq x \leq \bar{x} + 1.5(Q_3 - Q_1)$$

are rejected and removed from the data set. Table 13 details the number of potential outliers in each of the basis data sets.

Table 13. Potential Outliers

	AMTAD	RMSTAD	ASPOW	RSPOW
Sea State 0	1	1	0	0
Sea State 1	7	8	7	13
Sea State 2	3	2	14	9
Sea State 3	2	1	16	8
Sea State 4	0	0	2	2
Sea State 5	0	0	0	0
Sea State 6	0	0	0	0

3.C. Distribution Fitting

3.C.1. *Distributions*

Several continuous distribution models were considered for the development of model limits. The skewed and positive attributes of the data require specific considerations for distribution selection. The chosen distributions produce the sea state limitations needed for subsequent modeling. The ideal distribution will closely fit the survey data and provide enough separation between confidence boundaries inherent to each sea state that non-overlapping delineation of the individual sea states is possible. If an ideal distribution is not found, it may be possible to combine two or more of the distributions to create a more global image of the sea states being represented by the models.

The Matlab fitdata [12] function is used to determine the distribution parameters. These parameters (i.e., μ = mean and σ = standard deviation) are then used to define the distribution as it pertains to the data, and are detailed for all distributions in Appendix Section C.

Probability plots are used to determine the closeness of fit of the distribution to the data. Each probability plot displays the displacement MTAD data (x-axis) in comparison to a theoretical distribution in terms of probability. The closer the data is to the reference line, the more significantly it can be represented by that distribution.

The natural aspect of the data suggests that a Gaussian distribution will provide the closest fit with the benefit of efficient computation. This distribution is modeled using the probability density function in the form

$$G(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where μ is the mean of the distribution and σ is the standard deviation (Mendenhall et al., 1981; Magrab et al., 2011; Shchigolev, 1965). Figures 16a and 16b of scaled MTAD detail the closeness of fit for the Gaussian distribution.

Figure 16a. Gaussian Probability Plot of AMTAD data

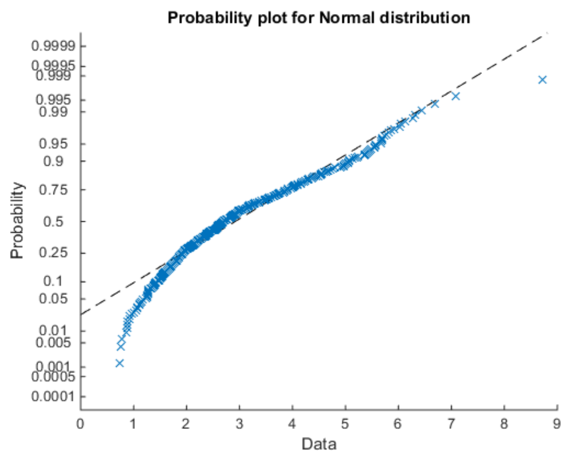
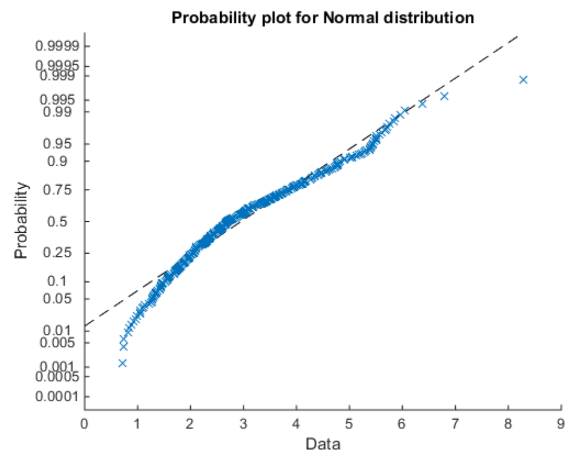


Figure 16b. Gaussian Probability Plot of RMSTAD Data



The Gaussian probability plots suggest that the MTAD data can be represented closely by this distribution for the central portion. However, values on the tail ends of the data do not fit well; these values are potential outliers and may be rejected using the 1.5-IQR rule.

Preliminary analysis of the MTAD data indicates that the distributions are positive and skewed right. Several non-Gaussian distributions work well with data in this form.

The Weibull distribution can be used with positive, right-skewed data, and is the interpolation between the less flexible exponential distribution and the Rayleigh distribution (Matlab, 2014; Weibull, 1951; Papoulis et al., 2002). It has the probability density function

$$W(x|\lambda, k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^{k}}$$

where k is the shape parameter and $k > 0$, and λ is the scale parameter and $\lambda > 0$. The nature of the absolute displacement and power spectrum data suggests that the $k > 1$ and that the density function increases until the mode is reached and then decreases thereafter (Papoulis et al., 2002). Figures 17a and 17b show the closeness of fit for MTAD data.

Figure 17a. Weibull Probability Plot of AMTAD Data

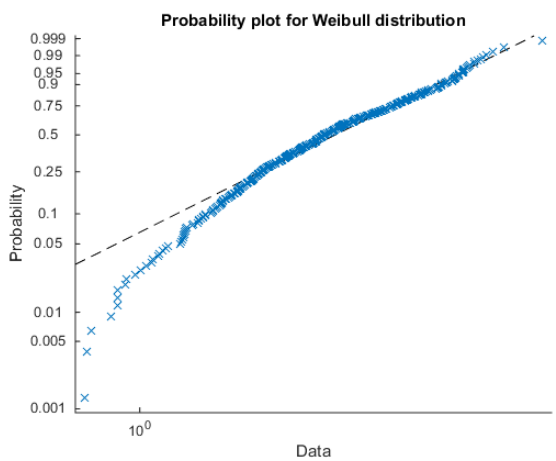
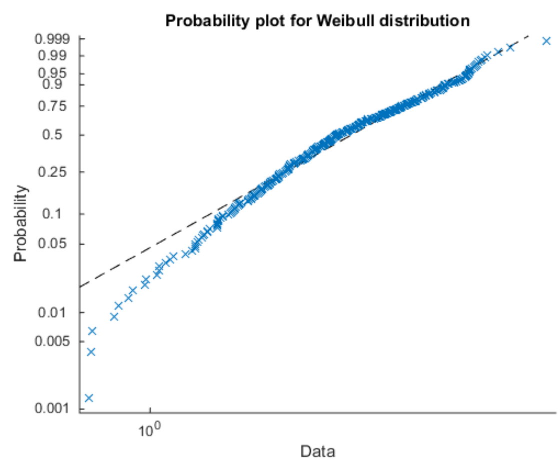


Figure 17b. Weibull Probability Plot of RMSTAD Data



A Rayleigh distribution, a special case of the Weibull distribution, can provide a distribution fitting for positive and skewed data given that the magnitude of the MTAD is related to displacement components. The Rayleigh probability density function (Matlab, 2014; Siddiqui, 1961) is modeled as

$$R(x|\sigma) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

where σ is the scale parameter of the distribution (generally considered the mode of the distribution). Notably, if k and λ are the parameters of the Weibull distribution, then the σ scale parameter of the Rayleigh distribution is considered equivalent to the Weibull distribution parameters by $\lambda = \sigma\sqrt{2}$ and $k = 2$. The probability plots in Figures 18a and 18b detail the closeness of fit for this distribution.

Figure 18a. Rayleigh Probability Plot of AMTAD Data

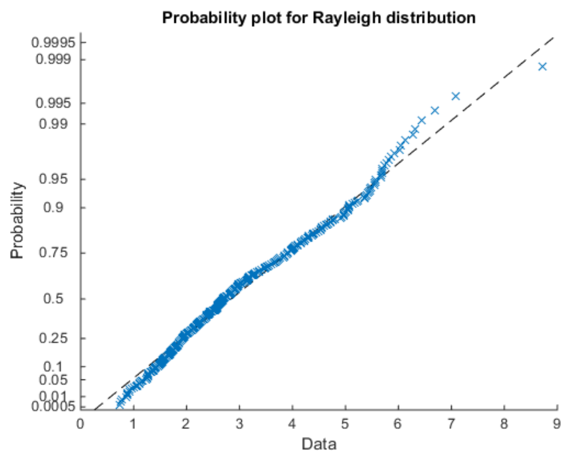
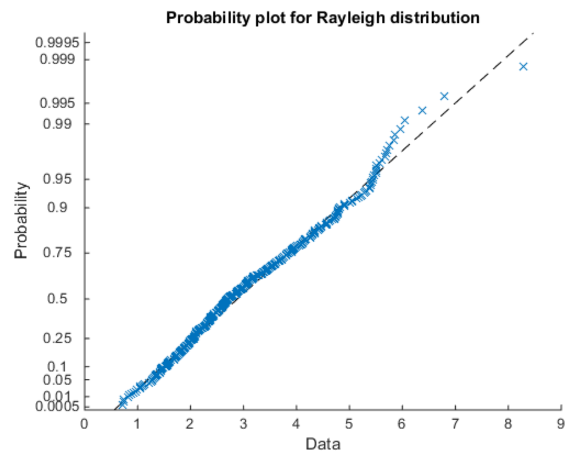


Figure 18b. Rayleigh Probability Plot of RMSTAD Data



The lognormal (Galton) distribution will be used given the variability-limited and positive nature of logarithmic data. The probability density function for a lognormal distribution is defined as

$$\mathrm{L}(x|\mu,\sigma) = \ln f(x|\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$$

with all $x > 0$ and μ and σ defined as the log mean and log standard deviation, respectively (Matlab, 2014; Crow and Shimizu, 1988; Johnson et al., 1994). The closeness of fit for this distribution is shown in Figures 19a and 19b.

Figure 19a. Lognormal Probability Plot of AMTAD Data

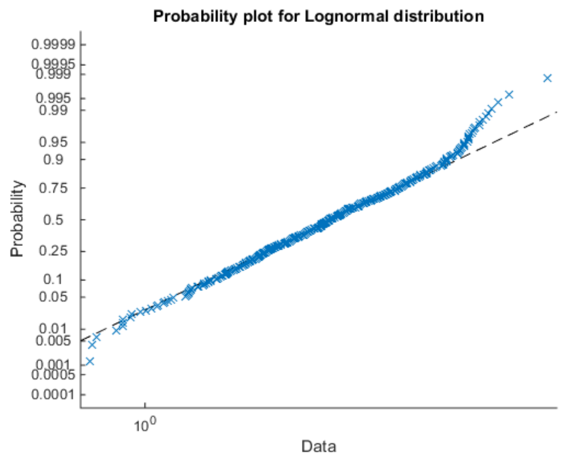
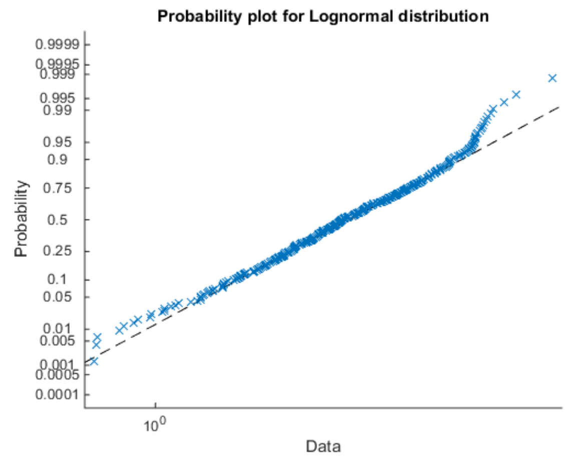


Figure 19b. Lognormal Probability Plot of RMSTAD Data



It is apparent that Weibull and Rayleigh distributions are closely representative of the Gaussian distribution. The Rayleigh distribution is following the skewed nature of the data, but is providing a wide confidence band which suggests that close to the mode, the Rayleigh will fit very well, but it will lose accuracy as it spreads out from this modal center. The lognormal shows a higher kurtosis and closely represents the skewed-right nature of the data.

Histograms of MTAD data for the sea states with the most data (1, 2, and 3) are provided in Figures 20-22. The bin widths are 0.1 meters. Associated plots of the distributions' probability density functions are displayed as well; these curves are scaled by a factor of 10 for display purposes only. RMSTAD data are not shown due to their approximate equivalency to AMTAD data.

Figure 20. Sea State 1 AMTAD and Associated Distributions

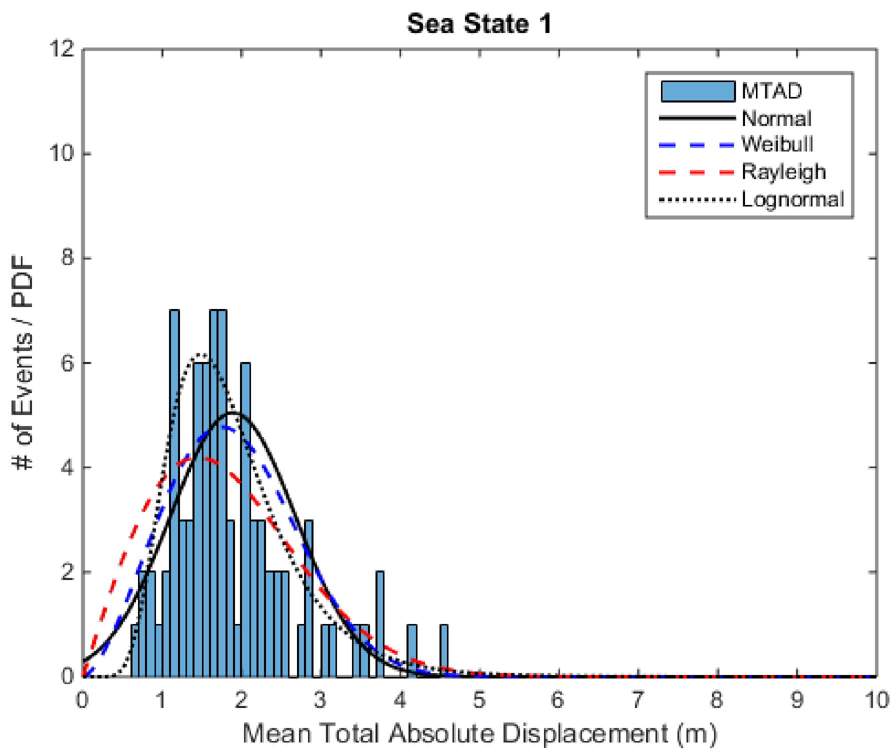


Figure 21. Sea State 2 AMTAD and Associated Distributions

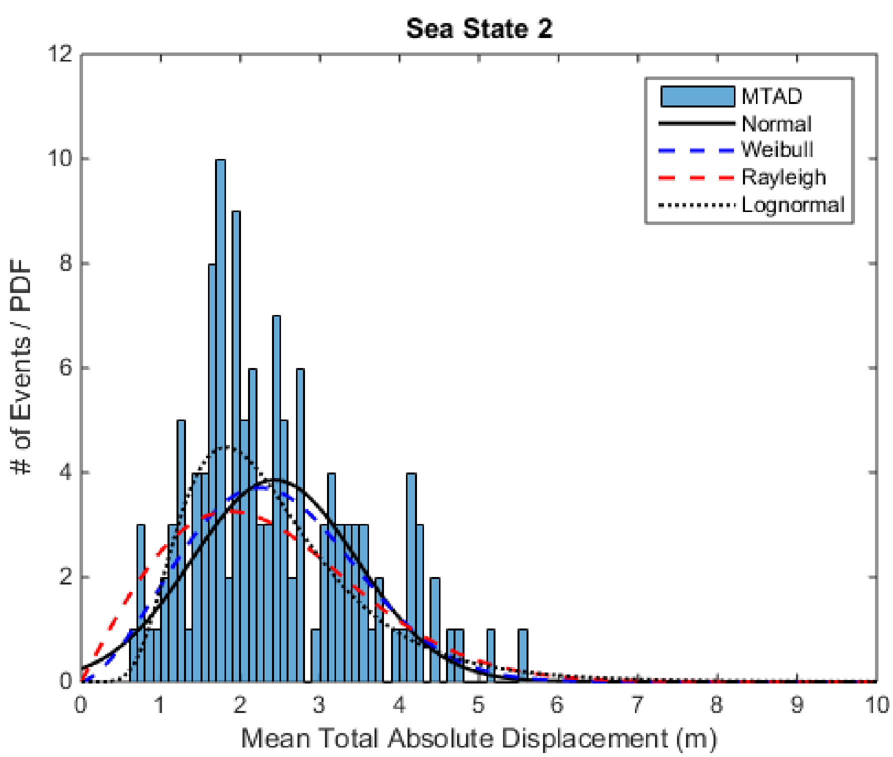
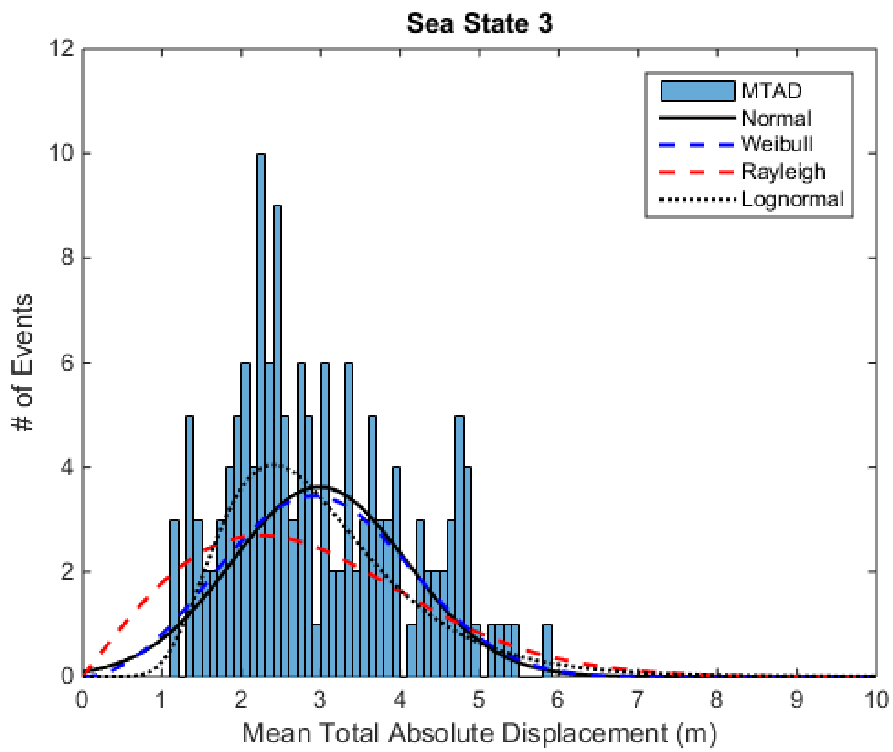


Figure 22. Sea State 3 AMTAD and Associated Distributions



3.C.2. *Tolerance and Confidence Intervals*

The models developed in this study are dependent on the parameters developed through the use of α-based tolerance and confidence intervals derived from the fitted distributions. The fitdist, paramci, and icdf functions in Matlab [12] are used to determine the various intervals. Sampling and operational requirements (NAVOCEANO Personnel, 2015) suggest using a minimum of $\alpha = 0.30$ and $\alpha = 0.20$, but for thorough analysis, intervals using $\alpha = 0.10,\ 0.05,$ and 0.01 are also produced. Ideally, the tolerance and confidence intervals for each sea state would not overlap and would have corresponding limits. In anticipation of less-than-ideal intervals, averaging of tails may be required.

It is important to note that while a 100(1-α)% *tolerance interval* represents the amount of data that lie within a 100(1-α)% interval, a 100(1-α)% *confidence interval* does not mean that 100(1-α)% of the sample data lie within the interval. It should be understood as an estimate of the possible values for the population parameter such as a population mean. Since the test statistic is a mean, arithmetic or geometric, the intervals are marking the bounds for which the test statistic is probabilistically within range of the distribution parameter. It can be said that there is a 100(1-α)% confidence that the population parameter will be within the confidence bounds.

The Gaussian cumulative distribution function (Mendenhall, 1981) can be defined using the generalized form

$$CDF(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{x-\mu}{\sigma}} e^{-\frac{t^2}{2}} dt$$

and utilized in the quantile-based function (inverse cdf)

$$CDF(\mu + z\sigma) - CDF(\mu - z\sigma) = \frac{1}{\sqrt{\pi}} \int_{-\frac{z}{\sqrt{2}}}^{\frac{z}{\sqrt{2}}} e^{-t^2} dt$$

to determine the tolerance intervals for a given confidence level. The Gaussian distribution two-sided α confidence interval can be developed from the relationship defined as

$$P\left(\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \leq \mu \leq \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right) = 1 - \alpha.$$

This produces the α confidence interval $\left[\bar{x} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right), \bar{x} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right]$.

The Weibull cumulative distribution function can be defined as

$$CDF(x) = 1 - e^{-(x/\lambda)^k}$$

and utilized as a basis for the quantile-based function

$$Q(CDF) = \lambda(-\ln(1 - CDF))^{1/k}$$

to determine tolerance intervals for percentage p, such that $0 \leq p < 1$. The Weibull distribution two-sided α confidence interval (Lloyd and Lipow, 1962; Nelson 1982) is defined as

$$P\left(\bar{x} - K_{\alpha/2}\left(\sqrt{\sigma^2}\right) \leq \lambda \leq \bar{x} + K_{\alpha/2}\left(\sqrt{\sigma^2}\right)\right) = 1 - \alpha$$

where K_α is defined as

$$\alpha = \frac{1}{\sqrt{2\pi}} \int_{K_\alpha}^{\infty} e^{-t^2/2}\, dt$$

This produces the α confidence interval $\left[\bar{x} - K_{\alpha/2}\left(\sqrt{\sigma^2}\right), \bar{x} + K_{\alpha/2}\left(\sqrt{\sigma^2}\right)\right]$.

The cumulative distribution function for the Rayleigh distribution (Papoulis and Pillai, 2002) can be defined as

$$CDF(x) = 1 - e^{-x^2/2\sigma^2}$$

and utilized in the quantile function

$$Q(CDF) = \sigma\sqrt{-\ln((1 - CDF)^2)}$$

to determine the tolerance bounds. The Rayleigh distribution two-sided α confidence interval can be defined from Siddiqui [24] such that if two numbers χ_1^2 and χ_2^2 corresponding to $2N$ degrees of freedom (N is the number of independent observations) are determined and applied to the system

$$P(\chi^2 \leq \chi_1^2) = \frac{\alpha}{2}$$

$$P(\chi^2 \leq \chi_2^2) = 1 - \frac{\alpha}{2}$$

then

$$P\left(\frac{2N\bar{x}}{\chi_1^2} \leq \sigma \leq \frac{2N\bar{x}}{\chi_2^2}\right) = 1 - \alpha$$

producing the α confidence interval $\left[\frac{2N\bar{x}}{\chi_1^2}, \frac{2N\bar{x}}{\chi_2^2}\right]$.

The lognormal cumulative distribution function (Crow and Shimizu, 1988; Johnson et al., 1994) is

$$\int_0^x \ln f(a|\mu,\sigma)da = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\frac{\ln x-\mu}{\sigma}} e^{-\frac{t^2}{2}}dt$$

with a corresponding quantile function

$$Q(p) = e^{\left(\mu+\sigma G^{-1}(p|\mu,\sigma)\right)}$$

where G is the Gaussian distribution at p such that

$$G(p|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(p-\mu)^2}{2\sigma^2}}$$

The lognormal distribution two-sided α confidence interval can be developed using the Cox method (Land, 1971), such that if X represents the original data that follows a lognormal distribution with an expected value $E(X) = \bar{x}$, if Y is the log-transformed representation of X, where $E(Y) = \mu$, $Var(Y) = \sigma^2$, the sample mean of Y is $\bar{Y}$, and sample variance of Y is s^2, the relationship can be defined as

$$P\left(\bar{Y}+\frac{s^2}{2}-z_{\alpha/2}\sqrt{\frac{s^2}{n}+\frac{s^4}{2(n-1)}} \le \bar{x} \le \bar{Y}+\frac{s^2}{2}+z_{\alpha/2}\sqrt{\frac{s^2}{n}+\frac{s^4}{2(n-1)}}\right) = 1-\alpha$$

This produces the α confidence interval $\left[\bar{Y}+\frac{s^2}{2}-z_{\alpha/2}\sqrt{\frac{s^2}{n}+\frac{s^4}{2(n-1)}}, \bar{Y}+\frac{s^2}{2}+z_{\alpha/2}\sqrt{\frac{s^2}{n}+\frac{s^4}{2(n-1)}}\right]$.

It is apparent from the tolerance and confidence intervals that only $\alpha = 0.30$ for tolerance intervals and $\alpha = 0.20$ for confidence intervals provide bounds that are useful for this study. Confidence levels higher than 70% in tolerance intervals and 80% in confidence intervals produce excessive overlap and bounds that are unrealistic (e.g., a lower bound of -461 in data that are exclusively positive). As a result, only 70% tolerance intervals and 80% confidence intervals are used in model development.

3.D. Modeling

The developed tolerance and confidence intervals produce boundaries that represent likelihoods of any sampled TAD or power spectrum mean to fall within what is expected to be the bounds of population data or population mean for a given confidence level. A deterministic relationship between displacement or power spectrum means and observed sea states is dependent on the results of the model runs. If a distribution parameter set matches observed sea states to modeled sea states, then that distribution becomes a candidate for the underlying basis in the final model application. If no ideal distribution parameter set exists, a comprehensive model utilizing more than one distribution, or more than one model, may be required.

Given sample sizes and confidence bounds, it is expected that representation of sea states 0, 5 and 6 will be limited. Sea state 4, although limited by sample size, had only a few outliers (or none), and may be a significant representation of displacement in this sea state. The only significant confidence will exist in the parameters developed for sea states 2 and 3. As a result of this consideration, the models are only predictive to sea state 4 with a limited confidence in sea state 0. Any data that produce values yielding sea states higher than 4 are considered sea state 5 or greater.

The following are descriptions of each of the models used in this study.

Model 1 utilizes tolerance intervals at a 70% confidence level. If the boundaries for each sea state are either overlapping or have no intersection they are arithmetically averaged. Since there are four distributions being used, each one creating a set of boundaries for AMTAD, RMSTAD, ASPOW, and RSPOW, there are a total of 16 distribution-based parameter sets yielding the same number of sea state predictions.

Model 2 is Model 1 with 1.5-IQR outliers removed from the data.

Model 3 uses confidence intervals at an 80% confidence level. The boundaries for each sea state, either overlapping or having no intersection, are arithmetically averaged. This model will produce 16 individual sea state predictions.

Model 4 is Model 3 with 1.5-IQR outliers removed from the data.

Model 5 is an experimental model that uses a probability parameterization. The magnitudes of AMTAD, RMSTAD, ASPOW and RSPOW are determined and then parsed using the sea state frequencies plotted in Figure 1. The boundaries for each partition represent the displacement and power spectrum limits for each individual sea state. For example, considering the non-logarithmic AMTAD data, if $\max(AMTAD) = 8.71267$, Table 14 details the partitions using the known sea state frequencies.

Table 14. Model 5 Boundaries for Non-logarithmic AMTAD Data

	Probability	Lower Boundary	Upper Boundary
Sea State 0	0.023256	0.00000	0.20262
Sea State 1	0.209300	0.20263	2.02618
Sea State 2	0.333330	2.02619	4.93038
Sea State 3	0.377260	4.93039	8.21732
Sea State 4	0.041344	8.21733	8.57754

Note that any MTAD values above the sea state 4 upper boundary are considered sea states 5 or 6 (or higher). Since the distributions in this research are not used in this model, only scaled MTAD and SPOW are used, in both non-logarithmic and logarithmic forms. Given this parameterization, there are only eight individual parameter sets used in this model.

3.E. Model Testing and Results

There are two model runs. "Trial A" represents use of all survey events from the data used to create the model parameters. This is done to determine model accuracy through exact and marginal tolerance matching of model sea states to the observed sea states. "Trial B" uses survey events not included in the model parameter development; these events were conducted during the period of the research and are used as a test to determine model accuracy.

The plots in Sections 3.E.1 and 3.E.3 detail the model results for randomly selected events in each sea state. Each graphic is composed of the observed, wind-generated and model mean sea states, the aggregate counts of model results, and the time-series ATAD in both non-logarithmic and logarithmic (LATAD) form. Note that the AMTAD and logarithmic AMTAD (LAMTAD) are plotted for reference.

TRIAL A

To represent the results of Trial A pseudo random number generator is used to pick four events within the surveys utilized in this study (with exception of sea state 5 that only has four events, and sea state 6 that only has two events). If an event without wind data was selected, another random event was taken as a replacement. Table 15 presents the randomly selected event numbers for each sea state.

Table 15. Randomly Selected Events for Trial A

Sea State	Events Randomly Selected
0	196, 218, 226, 275
1	038, 156, 278, 291
2	023, 078, 289, 372
3	080, 159, 332, 362
4	094, 208, 229, 356
5	065, 238, 345, 363
6	236, 237

TRIAL B

Two naval oceanographic surveys were recently conducted aboard survey ship 1. A total of 26 events were produced, and with all rejection criteria considered and applied to the selection of the events, 23 survey events were selected (events numbered 388 to 410). The counts for each sea state are detailed in Table 16. Events 388 to 392 are considered summer events using the meteorological standard in 2.C.2; the remaining events are fall events. Four out of the nine events in the first survey were considered good visibility due to daylight; 13 of the 14 events in the second survey are considered good visibility due to daylight. Resolution of the displacement data was 0.1 seconds.

Table 16. Selected Survey Event Count by Sea State

SS	n
0	1
1	3
2	5
3	11
4	2
5	1
6	0
Total	23

A similar random generation was done to determine the event samples to be used. Four randomly selected events were determined if there were more than four events available. If a sea state is represented by fewer than four events, then all available events for that sea state were used. Table 17 details the events selected.

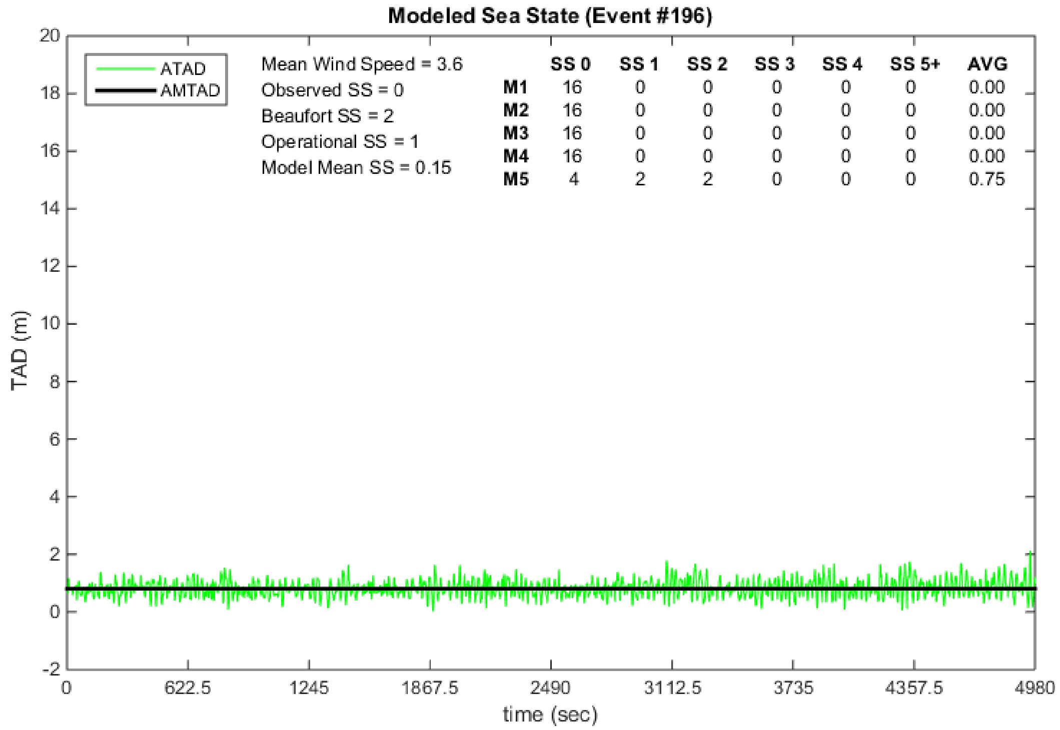
It should be noted that large swells from out-of-area typhoons were present during many of the events; the heave values may skew the data and significantly over value the sea state.

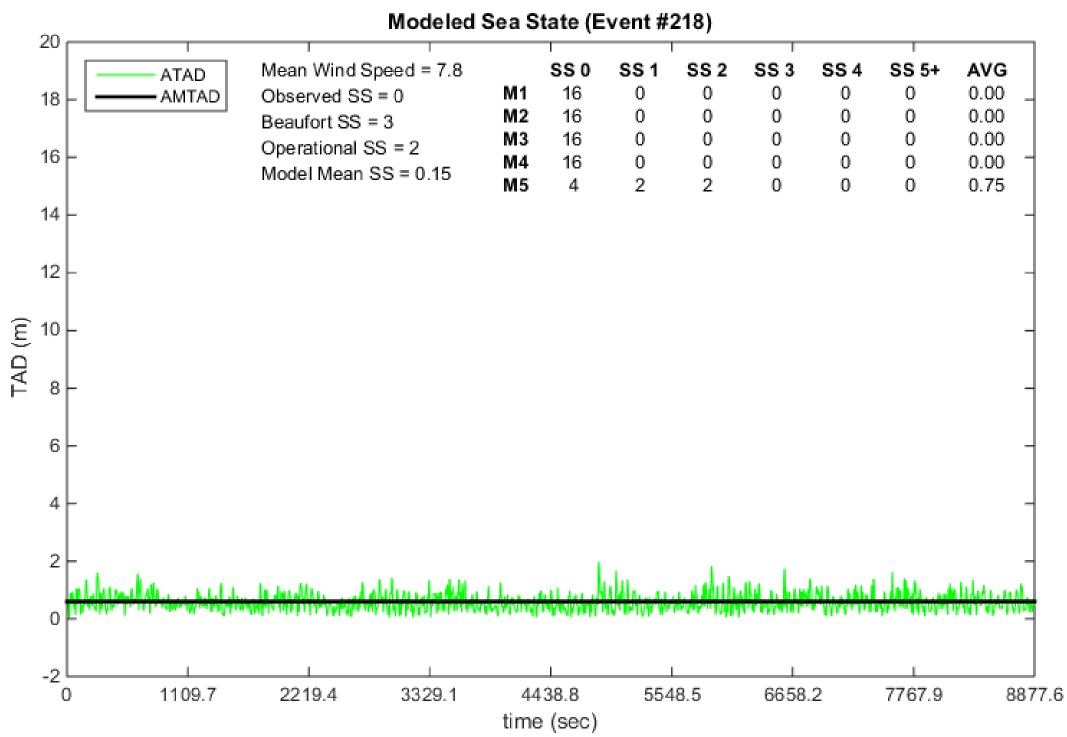
Table 17. Randomly Selected Events for Trial B

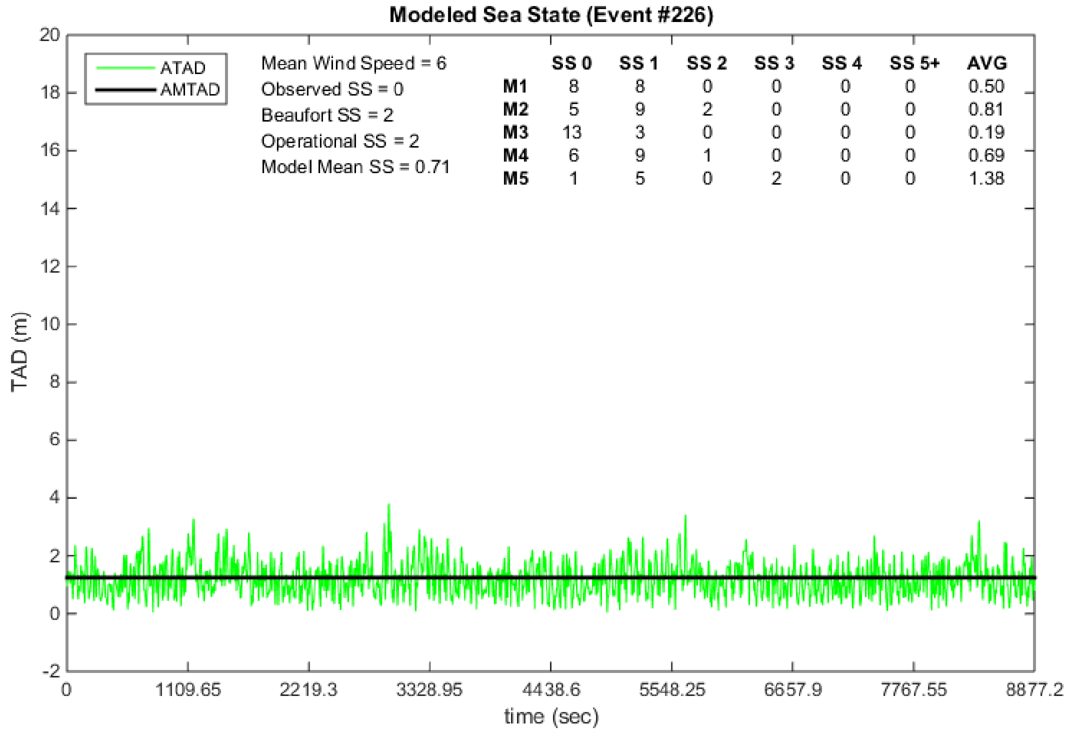
Sea State	Events Randomly Selected
0	395
1	388, 389, 392
2	393, 394, 396, 407
3	391, 399, 404, 406
4	409, 410
5	408
6	NA

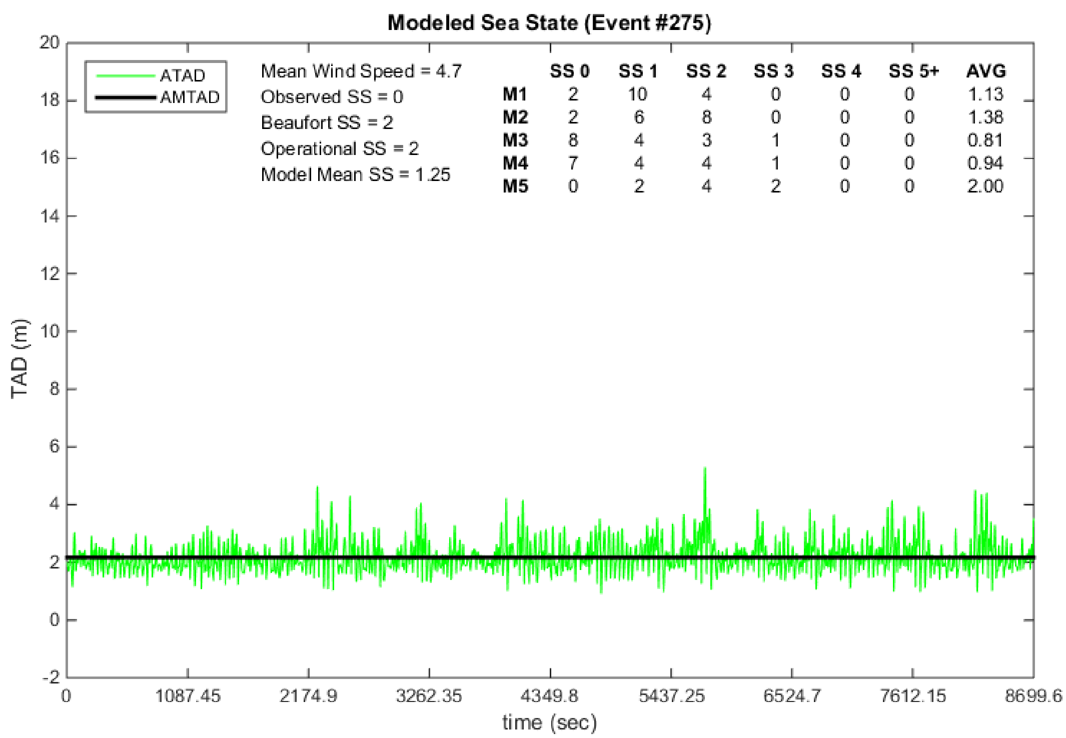
3.E.1. *Trial A Plots*

Figures 23(a-d). Trial A (Sea State 0)

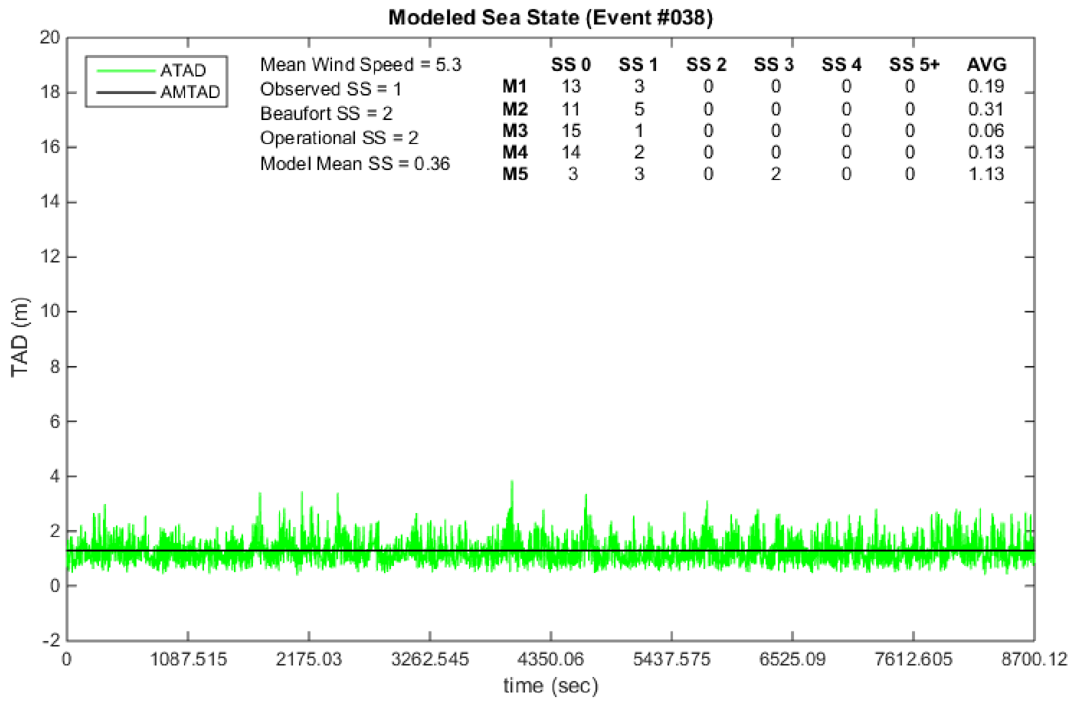


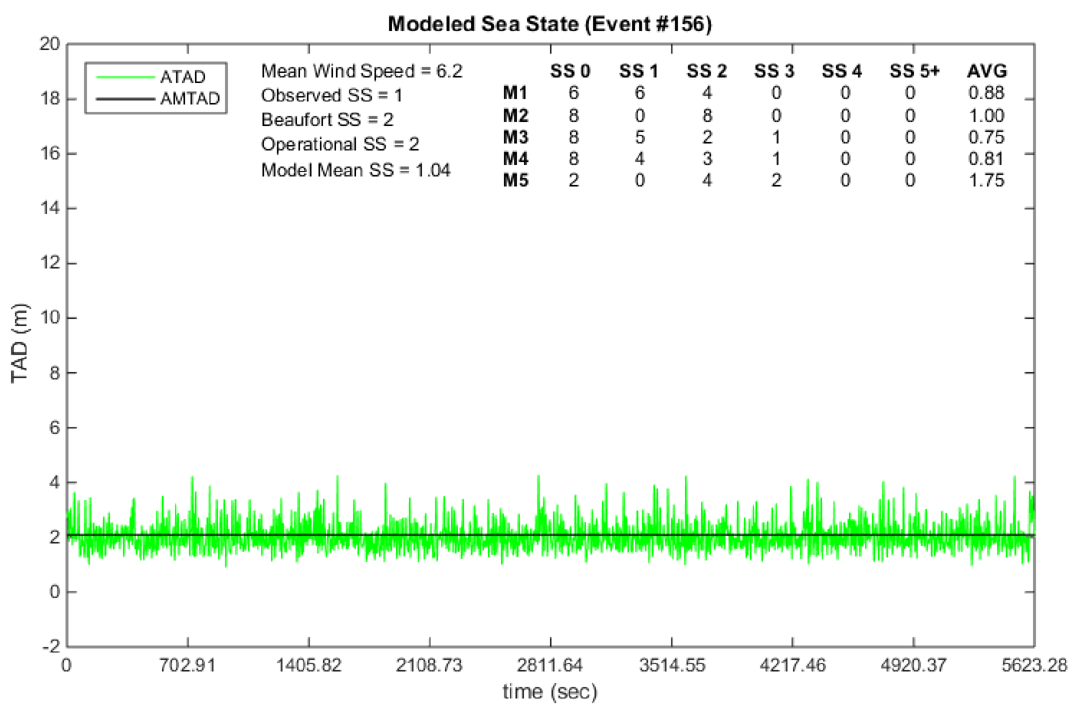


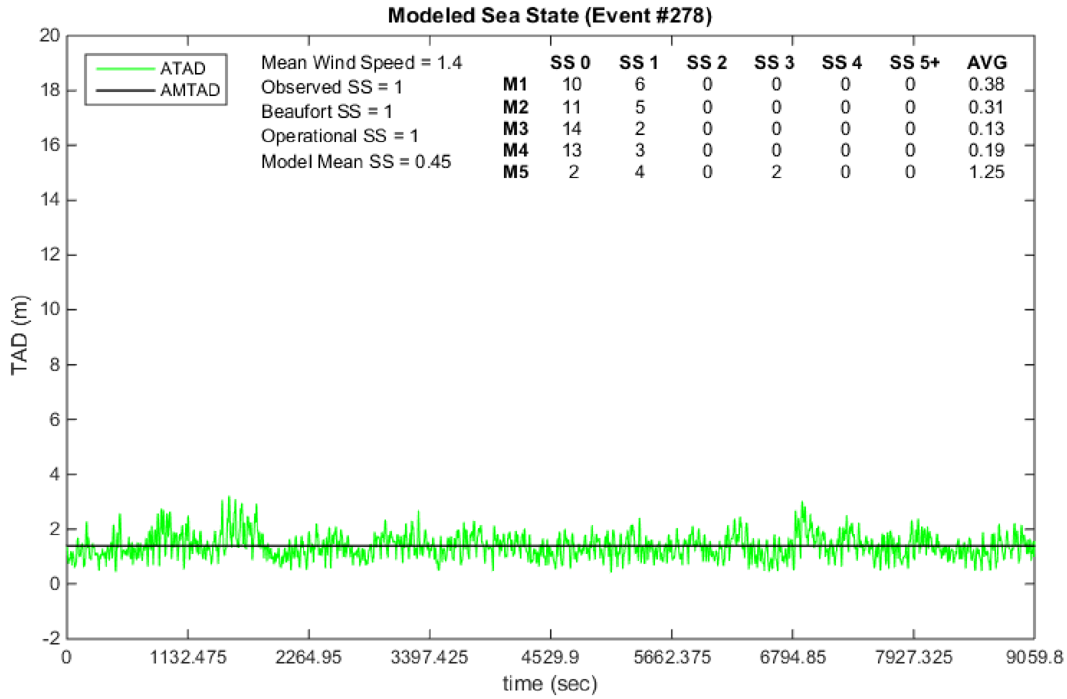

Modeled Sea State (Event #226)
ATAD
AMTAD
Mean Wind Speed = 6
Observed SS = 0
Beaufort SS = 2
Operational SS = 2
Model Mean SS = 0.71
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 8 8 0 0 0 0 0.50
M2 5 9 2 0 0 0 0.81
M3 13 3 0 0 0 0 0.19
M4 6 9 1 0 0 0 0.69
M5 1 5 0 2 0 0 1.38
TAD (m)
20
18
16
14
12
10
8
6
4
2
0
-2
0
1109.65
2219.3
3328.95
4438.6
5548.25
6657.9
7767.55
8877.2
time (sec)

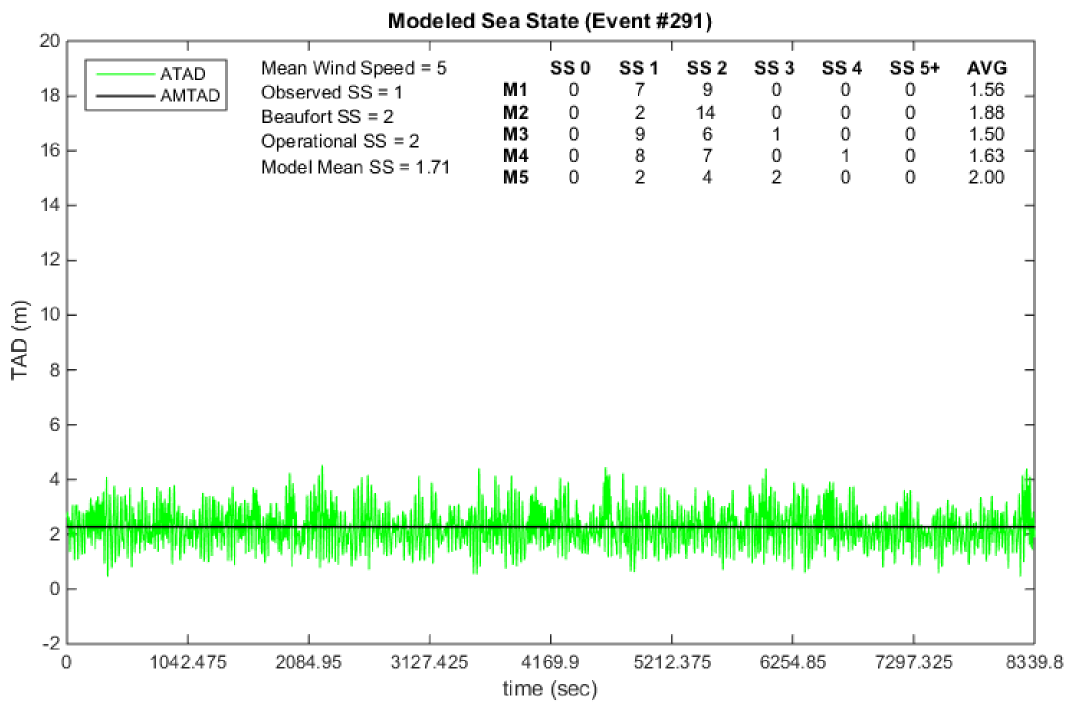

Modeled Sea State (Event #275)
ATAD
AMTAD
Mean Wind Speed = 4.7
Observed SS = 0
Beaufort SS = 2
Operational SS = 2
Model Mean SS = 1.25
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 2 10 4 0 0 0 1.13
M2 2 6 8 0 0 0 1.38
M3 8 4 3 1 0 0 0.81
M4 7 4 4 1 0 0 0.94
M5 0 2 4 2 0 0 2.00
TAD (m)
20
18
16
14
12
10
8
6
4
2
0
-2
0
1087.45
2174.9
3262.35
4349.8
5437.25
6524.7
7612.15
8699.6
time (sec)

Figures 24(a-d). Trial A (Sea State 1)

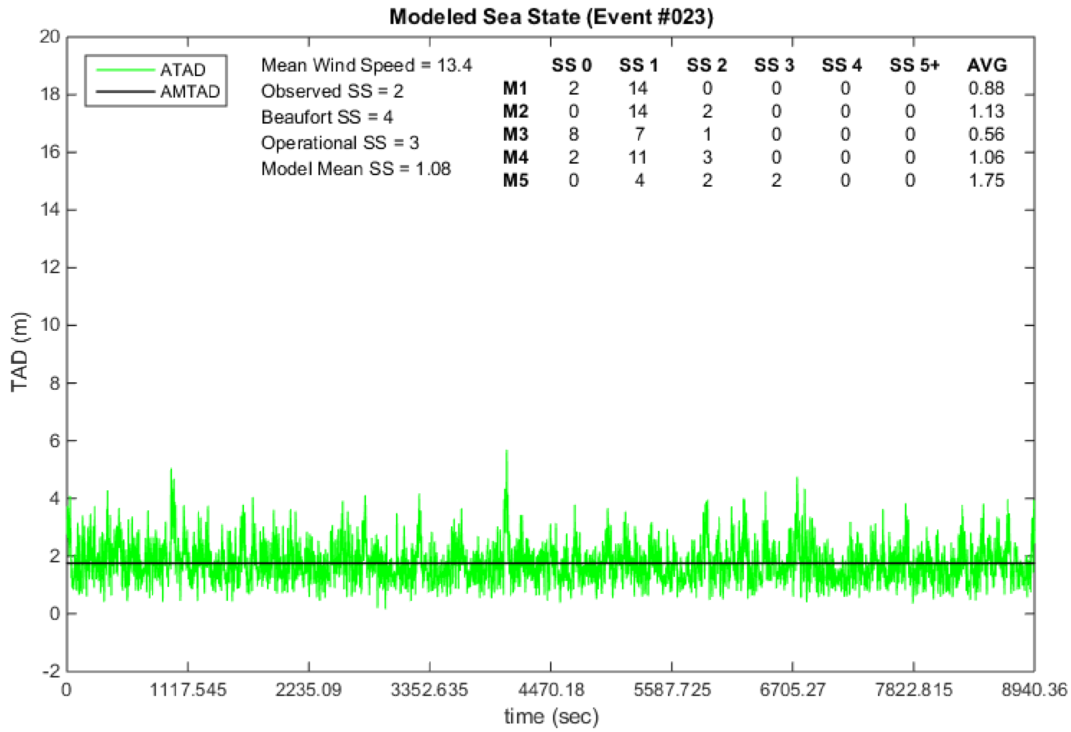


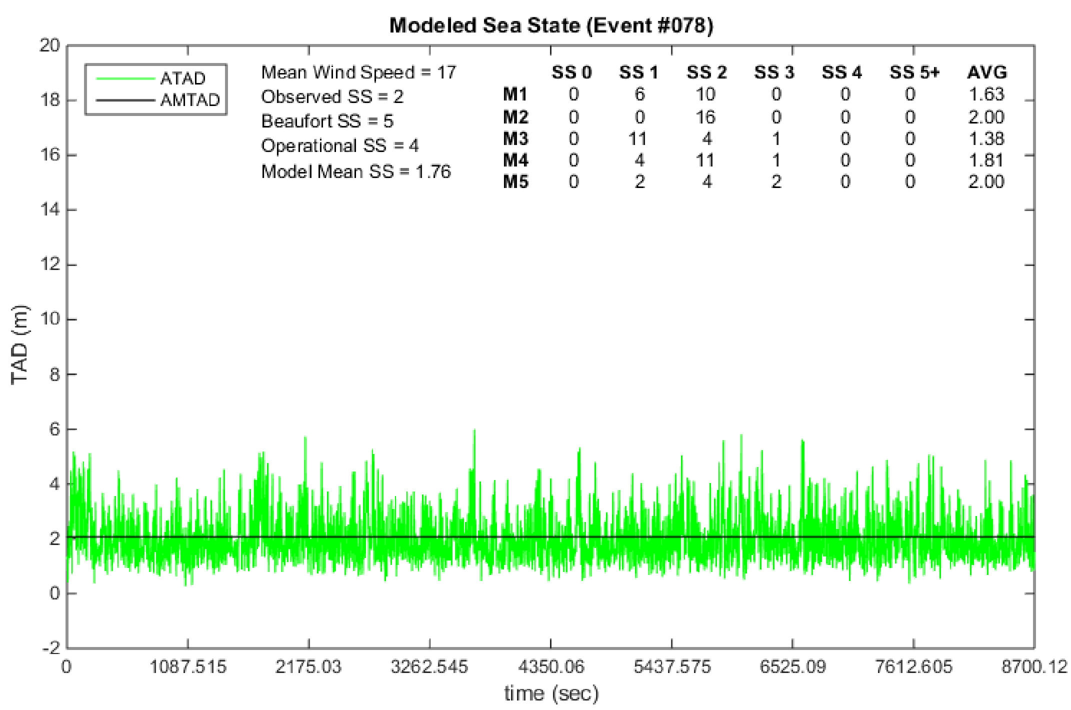


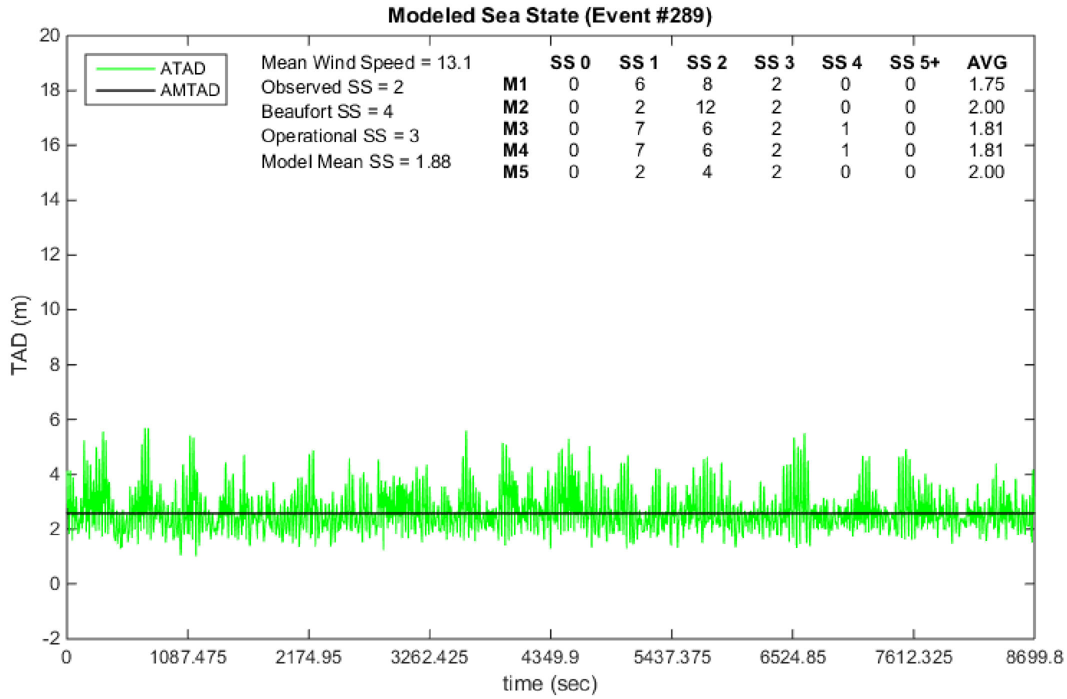

Modeled Sea State (Event #278)
ATAD
AMTAD
Mean Wind Speed = 1.4
Observed SS = 1
Beaufort SS = 1
Operational SS = 1
Model Mean SS = 0.45
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 10 6 0 0 0 0 0.38
M2 11 5 0 0 0 0 0.31
M3 14 2 0 0 0 0 0.13
M4 13 3 0 0 0 0 0.19
M5 2 4 0 2 0 0 1.25
TAD (m)
time (sec)
20
18
16
14
12
10
8
6
4
2
0
-2
0
1132.475
2264.95
3397.425
4529.9
5662.375
6794.85
7927.325
9059.8

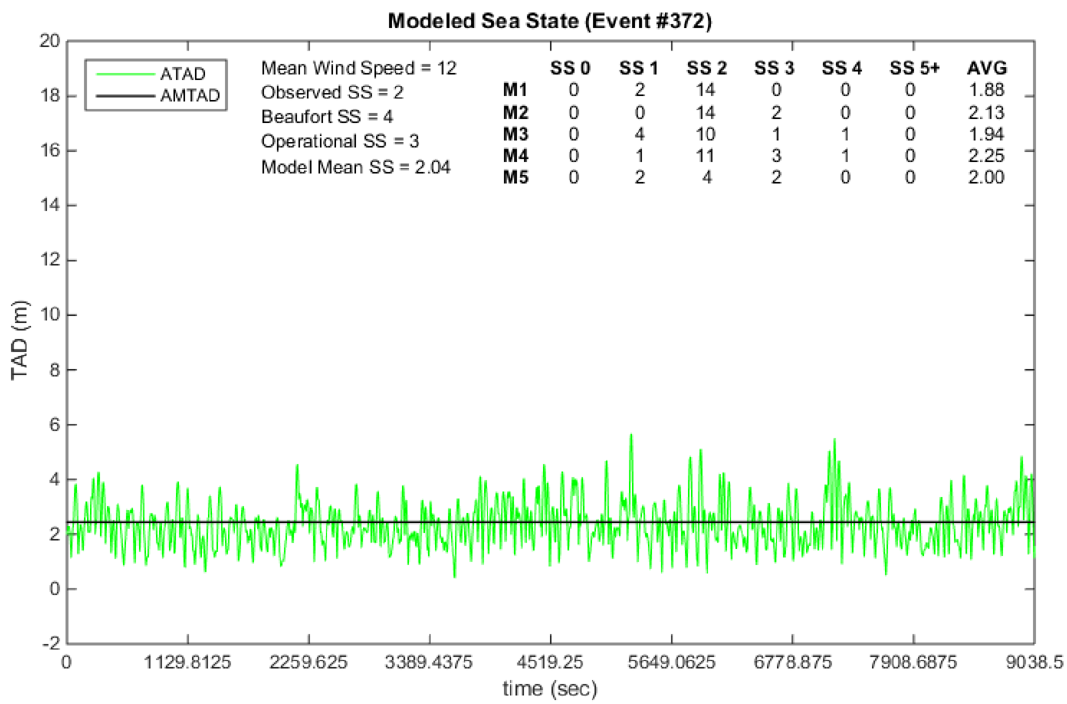

Modeled Sea State (Event #291)
ATAD
AMTAD
Mean Wind Speed = 5
Observed SS = 1
Beaufort SS = 2
Operational SS = 2
Model Mean SS = 1.71
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 7 9 0 0 0 1.56
M2 0 2 14 0 0 0 1.88
M3 0 9 6 1 0 0 1.50
M4 0 8 7 0 1 0 1.63
M5 0 2 4 2 0 0 2.00
TAD (m)
time (sec)
20
18
16
14
12
10
8
6
4
2
0
-2
0
1042.475
2084.95
3127.425
4169.9
5212.375
6254.85
7297.325
8339.8

Figures 25(a-d). Trial A (Sea State 2)

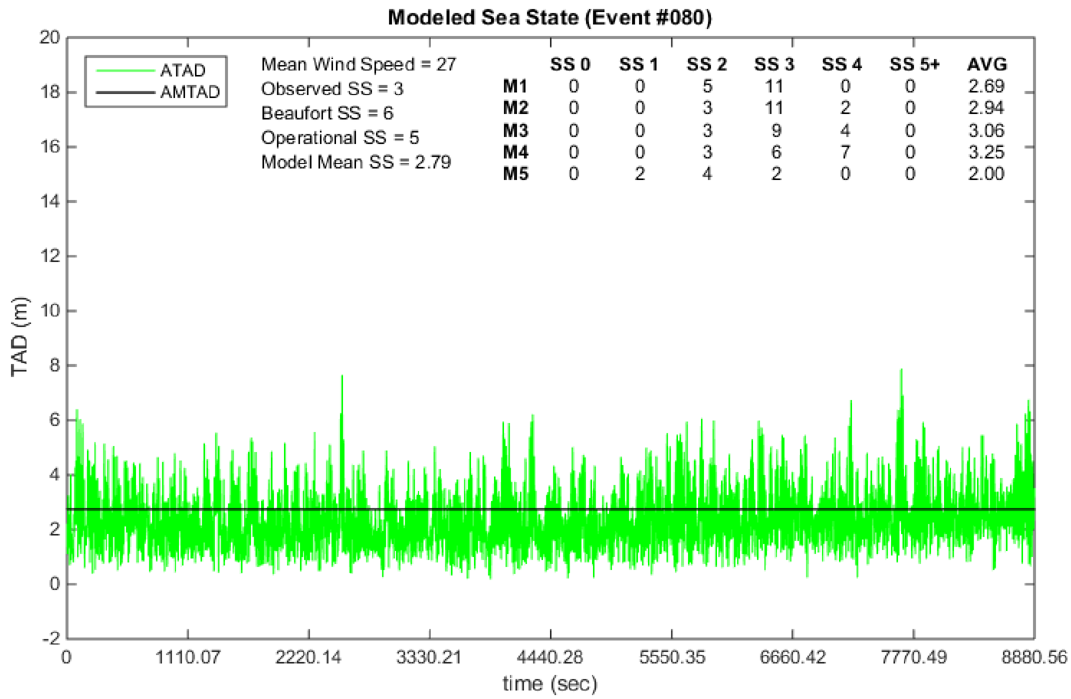


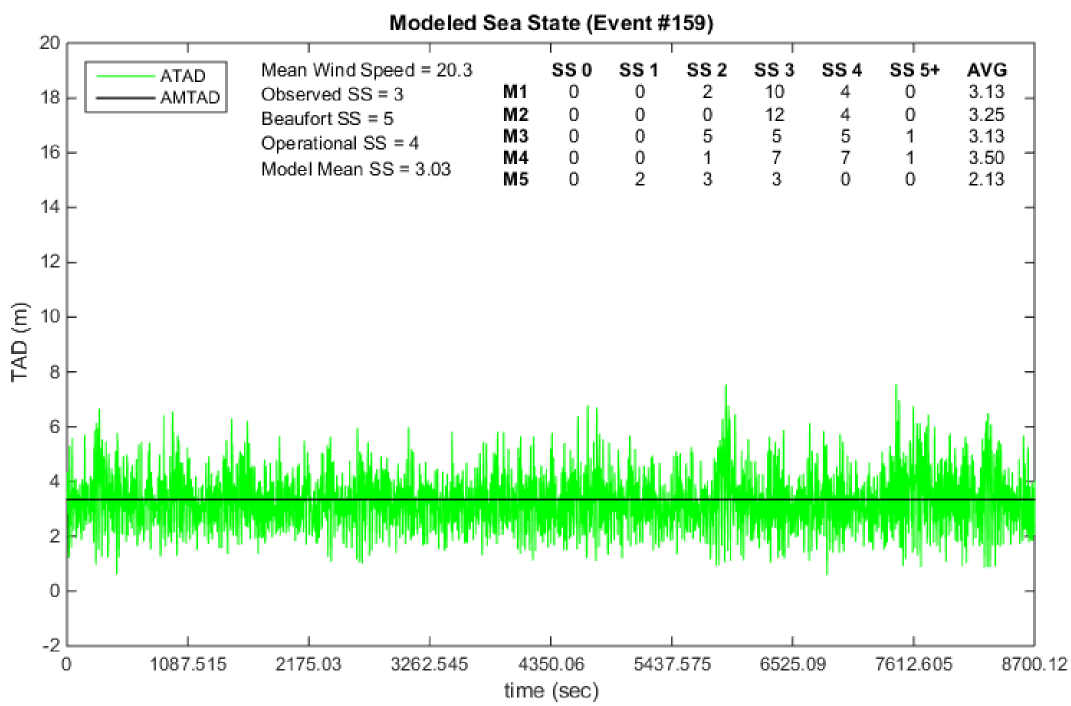


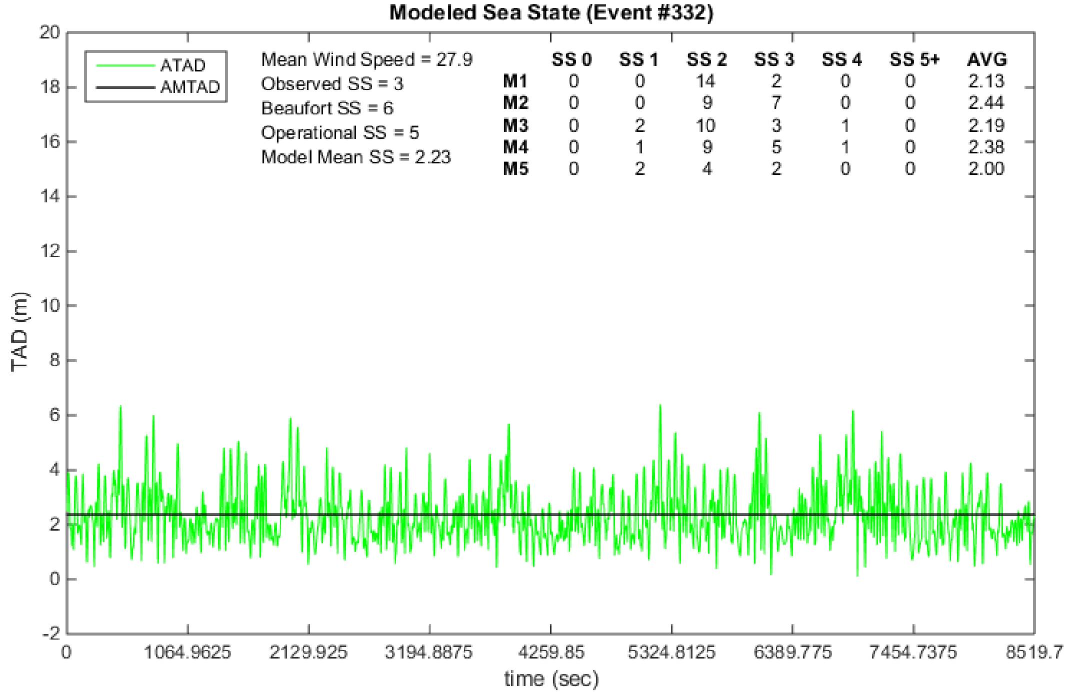

Modeled Sea State (Event #289)
ATAD
AMTAD
Mean Wind Speed = 13.1
Observed SS = 2
Beaufort SS = 4
Operational SS = 3
Model Mean SS = 1.88
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 6 8 2 0 0 1.75
M2 0 2 12 2 0 0 2.00
M3 0 7 6 2 1 0 1.81
M4 0 7 6 2 1 0 1.81
M5 0 2 4 2 0 0 2.00
TAD (m)
time (sec)
0
1087.475
2174.95
3262.425
4349.9
5437.375
6524.85
7612.325
8699.8

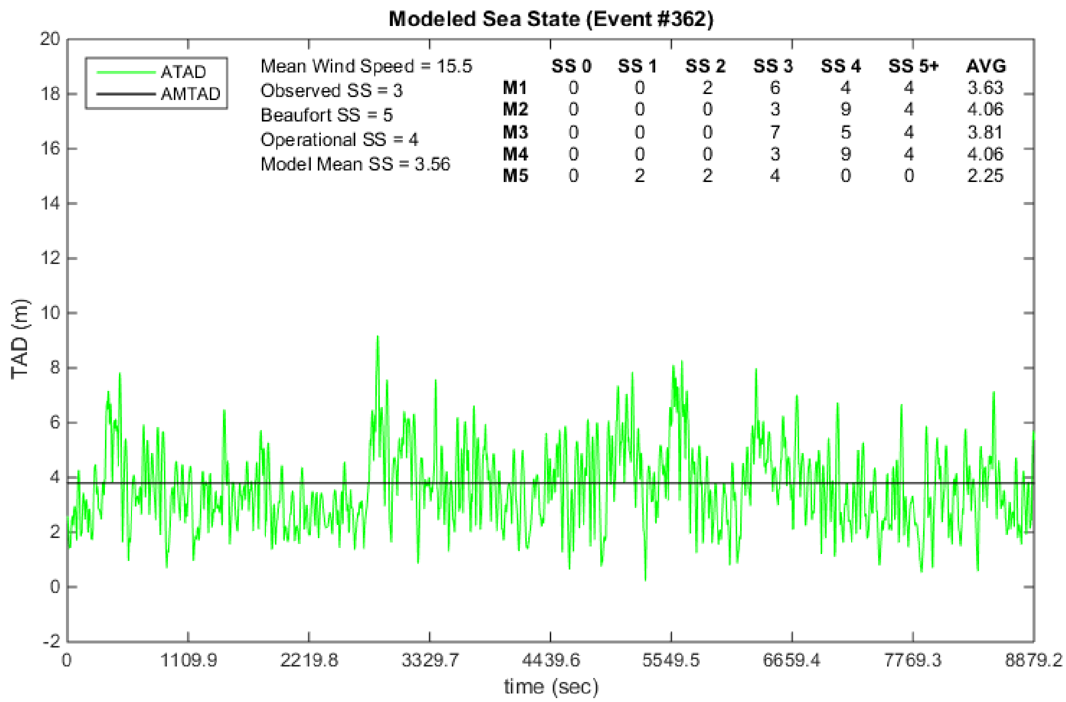

Modeled Sea State (Event #372)
ATAD
AMTAD
Mean Wind Speed = 12
Observed SS = 2
Beaufort SS = 4
Operational SS = 3
Model Mean SS = 2.04
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 2 14 0 0 0 1.88
M2 0 0 14 2 0 0 2.13
M3 0 4 10 1 1 0 1.94
M4 0 1 11 3 1 0 2.25
M5 0 2 4 2 0 0 2.00
TAD (m)
time (sec)
0
1129.8125
2259.625
3389.4375
4519.25
5649.0625
6778.875
7908.6875
9038.5

Figures 26(a-d). Trial A (Sea State 3)

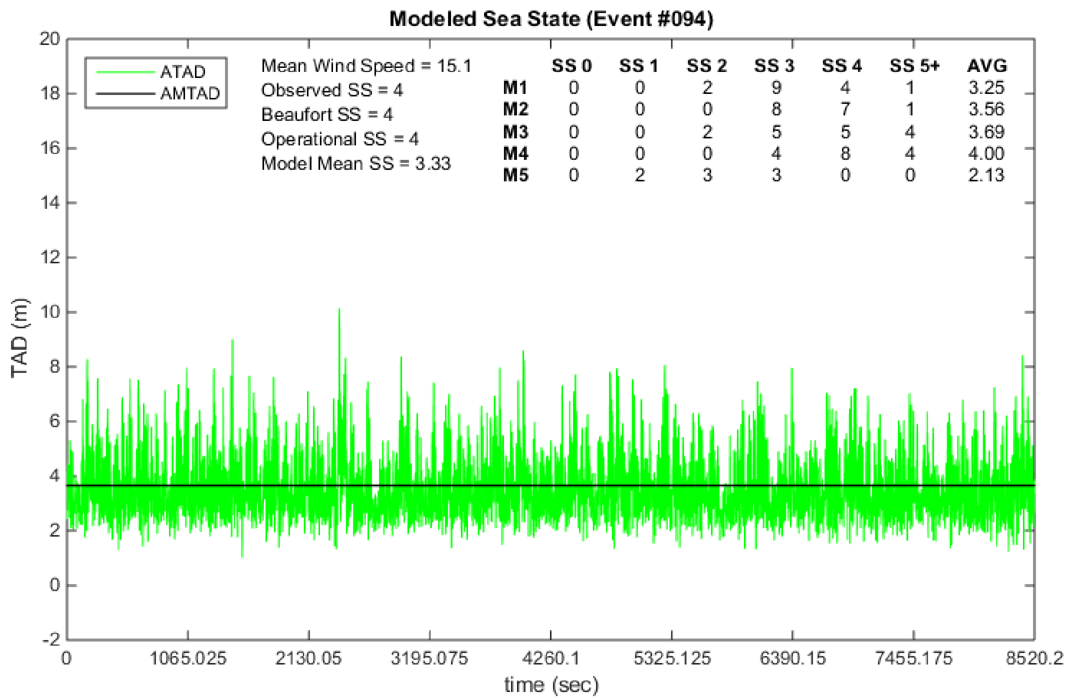


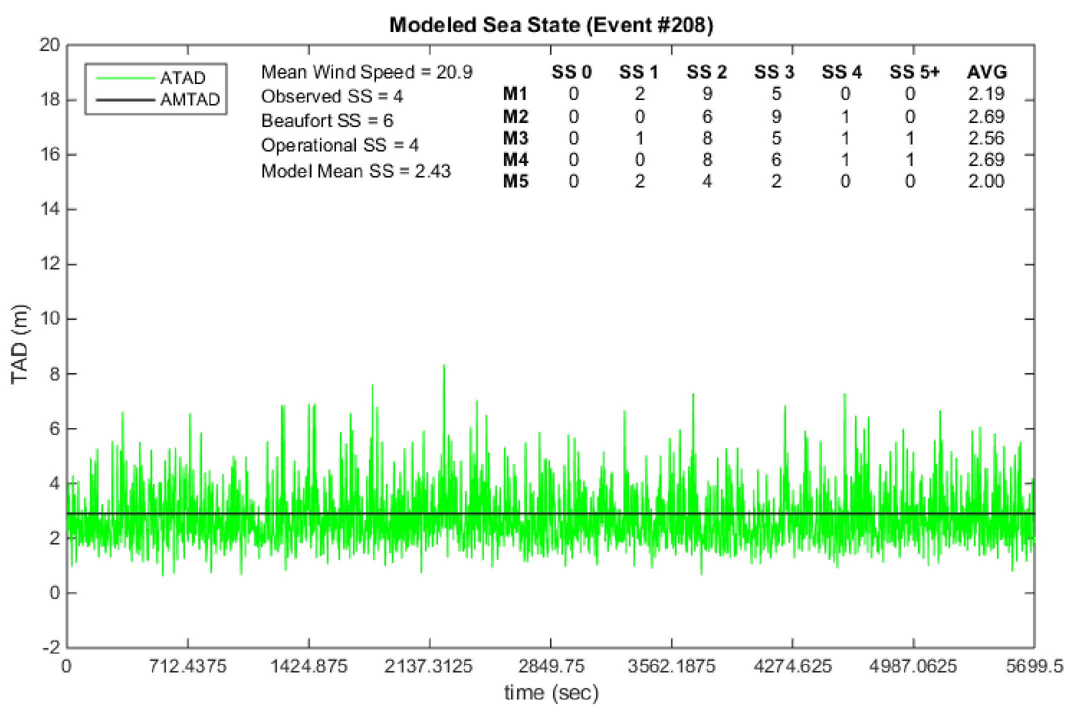


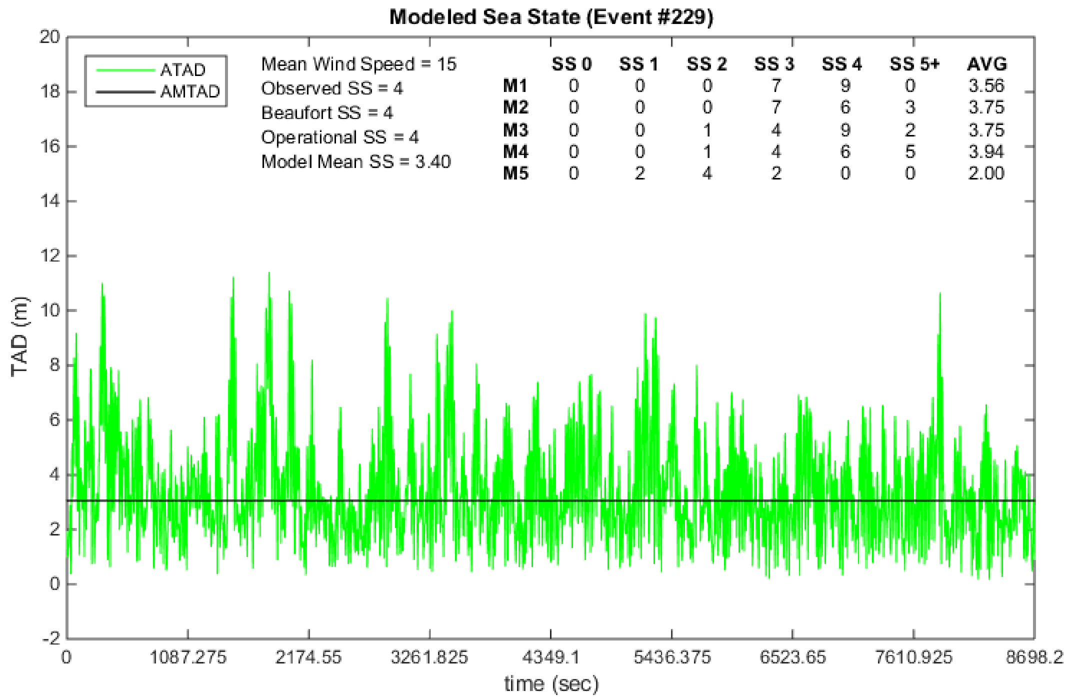

Modeled Sea State (Event #332)
ATAD
AMTAD
Mean Wind Speed = 27.9
Observed SS = 3
Beaufort SS = 6
Operational SS = 5
Model Mean SS = 2.23
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 0 14 2 0 0 2.13
M2 0 0 9 7 0 0 2.44
M3 0 2 10 3 1 0 2.19
M4 0 1 9 5 1 0 2.38
M5 0 2 4 2 0 0 2.00
TAD (m)
time (sec)


Modeled Sea State (Event #362)
ATAD
AMTAD
Mean Wind Speed = 15.5
Observed SS = 3
Beaufort SS = 5
Operational SS = 4
Model Mean SS = 3.56
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 0 2 6 4 4 3.63
M2 0 0 0 3 9 4 4.06
M3 0 0 0 7 5 4 3.81
M4 0 0 0 3 9 4 4.06
M5 0 2 2 4 0 0 2.25
TAD (m)
time (sec)

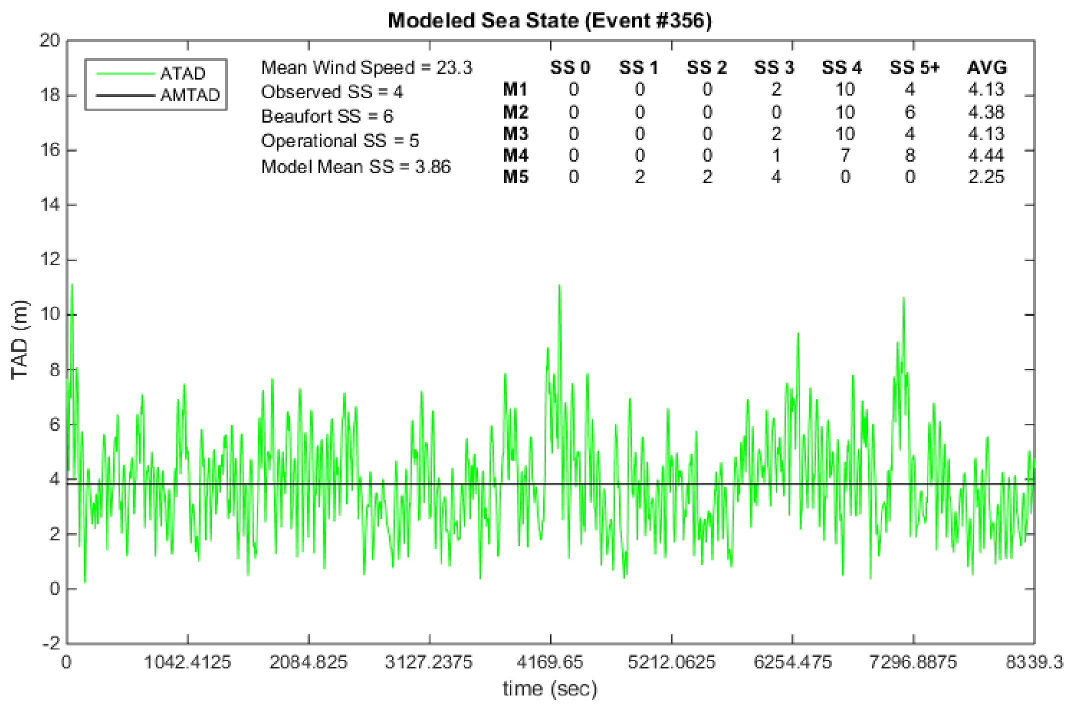
Figures 27(a-d). Trial A (Sea State 4)





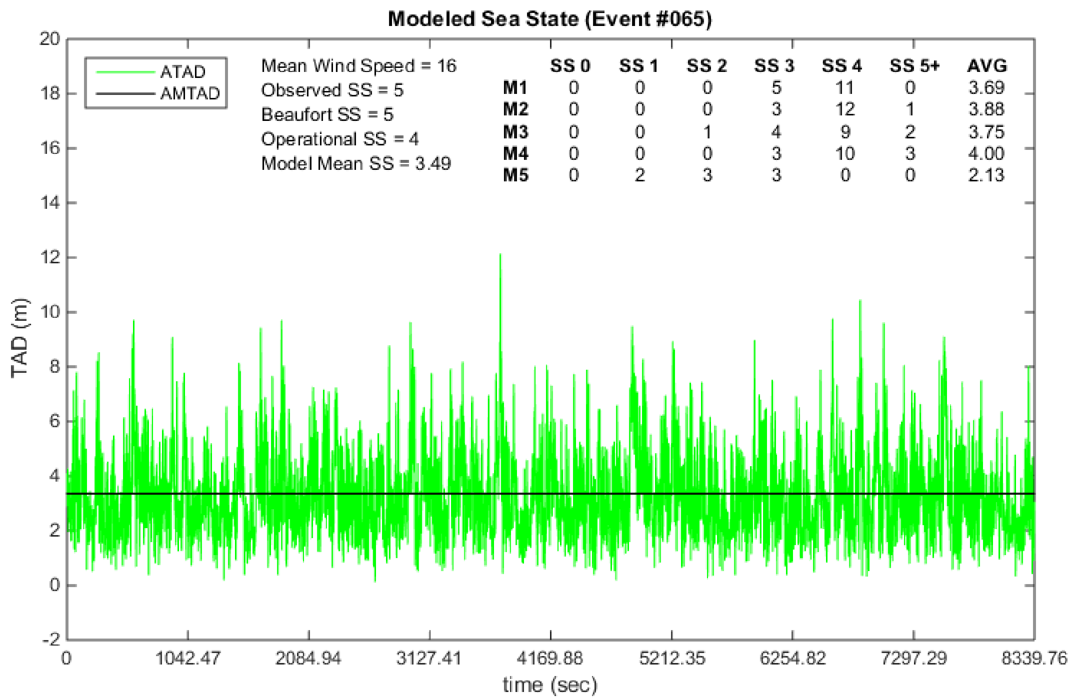


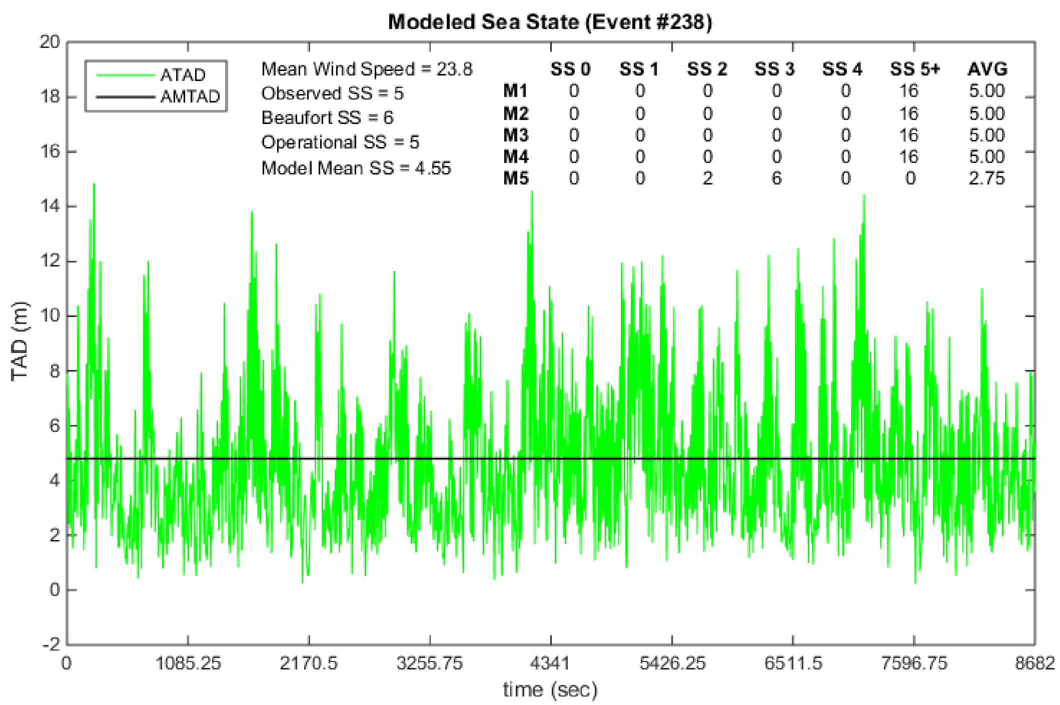
Modeled Sea State (Event #229)
ATAD
AMTAD
Mean Wind Speed = 15
Observed SS = 4
Beaufort SS = 4
Operational SS = 4
Model Mean SS = 3.40
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 0 0 7 9 0 3.56
M2 0 0 0 7 6 3 3.75
M3 0 0 1 4 9 2 3.75
M4 0 0 1 4 6 5 3.94
M5 0 2 4 2 0 0 2.00
TAD (m)
time (sec)
0
1087.275
2174.55
3261.825
4349.1
5436.375
6523.65
7610.925
8698.2

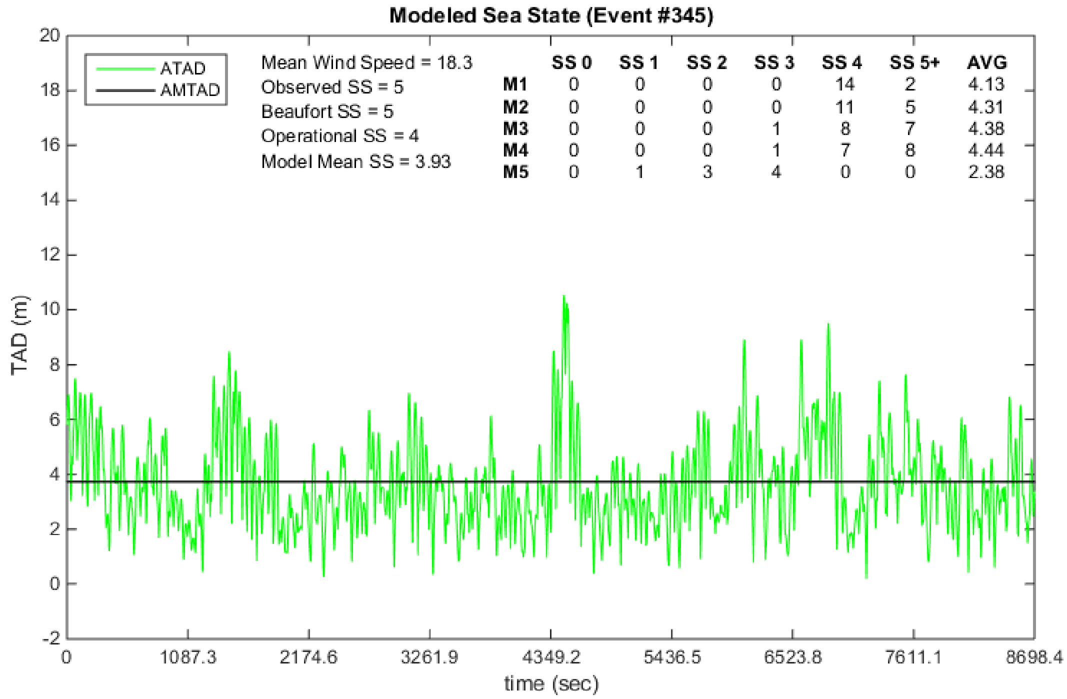


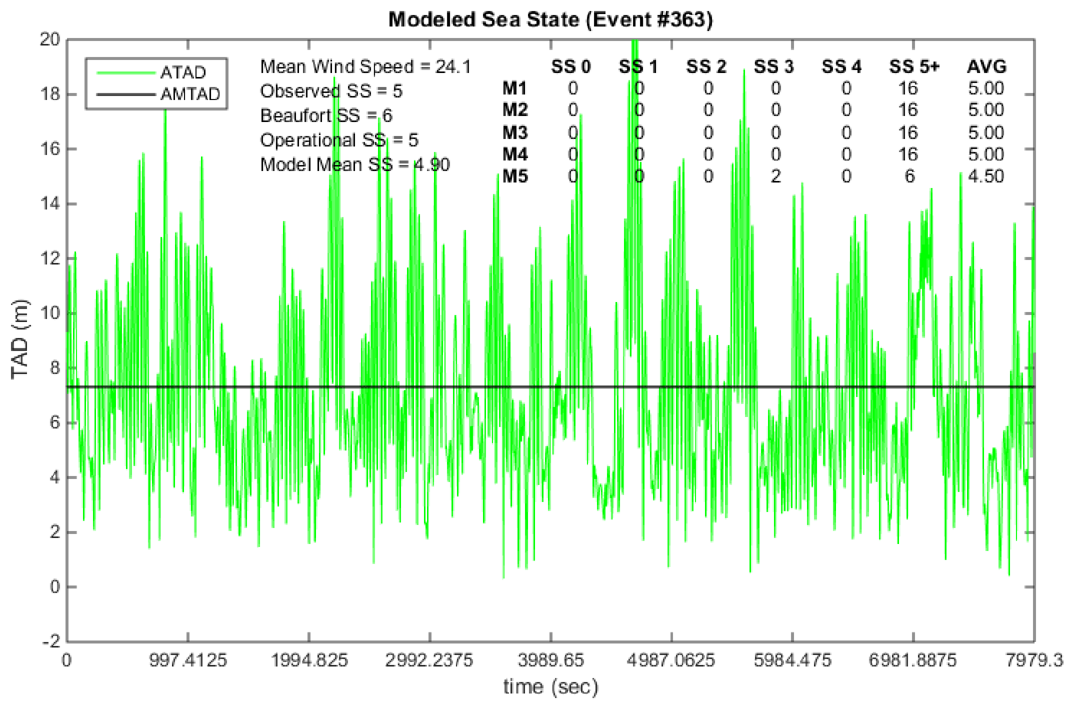
Modeled Sea State (Event #356)
ATAD
AMTAD
Mean Wind Speed = 23.3
Observed SS = 4
Beaufort SS = 6
Operational SS = 5
Model Mean SS = 3.86
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 0 0 2 10 4 4.13
M2 0 0 0 0 10 6 4.38
M3 0 0 0 2 10 4 4.13
M4 0 0 0 1 7 8 4.44
M5 0 2 2 4 0 0 2.25
TAD (m)
time (sec)
0
1042.4125
2084.825
3127.2375
4169.65
5212.0625
6254.475
7296.8875
8339.3

Figures 28(a-d). Trial A (Sea State 5)

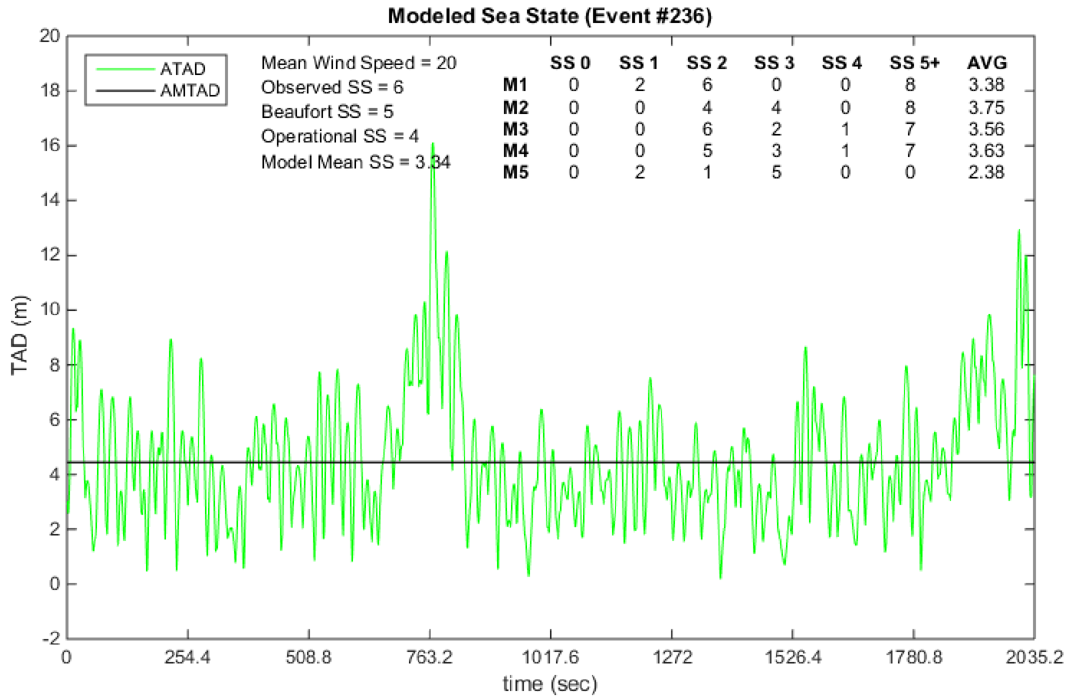


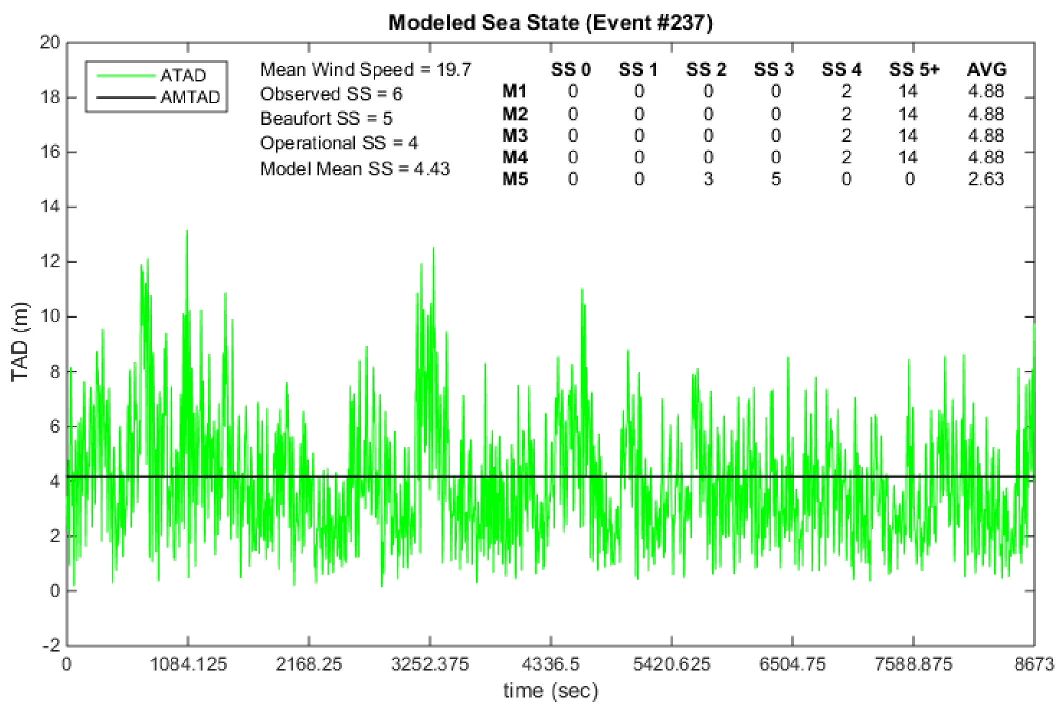



Modeled Sea State (Event #345)
ATAD
AMTAD
Mean Wind Speed = 18.3
Observed SS = 5
Beaufort SS = 5
Operational SS = 4
Model Mean SS = 3.93
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 0 0 0 14 2 4.13
M2 0 0 0 0 11 5 4.31
M3 0 0 0 1 8 7 4.38
M4 0 0 0 1 7 8 4.44
M5 0 1 3 4 0 0 2.38
TAD (m)
time (sec)
0
1087.3
2174.6
3261.9
4349.2
5436.5
6523.8
7611.1
8698.4
20
18
16
14
12
10
8
6
4
2
0
-2


Modeled Sea State (Event #363)
ATAD
AMTAD
Mean Wind Speed = 24.1
Observed SS = 5
Beaufort SS = 6
Operational SS = 5
Model Mean SS = 4.90
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 0 0 0 0 16 5.00
M2 0 0 0 0 0 16 5.00
M3 0 0 0 0 0 16 5.00
M4 0 0 0 0 0 16 5.00
M5 0 0 0 2 0 6 4.50
TAD (m)
time (sec)
0
997.4125
1994.825
2992.2375
3989.65
4987.0625
5984.475
6981.8875
7979.3
20
18
16
14
12
10
8
6
4
2
0
-2

Figures 29(a-b). Trial A (Sea State 6)





3.E.2. *Trial A Analysis*

Trial A is conducted using all 387 survey events. Sea state predictions from the individual parameter sets are then compared to associate observed, Beaufort, and Operational sea state measurements. The number of matching sea states is recorded in Appendix Section G, Tables G1a-G5a. The matching of exact sea states is detailed in Table 18 and suggests that the probability of a predicted sea state matching observations or wind-scale-based codes is low.

Table 18. Trial A - Model Sea State Matching

Model Sea State Matching									
	Observed SS			Beaufort SS			Operational SS		
	Matched	Out of	P	Matched	Out of	P	Matched	Out of	P
Model 1	1504	6192	0.243	994	5504	0.181	1005	5504	0.183
Model 2	1553	6192	0.251	1057	5504	0.192	1166	5504	0.212
Model 3	1199	6192	0.194	955	5504	0.174	900	5504	0.164
Model 4	1298	6192	0.210	1058	5504	0.192	1055	5504	0.192
Model 5	1043	3096	0.337	296	2752	0.108	504	2752	0.183

Model 2 provides the closest overall fit for all three sea state code types, but the limited probabilities make Model 2 an unlikely candidate for a final application. The results of the experimental model suggest greater connection to the observed sea states, but similarly low expectations for the wind-based scales. Model 5 has, on average, about a 10% higher prediction potential for the observed sea state.

The plots in Figures 30-34 feature the comparison between the observed sea state measurements and model sea state predictions. Moving averages of the observed and model sea states are used to smooth out any short-term variations and to emphasize trends. In these plots, the average is taken using an 80-point window.

Figure 30. Trial A - Model 1 Results

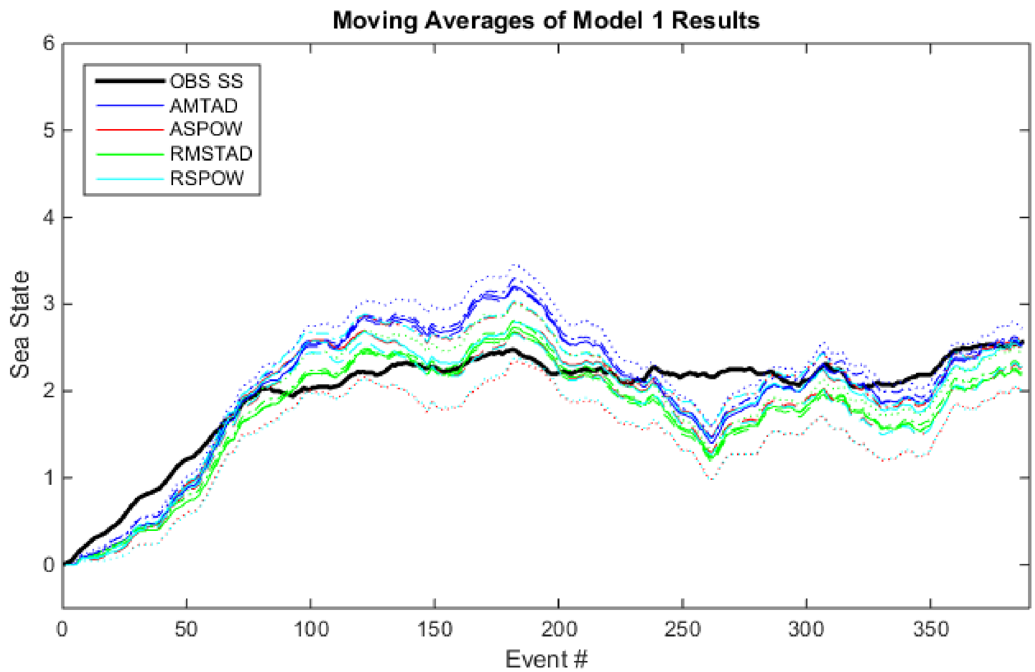


Figure 31. Trial A - Model 2 Results

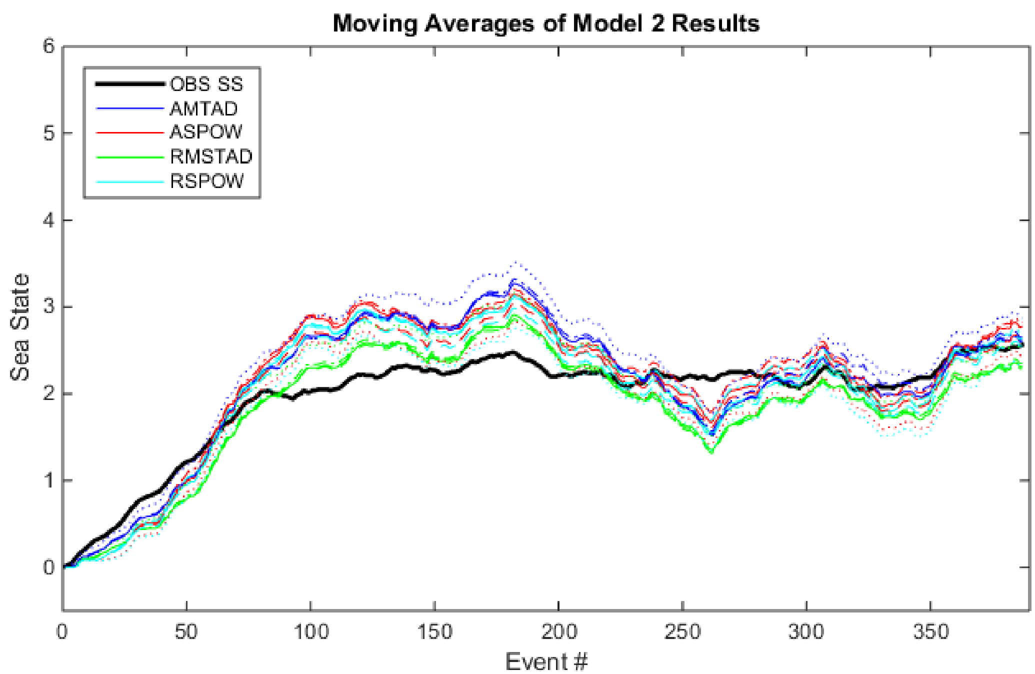


Figure 32. Trial A - Model 3 Results

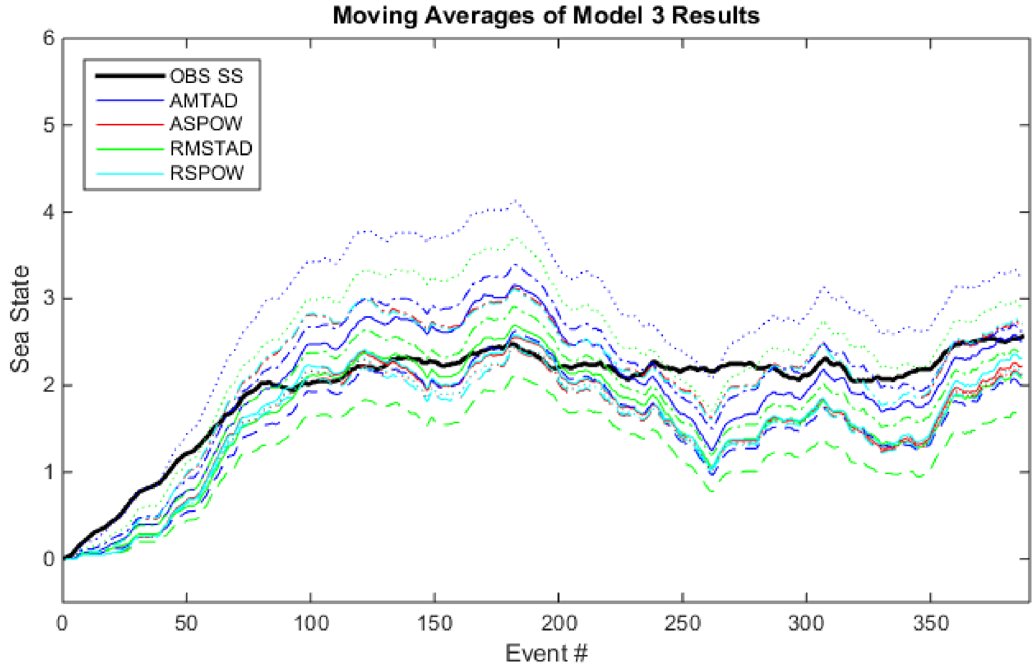


Figure 33. Trial A - Model 4 Results

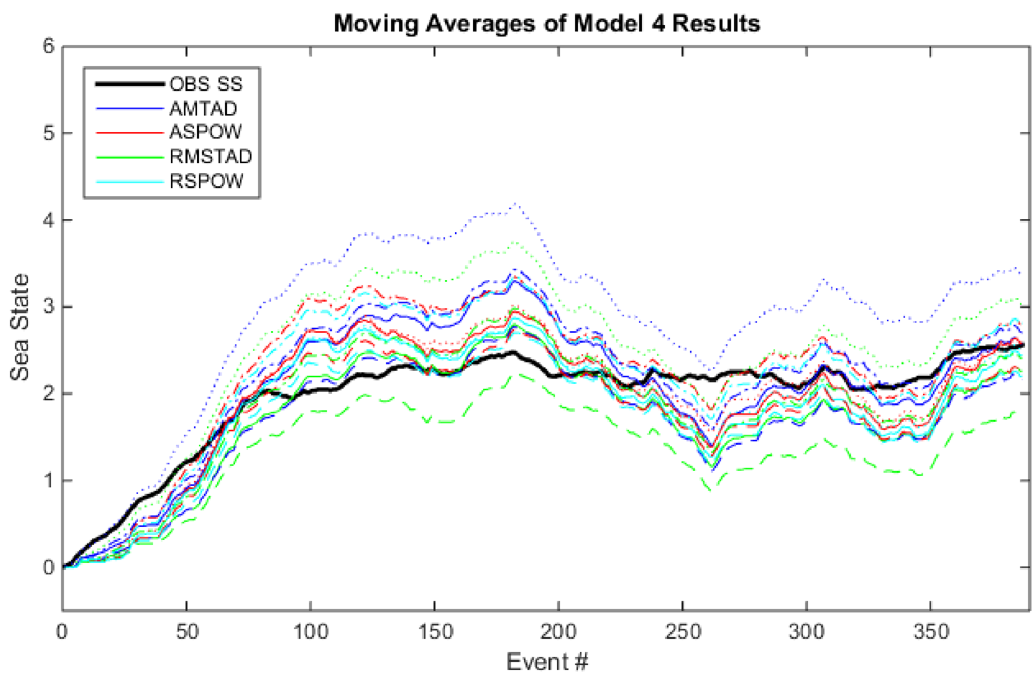
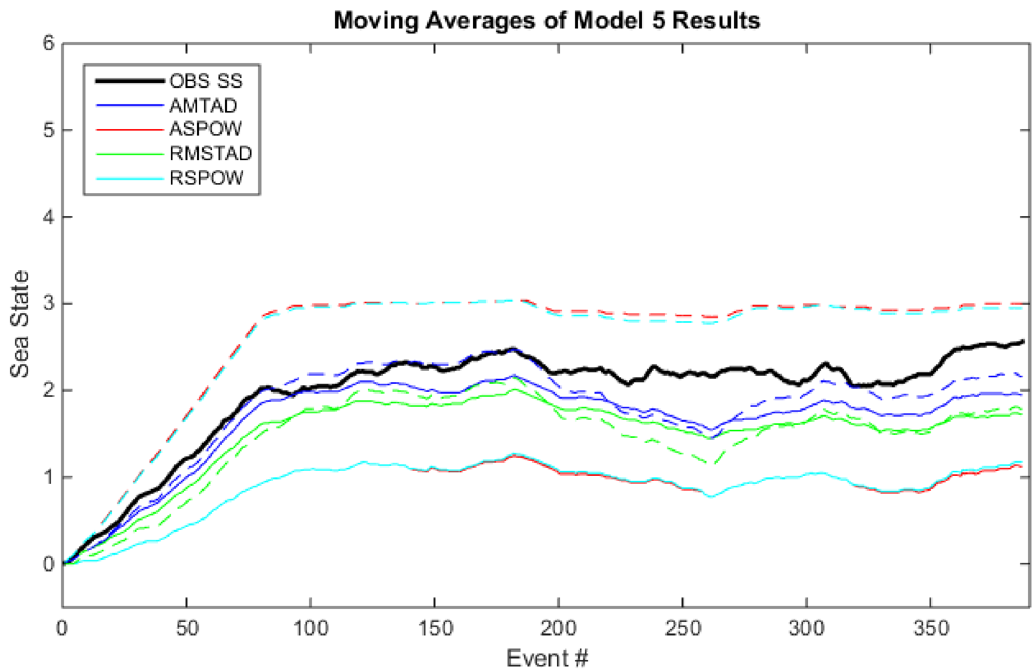


Figure 34. Trial A - Model 5 Results



If the counts of total predictions for any individual sea state are used as weighting factors, a weighted average of the predicted sea state is then produced for each model. To informally commute all model predictions into a universal model mean, the averages of all five models are combined to form a total model mean of the predicted sea state. This average is then compared to sea state marginal tolerances (i.e., within ±0.5 of a sea state, within ±1 of a sea state, etc.) for each event's observed, Beaufort, and Operational sea state measurement. Since the total model mean sea state is generally not integral it can be compared in terms of fractional sea states. The matching of the given marginal tolerances is detailed in Table 19. The probabilities in these comparisons suggest a strong likelihood of the model to predict the sea state within two sea state codes.

Table 19. Trial A – Total Model Mean Sea State Margin Matching

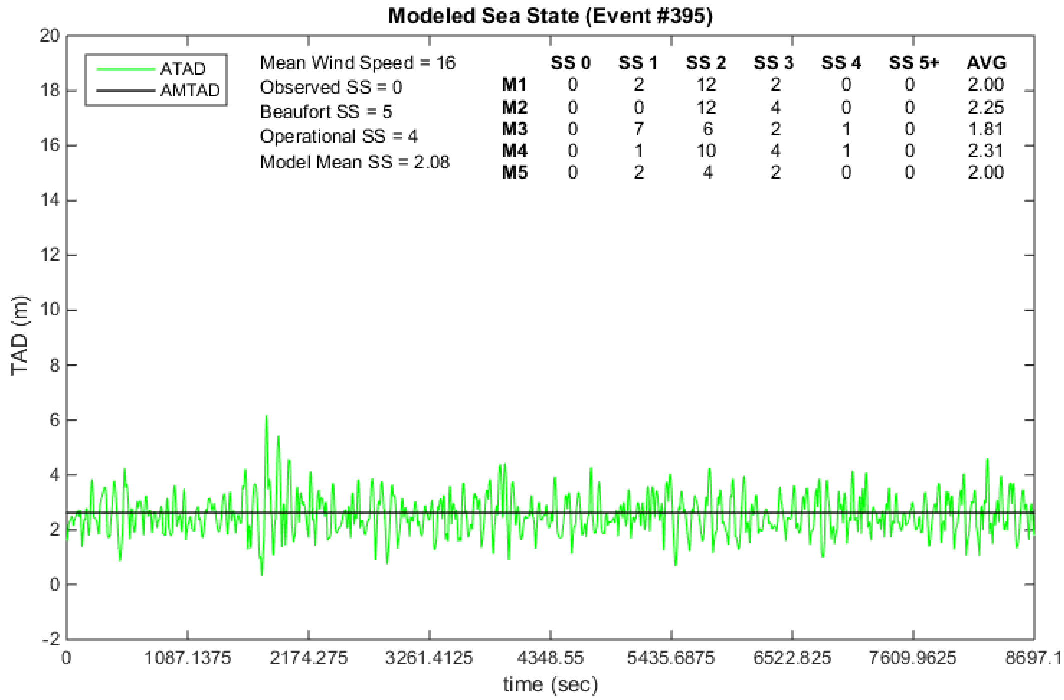
Total Model Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	108	0.279	58	0.169	74	0.215
± 1.0	208	0.537	105	0.305	140	0.407
± 1.5	295	0.762	158	0.459	212	0.616
± 2.0	351	0.907	205	0.596	250	0.727

Matching within the marginal tolerances is compared for individual models and the results are shown in Appendix Section G, Tables G1b-G5b. The results suggest that Models 1 through 4 will only be within one observed sea state approximately 50% of the time, whereas Model 5 can produce sea states that will be within one observed sea state approximately 80.6% of the time.

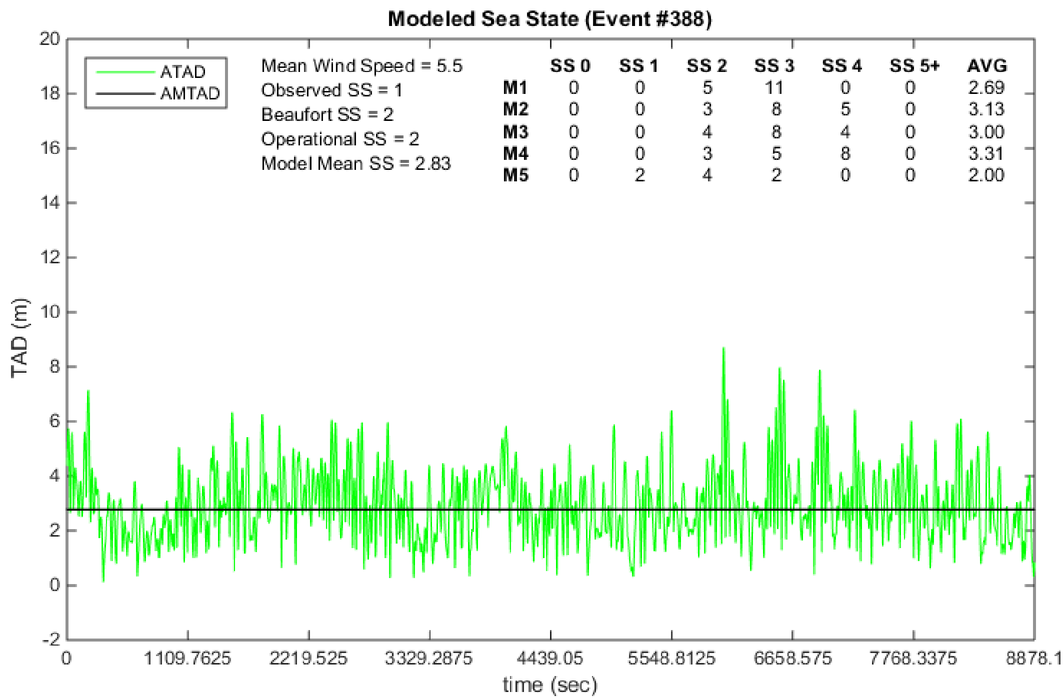
Although the total model mean is a better overall predictor within marginal tolerances compared to exact matching, Model 5 appears to be the best performer for modeling observed sea states in both exact matching and marginal estimates.

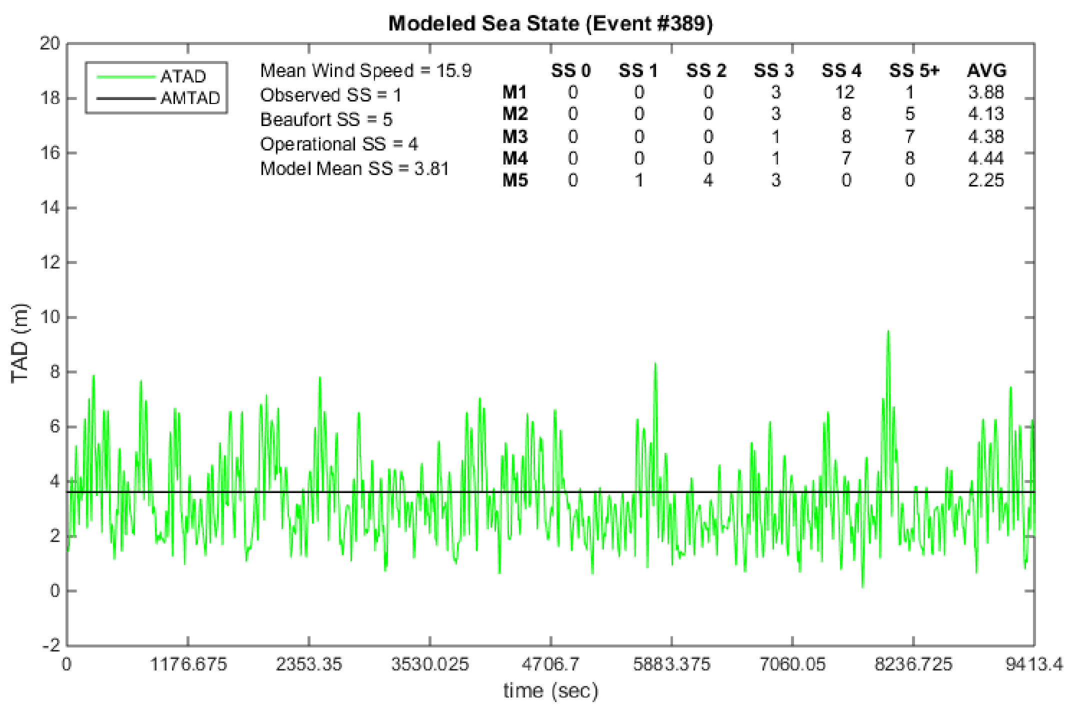
3.E.3. *Trial B*

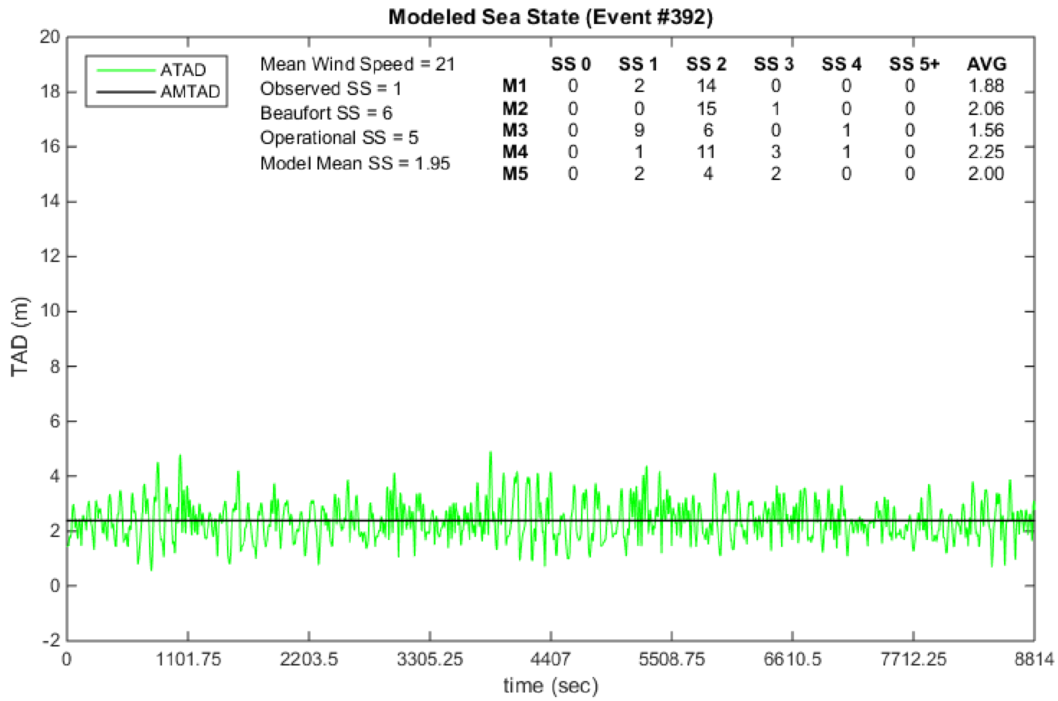
Figure 35. Trial B (Sea State 0)



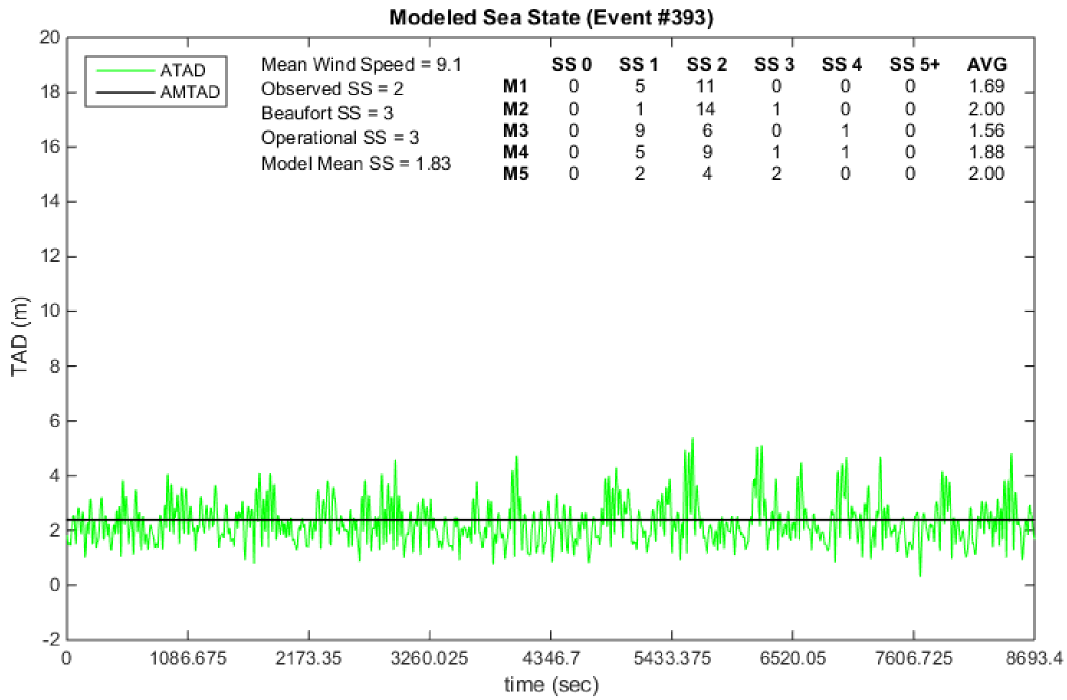
Figures 36(a-c). Trial B (Sea State 1)

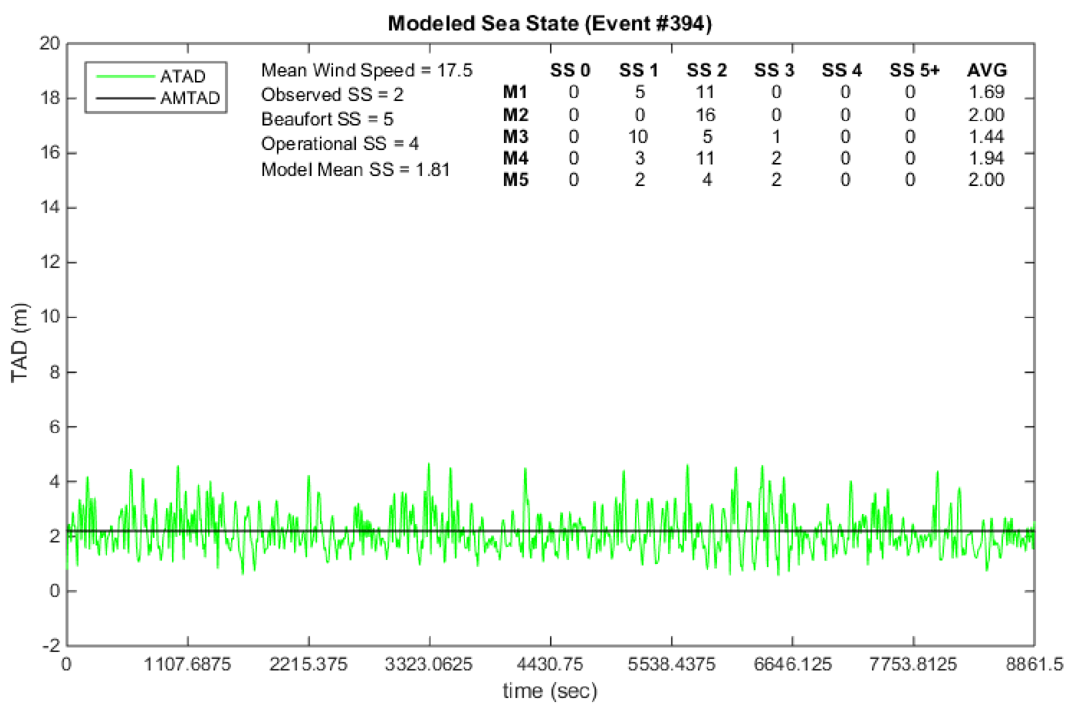


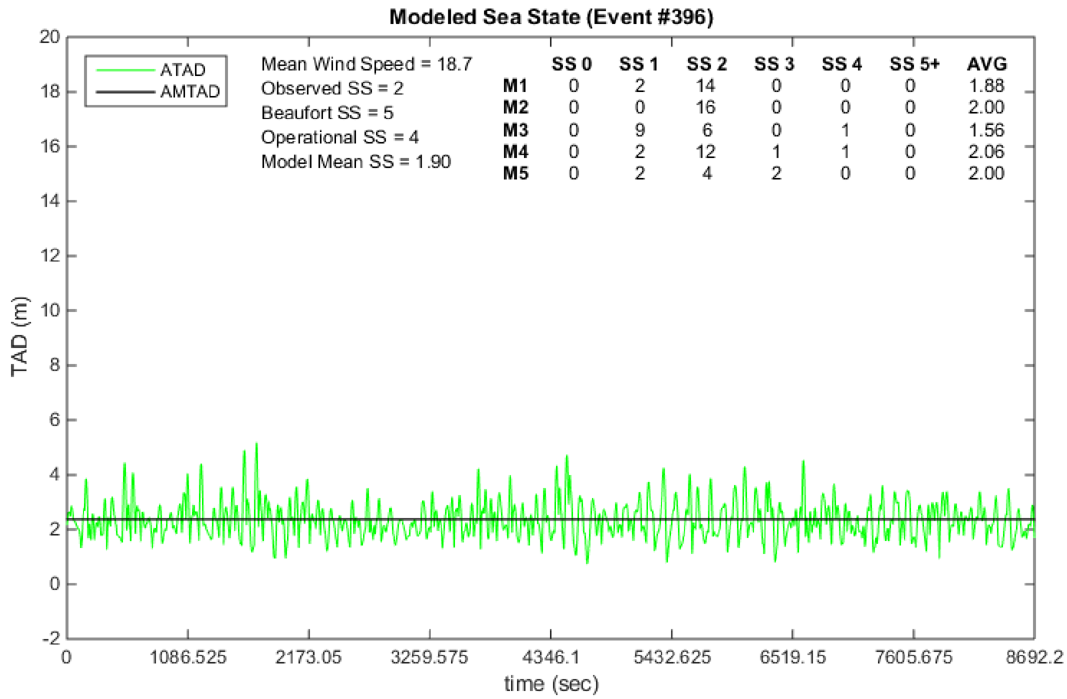


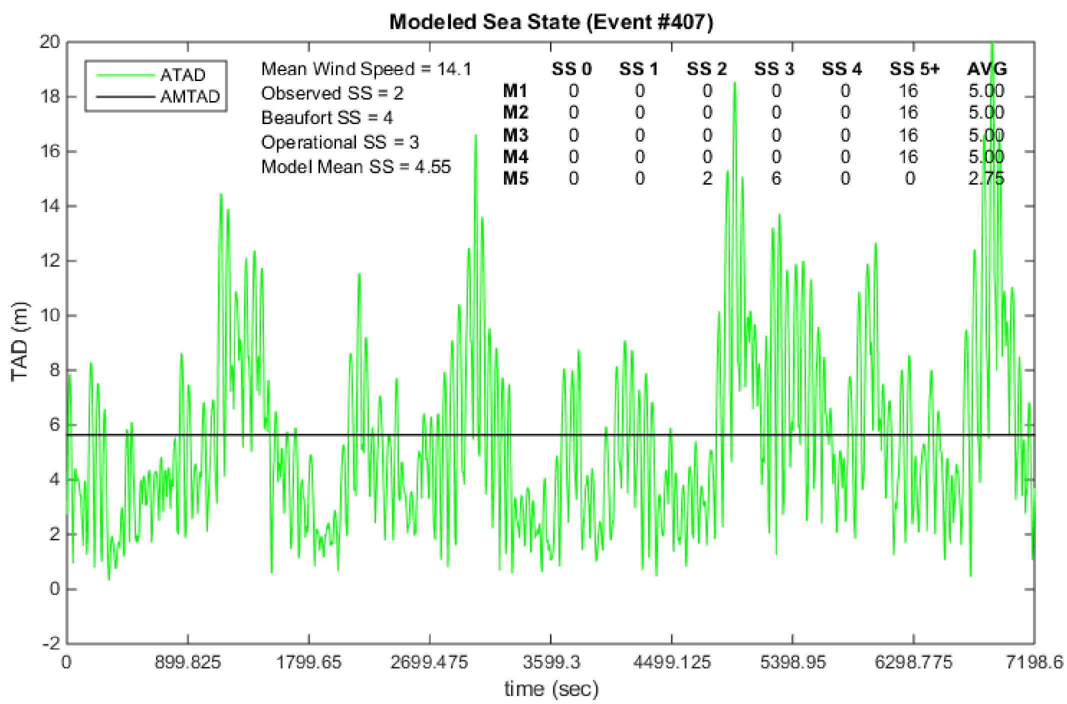

Modeled Sea State (Event #392)
ATAD
AMTAD
Mean Wind Speed = 21
Observed SS = 1
Beaufort SS = 6
Operational SS = 5
Model Mean SS = 1.95
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 2 14 0 0 0 1.88
M2 0 0 15 1 0 0 2.06
M3 0 9 6 0 1 0 1.56
M4 0 1 11 3 1 0 2.25
M5 0 2 4 2 0 0 2.00
TAD (m)
time (sec)
0
1101.75
2203.5
3305.25
4407
5508.75
6610.5
7712.25
8814

Figures 37(a-d). Trial B (Sea State 2)

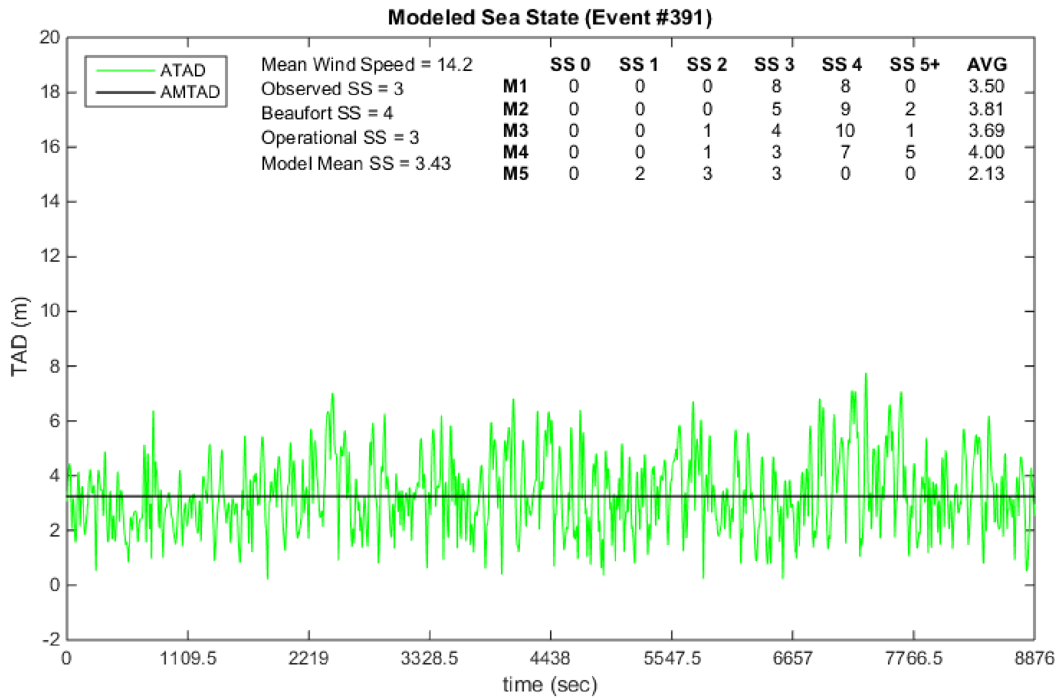


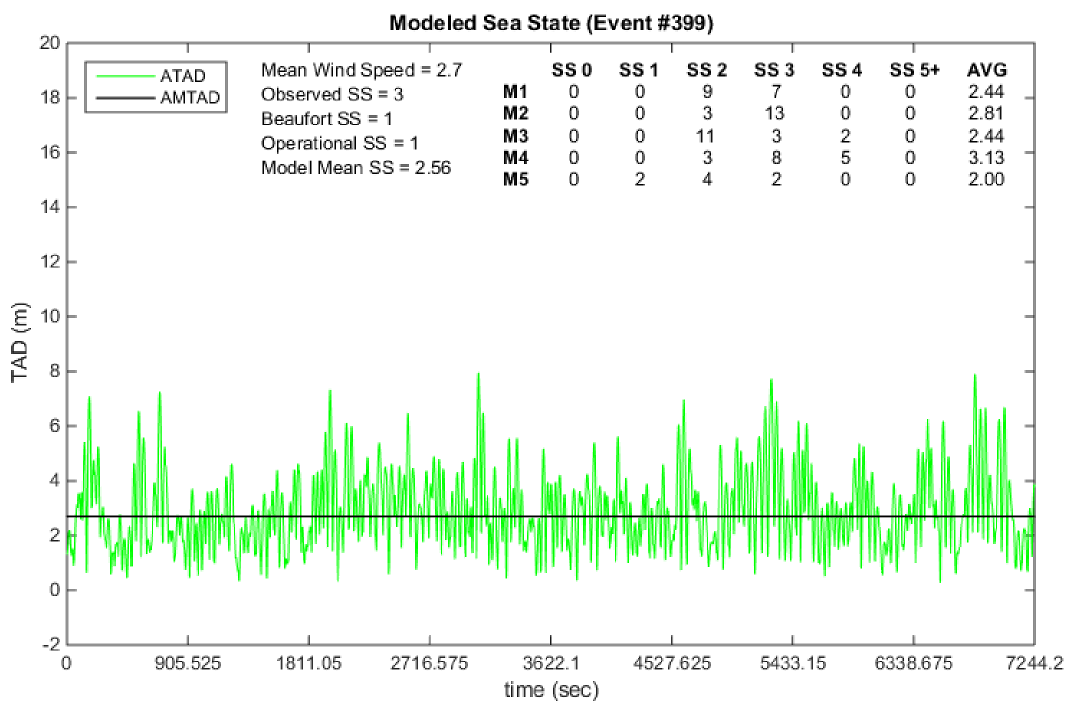


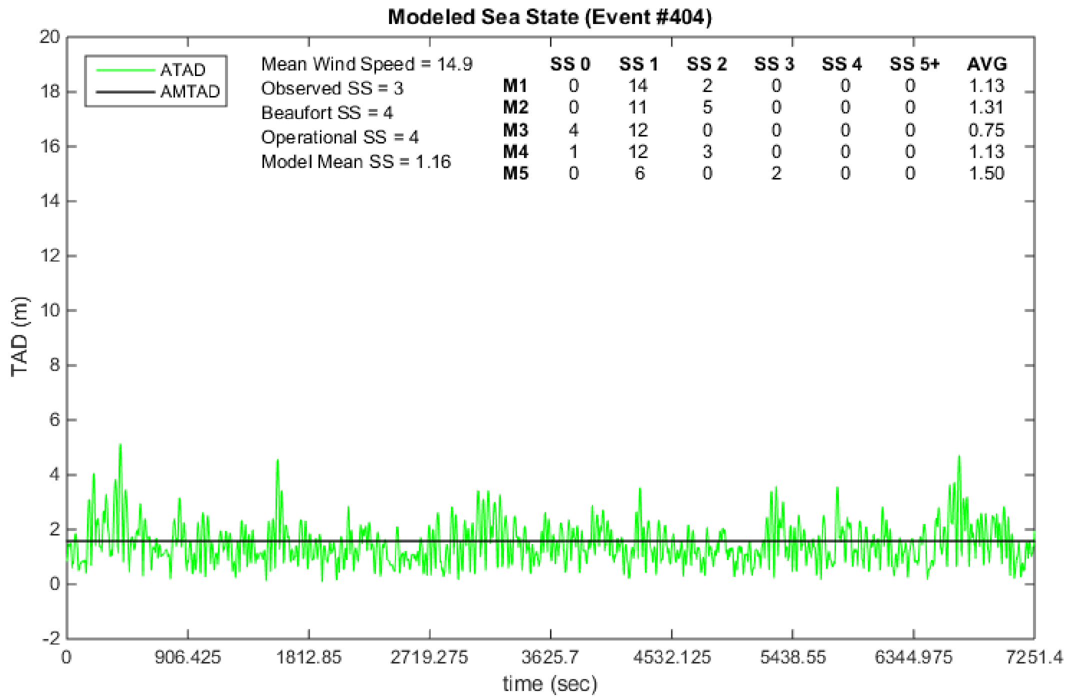

Modeled Sea State (Event #396)
ATAD
AMTAD
Mean Wind Speed = 18.7
Observed SS = 2
Beaufort SS = 5
Operational SS = 4
Model Mean SS = 1.90
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 2 14 0 0 0 1.88
M2 0 0 16 0 0 0 2.00
M3 0 9 6 0 1 0 1.56
M4 0 2 12 1 1 0 2.06
M5 0 2 4 2 0 0 2.00
TAD (m)
time (sec)
0
1086.525
2173.05
3259.575
4346.1
5432.625
6519.15
7605.675
8692.2

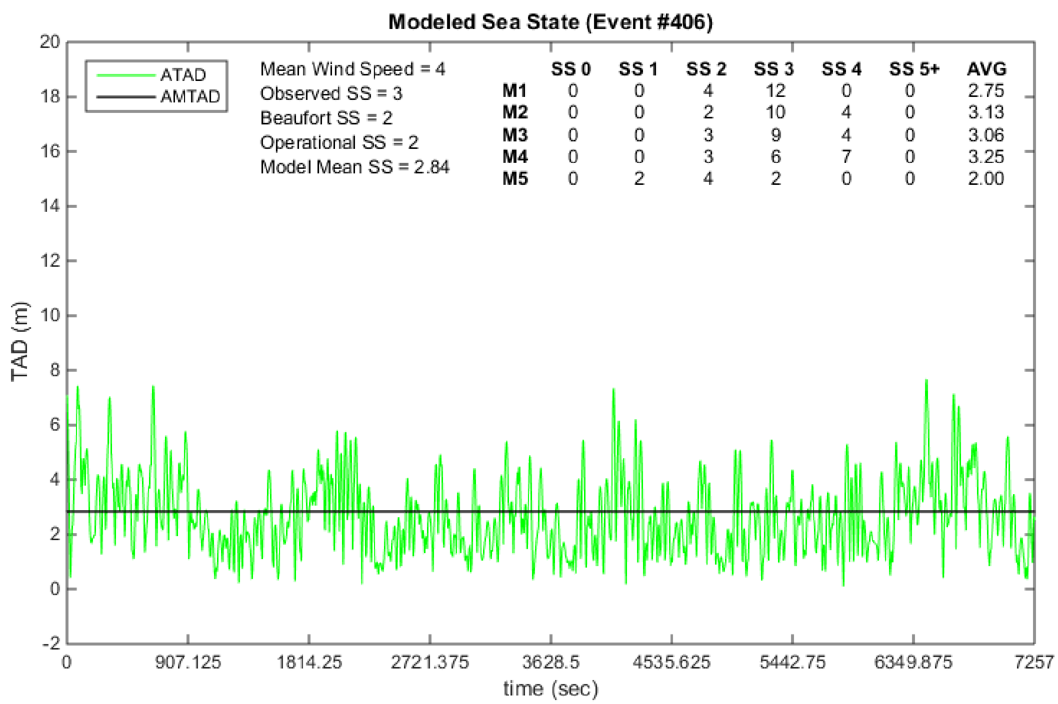

Modeled Sea State (Event #407)
ATAD
AMTAD
Mean Wind Speed = 14.1
Observed SS = 2
Beaufort SS = 4
Operational SS = 3
Model Mean SS = 4.55
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 0 0 0 0 16 5.00
M2 0 0 0 0 0 16 5.00
M3 0 0 0 0 0 16 5.00
M4 0 0 0 0 0 16 5.00
M5 0 0 2 6 0 0 2.75
TAD (m)
time (sec)
0
899.825
1799.65
2699.475
3599.3
4499.125
5398.95
6298.775
7198.6

Figures 38(a-d). Trial B (Sea State 3)

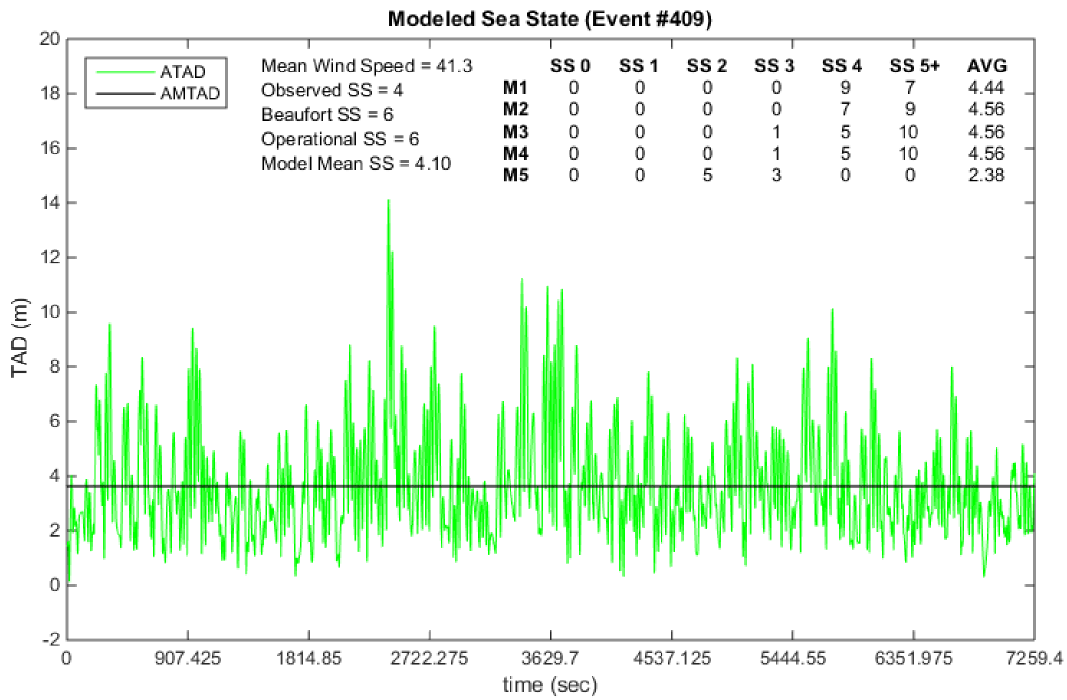





Modeled Sea State (Event #404)
ATAD
AMTAD
Mean Wind Speed = 14.9
Observed SS = 3
Beaufort SS = 4
Operational SS = 4
Model Mean SS = 1.16
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 14 2 0 0 0 1.13
M2 0 11 5 0 0 0 1.31
M3 4 12 0 0 0 0 0.75
M4 1 12 3 0 0 0 1.13
M5 0 6 0 2 0 0 1.50
TAD (m)
time (sec)
0
906.425
1812.85
2719.275
3625.7
4532.125
5438.55
6344.975
7251.4


Modeled Sea State (Event #406)
ATAD
AMTAD
Mean Wind Speed = 4
Observed SS = 3
Beaufort SS = 2
Operational SS = 2
Model Mean SS = 2.84
SS 0 SS 1 SS 2 SS 3 SS 4 SS 5+ AVG
M1 0 0 4 12 0 0 2.75
M2 0 0 2 10 4 0 3.13
M3 0 0 3 9 4 0 3.06
M4 0 0 3 6 7 0 3.25
M5 0 2 4 2 0 0 2.00
TAD (m)
time (sec)
0
907.125
1814.25
2721.375
3628.5
4535.625
5442.75
6349.875
7257

Figures 39(a,b). Trial B (Sea State 4)



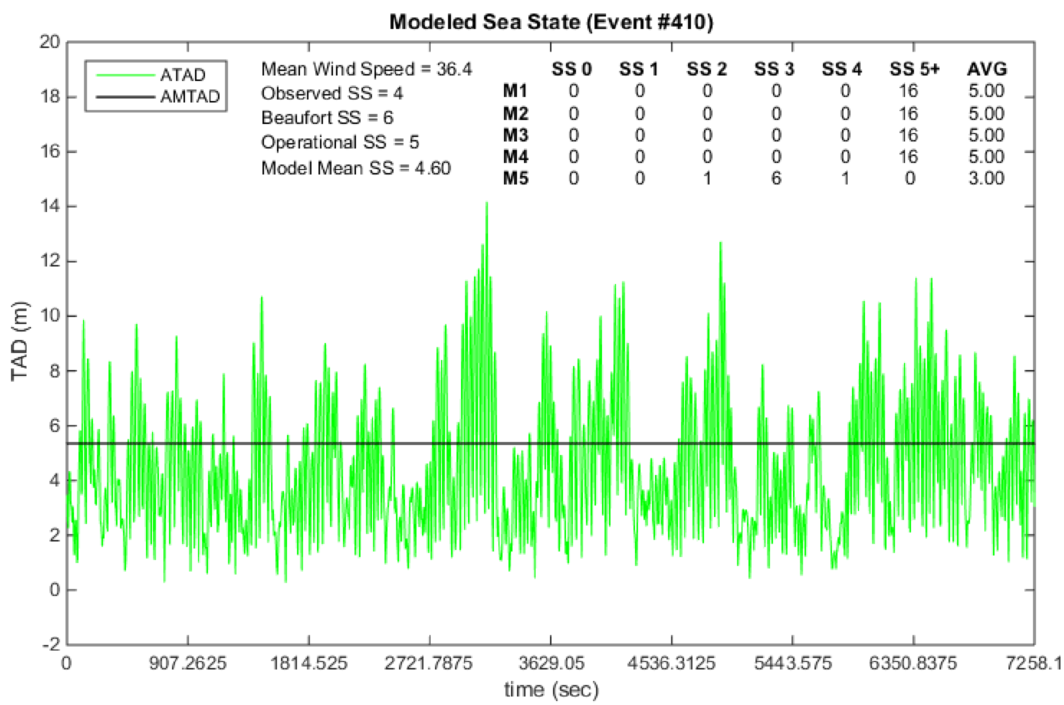
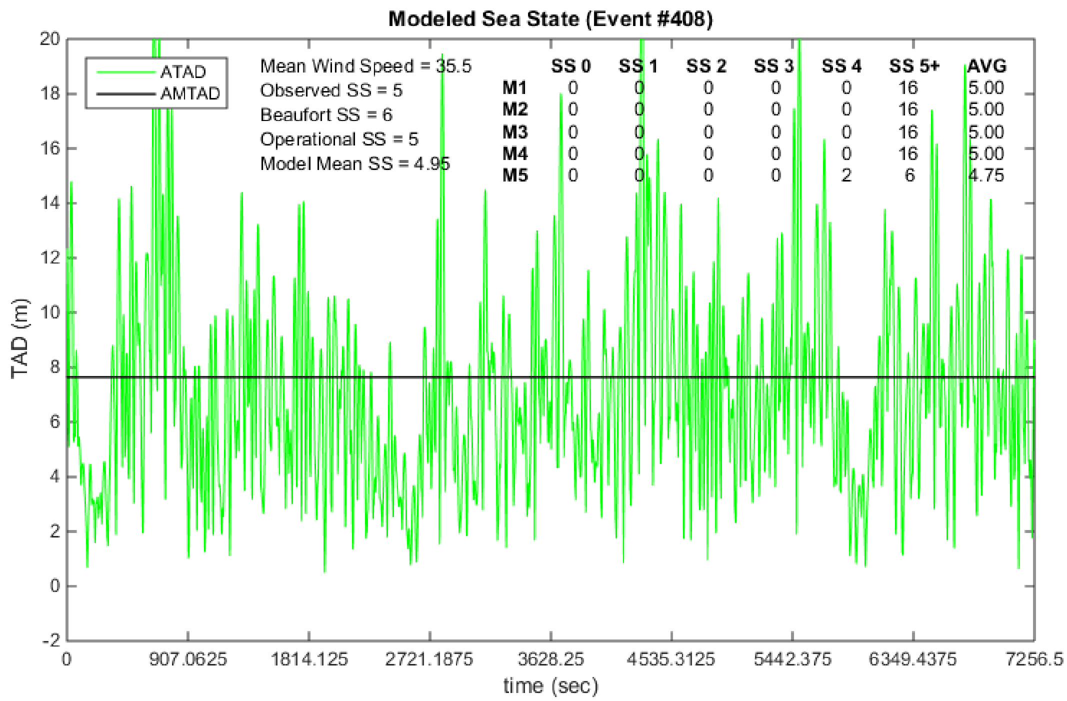


Figure 40. Model Run B (Sea State 5)



3.E.4. *Trial B Analysis*

Trial B is conducted using all 23 survey events. Sea state predictions from the individual parameter sets are then compared to associate observed, Beaufort, and Operational sea state measurements. The number of matching sea states is recorded in Appendix Section H, Tables H1a-H5a. The matching of exact sea states is detailed in Table 20 and suggests that the probability of a predicted sea state matching observations or wind-scale-based codes is low and comparable to the matching in Trial A (see Table 16).

Table 20. Trial B - Model Sea State Matching

Model Sea State Matching									
	Observed SS			Beaufort SS			Operational SS		
	Matched	Out of	P	Matched	Out of	P	Matched	Out of	P
Model 1	96	368	0.261	103	368	0.280	105	368	0.285
Model 2	110	368	0.299	107	368	0.291	97	368	0.264
Model 3	67	368	0.182	111	368	0.302	101	368	0.274
Model 4	80	368	0.217	111	368	0.302	97	368	0.264
Model 5	59	184	0.321	28	184	0.152	43	184	0.234

In this run, Model 2 provides the closest overall fit for all three sea state code types, but once again, limited probabilities make Model 2 an unlikely candidate for a final application. The results of the experimental model suggest greater connection to the observed sea states than Models 1 through 4, but have similarly low expectations for the wind-based scales. Model 5 has, on average, about a 10% higher prediction potential for the observed sea state.

Matching within marginal tolerances is shown in Table 21 and conveys comparable probabilities, albeit lower, to the marginal matching in Trial A. Note that there is about a 10% increase in matching within 0.5 of an observed sea state for Model 1, an almost 20% increase in matching within 1 Beaufort sea state code, and approximately 14% decrease in matching within 1.5 Operational sea states.

Table 21. Trial B – Total Model Mean Sea State Margin Matching

Total Model Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	9	0.391	4	0.174	7	0.304
± 1.0	11	0.478	11	0.478	10	0.435
± 1.5	13	0.565	13	0.565	11	0.478
± 2.0	19	0.826	14	0.609	16	0.696

Matching within the marginal tolerances is compared for individual models and the results are shown in Appendix Section H, Tables H1b-H5b. Probabilities are generally lower than those produced in Trial A. The results suggest that Models 1 through 4 will only be within one observed sea state approximately 50% of the time, whereas Model 5 can produce sea states that will be within one observed sea state 69.6% of the time.

Although the total model mean is a better overall predictor within marginal tolerances compared to exact matching, Model 5 appears to be the best performer for modeling marginal and exact observed sea states.

It is apparent from the plots and associated measurements that either the height of swells overvalued the heave displacement measurement (the presence of significant swells can be mistaken as sea roughness), or observer error is present. The observer in the second survey was highly qualified, but there is limited knowledge of the qualifications or source of the observations in the second survey.

No moving average plots of the model performance were produced due to the similarity of results from Trial A.

3.F. Additional Analysis

Visibility Comparisons

Comparisons were made with visibility conditions from events 001 to 387 and the marginal matching of sea state observations. Table 22 shows marginal counts for both lighting conditions.

Table 22. Visibility Margin Matching

Visibility Matching				
	Poor	P	Good	P
± 0.5	32	0.083	76	0.196
± 1.0	58	0.150	150	0.388
± 1.5	84	0.217	211	0.545
± 2.0	107	0.276	244	0.630

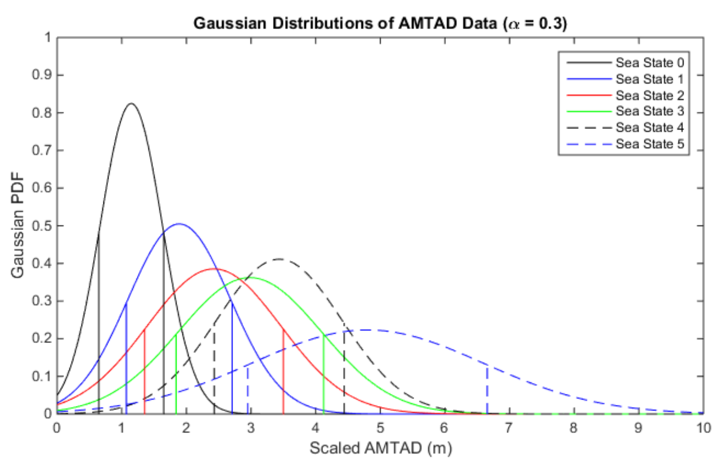
The probabilities suggest that there is a significantly greater chance that model results will predict observations made in the daytime. This infers that observations made in the daytime are more likely to be accurate.

CHAPTER 4 – CONCLUSIONS

The objective of this research was to determine modeling parameters that could produce data boundaries for individual sea states that could be used to predict the in-situ sea state using shipboard, displacement-based data. Several distribution types were used to provide comprehensive coverage of possible boundaries. Trial runs were used to test these boundaries and to provide an account of the number of matches made between modeled sea states and the observed as well as wind-based scale codes.

The results of the model runs suggest that all five models in this study are unable to exactly match and convert observed sea states through the use of ship's displacement values with a significant confidence. There is likelihood that low matching counts are a result of the inaccuracy of the sea state boundaries within each model; the averaging of boundary limits may provide too wide a variation for linking modeled sea state to observations. Figure 41 shows the overlapping of tolerance interval boundaries with $\alpha = 0.3$ for Gaussian distributions of AMTAD data.

Figure 41. Gaussian Tolerance Intervals of AMTAD Data



The experimental model provides greater predictability in both exact and marginal matching of observed sea states. Exact matching for the average of Models 1 through 4 predicts observed sea state approximately 22.4% of the time, whereas Model 5 will exactly match observed sea state about 33.7% of the time. Marginal tolerance matching of the total model mean implies that within 0.5 of an observed sea state, the Models 1-4 will predict the sea state approximately 24.2% of the time, whereas independently, Model 5 can marginally predict the sea state within 0.5 of an observed sea state approximately 42.6% of the time. Within one sea state, the average of Models 1-4 will predict the correct sea state approximately 51.0% of the time and Model 5 will predict the sea state approximately 80.6% of the time. Although this experimental model provides greater prediction accuracy, it is spatially and temporally dependent; data were collected in specific regions during specific times of the year.

While some distributions within the models were far better at predicting certain sea states, they were inconsistently able to perform well versus the alternate sea state measurements. This precludes any possibility of utilizing certain distributions exclusively to form a new model.

It is recommended that a final, shipboard model application should include TAD trend lines and an aggregate model result matrix similar to the model trial plots to provide a comprehensive decision aid.

CHAPTER 5 – FURTHER RESEARCH

Although the modeled sea states do not correlate well with the observed or wind-based sea states, it is apparent that there is a strong relationship between the modeled sea states and the time- and frequency-series data. In fact, without knowledge of the true state of the sea during any given period, the data suggests that the modeled sea states are more representative of the motion of the ship than the observed sea states. Further analysis of the time-series waveforms as a more precise measurement of sea state should be conducted. Pattern recognition algorithms may be useful.

It is strongly suggested that operational leaders create a protocol for taking periodic sea state measurements (similar to Expendable Bathythermograph (XBT) operations) to provide further data—especially in cases of low and high sea states.

To assist in the quality of acoustic products, further analysis should be conducted to connect observations to the corruption of acoustic signals created by wind-wave surface interactions and noise.

The sampling sizes of time-series data in this study were not uniform. Taking uniform samples may provide a better basis for comparison.

Corrections for swell heights, especially in cases where the state of the sea is clam and the swells are large, could provide better model boundary development and results. It may be necessary to remove heave as a displacement variable.

REFERENCES

[1] Black, P. and Adams, W. (1983), *Guidance for Estimating Surface Winds Based on Sea State Observations from Aircraft and Sea State Catalog*, Federal Coordinator for Meteorological Services and Supporting Research, U.S. Department of Commerce, NOAA, Washington D.C.

[2] Bowditch, N. (1995), *American Practical Navigator*, No. 9, Vol. 1, Defense Mapping Agency Hydrographic/Topographic Center

[3] Clayton, L., Ivey, H.D., and Teegardin, H.H. (1954), *Preliminary Study for the Development of a Sea State Meter*, Technical Report No. 6, Project Number 157-96, Georgia Institute of Technology

[4] Crow, E. L. and Shimizu, K. (1988), *Lognormal Distributions, Theory and Applications*, Statistics: Textbooks and Monographs No. 88, Marcel Dekker, Inc., New York

[5] Diede, G. and Thieme, H. (1950s), Sea State and Wind Research Problems, Final Report, The Martin Company, Baltimore

[6] Eschbach, O.W., Tapley, B.D., and Poston, T.R. (1990), *Eschbach's Handbook of Engineering Fundamentals*, Wiley, New York, Fourth Edition

[7] Johnson, N. L., Kotz, S., and Balakrishnan, N. (1994), *14: Lognormal Distributions*, Continuous univariate distributions. Vol. 1, Wiley Series in Probability and Mathematical Statistics: Applied Probability and Statistics (2nd ed.), John Wiley & Sons, New York

[8] King, A.D. (1998), *Inertial Navigation – Forty Years of Evolution*, GEC Review, Vol. 13, No. 3, pp. 140-149

[9] Land, C. E. (1971), *Confidence intervals for linear functions of the normal mean and variance*, Annals of Mathematical Statistics, Vol. 42, p. 1187-1205.

[10] Lloyd, D.K. and Lipow, M. (1962), Reliability*: Management, Methods, and Mathematics*, Prentice Hall, Englewood Cliffs, New Jersey

[11] Magrab, E., Azarm, S., Balachandran, B., Duncan, J., Herold, K., and Walsh, G. (2011), *An Engineer's Guide to Matlab*, Pearson Education (Prentice Hall), New Jersey, Third Edition

[12] Matlab, Matlab Software Documentation, Ver. 2014b, MathWorks Inc., Natick, MA

[13] Mendenhall, W., Scheaffer, R.L., and Wackerly, D.D. (1981), *Mathematical Statistics with Applications*, Duxbury Press, Boston, MA

[14] Meyer, S.L. (1975), *Data Analysis for Scientists and Engineers*, John Wiley & Sons, Inc., New York

[15] NAVOCEANO Operation Manuals, Stennis Space Center, MS

[16] Nelson, W. (1982), *Applied Life Data Analysis*, John Wiley & Sons, Inc., New York

[17] Nielsen, U. D. (2007), *Response-based Estimation of Sea State Parameters – Influence of Filter*, Ocean Engineering, Vol. 34, pp. 1797-1810

[18] NOAA (2013), *Meteorological Versus Astronomical Summer – What's the Difference?*, https://www.ncdc.noaa.gov/news/meteorological-versus-astronomical-summer%E2%80%94what%E2%80%99s-difference

[19] Papoulis, A.P. and Pillai, S.U. (2002), *Probability, Random Variables, and Stochastic Processes*, McGraw-Hill, Boston, Fourth Edition

[20] Shchigolev, B.M. (1965), *Mathematical Analysis of Observations*, Iliffe Books LTD, New York

[21] Shonting, D.H., Hebda, P.R., McCarthy, E.M., and Chaves, J.S. (1989), *Results of the Submarine-Deployed Sea State Sensor (SUDSS) Launch Tests; The First Direct Sea State Measurements Made From a Submerged*, *Moving Submarine*, NUSC Technical Document 6854, Naval Underwater Systems Center, Rhode Island/Connecticut

[22] Siddiqui, M. (1961), *Some Problems Connected with Rayleigh Distributions*, The Journal of Research of the National Bureau of Standards, Sec. D: Radio Propagation, Vol. 66D, No. 2, p. 167-174

[23] Technical Discussions with subject matter experts at the Naval Oceanographic Office, Stennis Space Center, MS

[24] Tukey, J. (1977), *Exploratory Data Analysis*, Addison-Wesley, pp. 43-44

[25] Varkey, M.J. (1993), *Interpretation of Nonlinearity in Wind Generated Ocean Surface Waves*, Indian Journal of Marine Sciences, Vol. 22, pp. 98-110

[26] Velazquez-Aviles, A.L., Owen, G.C., and Teichman, J. (1999-2014), *Documentation for Propagation Loss Acquisition and Processing of Underwater Signals (PLATYPUS) Software*, Version 8.2.1, Acoustics Division, Naval Oceanographic Office, SSC

[27] Waite, A. (2005), *SONAR for Practicing Engineers*, John Wiley & Sons, 3rd ed.

[28] Weibull, W. (1951), *A statistical distribution function of wide applicability*, J. Appl. Mech.-Trans, ASME 18 (3): pp. 293–297

[29] White, L. and Hanson, J. (2000), *An Automatic Sea State Calculator*, IEEE, pp. 1727-1734

[30] WOCE Data Products Committee (2002), WOCE International Project Office, WOCE Report No. 180/02, Southampton, UK, https://www.nodc.noaa.gov/woce/-woce_v3/wocedata_1/woceuot/document/wmocode.htm

APPENDICES

A. Matlab Pseudo Code and Descriptions

There were more than 60 Matlab scripts created for this research. Many of the initial programs were superseded by updated and often vastly different versions. Most provided small, ad hoc requests and are not included in this appendix. The programs listed below were the most significant. Since most of the code in each was repetitive declarations and array manipulations, only a description of the code's purpose is included. All scripts are available for further review upon request.

A1. proc2dispwind.m	Produces master list of displacement and wind data.
A2. proc2power.m	Produces master list of PSD calculations.
A3. outliers.m	Determines outlier data based on 1.5-IQR rule.
A4. stats.m	Determines the elementary statistics of data.
A5. seasonavg.m	Produces seasonal plots.
A6. distparams.m	Determines distribution parameters.
A7. histplots.m	Provides a variety of histograms and probability plots.
A8. datacorr.m	Used to determine Pearson correlation coefficients of data.
A9. modelrun.m	Produces the predictive results of the various models.
A10. modelfigure.m	Plots event time-series data and model results.

B. Data Statistics

Table B1. Pre-Scaled MAD Data Statistics

PRE-SCALED ROLL AMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.00673	0.00813	0.00991	0.01319	0.01647	0.02141	0.02463
Median	0.00570	0.00646	0.00872	0.01213	0.01681	0.01902	0.02463
Min	0.00410	0.00211	0.00164	0.00221	0.00917	0.01694	0.02432
Max	0.01312	0.02691	0.02775	0.03186	0.02599	0.03066	0.02493
Q1	0.00488	0.00492	0.00596	0.00867	0.01236	0.01779	0.02447
Q3	0.00734	0.01012	0.01233	0.01659	0.01859	0.02264	0.02478
Std Dev	0.00288	0.00483	0.00538	0.00626	0.00484	0.00629	0.00043
Variance	0.00001	0.00002	0.00003	0.00004	0.00002	0.00004	0.00000
Kurtosis	2.46338	2.81154	1.59263	0.12069	-0.54941	3.19164	#DIV/0!
Skewness	1.53679	1.61972	1.24681	0.69444	0.28071	1.77135	#DIV/0!

PRE-SCALED ROLL RMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.00820	0.01138	0.01371	0.01801	0.02125	0.02742	0.03110
Median	0.00639	0.00780	0.01043	0.01500	0.01938	0.02331	0.03110
Min	0.00488	0.00265	0.00205	0.00278	0.01141	0.02157	0.03099
Max	0.01339	0.03390	0.03462	0.04007	0.03271	0.03814	0.03121
Q1	0.00557	0.00610	0.00747	0.01082	0.01556	0.02246	0.03104
Q3	0.00887	0.01229	0.01522	0.02036	0.02362	0.02743	0.03115
Std Dev	0.00289	0.00586	0.00659	0.00778	0.00610	0.00776	0.00016
Variance	0.00001	0.00003	0.00004	0.00006	0.00004	0.00006	0.00000
Kurtosis	0.20515	3.71939	1.70080	0.31490	-0.49613	3.71615	#DIV/0!
Skewness	0.99164	1.79942	1.27781	0.77546	0.42119	1.91350	#DIV/0!

PRE-SCALED PITCH AMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.00300	0.00753	0.00919	0.00997	0.01021	0.01395	0.00973
Median	0.00215	0.00740	0.00936	0.00944	0.01042	0.01339	0.00973
Min	0.00100	0.00075	0.00196	0.00367	0.00411	0.00907	0.00933
Max	0.00750	0.01593	0.01819	0.02214	0.01677	0.01994	0.01012
Q1	0.00187	0.00612	0.00733	0.00756	0.00693	0.01147	0.00953
Q3	0.00360	0.00922	0.01071	0.01148	0.01232	0.01587	0.00992
Std Dev	0.00196	0.00316	0.00338	0.00366	0.00395	0.00458	0.00056
Variance	0.00000	0.00001	0.00001	0.00001	0.00002	0.00002	0.00000
Kurtosis	3.31980	1.23973	0.17707	1.77113	-1.01387	0.66164	#DIV/0!
Skewness	1.72183	0.55773	0.00066	1.11446	0.24949	0.65809	#DIV/0!

PRE-SCALED PITCH RMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.00413	0.00882	0.01099	0.01229	0.01304	0.01798	0.01225
Median	0.00266	0.00791	0.01019	0.01090	0.01221	0.01657	0.01225
Min	0.00126	0.00094	0.00225	0.00455	0.00516	0.01115	0.01180
Max	0.00821	0.01708	0.02064	0.02426	0.01958	0.02480	0.01270
Q1	0.00229	0.00656	0.00805	0.00884	0.00843	0.01399	0.01202
Q3	0.00452	0.01030	0.01231	0.01380	0.01494	0.01985	0.01247
Std Dev	0.00215	0.00325	0.00383	0.00419	0.00453	0.00579	0.00064
Variance	0.00000	0.00001	0.00001	0.00002	0.00002	0.00003	0.00000
Kurtosis	1.77398	0.95337	0.11370	1.02615	-1.12296	0.32033	#DIV/0!
Skewness	1.37008	0.42885	0.14260	0.98381	0.18962	0.63729	#DIV/0!

PRE-SCALED HEAVE AMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.06443	0.09141	0.13419	0.17883	0.21557	0.32849	0.28315
Median	0.07017	0.06899	0.11106	0.14851	0.18739	0.27657	0.28315
Min	0.03317	0.02199	0.01165	0.03560	0.09727	0.20776	0.27142
Max	0.10481	0.33121	0.42923	0.44403	0.40964	0.55308	0.29488
Q1	0.03957	0.05038	0.06378	0.10677	0.14655	0.21483	0.27729
Q3	0.07946	0.11553	0.20041	0.24590	0.29955	0.39023	0.28902
Std Dev	0.02435	0.06441	0.08974	0.09444	0.09678	0.16069	0.01659
Variance	0.00059	0.00415	0.00805	0.00892	0.00937	0.02582	0.00028
Kurtosis	-1.06217	2.81738	0.14520	-0.40307	-0.90708	1.04986	#DIV/0!
Skewness	0.14953	1.68084	0.87546	0.73175	0.52285	1.31587	#DIV/0!

PRE-SCALED HEAVE RMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.08588	0.14023	0.20230	0.25516	0.29389	0.44604	0.35589
Median	0.08814	0.08721	0.13909	0.18864	0.23344	0.34648	0.35564
Min	0.04145	0.02907	0.01486	0.04445	0.12176	0.25860	0.34221
Max	0.13167	0.41517	0.53217	0.55406	0.51417	0.69167	0.36906
Q1	0.04973	0.06327	0.08030	0.13749	0.18339	0.27033	0.34892
Q3	0.09962	0.14482	0.25146	0.30709	0.37272	0.48696	0.36235
Std Dev	0.03060	0.08063	0.11222	0.11779	0.12106	0.20063	0.01899
Variance	0.00094	0.00650	0.01259	0.01387	0.01466	0.04025	0.00036
Kurtosis	-1.05294	2.84410	0.09317	-0.43291	-0.86321	1.11972	#DIV/0!
Skewness	0.15106	1.68571	0.86021	0.70170	0.54198	1.32748	#DIV/0!

Table B2. Pre-Scaled MTAD Data Statistics

PRE-SCALED AMTAD							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.07416	0.10707	0.15329	0.20198	0.24225	0.36385	0.31750
Median	0.08080	0.08406	0.13038	0.17279	0.21863	0.30681	0.31750
Min	0.04102	0.03265	0.02283	0.04785	0.11769	0.23810	0.30507
Max	0.11618	0.36038	0.46818	0.47341	0.44357	0.60368	0.32993
Q1	0.04570	0.06514	0.08108	0.12416	0.17158	0.24192	0.31129
Q3	0.08970	0.13179	0.22539	0.27710	0.32581	0.42874	0.32372
Std Dev	0.02726	0.06884	0.09557	0.10043	0.10104	0.17121	0.01758
Variance	0.00074	0.00474	0.00913	0.01009	0.01021	0.02931	0.00031
Kurtosis	-1.47456	2.88054	0.14594	-0.45367	-0.90083	1.12840	#DIV/0!
Skewness	0.07618	1.68901	0.87034	0.69476	0.49873	1.33510	#DIV/0!
PRE-SCALED RMSTAD							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.09761	0.15846	0.22521	0.28346	0.32614	0.49086	0.39923
Median	0.10092	0.10410	0.16179	0.21553	0.27089	0.38386	0.39898
Min	0.05028	0.04061	0.02645	0.05790	0.14575	0.29630	0.38500
Max	0.14574	0.45210	0.58099	0.59077	0.55761	0.75461	0.41296
Q1	0.05709	0.07976	0.10009	0.16052	0.21497	0.30429	0.39199
Q3	0.11271	0.16421	0.28143	0.34534	0.40449	0.53423	0.40597
Std Dev	0.03392	0.08664	0.11989	0.12553	0.12669	0.21362	0.01978
Variance	0.00115	0.00751	0.01437	0.01576	0.01605	0.04563	0.00039
Kurtosis	-1.38017	2.92123	0.10487	-0.47336	-0.83696	1.22538	#DIV/0!
Skewness	0.08488	1.69978	0.85827	0.66966	0.52688	1.35360	#DIV/0!

Table B3. Scaling Factors

Arithmetic Scaling Factors			
	Roll	Pitch	Heave
Mean	0.01435	0.00908	0.18515
1/µ	69.67944	110.11395	5.40095
RMS Scaling Factors			
	Roll	Pitch	Heave
Mean	0.02028	0.01201	0.27904
1/µ	49.30800	83.23810	3.58367

Table B4. Scaled MAD Data Statistics

SCALED ROLL AMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.46871	0.56657	0.69063	0.91891	1.14749	1.49184	1.71586
Median	0.39717	0.45013	0.60760	0.84486	1.17096	1.32530	1.71586
Min	0.28569	0.14702	0.11427	0.15399	0.63896	1.18037	1.69460
Max	0.91419	1.87507	1.93360	2.21999	1.81097	2.13637	1.73711
Q1	0.34004	0.34282	0.41529	0.60429	0.86124	1.23942	1.70523
Q3	0.51145	0.70516	0.85915	1.15616	1.29499	1.57772	1.72648
Std Dev	0.20043	0.33643	0.37486	0.43623	0.33720	0.43843	0.03006
Variance	0.04017	0.11319	0.14052	0.19030	0.11370	0.19222	0.00090
Kurtosis	2.46338	2.81154	1.59263	0.12069	-0.54941	3.19164	#DIV/0!
Skewness	1.53679	1.61972	1.24681	0.69444	0.28071	1.77135	#DIV/0!

SCALED ROLL RMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.40422	0.56121	0.67582	0.88789	1.04767	1.35191	1.53340
Median	0.31508	0.38470	0.51442	0.73972	0.95544	1.14918	1.53339
Min	0.24057	0.13043	0.10115	0.13702	0.56278	1.06341	1.52795
Max	0.66031	1.67163	1.70723	1.97591	1.61262	1.88071	1.53883
Q1	0.27470	0.30082	0.36849	0.53327	0.76718	1.10750	1.53067
Q3	0.43732	0.60589	0.75023	1.00373	1.16476	1.35230	1.53611
Std Dev	0.14261	0.28899	0.32496	0.38346	0.30099	0.38284	0.00770
Variance	0.02034	0.08351	0.10560	0.14704	0.09060	0.14656	0.00006
Kurtosis	0.20515	3.71939	1.70080	0.31490	-0.49613	3.71615	#DIV/0!
Skewness	0.99164	1.79942	1.27781	0.77546	0.42119	1.91350	#DIV/0!

SCALED PITCH AMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.33022	0.82902	1.01178	1.09756	1.12475	1.53581	1.07086
Median	0.23674	0.81484	1.03067	1.03948	1.14739	1.47443	1.07086
Min	0.11011	0.08259	0.21582	0.40412	0.45257	0.99873	1.02736
Max	0.82585	1.75412	2.00297	2.43792	1.84661	2.19567	1.11435
Q1	0.20591	0.67390	0.80714	0.83274	0.76309	1.26301	1.04911
Q3	0.39641	1.01525	1.17932	1.26438	1.35688	1.74723	1.09261
Std Dev	0.21582	0.34803	0.37259	0.40355	0.43545	0.50393	0.06151
Variance	0.04658	0.12113	0.13882	0.16285	0.18962	0.25394	0.00378
Kurtosis	3.31980	1.23973	0.17707	1.77113	-1.01387	0.66164	#DIV/0!
Skewness	1.72183	0.55773	0.00066	1.11446	0.24949	0.65809	#DIV/0!

SCALED PITCH RMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.34371	0.73377	0.91515	1.02314	1.08543	1.49685	1.02001
Median	0.22113	0.65802	0.84839	0.90759	1.01644	1.37906	1.01932
Min	0.10480	0.07808	0.18691	0.37851	0.42933	0.92775	0.98190
Max	0.68364	1.42162	1.71818	2.01920	1.63018	2.06429	1.05675
Q1	0.19066	0.54609	0.66969	0.73584	0.70166	1.16469	1.00061
Q3	0.37619	0.85744	1.02479	1.14842	1.24359	1.65192	1.03803
Std Dev	0.17883	0.27044	0.31885	0.34858	0.37677	0.48173	0.05292
Variance	0.03198	0.07314	0.10166	0.12151	0.14195	0.23207	0.00280
Kurtosis	1.77398	0.95337	0.11370	1.02615	-1.12296	0.32033	#DIV/0!
Skewness	1.37008	0.42885	0.14260	0.98381	0.18962	0.63729	#DIV/0!

SCALED HEAVE AMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.34800	0.49371	0.72475	0.96583	1.16426	1.77417	1.52928
Median	0.37898	0.37261	0.59983	0.80210	1.01208	1.49371	1.52928
Min	0.17915	0.11877	0.06292	0.19227	0.52535	1.12210	1.46593
Max	0.56607	1.78885	2.31825	2.39818	2.21245	2.98716	1.59263
Q1	0.21372	0.27210	0.34447	0.57666	0.79150	1.16026	1.49760
Q3	0.42916	0.62397	1.08240	1.32807	1.61787	2.10763	1.56096
Std Dev	0.13150	0.34788	0.48470	0.51009	0.52273	0.86787	0.08959
Variance	0.01729	0.12102	0.23493	0.26019	0.27324	0.75320	0.00803
Kurtosis	-1.06217	2.81738	0.14520	-0.40307	-0.90708	1.04986	#DIV/0!
Skewness	0.14953	1.68084	0.87546	0.73175	0.52285	1.31587	#DIV/0!

SCALED HEAVE RMAD STATS							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	0.30776	0.50254	0.72497	0.91442	1.05321	1.59847	1.27539
Median	0.31585	0.31251	0.49846	0.67602	0.83656	1.24169	1.27448
Min	0.14853	0.10418	0.05326	0.15930	0.43634	0.92673	1.22637
Max	0.47185	1.48784	1.90711	1.98555	1.84262	2.47871	1.32259
Q1	0.17820	0.22673	0.28778	0.49274	0.65722	0.96878	1.25043
Q3	0.35700	0.51898	0.90114	1.10053	1.33571	1.74511	1.29854
Std Dev	0.10964	0.28897	0.40215	0.42211	0.43386	0.71901	0.06804
Variance	0.01202	0.08350	0.16172	0.17818	0.18823	0.51697	0.00463
Kurtosis	-1.05294	2.84410	0.09317	-0.43291	-0.86321	1.11972	#DIV/0!
Skewness	0.15106	1.68571	0.86021	0.70170	0.54198	1.32748	#DIV/0!

Table B5. Scaled MTAD Data Statistics (Outliers Included)

SCALED AMTAD							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	1.14693	1.88930	2.42716	2.98231	3.43649	4.80182	4.31599
Median	1.18882	1.70221	2.16056	2.75098	3.46727	4.26800	4.31599
Min	0.60622	0.60564	0.65038	1.11368	2.11439	3.35208	4.18789
Max	2.16780	4.51224	5.58096	5.84480	5.16014	7.31920	4.44409
Q1	0.81307	1.39939	1.69597	2.18269	2.52480	3.63725	4.25194
Q3	1.24342	2.24904	3.15206	3.78452	4.03372	5.43258	4.38004
Std Dev	0.48376	0.78524	1.03120	1.09762	0.97022	1.78725	0.18116
Variance	0.23403	0.61660	1.06338	1.20476	0.94133	3.19427	0.03282
Kurtosis	1.52606	1.50279	-0.04535	-0.63882	-1.10144	1.49042	#DIV/0!
Skewness	1.16990	1.17980	0.68130	0.47808	0.24133	1.36426	#DIV/0!

SCALED RMSTAD							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	1.03522	1.71935	2.24728	2.74962	3.10787	4.43045	3.82828
Median	1.01955	1.37920	1.84845	2.41416	2.95701	3.74190	3.82719
Min	0.52114	0.51827	0.55817	0.94286	1.79439	2.97396	3.73622
Max	1.69898	3.97113	4.88481	5.10095	4.59583	6.42370	3.91817
Q1	0.66672	1.14376	1.38593	1.87246	2.23186	3.21294	3.78171
Q3	1.08096	1.86076	2.71143	3.27750	3.54453	4.74932	3.87268
Std Dev	0.38184	0.69110	0.91276	0.96716	0.85934	1.55669	0.12866
Variance	0.14580	0.47762	0.83313	0.93540	0.73846	2.42328	0.01655
Kurtosis	0.08808	1.95048	0.04613	-0.60830	-0.94621	1.66872	#DIV/0!
Skewness	0.77186	1.32943	0.73459	0.48912	0.32949	1.40704	#DIV/0!

Table B6. Scaled MTAD Data Statistics (Outliers Removed)

SCALED AMTAD							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	8	74	126	144	16	4	2
Mean	1.01932	1.74635	2.36207	2.94563	3.43649	4.80182	4.31599
Median	1.02639	1.65757	2.13274	2.74741	3.46727	4.26800	4.31599
Min	0.60622	0.74502	0.65038	1.11368	2.11439	3.35208	4.18789
Max	1.53139	3.11255	4.60806	5.36640	5.16014	7.31920	4.44409
Q1	0.78892	1.39447	1.67819	2.17152	2.52480	3.63725	4.25194
Q3	1.20424	2.08805	3.03610	3.68581	4.03372	5.43258	4.38004
Std Dev	0.31617	0.55150	0.95071	1.05952	0.97022	1.78725	0.18116
Variance	0.09996	0.30415	0.90385	1.12259	0.94133	3.19427	0.03282
Kurtosis	-1.02625	-0.19626	-0.51910	-0.75800	-1.10144	1.49042	#DIV/0!
Skewness	0.26929	0.48845	0.49955	0.42200	0.24133	1.36426	#DIV/0!
SCALED RMSTAD							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	8	73	127	145	16	4	2
Mean	0.91914	1.52653	2.18664	2.72637	3.10787	4.43045	3.82828
Median	0.87813	1.35888	1.82739	2.41095	2.95701	3.74190	3.82719
Min	0.52114	0.64815	0.55817	0.94286	1.79439	2.97396	3.73622
Max	1.34919	2.65591	4.18844	4.81611	4.59583	6.42370	3.91817
Q1	0.65478	1.14376	1.38544	1.87246	2.23186	3.21294	3.78171
Q3	1.05268	1.78681	2.66022	3.22282	3.54453	4.74932	3.87268
Std Dev	0.28525	0.46315	0.85655	0.94737	0.85934	1.55669	0.12866
Variance	0.08137	0.21451	0.73368	0.89750	0.73846	2.42328	0.01655
Kurtosis	-1.01758	-0.23340	-0.42343	-0.70790	-0.94621	1.66872	#DIV/0!
Skewness	0.34940	0.56881	0.57401	0.44755	0.32949	1.40704	#DIV/0!

Table B7. Scaled SPOW Data Statistics (Outliers Included)

SCALED ASPOW							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	706.227	1623.975	3117.172	4643.196	6244.766	12646.000	6117.062
Median	759.286	810.779	1833.367	3120.219	4739.306	9203.542	6117.062
Min	177.141	118.608	36.196	277.409	2078.334	5339.404	2657.367
Max	1409.034	9906.044	19945.937	19112.659	17416.474	26837.511	9576.757
Q1	348.744	437.389	835.167	1795.616	2940.477	5770.401	4387.215
Q3	1016.089	1954.199	4058.414	6082.942	8046.293	16079.141	7846.910
Std Dev	434.265	1980.202	3463.599	3996.096	4501.744	10002.127	4892.748
Variance	188586.355	3921201.347	11996519.556	15968779.856	20265698.468	100042553.621	23938980.930
Kurtosis	-1.329	5.891	5.564	1.476	0.980	1.642	#DIV/0!
Skewness	0.263	2.381	2.134	1.406	1.304	1.441	#DIV/0!
SCALED RSPOW							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	81	129	146	16	4	2
Mean	406.701	1271.849	2332.457	3057.399	3772.499	7585.438	3475.106
Median	397.067	420.470	935.623	1594.484	2318.921	4672.769	3024.439
Min	90.424	61.068	18.599	145.806	1100.854	2661.351	1312.968
Max	697.669	4782.473	9813.514	9273.000	8566.383	13212.479	4735.909
Q1	176.346	229.103	443.319	909.430	1515.836	3003.963	2168.704
Q3	496.108	1008.225	1992.676	3086.847	4125.570	7973.648	3880.174
Std Dev	215.164	974.828	1723.240	1979.123	2208.325	4870.023	2420.384
Variance	46295.553	950289.563	2969554.417	3916927.073	4876700.843	23717123.216	5858260.470
Kurtosis	-1.367	5.413	5.308	1.451	0.910	1.647	#DIV/0!
Skewness	0.224	2.299	2.094	1.396	1.284	1.436	#DIV/0!

Table B8. Scaled SPOW Data Statistics (Outliers Removed)

SCALED ASPOW							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	74	115	130	14	4	2
Mean	706.227	1099.369	2141.597	3542.099	5058.215	12646.000	6117.062
Median	759.286	778.788	1629.956	2789.001	3684.643	9203.542	6117.062
Min	177.141	118.608	36.196	277.409	2078.334	5339.404	2657.367
Max	1409.034	3695.630	7054.182	10560.645	12193.009	26837.511	9576.757
Q1	348.744	392.434	760.714	1646.353	2826.203	5770.401	4387.215
Q3	1016.089	1517.790	2850.138	4868.984	5886.213	16079.141	7846.910
Std Dev	434.265	926.884	1813.769	2501.363	3160.747	10002.127	4892.748
Variance	188586.355	859113.039	3289759.274	6256818.709	9990321.437	100042553.621	23938980.930
Kurtosis	-1.329	0.392	0.145	0.272	1.168	1.642	#DIV/0!
Skewness	0.263	1.170	1.036	1.003	1.397	1.441	#DIV/0!
SCALED RSPOW							
	SS 0	SS 1	SS 2	SS 3	SS 4	SS 5	SS 6
n	9	68	120	138	14	4	2
Mean	406.701	761.875	1638.207	2548.569	2946.850	7585.438	3475.106
Median	397.067	419.685	863.337	1469.073	1834.152	4672.769	3024.439
Min	90.424	113.817	18.599	145.806	1100.854	2661.351	1312.968
Max	697.669	1889.592	4610.698	6286.935	5985.601	13212.479	4735.909
Q1	176.346	252.037	418.601	879.369	1459.666	3003.963	2168.704
Q3	496.108	851.688	1848.159	2805.497	2901.619	7973.648	3880.174
Std Dev	215.164	465.903	1094.369	1546.252	1565.608	4870.023	2420.384
Variance	46295.553	217065.482	1197643.963	2390896.138	2451128.254	23717123.216	5858260.470
Kurtosis	-1.367	0.296	0.827	0.510	1.019	1.647	#DIV/0!
Skewness	0.224	1.132	1.232	1.137	1.376	1.436	#DIV/0!

C. Distribution Parameters

Tables C1. Distribution Parameters (Outliers Included)

AMTAD							
	Gaussian		Weibull		Rayleigh	Lognormal	
	μ	σ	λ	k	σ	μ	σ
SS 0	1.14693	0.48376	1.29331	2.66274	0.87277	0.06428	0.39952
SS 1	1.88930	0.79013	2.13178	2.52532	1.44673	0.55620	0.40230
SS 2	2.42716	1.03523	2.74188	2.52061	1.86473	0.79347	0.44455
SS 3	2.98231	1.10140	3.35016	2.94405	2.24710	1.02275	0.38177
SS 4	3.43649	0.97022	3.79480	4.06524	2.51912	1.19605	0.28863
SS 5	4.80182	1.78725	5.36257	3.29783	3.56744	1.52162	0.34706
SS 6	4.31599	0.18116	4.37792	40.40803	3.05321	1.46189	0.04199

ASPOW							
	Gaussian		Weibull		Rayleigh	Lognormal	
	μ	σ	λ	k	σ	μ	σ
SS 0	706.227	434.265	796.099	1.793	577.230	6.349	0.733
SS 1	1623.975	1992.540	1578.879	0.948	1810.869	6.822	1.075
SS 2	3117.172	3477.103	3023.029	0.938	3294.942	7.421	1.261
SS 3	4643.196	4009.852	4988.107	1.228	4331.746	8.073	0.909
SS 4	6244.766	4501.744	7017.779	1.563	5384.988	8.524	0.661
SS 5	12646.000	10002.127	14216.039	1.574	10838.663	9.225	0.751
SS 6	6117.062	4892.748	6926.876	1.872	4969.303	8.526	0.907

RMSTAD							
	Gaussian		Weibull		Rayleigh	Lognormal	
	μ	σ	λ	k	σ	μ	σ
SS 0	0.97061	0.38184	1.09129	2.89901	0.73201	-0.09703	0.38838
SS 1	1.57434	0.69541	1.77992	2.39515	1.21577	0.36760	0.41413
SS 2	2.05357	0.91632	2.32339	2.40791	1.58907	0.61836	0.46223
SS 3	2.57391	0.97049	2.89429	2.88170	1.94427	0.87238	0.39045
SS 4	2.99442	0.85934	3.31026	3.95842	2.19759	1.05751	0.29113
SS 5	4.22036	1.55669	4.71078	3.31966	3.13280	1.39364	0.34258
SS 6	3.82719	0.12866	3.87138	50.45821	2.70700	1.34185	0.03362

RSPOW							
	Gaussian		Weibull		Rayleigh	Lognormal	
	μ	σ	λ	k	σ	μ	σ
SS 0	352.497	215.164	397.444	1.806	287.581	5.655	0.732
SS 1	816.891	980.902	799.926	0.960	899.333	6.147	1.067
SS 2	1571.878	1729.958	1532.396	0.948	1649.296	6.746	1.254
SS 3	2330.399	1985.936	2511.810	1.245	2161.907	7.392	0.898
SS 4	3108.028	2208.325	3497.608	1.585	2667.560	7.833	0.651
SS 5	6304.842	4870.023	7103.990	1.613	5363.715	8.539	0.733
SS 6	3024.439	2420.384	3424.725	1.870	2457.271	7.821	0.907

Tables C2. Distribution Parameters (Outliers Removed)

AMTAD							
	Gaussian		Weibull		Rayleigh	Lognormal	
	μ	σ	λ	k	σ	μ	σ
SS 0	1.01932	0.31617	1.12983	3.83384	0.75049	-0.02440	0.31865
SS 1	1.74635	0.55526	1.94425	3.39933	1.29497	0.50630	0.32755
SS 2	2.36207	0.95450	2.66312	2.68234	1.80045	0.77334	0.42977
SS 3	2.94563	1.06322	3.30514	3.02448	2.21351	1.01298	0.37516
SS 4	3.43649	0.97022	3.79480	4.06524	2.51912	1.19605	0.28863
SS 5	4.80182	1.78725	5.36257	3.29783	3.56744	1.52162	0.34706
SS 6	4.31599	0.18116	4.37792	40.40803	3.05321	1.46189	0.04199

ASPOW							
	Gaussian		Weibull		Rayleigh	Lognormal	
	μ	σ	λ	k	σ	μ	σ
SS 0	706.227	434.265	796.099	1.793	577.230	6.349	0.733
SS 1	1099.369	933.210	1181.912	1.234	1016.791	6.630	0.910
SS 2	2141.597	1821.707	2238.817	1.133	1984.465	7.195	1.140
SS 3	3542.099	2511.040	3929.153	1.472	3066.210	7.896	0.800
SS 4	5058.215	3160.747	5741.756	1.811	4175.061	8.375	0.555
SS 5	12646.000	10002.127	14216.039	1.574	10838.663	9.225	0.751
SS 6	6117.062	4892.748	6926.876	1.872	4969.303	8.526	0.907

RMSTAD							
	Gaussian		Weibull		Rayleigh	Lognormal	
	μ	σ	λ	k	σ	μ	σ
SS 0	0.87956	0.28525	0.97779	3.64527	0.64993	-0.17541	0.33044
SS 1	1.45457	0.46635	1.62070	3.35779	1.07942	0.32390	0.32375
SS 2	2.01189	0.85995	2.27343	2.52838	1.54619	0.60374	0.45069
SS 3	2.55648	0.95065	2.87281	2.93041	1.92784	0.86716	0.38666
SS 4	2.99442	0.85934	3.31026	3.95842	2.19759	1.05751	0.29113
SS 5	4.22036	1.55669	4.71078	3.31966	3.13280	1.39364	0.34258
SS 6	3.82719	0.12866	3.87138	50.45821	2.70700	1.34185	0.03362

RSPOW							
	Gaussian		Weibull		Rayleigh	Lognormal	
	μ	σ	λ	k	σ	μ	σ
SS 0	352.497	215.164	397.444	1.806	287.581	5.655	0.732
SS 1	602.817	469.367	664.585	1.379	538.727	6.110	0.779
SS 2	1219.048	1098.958	1256.006	1.082	1158.387	6.599	1.172
SS 3	2025.909	1551.885	2224.359	1.368	1802.111	7.303	0.841
SS 4	2531.379	1565.608	2874.722	1.830	2083.738	7.687	0.548
SS 5	6304.842	4870.023	7103.990	1.613	5363.715	8.539	0.733
SS 6	3024.439	2420.384	3424.725	1.870	2457.271	7.821	0.907

D. Tolerance Interval Tables

Tables D1. 70% and 80% AMTAD/ASPOW Tolerance Intervals (Outliers Included)

AMTAD 70% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.64554	1.64831	0.65367	1.64491	0.49759	1.70006	-0.34980	0.47836
SS 1	1.07039	2.70822	1.03817	2.74704	0.82481	2.81806	0.13924	0.97316
SS 2	1.35421	3.50010	1.33350	3.53490	1.06312	3.63228	0.33273	1.25422
SS 3	1.84078	4.12383	1.80732	4.16414	1.28112	4.37708	0.62707	1.41842
SS 4	2.43092	4.44206	2.42706	4.44218	1.43621	4.90695	0.89691	1.49520
SS 5	2.94946	6.65419	3.09097	6.51177	2.03387	6.94894	1.16192	1.88133
SS 6	4.12823	4.50376	4.18542	4.44784	1.74070	5.94729	1.41837	1.50540

ASPOW 70% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	256.140	1156.314	288.927	1137.878	329.091	1124.376	5.589	7.109
SS 1	-441.160	3689.110	232.104	3103.124	1032.414	3527.358	5.709	7.936
SS 2	-486.613	6720.958	436.018	5981.463	1878.515	6418.154	6.115	8.728
SS 3	487.252	8799.140	1136.027	8402.016	2469.619	8437.725	7.131	9.014
SS 4	1579.008	10910.523	2194.239	10571.659	3070.094	10489.315	7.839	9.210
SS 5	2279.461	23012.539	4480.992	21354.262	6179.348	21112.424	8.446	10.003
SS 6	1046.055	11188.069	2623.703	9752.738	2833.103	9679.610	7.587	9.466

AMTAD 80% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.52696	1.76689	0.55548	1.76903	0.40064	1.87294	-0.44773	0.57629
SS 1	0.87671	2.90190	0.87445	2.96604	0.66411	3.10464	0.04063	1.07177
SS 2	1.10046	3.75385	1.12284	3.81725	0.85599	4.00166	0.22376	1.36319
SS 3	1.57081	4.39380	1.55991	4.44733	1.03152	4.82220	0.53349	1.51200
SS 4	2.19310	4.67988	2.18162	4.65896	1.15639	5.40595	0.82616	1.56595
SS 5	2.51137	7.09228	2.71031	6.90569	1.63761	7.65560	1.07685	1.96640
SS 6	4.08383	4.54816	4.14077	4.46922	1.40156	6.55209	1.40808	1.51570

ASPOW 80% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	149.694	1262.761	226.877	1267.711	264.974	1238.716	5.409	7.288
SS 1	-929.568	4177.518	146.914	3806.843	831.268	3886.064	5.445	8.200
SS 2	-1338.914	7573.259	274.734	7352.836	1512.522	7070.833	5.806	9.037
SS 3	-495.636	9782.027	798.214	9837.430	1988.460	9295.780	6.908	9.237
SS 4	475.549	12013.983	1662.817	11966.558	2471.944	11556.001	7.677	9.372
SS 5	-172.242	25464.242	3402.297	24151.068	4975.419	23259.402	8.262	10.187
SS 6	-153.246	12387.371	2081.341	10816.164	2281.127	10663.955	7.364	9.688

Tables D2. 90% and 95% AMTAD/ASPOW Tolerance Intervals (Outliers Included)

AMTAD 90% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.35120	1.94265	0.42390	1.95279	0.27954	2.13632	-0.59287	0.72144
SS 1	0.58965	3.18895	0.65757	3.29180	0.46338	3.54123	-0.10553	1.21793
SS 2	0.72436	4.12995	0.84390	4.23732	0.59726	4.56440	0.06226	1.52469
SS 3	1.17067	4.79394	1.22156	4.86316	0.71973	5.50033	0.39480	1.65070
SS 4	1.84062	5.03237	1.82759	4.97053	0.80685	6.16618	0.72130	1.67081
SS 5	1.86205	7.74159	2.17884	7.47933	1.14262	8.73218	0.95076	2.09249
SS 6	4.01801	4.61398	4.06766	4.49842	0.97792	7.47349	1.39282	1.53095

ASPOW 90% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-8.076	1420.530	151.847	1468.158	184.882	1412.913	5.143	7.555
SS 1	-1653.462	4901.412	68.736	5025.309	580.006	4432.550	5.055	8.590
SS 2	-2602.152	8836.497	127.572	9733.088	1055.342	8065.183	5.348	9.495
SS 3	-1952.423	11238.815	444.182	12188.319	1387.421	10603.018	6.578	9.567
SS 4	-1159.944	13649.476	1049.086	14161.127	1724.765	13181.087	7.437	9.612
SS 5	-3806.036	29098.035	2153.427	28546.729	3471.531	26530.303	7.989	10.460
SS 6	-1930.792	14164.916	1416.800	12449.108	1591.625	12163.595	7.035	10.017

AMTAD 95% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.19877	2.09508	0.32517	2.11156	0.19639	2.37063	-0.71877	0.84733
SS 1	0.34068	3.43793	0.49718	3.57460	0.32555	3.92962	-0.23230	1.34470
SS 2	0.39815	4.45616	0.63774	4.60206	0.41961	5.06500	-0.07782	1.66477
SS 3	0.82361	5.14100	0.96108	5.21941	0.50565	6.10358	0.27450	1.77099
SS 4	1.53489	5.33809	1.53621	5.23164	0.56686	6.84245	0.63035	1.76176
SS 5	1.29887	8.30477	1.75891	7.96658	0.80276	9.68988	0.84139	2.20185
SS 6	3.96092	4.67106	3.99720	4.52165	0.68704	8.29315	1.37959	1.54418

ASPOW 95% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-144.917	1557.372	102.413	1648.905	129.890	1567.874	4.912	7.786
SS 1	-2281.332	5529.282	32.630	6259.582	407.488	4918.689	4.716	8.929
SS 2	-3697.823	9932.168	60.114	12150.122	741.440	8949.731	4.951	9.892
SS 3	-3215.969	12502.360	249.962	14439.362	974.745	11765.902	6.292	9.854
SS 4	-2578.490	15068.022	667.738	16178.426	1211.749	14626.721	7.228	9.820
SS 5	-6957.810	32249.809	1374.959	32583.112	2438.955	29440.009	7.752	10.697
SS 6	-3472.547	15706.672	971.561	13913.469	1118.210	13497.635	6.749	10.303

Tables D3. 99% AMTAD/ASPOW Tolerance Intervals (Outliers Included)

AMTAD 99% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-0.09917	2.39302	0.17699	2.41911	0.08739	2.84109	-0.96482	1.09339
SS 1	-0.14594	3.92455	0.26181	4.12567	0.14485	4.70947	-0.48007	1.59246
SS 2	-0.23941	5.09372	0.33543	5.31294	0.18671	6.07017	-0.35161	1.93855
SS 3	0.14530	5.81931	0.55443	5.90245	0.22499	7.31487	0.03938	2.00611
SS 4	0.93736	5.93562	1.03141	5.71897	0.25223	8.20037	0.45259	1.93952
SS 5	0.19817	9.40548	1.07637	8.89104	0.35719	11.61289	0.62765	2.41560
SS 6	3.84935	4.78263	3.84016	4.56234	0.30570	9.93896	1.35374	1.57004

ASPOW 99% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-412.366	1824.821	41.495	2017.949	57.795	1879.027	4.460	8.237
SS 1	-3508.469	6756.418	5.908	9172.153	181.314	5894.828	4.054	9.591
SS 2	-5839.250	12073.595	10.700	17871.241	329.907	10725.850	4.174	10.668
SS 3	-5685.497	14971.889	66.855	19390.482	433.718	14100.904	5.732	10.413
SS 4	-5350.958	17840.490	236.889	20396.284	539.174	17529.467	6.821	10.227
SS 5	-13117.773	38409.773	491.313	41011.688	1085.225	35282.528	7.290	11.160
SS 6	-6485.821	18719.945	408.934	16883.107	497.553	16176.309	6.191	10.861

Tables D4. 70% and 80% RMSTAD/RSPOW Tolerance Intervals (Outliers Included)

RMSTAD 70% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.57486	1.36636	0.58310	1.36103	0.41733	1.42587	-0.49956	0.30549
SS 1	0.85360	2.29508	0.83357	2.32546	0.69313	2.36817	-0.06162	0.79682
SS 2	1.10387	3.00327	1.09248	3.03120	0.90596	3.09531	0.13929	1.09743
SS 3	1.56806	3.57976	1.54068	3.61447	1.10847	3.78722	0.46770	1.27706
SS 4	2.10377	3.88506	2.09178	3.89149	1.25289	4.28065	0.75577	1.35925
SS 5	2.60696	5.83377	2.72514	5.71301	1.78608	6.10233	1.03858	1.74870
SS 6	3.69385	3.96054	3.73445	3.92082	1.54332	5.27291	1.30700	1.37670

RSPOW 70% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	129.494	575.500	145.340	566.560	163.956	560.173	4.897	6.414
SS 1	-199.748	1833.531	120.498	1558.712	512.729	1751.794	5.042	7.253
SS 2	-221.108	3364.864	225.314	3011.559	940.298	3212.632	5.446	8.046
SS 3	272.109	4388.689	583.539	4201.390	1232.549	4211.138	6.462	8.323
SS 4	819.246	5396.811	1111.563	5238.656	1520.831	5196.088	7.157	8.508
SS 5	1257.388	11352.296	2303.744	10564.786	3057.966	10447.878	7.780	9.299
SS 6	515.872	5533.006	1296.324	4822.995	1400.941	4786.471	6.881	8.762

RMSTAD 80% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.48126	1.45995	0.50213	1.45507	0.33602	1.57087	-0.59476	0.40069
SS 1	0.68314	2.46554	0.69560	2.52133	0.55809	2.60899	-0.16313	0.89833
SS 2	0.87926	3.22788	0.91252	3.28510	0.72945	3.41009	0.02599	1.21073
SS 3	1.33018	3.81764	1.32554	3.86577	0.89251	4.17235	0.37200	1.37276
SS 4	1.89313	4.09570	1.87484	4.08664	1.00879	4.71596	0.68440	1.43061
SS 5	2.22539	6.21534	2.39160	6.05627	1.43809	6.72289	0.95460	1.83267
SS 6	3.66231	3.99208	3.70251	3.93590	1.24263	5.80913	1.29876	1.38494

RSPOW 80% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	76.753	628.241	114.333	630.696	132.012	617.139	4.718	6.593
SS 1	-440.185	2073.967	76.716	1907.226	412.833	1929.939	4.780	7.514
SS 2	-645.152	3788.908	142.623	3694.435	757.099	3539.333	5.139	8.354
SS 3	-214.680	4875.478	411.964	4908.752	992.410	4639.380	6.242	8.543
SS 4	277.946	5938.111	845.636	5919.586	1224.526	5724.492	6.998	8.667
SS 5	63.656	12546.027	1761.063	11912.348	2462.179	11510.350	7.600	9.478
SS 6	-77.409	6126.286	1028.190	5349.269	1127.994	5273.220	6.659	8.984

Tables D5. 90% and 95% RMSTAD/RSPOW Tolerance Intervals (Outliers Included)

RMSTAD 90% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.34254	1.59868	0.39173	1.59334	0.23446	1.79178	-0.73585	0.54179
SS 1	0.43050	2.71818	0.51503	2.81414	0.38940	2.97589	-0.31358	1.04879
SS 2	0.54636	3.56077	0.67673	3.66448	0.50896	3.88964	-0.14194	1.37866
SS 3	0.97760	4.17022	1.03255	4.23542	0.62273	4.75909	0.23015	1.51462
SS 4	1.58094	4.40790	1.56311	4.36756	0.70387	5.37915	0.57863	1.53638
SS 5	1.65984	6.78089	1.92538	6.55591	1.00341	7.66831	0.83014	1.95713
SS 6	3.61557	4.03882	3.65007	3.95648	0.86703	6.62605	1.28654	1.39716

RSPOW 90% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-1.417	706.410	76.752	729.620	92.110	703.925	4.452	6.859
SS 1	-796.549	2430.331	36.241	2508.796	288.049	2201.340	4.393	7.902
SS 2	-1273.649	4417.406	66.734	4876.785	528.255	4037.059	4.683	8.809
SS 3	-936.174	5596.973	231.057	6064.336	692.440	5291.802	5.915	8.869
SS 4	-524.344	6740.400	536.976	6988.672	854.396	6529.510	6.761	8.904
SS 5	-1705.633	14315.317	1127.250	14022.669	1717.952	13129.016	7.334	9.745
SS 6	-956.739	7005.617	699.719	6157.456	787.043	6014.777	6.329	9.314

RMSTAD 95% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.22222	1.71900	0.30705	1.71194	0.16472	1.98829	-0.85823	0.66417
SS 1	0.21137	2.93731	0.38354	3.06962	0.27358	3.30227	-0.44408	1.17929
SS 2	0.25762	3.84951	0.50474	3.99532	0.35758	4.31623	-0.28759	1.52431
SS 3	0.67179	4.47603	0.80817	4.55264	0.43751	5.28104	0.10711	1.63765
SS 4	1.31015	4.67868	1.30775	4.60335	0.49451	5.96911	0.48689	1.62812
SS 5	1.16931	7.27142	1.55650	6.98010	0.70495	8.50933	0.72219	2.06509
SS 6	3.57502	4.07937	3.59935	3.97283	0.60914	7.35276	1.27595	1.40775

RSPOW 95% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-69.217	774.210	51.918	818.734	64.712	781.128	4.221	7.089
SS 1	-1105.641	2739.423	17.368	3116.264	202.371	2442.772	4.057	8.238
SS 2	-1818.777	4962.533	31.682	6074.442	371.131	4479.823	4.288	9.205
SS 3	-1561.963	6222.761	131.035	7168.014	486.480	5872.179	5.633	9.152
SS 4	-1220.210	7436.267	343.954	7969.337	600.264	7245.633	6.556	9.109
SS 5	-3240.228	15849.912	727.735	15953.418	1206.962	14568.939	7.103	9.976
SS 6	-1719.428	7768.305	479.704	6882.268	552.944	6674.446	6.044	9.599

Tables D6. 99% RMSTAD/RSPOW Tolerance Intervals (Outliers Included)

RMSTAD 99% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-0.01295	1.95416	0.17563	1.93967	0.07329	2.38288	-1.09742	0.90336
SS 1	-0.21691	3.36559	0.19505	3.57056	0.12173	3.95762	-0.69913	1.43434
SS 2	-0.30670	4.41384	0.25761	4.64360	0.15911	5.17281	-0.57226	1.80898
SS 3	0.07410	5.07372	0.46070	5.16214	0.19467	6.32909	-0.13335	1.87812
SS 4	0.78092	5.20792	0.86863	5.04426	0.22003	7.15371	0.30759	1.80742
SS 5	0.21060	8.23013	0.95558	7.78447	0.31367	10.19805	0.51121	2.27607
SS 6	3.49579	4.15860	3.48565	4.00144	0.27104	8.81195	1.25524	1.42846

RSPOW 99% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-201.729	906.723	21.178	1000.466	28.794	936.147	3.771	7.540
SS 1	-1709.744	3343.527	3.214	4544.110	90.046	2927.552	3.400	8.895
SS 2	-2884.198	6027.954	5.737	8900.521	165.136	5368.867	3.516	9.977
SS 3	-2785.032	7445.830	35.673	9587.816	216.462	7037.543	5.080	9.705
SS 4	-2580.241	8796.298	123.808	10014.434	267.090	8683.566	6.155	9.510
SS 5	-6239.506	18849.190	266.709	19966.896	537.044	17460.218	6.652	10.427
SS 6	-3210.058	9258.936	201.789	8352.302	246.035	7999.024	5.485	10.158

Tables D7. 70% and 80% AMTAD/ASPOW Tolerance Intervals (Outliers Removed)

AMTAD 70% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.69162	1.34701	0.70338	1.33521	0.42787	1.46187	-0.35465	0.30586
SS 1	1.17086	2.32185	1.13925	2.34726	0.73829	2.52245	0.16682	0.84579
SS 2	1.37279	3.35135	1.35272	3.38117	1.02647	3.50706	0.32792	1.21877
SS 3	1.84367	4.04758	1.81254	4.08449	1.26197	4.31166	0.62415	1.40181
SS 4	2.43092	4.44206	2.42706	4.44218	1.43621	4.90695	0.89691	1.49520
SS 5	2.94946	6.65419	3.09097	6.51177	2.03387	6.94894	1.16192	1.88133
SS 6	4.12823	4.50376	4.18542	4.44784	1.74070	5.94729	1.41837	1.50540

ASPOW 70% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	256.140	1156.314	288.927	1137.878	329.091	1124.376	5.589	7.109
SS 1	132.159	2066.580	270.936	1986.259	579.694	1980.587	5.687	7.574
SS 2	253.519	4029.676	450.607	3938.967	1131.385	3865.501	6.013	8.377
SS 3	939.574	6144.625	1143.718	6070.020	1748.110	5972.611	7.067	8.725
SS 4	1782.311	8334.119	2105.679	8176.705	2380.290	8132.521	7.801	8.950
SS 5	2279.461	23012.539	4480.992	21354.262	6179.348	21112.424	8.446	10.003
SS 6	1046.055	11188.069	2623.703	9752.738	2833.103	9679.610	7.587	9.466

AMTAD 80% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.61413	1.42451	0.62819	1.40440	0.34451	1.61053	-0.43276	0.38397
SS 1	1.03475	2.45795	1.00288	2.48489	0.59445	2.77896	0.08653	0.92607
SS 2	1.13882	3.58532	1.15090	3.63436	0.82648	3.86370	0.22258	1.32411
SS 3	1.58305	4.30820	1.57055	4.35462	1.01610	4.75013	0.53219	1.49376
SS 4	2.19310	4.67988	2.18162	4.65896	1.15639	5.40595	0.82616	1.56595
SS 5	2.51137	7.09228	2.71031	6.90569	1.63761	7.65560	1.07685	1.96640
SS 6	4.08383	4.54816	4.14077	4.46922	1.40156	6.55209	1.40808	1.51570

ASPOW 80% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	149.694	1262.761	226.877	1267.711	264.974	1238.716	5.409	7.288
SS 1	-96.588	2295.327	190.666	2323.981	466.751	2181.998	5.464	7.797
SS 2	-193.014	4476.209	307.415	4673.071	910.956	4258.593	5.734	8.657
SS 3	324.072	6760.126	852.059	6923.533	1407.524	6579.982	6.871	8.921
SS 4	1007.555	9108.875	1657.579	9099.566	1916.535	8959.539	7.665	9.086
SS 5	-172.242	25464.242	3402.297	24151.068	4975.419	23259.402	8.262	10.187
SS 6	-153.246	12387.371	2081.341	10816.164	2281.127	10663.955	7.364	9.688

Tables D8. 90% and 95% AMTAD/ASPOW Tolerance Intervals (Outliers Removed)

AMTAD 90% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.49926	1.53937	0.52066	1.50418	0.24038	1.83701	-0.54852	0.49973
SS 1	0.83303	2.65968	0.81149	2.68490	0.41477	3.16976	-0.03247	1.04507
SS 2	0.79205	3.93209	0.88001	4.00899	0.57667	4.40704	0.06644	1.48025
SS 3	1.19678	4.69447	1.23791	4.75049	0.70897	5.41812	0.39589	1.63006
SS 4	1.84062	5.03237	1.82759	4.97053	0.80685	6.16618	0.72130	1.67081
SS 5	1.86205	7.74159	2.17884	7.47933	1.14262	8.73218	0.95076	2.09249
SS 6	4.01801	4.61398	4.06766	4.49842	0.97792	7.47349	1.39282	1.53095

ASPOW 90% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-8.076	1420.530	151.847	1468.158	184.882	1412.913	5.143	7.555
SS 1	-435.625	2634.364	106.374	2876.636	325.669	2488.846	5.133	8.127
SS 2	-854.844	5138.039	162.893	5894.383	635.607	4857.467	5.320	9.071
SS 3	-588.194	7672.392	522.552	8278.573	982.081	7505.305	6.581	9.212
SS 4	-140.751	10257.181	1113.990	10522.469	1337.236	10219.492	7.463	9.288
SS 5	-3806.036	29098.035	2153.427	28546.729	3471.531	26530.303	7.989	10.460
SS 6	-1930.792	14164.916	1416.800	12449.108	1591.625	12163.595	7.035	10.017

AMTAD 95% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.39963	1.63900	0.43308	1.58810	0.16888	2.03849	-0.64893	0.60014
SS 1	0.65806	2.83465	0.65929	2.85443	0.29140	3.51741	-0.13568	1.14829
SS 2	0.49128	4.23286	0.67635	4.33245	0.40514	4.89038	-0.06898	1.61567
SS 3	0.86175	5.02950	0.98018	5.08891	0.49809	6.01236	0.27767	1.74828
SS 4	1.53489	5.33809	1.53621	5.23164	0.56686	6.84245	0.63035	1.76176
SS 5	1.29887	8.30477	1.75891	7.96658	0.80276	9.68988	0.84139	2.20185
SS 6	3.96092	4.67106	3.99720	4.52165	0.68704	8.29315	1.37959	1.54418

ASPOW 95% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-144.917	1557.372	102.413	1648.905	129.890	1567.874	4.912	7.786
SS 1	-729.690	2928.428	60.013	3405.376	228.802	2761.809	4.847	8.414
SS 2	-1428.883	5712.078	87.369	7082.571	446.551	5390.209	4.960	9.430
SS 3	-1379.449	8463.647	323.484	9535.695	689.970	8328.448	6.329	9.464
SS 4	-1136.735	11253.165	754.382	11803.803	939.487	11340.313	7.288	9.463
SS 5	-6957.810	32249.809	1374.959	32583.112	2438.955	29440.009	7.752	10.697
SS 6	-3472.547	15706.672	971.561	13913.469	1118.210	13497.635	6.749	10.303

Tables D9. 99% AMTAD/ASPOW Tolerance Intervals (Outliers Removed)

AMTAD 99% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.20491	1.83372	0.28386	1.74539	0.07514	2.44303	-0.84517	0.79638
SS 1	0.31609	3.17662	0.40942	3.17524	0.12966	4.21545	-0.33741	1.35001
SS 2	-0.09657	4.82071	0.36979	4.95856	0.18027	5.86090	-0.33366	1.88035
SS 3	0.20695	5.68430	0.57378	5.73608	0.22163	7.20554	0.04662	1.97933
SS 4	0.93736	5.93562	1.03141	5.71897	0.25223	8.20037	0.45259	1.93952
SS 5	0.19817	9.40548	1.07637	8.89104	0.35719	11.61289	0.62765	2.41560
SS 6	3.84935	4.78263	3.84016	4.56234	0.30570	9.93896	1.35374	1.57004

ASPOW 99% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-412.366	1824.821	41.495	2017.949	57.795	1879.027	4.460	8.237
SS 1	-1304.421	3503.160	16.145	4567.115	101.806	3309.905	4.286	8.975
SS 2	-2550.809	6834.004	20.930	9748.262	198.695	6459.923	4.258	10.133
SS 3	-2925.911	10010.109	107.672	12194.313	307.005	9981.271	5.836	9.956
SS 4	-3083.330	13199.760	308.509	14415.663	418.029	13590.856	6.947	9.804
SS 5	-13117.773	38409.773	491.313	41011.688	1085.225	35282.528	7.290	11.160
SS 6	-6485.821	18719.945	408.934	16883.107	497.553	16176.309	6.191	10.861

Tables D10. 70% and 80% RMSTAD/RSPOW Tolerance Intervals (Outliers Removed)

RMSTAD 70% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.58392	1.17521	0.59398	1.16556	0.37054	1.26599	-0.51789	0.16706
SS 1	0.97123	1.93792	0.94340	1.96121	0.61540	2.10258	-0.01165	0.65944
SS 2	1.12062	2.90317	1.10812	2.92867	0.88151	3.01179	0.13662	1.07085
SS 3	1.57120	3.54177	1.54536	3.57443	1.09910	3.75519	0.46642	1.26790
SS 4	2.10377	3.88506	2.09178	3.89149	1.25289	4.28065	0.75577	1.35925
SS 5	2.60696	5.83377	2.72514	5.71301	1.78608	6.10233	1.03858	1.74870
SS 6	3.69385	3.96054	3.73445	3.92082	1.54332	5.27291	1.30700	1.37670

RSPOW 70% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	129.494	575.500	145.340	566.560	163.956	560.173	4.897	6.414
SS 1	116.350	1089.285	177.939	1057.389	307.140	1049.377	5.303	6.917
SS 2	80.052	2358.045	234.094	2270.491	660.421	2256.401	5.384	7.813
SS 3	417.484	3634.335	589.511	3551.823	1027.421	3510.297	6.432	8.175
SS 4	908.730	4154.027	1064.845	4079.424	1187.983	4058.873	7.119	8.254
SS 5	1257.388	11352.296	2303.744	10564.786	3057.966	10447.878	7.780	9.299
SS 6	515.872	5533.006	1296.324	4822.995	1400.941	4786.471	6.881	8.762

RMSTAD 80% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.51400	1.24513	0.52740	1.22917	0.29835	1.39473	-0.59889	0.24806
SS 1	0.85692	2.05223	0.82917	2.07767	0.49550	2.31640	-0.09100	0.73879
SS 2	0.90983	3.11396	0.93356	3.16185	0.70977	3.31807	0.02615	1.18132
SS 3	1.33817	3.77479	1.33290	3.81868	0.88496	4.13707	0.37164	1.36268
SS 4	1.89313	4.09570	1.87484	4.08664	1.00879	4.71596	0.68440	1.43061
SS 5	2.22539	6.21534	2.39160	6.05627	1.43809	6.72289	0.95460	1.83267
SS 6	3.66231	3.99208	3.70251	3.93590	1.24263	5.80913	1.29876	1.38494

RSPOW 80% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	76.753	628.241	114.333	630.696	132.012	617.139	4.718	6.593
SS 1	1.299	1204.335	129.946	1216.864	247.299	1156.090	5.112	7.108
SS 2	-189.323	2627.419	156.803	2715.804	531.750	2485.860	5.097	8.101
SS 3	37.088	4014.730	429.463	4091.959	827.247	3867.268	6.226	8.381
SS 4	524.972	4537.786	840.242	4535.010	956.527	4471.631	6.985	8.388
SS 5	63.656	12546.027	1761.063	11912.348	2462.179	11510.350	7.600	9.478
SS 6	-77.409	6126.286	1028.190	5349.269	1127.994	5273.220	6.659	8.984

Tables D11. 90% and 95% RMSTAD/RSPOW Tolerance Intervals (Outliers Removed)

RMSTAD 90% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.41036	1.34876	0.43289	1.32119	0.20817	1.59087	-0.71894	0.36811
SS 1	0.68749	2.22166	0.66917	2.24705	0.34573	2.64215	-0.20862	0.85641
SS 2	0.59741	3.42638	0.70226	3.50868	0.49523	3.78468	-0.13759	1.34506
SS 3	0.99280	4.12016	1.04260	4.17747	0.61747	4.71885	0.23117	1.50315
SS 4	1.58094	4.40790	1.56311	4.36756	0.70387	5.37915	0.57863	1.53638
SS 5	1.65984	6.78089	1.92538	6.55591	1.00341	7.66831	0.83014	1.95713
SS 6	3.61557	4.03882	3.65007	3.95648	0.86703	6.62605	1.28654	1.39716

RSPOW 90% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-1.417	706.410	76.752	729.620	92.110	703.925	4.452	6.859
SS 1	-169.222	1374.857	77.098	1472.743	172.550	1318.668	4.829	7.391
SS 2	-588.577	3026.673	80.595	3463.930	371.022	2835.439	4.671	8.526
SS 3	-526.715	4578.534	253.775	4959.735	577.201	4411.110	5.920	8.686
SS 4	-43.817	5106.575	566.933	5236.561	667.403	5100.463	6.786	8.587
SS 5	-1705.633	14315.317	1127.250	14022.669	1717.952	13129.016	7.334	9.745
SS 6	-956.739	7005.617	699.719	6157.456	787.043	6014.777	6.329	9.314

RMSTAD 95% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.32048	1.43864	0.35666	1.39882	0.14625	1.76535	-0.82307	0.47224
SS 1	0.54054	2.36861	0.54227	2.39074	0.24289	2.93192	-0.31064	0.95843
SS 2	0.32643	3.69736	0.53115	3.80973	0.34793	4.19976	-0.27960	1.48708
SS 3	0.69324	4.41972	0.81937	4.48497	0.43381	5.23639	0.10933	1.62499
SS 4	1.31015	4.67868	1.30775	4.60335	0.49451	5.96911	0.48689	1.62812
SS 5	1.16931	7.27142	1.55650	6.98010	0.70495	8.50933	0.72219	2.06509
SS 6	3.57502	4.07937	3.59935	3.97283	0.60914	7.35276	1.27595	1.40775

RSPOW 95% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-69.217	774.210	51.918	818.734	64.712	781.128	4.221	7.089
SS 1	-317.125	1522.759	46.202	1712.703	121.226	1463.293	4.583	7.636
SS 2	-934.870	3372.966	41.956	4198.997	260.664	3146.415	4.302	8.896
SS 3	-1015.730	5067.549	151.476	5774.593	405.517	4894.898	5.655	8.951
SS 4	-537.156	5599.914	385.416	5867.498	468.890	5659.855	6.613	8.760
SS 5	-3240.228	15849.912	727.735	15953.418	1206.962	14568.939	7.103	9.976
SS 6	-1719.428	7768.305	479.704	6882.268	552.944	6674.446	6.044	9.599

Tables D12. 99% RMSTAD/RSPOW Tolerance Intervals (Outliers Removed)

RMSTAD 99% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.14480	1.61432	0.22872	1.54489	0.06507	2.11569	-1.02657	0.67574
SS 1	0.25332	2.65582	0.33477	2.66294	0.10808	3.51378	-0.51002	1.15781
SS 2	-0.20318	4.22697	0.27992	4.39628	0.15481	5.03323	-0.55717	1.76464
SS 3	0.10777	5.00520	0.47147	5.07480	0.19303	6.27558	-0.12880	1.86312
SS 4	0.78092	5.20792	0.86863	5.04426	0.22003	7.15371	0.30759	1.80742
SS 5	0.21060	8.23013	0.95558	7.78447	0.31367	10.19805	0.51121	2.27607
SS 6	3.49579	4.15860	3.48565	4.00144	0.27104	8.81195	1.25524	1.42846

RSPOW 99% Tolerance Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	-201.729	906.723	21.178	1000.466	28.794	936.147	3.771	7.540
SS 1	-606.192	1811.826	14.274	2227.001	53.940	1753.691	4.104	8.116
SS 2	-1611.679	4049.776	9.385	5868.590	115.984	3770.837	3.580	9.617
SS 3	-1971.482	6023.301	46.375	7523.905	180.437	5866.316	5.138	9.469
SS 4	-1501.360	6564.118	159.030	7151.550	208.635	6783.082	6.276	9.097
SS 5	-6239.506	18849.190	266.709	19966.896	537.044	17460.218	6.652	10.427
SS 6	-3210.058	9258.936	201.789	8352.302	246.035	7999.024	5.485	10.158

E. Confidence Interval Tables

Table E1. 70% and 80% AMTAD/ASPOW Confidence Intervals (Outliers Included)

AMTAD 70% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.96823	1.32562	1.12684	1.48438	0.75341	1.07132	-0.08329	0.21186
SS 1	1.79772	1.98089	2.03106	2.23749	1.36995	1.53753	0.50957	0.60283
SS 2	2.33230	2.52201	2.63882	2.84897	1.78500	1.95579	0.75274	0.83421
SS 3	2.88749	3.07712	3.24843	3.45507	2.15644	2.34981	0.98988	1.05561
SS 4	3.17610	3.69688	3.54739	4.05946	2.24599	2.91832	1.11859	1.27352
SS 5	3.68499	5.91866	4.53745	6.33772	2.90952	4.99651	1.30475	1.73850
SS 6	4.06458	4.56741	4.29488	4.46256	2.35126	5.22379	1.40362	1.52016

ASPOW 70% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	545.818	866.637	649.764	975.391	498.285	708.548	6.078	6.620
SS 1	1393.018	1854.931	1387.807	1796.258	1714.765	1924.518	6.698	6.947
SS 2	2798.586	3435.759	2727.952	3350.024	3154.060	3455.831	7.306	7.537
SS 3	4298.013	4988.379	4632.611	5370.883	4156.984	4529.741	7.994	8.151
SS 4	5036.575	7452.957	5884.367	8369.503	4801.123	6238.331	8.347	8.702
SS 5	6395.780	18896.220	10020.418	20168.397	8839.765	15180.514	8.755	9.694
SS 6	-672.972	12907.096	4581.159	10473.684	3826.822	8502.062	7.268	9.784

AMTAD 80% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.92168	1.37217	1.09072	1.53354	0.72634	1.12337	-0.12174	0.25030
SS 1	1.77586	2.00275	2.00795	2.26325	1.35216	1.55959	0.49843	0.61396
SS 2	2.30974	2.54457	2.61501	2.87491	1.76646	1.97778	0.74305	0.84389
SS 3	2.86496	3.09966	3.22483	3.48036	2.13534	2.37458	0.98207	1.06342
SS 4	3.11132	3.76166	3.49128	4.12470	2.18372	3.01966	1.09932	1.29279
SS 5	3.33829	6.26535	4.36166	6.59316	2.76040	5.40154	1.23742	1.80582
SS 6	3.92174	4.71025	4.27547	4.48282	2.18934	5.92097	1.37051	1.55326

ASPOW 80% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	504.031	908.423	619.289	1023.389	480.382	742.971	6.007	6.690
SS 1	1337.885	1910.065	1346.109	1851.900	1692.493	1952.132	6.668	6.977
SS 2	2722.800	3511.544	2662.487	3432.394	3121.291	3494.683	7.278	7.564
SS 3	4215.956	5070.435	4552.310	5465.624	4116.297	4577.487	7.976	8.169
SS 4	4736.000	7753.531	5644.265	8725.534	4668.020	6454.964	8.303	8.746
SS 5	4455.536	20836.464	9224.928	21907.573	8386.718	16411.061	8.609	9.840
SS 6	-4530.785	16764.909	4154.403	11549.580	3563.291	9636.771	6.553	10.499

Table E2. 90% and 95% AMTAD/ASPOW Confidence Intervals (Outliers Included)

AMTAD 90% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.84706	1.44679	1.03929	1.60943	0.68916	1.20835	-0.18336	0.31193
SS 1	1.74321	2.03540	1.97417	2.30198	1.32649	1.59326	0.48181	0.63058
SS 2	2.27614	2.57817	2.58013	2.91378	1.73956	2.01113	0.72862	0.85832
SS 3	2.83141	3.13320	3.19016	3.51819	2.10468	2.41209	0.97044	1.07505
SS 4	3.01128	3.86170	3.40974	4.22334	2.09667	3.18075	1.06956	1.32255
SS 5	2.69880	6.90485	4.11355	6.99083	2.56232	6.10394	1.11324	1.93001
SS 6	3.50720	5.12479	4.24686	4.51301	1.98247	7.24331	1.27444	1.64934

ASPOW 90% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	437.048	975.406	576.730	1098.909	455.792	799.175	5.894	6.803
SS 1	1255.549	1992.401	1286.600	1937.556	1660.365	1994.280	6.624	7.021
SS 2	2609.943	3624.402	2568.336	3558.220	3073.762	3553.610	7.237	7.605
SS 3	4093.829	5192.563	4435.844	5609.127	4057.211	4649.802	7.948	8.197
SS 4	4271.820	8217.712	5306.300	9281.274	4481.937	6799.318	8.234	8.814
SS 5	876.679	24415.320	8160.558	24764.946	7784.889	18545.124	8.341	10.109
SS 6	-15726.593	27960.718	3593.904	13350.831	3226.594	11788.954	4.479	12.573

AMTAD 95% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.77507	1.51878	0.99665	1.67828	0.65948	1.29068	-0.24282	0.37138
SS 1	1.71459	2.06402	1.94533	2.33610	1.30489	1.62345	0.46724	0.64515
SS 2	2.24681	2.60750	2.55025	2.94792	1.71678	2.04081	0.71603	0.87092
SS 3	2.80215	3.16247	3.16039	3.55132	2.07869	2.44543	0.96030	1.08519
SS 4	2.91950	3.95349	3.34056	4.31080	2.02585	3.33203	1.04225	1.34985
SS 5	1.95791	7.64574	3.90981	7.35511	2.40965	6.83439	0.96937	2.07388
SS 6	2.68832	5.94366	4.22221	4.53936	1.82928	8.77358	1.08465	1.83912

ASPOW 95% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	372.421	1040.033	542.192	1168.910	436.162	853.622	5.785	6.912
SS 1	1183.388	2064.562	1237.120	2015.051	1633.323	2032.064	6.585	7.060
SS 2	2511.419	3722.926	2489.375	3671.085	3033.515	3606.061	7.202	7.641
SS 3	3987.292	5299.099	4337.244	5736.642	4007.108	4714.066	7.924	8.221
SS 4	3845.956	8643.576	5029.598	9791.882	4330.549	7122.686	8.172	8.877
SS 5	-3269.617	28561.617	7337.352	27543.420	7321.051	20764.395	8.029	10.420
SS 6	-37842.533	50076.657	3169.375	15139.138	2977.274	14279.568	0.381	16.671

Table E3. 99% AMTAD/ASPOW Confidence Intervals (Outliers Included)

AMTAD 99% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.60585	1.68800	0.91830	1.82148	0.60746	1.47939	-0.38257	0.51114
SS 1	1.65765	2.12096	1.89018	2.40426	1.26434	1.68522	0.43825	0.67415
SS 2	2.18883	2.66548	2.49284	3.01580	1.67369	2.10095	0.69113	0.89582
SS 3	2.74439	3.22023	3.10301	3.61699	2.02943	2.51282	0.94028	1.10521
SS 4	2.72175	4.15123	3.20938	4.48700	1.89872	3.66309	0.98342	1.40868
SS 5	-0.41777	10.02141	3.54029	8.12280	2.15345	8.70232	0.50804	2.53521
SS 6	-3.83846	12.47045	4.17444	4.59131	1.58407	13.42188	-0.42803	3.35180

ASPOW 99% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	220.518	1191.937	480.547	1318.857	401.761	978.434	5.529	7.169
SS 1	1039.786	2208.163	1145.840	2175.575	1582.570	2109.389	6.507	7.138
SS 2	2316.677	3917.667	2341.990	3902.112	2957.371	3712.327	7.131	7.712
SS 3	3776.994	5509.398	4150.817	5994.294	3912.141	4843.969	7.876	8.269
SS 4	2928.429	9561.102	4529.766	10872.356	4058.795	7830.372	8.037	9.011
SS 5	-16564.760	41856.760	5960.494	33905.876	6542.661	26439.567	7.031	11.418
SS 6	-214115.86	226349.980	2478.991	19355.296	2578.174	21844.983	-32.278	49.330

Table E4. 70% and 80% RMSTAD/RSPOW Confidence Intervals (Outliers Included)

RMSTAD 70% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.82956	1.11165	0.96176	1.23827	0.63190	0.89854	-0.24049	0.04643
SS 1	1.49374	1.65495	1.69132	1.87316	1.15124	1.29207	0.31960	0.41561
SS 2	1.96961	2.13753	2.23203	2.41849	1.52112	1.66666	0.57601	0.66071
SS 3	2.49037	2.65745	2.80453	2.98691	1.86583	2.03314	0.83877	0.90599
SS 4	2.76379	3.22505	3.08867	3.54773	1.95932	2.54584	0.97937	1.13564
SS 5	3.24761	5.19312	3.99020	5.56149	2.55504	4.38777	1.17956	1.60771
SS 6	3.64864	4.00575	3.81246	3.93120	2.08464	4.63145	1.29519	1.38851
RSPOW 70% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	273.019	431.974	324.898	486.188	248.250	353.005	5.385	5.926
SS 1	703.194	930.588	704.287	908.552	851.605	955.775	6.024	6.271
SS 2	1413.372	1730.384	1384.258	1696.387	1578.777	1729.830	6.631	6.861
SS 3	2159.442	2501.356	2335.125	2701.864	2074.686	2260.724	7.315	7.470
SS 4	2515.352	3700.705	2939.956	4161.037	2378.330	3090.280	7.658	8.007
SS 5	3261.618	9348.066	5050.658	9992.099	4374.523	7512.361	8.081	8.997
SS 6	-334.511	6383.388	2264.334	5179.777	1892.325	4204.185	6.563	9.080
RMSTAD 80% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.79282	1.14839	0.93345	1.27583	0.60919	0.94219	-0.27786	0.08380
SS 1	1.47449	1.67419	1.67102	1.89592	1.13629	1.31061	0.30814	0.42706
SS 2	1.94964	2.15750	2.21095	2.44154	1.50532	1.68540	0.56593	0.67078
SS 3	2.47051	2.67731	2.78372	3.00924	1.84757	2.05457	0.83078	0.91398
SS 4	2.70641	3.28242	3.03847	3.60634	1.90500	2.63425	0.95993	1.15508
SS 5	2.94564	5.49509	3.83658	5.78418	2.42409	4.74345	1.11311	1.67417
SS 6	3.54720	4.10719	3.79866	3.94549	1.94108	5.24958	1.26868	1.41502
RSPOW 80% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	252.315	452.678	309.775	509.924	239.330	370.154	5.315	5.996
SS 1	676.053	957.729	683.394	936.328	840.544	969.489	5.994	6.301
SS 2	1375.667	1768.089	1351.371	1737.671	1562.375	1749.277	6.604	6.889
SS 3	2118.803	2541.995	2295.190	2748.875	2054.380	2284.553	7.297	7.488
SS 4	2367.905	3848.152	2821.639	4335.518	2312.395	3197.593	7.614	8.051
SS 5	2316.916	10292.768	4659.177	10831.671	4150.324	8121.320	7.939	9.139
SS 6	-2242.925	8291.802	2053.263	5712.246	1762.012	4765.288	5.847	9.796

Table E5. 90% and 95% RMSTAD/RSPOW Confidence Intervals (Outliers Included)

RMSTAD 90% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.73392	1.20729	0.89301	1.33361	0.57801	1.01347	-0.33777	0.14370
SS 1	1.44576	1.70292	1.64138	1.93016	1.11472	1.33890	0.29103	0.44418
SS 2	1.91990	2.18724	2.18008	2.47611	1.48240	1.71382	0.55093	0.68579
SS 3	2.44095	2.70687	2.75315	3.04266	1.82105	2.08703	0.81889	0.92587
SS 4	2.61780	3.37103	2.96557	3.69500	1.82906	2.77478	0.92991	1.18510
SS 5	2.38864	6.05209	3.61969	6.13077	2.25014	5.36028	0.99053	1.79675
SS 6	3.25279	4.40160	3.77829	3.96675	1.75767	6.42197	1.19174	1.49196
RSPOW 90% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	219.127	485.866	288.645	547.251	227.080	398.156	5.202	6.109
SS 1	635.520	998.263	653.562	979.067	824.588	990.421	5.950	6.345
SS 2	1319.517	1824.239	1304.059	1800.714	1538.584	1778.774	6.563	6.929
SS 3	2058.317	2602.481	2237.252	2820.062	2024.891	2320.644	7.269	7.515
SS 4	2140.202	4075.855	2654.977	4607.673	2220.215	3368.176	7.547	8.118
SS 5	574.375	12035.309	4134.057	12207.540	3852.498	9177.401	7.677	9.402
SS 6	-7781.359	13830.236	1776.067	6603.773	1595.519	5829.520	3.772	11.871
RMSTAD 95% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.67710	1.26412	0.85935	1.38583	0.55312	1.08252	-0.39556	0.20150
SS 1	1.42057	1.72811	1.61609	1.96035	1.09657	1.36427	0.27603	0.45918
SS 2	1.89394	2.21320	2.15365	2.50650	1.46299	1.73911	0.53783	0.69888
SS 3	2.41516	2.73265	2.72691	3.07194	1.79856	2.11588	0.80851	0.93625
SS 4	2.53651	3.45232	2.90376	3.77366	1.76728	2.90674	0.90237	1.21264
SS 5	1.74333	6.69740	3.44153	6.44815	2.11607	6.00174	0.84851	1.93876
SS 6	2.67122	4.98317	3.76072	3.98529	1.62185	7.77871	1.03975	1.64395
RSPOW 95% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	187.107	517.886	271.489	581.833	217.300	425.282	5.093	6.218
SS 1	599.996	1033.786	628.744	1017.713	811.158	1009.185	5.912	6.383
SS 2	1270.498	1873.258	1264.367	1857.245	1518.438	1805.028	6.528	6.965
SS 3	2005.554	2655.245	2188.186	2883.298	1999.886	2352.717	7.245	7.539
SS 4	1931.295	4284.762	2518.415	4857.524	2145.222	3528.362	7.485	8.180
SS 5	-1444.451	14054.135	3726.770	13541.665	3622.959	10275.649	7.373	9.705
SS 6	-18721.852	24770.729	1566.134	7488.977	1472.232	7061.104	-0.329	15.972

Table E6. 99% RMSTAD/RSPOW Confidence Intervals (Outliers Included)

RMSTAD 99% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.54353	1.39768	0.79719	1.49389	0.50949	1.24080	-0.53142	0.33735
SS 1	1.37046	1.77823	1.56780	2.02074	1.06249	1.41618	0.24618	0.48902
SS 2	1.84262	2.26452	2.10292	2.56696	1.42627	1.79036	0.51194	0.72477
SS 3	2.36427	2.78355	2.67634	3.12998	1.75594	2.17418	0.78804	0.95673
SS 4	2.36136	3.62747	2.78664	3.93226	1.65638	3.19555	0.84304	1.27198
SS 5	-0.32587	8.76660	3.11824	7.11667	1.89109	7.64209	0.39314	2.39413
SS 6	-1.96411	9.61850	3.72660	4.02178	1.40445	11.89993	-0.17163	2.85533
RSPOW 99% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	111.844	593.150	240.847	655.858	200.161	487.464	4.837	6.474
SS 1	529.303	1104.480	582.927	1097.705	785.953	1047.587	5.835	6.460
SS 2	1173.609	1970.147	1190.244	1972.905	1480.324	1858.220	6.458	7.035
SS 3	1901.400	2759.398	2095.373	3011.010	1952.489	2417.550	7.198	7.586
SS 4	1481.203	4734.854	2271.462	5385.635	2010.604	3878.929	7.353	8.312
SS 5	-7917.839	20527.523	3042.965	16584.705	3237.758	13084.114	6.399	10.679
SS 6	-105922.18	111971.054	1224.777	9576.226	1274.882	10802.127	-33.011	48.654

Table E7. 70% and 80% AMTAD/ASPOW Confidence Intervals (Outliers Removed)

AMTAD 70% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.89421	1.14442	1.02121	1.25000	0.64305	0.93497	-0.15048	0.10169
SS 1	1.67897	1.81373	1.87265	2.01859	1.22333	1.38035	0.46656	0.54605
SS 2	2.27357	2.45057	2.56793	2.76184	1.72261	1.88948	0.73350	0.81319
SS 3	2.85346	3.03779	3.20677	3.40653	2.12363	2.31544	0.98046	1.04550
SS 4	3.17610	3.69688	3.54739	4.05946	2.24599	2.91832	1.11859	1.27352
SS 5	3.68499	5.91866	4.53745	6.33772	2.90952	4.99651	1.30475	1.73850
SS 6	4.06458	4.56741	4.29488	4.46256	2.35126	5.22379	1.40362	1.52016

ASPOW 70% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	545.818	866.637	649.764	975.391	498.285	708.548	6.078	6.620
SS 1	986.129	1212.610	1065.813	1310.658	960.541	1083.828	6.520	6.741
SS 2	1964.729	2318.466	2046.630	2449.052	1894.936	2087.516	7.085	7.306
SS 3	3312.921	3771.277	3681.024	4194.009	2935.581	3215.313	7.823	7.969
SS 4	4146.339	5970.091	4879.995	6755.696	3696.720	4893.039	8.215	8.536
SS 5	6395.780	18896.220	10020.418	20168.397	8839.765	15180.514	8.755	9.694
SS 6	-672.972	12907.096	4581.159	10473.684	3826.822	8502.062	7.268	9.784

AMTAD 80% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.86115	1.17748	0.99709	1.28024	0.61871	0.98374	-0.18380	0.13501
SS 1	1.66288	1.82983	1.85611	2.03658	1.20674	1.40112	0.45706	0.55555
SS 2	2.25252	2.47163	2.54591	2.78572	1.70451	1.91098	0.72402	0.82267
SS 3	2.83155	3.05970	3.18393	3.43096	2.10271	2.34002	0.97272	1.05323
SS 4	3.11132	3.76166	3.49128	4.12470	2.18372	3.01966	1.09932	1.29279
SS 5	3.33829	6.26535	4.36166	6.59316	2.76040	5.40154	1.23742	1.80582
SS 6	3.92174	4.71025	4.27547	4.48282	2.18934	5.92097	1.37051	1.55326

ASPOW 80% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	504.031	908.423	619.289	1023.389	480.382	742.971	6.007	6.690
SS 1	959.072	1239.666	1040.067	1343.103	947.515	1100.140	6.494	6.767
SS 2	1922.625	2360.570	2003.644	2501.594	1874.129	2112.435	7.058	7.332
SS 3	3258.407	3825.792	3624.670	4259.214	2905.195	3251.316	7.806	7.987
SS 4	3917.664	6198.766	4695.873	7020.582	3587.823	5076.455	8.175	8.576
SS 5	4455.536	20836.464	9224.928	21907.573	8386.718	16411.061	8.609	9.840
SS 6	-4530.785	16764.909	4154.403	11549.580	3563.291	9636.771	6.553	10.499

Table E8. 90% and 95% AMTAD/ASPOW Confidence Intervals (Outliers Removed)

AMTAD 90% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.80753	1.23110	0.96238	1.32641	0.58541	1.06391	-0.23783	0.18904
SS 1	1.63882	1.85389	1.83186	2.06354	1.18284	1.43288	0.44287	0.56974
SS 2	2.22116	2.50298	2.51364	2.82149	1.67827	1.94361	0.70990	0.83679
SS 3	2.79894	3.09231	3.15039	3.46749	2.07232	2.37725	0.96122	1.06473
SS 4	3.01128	3.86170	3.40974	4.22334	2.09667	3.18075	1.06956	1.32255
SS 5	2.69880	6.90485	4.11355	6.99083	2.56232	6.10394	1.11324	1.93001
SS 6	3.50720	5.12479	4.24686	4.51301	1.98247	7.24331	1.27444	1.64934

ASPOW 90% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	437.048	975.406	576.730	1098.909	455.792	799.175	5.894	6.803
SS 1	918.636	1280.102	1003.046	1392.674	928.748	1125.072	6.454	6.807
SS 2	1859.889	2423.306	1941.588	2581.548	1843.988	2150.283	7.019	7.372
SS 3	3177.228	3906.970	3542.728	4357.728	2861.119	3305.917	7.780	8.013
SS 4	3562.228	6554.203	4435.678	7432.407	3436.146	5369.584	8.113	8.638
SS 5	876.679	24415.320	8160.558	24764.946	7784.889	18545.124	8.341	10.109
SS 6	-15726.593	27960.718	3593.904	13350.831	3226.594	11788.954	4.479	12.573

AMTAD 95% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.75499	1.28364	0.93326	1.36780	0.55894	1.14219	-0.29079	0.24200
SS 1	1.61771	1.87500	1.81108	2.08721	1.16275	1.46139	0.43042	0.58219
SS 2	2.19378	2.53036	2.48597	2.85289	1.65604	1.97266	0.69757	0.84912
SS 3	2.77049	3.12076	3.12158	3.49949	2.04657	2.41035	0.95118	1.07477
SS 4	2.91950	3.95349	3.34056	4.31080	2.02585	3.33203	1.04225	1.34985
SS 5	1.95791	7.64574	3.90981	7.35511	2.40965	6.83439	0.96937	2.07388
SS 6	2.68832	5.94366	4.22221	4.53936	1.82928	8.77358	1.08465	1.83912

ASPOW 95% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	372.421	1040.033	542.192	1168.910	436.162	853.622	5.785	6.912
SS 1	883.162	1315.577	972.005	1437.149	912.973	1147.459	6.420	6.841
SS 2	1805.076	2478.119	1889.322	2652.964	1818.499	2184.026	6.985	7.406
SS 3	3106.363	3977.835	3473.157	4445.018	2823.792	3354.512	7.758	8.035
SS 4	3233.254	6883.176	4221.701	7809.118	3313.243	5646.568	8.055	8.696
SS 5	-3269.617	28561.617	7337.352	27543.420	7321.051	20764.395	8.029	10.420
SS 6	-37842.533	50076.657	3169.375	15139.138	2977.274	14279.568	0.381	16.671

Table E9. 99% AMTAD/ASPOW Confidence Intervals (Outliers Removed)

AMTAD 99% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.62813	1.41050	0.87885	1.45247	0.51282	1.32383	-0.41864	0.36985
SS 1	1.57563	1.91707	1.77115	2.13427	1.12511	1.51987	0.40560	0.60701
SS 2	2.13964	2.58450	2.43278	2.91527	1.61402	2.03154	0.67320	0.87349
SS 3	2.71432	3.17693	3.06604	3.56288	1.99775	2.47727	0.93136	1.09459
SS 4	2.72175	4.15123	3.20938	4.48700	1.89872	3.66309	0.98342	1.40868
SS 5	-0.41777	10.02141	3.54029	8.12280	2.15345	8.70232	0.50804	2.53521
SS 6	-3.83846	12.47045	4.17444	4.59131	1.58407	13.42188	-0.42803	3.35180
ASPOW 99% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	220.518	1191.937	480.547	1318.857	401.761	978.434	5.529	7.169
SS 1	812.445	1386.294	914.084	1528.216	883.417	1193.378	6.351	6.910
SS 2	1696.585	2586.610	1791.199	2798.295	1770.366	2252.543	6.917	7.474
SS 3	2966.305	4117.893	3341.104	4620.701	2753.166	3452.951	7.713	8.080
SS 4	2513.610	7602.821	3832.841	8601.391	3093.748	6258.345	7.929	8.822
SS 5	-16564.760	41856.760	5960.494	33905.876	6542.661	26439.567	7.031	11.418
SS 6	-214115.86	226349.980	2478.991	19355.296	2578.174	21844.983	-32.278	49.330

Table E10. 70% and 80% RMSTAD/RSPOW Confidence Intervals (Outliers Removed)

RMSTAD 70% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.76669	0.99243	0.87913	1.08752	0.55689	0.80969	-0.30616	-0.04467
SS 1	1.39759	1.51156	1.55984	1.68394	1.01933	1.15112	0.28434	0.36345
SS 2	1.93248	2.09131	2.18761	2.36261	1.47959	1.62232	0.56212	0.64536
SS 3	2.47436	2.63860	2.78491	2.96350	1.84981	2.01628	0.83376	0.90056
SS 4	2.76379	3.22505	3.08867	3.54773	1.95932	2.54584	0.97937	1.13564
SS 5	3.24761	5.19312	3.99020	5.56149	2.55504	4.38777	1.17956	1.60771
SS 6	3.64864	4.00575	3.81246	3.93120	2.08464	4.63145	1.29519	1.38851

RSPOW 70% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	273.019	431.974	324.898	486.188	248.250	353.005	5.385	5.926
SS 1	543.364	662.270	603.362	732.021	507.748	575.929	6.011	6.209
SS 2	1114.617	1323.479	1145.437	1377.249	1107.155	1217.183	6.488	6.710
SS 3	1888.471	2163.348	2077.745	2381.318	1727.455	1886.993	7.229	7.378
SS 4	2079.701	2983.057	2447.179	3376.961	1845.002	2442.075	7.529	7.845
SS 5	3261.618	9348.066	5050.658	9992.099	4374.523	7512.361	8.081	8.997
SS 6	-334.511	6383.388	2264.334	5179.777	1892.325	4204.185	6.563	9.080

RMSTAD 80% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.73686	1.02226	0.85729	1.11523	0.53581	0.85193	-0.34072	-0.01011
SS 1	1.38398	1.52517	1.54578	1.69925	1.00542	1.16857	0.27489	0.37291
SS 2	1.91359	2.11020	2.16780	2.38421	1.46411	1.64071	0.55221	0.65526
SS 3	2.45484	2.65812	2.76451	2.98536	1.83164	2.03761	0.82582	0.90850
SS 4	2.70641	3.28242	3.03847	3.60634	1.90500	2.63425	0.95993	1.15508
SS 5	2.94564	5.49509	3.83658	5.78418	2.42409	4.74345	1.11311	1.67417
SS 6	3.54720	4.10719	3.79866	3.94549	1.94108	5.24958	1.26868	1.41502

RSPOW 80% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	252.315	452.678	309.775	509.924	239.330	370.154	5.315	5.996
SS 1	529.146	676.489	589.728	748.945	500.580	584.994	5.988	6.232
SS 2	1089.764	1348.332	1120.743	1407.595	1095.244	1231.394	6.461	6.737
SS 3	1855.789	2196.029	2044.509	2420.030	1710.082	1907.475	7.211	7.396
SS 4	1966.432	3096.326	2355.739	3508.041	1790.652	2533.616	7.489	7.884
SS 5	2316.916	10292.768	4659.177	10831.671	4150.324	8121.320	7.939	9.139
SS 6	-2242.925	8291.802	2053.263	5712.246	1762.012	4765.288	5.847	9.796

Table E11. 90% and 95% RMSTAD/RSPOW Confidence Intervals (Outliers Removed)

RMSTAD 90% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.68849	1.07063	0.82591	1.15759	0.50697	0.92136	-0.39676	0.04593
SS 1	1.36362	1.54553	1.52518	1.72220	0.98538	1.19525	0.26076	0.38703
SS 2	1.88545	2.13834	2.13876	2.41658	1.44164	1.66860	0.53747	0.67001
SS 3	2.42578	2.68718	2.73456	3.01806	1.80526	2.06991	0.81400	0.92032
SS 4	2.61780	3.37103	2.96557	3.69500	1.82906	2.77478	0.92991	1.18510
SS 5	2.38864	6.05209	3.61969	6.13077	2.25014	5.36028	0.99053	1.79675
SS 6	3.25279	4.40160	3.77829	3.96675	1.75767	6.42197	1.19174	1.49196

RSPOW 90% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	219.127	485.866	288.645	547.251	227.080	398.156	5.202	6.109
SS 1	507.881	697.754	570.084	774.752	490.264	598.869	5.952	6.267
SS 2	1052.741	1385.356	1085.120	1453.805	1077.982	1252.966	6.422	6.776
SS 3	1807.136	2244.683	1996.222	2478.568	1684.866	1938.516	7.185	7.422
SS 4	1790.374	3272.384	2226.458	3711.738	1714.952	2679.914	7.427	7.946
SS 5	574.375	12035.309	4134.057	12207.540	3852.498	9177.401	7.677	9.402
SS 6	-7781.359	13830.236	1776.067	6603.773	1595.519	5829.520	3.772	11.871

RMSTAD 95% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.64109	1.11804	0.79963	1.19564	0.48405	0.98915	-0.45167	0.10084
SS 1	1.34577	1.56338	1.50753	1.74236	0.96853	1.21920	0.24836	0.39943
SS 2	1.86088	2.16291	2.11388	2.44502	1.42263	1.69344	0.52459	0.68288
SS 3	2.40044	2.71253	2.70885	3.04671	1.78290	2.09862	0.80369	0.93063
SS 4	2.53651	3.45232	2.90376	3.77366	1.76728	2.90674	0.90237	1.21264
SS 5	1.74333	6.69740	3.44153	6.44815	2.11607	6.00174	0.84851	1.93876
SS 6	2.67122	4.98317	3.76072	3.98529	1.62185	7.77871	1.03975	1.64395

RSPOW 95% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	187.107	517.886	271.489	581.833	217.300	425.282	5.093	6.218
SS 1	489.206	716.428	553.577	797.854	481.604	611.346	5.921	6.298
SS 2	1020.403	1417.693	1055.140	1495.112	1063.378	1272.186	6.387	6.811
SS 3	1764.680	2287.138	1955.265	2530.487	1663.497	1966.120	7.162	7.445
SS 4	1627.424	3435.334	2120.086	3897.969	1653.612	2818.155	7.370	8.003
SS 5	-1444.451	14054.135	3726.770	13541.665	3622.959	10275.649	7.373	9.705
SS 6	-18721.852	24770.729	1566.134	7488.977	1472.232	7061.104	-0.329	15.972

Table E12. 99% RMSTAD/RSPOW Confidence Intervals (Outliers Removed)

RMSTAD 99% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	0.52663	1.23249	0.75066	1.27365	0.44411	1.14645	-0.58425	0.23342
SS 1	1.31016	1.59899	1.47363	1.78244	0.93699	1.26837	0.22364	0.42415
SS 2	1.81232	2.21147	2.06610	2.50157	1.38666	1.74377	0.49914	0.70833
SS 3	2.35040	2.76257	2.65928	3.10349	1.74051	2.15667	0.78334	0.95098
SS 4	2.36136	3.62747	2.78664	3.93226	1.65638	3.19555	0.84304	1.27198
SS 5	-0.32587	8.76660	3.11824	7.11667	1.89109	7.64209	0.39314	2.39413
SS 6	-1.96411	9.61850	3.72660	4.02178	1.40445	11.89993	-0.17163	2.85533
RSPOW 99% Confidence Intervals								
	Gaussian		Weibull		Rayleigh		Lognormal	
	Lo	Hi	Lo	Hi	Lo	Hi	Lo	Hi
SS 0	111.844	593.150	240.847	655.858	200.161	487.464	4.837	6.474
SS 1	451.912	753.722	522.682	845.015	465.407	636.998	5.859	6.360
SS 2	956.431	1481.665	998.916	1579.264	1035.779	1311.180	6.319	6.879
SS 3	1680.825	2370.994	1877.627	2635.120	1623.027	2021.978	7.116	7.490
SS 4	1270.963	3791.794	1926.637	4289.355	1544.064	3123.488	7.246	8.127
SS 5	-7917.839	20527.523	3042.965	16584.705	3237.758	13084.114	6.399	10.679
SS 6	-105922.18	111971.054	1224.777	9576.226	1274.882	10802.127	-33.011	48.654

F. Model Boundaries (only upper limits are shown)

Table F1. Model 1 Sea State Boundaries

MODEL 1 (70% Tolerance Intervals with Boundaries Averaged - Outliers Included)										
AMTAD	Gaussian	Weibull	Rayleigh	Lognormal		RMSTAD	Gaussian	Weibull	Rayleigh	Lognormal
SS 0	1.35935	1.34154	1.26243	0.30880		SS 0	1.10998	1.09730	1.05950	0.12194
SS 1	2.03122	2.04027	1.94059	0.65294		SS 1	1.69948	1.70897	1.63706	0.46806
SS 2	2.67044	2.67111	2.45670	0.94064		SS 2	2.28567	2.28594	2.10189	0.78257
SS 3	3.27737	3.29560	2.90664	1.15766		SS 3	2.84176	2.85313	2.52005	1.01641
SS 4	3.69576	3.76658	3.47041	1.32856		SS 4	3.24601	3.30831	3.03336	1.19891
ASPOW	Gaussian	Weibull	Rayleigh	Lognormal		RSPOW	Gaussian	Weibull	Rayleigh	Lognormal
SS 0	357.577	684.991	1078.395	6.409		SS 0	187.876	343.529	536.451	5.728
SS 1	1601.249	1769.571	2702.937	7.026		SS 1	806.211	892.013	1346.046	6.350
SS 2	3604.105	3558.745	4443.887	7.929		SS 2	1818.487	1797.549	2222.591	7.254
SS 3	5189.074	5298.127	5753.910	8.427		SS 3	2603.968	2656.477	2865.985	7.740
SS 4	6594.992	7526.325	8334.332	8.828		SS 4	3327.099	3771.200	4127.027	8.144

Table F2. Model 2 Sea State Boundaries

MODEL 2 (70% Tolerance Intervals with Boundaries Averaged- Outliers Removed)										
AMTAD	Gaussian	Weibull	Rayleigh	Lognormal		RMSTAD	Gaussian	Weibull	Rayleigh	Lognormal
SS 0	1.25893	1.23723	1.10008	0.23634		SS 0	1.07322	1.05448	0.94070	0.07771
SS 1	1.84732	1.84999	1.77446	0.58685		SS 1	1.52927	1.53466	1.49205	0.39803
SS 2	2.59751	2.59685	2.38451	0.92146		SS 2	2.23718	2.23702	2.05545	0.76863
SS 3	3.23925	3.25577	2.87393	1.14936		SS 3	2.82277	2.83310	2.50404	1.01183
SS 4	3.69576	3.76658	3.47041	1.32856		SS 4	3.24601	3.30831	3.03336	1.19891
ASPOW	Gaussian	Weibull	Rayleigh	Lognormal		RSPOW	Gaussian	Weibull	Rayleigh	Lognormal
SS 0	644.237	704.407	852.035	6.398		SS 0	345.925	372.249	433.657	5.858
SS 1	1160.050	1218.433	1555.986	6.794		SS 1	584.668	645.741	854.899	6.151
SS 2	2484.625	2541.342	2806.805	7.722		SS 2	1387.764	1430.001	1641.911	7.123
SS 3	3963.468	4087.849	4176.450	8.263		SS 3	2271.533	2308.334	2349.140	7.647
SS 4	5306.790	6328.848	7155.935	8.698		SS 4	2705.707	3191.584	3558.420	8.017

Table F3. Model 3 Sea State Boundaries

MODEL 3 (80% Confidence Intervals with Boundaries Averaged - Outliers Included)										
AMTAD	Gaussian	Weibull	Rayleigh	Lognormal		RMSTAD	Gaussian	Weibull	Rayleigh	Lognormal
SS 0	1.57401	1.77075	1.23777	0.37437		SS 0	1.31144	1.47342	1.03924	0.19597
SS 1	2.15625	2.43913	1.66302	0.67851		SS 1	1.81191	2.05344	1.40796	0.49650
SS 2	2.70476	3.04987	2.05656	0.91298		SS 2	2.31400	2.61263	1.76648	0.75078
SS 3	3.10549	3.48582	2.27915	1.08137		SS 3	2.69186	3.02386	1.97979	0.93696
SS 4	3.54998	4.24318	2.89003	1.26511		SS 4	3.11403	3.72146	2.52917	1.13410
ASPOW	Gaussian	Weibull	Rayleigh	Lognormal		RSPOW	Gaussian	Weibull	Rayleigh	Lognormal
SS 0	1123.154	1184.749	1217.732	6.679		SS 0	564.366	596.659	605.349	5.995
SS 1	2316.432	2257.194	2536.712	7.128		SS 1	1166.698	1143.850	1265.932	6.452
SS 2	3863.750	3992.352	3805.490	7.770		SS 2	1943.446	2016.430	1901.829	7.093
SS 3	4903.217	5554.945	4622.754	8.236		SS 3	2454.950	2785.257	2298.474	7.551
SS 4	6104.534	8975.231	7420.841	8.678		SS 4	3082.534	4497.347	3673.959	7.995

Table F4. Model 4 Sea State Boundaries

MODEL 4 (80% Confidence Intervals with Boundaries Averaged - Outliers Removed)										
AMTAD	Gaussian	Weibull	Rayleigh	Lognormal		RMSTAD	Gaussian	Weibull	Rayleigh	Lognormal
SS 0	1.42018	1.56817	1.09524	0.29603		SS 0	1.20312	1.33050	0.92867	0.13239
SS 1	2.04117	2.29125	1.55282	0.63978		SS 1	1.71938	1.93352	1.31634	0.46256
SS 2	2.65159	2.98483	2.00684	0.89770		SS 2	2.28252	2.57436	1.73618	0.74054
SS 3	3.08551	3.46112	2.26187	1.07627		SS 3	2.68227	3.01192	1.97130	0.93422
SS 4	3.54998	4.24318	2.89003	1.26511		SS 4	3.11403	3.72146	2.52917	1.13410
ASPOW	Gaussian	Weibull	Rayleigh	Lognormal		RSPOW	Gaussian	Weibull	Rayleigh	Lognormal
SS 0	933.748	1031.728	845.243	6.592		SS 0	490.912	549.826	435.367	5.992
SS 1	1581.146	1673.374	1487.134	6.913		SS 1	883.126	934.844	840.119	6.347
SS 2	2809.488	3063.132	2508.815	7.569		SS 2	1602.061	1726.052	1470.738	6.974
SS 3	3871.728	4477.544	3419.569	8.081		SS 3	2081.231	2387.885	1849.064	7.442
SS 4	5327.151	8122.755	6731.586	8.593		SS 4	2706.621	4083.609	3341.970	7.912

Table F5. Model 5 Sea State Boundaries

MODEL 5 (Experimental - SS Probability Boundaries)						
AMTAD	Non-Log	Log		RMSTAD	Non-Log	Log
SS 0	0.17022	0.04629		SS 0	0.14939	0.04326
SS 1	1.70212	0.46290		SS 1	1.49387	0.43255
SS 2	4.14183	1.12640		SS 2	3.63508	1.05254
SS 3	6.90308	1.87733		SS 3	6.05849	1.75425
SS 4	7.20568	1.95963		SS 4	6.32407	1.83115
ASPOW	Non-Log	Log		RSPOW	Non-Log	Log
SS 0	624.133	0.237		SS 0	307.269	0.221
SS 1	6241.224	2.372		SS 1	3072.641	2.207
SS 2	15186.972	5.771		SS 2	7476.757	5.370
SS 3	25311.691	9.618		SS 3	12461.297	8.949
SS 4	26421.261	10.039		SS 4	13007.553	9.342

G. Trail A Matching Tables

Table G1a. Model 1 – Model Sea State Matching

MODEL 1								
AMTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	97	0.251	97	0.251	97	0.251	105	0.271
BF SS	74	0.215	75	0.218	73	0.212	74	0.215
OP SS	70	0.203	70	0.203	83	0.241	72	0.209
ASPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	106	0.274	84	0.217	62	0.160	100	0.258
BF SS	62	0.180	54	0.157	54	0.157	61	0.177
OP SS	66	0.192	58	0.169	41	0.119	72	0.209
RMSTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	103	0.266	106	0.274	94	0.243	102	0.264
BF SS	61	0.177	54	0.157	67	0.195	56	0.163
OP SS	53	0.154	56	0.163	60	0.174	62	0.180
RSPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	103	0.266	85	0.220	64	0.165	99	0.256
BF SS	59	0.172	55	0.160	54	0.157	61	0.177
OP SS	66	0.192	60	0.174	42	0.122	74	0.215

Table G1b. Model 1 – Model Mean Sea State Margin Matching

Model 1 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	103	0.266	61	0.177	68	0.198
± 1.0	195	0.504	103	0.299	137	0.398
± 1.5	264	0.682	151	0.439	191	0.555
± 2.0	348	0.899	200	0.581	243	0.706

Table G2a. Model 2 – Model Sea State Matching

MODEL 2								
AMTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	108	0.279	107	0.276	106	0.274	107	0.276
BF SS	74	0.215	75	0.218	78	0.227	74	0.215
OP SS	72	0.209	72	0.209	92	0.267	77	0.224
ASPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	90	0.233	88	0.227	78	0.202	96	0.248
BF SS	69	0.201	59	0.172	55	0.160	69	0.201
OP SS	74	0.215	74	0.215	72	0.209	80	0.233
RMSTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	106	0.274	106	0.274	101	0.261	109	0.282
BF SS	65	0.189	60	0.174	71	0.206	63	0.183
OP SS	64	0.186	67	0.195	74	0.215	71	0.206
RSPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	95	0.245	91	0.235	74	0.191	91	0.235
BF SS	66	0.192	59	0.172	54	0.157	66	0.192
OP SS	70	0.203	70	0.203	64	0.186	73	0.212

Table G2b. Model 2 – Model Mean Sea State Margin Matching

Model 2 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	100	0.258	66	0.192	80	0.233
± 1.0	209	0.540	126	0.366	159	0.462
± 1.5	270	0.698	164	0.477	201	0.584
± 2.0	345	0.891	223	0.648	262	0.762

Table G3a. Model 3 – Model Sea State Matching

MODEL 3								
AMTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	83	0.214	74	0.191	82	0.212	87	0.225
BF SS	65	0.189	51	0.148	105	0.305	72	0.209
OP SS	58	0.169	49	0.142	67	0.195	82	0.238
ASPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	63	0.163	68	0.176	55	0.142	83	0.214
BF SS	51	0.148	55	0.160	50	0.145	60	0.174
OP SS	45	0.131	44	0.128	46	0.134	70	0.203
RMSTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	76	0.196	75	0.194	92	0.238	81	0.209
BF SS	57	0.166	27	0.078	78	0.227	63	0.183
OP SS	48	0.140	44	0.128	81	0.235	59	0.172
RSPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	66	0.171	72	0.186	57	0.147	85	0.220
BF SS	53	0.154	54	0.157	52	0.151	62	0.180
OP SS	45	0.131	43	0.125	45	0.131	74	0.215

Table G3b. Model 3 – Model Mean Sea State Margin Matching

Model 3 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	79	0.204	60	0.174	64	0.186
± 1.0	190	0.491	106	0.308	136	0.395
± 1.5	248	0.641	138	0.401	172	0.500
± 2.0	339	0.876	204	0.593	245	0.712

Table G4a. Model 4 – Model Sea State Matching

MODEL 4								
AMTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	91	0.235	91	0.235	83	0.214	97	0.251
BF SS	68	0.198	55	0.160	112	0.326	74	0.215
OP SS	66	0.192	54	0.157	74	0.215	82	0.238
ASPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	72	0.186	72	0.186	71	0.183	82	0.212
BF SS	64	0.186	56	0.163	63	0.183	81	0.235
OP SS	64	0.186	58	0.169	70	0.203	74	0.215
RMSTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	89	0.230	85	0.220	99	0.256	89	0.230
BF SS	59	0.172	33	0.096	79	0.230	67	0.195
OP SS	51	0.148	51	0.148	89	0.259	65	0.189
RSPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	62	0.160	68	0.176	68	0.176	79	0.204
BF SS	61	0.177	55	0.160	57	0.166	74	0.215
OP SS	59	0.172	57	0.166	68	0.198	73	0.212

Table G4b. Model 4 – Model Mean Sea State Margin Matching

Model 4 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	92	0.238	66	0.192	77	0.224
± 1.0	196	0.506	121	0.352	145	0.422
± 1.5	255	0.659	158	0.459	192	0.558
± 2.0	343	0.886	215	0.625	254	0.738

Table G5a. Model 5 – Model Sea State Matching

MODEL 5				
AMTAD	Non-Log	P	Log	P
OBS SS	152	0.393	157	0.406
BF SS	33	0.096	37	0.108
OP SS	62	0.180	71	0.206
ASPOW	Non-Log	P	Log	P
OBS SS	71	0.183	157	0.406
BF SS	9	0.026	75	0.218
OP SS	19	0.055	113	0.328
RMSTAD	Non-Log	P	Log	P
OBS SS	139	0.359	134	0.346
BF SS	26	0.076	27	0.078
OP SS	55	0.160	54	0.157
RSPOW	Non-Log	P	Log	P
OBS SS	74	0.191	159	0.411
BF SS	9	0.026	80	0.233
OP SS	19	0.055	111	0.323

Table G5b. Model 5 – Model Mean Sea State Margin Matching

Model 5 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	165	0.426	37	0.108	71	0.206
± 1.0	312	0.806	92	0.267	149	0.433
± 1.5	357	0.922	130	0.378	200	0.581
± 2.0	382	0.987	179	0.520	272	0.791

H. Trial B Matching Tables

Table H1a. Model 1 – Model Sea State Matching

MODEL 1								
AMTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	9	0.391	9	0.391	7	0.304	8	0.348
BF SS	5	0.217	4	0.174	8	0.348	5	0.217
OP SS	6	0.261	7	0.304	5	0.217	6	0.261
ASPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	5	0.217	4	0.174	4	0.174	7	0.304
BF SS	7	0.304	7	0.304	7	0.304	7	0.304
OP SS	6	0.261	6	0.261	8	0.348	6	0.261
RMSTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	5	0.217	5	0.217	7	0.304	7	0.304
BF SS	7	0.304	7	0.304	5	0.217	6	0.261
OP SS	8	0.348	8	0.348	6	0.261	7	0.304
RSPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	4	0.174	4	0.174	4	0.174	7	0.304
BF SS	7	0.304	7	0.304	7	0.304	7	0.304
OP SS	6	0.261	6	0.261	8	0.348	6	0.261

Table H1b. Model 1 – Model Mean Sea State Margin Matching

Model 1 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	7	0.304	6	0.261	6	0.261
± 1.0	11	0.478	10	0.435	10	0.435
± 1.5	12	0.522	14	0.609	12	0.522
± 2.0	19	0.826	14	0.609	15	0.652

Table H2a. Model 2 – Model Sea State Matching

MODEL 2									
AMTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	8	0.348	9	0.391	6	0.261	8	0.348
BF SS	6	0.261	4	0.174	9	0.391	5	0.217
OP SS	5	0.217	7	0.304	6	0.261	6	0.261
ASPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	6	0.261	6	0.261	6	0.261	6	0.261
BF SS	7	0.304	8	0.348	7	0.304	8	0.348
OP SS	5	0.217	5	0.217	6	0.261	5	0.217
RMSTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	6	0.261	6	0.261	9	0.391	7	0.304
BF SS	7	0.304	7	0.304	5	0.217	6	0.261
OP SS	8	0.348	8	0.348	6	0.261	7	0.304
RSPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	7	0.304	7	0.304	6	0.261	7	0.304
BF SS	7	0.304	7	0.304	7	0.304	7	0.304
OP SS	5	0.217	6	0.261	6	0.261	6	0.261

Table H2b. Model 2 – Model Mean Sea State Margin Matching

Model 2 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	7	0.304	7	0.304	6	0.261
± 1.0	11	0.478	10	0.435	9	0.391
± 1.5	13	0.565	13	0.565	12	0.522
± 2.0	18	0.783	15	0.652	19	0.826

Table H3a. Model 3 Sea State Matching

MODEL 3									
AMTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	5	0.217	4	0.174	2	0.087	7	0.304
BF SS	8	0.348	7	0.304	7	0.304	8	0.348
OP SS	5	0.217	9	0.391	7	0.304	5	0.217
ASPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	3	0.130	4	0.174	3	0.130	6	0.261
BF SS	8	0.348	5	0.217	7	0.304	8	0.348
OP SS	5	0.217	6	0.261	6	0.261	5	0.217
		0.000						
RMSTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	4	0.174	2	0.087	4	0.174	6	0.261
BF SS	7	0.304	7	0.304	8	0.348	5	0.217
OP SS	9	0.391	7	0.304	5	0.217	8	0.348
RSPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	3	0.130	5	0.217	3	0.130	6	0.261
BF SS	7	0.304	4	0.174	7	0.304	8	0.348
OP SS	6	0.261	7	0.304	6	0.261	5	0.217

Table H3b. Model 3 – Model Mean Sea State Margin Matching

Model 3 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	4	0.174	7	0.304	6	0.261
± 1.0	11	0.478	10	0.435	9	0.391
± 1.5	11	0.478	14	0.609	12	0.522
± 2.0	18	0.783	14	0.609	14	0.609

Table H4a. Model 4 – Model Sea State Matching

MODEL 4								
AMTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	7	0.304	5	0.217	2	0.087	7	0.304
BF SS	8	0.348	7	0.304	7	0.304	8	0.348
OP SS	5	0.217	9	0.391	7	0.304	5	0.217
ASPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	6	0.261	7	0.304	4	0.174	4	0.174
BF SS	7	0.304	6	0.261	7	0.304	7	0.304
OP SS	5	0.217	5	0.217	6	0.261	5	0.217
RMSTAD	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	5	0.217	2	0.087	2	0.087	8	0.348
BF SS	7	0.304	7	0.304	9	0.391	4	0.174
OP SS	9	0.391	7	0.304	6	0.261	7	0.304
RSPOW	Gaussian	P	Weibull	P	Rayleigh	P	Lognormal	P
OBS SS	6	0.261	6	0.261	5	0.217	4	0.174
BF SS	7	0.304	6	0.261	7	0.304	7	0.304
OP SS	5	0.217	5	0.217	6	0.261	5	0.217

Table H4b. Model 4 – Model Mean Sea State Margin Matching

Model 4 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	7	0.304	7	0.304	6	0.261
± 1.0	10	0.435	9	0.391	8	0.348
± 1.5	13	0.565	13	0.565	12	0.522
± 2.0	17	0.739	14	0.609	16	0.696

Table H5a. Model 5 – Model Sea State Matching

MODEL 5				
AMTAD	Non-Log	P	Log	P
OBS SS	6	0.261	6	0.261
BF SS	3	0.130	5	0.217
OP SS	5	0.217	6	0.261
ASPOW	Non-Log	P	Log	P
OBS SS	6	0.261	11	0.478
BF SS	3	0.130	5	0.217
OP SS	4	0.174	8	0.348
RMSTAD	Non-Log	P	Log	P
OBS SS	6	0.261	7	0.304
BF SS	2	0.087	2	0.087
OP SS	4	0.174	4	0.174
RSPOW	Non-Log	P	Log	P
OBS SS	7	0.304	10	0.435
BF SS	3	0.130	5	0.217
OP SS	4	0.174	8	0.348

Table H5b. Model 5 – Model Mean Sea State Margin Matching

Model 5 Mean Sea State Margin Matching						
	Observed SS	P	Beaufort SS	P	Operational SS	P
± 0.5	6	0.261	3	0.130	5	0.217
± 1.0	16	0.696	7	0.304	9	0.391
± 1.5	21	0.913	9	0.391	13	0.565
± 2.0	23	1.000	12	0.522	19	0.826

VITA

The author was born in Mt. Pleasant, Michigan. He obtained his Bachelor's degree in Applied Mathematics from Ferris State University in 2002. In 2013, he joined the University of New Orleans applied physics graduate program to pursue a M.S. in applied physics with concentrations in Acoustics, Spectral Analysis, and Mathematical Physics. He is currently employed as a mathematician at the Naval Oceanographic Office.