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R. M.A. Azzam
University of New Orleans, razzam@uno.edu

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R. M. A. Azzam

Department of Electrical Engineering, University of New Orleans,
New Orleans, Louisiana 70148, USA (razzam@uno.edu)

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An analytically tractable design procedure is presented for a polarizing beam splitter (PBS) that uses frustrated total internal reflection and optical tunneling by a symmetric LHL trilayer thin-film stack embedded in a high-index prism. Considerable simplification arises when the refractive index of the high-index center layer H matches the refractive index of the prism and its thickness is quarter-wave. This leads to a cube design in which zero reflection for the p polarization is achieved at a 45° angle of incidence independent of the thicknesses of the identical symmetric low-index tunnel layers L and L. Arbitrarily high reflectance for the s polarization is obtained at subwavelength thicknesses of the tunnel layers. This is illustrated by an IR Si-cube PBS that uses an embedded ZnS–Si–ZnS trilayer stack. © 2011 Optical Society of America

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1. Introduction

Thin-film polarizing beam splitters (PBSs) are versatile optical elements that use destructive interference of light for one linear polarization (p or s) and nearly full constructive interference for the orthogonal polarization in a multilayer stack at oblique incidence [1–10].

This paper follows up on previous work by Azzam and Perla [11,12] that dealt with the polarizing properties of a symmetric LHL trilayer stack (that consists of a high-index center-layer H of refractive index n_2 and thickness d_2 sandwiched between two identical low-index tunnel layers L of refractive index n_1 and thickness d_1), which is embedded in a prism of high refractive index $n_0 > n_1$, Fig. 1. All media are considered to be transparent and optically isotropic and are separated by parallel-plane boundaries. Light is incident from medium 0 at an angle $\phi_0 >$ the critical angle $\phi_{c01} = \arcsin(n_1/n_0)$ of the 01 interface, so that frustrated total internal reflection (FTIR) and optical tunneling through the trilayer stack take place. The optical field is evanescent in the low-index tunnel layers and is propagating in the prism and center layer. In Sections 2 and 3 we consider an important and analytically tractable special case in which the refractive index of the center layer is deliberately matched to that of the prism, i.e., $n_2 = n_0$. This simple design employs only two optical materials and depends on fewer design parameters, namely only one index ratio $N = n_0/n_1$, angle of incidence ϕ_0, and film thicknesses d_1, d_2. A novel PBS cube ($\phi_0 = 45°$) is introduced in Section 4 that uses index ratio $N = (\sqrt{2}+1)^{1/2} = 1.55377$ and center layer of quarter-wave optical thickness. This PBS, which uses tunnel layers of subwavelength thicknesses, fully transmits the p polarization and nearly totally reflects the s polarization. Section 5 provides a brief summary of this paper.

2. Constraint on Film Thicknesses for Zero Reflection of the *p* or *s* Polarization

In [11] it was shown that the condition of zero reflection of the ν polarization ($\nu = p, s$) by an embedded symmetric trilayer stack can be put in the form

$$X_2 = \frac{\ell + mX_1 - nX_1^2}{-n + mX_1 + \ell X_1^2}. \quad (1)$$

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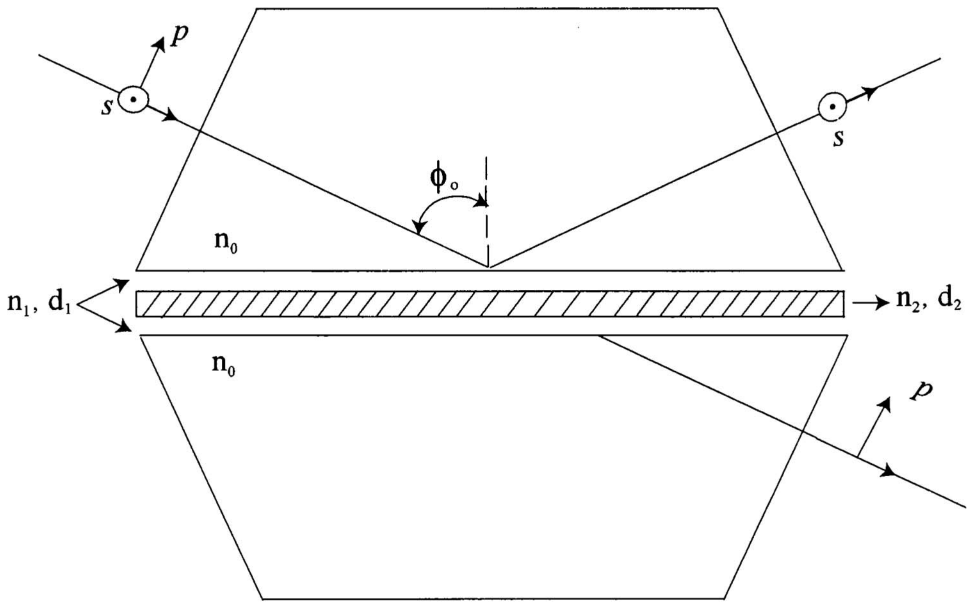

Fig. 1. Embedded symmetric trilayer stack as a PBS operating under conditions of FTIR. p and s are the linear polarizations parallel and perpendicular to the plane of incidence, respectively, and ϕ_0 is the angle of incidence.

When index matching ($n_2 = n_0$) is satisfied, ℓ, m, and n in Eq. (1) are determined by only one Fresnel reflection coefficient $r_{01\nu}$ at the 01 interface:

$$\ell = r_{01\nu}, \qquad m = -r_{01\nu}(1 + r_{01\nu}^2), \qquad n = -r_{01\nu}^3. \quad (2)$$

Upon substitution of ℓ, m, and n from Eqs. (2) in Eq. (1), the condition of zero reflection of the ν polarization reduces to

$$X_2 = \frac{1 - r_{01\nu}^2 X_1}{r_{01\nu}^2 - X_1}. \quad (3)$$

In Eqs. (1) and (3) X_1 and X_2 are exponential functions of film thicknesses given by

$$X_i = \exp[-j4\pi(n_i d_i/\lambda)\cos\phi_i], \qquad i = 1, 2. \quad (4)$$

In Eq. (4), ϕ_i is the angle of refraction in the ith layer, and λ is the vacuum wavelength of light.

Since FTIR takes place at the 01 interface at ϕ_0, the light field is evanescent in the low-index tunnel layers, $\cos\phi_1$ is pure imaginary, and X_1 is pure real in the range $0 \le X_1 \le 1$. Also, because $n_2 = n_0$, the angle of refraction in the high-index center layer $\phi_2 = \phi_0$ and X_2 is a pure phase factor, i.e., $|X_2| = 1$.

To further clarify the thickness constraint for zero reflection of the ν polarization, we substitute

$$r_{01\nu} = \exp(j\delta), \qquad X_2 = \exp(-j\theta_2), \quad (5)$$

in Eq. (3) and solve for θ_2 in terms of δ, X_1:

$$\theta_2 = 2\delta + 2\arg[1 - X_1 \exp(-j2\delta)]. \quad (6)$$

Equation (6) is significant in that it is equally valid for the p and s polarizations (for simplicity the subscript ν is dropped from δ) and depends only implicitly on the index ratio N and angle of incidence ϕ_0 via the 01-interface reflection phase shift δ.

From the round-trip phase thickness θ_2 of the high-index center layer, the corresponding metric thickness d_2 is determined by

$$d_2/\lambda = [(4\pi n_0)^{-1}\sec\phi_0]\theta_2. \quad (7)$$

Likewise, the metric thickness of the low-index tunnel layers is obtained from X_1 by

$$d_1/\lambda = \frac{-\ln X_1}{(4\pi n_1)(N^2 - \sin^2\phi_0)^{1/2}}. \quad (8)$$

Figure 2 shows a family of curves of the normalized center-layer phase thickness θ_2/π as a function of the tunnel-layer thickness parameter X_1 in the range $0 \le X_1 \le 1$ as calculated from Eq. (6) for discrete values of the interface reflection phase shift $\delta = q\pi/8$, $q = 0, 1, 2, \ldots, 8$ as a parameter. Limiting cases are identified from Eq. (6) and Fig. 2 as follows.

1. When $\delta = 0$, $\theta_2 = 0$; and when $\delta = \pi$, $\theta_2 = 2\pi$ independent of X_1. These limiting values of δ occur at the critical angle and grazing incidence [13], respectively, and are of little or no practical interest.
2. In the limit as $X_1 \to 0$, i.e., for tunnel-layer thickness of the order of a wavelength or greater, $\theta_2 \to 2\delta$. Such a condition is desirable for achieving high reflectance of the unextinguished orthogonal polarization as discussed in Section 3.
3. When $\delta = \pi/2$ (01-interface reflection phase shift is quarter-wave), $\theta_2 = \pi$ (center layer is of quarter-wave optical thickness at oblique incidence) independent of X_1. This interesting special case provides the basis of the unique PBS cube design described in Section 4.

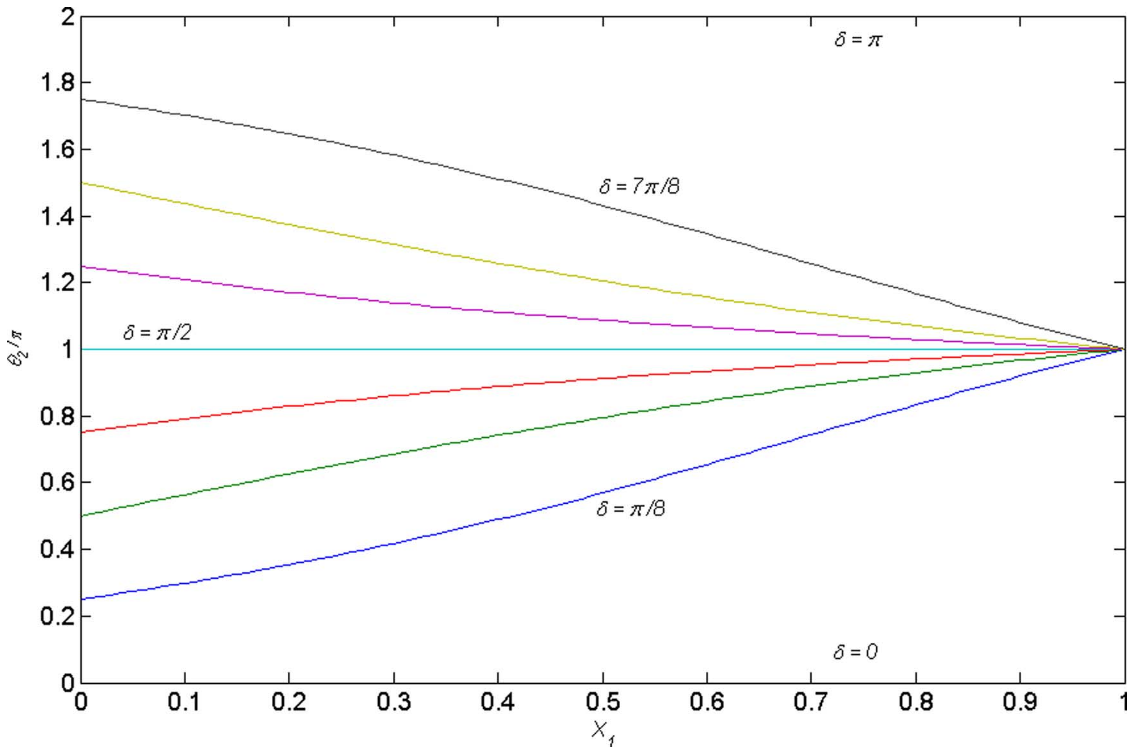

Fig. 2. (Color online) Normalized center-layer phase thickness θ_2/π is plotted as a function of the tunnel-layer thickness parameter X_1 using Eq. (6) at discrete values of the 01-interface reflection phase shift $\delta = q\pi/8, q = 0, 1, 2, \ldots, 8$ such that zero reflection is achieved for the p or s polarization.

3. Embedded Symmetric Trilayer with Index-Matched Quarter-Wave-Thick Center Layer: Reflectance of the Orthogonal Polarization

For an embedded symmetric trilayer stack with index-matched quarter-wave center layer (i.e., $n_2 = n_0, \theta_2 = \pi, X_2 = -1$) the complex-amplitude reflection coefficient for the orthogonal polarization (o) is given by

$$R_o = \frac{(1 - X_1)^2 \exp(j\delta_o)}{1 - 2X_1 \sec\delta_o \exp(j\delta_o) + X_1^2 \exp(j2\delta_o)}. \quad (9)$$

In Eq. (9) δ_o is the 01-interface reflection phase shift for the orthogonal polarization. The associated intensity reflectance is obtained from Eq. (9) as

$$|R_o|^2 = \frac{(1 - X_1)^4}{(1 - X_1)^4 + 4(\sec\delta_o - \cos\delta_o)^2 X_1^2}. \quad (10)$$

Figure 3 shows a family of curves of $|R_o|^2$ as a function of X_1 in the range $0 \le X_1 \le 0.1$ calculated from Eq. (10) at selected values of $\delta_o = q\pi/8$, $q = 0, 1, 2, 3, 5, 6, 7, 8$. In Fig. 3 $\delta_o = \pi/2$ is excluded as it renders $|R_o|^2 = 0$, which is not physically acceptable. Furthermore, $\delta_o = 0, \pi$ correspond to total reflection, $|R_o|^2 = 1$, at the critical angle and grazing incidence, respectively, and are not of interest.

4. PBS Cube Using Embedded Symmetric Trilayer with Index-Matched, Quarter-Wave-Thick Center Layer

The p polarization can be totally suppressed in reflection (hence is fully transmitted) by a symmetric trilayer stack with index-matched center layer $n_2 = n_0$ that is embedded in a cube prism of refractive index n_0. The 01-interface reflection phase shift $\delta_p = \pi/2$ is achieved at $\phi_0 = 45°$ if the index ratio N is selected such that [13]

$$\sin^2\phi_0 = 1/2 = (N^2 + 1)/(N^4 + 1). \quad (11)$$

Equation (11) yields

$$N = n_0/n_1 = \left(\sqrt{2} + 1\right)^{1/2} = 1.55377. \quad (12)$$

It is interesting to note that $N = 1.55377$ causes maximum separation between the critical angle and the angle at which $\delta_p - \delta_s$ is maximum [13].

Table 1 lists some prism and film refractive indices that satisfy Eq. (12). For a given prism index n_0, the refractive index n_1 of the tunnel layers is calculated from Eq. (12) and rounded to three decimal places.

As already noted in Section 2, zero reflection for the p polarization is achieved when $\delta_p = \pi/2$ *and* $\theta_2 = \pi$ (center layer of quarter-wave optical thickness) *independent of the thickness of the symmetric tunnel layers*.

As a specific design, consider an IR-transparent ZnS–Si–ZnS trilayer that is embedded in a Si cubical prism with refractive indices $n_0 = n_2 = 3.4$, $n_1 = 2.188$. Substitution of $n_0 = 3.4$, $\phi_0 = 45°$, $\theta_2 = \pi$ in Eq. (7) gives the required metric thickness of the center layer

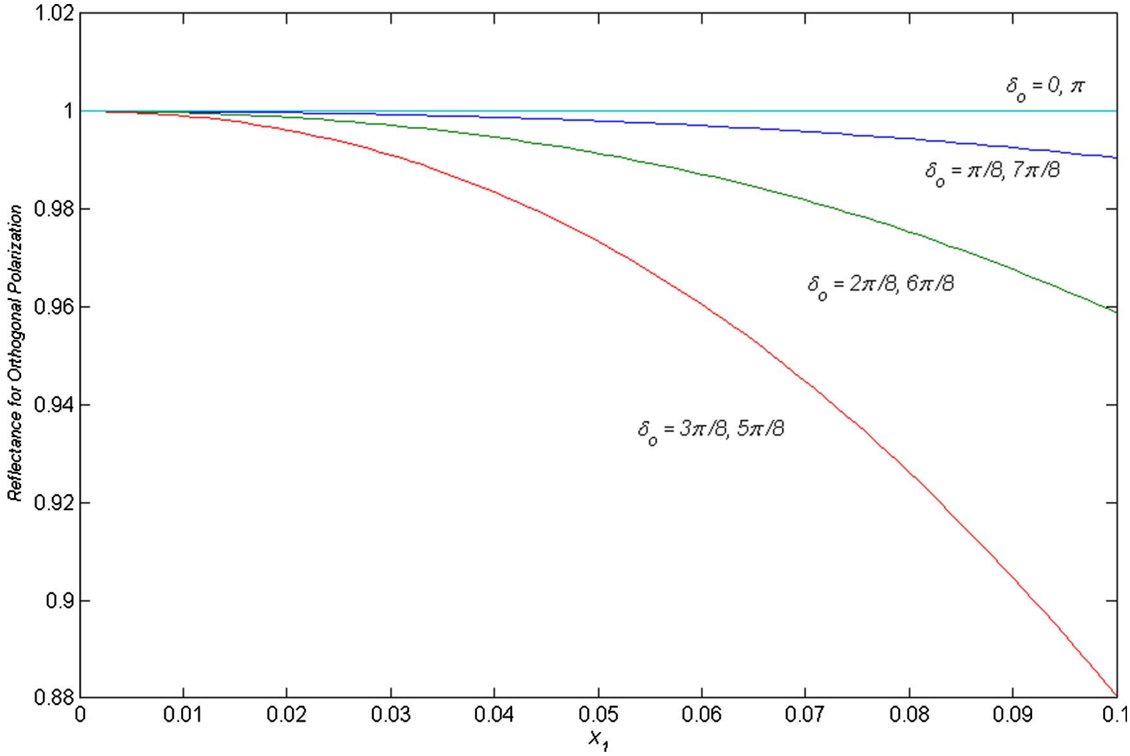

Fig. 3. (Color online) Intensity reflectance for the orthogonal polarization $|R_o|^2$ is plotted as a function of X_1 using Eq. (10) at selected values of $\delta_o = q\pi/8$, $q = 0, 1, 2, 3, 5, 6, 7, 8$.

$$d_2 = 0.10399\lambda. \tag{13}$$

To calculate the reflectance for the orthogonal ($o \equiv s$) polarization, Abelès's condition [14] at $\phi_0 = 45°$ is used to obtain

$$\delta_o = \delta_s = \delta_p/2 = \pi/4. \tag{15}$$

Substitution of $\delta_o = \pi/4$ in Eq. (10) gives

$$|R_s|^2 = \frac{(1 - X_1)^4}{(1 - X_1)^4 + 2X_1^2}. \tag{16}$$

From Eq. (16) it is apparent that the reflectance for the s polarization can be made as high as possible, without affecting the condition of zero reflection for the p polarization, by increasing the thickness of the tunnel layers to make X_1 as small as one wishes. For example, if $X_1 = 0.001$ is substituted in Eq. (16), $|R_s|^2 = 0.999998$ is obtained. The metric thickness of the tunnel layers is calculated by substituting $X_1 = 0.001$, $N = (\sqrt{2} + 1)^{1/2}$, $n_1 = 2.188$, $\phi_0 = 45°$ in Eq. (8); this gives

$$d_1 = 0.18159\lambda. \tag{17}$$

From Eqs. (16) and (17) the total thickness of the trilayer stack,

$$d_{\text{tot}} = d_2 + 2d_1 = 0.46717\lambda. \tag{18}$$

For a CO_2-laser wavelength $\lambda = 10.6\,\mu\text{m}$, $d_{\text{tot}} = 4.952\,\mu\text{m}$.

The design presented in this section is intended for operation at a single (e.g., laser) wavelength or within a narrow bandwidth centered at a given λ. This is obvious from Eqs. (13), (17), and (18), which show that the required layer thicknesses are specific fractions of the wavelength of incident light. When a collimated laser source is used, the limited field of view of the PBS is not a significant concern. As is well known, the design of broadband and wide-field-of-view PBS requires the use of a large number of thin films (see, e.g., [8,9,15]).

5. Summary

Considerable simplification is achieved in the design of PBSs that employ symmetric LHL trilayers embedded in a high-index prism when the refractive indices of the high-index center layer and the prism are matched. The p polarization is suppressed in reflection independent of the thicknesses of the tunnel

Table 1. Selected Prism and Film Refractive Indices That Satisfy Eq. (12)[a]

n_0 (prism)	1.55377 (glass)	2.2 (ZnS)	2.35 (ZnS)	3.4 (Si)	4 (Ge)
n_1 (thin film)	1	1.416	1.512	2.188	2.574

[a]Refractive indices of ZnS listed in the third and fourth columns are appropriate to the IR and visible spectrum, respectively.

layers at a 45° angle of incidence (PBS cube) by using an index-matched center layer of quarter-wave thickness. High reflectance of the *s* polarization is obtained by suitable choice of the thickness of the tunnel layers. One such Si-cube PBS is described that uses an embedded ZnS–Si–ZnS stack in the IR.

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