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Distributary Channel Networks as Moving Boundaries: Causes and Morphodynamic Effects

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1. Introduction

The morphology of a distributary channel network sets the structure by which water, sediment, and nutrients are spread across a river delta and into its receiving basin. Network structures found on river deltas vary widely, from a single channel on some wave-dominated deltas to many hundreds of interconnected channels found on large tide-dominated deltas (Coleman & Wright, 1975; Syvitski, 2005). When restricted to highly progradational channel networks, one particularly common structure consists of a branching network of coeval distributary channels with few confluences (Figure 1). This distributary network morphology has been interpreted to be the result of dominant fluvial sediment input (Bates, 1953; Galoway, 1975), with a median sediment grain size of fine sand to silt (Orton & Reading, 1993). The structure is important because it is the prototype for controlled sediment diversions designed to mitigate sea level rise (Kim et al., 2009; Paola et al., 2011) and because it sets stratigraphic facies distributions for river-dominated deltas in the rock record (Aschoff et al., 2016; Fidolini & Ghinassi, 2016; Jerrett et al., 2016; Olariu & Bhattacharya, 2006; Zhang et al., 2018). These distributary networks are the subject of this modeling study.

The modeling of branching distributary channel networks is a classic problem in coastal morphodynamics. The model of channel mouth deposition from sediment-laden turbulent jets (“the jet model” here) is focused on the processes that control sedimentation in front of channels (Bates, 1953; Fagherazzi et al., 2016; Wright, 1977). It is argued that at a given channel mouth, channel bifurcation is favored when sedimentation is focused along the channel axis forcing flow around the obstruction (Edmonds & Slingerland, 2007), and channel extension is favored when sedimentation is focused along channel margins producing confining levees (Canestrelli et al., 2014; Falcini & Jerolmack, 2010; Rowland et al., 2010). Distributary channel networks have also been investigated using complex morphodynamic models run over for delta-building timescales, with emphasis on the control of sediment grain size and cohesion (Burpee et al., 2015; Caldwell & Edmonds, 2014; Edmonds & Slingerland, 2010).

Here we present a new model for distributary channel network growth that honors a growing body of measurements of prograding, branching river deltas. The Wax Lake Delta (WLD) in coastal Louisiana will serve as the model’s primary inspiration because its morphology, sedimentology, and hydrology have been studied in detail for decades (Roberts et al., 1980, 1997, 2015; Wellner et al., 2005) and will be introduced in detail...
in section 2. While the WLD is a unique instance of a wide array of distributary channel networks, its morphology is similar to many progradational modern deltas (Cahoon et al., 2011; Edmonds et al., 2011; Welder, 1959) as well as ancient delta stratigraphy (Figure 1). Four key observations from these systems motivate our modeling approach. First, the distributary channel network is composed of many branching distributary channels separating shallow unchannelized interdistributary regions composed of islands, low-lying natural levees, marshes, shallow bays, and a gradually basinward sloping delta front (Bevington & Twilley, 2018; Edmonds et al., 2011; Shaw et al., 2013, 2016). Second, repeat bathymetric surveys have shown that relatively deep distributary channels prograde by eroding into relatively shallow delta front deposits (Shaw & Mohrig, 2014). These two conditions contrast with initial conditions assumed in the jet model, which often assume an empty basin with a gentle bed slope or uniform depth. Third, the distributary channel network and the unchannelized islands and delta front have been shown to be tightly hydraulically connected on this type of delta (Hiatt & Passalacqua, 2015, 2017; Shaw et al., 2016). Fourth, the friction-dominated flows over deltas can be described as potential flow (Coffey & Shaw, 2017), and this simplifying assumption has been helpful for modeling channel network growth in tidal flats and deltas (D’Alpaos et al., 2005; Fagherazzi, 2008; Rinaldo et al., 1999).

This final observation is particularly intriguing, because Laplace’s equation is used to understand a wide variety of processes that produce branching, fingering, shapes (potential theory). These include the viscous fingering of low-viscosity water injected into high-viscosity oil (Chuoke et al., 1959; Lajeunesse & Couder, 2000; Saffman & Taylor, 1958), metal solidification (Voller, 2008), lightening branching (Arrayás et al., 2002), and the growth of groundwater seepage networks (Devauchelle et al., 2012). Models of tidal channel network growth often (successfully) rely on the related Poisson equation, but a model of quasi-steady hydrodynamics on a prograding river delta has never been attempted. Laplace’s equation can also be solved efficiently on domains of arbitrary shape, making it ideal for modeling moving boundary problems (Brebbia & Wrobel Luiz, 1984; Becker, 1983). Laplace’s equation is introduced in further detail in section 3.

In this paper, we propose a new model for distributary channel network growth that distills a river delta into a channelized domain, an unchannelized domain, and the channel network boundary between them. While exploratory, this model honors the observations described above and has the potential to isolate the controls on network initiation, channel extension, and channel bifurcation. In section 2, we review the WLD, which serves as a prototype for our study. In section 3, we describe the model with detailed justification of the moving boundary formulation in supporting information Text S2. In section 4, parameters and boundary conditions are chosen for the model. Section 5 presents results of the model runs, with special attention to the influence exerted by parameters within the sediment transport equation and discharge regimes. In section 6, we discuss the controls on network growth gathered from the model.

2. Field Comparison

The WLD in coastal Louisiana serves as the prototype of a prograding distributary channel network (Figure 1a). We highlight several key aspects of the delta that are important for our analysis. The WLD has
built into the shallow (~2.5 m) Atchafalaya Bay from a deep (~20 m) flood control channel dredged from Atchafalaya River in 1942. Through the Atchafalaya River, the WLD receives water and sediment from the Mississippi River. U.S. Geological Survey records in the flood control channel (gage #07381590) reveal that the median water discharge is 1.912 m$^3$/s and also reveal that the median water discharge for each water year has increased approximately linearly between 1985 and 2015 at a rate of 31 m$^3$/s per year (Figure 2), possibly due to erosion causing the flood control channel to deepen and widen over time (Shaw et al., 2013).

A well-defined distributary channel network emerged in the early 1980s with the transport of significant sand to Atchafalaya Bay (Roberts et al., 1980). Since then, the WLD has grown to cover ~50 km$^2$ of subaerially exposed land and marsh (Olliver & Edmonds, 2017), with an additional 80 km$^2$ of significant subaqueous deposits (Shaw et al., 2016). Decadally averaged progradation rates for primary distributary channels were remarkably consistent at each channel between 1974 and 2016 and were 69–116 m/year for an individual channel tip (Shaw, Estep, et al., 2018). Progradation of primary distributary channels that are 3 m deep into the delta front of recently deposited sediments that is about 1 m deep is accomplished by erosion along the tips and margins of the subaqueous distributary network (Shaw & Mohrig, 2014).

The WLD has seven primary distributary channels that extend quasi-radially from the delta apex into the bay and branch into 11 significant distributary channel tips in their distal, subaqueous reaches. Channel widths were measured throughout the delta using Sentinel 2 imagery in subaerial parts and using a bathymetric digital elevation model from February 2015 (Shaw et al., 2016; Figure 1a). In the subaerial region of the delta, channel widths are simply bank-to-bank measurements. In the subaqueous region, widths are measured by finding the maximum channel depth (minimum elevation) and measuring from 1 m higher elevation on either side of this point. Primary distributary channels have quasi-uniform widths where they are subaerially exposed but grow progressively narrower in their subaqueous reaches (Figure 1a). Near distributary channel tips, widths are 32 to 242 m, with a mean of 105 m and standard deviation of 53 m. There are also several tens of secondary channels that branch from the primary channels but are shallower (≤1 m) and connect primary channels to island interiors. These secondary channels are neglected in this analysis. Distributary channel bifurcation angles measured on the delta range between 26° and 107°, with a mean of 70.8° and standard deviation of 19.3° (Coffey & Shaw, 2017).

Distributary channels are hydraulically connected to interdistributary bays, allowing a significant fraction of water and sediment to leave the distributary channels laterally and flow to the basin through shallow unchannelized island interiors and the delta front (Hiatt & Passalacqua, 2015; Shaw et al., 2016). While levees and proximal island interiors are frequently exposed, the entire delta (neglecting vegetation) is frequently inundated to at least decimeter scale during floods and storms. Hiatt and Passalacqua (2017) modeled flow across the channel network boundary using the shallow water equations and found that the angle between flow along a channel centerline and flow directly outside the channel ($\Psi$) was predominantly >45°, although this angle dropped to ~35° near channel tips. The ratio of cross-boundary to along stream water surface slopes should approximate tan($\Psi$), meaning that cross-boundary water surface slopes are predominately greater than along channel slopes. This is possibly due to increased friction on the delta front relative to within channels produced by shallower water depths or increased roughness from sediment or vegetation (García, 2008; Hiatt & Passalacqua, 2017).

The grain size of delta front deposits on the WLD ranges from sand over 200 μm in diameter down to ~2-μm silt and clay. While variable, a median grain diameter of 100 μm is predominant on the bed surface near distributary channels (Shaw & Mohrig, 2014; Wellner et al., 2005).
3. Moving Boundary Model for Distributary Channel Networks: MB_DCN

Our aim is to build an exploratory model that resolves the complex geometry and kinematics of distributary channel networks. The exploratory approach involves significant simplifying assumptions but includes sufficient physics to reveal possible controls on network geometry and kinematics (Larsen et al., 2016; Murray, 2007). We begin with the fundamental observation that deltas are composed of two domains, one that is channelized that acts as a focused conduit for water and sediment transport and one that is unchannelized where transport is diffuse. These domains are separated by the channel network boundary ($\Gamma_c$; see Figure 3). We construct a model to describe the evolution of this boundary. Moving boundary models have been successfully used to explore delta shoreline evolution by decomposing deltas into the domains landward and seaward of a shoreline, neglecting channels (Ke & Capart, 2015; Lorenzo-Trueba et al., 2009; Swenson et al., 2000; Wolinsky, 2009). We will model the channel network with this approach for the first time and name the model MB_DCN for Moving Boundary of a Distributary Channel Network.

As with all morphodynamic models, this one requires three ingredients: morphology, hydrodynamics, and sediment dynamics. In section 3.1, the model domain is described, with all complexity stemming from the morphology of the channel network boundary. Hydrodynamics (section 3.2) are modeled using Laplace’s equation, which calculates shear stress along the network boundary. Sediment dynamics (section 3.3) are a nonlinear function of this Shields stress.
3.1. Morphology

Consider a shallow delta front of constant elevation \( z_b \) that receives water discharge \( Q \) from a deep river channel network with constant bed elevation \( z_c \) (Figure 3a). It is bounded on its sides by no-flux boundaries set by the coastline and on the basinward end by a far-field outflow boundary. On the upstream boundary, a channel network progrades into the basin. The delta front domain is composed of sediments of a uniform thickness \( h_c = z_b - z_c \) and a characteristic grain diameter \( D \) (Figure 3b). This morphology does not describe the initial deposition of river-derived sediments in a basin, which may have a focused depocenter (Fagherazzi et al., 2016; Wright, 1977). However, it closely resembles the emergent morphology of the modern WLD and other deltas with branching networks and significant delta front deposits (Edmonds et al., 2011; Shaw & Mohrig, 2014; Shaw, Estep, et al. 2018). As the channel network extends by eroding into the delta front (Figure 3c), it erodes sediment of thickness \( h_c \). The unchannelized and channelized domains are separated by a sharp interface that is defined as the channel network boundary \( \Gamma_c \) described by the set of coordinates \( \Gamma_c = s(x, y, t) \). The eventual deposition of this eroded sediment or any incoming sediment is not resolved. It is assumed that this material is deposited far from the channel or outside the model domain with low relief, which allows the simple geometry of the delta front to be maintained. Analogous simplifying assumptions are made in detachment-limited morphodynamic models of tributary networks (Izumi & Parker, 1995; Perron et al., 2008, 2009; Tucker & Bras, 1998) and tidal networks D’Alpaos et al. (2005).

3.2. Hydrodynamics

Fluid flow across shallow coastal domains can be described by the depth-averaged shallow water equations:

\[
\frac{\partial h_w}{\partial t} + \frac{\partial}{\partial x}(h_w u) + \frac{\partial}{\partial y}(h_w v) = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -g \frac{\partial h_w}{\partial x} - g \frac{\partial z_b}{\partial x} - C_f \sqrt{u^2 + v^2} \frac{u}{h_w}, \tag{2}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -g \frac{\partial h_w}{\partial y} - g \frac{\partial z_b}{\partial y} - C_f \sqrt{u^2 + v^2} \frac{v}{h_w}, \tag{3}
\]

where \( h_w \) is the water depth, \( z_b \) is the bed elevation, \( u \) and \( v \) are the depth-averaged flow velocity in the \( x \) and \( y \) directions, respectively, \( C_f \) is a quadratic friction coefficient, and \( g \) is the gravitational acceleration. While general, these equations are nonlinear, making them computationally expensive and susceptible to different scaling regimes. However, the study of hydrodynamics on tidal flats and marshes reveals that two assumptions greatly simplify the shallow water equations (Rinaldo et al., 1999), making them tractable for moving boundary analysis. The first assumption is that flow on deltas is friction dominated, which is to say that the pressure gradient applied by the water surface slope is counteracted entirely by bed friction, with pressure gradients across bed slopes and flow inertia playing insignificant roles in the force balance:

\[
\frac{\partial h_w}{\partial x} \approx -C_f \sqrt{u^2 + v^2} \frac{u}{gh_w}, \tag{4}
\]

\[
\frac{\partial h_w}{\partial y} \approx -C_f \sqrt{u^2 + v^2} \frac{v}{gh_w}. \tag{5}
\]

Friction-dominated flow occurs when variation in bed elevation is small relative to water depth, and

\[
\frac{H}{C_f L} \ll 1, \tag{6}
\]

where \( H \) and \( L \) are flow depth and horizontal length scales. For river delta fronts with \( H \sim 1 \) m, \( C_f \sim 0.007 \), and \( L > 1,000 \) m, this condition is satisfied. Equation (6) is the inverse of the stability parameter Socolofsky and Jirka (2004), which indicates stable flow with minimal lateral turbulent diffusion known for producing many distributary channels for these conditions (Canestrelli et al., 2014).

Friction-dominated flow is still nonlinear, because water surface slope scales with velocity squared (right-hand side of equations (4) and (5)) and is inversely proportional to flow depth. The second
simplifying assumption is that friction can be linearized by assuming a scale water depth \((H)\) and velocity \((U)\), and by choosing a linear friction coefficient \((\Lambda)\) such that the work performed by the linearized and quadratic friction laws is similar for an applied external forcing (Zimmerman, 1982):

\[- \frac{\Lambda}{gH} u = \nabla h_w.\]  

(7)

In tidal studies where this assumption is commonly made, the external forcing is that of a periodic tidal wave, and \(\Lambda \sim UC_f\). Several studies have shown that this linearization performs well relative to fully nonlinear flow across simple and complex domains (Marani et al., 2003; Van Oyen et al., 2012, 2014), although significant errors do arise from spatially variable friction and very shallow flows. An important property of this linear model is that it conforms to Poisson’s equation, \(\nabla^2 h_w = \frac{\Lambda}{gH^2} \frac{\partial h_w}{\partial t}\) (Rinaldo et al., 1999; note that \(\partial h_w/\partial t\) signifies the changing spatially averaged tidal water depth). While caution is recommended in employing these linearizing assumptions, models employing spatiotemporally constant linear friction coefficients have been successfully used to simulate the long-term dynamics of tidal marshes and channels (D’Alpaos et al., 2005, 2007; Di Silvio et al., 2010; Mariotti & Murshid, 2018).

To our knowledge, this study is the first to apply this type of linearization to quasi-steady, unidirectional flow across a delta front. However, the delta front, assumed flat and below sea level, is similar to the steady tidal model, with water driven across the delta front solely by a difference in water surface elevation between the upstream and downstream boundaries. Friction is modeled as constant, and the delta front is modeled with a significant flow depth, ensuring that the common pitfalls of tidal linearization do not occur. We compare the fully nonlinear shallow water equations and the linearized flow model in supporting information Text S1 with \(C_f/U = \Lambda = 0.0035\) and find that the differences between the linear and nonlinear models are small and produce similar stress fields across the model domain.

For this quasi-steady deltaic case \((\partial/\partial t \approx 0)\), equation (7) can be applied to the velocity divergence in potential flow \(\nabla \cdot u = 0\) to produce Laplace’s equation:

\[\nabla^2 h_w(x) = 0.\]  

(8)

This simplification is especially powerful, because a unique solution exists for any domain with appropriate boundary conditions (Becker, 1983; Brebbia & Wrobel Luiz, 1984). It is the use of Laplace’s equation that allows us to solve for channel network growth as a moving boundary.

The water surface elevation along the channel network boundary \(\Gamma_c\) \((h_w^i; \text{ with } t \text{ indicating the time step})\) is set as constant for a given time step, which effectively assumes along-channel water surface slopes are negligible relative to cross-boundary slopes. This is another simplifying assumption employed by models of tidal network growth (D’Alpaos et al., 2005). As discussed in section 2, cross-boundary slopes are predominantly subequal or larger than along-channel slopes \((\Psi > 45^\circ)\). We employ this assumption in order to simplify the distribution of flow across the network boundary.

The most tractable upstream boundary condition on deltas is water discharge from upstream \((Q)\) rather than water depth \(h_w^i\). Discharge is the integral of water flux \(q_w = uH\) along the channel network boundary \(\Gamma_c\):

\[q_w = -\frac{gH^2}{\Lambda} \frac{\partial h_w}{\partial n},\]  

(9)

\[Q = -\frac{gH^2}{\Lambda} \int_{\Gamma_c} \frac{\partial h_w}{\partial n} ds.\]  

(10)

Water flux increases with increasing water surface slopes, scale velocities and depths, and decreases with increasing friction. Equation (10) shows that for a constant \(Q\), the average water surface slope must decrease as the length of the channel network boundary \(\Gamma_c\) grows. Because water surface slopes are linearly related to flux (equation (9)), \(Q\) must scale linearly with the water surface elevation on \(\Gamma_c\); \(h_w^i\). At each time step, equation (8) is solved with an arbitrary \(\Delta h_w\) and then linearly scaled to satisfy equation (10).
3.3. Sediment Dynamics

The water surface slope ($\frac{\partial h_w}{\partial n}$) is used to calculate Shields stress ($\tau_b^*$) along the channel network boundary, which is boundary shear stress normalized to sediment weight:

$$\tau_b^* = \frac{h_w \frac{\partial h_w}{\partial n} R D}{\rho D},$$

where $D$ is the average sediment particle diameter, $R = \frac{\rho_{ss} - \rho}{\rho}$ is the submerged specific gravity of sediment, and $\rho$ and $\rho_s$ are the density of water and sediment, respectively. Sediment flux $q_c$ is calculated with a generalized nonlinear sediment transport formula because the details of the formula should vary between deltas and deltaic environments:

$$q_c = \begin{cases} CD \sqrt{gRD}(\tau_b^* - \tau_{cr}^*)^{\alpha}, & \tau_b^* \geq \tau_{cr}^* \\ 0, & \tau_b^* < \tau_{cr}^* \end{cases}$$

(12)

in which $\tau_{cr}^* = \frac{\tau_{cr}}{\rho D}$ is the critical Shields stress associated with the onset of sediment motion, $C$ is an empirical coefficient, and $\alpha$ is a sediment transport nonlinearity. Values of $C$, $\tau_{cr}^*$, and $\alpha$ used in the model are discussed in section 4.4. The network boundary migration rate $u_c$ is then the sediment flux along the channel network boundary divided by the thickness of the eroded sediment $h_c$:

$$u_c = \frac{q_c}{h_c}.$$  

(13)

Preliminary model runs were dominated by many narrow channels that scaled with the segment length chosen in the method, which we deemed unreasonable. This is a common behavior in models controlled by Laplace's equation, where all wavelengths are unstable and grow, sometimes called the “ultraviolet crisis” (Devauchelle et al., 2017; Pecelerowicz & Szymczak, 2016). It is also important to note that for a given channel depth as modeled here, channel widths cannot be arbitrarily narrow. Width to depth ratios in channelized systems are typically on the order of 100:1 (e.g., Yalin, 1992). This is because the competing processes transporting sediment toward and away from a channel bank are in equilibrium for roughly this ratio (Parker, 1978).

We note briefly that the theory developed by Parker (1978) and subsequent studies for channel dimensions generally focuses on bank-full flows and does not consider water or sediment flux across channel boundaries, which is central to the network dynamics discussed here. However, these fluxes can be included in the theory. If cross-boundary flux of water and sediment is considered, flux toward and across channel boundaries may have an advective component associated with cross-boundary water flux in addition to sediment diffusivity. Flux away from channel banks is the sum of slope driven sediment transport toward the channel center plus any transport of sediment across the channel boundary. Given that (a) delta distributary channels at disequilibrium are often erosional at their base and their banks (Shaw & Mohrig, 2014) and (b) we assume cross boundary slopes to exceed along channel slopes, we can assume that sediment flux across the channel boundary dominates this removal. While details may change slightly, an updated theory will have equilibrium channels with Shields stress at the boundary equal to $\tau_{cr}^*$, and channel widening when Shields stress at the boundary exceeds $\tau_{cr}^*$, consistent with the existing theory and equation (12).

To enforce a minimum width to depth ratio, we average the Shields stress at a point with a smoothing length $S_L = 100h_c$:

$$\bar{\tau}_b^* = \frac{h_w}{R D S_L} \int_{-S_{L}/2}^{S_{L}/2} \frac{\partial h_w}{\partial n} ds.$$  

(14)

The smoothed prograding speed of the channel network boundary is now

$$u_c = \frac{D \sqrt{gRD}}{h_c} C (\bar{\tau}_b^* - \tau_{cr}^*)^\alpha.$$  

(15)

This migration rate causes boundary translation in the direction normal to the boundary ($\hat{n}$)

$$u_c = \frac{dx}{dt} = (u, v) = \hat{u}_c \hat{n}.$$  

(16)

The migration updates the geometry of the channel network boundary ($\Gamma_c$), which changes the flow field in the next time step. This completes the morphodynamic cycle describing the evolution of the channel network boundary.
4. Numerical Experiments

We present several numerical experiments applying the boundary element method to investigate MB_DCN. The branching network of the WLD is our inspiration (Figure 1a), so we choose boundary conditions to resemble this delta when they are reasonably constrained and choose a range of possible parameters for discharge and sediment transport models that are presently unconstrained in this system. The numerical experiments are grouped into two sets. The first set examines the influence of the critical Shields stress ($\tau^{*}_{cr}$) and the nonlinearity ($\alpha$). The second set examines the influence of the flow discharge ($Q$).

4.1. Computational Domain

We build the computational domain to resemble the delta front of the WLD (Figure 1a). The unchannelized domain is bounded by the moving channel boundary $\Gamma_{c}$, the fixed far-field boundary $\Gamma_{w}$, and two fixed no-flux boundaries $\Gamma_{w}$. The initial computational domain is set as a semi-ring (Figure 3). The initial channel boundary starts from a semicircular arc with radius $R_0 = 100$ m, and the initial water depth is $h_{w,0}^{\infty}$. Thus, the initial channel boundary length is $R_0 \pi \approx 314$ m. The initial channel boundary is divided by 16 segments, in which the length of each segment is $\Delta L = 25\pi/4 \approx 20$ m. Although channels evolved from this boundary in preliminary runs from the amplification of numerical artifacts associated with this discretization, small random perturbations were added to the location of each initial segment node (the same for each run initiation) to represent local heterogeneity and promote asymmetric growth.

The far-field boundary is a fixed boundary that is a semicircular arc with $R_{\infty} = 10,000$ m with a constant water depth $h_{w,\infty}$. The no-flux boundaries are straight boundaries connecting the endpoints of the channel boundary and far-field boundary. In all model runs of the numerical experiments, we apply the identical initial computational domain and boundary.

4.2. Flow Conditions

With the reference of WLD, the characteristic scale applied in the model computation are chosen to be $H = 1$ m, $U = 0.5$ m/s, and $\Lambda = 0.0035$. The delta bed elevation was set as $z_b = -0.1$ m, and the water elevation at the far-field boundary was set as $z_{w,\infty} = 0$ m for all model runs. Thus, the water depth at the far-field boundary is $h_{w,\infty} = z_{w,\infty} - z_b = 0.1$ m. In the first set of numerical experiments, we apply $z_{w,0}^{\infty} = 1$ m, and the corresponding water depth is $h_{w,0}^{\infty} = 1.1$ m and water discharge is $Q = 1,912$ m$^3$/s. In the second set, three water discharges are applied in the model runs: (1) the same discharge applied in the first set $Q = 1,912$ m$^3$/s; (2) the water elevation at channel boundary is constant $z_{w,0}^{l} = 1$ m throughout the run, and the corresponding water discharge can be calculated by equation (10) as the channel boundary evolves; (3) the discharge is dependent on time ($t$ in years since 1974): $Q = 31t + 1,552$ as shown in Figure 9. In this case, Wash initial water elevation at the channel boundary $z_{w,0}^{l} = 0.55$ m.

4.3. Sediment Transport Parameters

The basic parameters for the sediment applied in the model runs are the mean sediment grain diameter $D = 0.1$ mm, the submerged specific gravity $R = 1.65$, and $C = 4$. In section 3.3, the Shields stress $\tau_{cr}$ and the sediment transport nonlinearity $\alpha$ are introduced. In the first set of numerical experiments, we choose a range of $\tau_{cr}$ and $\alpha$ to assess their impact on the resulting network. We apply $\tau_{cr} = 0$, 0.05, 0.1, 0.2, and 0.4 to include a threshold-free base case, 0.05 and 0.1 for cohesionless fine and very fine sand determined using the Brownlie equation (Brownlie, 1981; García, 2008), and 0.2 and 0.4 for sediment with significant cohesion that could be produced by either indurated clay particles or vegetation that resists erosion. We model $\alpha = 1, 1.5, 2,$ and 2.5 to include a linear base case and 1.5, which is suggested by the Meyer-Peter and Muller equation and a majority of bed material sediment transport relations (García, 2008). Gravel entrainment experiments by Wong and Parker (2006) suggest $\alpha = 2$ and $\alpha = 2.5$ is used by the Engelund-Hansen transport formula where significant bed load and suspended load occur simultaneously (García, 2008). In the second set experiments where discharge is varied, we apply $\tau_{cr} = 0.05$ and $\alpha = 1.5$ to all runs.

4.4. Boundary Element Method and Moving Boundary Problem

The boundary element method (Becker, 1983; Brebbia & Wrobel Luiz, 1984) is used to solve Laplace’s equation across the delta front domain ($Q$) in our numerical experiments. When the boundary element method is applied (details are described in supporting information Text S2), the gradient of water depth $\partial h_{w}/\partial n$ and prograding velocity $\vec{v}_p$ can be solved for each channel boundary segment. A smoothing
Figure 4. The results of numerical experiments with $\alpha = 1$ (a, b, c, d, e) and $\alpha = 1.5$ (f, g, h, i, j). Transport nonlinearity ($\alpha$), critical Shields stress ($\tau^*_c$), segment length ($\Delta L$), and smoothing length ($S_L$) are given in the bottom right of each run. Color indicates network location at time according to color bar.
Figure 5. The results of numerical experiments with high $\alpha = 2$ (a, b, c, d, e) and 2.5 (f, g, h, i, j). Sediment transport nonlinearity ($\alpha$), critical Shields stress ($\tau^*_{cr}$), segment length ($\Delta L$), and smoothing length ($S_L$) are given in the bottom right of each run. Color indicates network location at time according to color bar.
length $S_L = 100\, \text{m}$ and channel depth $h_c = 1\, \text{m}$ are used in all runs. The evolving boundary can be calculated numerically by the forward Euler scheme:

$$x^{t+\Delta t} = x^t + u_c \Delta t.$$  \hspace{1cm} (17)

For computational stability, we require the maximum progradation distance at a given time step be smaller than the initial segment length. Thus, the interval time step $\Delta t$ must be

$$\Delta t = C_r \frac{\Delta L}{\max(\|u_c\|)}$$ \hspace{1cm} (18)

in which $C_r = 0.5$. Now the channel boundary ($\Gamma_c$) progardes to new location $x^{t+\Delta t}$. If points are spaced too widely or too close together on $x^{t+\Delta t}$, new points are added or existing points are subtracted (see supporting information Text S2). To achieve the constant or specified water discharge, the water depth at channel boundary must correspond to the new domain, which is accomplished through linear scaling described in section 3.2. Then the evolution of channel boundary continues to the next time step. Numerical experiments were run for 50 years ($T_{\text{Final}}$) to be similar to the development time of the modern WLD (Roberts et al., 1980, Shaw, Estep, et al., 2018). $Q$ was set as the median discharge 1985 and 2015 (Figure 2), so no intermittency factor was necessary to relate simulation time and morphological time. The constant $h_{w,0}$ case had $T_{\text{Final}} = 1.5$ years because it grew so quickly.

### 5. Results

#### 5.1. Overview of Model Runs

When the MB_DCN model was applied to a unchannelized initial basin geometry, distributary channel networks invariably formed (Figures 4 and 5). Over the first few time steps, small disparities in network boundary water surface slopes ($\partial h_w/\partial n$) and associated channel network extension rate ($u_c$) grew, with fingers forming at local maxima in $u_c$. As the fingers grew, they came to resemble prograding distributary channels. The largest water surface slopes and nearly all channel extension was focused at distributary channel tips. Smaller $\partial h_w/\partial n$ along channel margins near tips served to widen channels up to a point, but $\partial h_w/\partial n$ trended toward 0 far from channel tips, producing channel boundaries that were eventually stable (locally at equilibrium) regardless of whether the critical Shields stress $\tau_*^c$ was significant or not. Concave regions of the network boundary had the smallest $u_c$, and came to resemble interdistributary bays or islands between the channels.

Within five model years of the initial semicircular domain, between two and seven distributary channels had extended radially into the basin at an apical furcation, the morphology of multiple branches from a single node at the delta apex (Shaw, Miller, & McElroy, 2018; Chamberlain et al., 2018). Through the remaining model duration, some deltas continued the process of channel bifurcation at channel tips. Bifurcation occurred where the channel tip stopped being the local maximum of $u_c$. When this occurred, progradation on either side of the tip quickly made the one-time tip a concave region, with two new tips growing on either side. In some cases, the branches had quasi-symmetric lengths. In other cases, the bifurcations were asymmetric, with one downstream branch extending less than one channel width while the other extended many tens of channel widths. In still other cases, some models never branched again after the apical furcation.

Once a set of radially extending channels were established, a phase of competition between neighboring distributary channels appeared to take hold through the following feedback process: The channels with the furthest distal tips had the largest channel tip progradation rates. Channel tips that were behind their distal neighbors had smaller $u_c$, and subsequently prograded more slowly, making them even more sheltered. Many emergent distributary channels stagnated in this way shortly after they initiated and ceased to grow for the remainder of the model run.

#### 5.2. Influence of Critical Shields Stress $\tau_*^c$ and Sediment Transport Nonlinearity $\alpha$

A suite of deltas were modeled to investigate the influence of critical Shields stress ($\tau_*^c$) and sediment transport nonlinearity ($\alpha$) on the network morphology, holding initial basin and network morphology and
Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$K_3$</th>
<th>$K_4$</th>
<th>$K_5$</th>
<th>$K_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_C$</td>
<td>13.53</td>
<td>1.035</td>
<td>-9.345</td>
<td>13.14</td>
<td>-2.486</td>
<td>0.002263</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.5895</td>
<td>4.83</td>
<td>2.894</td>
<td>2.936</td>
<td>3.43</td>
<td>-2.728</td>
</tr>
<tr>
<td>$W_0$</td>
<td>6.843</td>
<td>0.6622</td>
<td>-8.434</td>
<td>42.8</td>
<td>-2.369</td>
<td>49.41</td>
</tr>
<tr>
<td>$T$</td>
<td>2.0953</td>
<td>0.7160</td>
<td>-1.9998</td>
<td>1.0004</td>
<td>0.2603</td>
<td>-5.1447</td>
</tr>
</tbody>
</table>

The Coefficients of Fitting Function Describing the Morphometric $f$ as a Function of $\tau^*$ and $\alpha$

The number of distributary channels ($N_C$) varied from 2 to 26. A channel was counted if a perturbation along the network boundary was longer than it was wide (i.e., a circle could be circumscribed at a channel tip; Figure 6). Both increasing $\tau^*$ and increasing $\alpha$ tended to reduce the number of channels (Figure 7a). The power law function fit to the modeled channel numbers reveals a critical Shields stress influence of $\tau^*$ and nonlinearity influence of $\alpha^{-2.486}$, showing that $\tau^*$ is particularly important for controlling channel numbers. The modern WLD has 11 deep distributary channels, which is similar to the number predicted for $\tau^* = 0.05 - 0.1$ and $\alpha = 1.5 - 2.0$ in the modeled deltas.

Modeled interdistributary regions had a complex network boundary shape with a rounded shape at channel bifurcations. This contrasts with many natural interdistributary islands that have a pointed upstream tip (Figures 4 and 5). The bifurcation angle between the downstream branches ($\theta$) was measured at each bifurcation by drawing straight lines from the island tip to the channel network boundary (Coffey and Shaw, 2017; Figure 6). The straight lines were set to 200-m length to correspond with the initial channel width. The average angle in the numerical experiments increased primarily with increasing $\alpha$, with a scaling exponent of 3.43, and also increased with $\tau^*$ (exponent 2.894, Figure 7c). On the modern WLD, measured bifurcation angles had a mean of 70.8°. Model runs with $\tau^* < 0.1$ and $\alpha = 1.5$ or $\tau^* > 0.2$ and $\alpha = 1$ had average bifurcation angles were also roughly 72°, consistent with the average $\theta$ for the WLD and in line with the theoretical predictions of Devauchelle et al. (2012).

The width of each channel tip ($W_0$) was measured as the width of one inscribed circle's distance from the channel tip (Figure 6). Similar to the number of channels, the channel width was reduced with increasing $\tau^*$ and increasing $\alpha$, with similar scaling exponents showing $\tau^*$ as the primary control (Figure 8b). The average channel width was generally larger than the smoothing length $\Delta L$ set at 100 m although this width was approached for $\tau^* > 0.2$.

Interestingly, $\alpha$ and $\tau^*$ affected channel tip shape in different ways. For large $\tau^*$ and small $\alpha$, channels tapered to a narrow point (Figure 4e and

Figure 6. Definitions of geometric properties measured in numerical experiments. The lines on the left channel represent the width measurements used to calculate $T$. 

$\text{hydrology constant (Figures 4 and 5). We found that controls on sediment dynamics significantly influenced the number of distributary channels, the bifurcation angle between channels, the channel width, and whether the channel tapered in a modeled network. For each parameter, a power law surface was fit to determine the first-order influences of $\tau^*$ and $\alpha$ on parameter$f$ of the form}$

$$f = K_1 (\tau^* + K_2)^{K_3} + K_4 \alpha^{K_5} + K_6,$$

with $K$ coefficients presented in Table 1. Importantly, there is significant variation within each model run for these morphometric parameters (Figure 7 and 8), but these power law regressions fit the general trend of the data reasonably well and give an empirical prediction of mean channel network characteristics as a function of $\tau^*$ and $\alpha$. 

Interestingly, $\alpha$ and $\tau^*$ affected channel tip shape in different ways. For large $\tau^*$ and small $\alpha$, channels tapered to a narrow point (Figure 4e and
Figure 7. (a) Power law model fit of channel number ($N_C$) from numerical experiments. (b) Model results compared to $N_C$ for Wax Lake Delta. (c) Power law model fit of channel bifurcation angle ($\theta$) from numerical experiments. (d) Model results compared to average $\theta$ for Wax Lake Delta (error bars are one standard deviation). The dashed line is the theoretical prediction $\theta = 72^\circ$. See Table 1 for fitting coefficients.

similar), while for large $\alpha$ and small $\tau_{cr}^*$, channels maintained a relatively constant width up to the channel tip (Figure 5b, and similar). We quantitatively define channel taper ($T$) as

$$T = \frac{L}{W_0} \frac{dW}{dx},$$

where $dW/dx$ is the linear fit of width change with distance toward a channel tip and $L$ was the channel length (see Figure 6). Regression shows that $\tau_{cr}^*$ is the primary control on mean $T$ for a given model run, although there is significant variation among distributary channels for each model run. The taper measured at 11 distributary channels on the WLD had a mean of $T = -1.37$, with significant standard deviation. This corresponds roughly with model runs with $\tau_{cr}^* = 0.2$.

5.3. Influence of Discharge
The constant water discharge case (Figure 4g) is compared to a constant depth difference (constant $\Delta h_w$) and specified discharge case (Figure 9) to investigate controls on progradation rate, quantified as $\bar{u}_c$ of the furthest point from the initial semiring channel network boundary at each time step. Given their similar initial conditions, the initial progradation rate began with similarly large magnitudes ($10^4$ m/year) and decreased rapidly. The progradation rate of the constant water depth case stabilized at about 4,000 m/year, and the discharge increased exponentially to values that were unrealistic for the WLD ($\sim 10,000$ m$^3$/year; Figure 10b). The constant discharge case and linearly increasing discharge cases both decreased rapidly to growth rates on the order of 200 m/year after just 5 years. From this point, progradation rates decreased exponentially for the constant discharge case to $<30$ m/year at $t = 50$ years (Figure 10d). The linearly increasing discharge case, in contrast, stabilized its growth rate, and remained between 80 and 120 m/year from $t = 10–50$ years. This fell in the range of progradation rates measured at the WLD (69–116 m/year) over the past several decades (Figure 10d).
6. Discussion

6.1. A New Conceptual Model for Distributary Network Growth and Dynamics

The MB_DCN model proposed here describes complex distributary channel networks as controlled by three factors: (a) the shape of the channel network boundary and receiving basin, (b) upstream water discharge and Laplace’s equation that control shear stress along the channel margin, and (c) the sediment transport relation controlling erosion along the channel network boundary. Flow patterns are a nonlocal function of the geometry of the entire distributary network and basin boundary. For any boundary configuration and applied $Q$ where $\tau^* > \tau^*_c$ at some boundary section, this section is in disequilibrium and must evolve through channel widening or progradation. Distributary channels extend because the $\tau^*$ at their tips is far larger.

![Figure 8](image1.png)

Figure 8. (a) The fitting results of channel tip width ($W_0$). (b) The averaged channel width compared to the average channel tip width of the Wax Lake Delta (error bars are one standard deviation). (c) The fitting results of channel taper ($T$). (d) The averaged channel taper compared to the average taper on the Wax Lake Delta (error bars are one standard deviation). See Table 1 for fitting coefficients.

![Figure 9](image2.png)

Figure 9. The results of numerical experiments varying the hydrodynamic conditions at the channel network boundary. (a) The linear increasing water discharge. (b) The constant water depth of distributary channel boundary. Compare to Figure 4g for the constant $Q$ case.
than along their lateral margins. Distributary channels branch when $\tau^*$ has two maxima along the network boundary instead of one. Channel progradation at a given channel tip can also be reduced if neighboring channels that are more distal cause shear stress at its tip to be reduced. Concave regions of the network quickly cease all progradation and become stable.

The primary differences between the jet model (reviewed in section 1) and the network boundary model lie in their objectives and what the model considers to be simple and complex. The jet model is designed to characterize initial fluvially derived depositional patterns in front of a discrete channel mouth, while the network boundary model is designed to characterize the evolution of a delta deposit with well-developed channels and delta front. In terms of complexity, the jet model assumes a simple channel mouth and expects variation to stem from complex hydrodynamics and sediment dynamics controlled by the channel mouth morphology and discharge. In contrast, the network boundary model assumes that Laplace’s equation adequately describes the hydrodynamics and that complexity is driven by the network and basin geometry and the sediment transport equation.

6.1.1. Channel Network and Receiving Basin

The shape of the distributary channel network sets the flow field and therefore is the primary control on evolution at any given time. Even for a channel tip of set geometry, the proximity and relative location of neighboring channels helps set the growth rate for a channel tip and whether branches will form. Nearly all channel extension is focused at distributary channel tips that are the most distal within the network. The width of these channels is set by $\alpha$ and $\tau^*_{*}$ in the sediment transport relation, and narrower channels tend to extend and wider channels tend to branch, similar to the predictions of previous models (Canestrelli et al., 2014; Falcini & Jerolmack, 2010; Rowland et al., 2010). Upstream of these channel tips, the water surface slopes are greatly reduced, preventing channels from extending or initiating. The application of Laplace’s equation allows shear stresses to be controlled by only the discharge and the network geometry. Discharge in

Figure 10. (a) The variation of water elevation of the three model runs with varying hydrology (see section 4.2). (b) Discharge ($Q$) time series. Note that for the constant water level model, $T_{\text{final}} = 1.5$ years, while $T_{\text{final}} = 50$ years for the other two runs. (c) The distance of the furthest point on the network over time. (d) The progradation rate of channel boundary. Note that the range of progradation rates estimated for the Wax Lake Delta is shaded in gray.
6.1.2. Hydrodynamics

Water discharge ($Q$) is linearly related to shear stress on the network boundary through equations (10) and (11). Simulations with constant $Q$ showed a reduction in progradation rate to unreasonably small magnitudes for the WLD ($<30$ m/year). The sensitivity to input discharge within the model is because discharge is distributed across the network boundary which grows quasi-exponentially in length. By applying the gradually increasing fluid discharge measured upstream of the WLD (Figure 2), we found that the constant progradation rates measured over decades in the system were well characterized. The constant $\Delta h_{in}$, commonly applied to viscous fingering studies (Praud & Swinney, 2005) forced discharges that increased unreasonably over time in the moving boundary model (Figures 9b and 10). This condition appears inappropriate for modeling delta networks. Further study is required to improve the understanding of how discharge changes influence network morphology and growth. However, the close relationship between progradation rate and discharge suggests that information about a distributary network’s growth could be preserved within the network’s structure.

Laplace’s equation is a strong simplification of the shallow water equations. In particular, the linearization of friction (equation (7)) introduces some error relative to quadratic friction models that are more realistic. Future work investigating the differences in complex network boundary dynamics under linear and quadratic friction models may reveal important nonlinear hydrodynamic control that augment these results. Even so, the branching of the linear-friction channel network described here occurred spontaneously, with emergent properties that resembled the field prototype. Therefore, we conclude that the simplified hydrodynamics described by Laplace’s equation are sufficient to produce realistic distributary channel dynamics.

6.1.3. Sediment Transport Control

The details of the model’s sediment transport formula were shown to have a strong impact on the network morphology. Increasing sediment transport nonlinearity ($\alpha$) tended to produce fewer channels with increased branching angles and quasi-uniform widths ($T > 0$), while an increased $\tau^*_{cr}$ produced fewer, narrower channels that tapered to their tip ($T < 0$). Increases in $\tau^*_{cr}$ and $\alpha$ influence the network in the same qualitative way, because $\tau^*_{cr}$ nullifies small values of sediment transport ($q_c$) relative to a linear relation, while $\alpha > 1$ reduces it for small $\tau^*_{cr}$ relative to large $\tau^*_{cr}$ (Figure 11). Positive values of $\tau^*_{cr}$ cause progradation to increase on the margins of channels, producing tapering channels. In contrast, $\alpha > 1$ merely makes the channel widening process slow but eventually produces channels with relatively uniform widths.

The empirical analyses of network geometry are consistent with many previous analyses of distributary networks and tidal networks. Several modeling studies using Delft3D, a grid-based model that explicitly resolves the depth-averaged flow field, sediment transport of multiple grain sizes, and bed evolution, have shown that increasing critical Shields stress leads to fewer active channels on a delta (Burpee et al., 2015; 2015; Caldwell & Edmonds, 2014). The numerical experiments showed that channel bifurcations occurred when two local maxima of $u_c$ occurred on a single channel tip. Tidal network models controlled by the unsteady Poisson equation instead of Laplace’s equation also reveal a reduction in branching with increasing $\tau^*_{cr}$ (D’Alpaos et al., 2005; Fagherazzi & Sun, 2004). We cannot yet provide an analytical explanation for the reduced branching in networks with large $\alpha$ and $\tau^*_{cr}$ to compare to solutions for the linear case (Chuoke et al., 1959; Lajeunesse & Couder, 2000; Saffman & Taylor, 1958). However, it is these nonlinearities that increase $u_c$ at the channel tip relative to the channel margins, hindering the branching process.

The sediment transport parameters of $\tau^*_{cr} \approx 0.1$ and $\alpha = 1.5–2$ proved the best for modeling channel number, width, bifurcation angle and channel taper of the WLD, although predictions based on each network metric varied. These values are reasonable given our knowledge of bed material sediment transport near the threshold of motion with minimal cohesion, as exemplified by the Meyer-Peter and Muller sediment transport formula for very fine sand, or the Wong and Parker (2006) formulation. Very nonlinear ($\alpha = 2.5$)
models that are used for fully entrained sediments appear to be an unreasonable comparison. Despite this first-order resemblance, sediment transport models are generally developed for a relatively flat bed and uniform flows, which are not the case at a rapidly shallowing channel network boundary. Given the importance of the sediment transport formula, we recommend direct measurements of sediment transport at the channel network boundary to remove an important degree of freedom from future modeling efforts. With further validation and fine-tuning of metrics, it is conceivable that network structure characterized in geologic or planetary remote sensing could be used to estimate the sediment transport parameters in deltas where these data are unavailable.

This study shows that distributary channel networks are a member of the broad family of branching processes well described by diffusion of a scalar field (Laplace’s equation) coupled to a moving boundary (Couder, 2001). However, the boundary evolution in most previously described cases is a simple linear relation between stress and boundary evolution, because mass conservation on one side of the interface requires linear movement as a function of the gradient. This is identical to the sediment transport conditions of $\tau_{cr}^* = 0$ and $\alpha = 1$ (Figure 4a), and we note the qualitative resemblance of this run to viscous fingering studies (e.g., Lajeunesse and Couder, 2000). However, in the case of distributary channel networks, water is passing over the network boundary eroding sediment as it goes, which means sediment mass conservation on one side of the boundary is not required. The nonlinear boundary evolution investigated here exerts a previously unexplored control on the geometry of the one such fingering process. The channelization is qualitatively reminiscent of groundwater-fed tributary network geometries, which are also described by Laplace’s equation coupling a channelized and unchannelized (groundwater) domain. These networks must also involve nonlinear sediment transport laws, and these nonlinearities could have important influence on the geometric characteristics such as channel spacing and channel width (Petroff et al., 2013; Seybold et al., 2016, 2018). We urge further study of nonlinear interface movement to further generalize moving boundary models of processes controlled by Laplace’s equation.

**6.1.4. Second-Order Controls**

The simplicity of MB_DCN gives indications of the primary controls on network formation. Depositional feedbacks and sediment discharge are notably absent from the model and can therefore be considered of second-order in this formulation. Our exploratory approach cannot rule out the possibility that these (or other) aspects of the system could dominate controls on network structure. We discuss these second-order controls here.

In the jet model, it was argued that a depositional obstacle or channel confinement provided by a mouth bar or levees controlled the manner in which a distributary channel branched or extended (Wright, 1977). The moving boundary model results presented here assume a delta front of uniform elevation, and yet the propagating channel network boundary still produces branching patterns as well as reaches that do not branch for great lengths ($\sim 10$ channel widths for the run in Figure 4g). In reality, channel erosion and delta front deposition occur simultaneously, and mouth bars would enhance spreading at channel tips and levees would reduce lateral flow, reinforcing the flow patterns modeled by MB_DCN in each case. However, the model results suggest that these depocenters need not be significant and that prediction of branching or channel extension should focus on boundary erosion rather than deposition.

Sediment discharge to the delta is not involved in the moving boundary model. This is surprising because sediment discharge is a necessity when modeling the volume of a delta deposit over time. However, it is not necessary here because a large delta front is assumed, and the channel network itself is devoid of sediment. The sedimentary volume of a delta must be controlled in part by sediment input, so it is conceivable that channel network growth and deposit accumulation could be somewhat decoupled on river deltas. On one hand, a channel network could outgrow the delta front if sediment discharge was too small relative to channel progradation rates. The contrast between constant channel progradation rates and a gradually declining marsh area creation rate on the WLD (Shaw, Estep, et al., 2018) is potentially the result of this scenario. On the other hand, the delta front could become clogged with sediment if the sediment discharge was too large compared to the network progradation rates. We imagine that a clogged delta front would bear resemblance to the morphodynamic backwater (Edmonds et al., 2009; Hoyal & Sheets, 2009), where deposition within a delta network is the first step in the process of channel abandonment and avulsion (Reitz & Jerolmack, 2012). We do not attempt to model these feedbacks here, although a moving far-field boundary could be added to an updated version of MB_DCN. However, we note that prograding distributary channel networks may grow independently of delta accumulation or deposit progradation in certain cases.
7. Conclusions

We present a model of distributary channel network growth on river deltas that reproduces key features of natural distributary networks despite its simplicity. The shape of the channel network boundary is the only morphological information that evolves, suggesting that this shape is the primary control on future growth. The simplification of shallow water flow to Laplace’s equation on the delta front shows that shear stress patterns are directly related to geometry and upstream discharge and highlights the essential feedbacks between protruding channel tips that extend and concave interdistributary regions that are stable. This model produces a spontaneously branching network, although a smoothing length associated with an equilibrium channel width-to-depth ratio is required to limit arbitrarily small branches. The morphology of networks produced with this model is controlled by the nonlinear sediment transport relationship that converts shear stress along the boundary into erosion and network extension. A threshold for sediment motion ($\tau^{\ast}_{cr}$) controls channel width, the number of channel bifurcations, and the taper of channel tips. Sediment transport nonlinearity ($\alpha$) influences channel bifurcation angle and is a second-order control on channel width. Comparison to the Wax Lake Delta suggests that common values of these terms produce a network with similar geometric features. A gradually increasing water discharge as measured upstream of the Wax Lake Delta produces reasonable channel tip progradation rates. Flow-altering sedimentary depocenters and sediment discharge to the delta were not included in the model and deemed of secondary importance for understanding the emergent networks. The controls on channel network dynamics demonstrated here may improve predictions of future channel network growth or aid in the interpretation of channel networks where only the structure is preserved.

Notation

\begin{align*}
\alpha & \quad \text{Sediment transport nonlinearity (–)} \\
C & \quad \text{Sediment transport coefficient (–)} \\
C_f & \quad \text{Dimensionless friction coefficient (–)} \\
C_t & \quad \text{Numerical stability condition (–)} \\
D & \quad \text{Characteristic grain size (L)} \\
\Delta & \quad \text{Dirac delta function (–)} \\
Fr^2 & \quad \text{Froude number squared (–)} \\
g & \quad \text{gravitational acceleration (L/T^2)} \\
G & \quad \text{Green function (–)} \\
\Gamma_c & \quad \text{Channel boundary (–)} \\
\Gamma_w & \quad \text{No-flux boundary (–)} \\
\Gamma_\infty & \quad \text{Far-field boundary (–)} \\
h_c & \quad \text{Channel depth (L)} \\
h_w & \quad \text{Water depth (L)} \\
h_{w,0} & \quad \text{Initial water depth of channel boundary (L)} \\
h_{w,\infty} & \quad \text{Water depth of far-field boundary (L)} \\
H & \quad \text{Depth scale (L)} \\
K_i, i = 1 \sim 6 & \quad \text{Fitting coefficient (–)} \\
\Delta L & \quad \text{Segment length (L)} \\
L & \quad \text{Length scale (L)} \\
h & \quad \text{Normal vector of channel boundary (–)} \\
N_C & \quad \text{Channel number (–)} \\
w_k & \quad \text{Weight of Gaussian quadrature (–)} \\
\omega & \quad \text{Tidal frequency (L)} \\
\Omega & \quad \text{Flow domain (–)} \\
q_c & \quad \text{Sediment flux (L^2/T)} \\
q_w & \quad \text{Water flux along the channel boundary (L^2/T)} \\
Q & \quad \text{Water discharge (L^3/T)} \\
r & \quad \text{Radial distance (L)} \\
R & \quad \text{Submerged specific gravity (–)} \\
R_0 & \quad \text{Radius of initial channel boundary (L)}
\end{align*}
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