

3-7-2008

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### Recommended Citation

R. M. A. Azzam and F. F. Sudradjat, "Reflection coefficients of p- and s-polarized light by a quarter-wave layer: explicit expressions and application to beam splitters," *Appl. Opt.* 47, 1103-1108 (2008).

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# Reflection coefficients of $p$ - and $s$ -polarized light by a quarter-wave layer: explicit expressions and application to beam splitters

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Received 31 October 2007; accepted 13 December 2007;  
posted 9 January 2008 (Doc. ID 89186); published 7 March 2008

The complex-amplitude reflection coefficients of  $p$ - and  $s$ -polarized light by a transparent freestanding, embedded, or deposited quarter-wave layer (QWL) are derived as explicit functions of the angle of incidence and layer refractive index. This provides the basis for the design of 50%–50% beam splitters for incident  $s$ -polarized or unpolarized light that use a high-index (e.g., TiO<sub>2</sub> or Ge) QWL embedded in a glass cube in the visible and near infrared spectral range. These simple devices have good angular and spectral response and are insensitive to small film thickness errors to the first order. © 2008 Optical Society of America

OCIS codes: 230.1360, 260.5430, 310.6860.

## 1. Introduction

The reflection of linearly polarized light, with the electric field vector parallel ( $p$ ) and perpendicular ( $s$ ) to the plane of incidence, by a film–substrate system, Fig. 1, is governed by complex-amplitude reflection coefficients  $R_p$  and  $R_s$  that are given by

$$R_\nu = \frac{r_{01\nu} + r_{12\nu}X}{1 + r_{01\nu}r_{12\nu}X}, \quad \nu = p, s, \quad (1a)$$

$$X = \exp(-j2\pi d/D_\phi). \quad (1b)$$

In Eqs. (1a) and (1b)  $r_{01\nu}$ ,  $r_{12\nu}$  are the Fresnel reflection coefficients at the ambient–film (01) and film–substrate (02) interfaces for the  $\nu$  polarization,  $d$  is the metric film thickness,

$$D_\phi = (\lambda/2)(\epsilon_1 - \epsilon_0 \sin^2 \phi)^{-1/2} \quad (2)$$

is the film-thickness period,  $\lambda$  is the vacuum wavelength of light,  $\phi$  is the angle of incidence in medium 0, and  $\epsilon_0$ ,  $\epsilon_1$  are the wavelength-dependent dielectric functions of the transparent ambient and film, respectively. All media are assumed to be optically isotropic, homogeneous, and separated by parallel-plane boundaries. The reflection coefficients of Eqs. (1a) and (1b) play a key role in ellipsometry [1], and their properties have been studied in detail [1–3].

The special case of a freestanding or embedded layer, when the ambient and substrate are the same, and the layer thickness is one or an odd multiple of the quarter-wave optical thickness at oblique incidence, i.e.,

$$\epsilon_2 = \epsilon_0, \quad (3)$$

$$d = D_\phi/2 = (\lambda/4)(\epsilon_1 - \epsilon_0 \sin^2 \phi)^{-1/2}, \quad (4)$$

is of particular interest. Quarter-wave layers (QWLs) or sections are widely used in optical interference coatings [4] and transmission lines [5].

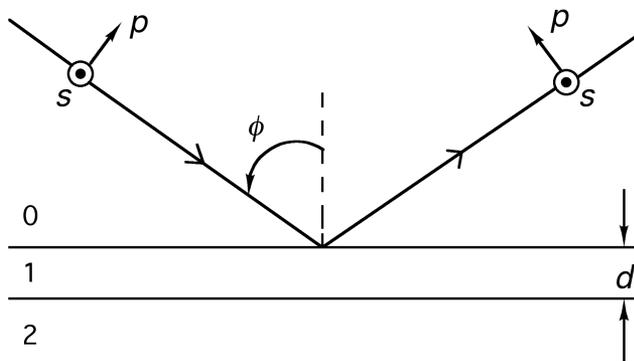


Fig. 1. Reflection of  $p$ - and  $s$ -polarized light at an angle  $\phi$  by a film-substrate system. Media 0, 1, and 2 are the ambient, film, and substrate, respectively, and  $d$  is the film thickness.

In Section 2 explicit expressions are derived for  $R_p$  and  $R_s$  as functions of the angle of incidence  $\phi$  and the layer-to-ambient relative dielectric function:

$$\varepsilon = \varepsilon_1/\varepsilon_0. \quad (5)$$

These expressions provide the basis for the design of 50%–50% beam splitters (BSs) for incident  $s$ -polarized or unpolarized light that use a high-index QWL, which is embedded in a glass cube ( $\phi = 45^\circ$ ). In Section 3 an  $s$ -polarization BS is described that uses a  $\text{TiO}_2$  thin film embedded in a glass cube for visible light. In Section 4 a near-IR BS for incident unpolarized light is presented that uses a Ge QWL embedded in an N-LAK12 Schott-glass cube. The angular, spectral, and film-thickness sensitivities of these designs are evaluated. In Section 5 the reflection coefficients of a QWL that is deposited on a substrate are obtained in terms of (1) the corresponding reflection coefficients of the same QWL embedded in the ambient, and (2) the ambient-substrate Fresnel reflection coefficient evaluated at the same angle of incidence. Section 6 is a brief summary of the paper.

## 2. Complex-Amplitude Reflection Coefficients $R_p$ and $R_s$ of an Embedded Quarter-Wave Layer

When the ambient and substrate have the same optical properties, Eq. (3), we have

$$r_{12\nu} = r_{10\nu} = -r_{01\nu}, \quad \nu = p, s. \quad (6)$$

For a QWL ( $d = D_\phi/2$ ), Eq. (1b) gives

$$X = -1. \quad (7)$$

Substitution of Eqs. (6) and (7) in Eq. (1a) gives

$$R_\nu = \frac{2r_{01\nu}}{1 + r_{01\nu}}, \quad \nu = p, s. \quad (8)$$

The interface Fresnel coefficient  $r_{01\nu}$  of partial external reflection for the  $\nu$  polarization can be written as the tangent of an angle:

$$r_{01\nu} = \tan \alpha_\nu, \quad \nu = p, s, \quad -45^\circ \leq \alpha_p \leq 45^\circ, \\ -45^\circ \leq \alpha_s \leq 0. \quad (9)$$

The ranges of  $\alpha_p$ ,  $\alpha_s$  given by Eq. (9) are consistent with the Nebraska-Muller conventions [1,6], which we adopt here. From Eqs. (8) and (9), we obtain

$$R_\nu = \sin(2\alpha_\nu), \quad \nu = p, s. \quad (10)$$

Equation (10) is probably the simplest form of the reflection coefficients of an embedded QWL for the  $p$  and  $s$  polarizations at oblique incidence.

Alternatively, substitution of the standard Fresnel reflection coefficients

$$r_{01p} = \frac{\varepsilon \cos \phi - (\varepsilon - \sin^2 \phi)^{1/2}}{\varepsilon \cos \phi + (\varepsilon - \sin^2 \phi)^{1/2}}, \quad (11)$$

$$r_{01s} = \frac{\cos \phi - (\varepsilon - \sin^2 \phi)^{1/2}}{\cos \phi + (\varepsilon - \sin^2 \phi)^{1/2}}, \quad (12)$$

in Eq. (8) yields

$$R_p = \frac{\varepsilon^2 \cos^2 \phi - \varepsilon + \sin^2 \phi}{\varepsilon^2 \cos^2 \phi + \varepsilon - \sin^2 \phi}, \quad (13)$$

$$R_s = \frac{1 - \varepsilon}{\cos 2\phi + \varepsilon}. \quad (14)$$

Equations (13) and (14) are the desired explicit expressions of the reflection coefficients for the  $p$  and  $s$  polarizations of a freestanding or embedded QWL in terms of the layer's relative dielectric function  $\varepsilon$  and angle of incidence  $\phi$ .

At normal incidence,  $\phi = 0$ , Eqs. (13) and (14) simplify to

$$R_p(0) = -R_s(0) = \frac{\varepsilon - 1}{\varepsilon + 1}. \quad (15)$$

To attain 50% reflectance for incident light of any polarization by an embedded QWL or pellicle at and near normal incidence, we set  $R_s(0) = -1/\sqrt{2}$  in Eq. (15). This gives

$$\varepsilon = (\sqrt{2} + 1)^2, \quad n = (\sqrt{2} + 1) = 2.4142. \quad (16)$$

The refractive index given by Eq. (16) is that of a diamond or ZnSe [7] pellicle in air for visible light. It is also the relative index of a Si layer embedded in  $\text{SiO}_2$  in the near-IR [8].

In the limit of the grazing incidence,  $\phi = 90^\circ$ , Eqs. (13) and (14) reduce to  $R_p(90^\circ) = R_s(90^\circ) = -1$  independent of  $\varepsilon$ .

At  $\phi = 45^\circ$ , Eqs. (13) and (14) become

$$R_p(45^\circ) = \frac{\varepsilon^2 - 2\varepsilon + 1}{\varepsilon^2 + 2\varepsilon - 1}, \quad (17)$$

$$R_s(45^\circ) = -1 + (1/\varepsilon). \quad (18)$$

Equation (18) indicates that  $R_s(45^\circ)$  of an embedded QWL differs from  $-1$  by the reciprocal of the relative dielectric function  $\varepsilon$  and provides one of the simplest explicit relations between an external reflection property and an intrinsic optical property. Figure 2 shows a family of curves of the function  $R_p(\phi, \varepsilon)$  [Eq. (13)] versus  $\phi$  for constant values of the relative refractive index  $n = \sqrt{\varepsilon}$  from 1.5 to 6.0 in steps of 0.5. Note that  $R_p = 0$  (dashed line) at the Brewster angle  $\phi_B = \tan^{-1}n$  of the 01 interface. Figure 3 shows the corresponding family of curves of the function  $R_s(\phi, \varepsilon)$  [Eq. (14)] for the  $s$  polarization. In Fig. 3 it is evident that Eq. (18) is satisfied at point  $P$  on the  $n = 2$  curve at  $\phi = 45^\circ$ .

### 3. Visible Beam Splitter for $s$ -Polarized Light using a $\text{TiO}_2$ Quarter-Wave Layer Embedded in a Glass Cube

BSs for  $s$ -polarized light are important in interferometry, holography, and the recording of fiber-Bragg gratings. The desired 50%–50% split can be obtained by light reflection at the Brewster angle of an air-dielectric or dielectric–dielectric interface [9]. To attain 50% reflectance for incident  $s$ -polarized light by a QWL embedded in a cube, Fig. 4,  $R_s(45^\circ) = -1/\sqrt{2}$  is substituted in Eq. (18). This gives

$$\varepsilon = \sqrt{2} + 2, \quad n = 1.84776. \quad (19)$$

The relative refractive index of Eq. (19) is achievable in the visible by a  $\text{TiO}_2$  layer embedded in a glass

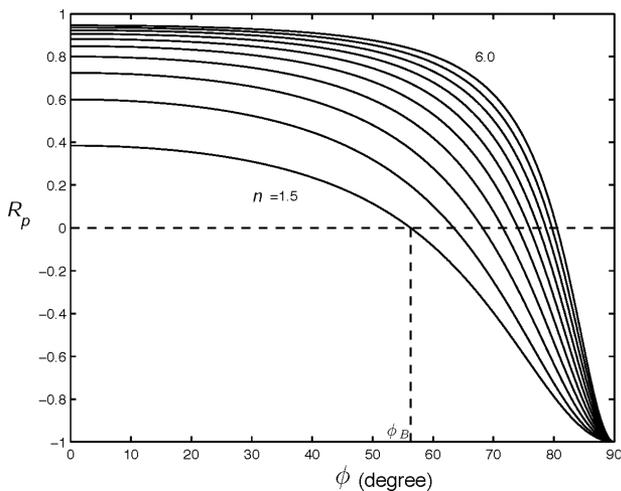


Fig. 2. Family of curves of the reflection coefficient  $R_p(\phi, \varepsilon)$  of a QWL for the  $p$  polarization, Eq. (13), as a function of the angle of incidence  $\phi$  for constant values of layer-to-ambient relative refractive index  $n = \sqrt{\varepsilon}$  from 1.5 to 6.0 in steps of 0.5. Note that  $R_p = 0$  (dashed line) at the Brewster angle  $\phi_B = \tan^{-1}n$  of the 01 interface.

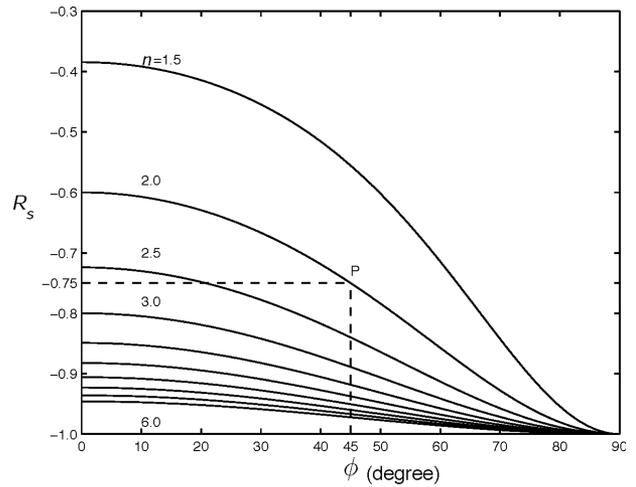


Fig. 3. Family of curves of the reflection coefficient  $R_s(\phi, \varepsilon)$  of a QWL for the  $s$  polarization, Eq. (14), as a function of the angle of incidence  $\phi$  for constant values of layer-to-ambient relative refractive index  $n = \sqrt{\varepsilon}$  from 1.5 to 6.0 in steps of 0.5. Note that Eq. (18) is satisfied at point  $P$  on the  $n = 2$  curve at  $\phi = 45^\circ$ .

( $\text{SiO}_2$ ) cube. Single-crystal  $\text{TiO}_2$  is uniaxially anisotropic, and the dispersion of its ordinary ( $o$ ) and extraordinary ( $e$ ) refractive indices is given by [10]

$$n_i^2 = A_i + \frac{B_i}{\lambda^2 - C_i}, \quad i = o, e. \quad (20)$$

In Eq. (20)

$$(A_o, B_o, C_o) = (5.913, 0.2441, 0.0803),$$

$$(A_e, B_e, C_e) = (7.197, 0.3322, 0.0843),$$

for the  $o$  and  $e$  indices, respectively, and the wavelength  $\lambda$  is in the range  $0.425 \leq \lambda \leq 1.5 \mu\text{m}$ . A vacuum-deposited thin film is polycrystalline with random orientation of the optic axis and hence is essentially optically isotropic with an average refractive index that can be approximated by  $n_a = (n_o + n_e)/2$ . The average of the two principal indices of  $\text{TiO}_2$  is fitted

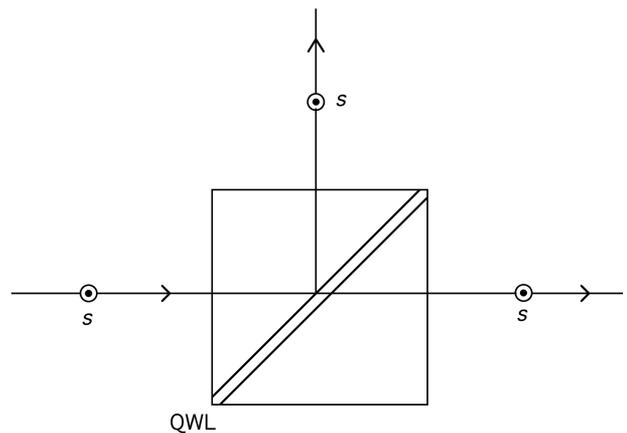


Fig. 4. Cross section of an  $s$ -polarization BS that uses a high-index QWL embedded in a cube.

accurately by a dispersion formula of the same form as Eq. (20) with coefficients given by

$$(A_a, B_a, C_a) = (6.5390, 0.2866, 0.08252), \quad (21)$$

and residual root-mean-square error of  $\leq 2.2 \times 10^{-6}$ .

Fused-silica ( $\text{SiO}_2$ ) also has a dispersion relation of the form of Eq. (20) with coefficients [11]:

$$(A, B, C) = [1, 1.09877, (0.0924317)^2]. \quad (22)$$

The relative dielectric function  $\epsilon = n^2(\text{TiO}_2)/n^2(\text{SiO}_2)$  is calculated using Eqs. (20)–(22) and is plotted versus  $\lambda$  in the spectral range  $0.425 \leq \lambda \leq 1.5 \mu\text{m}$  in Fig. 5.

The dashed lines in Fig. 5 show that  $\epsilon = \sqrt{2} + 2$ ,  $n = 1.84776$  at  $\lambda = 704.6 \text{ nm}$ . The refractive indices of  $\text{TiO}_2$  and  $\text{SiO}_2$  at this wavelength are 2.6891 and 1.4553, respectively, and the QWL thickness is 70.9 nm.

The wavelength at which an embedded  $\text{TiO}_2$  QWL achieves 50%–50% split for incident  $s$ -polarized light at  $\phi = 45^\circ$  can be shifted downward by replacing the fused-silica prism with an appropriate Schott glass. This is also shown in Fig. 5, where fused silica is replaced by N-FK5 Schott glass. The published dispersion relation for this glass [12] is used to generate the second curve in Fig. 5, which intersects the line  $\epsilon = \sqrt{2} + 2$  at  $\lambda = 605.4 \text{ nm}$ . The refractive indices of  $\text{TiO}_2$  and N-FK5 Schott glass at this wavelength are 2.7474 and 1.4869, respectively, and the QWL thickness is 59.6 nm. The angular, spectral, and film-thickness sensitivity of the  $s$ -polarization BS that uses a  $\text{TiO}_2$  QWL layer embedded in a fused silica cube is now considered.

Figure 6 shows the intensity reflectances  $\mathcal{R}_\nu = R_\nu^2$  ( $\nu = p, s, a$ ) for incident  $p$ - and  $s$ -polarized light, and their average as the internal angle of incidence  $\phi$  is increased from  $40^\circ$  to  $50^\circ$ . In this calculation the

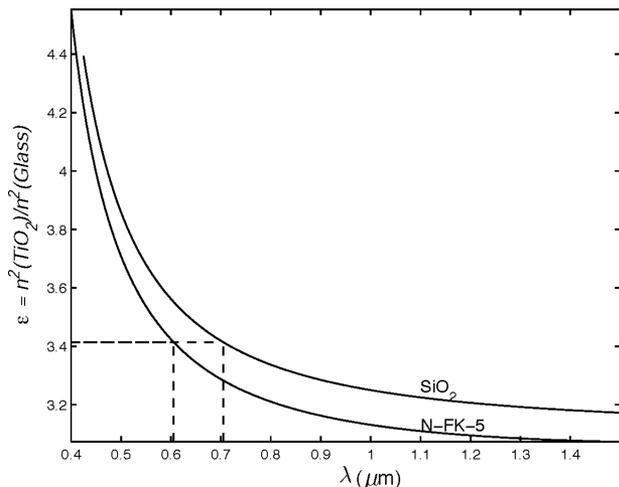


Fig. 5. Relative dielectric function  $\epsilon = n^2(\text{TiO}_2)/n^2$  (glass) is plotted versus wavelength  $\lambda$  in the spectral range  $0.4 \leq \lambda \leq 1.5 \mu\text{m}$  for a  $\text{TiO}_2$  layer embedded in fused silica ( $\text{SiO}_2$ ) and N-FK5 Schott glass. The dashed lines show that  $\epsilon = \sqrt{2} + 2$ ,  $n = 1.84776$  at  $\lambda = 704.6 \text{ nm}$  and  $\lambda = 605.4 \text{ nm}$  for the  $\text{SiO}_2$  and N-FK5 Schott glass substrates, respectively.

wavelength and metric film thickness are held constant at  $\lambda = 704.6 \text{ nm}$ ,  $d = 70.9 \text{ nm}$ . In Fig. 6 the  $s$  reflectance increases approximately linearly at the rate of 1% per degree change of  $\phi$ . The reflectance for the  $p$  polarization decreases at almost the same rate, which leaves the average reflectance nearly constant with respect to  $\phi$  at  $\approx 30\%$ . Therefore, the device performs as a wide-angle 30%–70% BS for incident unpolarized light.

Figure 7 shows the reflectance response over the 650–750 nm wavelength range. Here the angle of incidence and metric film thickness are constant ( $\phi = 45^\circ$ ,  $d = 70.9 \text{ nm}$ ),  $\lambda$  is varied, and the material dispersion is taken into consideration. In Fig. 7 all reflectances are nearly constant with respect to  $\lambda$ ;

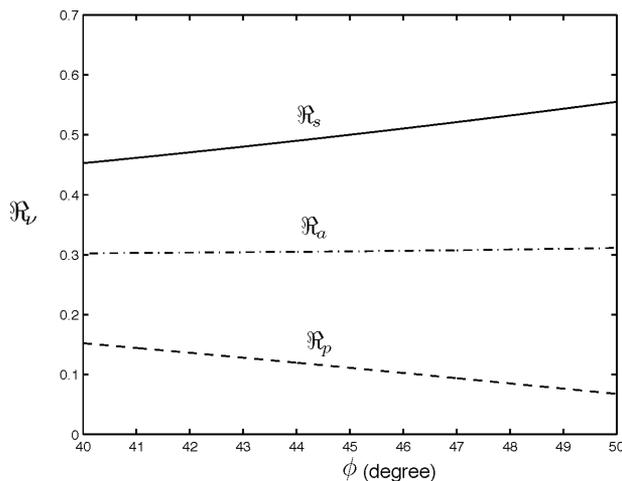


Fig. 6. Intensity reflectances  $\mathcal{R}_\nu = R_\nu^2$  ( $\nu = p, s, a$ ) for incident  $p$ - and  $s$ -polarized light and their average as functions of the internal angle of incidence  $\phi$  from  $40^\circ$  to  $50^\circ$  of an  $s$ -polarization BS that uses a QWL of  $\text{TiO}_2$  embedded in a fused-silica cube. The wavelength of light and the metric thickness of the  $\text{TiO}_2$  thin film are kept constant at  $\lambda = 704.6 \text{ nm}$  and  $d = 70.9 \text{ nm}$ , respectively.

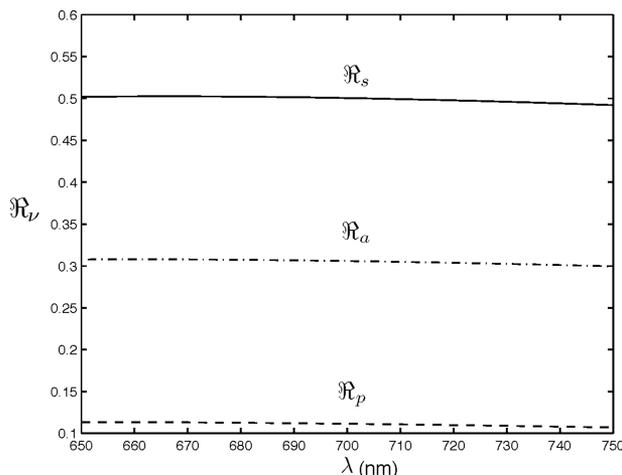


Fig. 7. Intensity reflectances  $\mathcal{R}_\nu = R_\nu^2$  ( $\nu = p, s, a$ ) for incident  $p$ - and  $s$ -polarized light and their average as functions of wavelength  $\lambda$  from 650 to 750 nm of an  $s$ -polarization BS that uses a QWL of  $\text{TiO}_2$  embedded in a fused-silica cube. The angle of incidence and the metric thickness of the  $\text{TiO}_2$  thin film are fixed at  $\phi = 45^\circ$  and  $d = 70.9 \text{ nm}$ , respectively.

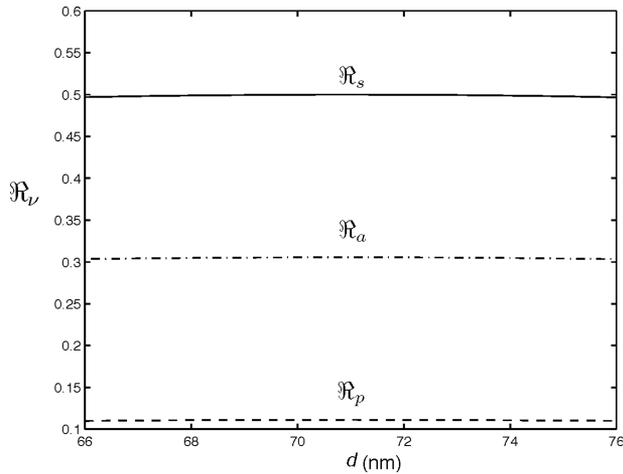


Fig. 8. Intensity reflectances  $\mathcal{R}_\nu = R_\nu^2$  ( $\nu = p, s, a$ ) for incident  $p$ - and  $s$ -polarized light and their average as functions of the thickness  $d$  of a  $\text{TiO}_2$  layer, which is embedded in a fused-silica cube. The thickness  $d$  is varied by  $\pm 5$  nm around 71 nm, while the angle of incidence  $\phi = 45^\circ$  and wavelength  $\lambda = 704.6$  nm are fixed. The device performance as a 50%–50%  $s$ -polarization BS is essentially independent of small film thickness changes to first order.

hence the device is essentially achromatic over the 650–750 nm range.

Figure 8 shows the effect of shifting the film thickness  $d$  by  $\pm 5$  nm around 71 nm, while keeping  $\phi$  and  $\lambda$  fixed at their designated values. The reflectance shifts in Fig. 8 are of second order (quadratic with respect to  $\Delta d$ ) and negligible, which is expected for operation near the quarter-wave condition. The angular, spectral, and film-thickness sensitivity of the  $s$ -polarization BS that uses a  $\text{TiO}_2$  QWL embedded in N-FK5 Schott-glass cube is similar to that described above and is not repeated here.

#### 4. Infrared Beam Splitter for Incident Unpolarized Light using a Ge Quarter-Wave Layer Embedded in a Glass Cube

To achieve 50% reflectance for incident unpolarized light by an embedded QWL at  $\phi = 45^\circ$ , it is required that

$$R_s^2(45^\circ) + R_p^2(45^\circ) = 1. \quad (23)$$

Substitution of Eqs. (17) and (18) into Eq. (23) leads to a sixth-degree equation:

$$\zeta^6 - 16\zeta^4 - 48\zeta^3 - 52\zeta^2 - 24\zeta - 4 = 0, \quad \zeta = \epsilon - 1, \quad (24)$$

which has only one acceptable root:

$$\zeta = 5.2223, \quad \epsilon = 6.2223. \quad (25)$$

This value of the relative dielectric function  $\epsilon = 6.2223$  can be realized by an embedded Ge film in N-LAK12 Schott glass. From the known dispersion of these two optical materials [12,13],  $\epsilon = 6.2223$  is obtained at  $\lambda = 1.8584 \mu\text{m}$ . The refractive indices of Ge and N-LAK12 Schott glass at this wavelength are 4.1225 and 1.6527, respectively, and the thickness of the Ge QWL is  $d = 117.5$  nm. The extinction

coefficient of Ge is  $< 0.0001$  at and near  $\lambda = 1.86 \mu\text{m}$ , which is negligible [13].

The reflectance of this BS for incident unpolarized light varies by  $\approx \mp 1\%$  as  $\phi$  is shifted by  $\pm 5^\circ$  around  $45^\circ$ , while  $\lambda$  and  $d$  are kept constant. With  $\phi = 45^\circ$  and  $d = 117.5$  nm, the reflectance for incident unpolarized light deviates from 50% by  $< 0.1\%$  over the 1.8–1.9  $\mu\text{m}$  spectral range (e.g., of an InGaAsP laser [14]). Changes of the thickness  $d$  of the Ge film by  $\pm 5$  nm around 117.5 nm, while keeping  $\phi$  and  $\lambda$  fixed, reduce the reflectance by  $\approx 1\%$ . Therefore, this BS has good angular and spectral responses and is insensitive to small film-thickness errors to the first order.

The results of Section 4 complement those published recently [15] regarding QWL that produce 50% reflectance for incident unpolarized light at any angle of incidence. If the incident light is totally polarized (instead of being unpolarized) and its  $p$  and  $s$  components have equal amplitudes and an arbitrary but fixed phase difference, the two beams that are reflected and transmitted by this QWL buried-in-a-cube BS become orthogonally polarized. Further details regarding this interesting function appear elsewhere [15,16]. The cube BS has an important practical advantage over the previous prism design [15].

#### 5. Complex-Amplitude Reflection Coefficients of a Quarter-Wave Layer Deposited on a Substrate

For a transparent QWL ( $X = -1$ ) that is deposited on an optically isotropic substrate which may be absorbing, Eq. (1a) gives

$$R_{012\nu} = \frac{r_{01\nu} - r_{12\nu}}{1 - r_{01\nu}r_{12\nu}}, \quad \nu = p, s. \quad (26)$$

For a freestanding or embedded QWL (i.e., if the ambient and substrate have the same refractive index), Eq. (26) becomes

$$R_{010\nu} = \frac{2r_{01\nu}}{1 + r_{01\nu}^2}, \quad \nu = p, s. \quad (27)$$

Except for a slight change of notation, Eq. (27) is the same as Eq. (8).

In the absence of a film ( $d = 0$ ,  $X = 1$ ), Eq. (1a) gives the identity

$$R_{012\nu} = r_{02\nu} = \frac{r_{01\nu} + r_{12\nu}}{1 + r_{01\nu}r_{12\nu}}, \quad \nu = p, s. \quad (28)$$

Equation (28) can be solved for the Fresnel coefficient of internal reflection at the film–substrate interface  $r_{12\nu}$  in terms of the Fresnel coefficients of external reflection  $r_{01\nu}$  and  $r_{02\nu}$  at the ambient–film and ambient–substrate interfaces, respectively, at the same angle of incidence  $\phi$ . This gives

$$r_{12\nu} = \frac{r_{02\nu} - r_{01\nu}}{1 - r_{01\nu}r_{02\nu}}, \quad \nu = p, s. \quad (29)$$

Finally, if  $r_{12\nu}$  of Eq. (29) is substituted into Eq. (26) we obtain

$$R_{012\nu} = \frac{R_{010\nu} - r_{02\nu}}{1 - r_{02\nu}R_{010\nu}}, \quad \nu = p, s. \quad (30)$$

Equation (30) provides a direct relation between the complex reflection coefficients  $R_{012\nu}$ ,  $\nu = p, s$  of a transparent QWL that is deposited on a transparent or absorbing substrate and the corresponding real reflection coefficients  $R_{010\nu}$ ,  $\nu = p, s$  of the same QWL embedded in the same ambient at the same angle of incidence. This enables the application of the results obtained in Section 2 to the more general case of a QWL between dissimilar ambient and substrate media.

As an example, consider the reflection of  $p$ -polarized light by a QWL that is deposited on a transparent substrate at the Brewster angle of the ambient–substrate interface,  $\phi_{B02} = \tan^{-1}(\varepsilon_2/\varepsilon_0)$ . In this case,  $r_{02p} = 0$ , and Eq. (30) reduces to

$$R_{012p} = R_{010p}. \quad (31)$$

Equation (31) shows that  $p$ -polarized light is reflected by the QWL at the Brewster angle  $\phi_{B02}$  as if the substrate were not present.

It is also of interest to determine the reflection coefficient of the QWL for incident  $s$ -polarized light at the same Brewster angle  $\phi_{B02}$ . For the  $s$  polarization

$$r_{02s} = \cos(2\phi_{B02}), \quad (32)$$

$$R_{010s} = \frac{1 - \varepsilon_1}{\cos 2\phi_{B02} + \varepsilon_1}. \quad (33)$$

From Eqs. (30), (32), and (33) we obtain

$$R_{012s} = \frac{1 - \cos(2\phi_{B02}) - \varepsilon_1}{\varepsilon_1}. \quad (34)$$

Finally, by substituting

$$\cos(2\phi_{B02}) = -(\varepsilon_2 - 1)/(\varepsilon_2 + 1) \quad (35)$$

into Eq. (34), we get

$$R_{012s}(\phi_{B02}) = \frac{2\varepsilon_2 - \varepsilon_1\varepsilon_2 - \varepsilon_1}{\varepsilon_1(\varepsilon_2 + 1)}. \quad (36)$$

Equation (36) predicts that the reflection coefficient  $R_{012s}$  of the coated substrate for  $s$ -polarized light at the Brewster angle  $\phi_{B02}$  is zero if

$$\varepsilon_1 = 2\varepsilon_2/(\varepsilon_2 + 1), \quad n_1 = \sqrt{2}n_2/(n_2^2 + 1)^{1/2}. \quad (37)$$

In Eqs. (33)–(37),  $\varepsilon_1$  and  $\varepsilon_2$  are used to represent  $\varepsilon_1/\varepsilon_0$  and  $\varepsilon_2/\varepsilon_0$ , respectively. The above analysis provides an alternative approach to the conditions for achieving spatial (or temporal) binary polarization modulation based on light reflection by a coated surface [17,18].

## 6. Summary

An analytical procedure is presented for the design of 50%–50% cube BS for incident  $s$ -polarized or unpolarized light that uses a high-index (e.g.,  $\text{TiO}_2$  or Ge) QWL that is embedded in glass. The approach is based on new explicit expressions for the complex-amplitude reflection coefficients of the QWL for the  $p$  and  $s$  polarizations at oblique incidence. These BSs exhibit small shifts from ideal performance over an internal field of view of  $\pm 5^\circ$ , a 100 nm spectral bandwidth, or in the presence of  $\pm 5\%$  film-thickness errors. A direct relation has also been obtained between the reflection coefficients of a QWL that is embedded between similar and dissimilar bulk media.

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