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Determination of the refractive index and thickness of transparent pellicles by use of the polarization-independent absentee-layer condition

Y. Cui and R. M. A. Azzam

The refractive index and the thickness of a transparent pellicle are determined when the pellicle is placed between two vertical crossed polarizers and rotated in the horizontal plane. The transmission axes of the polarizers are neither parallel nor perpendicular to the plane of incidence. The light transmitted through the crossed polarizers reaches a minimum when the pellicle satisfies the absentee-layer condition. The refractive index and the film thickness are obtained from the pellicle orientation angles under such a condition. © 1996 Optical Society of America

1. Introduction

Simple and accurate methods for the determination of the refractive index and the thickness of thin films are important to industry and the research laboratory. Many methods have been developed to measure the refractive index and the thickness of thin films, e.g., variable-angle monochromatic fringe observation,¹ constant-angle reflection interference spectroscopy,² polarization-independent reflectance matching³ (PIRM), polarized reflectance measurement technique for thickness and index,^{4,5} and ellipsometry.⁶ Each method has its own advantages and limitations.

The PIRM technique³ utilizes the interference phenomena in the reflected and the transmitted light beams and the angles that satisfy the absentee-layer condition (in which the light is reflected from and transmitted through the film as if the film does not exist) to determine the refractive index and the thickness of the thin film simultaneously. It provides a simple and accurate nondestructive method to determine the parameters of the thin film.

In this paper we report the experimental results

from the measurement of the refractive index and the thickness of two thin-film pellicles at two different wavelengths by using a novel implementation of the PIRM technique.

2. Polarization-Independent Reflectance Matching Theory

For the thin-film structure shown in Fig. 1, the reflection coefficient is given by

$$R_v = \frac{r_{01v} + r_{12v} \exp(-j2\pi d/D_\varphi)}{1 + r_{01v}r_{12v} \exp(-j2\pi d/D_\varphi)}, \quad (1)$$

where d is the film thickness, $D_\varphi = (\lambda/2)(N_1^2 - N_0^2 \sin^2 \varphi)^{-1/2}$ is the film-thickness period,³ and r_{01v} and r_{12v} are the Fresnel reflection coefficients of the 0-1 and 1-2 interfaces for the v polarization ($v = p$ or s for the linear polarization parallel or perpendicular to the plane of incidence). When $\exp(-j2\pi d/D_\varphi) = 1$, or

$$N_1^2 = N_0^2 \sin^2 \varphi_m + m^2 \left(\frac{\lambda}{2d} \right)^2, \quad (2)$$

light is reflected and transmitted as if there is no film (medium 1) present, regardless of the incident linear polarization.³ This is the absentee-layer condition,³ in which m is an integer and λ is the wavelength of light. In the discussion below we assume that medium 2 is the same as medium 0, i.e., the case of a pellicle in air. Under the absentee-layer condition, the light transmitted by the thin film has exactly the same polarization state as the incident light. If the pellicle is placed between two crossed polarizers,

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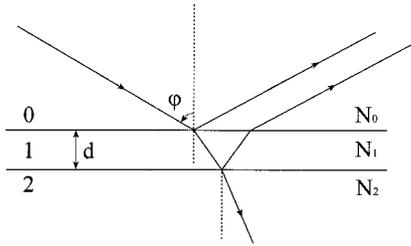


Fig. 1. Thin-film structure.

there will be no transmitted light under the absentee-layer condition. From the angles of successive minimum transmission $\varphi_{m'}$ and $\varphi_{m''}$ and the corresponding order numbers m' and m'' , the film refractive index N_1 and the film thickness d can be obtained by³

$$N_1 = \left[\sin^2 \varphi_m + m^2 \left(\frac{\sin^2 \varphi_{m''} - \sin^2 \varphi_{m'}}{m'^2 - m''^2} \right) \right]^{1/2}$$

$$(m = m' \text{ or } m''), d = \frac{\lambda}{2} \left(\frac{m'^2 - m''^2}{\sin^2 \varphi_{m''} - \sin^2 \varphi_{m'}} \right)^{1/2}. \quad (3)$$

3. Novel Implementation of Polarization-Independent Reflectance Matching

The experimental setup is shown in Fig. 2(a). The x axis is in the laboratory horizontal direction. The light from a 1-mW, 632.8-nm (or 543.5-nm) He-Ne laser source L passes through aperture A , polarizer P_1 , a transparent thin-film (pellicle) sample to be measured, S , and a second polarizer P_2 . The transmitted light is detected by a photomultiplier tube (PMT). The polarizers' transmission axes t_1 of P_1 and t_2 of P_2 are orthogonal, as shown in Fig. 2(b). The angle γ between t_1 and the y axis is neither 0° nor 90° . The angle between the surface normal of sample S and the optical axis is the angle of incidence φ , which varies from 0° to 180° when the sample is rotated around its y axis. There is no transmitted

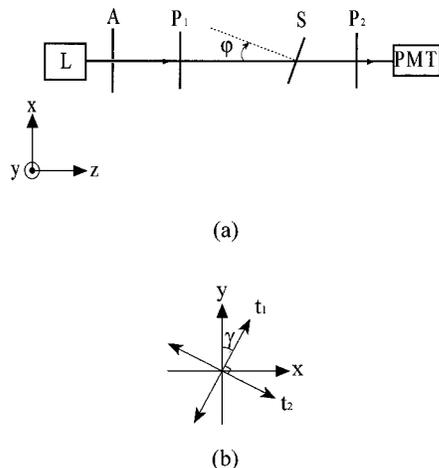


Fig. 2. (a) Experimental setup of the PIRM, (b) relative orientation of the transmission axes t_1 and t_2 of the crossed polarizers.

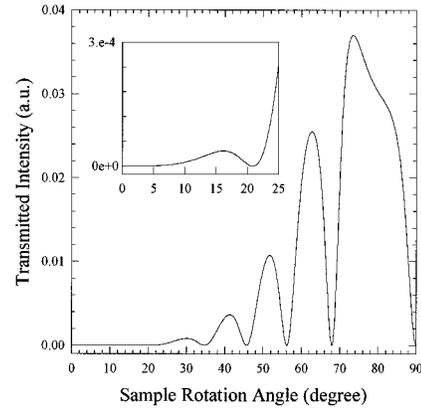


Fig. 3. Theoretically calculated result of the transmitted intensity after P_2 in the setup in Fig. 2(a) versus the sample rotation angle φ . The inset is the enlargement of the first minimum transmission angle. A pellicle of thickness $4.56 \mu\text{m}$ and a refractive index of 1.5 at the 632.8-nm wavelength are assumed.

light after P_2 when $\varphi = 0^\circ$ or when no sample is present. The rotation stage can be accurately controlled to within 0.5 arc sec or 0.008° .

To determine the pellicle orientation angle of minimum transmission, we first determine the angle of minimum output current from the PMT by varying the angle of incidence φ near zero and by using the method of swings.⁷ After rotating the pellicle by 180° , we determine the symmetric angle on the other side by using the same method. From these two angles we determine the goniometer scale reading that corresponds to normal incidence (i.e., $\varphi = 0$) as the average of the two measured angles of minimum transmission. The same procedure is applied to the m th-order angle of minimum transmission φ_m , where m is to be determined. The two symmetric angles determined for different orders are consistent to within $\sim 0.03^\circ$. The uncertainty of each angle reading when the method of swings is used is of the order of 0.05° . Thus the overall uncertainty of the angle measurement is of the order of 0.08° . The angles of lower-order number m (i.e., higher angle of incidence φ) have relatively high accuracy because the lower orders have narrower dips, as shown in Fig. 3. This is a theoretically calculated result for a $4.56\text{-}\mu\text{m}$ pellicle with refractive index $N_1 = 1.5$ at 632.8 nm. However, this observation is true for any thickness at any wavelength. The measured angles of minimum

Table 1. Measured Angles of Minimum Transmission φ_m and the Corresponding Calculated Order Numbers m , Refractive Index N_1 , and the Thickness d of Pellicle Sample 1 at Two Wavelengths

λ (nm)	m	$\varphi_m (\pm 0.08^\circ)$	$N_1 (\pm 0.003)$	$d (\mu\text{m})$ (± 0.004)
543.5	8	36.76°	1.505	1.574 ± 0.004
	7	63.81°		
632.8	7	31.74°	1.503	1.574 ± 0.004
	6	63.60°		

Table 2. Measured Angles of Minimum Transmission φ_m and the Corresponding Calculated Order Numbers m , Refractive Index N_1 , and the Thickness d of Pellicle Sample 2 at Two Wavelengths

λ (nm)	m	φ_m ($\pm 0.08^\circ$) (deg)	N_1	d (μm) (± 0.04)
543.5	25	14.45	1.51 ± 0.01	4.56
	24	28.72		
	23	39.10		
	22	48.33		
	21	57.48		
	20	67.73		
632.8	21	22.01	1.504 ± 0.009	4.56
	20	35.35		
	19	46.29		
	18	56.86		
	17	68.84		

transmission at two wavelengths ($\lambda = 543.5$ and 632.8 nm) are listed in Tables 1 and 2 for samples 1 and 2, respectively.⁸

4. Results and Discussion

The order number m can be determined by the following two methods. If the number of angles of minimum transmission (other than $\varphi = 0$) is >2 , the following formula can be used to calculate the order number m :

$$m = \frac{\sin^2 \varphi_{m-1} - \sin^2 \varphi_{m+1}}{4 \sin^2 \varphi_m - 2(\sin^2 \varphi_{m-1} + \sin^2 \varphi_{m+1})}. \quad (4)$$

This is the easiest way to determine m . But errors in the measured angles φ_m can lead to an unacceptable value of m as obtained from Eq. (4). Use of lower-order number m 's in Eq. (4) gives a better result, which is close to the actual value of m , because of the higher measurement accuracy. This method cannot determine the order number m when only two minimum transmission angles exist.

The second method involves a combination of graphic and numerical calculations. This works for any number of minimum transmission angles. Because the pellicle under test is of a known material (nitrocellulose), the refractive index is ~ 1.5 . From Eq. (2), we plot a graph of φ_m versus d for different order numbers m , assuming that $N_0 = 1.0$ and $N_1 = 1.5$ (or some reasonable value) at 632.8 nm, as shown in Fig. 4. By using the values and the total number of minimum transmission angles listed in Tables 1 and 2 and Fig. 4, we can determine the order number m to be 6 and 7 for sample 1 and 17–21 for sample 2.

Once m is determined, the refractive index N_1 and the film thickness d are obtained through Eqs. (3). The calculated values of N_1 and d for both samples are given in Tables 1 and 2. The refractive indices at each wavelength obtained from the two samples agree very well with each other. The measured film thickness at the two different wavelengths gives the same result of $1.57 \mu\text{m}$ for sample 1 and $4.56 \mu\text{m}$ for sample 2.

Theoretical calculations show that an orientation

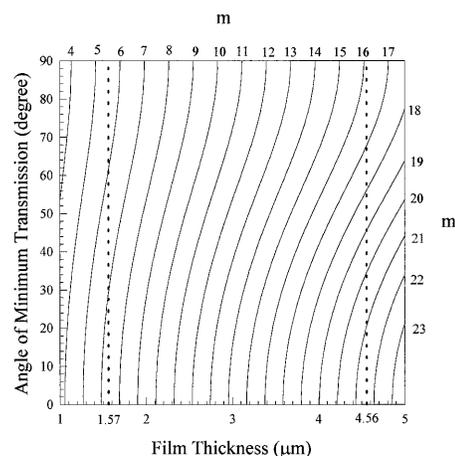


Fig. 4. Theoretically calculated result of the angle of minimum transmission versus the thin-film thickness for different orders. The vertical dotted lines correspond to the two samples tested. A refractive index of 1.5 is assumed. The order number m is labeled on the top and right-hand side of the graph.

error of the two crossed polarizers (i.e., t_1 and t_2 are not exactly 90° apart) and the limited extinction ratio of the two polarizers will affect the measured angles. The selection of polarizers is important; the lower the extinction ratio, the wider the usable range of angle γ . For a polarizer with an extinction ratio of 10^{-4} , the usable γ values are from 35° to 55° with the polarizers crossed to within 0.4° . At other angles γ , the measurement result has significant errors.

The limitations of this technique are that (i) when only one minimum transmission angle exists, there are multiple solutions and it is impossible to determine the correct thickness (this corresponds to the lower limit of the thickness measurement); and (ii) when successive adjacent minima are too close, the detector cannot resolve them apart (this corresponds to the upper limit of the thickness measurement). For the setup described above (with 0.08° measurement accuracy) the predicted lower and upper limits of the thickness measurement are 0.95 and $475 \mu\text{m}$, respectively.

The present setup can be further expanded to include a spectrometer to replace the PMT. In this way, the thin-film thickness and the refractive-index dispersion across the spectrum can be obtained from the spectral minima of different orders.

5. Conclusion

PIRM and the absentee-layer condition provide a simple and nondestructive technique to determine the refractive index and the thickness of transparent thin films with high accuracy. Measurements can be performed at different wavelengths to determine the dispersion of a thin-film sample. A novel implementation and application of this method to pellicles have been reported in this paper.

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