

10-1-1988

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Recommended Citation

R. M. A. Azzam and A. M. El-Saba, "Reflectance of an absorbing substrate for incident light of arbitrary polarization: appearance of a secondary maximum at oblique incidence," *Appl. Opt.* 27, 4034-4037 (1988)

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Reflectance of an absorbing substrate for incident light of arbitrary polarization: appearance of a secondary maximum at oblique incidence

R. M. A. Azzam and A. M. El-Saba

The reflectance of an absorbing substrate $R_\theta(\phi)$ is considered as a function of the angle of incidence ϕ and an incident polarization parameter θ , where $\cos^2\theta$ and $\sin^2\theta$ give the power fractions of incident radiation that are p - and s -polarized, respectively. Taking GaAs as an example, we find that at certain wavelengths (e.g., 0.248 and 0.620 μm), the R_θ vs ϕ curve becomes oscillatory in a narrow range of $\theta > 45^\circ$ with an unexpected secondary maximum appearing at oblique incidence. The extrema of the function $R_\theta(\phi)$ are determined numerically, and their angular positions and reflectance levels are plotted vs θ for GaAs at photon energies of 1, 2, and 5 eV.

I. Introduction

The reflection of a monochromatic collimated light beam at the planar interface between a transparent medium of incidence (usually air, $\epsilon_0 = 1$) and an absorbing medium of refraction (ϵ_1 complex) is governed by the Fresnel coefficients¹

$$r_p = [\epsilon \cos\phi - (\epsilon - \sin^2\phi)^{1/2}] / [\epsilon \cos\phi + (\epsilon - \sin^2\phi)^{1/2}], \quad (1)$$

$$r_s = [\cos\phi - (\epsilon - \sin^2\phi)^{1/2}] / [\cos\phi + (\epsilon - \sin^2\phi)^{1/2}],$$

where p and s identify the linear polarizations parallel and perpendicular to the plane of incidence, respectively, ϕ is the angle of incidence, and $\epsilon = \epsilon_1/\epsilon_0$ is the complex ratio of dielectric constants of the two media. The intensity (or power) reflectance is given by

$$R_\nu = r_\nu r_\nu^*, \quad \nu = p, s. \quad (2)$$

The behavior of the functions $R_p(\phi)$ and $R_s(\phi)$ for a given complex ϵ is well known. As ϕ increases from 0, $R_p(\phi)$ first decreases to a minimum at the pseudo-Brewster angle ϕ_{pB} , then increases to 1 as $\phi \rightarrow 90^\circ$. On the other hand, $R_s(\phi)$ increases monotonically from a minimum at normal incidence ($\phi = 0$) to 1 at grazing

incidence ($\phi = 90^\circ$). This behavior is invariably the same for any complex ϵ .

We consider the interface reflectance for incident light of an arbitrary state of polarization. If q ($0 \leq q \leq 1$) represents the fraction of incident radiation which is p -polarized, the interface reflectance is given by

$$R_q = qR_p + (1 - q)R_s, \quad (3)$$

The case of $q = 1/2$,

$$R_{0.5} = (R_p + R_s)/2, \quad (4)$$

received attention previously,²⁻⁴ as it corresponds to incident unpolarized light (or, more generally, to light whose second Stokes parameter referenced to the p and s directions is zero). Dependent on ϵ , the angular variation of $R_{0.5}(\phi)$ resembles either that of $R_p(\phi)$, with a minimum at oblique incidence, or that of $R_s(\phi)$ with a monotonic rise of reflectance. Our objective is to investigate the nature of the curve of $R_q(\phi)$ for an arbitrary q and to determine the angular positions and reflectance values of its extrema. An interesting and previously unknown behavior is discovered, namely, that $R_q(\phi)$, for a narrow range of q and certain ϵ , is oscillatory with a secondary maximum at oblique incidence. This is shown for one absorbing substrate, GaAs, at different wavelengths as a specific example.

For convenience, and without loss of generality, we substitute

$$q = \cos^2\theta, \quad (5)$$

which puts Eq. (3) in the form

$$R_\theta = (\cos^2\theta)R_p + (\sin^2\theta)R_s. \quad (6)$$

If the incident light is totally linearly polarized, θ rep-

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Received 12 April 1988.

0003-6935/88/194034-04\$02.00/0.

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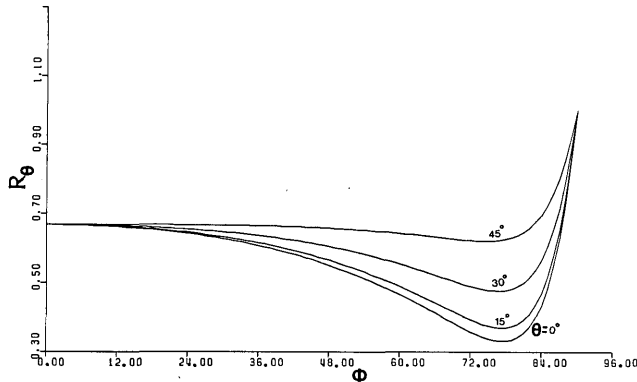


Fig. 1. Reflectance R_θ vs angle of incidence ϕ for a GaAs substrate at $\lambda = 0.248 \mu\text{m}$ ($\epsilon = -11.5125 - j18.5659$) and for $\theta = 0, 15, 30,$ and 45° . $\cos^2\theta$ and $\sin^2\theta$ give the proportions of incident light that are p - and s -polarized, respectively. For each θ in this range, R_θ exhibits only a minimum at oblique incidence.

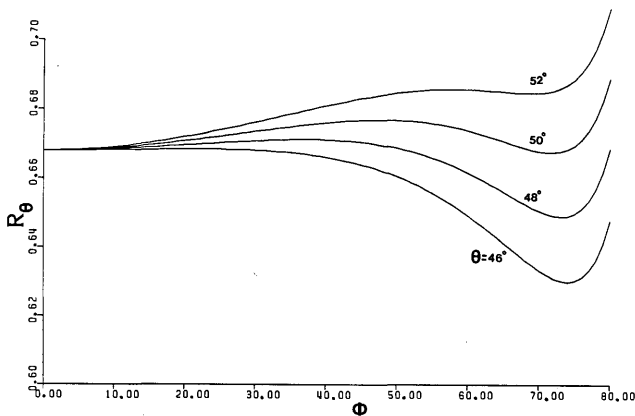


Fig. 2. Same as in Fig. 1 except that now $\theta = 46, 48, 50,$ and 52° . A secondary maximum and a minimum appear in this range of θ .

resents the azimuth of its electric vector measured from the plane of incidence.

The extrema of $R_\theta(\phi)$ are determined by

$$\partial R_\theta / \partial \phi = 0. \quad (7)$$

If we substitute from Eq. (2) into Eq. (6) and apply Eq. (7), the result can be cast in the form

$$\text{Re}[(\cos^2\theta)r'_p r_p^* + (\sin^2\theta)r'_s r_s^*] = 0, \quad (8)$$

where the derivatives $r'_v = \partial r_v / \partial \phi$, $v = p, s$, are given elsewhere⁵ and are not repeated here. r_v^* is the complex conjugate of r_v , as usual. For a given complex ϵ , Eq. (8) is solved numerically, and iteration on ϕ is stopped when the left-hand side of Eq. (8) is $<10^{-6}$.

II. Reflection by a GaAs Substrate

As a specific example of an absorbing medium, we take a GaAs substrate with ϵ at different wavelengths λ as given by Aspnes and Studna.⁶ We assume that the light is incident from air.

At $\lambda = 0.248 \mu\text{m}$ in the near UV (photon energy $E = h\nu = 5 \text{ eV}$), $\epsilon = -11.5125 - j18.5659$. We start with this complex ϵ as it clearly demonstrates the existence of oscillation of the $R_\theta(\phi)$ vs ϕ curves. The range of θ is

divided into three parts with distinctly different behavior of $R_\theta(\phi)$.

Figure 1 shows $R_\theta(\phi)$ vs ϕ for $\theta = 0, 15, 30,$ and 45° . $\theta = 0$ represents purely p -polarized incident light, $R_0 = R_p$, and the reflectance has a distinct minimum, $R_{p\text{min}} = 0.3315$ at $\phi_{pB} = 77.5358^\circ$. As θ increases, the proportion of s -polarized radiation increases. This is accompanied by a small monotonic shift of the angle of minimum reflectance $\phi_{\theta\text{min}}$ and a significant rise of the reflectance level at the minimum, $R_{\theta\text{min}}$. At $\theta = 45^\circ$ (equal power of the p - and s -polarized components), we have $\phi_{45\text{min}} = 74.4177^\circ$ and $R_{45\text{min}} = 0.6204$. Thus $R_{\theta\text{min}}$ is almost doubled, and $\phi_{\theta\text{min}}$ is downshifted by $\sim 3^\circ$, as θ increases from 0 (p -polarized light) to 45° (equal p and s components).

Figure 2 shows $R_\theta(\phi)$ vs ϕ for $\theta = 46, 48, 50,$ and 52° . Each curve shows a secondary maximum followed by a

Table I. Extrema of the R_θ vs ϕ Curves^a (Fig. 2) for Four values of θ

θ	ϕ_{min}	R_{min}	ϕ_{max}	R_{max}
46	21.4897	0.6684	74.0664	0.6301
48	36.9288	0.6712	73.1664	0.6492
50	47.7301	0.6769	71.7698	0.6675
52	57.9185	0.6859	68.8525	0.6848

^a GaAs, $\lambda = 0.248 \mu\text{m}$, $\epsilon = 11.5125 - j18.5659$. All angles are in degrees.

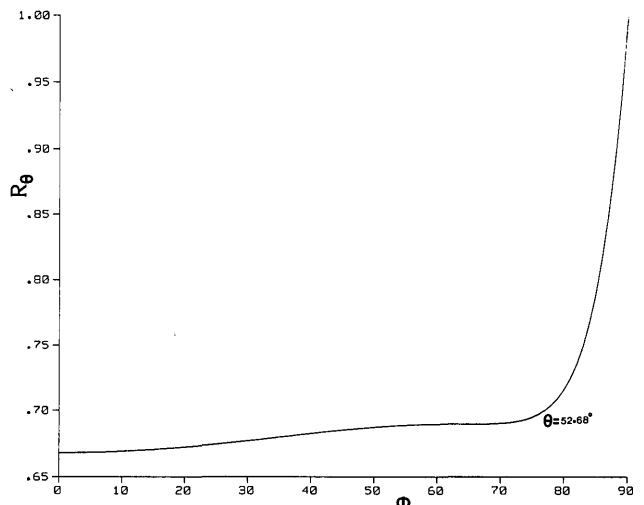


Fig. 3. Same as in Fig. 1 but for $\theta = 52.68^\circ$. This is a limiting case that separates the oscillatory (Fig. 2) and monotonic (Fig. 4) regimes of the R_θ vs ϕ curve.

minimum. The reflectance levels and angular positions of these extrema are listed in Table I. The secondary maximum appears as θ increases just above 45° and disappears as θ increases above 52.68° . Thus at $\theta = 45.001^\circ$ we find the secondary maximum located at $\phi = 0.573^\circ$, and its level is above the normal incidence reflectance by $<10^{-5}$. Figure 3 shows $R_\theta(\phi)$ vs ϕ for the upper limit $\theta_u = 52.68^\circ$. This monotonic curve shows the merging of the maximum and minimum which gives rise to a reflectance plateau in the 55 – 75° range.

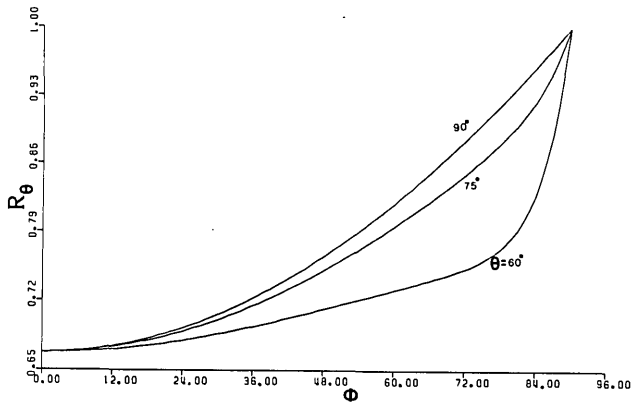


Fig. 4. Same as in Fig. 1 except that $\theta = 60, 75,$ and 90° . All curves in this range of θ indicate a monotonic rise of R_θ with ϕ .

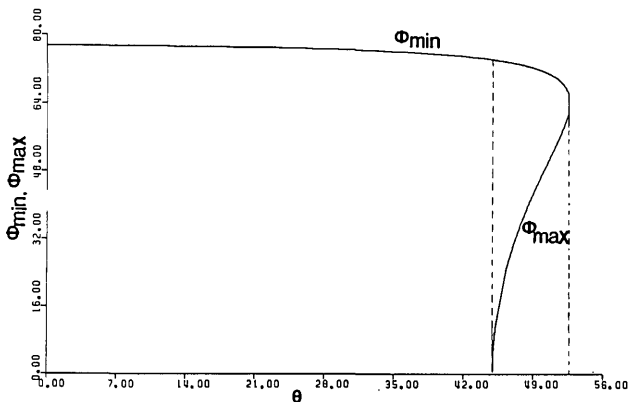


Fig. 5. Angle of minimum reflectance ϕ_{\min} as a function of θ . ($\cos^2\theta$ and $\sin^2\theta$ are the power fractions of incident light that are p - and s -polarized, respectively.) A GaAs substrate is assumed at $\lambda = 0.248 \mu\text{m}$ ($\epsilon = -11.5125 - j18.5659$). For $\theta > 52.68^\circ$, $\phi_{\min} = 0$. In the interval $45^\circ < \theta < 52.68^\circ$ a secondary maximum appears (see Fig. 2) whose angular position ϕ_{\max} is also indicated as a function of θ .

Figure 4 shows $R_\theta(\phi)$ vs ϕ for $\theta = 60, 75,$ and 90° . All curves show a monotonic increase of reflectance with incidence angle, but the curve for $\theta = 90^\circ$ (purely s -polarized incident light) is significantly different from that of $\theta = 60^\circ$ (25% p -polarized, 75% s -polarized incident light).

The oscillatory $R_\theta(\phi)$ vs ϕ curves for a narrow range of $\theta, 45^\circ < \theta < 52.68^\circ$, Fig. 2, represent the most interesting finding of this work. Such behavior is typically associated with interference in a thin film of thickness comparable to the wavelength of light. It is remarkable that one can get similar oscillation by reflection at a single interface when the incident light has the right mix of the p and s polarizations (being slightly more s than p -polarized).

Figure 5 shows the variation of ϕ_{\min} (upper branch) and ϕ_{\max} (lower branch) of minimum and maximum reflectance with the (azimuth) angle θ in the $0 \leq \theta \leq 52.68^\circ$ range. The secondary maximum appears only in the $45^\circ < \theta < 52.68^\circ$ interval and merges with the minimum at the upper limit $\theta_u = 52.68^\circ$. For $\theta >$

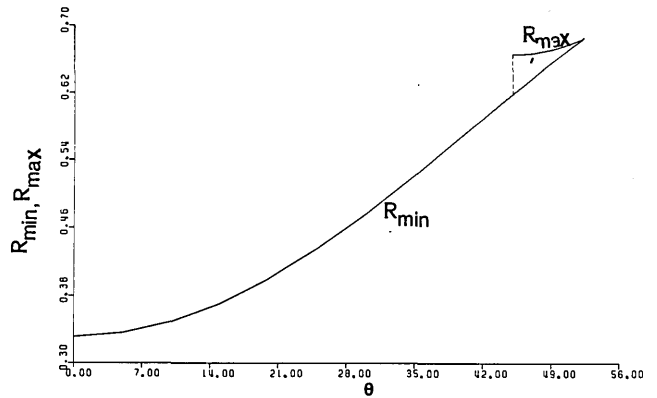


Fig. 6. Minimum and maximum reflectances R_{\min} and R_{\max} associated with the angles ϕ_{\min} and ϕ_{\max} in Fig. 5.

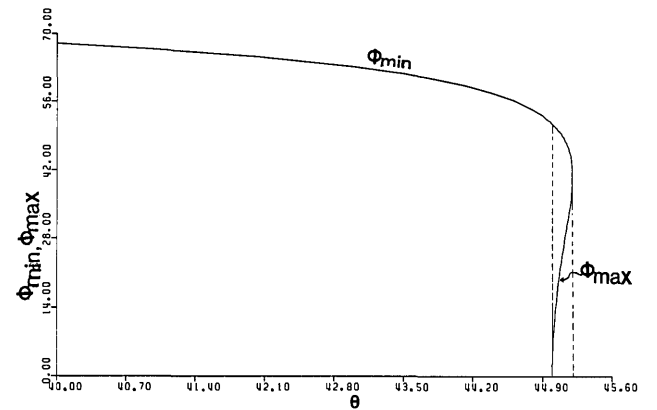


Fig. 7. Same as in Fig. 5 except that now $\lambda = 0.620 \mu\text{m}$ and $\epsilon = 14.9944 - j1.6265$ of GaAs. In this case, the secondary maximum appears only in the brief interval of $45^\circ < \theta < 45.20^\circ$.

52.68° , $\phi_{\min} = 0$ and $R_\theta(\phi)$ becomes monotonic with ϕ as already noted in Fig. 4.

Figure 6 shows the associated variation of R_{\min} and R_{\max} with θ for GaAs at the same photon energy $h\nu = 5$ eV and $\lambda = 0.248 \mu\text{m}$.

III. Results for GaAs at Two Other Wavelengths

To illustrate the way in which the results obtained in Sec. II change as complex ϵ is changed, we consider the same GaAs substrate at two new wavelengths, $\lambda = 0.620 \mu\text{m}$ in the visible and $\lambda = 1.24 \mu\text{m}$ in the near IR.

At $\lambda = 0.620 \mu\text{m}$ ($h\nu = 2$ eV), $\epsilon = 14.9944 - j1.6365$. Figure 7 shows ϕ_{\min} and ϕ_{\max} as functions of θ for $40^\circ < \theta < 46^\circ$. In the $0 \leq \theta \leq 40^\circ$ range, not shown, only ϕ_{\min} exists, and it drops monotonically from 75.5575° at $\theta = 0$ to 68.1467° at $\theta = 40^\circ$. The secondary maximum appears only in the very brief interval $45^\circ < \theta < 45.20^\circ$, and its angular position moves rapidly from 2.4983° at $\theta = 45.001 - 38.9086^\circ$ at $\theta = 45.20^\circ$. Figure 8 shows the associated R_{\min} and R_{\max} vs θ .

At $\lambda = 1.24 \mu\text{m}$ ($h\nu = 1$ eV) GaAs becomes transparent with a real dielectric constant $\epsilon = 11.7183 - j0$. For this ϵ , the secondary maximum no longer exists at any θ . Only a minimum appears for $0 \leq \theta < 45^\circ$, and its

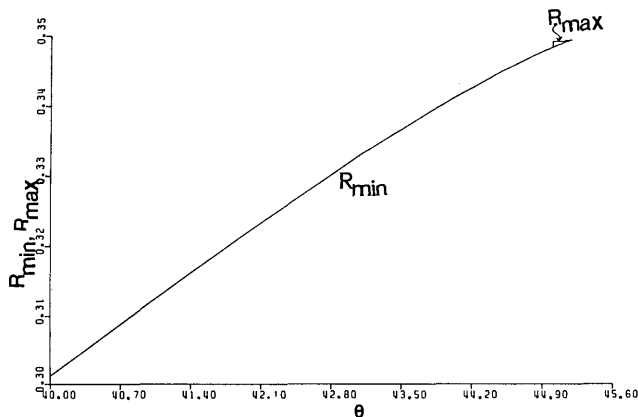


Fig. 8. Minimum and maximum reflectances R_{\min} and R_{\max} at the angles ϕ_{\min} and ϕ_{\max} given in Fig. 7 (GaAs, $\lambda = 0.620 \mu\text{m}$).

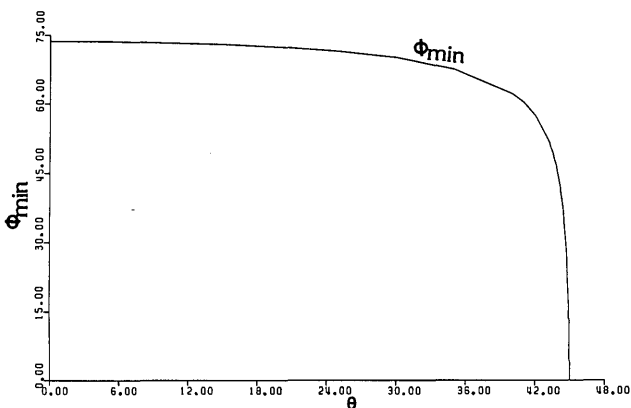


Fig. 9. Same as in Fig. 5 except that now $\lambda = 1.24 \mu\text{m}$ and $\epsilon = 11.7183 - j0$ of GaAs. No secondary maximum exists in the R_{θ} vs θ curve at any θ ; only a minimum occurs at ϕ_{\min} when $0 \leq \theta < 45^\circ$. For $\theta > 45^\circ$, $\phi_{\min} = 0$.

angular position and reflectance level are given by Figs. 9 and 10, respectively, as a function of θ . For $\theta > 45^\circ$, $R_{\theta}(\phi)$ is a monotonically increasing curve from normal ($\phi = 0$) to grazing ($\phi = 90^\circ$) incidence.

IV. Conclusions

We have considered the reflectance of an absorbing substrate as a function of angle of incidence $R_{\theta}(\phi)$ when the incident light is in an arbitrary admixture of the p - and s -polarization states, as represented by the power fractions $\cos^2\theta$ and $\sin^2\theta$, respectively. By taking GaAs as an example, we observed that $R_{\theta}(\phi)$ exhib-

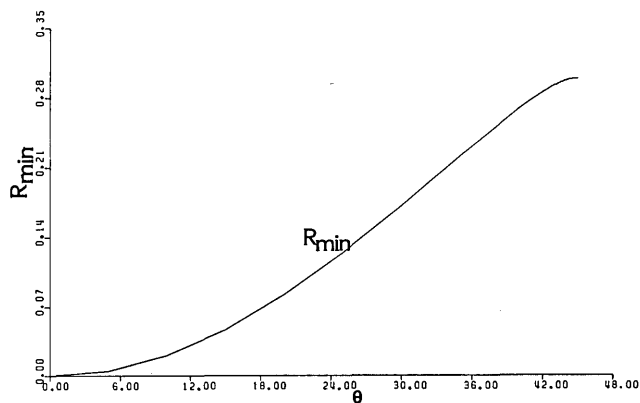


Fig. 10. Minimum reflectance R_{\min} at the angle ϕ_{\min} given in Fig. 9 (GaAs, $\lambda = 1.24 \mu\text{m}$).

its an unexpected secondary maximum at oblique incidence (Fig. 2) when θ is in a certain narrow range $45^\circ < \theta < \theta_u$ and at certain wavelengths. At $\lambda = 0.248$ and $0.620 \mu\text{m}$, $\theta_u - 45^\circ$ equals 7.68 and 0.20° , respectively. The incident radiation for which this interesting oscillatory reflectance occurs is slightly predominantly s -polarized. The extrema of the reflectance curve $R_{\theta}(\phi)$ are determined by solving Eq. (8) numerically, and the results are presented in Figs. 5–10 and Table I.

It would be interesting to explore the complex ϵ plane (without reference to any specific material) and define the domains for which the different types of $R_{\theta}(\phi)$ vs ϕ curves (monotonic, single minimum, and secondary maximum and minimum) do occur. This, however, lies outside the scope of this paper.

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