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Reflectance of an absorbing substrate for incident light of arbitrary polarization: appearance of a secondary maximum at oblique incidence

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Reflectance of an absorbing substrate for incident light of arbitrary polarization: appearance of a secondary maximum at oblique incidence

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The reflectance of an absorbing substrate $R_\theta(\phi)$ is considered as a function of the angle of incidence ϕ and an incident polarization parameter θ, where $\cos^2\theta$ and $\sin^2\theta$ give the power fractions of incident radiation that are p- and s-polarized, respectively. Taking GaAs as an example, we find that at certain wavelengths (e.g., 0.248 and 0.620 μm), the R_θ vs ϕ curve becomes oscillatory in a narrow range of $\theta > 45°$ with an unexpected secondary maximum appearing at oblique incidence. The extrema of the function $R_\theta(\phi)$ are determined numerically, and their angular positions and reflectance levels are plotted vs θ for GaAs at photon energies of 1, 2, and 5 eV.

I. Introduction

The reflection of a monochromatic collimated light beam at the planar interface between a transparent medium of incidence (usually air, $\epsilon_0 = 1$) and an absorbing medium of refraction (ϵ_1 complex) is governed by the Fresnel coefficients[1]

$$r_p = [\epsilon \cos\phi - (\epsilon - \sin^2\phi)^{1/2}]/[\epsilon \cos\phi + (\epsilon - \sin^2\phi)^{1/2}],$$
$$r_s = [\cos\phi - (\epsilon - \sin^2\phi)^{1/2}]/[\cos\phi + (\epsilon - \sin^2\phi)^{1/2}], \quad (1)$$

where p and s identify the linear polarizations parallel and perpendicular to the plane of incidence, respectively, ϕ is the angle of incidence, and $\epsilon = \epsilon_1/\epsilon_0$ is the complex ratio of dielectric constants of the two media. The intensity (or power) reflectance is given by

$$R_\upsilon = r_\upsilon r_\upsilon^*, \qquad \upsilon = p,s. \quad (2)$$

The behavior of the functions $R_p(\phi)$ and $R_s(\phi)$ for a given complex ϵ is well known. As ϕ increases from $0, R_p(\phi)$ first decreases to a minimum at the pseudo-Brewster angle ϕ_{pB}, then increases to 1 as $\phi \to 90°$. On the other hand, $R_s(\phi)$ increases monotonically from a minimum at normal incidence ($\phi = 0$) to 1 at grazing incidence ($\phi = 90°$). This behavior is invariably the same for any complex ϵ.

We consider the interface reflectance for incident light of an arbitrary state of polarization. If $q(0 \le q \le 1)$ represents the fraction of incident radiation which is p-polarized, the interface reflectance is given by

$$R_q = qR_p + (1 - q)R_s. \quad (3)$$

The case of $q = 1/2$,

$$R_{0.5} = (R_p + R_s)/2, \quad (4)$$

received attention previously,[2–4] as it corresponds to incident unpolarized light (or, more generally, to light whose second Stokes parameter referenced to the p and s directions is zero). Dependent on ϵ, the angular variation of $R_{0.5}(\phi)$ resembles either that of $R_p(\phi)$, with a minimum at oblique incidence, or that of $R_s(\phi)$ with a monotonic rise of reflectance. Our objective is to investigate the nature of the curve of $R_q(\phi)$ for an arbitrary q and to determine the angular positions and reflectance values of its extrema. An interesting and previously unknown behavior is discovered, namely, that $R_q(\phi)$, for a narrow range of q and certain ϵ, is oscillatory with a secondary maximum at oblique incidence. This is shown for one absorbing substrate, GaAs, at different wavelengths as a specific example.

For convenience, and without loss of generality, we substitute

$$q = \cos^2\theta, \quad (5)$$

which puts Eq. (3) in the form

$$R_\theta = (\cos^2\theta)R_p + (\sin^2\theta)R_s. \quad (6)$$

If the incident light is totally linearly polarized, θ rep-

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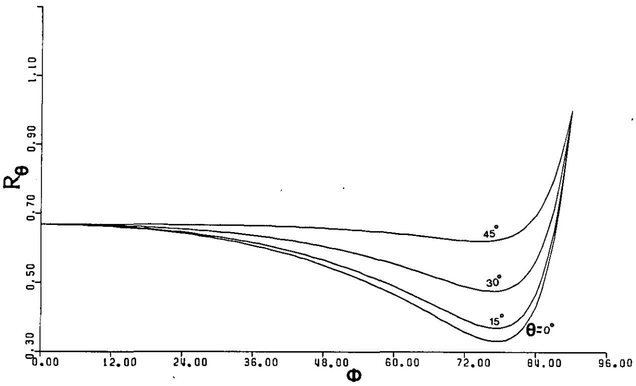

Fig. 1. Reflectance R_θ vs angle of incidence ϕ for a GaAs substrate at $\lambda = 0.248\ \mu$m ($\epsilon = -11.5125$–$j18.5659$) and for $\theta = 0$, 15, 30, and 45°. $\cos^2\theta$ and $\sin^2\theta$ give the proportions of incident light that are p- and s-polarized, respectively. For each θ in this range, R_θ exhibits only a minimum at oblique incidence.

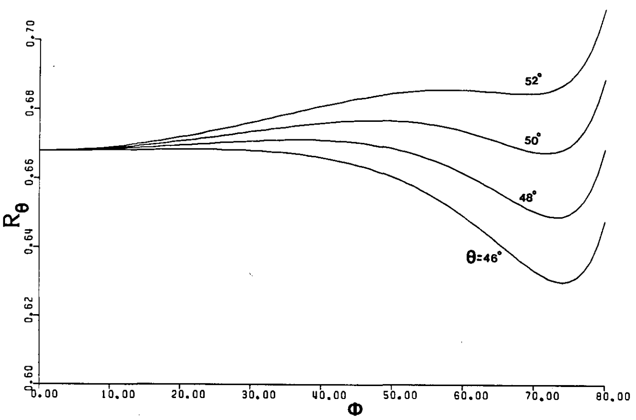

Fig. 2. Same as in Fig. 1 except that now $\theta = 46$, 48, 50, and 52°. A secondary maximum and a minimum appear in this range of θ.

resents the azimuth of its electric vector measured from the plane of incidence.

The extrema of $R_\theta(\phi)$ are determined by

$$\partial R_\theta / \partial \phi = 0. \tag{7}$$

If we substitute from Eq. (2) into Eq. (6) and apply Eq. (7), the result can be cast in the form

$$\mathrm{Re}[(\cos^2\theta) r_p' r_p^* + (\sin^2\theta) r_s' r_s^*] = 0, \tag{8}$$

where the derivatives $r_\nu' = \partial r_\nu / \partial \phi, \nu = p,s$, are given elsewhere[5] and are not repeated here. r_ν^* is the complex conjugate of r_ν, as usual. For a given complex ϵ, Eq. (8) is solved numerically, and iteration on ϕ is stopped when the left-hand side of Eq. (8) is $<10^{-6}$.

II. Reflection by a GaAs Substrate

As a specific example of an absorbing medium, we take a GaAs substrate with ϵ at different wavelengths λ as given by Aspnes and Studna.[6] We assume that the light is incident from air.

At $\lambda = 0.248\ \mu$m in the near UV (photon energy $E = h\nu = 5$ eV), $\epsilon = -11.5125 - j18.5659$. We start with this complex ϵ as it clearly demonstrates the existence of oscillation of the $R_\theta(\phi)$ vs ϕ curves. The range of θ is divided into three parts with distinctly different behavior of $R_\theta(\phi)$.

Figure 1 shows $R_\theta(\phi)$ vs ϕ for $\theta = 0, 15, 30$, and 45°. $\theta = 0$ represents purely p-polarized incident light, $R_0 = R_p$, and the reflectance has a distinct minimum, $R_{p\min} = 0.3315$ at $\phi_{pB} = 77.5358°$. As θ increases, the proportion of s-polarized radiation increases. This is accompanied by a small monotonic shift of the angle of minimum reflectance $\phi_{\theta\min}$ and a significant rise of the reflectance level at the minimum, $R_{\theta\min}$. At $\theta = 45°$ (equal power of the p- and s-polarized components), we have $\phi_{45\min} = 74.4177°$ and $R_{45\min} = 0.6204$. Thus $R_{\theta\min}$ is almost doubled, and $\phi_{\theta\min}$ is downshifted by ~3°, as θ increases from 0 (p-polarized light) to 45° (equal p and s components).

Figure 2 shows $R_\theta(\phi)$ vs ϕ for $\theta = 46, 48, 50$, and 52°. Each curve shows a secondary maximum followed by a minimum. The reflectance levels and angular positions of these extrema are listed in Table I. The secondary maximum appears as θ increases just above 45° and disappears as θ increases above 52.68°. Thus at $\theta = 45.001°$ we find the secondary maximum located at $\phi = 0.573°$, and its level is above the normal incidence reflectance by $<10^{-5}$. Figure 3 shows $R_\theta(\phi)$ vs ϕ for the upper limit $\theta_u = 52.68°$. This monotonic curve shows the merging of the maximum and minimum which gives rise to a reflectance plateau in the 55–75° range.

Table I. Extrema of the R_θ vs ϕ Curves[a] (Fig. 2) for Four values of θ

θ	$\phi_{\min}$	$R_{\min}$	$\phi_{\max}$	$R_{\max}$
46	21.4897	0.6684	74.0664	0.6301
48	36.9288	0.6712	73.1664	0.6492
50	47.7301	0.6769	71.7698	0.6675
52	57.9185	0.6859	68.8525	0.6848

[a] GaAs, $\lambda = 0.248\ \mu$m, $\epsilon = 11.5125$–$j18.5659$. All angles are in degrees.

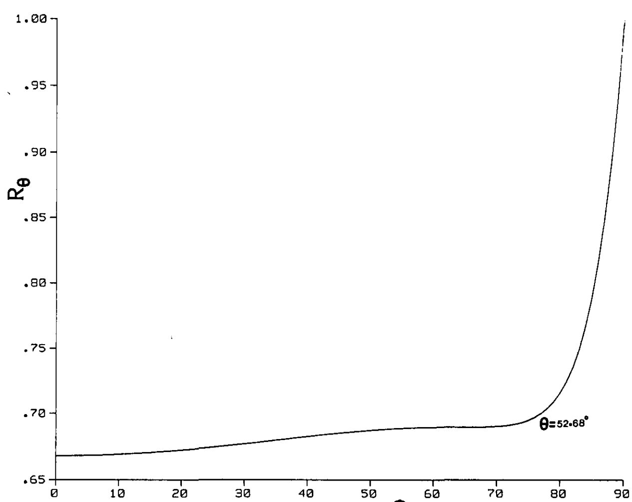

Fig. 3. Same as in Fig. 1 but for $\theta = 52.68°$. This is a limiting case that separates the oscillatory (Fig. 2) and monotonic (Fig. 4) regimes of the R_θ vs ϕ curve.

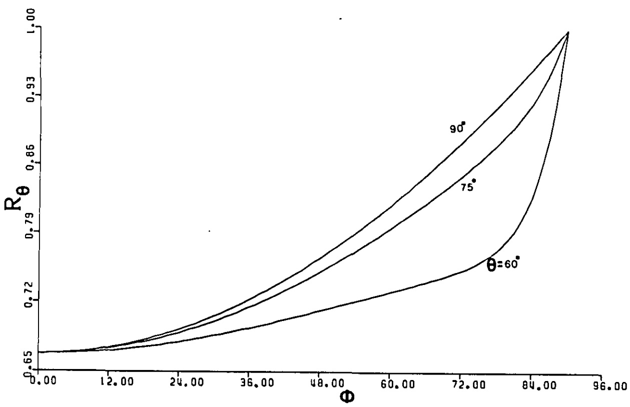

Fig. 4. Same as in Fig. 1 except that θ = 60, 75, and 90°. All curves in this range of θ indicate a monotonic rise of R_θ with ϕ.

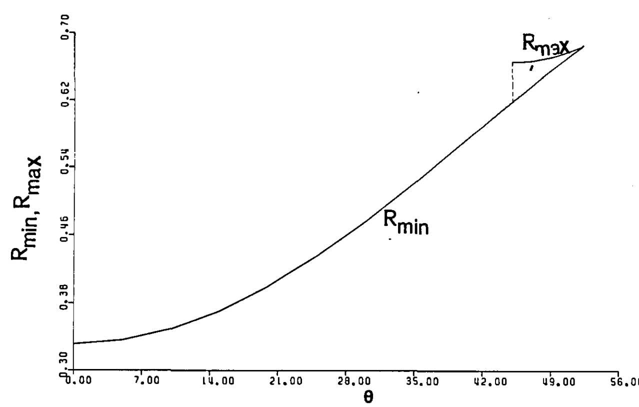

Fig. 6. Minimum and maximum reflectances R_{min} and R_{max} associated with the angles ϕ_{min} and ϕ_{max} in Fig. 5.

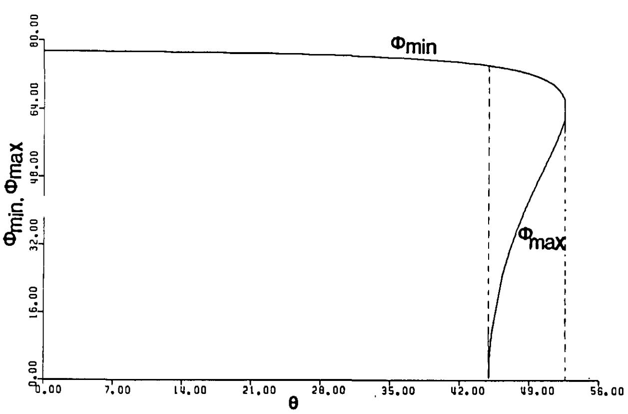

Fig. 5. Angle of minimum reflectance ϕ_{min} as a function of θ. ($\cos^2\theta$ and $\sin^2\theta$ are the power fractions of incident light that are p- and s-polarized, respectively.) A GaAs substrate is assumed at λ = 0.248 μm (ϵ = −11.5125–j18.5659). For θ > 52.68°, ϕ_{min} = 0. In the interval 45° < θ < 52.68° a secondary maximum appears (see Fig. 2) whose angular position ϕ_{max} is also indicated as a function of θ.

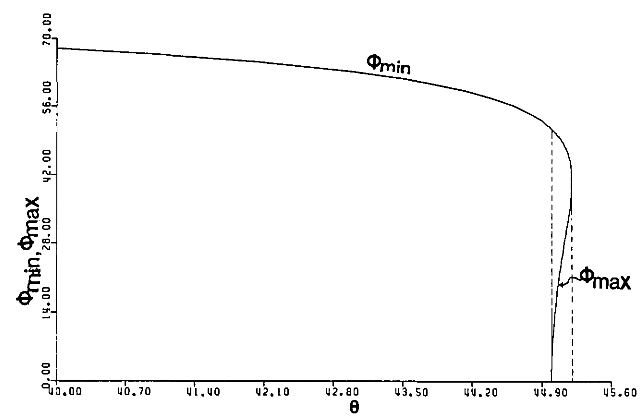

Fig. 7. Same as in Fig. 5 except that now λ = 0.620 μm and ϵ = 14.9944–j1.6265 of GaAs. In this case, the secondary maximum appears only in the brief interval of 45° < θ < 45.20°.

Figure 4 shows $R_\theta(\phi)$ vs ϕ for θ = 60, 75, and 90°. All curves show a monotonic increase of reflectance with incidence angle, but the curve for θ = 90° (purely s-polarized incident light) is significantly different from that of θ = 60° (25% p-polarized, 75% s-polarized incident light).

The oscillatory $R_\theta(\phi)$ vs ϕ curves for a narrow range of θ,45° < θ < 52.68°, Fig. 2, represent the most interesting finding of this work. Such behavior is typically associated with interference in a thin film of thickness comparable to the wavelength of light. It is remarkable that one can get similar oscillation by reflection at a single interface when the incident light has the right mix of the p and s polarizations (being slightly more s than p-polarized).

Figure 5 shows the variation of ϕ_{min} (upper branch) and ϕ_{max} (lower branch) of minimum and maximum reflectance with the (azimuth) angle θ in the 0 ≤ θ ≤ 52.68° range. The secondary maximum appears only in the 45° < θ < 52.68° interval and merges with the minimum at the upper limit θ_u = 52.68°. For θ > 52.68°, ϕ_{min} = 0 and $R_\theta(\phi)$ becomes monotonic with ϕ as already noted in Fig. 4.

Figure 6 shows the associated variation of R_{min} and R_{max} with θ for GaAs at the same photon energy $h\nu$ = 5 eV and λ = 0.248 μm.

III. Results for GaAs at Two Other Wavelengths

To illustrate the way in which the results obtained in Sec. II change as complex ϵ is changed, we consider the same GaAs substrate at two new wavelengths, λ = 0.620 μm in the visible and λ = 1.24 μm in the near IR.

At λ = 0.620 μm ($h\nu$ = 2 eV), ϵ = 14.9944–j1.6365. Figure 7 shows ϕ_{min} and ϕ_{max} as functions of θ for 40° < θ < 46°. In the 0 ≤ θ ≤ 40° range, not shown, only ϕ_{min} exists, and it drops monotonically from 75.5575° at θ = 0 to 68.1467° at θ = 40°. The secondary maximum appears only in the very brief interval 45° < θ < 45.20°, and its angular position moves rapidly from 2.4983° at θ = 45.001–38.9086° at θ = 45.20°. Figure 8 shows the associated R_{min} and R_{max} vs θ.

At λ = 1.24 μm ($h\nu$ = 1 eV) GaAs becomes transparent with a real dielectric constant ϵ = 11.7183–j0. For this ϵ, the secondary maximum no longer exists at any θ. Only a minimum appears for 0 ≤ θ < 45°, and its

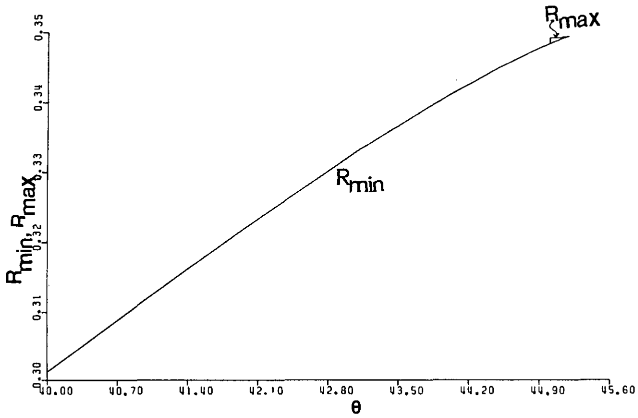

Fig. 8. Minimum and maximum reflectances R_{min} and R_{max} at the angles ϕ_{min} and ϕ_{max} given in Fig. 7 (GaAs,λ = 0.620 μm).

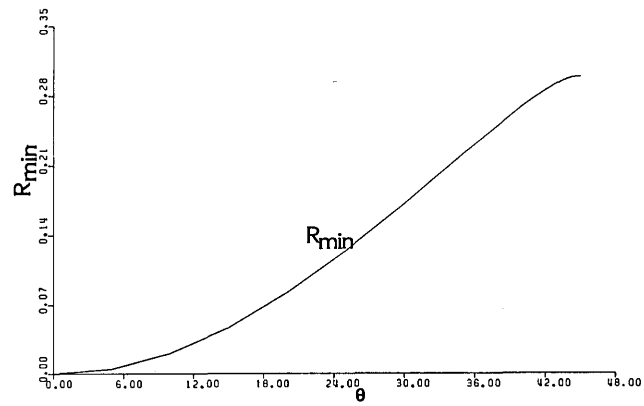

Fig. 10. Minimum reflectance R_{min} at the angle ϕ_{min} given in Fig. 9 (GaAs,λ = 1.24 μm).

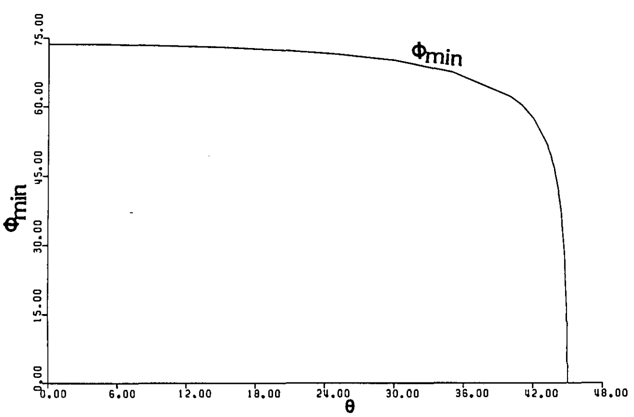

Fig. 9. Same as in Fig. 5 except that now λ = 1.24 μm and ϵ = 11.7183–j0 of GaAs. No secondary maximum exists in the R_θ vs θ curve at any θ; only a minimum occurs at ϕ_{min} when $0 \le \theta < 45°$. For $\theta > 45°$,$\phi_{min} = 0$.

angular position and reflectance level are given by Figs. 9 and 10, respectively, as a function of θ. For $\theta >$ 45°,$R_\theta(\phi)$ is a monotonically increasing curve from normal (ϕ = 0) to grazing (ϕ = 90°) incidence.

IV. Conclusions

We have considered the reflectance of an absorbing substrate as a function of angle of incidence $R_\theta(\phi)$ when the incident light is in an arbitrary admixture of the p- and s-polarization states, as represented by the power fractions $\cos^2\theta$ and $\sin^2\theta$, respectively. By taking GaAs as an example, we observed that $R_\theta(\phi)$ exhibits an unexpected secondary maximum at oblique incidence (Fig. 2) when θ is in a certain narrow range $45° < \theta < \theta_u$ and at certain wavelengths. At λ = 0.248 and 0.620 μm, $\theta_u - 45°$ equals 7.68 and 0.20°, respectively. The incident radiation for which this interesting oscillatory reflectance occurs is slightly predominantly s-polarized. The extrema of the reflectance curve $R_\theta(\phi)$ are determined by solving Eq. (8) numerically, and the results are presented in Figs. 5–10 and Table I.

It would be interesting to explore the complex ϵ plane (without reference to any specific material) and define the domains for which the different types of $R_\theta(\phi)$ vs ϕ curves (monotonic, single minimum, and secondary maximum and minimum) do occur. This, however, lies outside the scope of this paper.

References

1. See, e.g., R. M. A. Azzam and N. M. Bashara, *Ellipsometry and Polarized Light* (North-Holland, Amsterdam, 1977).
2. W. R. Hunter, "Errors in Using Reflectance vs Angle of Incidence Method for Measuring Optical Constants," J. Opt. Soc. Am. **55**, 1197 (1965).
3. H. B. Holl, "Specular Reflection and Characteristics of Reflected Light," J. Opt. Soc. Am. **57**, 683 (1967).
4. J. M. Bennett and H. E. Bennett, "Polarization," in *Handbook of Optics*, W. G. Driscoll and W. Vaughan, Eds. (McGraw-Hill, New York, 1978).
5. R. M. A. Azzam, "Stationary Property of Normal-Incidence Reflection from Isotropic Surfaces," J. Opt. Soc. Am. **72**, 1187 (1982).
6. D. E. Aspnes, and A. A. Studna, "Dielectric Functions and Optical Parameters of Si, Ge, GaP, GaAs, GaSb, InP, InAs, and InSb from 1.5 to 6.0 eV," Phys. Rev. B **27**, 985 (1983).