5-2014

A Semantic Conception of Truth

Jonathan Lumpkin
University of New Orleans

A Semantic Conception of Truth

An Honors Thesis

Presented to

the Department of Philosophy

of the University of New Orleans

In Partial Fulfillment

of the Requirements for the Degree of

Bachelor of Arts, with University Honors

And Honors in Philosophy

by

Jonathan Lumpkin

May 2014

Table of Contents

Abstract

I explore three main points in Alfred Tarski's *Semantic Conception of Truth and the Foundation of Theoretical Semantics*: (1) his physicalist program, (2) a general theory of truth, and (3) the necessity of a metalanguage when defining truth. Hartry Field argued that Tarski's theory of truth failed to accomplish what it set out to do, which was to ground truth and semantics in physicalist terms. I argue that Tarski has been adequately defended by Richard Kirkham. Development of logic in the past three decades has created a shift away from Fregean and Russellian understandings of quantification to an independent conception of quantification in independence-friendly first-order logic. This shift has changed some of the assumptions that led to Tarski's Impossibility Theorem.

Keywords: Alfred Tarski's Impossibility Theorem, Game Theory Semantics, and Independence-Friendly First-Order Logic, Semantics, Truth.

1.0: The Issue

‘Truth’ is central to a rational life: knowing whether a belief, idea, of hypothesis is true or not is an important part of the development of knowledge. It therefore becomes necessary to develop an understanding of truth and to not treat it as a primitive term. Creating precise conditions for ‘truth’ allows for a systematic evaluation of the truth-value of declarative sentences. Declarative sentences are of particular importance because they are the central linguistic entity for predictive sciences. Declarative sentences can assert a testable hypothesis that can be evaluated against the truth-conditions. ‘Truth’ is necessary to the furtherance of knowledge. Thus, how we characterize truth is of the upmost importance to the progress of science.

In the history of philosophy there have been a number of theories of truth that have fallen short in some respect or another. For example, perhaps the easiest targets are “coherence” theories of truth. Coherence theories of truth assert that the truth of declarative sentences is measured by their coherence (non-contradiction) with some ideal knowledge, which can be the “mind of god” or an “ideal and complete science.” A pragmatist will surely object that such a theory has no use. Coherence theories cannot produce a workable set of truth-conditions because the ideal set of true statements is only producible through the development of true statements. As such, coherence theories of truth do not supply the conditions necessary to provide a metric to test declarative sentences in science.

The twentieth century saw the rise of an obsession within philosophy of ridding itself of the mystical and finding solid grounding within the logical and physical world. I suspect much of the impetus for this came from the unsettling ideas of Georg Cantor, Ludwig Boltzmann, and Kurt Gödel who showed that mathematics rested on a foundation of sand. Certainty began to appear to be slipping away from European thought in a way that contradicted the Enlightenment foundations of modern Europe. Revolutions in logic and mathematics spurred a search for a new foundation on which to rest rationality.

The so-called Vienna Circle championed “logical positivism” which asserted that the only notions that were sensible (not nonsense) must be verifiable by some possible experience. Or “There is a teapot that orbits the sun” is verifiable by the possible experience of observing it, though the lack of contradictory evidence is not proof of its existence. For a brief and fleeting time, certainty appeared restored, however it was soon pointed out that the verification principle could not itself be verified by some possible experience. Chaos seemed poised to reign again.

This concern that preoccupied twentieth century philosophy is summed up well by Wittgenstein in his posthumously published work *On Certainty*, “At the foundation of well-founded beliefs lies belief that is not well-founded.”[1] Wittgenstein understood certainty as a psychological phenomenon. It is impossible to get meta-knowledge or extra-belief to be *sure* that some proposition is correct. Epistemology is a futile pursuit. The philosophical “to know” i.e. grounded certainty, must be given up. “I know” can only be used in the unrefined sense meaning something like “I can play this game.” How then should we overcome an infinite regress of foundations?

I believe that Wittgenstein conflated the possibility of obtaining *Truth* as such that is to say (in terms of coherence theories) the mind of god, with specifying precise truth-conditions. He was quite right to assert that this *Truth* was beyond our reach but wrong to believe that the truth of such-and-such a declarative sentence was akin to its functionality.

A half-cousin to logical positivism is physicalism. The physicalist project was to reduce all meaningful terms to logical, mathematical, and physical terms that were prima facie apparent to

[1] Ludwig Wittgenstein, *On Certainty*, 253.

rational observers. Science, thus, seemed the best place to build and develop knowledge, yet science itself could not deliver a precise formulation of truth by which to judge the predictions of science.

Alfred Tarski is one of the most important logicians of the 20th century. His work before the Second World War revolved around formalizing a notion of truth in a precise way to determine wherein truth lies. While not universally accepted by philosophers, Tarski's semantic conception of truth is by far the best formulation of truth because of its mathematical precision. I will consider some objections to Tarski's conception of truth after I explicate its character and then offer an apology for Tarskian truth.

Alfred Tarski set before himself the task to reduce semantic concepts to physical concepts in order to make semantics a respectable science. This goal led him to create the notion of satisfaction through a recursive technique that is motivated by a commitment to compositionality. Compositionality contends that the truth of a statement can be determined from its constituent parts and context is irrelevant. I contend accepting compositionality is unnecessary and overly restrictive, though not strictly incorrect. I intend to demonstrate this within the context of the physicalist program and explore its consequences for a general theory of truth.

2.0: Method and Presuppositions

In order to have language there must be some agreement of definitions and judgments.[2] Since language is public an agreement on what a sign means is needed for it to be a sign.[3] Customs form the agreement allowing language to exist and be meaningful. Without customs we cannot have language. Thus, I will generally proceed on pragmatic grounds.

I begin by reconstructing Alfred Tarski's pioneering conception of truth as a semantic concept. I will examine the physicalist program and how Hartry Field and Richard Kirkham have debated whether Tarski's semantic theory of truth lived up to its intensions. Then I study Jaakko Hintikka's use of the Skolem function to define a truth predicate in a first-order language, thereby avoiding the necessity to appeal to a richer metalanguage. Then I will show by analogy why Hintikka's first-order formulation of satisfaction cannot create a general theory of truth.

Unlike Wittgenstein I do not take this negative result to indicate that determining the truth of an individual declarative sentence is tantamount to believing it strongly. A general theory of truth is unnecessary to determining the truth of particular declarative sentences; *Truth* as such is a vestige of Platonic thought. We only need truth-conditions of a declarative sentence to determine its truth-value, and this fact does not necessarily entail a general theory of truth. A number is even if it is an integer that can be divided exactly by two – "evenness" does not enter into the picture.

[2] Ludwig Wittgenstein, *Philosophical Investigations*, 242.
[3] Ibid., 198.

3.0: *The Semantic Conception of Truth and the Foundations of Semantics*

The *Angst* of uncertainty in nineteenth and twentieth century thought led to a preoccupation with language. It became apparent that language could conjure up false problems, obscure and obfuscate the world. It then became apparent that to achieve a “high resolution” picture of the world, one must use precise language. One of the first philosophers to focus attention on language was Gottlob Frege. He explored how a term possessed a number of different and discrete qualities. As the “linguistic turn” further developed in Western philosophy the principle role of semantics became evident.

In “The Semantic Conception of Truth and the Foundations of Semantics,” Alfred Tarski's purpose is to identify the necessary and sufficient conditions for a sentence to be true, and to ground semantics in logical notions. Semantics is not a panacea for philosophical problems à la Wittgenstein, but a “modest science” concerning the relation between linguistic entities and the world. By defining semantic concepts in logic, we can be more convinced that our language can be the best mirror to the world possible; we would not inadvertently build our sciences upon meaningless linguistic concepts.

A satisfactory definition of truth must be materially adequate, meaning that it must capture the “old notion” of the term formalized by Aristotle.[4] Tarski considered the Aristotelian notion of truth to capture a primordial understanding of the term, if imprecise enough for Tarski’s project. An adequate theory of truth must also be formally correct by specifying the formal structure of the language and the formal rules used to apply the definition.[5]

[4] Alfred Tarski, “Semantic Conception” 341.
[5] Ibid., 342.

(3.1) Extension

Tarski limits the extension of the term 'true' to sentences within a given language. A term’s extension is all the “things” to which it refers. The term ‘voter’ refers to all things that vote. This is discrete from the meaning, intension, of that term. ‘Voter’ does not mean each and every thing to which it refers, it means “a person who votes or has the right to vote in an election.” Thus, for Tarski, the things to which ‘true’ refers are declarative sentences.

He does not include propositions because he believes their exact nature to be too vague. Propositions are the mental content communicated by a sentence. Ontologically, a proposition exists “over and above” a sentence. While Tarski does not directly dispute the existence of propositions, he considers their questionable ontological status as an unnecessary risk upon which to ground his conception of truth.[6]

‘True’ must be restricted to a given language, because a sentence true in one language may not be true in another language. The sentence ‘p’ may have two discrete intensions in two different languages, where one expresses an existing state of affairs and another does not. This leads to the contradiction where ‘p’ is true and not true (false or meaningless) at the same time.[7] For example, “‘Dog’ is a three-letter word” is true, while “‘Hund’ ist ein Drei-Buchstaben-Wort” is false. The meaning of individual sentences is dependent on the language in which it occurs.

We can solve this confusion by further restricting ‘true’ to declarative sentence-tokens. A sentence-token is a particular physical representative of a sentence. The air vibrations when someone says, “Hello, good morning” are a token. When a token is written down, then the words expressed in ink is another token. The sentence type is a linguistic entity and non-physical. Tokens can only be in one language at a time. If sentence tokens are used exclusively as truth bearers for

[6] Tarski, “Semantic Conception”, 342.
[7] Ibid., 342.

the extensional analysis of ‘truth’, then “truth” is restricted to whatever language in which the truth bearer occurs. Thus, the problem of restricting “truth” to “truth-in-L” is avoided in the first place.[8]

(3.2) Intension

The intension of the concept of 'true' should adhere to the Aristotelian conception of truth.[9]

(1) “To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, or of what is not that it is not, is true”[10] (Aristotle's *Metaphysics*).

Tarski paraphrases Aristotle into modern philosophical terminology, producing a correspondence theory of truth,

(2) “The truth of a sentence consists in its agreement with (or correspondence to) reality.”[11]

Alternatively,

(3) “A sentence is true if it designates an existing state of affairs.”[12]

Tarski does not *reject* these three formulations of the intensions of ‘true.’ Rather, these definitions should be consistent with the semantic conception of truth and are the intuitive sense of what ‘truth’ should mean.[13] They are just not precise and clear enough for the physicalist program[14] and for a modal theory for predicate logic.[15]

Taski’s semantic conception of truth is, he believed, the essence of the correspondence theory. We can see this intent in the material adequacy condition. A correct definition of truth should capture the intuitive sense shown in Aristotle’s metaphysics; that is, be materially adequate.

[8] Richard Kirkham, Theories of Truth, p. 68.
[9] Tarski, “Semantic Conception”, 342.
[10] Ibid., 343.
[11] Ibid.
[12] Ibid.
[13] Tarski, “Semantic Conception”, 343.
[14] Ibid.
[15] Kirkham, *Theories*, p. 142.

Aristotle's definition (1) is reformulated into modern terminology in the guise of a correspondence theory (2) and (3).

(3.3) Material Adequacy

A criterion for the material adequacy of the definition is based on the classical Aristotelian definition of truth. Using the classical notion (1) as a standard, we ask what would make 'snow is white' true or false?

(4) 'Snow is white' is true, if snow is white. ("…to say of what is that it is, or of what is not that it is not, is true.")

(5) 'Snow is white' is true, only if snow is white. ("To say of what is that it is not, or of what is not that it is, is false…")

Each of these two conditionals expresses a part of the Aristotelian definition of truth in a more formalized phrasing, which can be conjoined to form a biconditional.

(6) 'Snow is white' is true, if and only if snow is white.

This sentence shows wherein the truth of 'Snow is white' consists – the world. Tarski generalizes this methodology where 'p' is a sentence token and 'X' is the name of the sentence token 'p'.[16]

(7) (T) X is true if, and only if, p.

This is an equivalence of the form (T) and expresses the definition of truth Tarski believed more intuitively correct than either formulation of correspondence theory (2) or (3). Aristotle's definition (1) is better than the others because it can be translated into a formal expression, where "A sentence is true if it designates an existing state of affairs" cannot be translated into a form suitable for Tarski's purpose.

Tarski proposes that his conception of truth be called "the semantic conception of truth."

[16] Tarski, "Semantic Conception", 344.

Semantic concepts express relations between language and objects referred to by a linguistic phrase. The easiest way to set up a notion of true is through the semantic notion of satisfaction. Snow satisfies the condition 'x is white.' I return to the notion of satisfaction later. Because truth is understood semantically, the problem of theoretical semantics is inextricably tied to the problem of a satisfactory definition of truth.

(3.4) The Antinomy of the Liar

The Antinomy of the Liar demonstrates the contradiction resulting from using ‘truth’ within in the language that it is being applied.

(8) The sentence at 8 is not true.

(9) Let 'The sentence at 8 is not true.' be replaced by 's'

Using the equivalence of the form (T) we can assert,

(10) 's' is true if and only if, the sentence at 8 is not true.

Also 's' was established empirically such that,
(9) 's' is identical with the sentence at 1.

We can then replace 'the sentence at 8' as it occurs at (8) with 's' from the identity at (9) to deduce the antinomy,

(11) 's' is true if and only if, 's' is not true.

The antinomy of the liar is a reductio ad absurdum argument that shows that at least one presupposition must be false. Assumptions of the antinomy of the liar:

(12) We assumed that a semantically closed language can ascribe semantic properties such as 'true' to sentences in that language.

(13) “We have assumed that in this language the ordinary laws of logic hold.”[17]

[17] Tarski, “Semantic Conception”, 348.

(14) “We have assumed that we can formulate and assert in our language an empirical premise such as the statement (9) which has occurred in our argument.”[18]

The antinomy can be constructed without the help of premise (14). Thus, either (12) or (13), or both, must be rejected. Tarski thinks it would be too much to consider rejecting premise (13), so he considers (12) more closely to see if the problem lies there. It follows from rejecting premise (12) that languages must be semantically open to assert semantic properties such as ‘true.’

The issue here turns on the distinction between use and mention. Consider the term ‘running’ in the following to sentences.

(15) ‘Running’ is a gerund.

(16) I am running.

(15) mentions the term ‘running’ and (16) uses the term. If we conflate the use/mention distinction, confusion and contradiction can easily follow. Notably the confusion often arises when semantic properties such as, definition, meaning, or truth. We can avoid this confusion by never applying semantic concepts to terms in the same “language” in which they occur. We denote the separation between these two languages by single quotation marks. This notation is why Tarski’s semantic conception of truth is often referred to as “truth by disquotation.”

The two languages we use to make explicit the use/mention distinction for semantic concepts are call the object language and the metalanguage.[19] The object language is the language used to mention a term. The metalanguage is where one talks about an object language.

$$\overbrace{\text{'Snow is white}}^{\text{Object language}}\ \overbrace{\text{is true if and only if snow is white.}}^{\text{Metalanguage}}$$

[18] Tarski, “Semantic Conception”, 348.

[19] ‘Meta’ is from the Greek meaning after, over, or above, and should not necessarily be associated with metaphysics.

The occurrence of 'snow is white' in the metalanguage is the sentence itself, and the occurrence in the object language functions as a name of the sentence. The name of the sentence must be used as the subject term because the subject of a sentence must either be a noun or a phrase functioning as a noun. Also when speaking of an object, a sentence designates (denotes) that object with its name and not the object itself.

(17) (T) X is true if, and only if, p.[20]

A sufficient metalanguage must: contain the object language as a part so it can express 'p' in (T), be able to express and name 'X' in (T), and have a general logical structure to express 'if, and only if,' and semantic terms (viz., 'true').[21] An essentially richer language has variables of a higher type than another language. A metalanguage is essentially richer than an object language because it contains all of the object language and more terms that cannot be fully expressed in the object language.

Truth can be defined in terms of the semantic notion of satisfaction, which is a relation between an arbitrary object and a predicate function, e.g. 'x is white' (Tarski refers to these functions as sentential functions). The predicate function 'x is white' is not a sentence because it contains the unbound variable 'x.'

> A sentence can now be defined simply as a sentential function which contains no free variables....*a sentence is true if it is satisfied by all objects, and false otherwise*.[22]

---

[20] This sentence form is called a "Schema-T", "T Schema", or "Sentence of the form Schema T."
[21] Tarski, "Semantic Conception," 350-351.
[22] Ibid., 353.

The sentence meets the satisfaction condition when each term in the object language is matched with a thing or set of things in the world. 'Snow is white' is true if and only if snow is white.

4.0: Analysis of "The Semantic Conception of Truth and the Foundations of Semantics"

(4.1) Satisfaction and Physicalism

Physicalism is the project whereby all "intellectually respectable" concepts are reduced to physical or logico-mathematical concepts.[23] One of Tarski's goals is to make the science of semantics intellectually respectable by defining all semantic concepts, except for satisfaction, in terms of truth and then truth defined in terms of satisfaction.[24] Satisfaction was then defined in logical terms through recursive definition.[25] Thus, the physicalist program is fulfilled by grounding theoretical semantics in logic.

An extensional definition of truth in a language with a finite number of sentences and without logical operators can be materially adequate by combining all possible equivalences of the form T. Consider language L with three sentences: "The ball is red.", "The sky is blue.", and "All mice are white". The definition of truth in this language would be,

(18) (s)(s is true if and only if[26] (s = 'The ball is red' and the ball is red)
or (s = 'The sky is blue' and the sky is blue)
or (s = 'All mice are white' and all mice are white))[27]

(18) provides the definition of satisfaction in language L. The logical conjunction of all T-sentences of a language is an adequate extensional analysis of truth in language L.[28] The issue of whether or not such an extensional analysis captures a general theory of truth is explored later.

[23] Richard Kirkham, "Tarski's Physicalism" 289.
[24] Tarski, "Semantic Conception", 345, 364-366.
[25] Ibid., 352-353.
[26] "If and only if" sometimes hereafter is shortened to "iff."
[27] Kirkham, *Theories*, p. 145.
[28] Tarski, "Semantic Conception", 344-345.

If a language has logical operators, then an infinite number of compound sentences can be formed from a finite number of atomic (basic) sentences. When logical connectives (negation and disjunction) are introduced to language L, then we may, form an infinite number of sentences. For example, "It is false that the ball is red." or "The sky is blue or all mice are white." (This new artificial language will be referred to as L_1.)

The methodology for the definition of truth (18), while adequate for language L, fails to capture all extensions of truth in L_1. Quite simply, it is impossible to list an infinite number of sentences. A "recursive definition" can capture an adequate definition of truth in language L_1.[29] Logical connectives function as rules for computing the truth value of their atomic parts. "It is false that the ball is red" is true iff the atomic component "The ball is red" is false. Recursion uses these rules to define all possible sentences in Language L_1 despite the fact that there are an infinite number of possible sentences.

(19) (s)(s is true iff either (s = 'The ball is red' and the ball is red)
or (s = 'The sky is blue' and the sky is blue)
or (s = 'All mice are white' and all mice are white)
or (s = 'not p' and it is not true that p)
or (s = 'p or q' and either it is true that p or it is true that q))

Thus, the definition at (2) is able to capture all extensions of true in Language L_1 through recursion at (19).[30] There must, however, be a finite number of basic "rules" for combining sentences for this method to work.

Tarski is not interested in languages such as L_1 or L_2. The goal for his semantic conception of truth is to provide a model theory for first-order predicate logic.[31] Predicate logic offers new complications to an adequate extensional analysis of truth because two non-truth functional

[29] Kirkham, *Theories,* p. 146.
[30] Ibid., p. 149.
[31] Ibid., p. 150.

components can be combined to create truth-functional sentences.

Open sentences are non-truth functional phrases in predicate logic where there is an free variable, e.g. “x is red”. Open sentences do not have a truth value because they do not assert anything. If “ball = x” is conjoined with “x is red”, then the conjunction has a truth value. Alternatively, we could bind the variable “x” with a quantifier e.g. “(x) (x is red)”, which has a truth value. The problem is there are an unlimited number of quantified sentences whose parts are open sentences without a truth value.

Recursion captures the truth of compound sentences by computing the truth value of its component parts by following the rules of the truth-functional logical connectives. Given that open sentences have no truth value, then recursion cannot capture all possible extensions of ‘true.’ The inability of the recursive technique requires Tarski to find a new solution to his extensional analysis.

Satisfaction is a property of open sentences and truth-functional sentences determined either by the truth value of a sentence’s components or by the satisfaction of an open sentence’s components. Truth can be computed from whether or not a sentence’s components possess satisfaction.[32] Using satisfaction with recursion, we can adequately define ‘satisfaction’ for a language with quantification and then define truth by satisfaction.[33] “Satisfaction” constructed as such allows Tarski to recursively define open sentences and quantifications while still meeting the material adequacy condition. Kirkham formalizes Tarski’s truth by satisfaction, “A sentence is true if it is satisfied by all objects, and false otherwise”[34] as,

(20) (s) (s is true iff s is satisfied by all sequences of objects)[35]

---

[32] Kirkham, *Theories*, p. 152.
[33] Ibid., p. 158.
[34] Tarski, “Semantic Conception”, 353.
[35] Kirkham, *Theories*, p. 153.

‘s’ ranges over sentences.” Satisfaction is a relational property between language and the world, a semantic concept. “x is red” is satisfied by any red thing because every red thing satisfies the predicate in the open sentence. A red ball satisfies “x is red” because the ball is red. When there is more than one variable in an open sentence, there must be more than one object that satisfies every variable.

Expressing relational predicates formally requires at least two variables, and for those variables to be in a certain order. A sequence of individuals is therefore needed. ‘x climbed y’ is satisfied by the sequence {Tenzing Norgay, Mount Everest} etc., but not by the sequence {Mount Everest, Tenzing Norgay} etc.

Universally quantified sentences compute their satisfaction by deleting their quantifier to create an open sentence. ‘(x_1) (x_1 is red)’ becomes ‘x_1 is red’. If the open sentence is satisfied by every sequence, then the universal quantification is satisfied.‘{blood, water, etc.}’ satisfies ‘x_1 is red’ but ‘{water, blood, etc.} fails to satisfy ‘x_1 is red’ and therefore ‘(x_1) (x_1 is red)’ is not satisfied.

An adequate definition of truth would have to define the satisfaction conditions for every predicate within a language. Kirkham points out that this was partly hidden within Tarski’s work which focused on set theory, a language with only one predicate; namely, “x is in y.” The problem that this poses occurs when the semantic theory of truth is applied to a natural language that has potentially unlimited predicates. This issue cannot be circumvented by recursion because each predicate acts as the “atomic” building block for a sentence’s satisfaction and therefore truth. It may be a more enlightened position to take that a complete definition of *Truth* is impossible to accomplish, though unnecessary as I intend to show later.

Physicalism à la Otto Neurath and Rudolf Carnap sought to reduce the "non-observable" to "observable."[36] This reduction would create a language for unified science able to formulate predictive sentences with clear empirical content.[37] The goal of this conception of physicalism is to predict but not explain, as Hartry Field characterized physicalism in "Tarski's Theory of Truth."[38]

The biconditional logical operator in the Schema-T expresses an extensional equivalence and not an explanation of the left bijunct, "…a Tarskian definition of truth is in no sense an explanation of its definiendum."[39] We can see that this is the case from the material adequacy condition stipulating that all equivalence of the form Schema-T follow from it.[40] Fields expects the recursive definition to do the work of theoretical physics. However, an extensional equivalence is sufficient because it ensures an equivalent truth value between the bijuncts.

Tarski grounds semantics in physicalist terms in the following way. Semantic terms are defined in terms of truth, and truth is defined in terms of satisfaction. Satisfaction is in turn defined through recursion and the definition of a language's predicates.[41]

> Consider a Tarskian definition of satisfaction for a language L, where L contains (1) an infinite series of variables, $x_1 \ldots x_i$; (2) a universal quantifier; (3) two truth functional operators, for negation and disjunction; (4) only the predicates 'is an electron', 'is round', and 'is a subset of'; and (5) no names at all (so its only closed sentences are quantified sentences). In what follows, 'z', 'ψ', and 'ϕ' all range over both genuine sentences and open sentences.
>
> (z) [z is satisfied by an infinite sequence S ≡
> (z = 'x_k is an electron', for some k, and the k^{th} object in S is an

[36] Kirkham, *Theories*, p. 289.
[37] Tarski, "Semantic Conception", 347.
[38] Kirkham, *Theories*, p. 289-291
[39] Ibid., 291.
[40] Tarski, "Semantic Conception", 344.
[41] Kirkham, *Theories*, p. 173.

electron)
or
(z = ‘x_k is round’, for some k, and the k^{th} object in S is round)
or
(z = ‘x_k is a subset of x_j’, for some k & j, and the k^{th} object in S is included in the jth object in S)
or
(z = ‘~ϕ’ and S does not satisfy ϕ)
or
(z = ‘ψ or ϕ’ and either S satisfies ψ or S satisfies ϕ)[42]

Truth as satisfaction is hereby grounded in the definition of the predicates in L. The once vague term ‘true’ is given a precise definition in terms of the physical concepts roundness, electronness, and subsetness.

In order to avoid antinomies, we must create a formal language whose structure is exactly specified: (a) Express exactly the words and phrases that are meaningful, (b) Enumerate all undefined terms, (c) Give rules of definition to introduce new defined terms, (d) Express what arrangement of terms which makes a sentence, (e) expresses in conditions what a sentence can be asserted, (f) give all axioms, (g) give all rules of inference.[43] Tarski suggests that if a non-formalized language can obtain such a specified structure, then it can “replace everyday language in scientific discourse.”[44]

Tarski’s semantic conception of truth allows a formulation of predictive sentence tokens that have precise conditions for being true or false. “‘Snow is white’ is true if and only if snow is white.” is not a vacuous sentence as it seems on its face. An important basis for the recursive approach is the notion of compositionality. Compositionality asserts that the parts of a complex expression can be “computed” to discern the semantic attributes of the whole expression. This is why recursion defines from the inside out.

[42] Kirkham, “Physicalism”, 290
[43] Tarski, “Semantic Conception”, 346.
[44] Ibid., 347.

(4.2) – Alfred Tarski's Impossibility Theorem, Game-Theory Semantics, and Independence-Friendly First-Order Logic

Tarski's impossibility theorem states that a truth predicate cannot be defined within a first-order language, but must be defined within a "richer" second-order metalanguage.[45] Thus, truth must be characterized is the form Schema-T, when 'X' is the name of a sentence token and 'p' is the sentence token itself.[46]

(21) 'X' is true ≡ p.

This prevented using Tarski's theory of truth in natural languages such as English, French or German.[47] "[W]e do not have any richer metalanguage available to us beyond our actual working language, Quine's "home language."[48] If we adhere strictly to Tarski's precepts, then truth cannot be formulated in English, because it is a first-order language and truth must be defined in a richer metalanguage.[49]

If we accept the material adequacy condition for Schema T, then the semantic conception of truth can only be applied to formal languages which are semantically open, have a finite number of predicates, and no names. Such languages include mathematics, set theory, and predicate logic. Indeed, having a definition of truth for set theory is an important accomplishment for the mathematician. However, Tarskian truth (according to Tarski) cannot be applied to what is often the most interesting of sentences, those which occur in natural languages.

[45] Tarski, "Semantic Conception", 351.
[46] Ibid., 343-344.
[47] Hintikka, "Assumptions", 354.
[48] Ibid., 354-355.
[49] However, this is not the exact rational Tarski himself used to justify the impossibility of defining 'true' in a natural language but it still follows from his premises.

A necessary condition for a well formed T-schema sentence is that the corresponding M-schema sentence has a one to one correspondence between terms. For example, "'Snow is white' means that snow is white." In a natural languages many sentences lack this correspondence due to subtle aspects of natural language such as irony, sarcasm, humor, etc. which do not occur in formal languages. For example, consider the sentence, "I could eat a horse" the corresponding T-schema would be

(22) 'I could eat a horse' is true, iff I could eat a horse.

However this partial definition of truth is false when it is possible that "I could eat a horse" is true when one could not eat a horse. The reason for this malfunction is that the corresponding M-sentence does not have a one-to-one correspondence between the two sides of the statement.

(23) 'I could eat a horse' means I am very hungry.

We can easily solve this problem by stipulating that the left side of the T-schema is the sentence as it appears and that the right side be the right side of the M-sentence of that sentence. But this solution disrupts the entire project Tarski was seeking if it was applied to natural languages. We are left asking what does 'x' really mean, obscuring the condition for truth. These issues seen above actually point to the problem of compositionality. We cannot determine the truth-value of these declarative sentences from their constituent parts alone; we must include the context of the messy natural language.

> By substituting "anyone can become a millionaire" for 'p' and its quote for 'X' and by taking only one half of the equivalence we can see that [it] entails

(24) "Anyone can become a millionaire" is true if anyone can become a millionaire

But [24] is not true: one person making a cool million does not imply that everyone can do it.[50]

'Anyone' functions both as a universal and existential quantifier in [24]."[51] What Hintikka demonstrates here is that natural languages are not compositional and cannot therefore use Tarskian truth.[52]

Tarski's impetus for this prohibition is the unnecessarily restrictive understanding of quantifiers found in Frege and Russles first-order logic (FOL).[53] "Frege's folly" is to assume that in a string of quantifiers each subsequent quantifier is dependent of all preceding quantifiers.[54] Another rationale for the impossibility theorem is that truth in FOL is compositional - that is, dependent only on its constituent parts and not on any context.[55] "Hintikka shows how such a truth definition can even be formulated in the language of [independence friendly logic] itself, thus avoiding Tarski's famous impossibility results."[56] English or any natural language, on the other hand, was not thought to be compositional by Tarski.[57]

Hintikka develops Game-Theory Semantics (GTS)[58] to demonstrate that such dependence is unnecessary in terms of truth.[59] So-called *independence-friendly first-order logic* (IFL) "allows

[50] Hintikka, *The Principles of Mathematics Revisited*,p. 139.
[51] Kirkham, *Theories*, p. 244.
[52] Jaakko Hintikka, "Post-Tarskian Truth," Synthese, Vol. 126, No. 1/2, (2001), 22-23
[53] Ibid., 21-22.
[54] Robert Reed, Review of *The Principles of Mathematics Revisited* by Jaakko Hintikka p. 134.
[55] Hintikka, "Assumptions", 356.
[56] Reed, Review of *Principles*, p. 135.
[57] Ibid., 356.
[58] Hintikka shows on page 142 of *The Principles of Mathematics Revisited* how GTS and IFL can avoid the Antinomy of the Liar.
[59] Ibid., p. 134.

arbitrary patterns of dependence between normal (that is otherwise familiar) quantifiers."[60] '$(\forall x)$ $(\exists y)$ $F(x, y)$' can be written as '$(\forall x//\exists y)$' to demonstrate independence between quantifiers. GTS works by two players interacting to unravel an expression, this outside-in approach is effectively the opposite of the recursive technique used by Tarski due to his assumption of compositionality. GST requires the "conversational" context in order to ascertain the semantic attributes of an expression and is therefore non-compositional and a counter-example to Tarskian truth.[61]

IFL allows the use of a Skolem function to create a truth predicate within a first-order language, provided that that language can express its own syntax through Gödel numbering.[62] Tarski's use/mention distinction can be replaced by Gödel numbering, 'g (S)' is the Gödel number for 'S'. The reduction to Skolem normal form removes existential quantifiers leaving only universal quantifiers.

(25) $(\forall\alpha)\ (\exists\beta)\ (\forall\gamma)\ P\ (\alpha, \beta, \gamma) \leftrightarrow (\forall\alpha)\ (\forall\gamma)\ P\ (\alpha, f\ (\alpha), \gamma)$

The existential quantifier '$\exists\beta$' is replaced by '$f\ (\alpha)$' where '$f\ (\alpha)$' maps α to β. The equivalency expressed by the biconditional is one of satisfaction, that is, the "witness expression" is satisfied if and only if the original form is satisfied but not equivalent in all respects.

If S is

(26) $(\forall x)\ (\exists y)\ F(x, y)$

The SFN of "$(\forall x)\ (\exists y)\ F(x, y)$" is,

(27) $(\forall x)\ F\ [x, f(x)]$.[63]

---

[60] Hintikka, "Post-Tarskian Truth", 22.
[61] Hintikka, *The Principles of Mathematics Revisited*, 110.
[62] Ibid., 22.
[63] Hintikka, "Post-Tarskian Truth", 30.

“In order words”, says Hintikka, “the function f yields as its value for different individuals x precisely the individuals f (x) the likes of which have to exist in order for S to be true.”[64]

Rather than using a sentence of the form Schema-T that must appeal to a richer metalanguage, “the truth- condition [sic] of a first-order sentence in a negation normal form is the existence of a full array of what are known as its Skolem functions.”[65] Hintikka’s IFL allows a truth predicate to be defined, in a first order language,[66] by the equivalence,

(28) $(\forall\alpha)(\exists\ \beta)\ S\ [\alpha, \beta] \leftrightarrow (\exists\ f)\ (\forall\alpha)\ S\ [\alpha, f(\alpha)]$.[67]

Hintikka points out that this equivalence is very similar to the axiom of choice in set theory.

Hintikka is not without critics. Jouko Vaananen argues that independence-friendly logic is tantamount to a second-order logic.[68] If Vaananen is correct, then satisfaction remains in the domain of the second-order. But this does not affect the important result from Hintikka’s work, which is to demonstrate that compositionality is neither necessary nor always helpful to developing a characterization of the truth-conditions for declarative sentences. Sentences in natural languages are often dependent on their context within a conversation, inflection, and etc. The semantic conception of truth can survive just fine without aligning itself to such an unwieldy assumption as compositionality.

By casting it aside, Tarski’s conception of truth can be more broadly applied without losing too much precision. We must resort to a more pragmatic understanding of language when operating

[64] Ibid., 30 – 31.
[65] Hintikka, “Assumptions”, 357.
[66] Hintikka, *The Principles of Mathematics Revisited*, p. 110-120.
[67] Ibid., 357.
[68] Jouko Vaananen, “Second-Order Logic and Foundations of Mathematics,” The Bulletin of Symbolic Logic, Vol. 7, No. 4 (2001).

with natural languages. There is an initial hesitance that the messiness of natural languages disrupts the physicalist program and returns thought to the mercy of relativism, but this is not so. Precision is only useful when its application is itself useful.

(4.3) – A General Theory of Truth

Tarski suggests that a conjunction of all sentences instantiated in Schema-T would, in a sense, be a general theory of truth.[69] I would like to explore why Tarski understood that such a conjunction would be a general theory of truth in a very weak sense. Each instance of an equivalence of the form (T) can be regarded as a partial truth, where the definition of truth would be a conjunction of all individual partial definitions. This would be impossible because semantic predicates such as 'is true' and 'is false' cannot be used by an object language to describe itself. Only a metalanguage may ascribe these predicates to a proposition in an object language. Thus, a conjunction of all T-sentences, in all possible languages, would require a violation of Tarski's distinction between object language and metalanguage by creating a semantically closed set of languages.

However, if Hintikka is correct in asserting that satisfaction can be defined in a first-order language, then is a general theory of truth possible? I will show by a mathematical analogy how the nature of such an infinite conjunction of all Schema-T sentences would not constitute *Truth* as such. Rather such a conjunction stands in contrast to some infinite series in mathematics that are countable, well defined, reducible to finite solutions.

A geometric series is an infinite series that is well defined.

[69] Hintikka, *The Principles of Mathematics Revisited*, p. 344.

(29) 1/2 + 1/4 + 1/8 + 1/16… = 1

This property arises from the regularity of the series. This geometric series defines '1'.[70] Other series will quickly run off to infinity. A conjunction of all propositions lacks the properties that allow a geometric series to be well defined.

Riemann zeta function

$$\zeta(s) = \sum_{n=1}^{\infty} n^{-s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots \qquad \sigma = \Re(s) > 1.$$

where s = -1,

(30) 1 + 2 + 3 + 4 + 5… = -1/12.

This zeta function demonstrates how a series that appears to expand towards infinity can uniquely define a finite number. One could imagine a similar process for all sentences that could somehow arrive at *Truth*. I think the final nail in the coffin is however the uncountability of the set of all sentences.

Countable infinite series of numbers,

Whole numbers	[1, 2, 3, 4, 5, 6…]
Integers	[0, -1, 1, -2, 2, -3, 3, -4, 4…]
Fractions	[1/1, 1/2, 1/3, 1/4…
	2/1, 2/2, 2/3, 2/4…
	3/1, 3/2, 3/3, 3/4…
	⋮]

Uncountable infinite series of numbers,

Real Numbers	[.01245714,
	.87298712,
	.00015792,
	⋮]

[70] Tarski, "Semantic Conception", 345.

Georg Cantor's diagonal line argument shows that real numbers are an uncountable set. If we take the digit in the tenth place of the first number, hundredth place of the second number, and so forth, and change every '0' to '5' and any other number to '1', then we will have a number unique to the infinite list.

A language can have an uncountable infinite number of sentences simply by having a sentence saying each and every decimal is a number. If the number of decimals is an uncountable infinite, then the number of sentences about each and every decimal is an uncountable infinite. Thus, even given an infinite amount of time it would be impossible to list every sentence.

What I intend to illustrate here by mathematical analogy is that an infinite series is not necessarily a prohibition of arriving at some finite definition. However, the type of infinity that an infinite conjunction of all instantiated Schema-T could not be reduced to a finite general theory of truth. *Truth*, as such, is not obtainable in the same sense that an infinite conjunction of all just acts would not constitute *Justice* as such. It would capture the extensional definition of truth-in-L, but not an intensional definition which is what Socrates sought exactly what cannot be found. A positive result of this analogy is that it is unnecessary to have a general theory of truth in order to use the term 'true' properly.

5.0: Conclusion

Hintikka's discoveries in *The Principles of Mathematics Revisited* and subsequent articles[71] does not disprove Tarski's impossibility theorem, but rather avoids the need to appeal to an essentially richer metalanguage. Using GTS and IFL, a truth condition can be defined within

[71] Hintikka, "Post-Tarskian," 2001. & Jaakko Hintikka, "On Tarski's Assumptions," Synthese, Vol. 142, No. 3, (2004).

the language in which truth is being predicated. Furthermore, this new definition of a truth predicate does not upset the physicalist program pursued by the semantic conception of truth. A truth predicate contained within a first-order logic is a more elegant and satisfying solution and still reduces semantics to logical concepts. However, a first-order definition of satisfaction cannot be used to create a "well defined" general theory of truth.

6.0: Bibliography

Feferman, Anita and Feferman, Solomon. *Alfred Tarski: Life and Logic*. Cambridge University Press, 2008.

Field, Hartry. “Tarski's Theory of Truth.” *The Journal of Philosophy* Vol. 69, No. 13 (1972): pp. 347-375. http://www.jstor.org/stable/2024879

Hintikka, Jaakko, “On Tarski’s Assumptions”, Synthese, Vol. 142, No. 3, (2004). pp. 353-369.

Hintikka, Jaakko, “Post-Tarskian Truth”, Synthese, Vol. 126, No. 1/2, (2001). pp. 17-36.

Hintikka, Jaakko. *The Principles of Mathematics Revisited*. Cambridge: Cambridge UP, 1996. http://philosophy.ru/library/philmath/eng/hintikka_principle_math_rev.pdf

Kirkham, Richard. “Tarski's Physicalism” *Erkenntnis*, Vol. 38, No. 3 (May, 1993), pp. 289-302 http://www.jstor.org/stable/20012479

Kirkham, Richard. *Theories of Truth: A Critical Introduction.* Combridge, MA: MIT Press. 1995.

Niiniluoto, Ilkka. “Tarski's Definition and Truth-makers.” *Annals of Pure and Applied Logic, Vol.* 126, (2004):57-76. http://www.sciencedirect.com/science/article/pii/S0168007203000939

Robert Reed, Review of *The Principles of Mathematics Revisited* by Jaakko Hintikkam Cambridge, Cambridge University Press, 1996.

Tarski, Alfred. “The Semantic Conception of Truth: and the Foundation of Semantics” *Philosophy and Phenomenological Research*, Vol. 4, No. 3 (1944): pp.341-376.

Jouko Vaananen, “Second-Order Logic and Foundations of Mathematics,” The Bulletin of Symbolic Logic, Vol. 7, No. 4 (2001).

Wittgenstein, Ludwig. *On Certainty*. Ed. G. E. M. Anscombe and G. H. Von Wright. Trans. Denis Paul and G. E. M. Anscombe. New York: Harper & Row, 1972. Print.

Wittgenstein, Ludwig. *Philosophical Investigations* Trans. G. E. M. Anscombe. Third ed. Oxford: Blackwell, 2001.

APPROVAL SHEET

This is to certify that Jonathan Deyo Lumpkin has successfully completed his Senior Honors Thesis, entitled:

A Semantic Conception of Truth

Robert Stufflebeam — Director of Thesis

Edward R. Johnson — for the Department

Abu Kabir Mostofa Sarwar — for the University Honors Program

April 30, 2014
Date