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The optical transmissivity of a mirrorless, nonlinear, absorbing dielectric thin film is investigated numerically. The dielectric function in the film region is dependent on the intensity of the electromagnetic field. Multivalued solutions of transmissivity as a function of incident power are calculated for the steady-state wave equation. The numerical solution is applied to two different model dielectric functions. As the absorption parameter is increased, larger values of incident intensity are required to switch the systems between stable output states. Also, the peak values of transmissivity are reduced as the absorption is increased.

In recent years much attention has been focused on the study of optically bistable systems, i.e., systems that can be switched optically between at least two stable states.[1] The earliest bistable systems used either optical feedback (in the form of a Fabry–Perot resonator) or electrical feedback to create bistability.[2] Optical bistability has been observed experimentally by using devices composed of a nonlinear semiconductor (GaAs) in a Fabry–Perot resonator[3] or by using polished parallel faces of a semiconductor (InSb) to form a resonator.[4] An optically bistable system using CuCl in a resonator was studied theoretically by Sung *et al.*[5] However, semiconductors may exhibit bistability without any external reflecting surfaces.[6] Chen and Mills[7] recently presented a theoretical study of the optical transmissivity of a mirrorless, nonlinear dielectric thin film. Their analysis considers only one type of nonlinearity and does not include the effects of absorption. Any real system will lose some energy to absorption. For a single thin film this energy loss may be small, but a practical logic system might consist of a cascade of bistable devices in which the output of one device would serve as the input for the next. The small amount of absorption in each device would need to be considered.

In this Letter we present a numerical solution for the transmissivity of a mirrorless, nonlinear, absorbing thin film, for which no closed-form solution is known to exist. The nonlinear effects are treated classically by using complex dielectric functions in the film region that are dependent on electromagnetic intensity. The numerical solution is tested for the nonabsorbing system studied by Chen and Mills.[7] In addition, the formulation is applied to a model semiconductor thin film (CuCl), which includes an excitonic nonlinearity,[5] and mirrorless bistability is investigated for this type of nonlinearity.

The geometry of the problem is shown in Fig. 1. A plane electromagnetic wave is normally incident upon a thin film of nonlinear material, of thickness d, which is surrounded by vacuum. The amplitude of the incident wave is E_0, the amplitude of the reflected wave is RE_0, and the amplitude of the transmitted wave is TE_0. Inside the film, the electric field obeys the nonlinear wave equation

$$\frac{d^2E}{dz^2} + k_0^2 n^2(I) E = 0, \tag{1}$$

where k_0 is the vacuum wave number equal to ω/c, $n(I)$ is the refractive index of the nonlinear film, and $I = |E|^2$ is the field intensity. Since the solution of the wave equation in an inhomogeneous medium cannot be uniquely divided into the sum of a direct and reflected wave,[8] the electric field is written as $E(z) = E_0 A(z) \exp[i\phi(z)]$, where both $A(z)$ and $\phi(z)$ are real. Substitution of this expression for $E(z)$ into Eq. (1) and separation into real and imaginary parts yields two coupled, second-order, nonlinear differential equations,

$$\frac{d^2A}{dz^2} - A\left(\frac{d\phi}{dz}\right)^2 + k_0^2 \epsilon_r(I) A = 0 \tag{2}$$

and

$$A\frac{d^2\phi}{dz^2} + 2\frac{d\phi}{dz}\frac{dA}{dz} + k_0^2 \epsilon_i(I) A = 0. \tag{3}$$

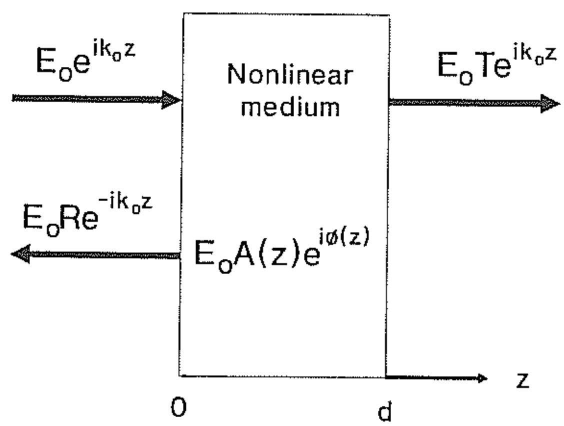

Fig. 1. Electromagnetic wave normally incident upon a nonlinear thin film is partially reflected and partially transmitted.

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In these equations, the index of refraction has been written in terms of the intensity-dependent complex dielectric function, $n^2(I) = \epsilon_r(I) + i\epsilon_i(I)$. Equation (3) is then written as

$$\frac{dW(z)}{dz} + k_0^2\epsilon_i(I)A^2(z) = 0, \tag{4}$$

where

$$W(z) = A^2(z)\,\frac{d\phi(z)}{dz}. \tag{5}$$

Using Poynting's theorem for a time-harmonic electromagnetic field with no current flow,[9] it can be shown that Eq. (4) is the equation that governs the flow of electromagnetic energy through the film and that the quantity $W(z)$ is proportional to the Poynting flux.[10]

To solve for the transmissivity of the nonlinear film requires the integration of the second-order nonlinear Eq. (2), coupled with the first-order nonlinear Eq. (4), and for the boundary conditions at both ends of the film to be simultaneously satisfied. If the dielectric function is real (no absorption), Eq. (4) says that $W(z)$ is constant; i.e., the Poynting flux is independent of the z coordinate. Equation (5) can then be used to eliminate $d\phi/dz$ from Eq. (2), uncoupling the two equations. Chen and Mills[7] show that Eq. (2) has an analytical solution for the real dielectric function, $n^2(I) = n_{\text{lin}}^2(1 + \alpha I)$, where n_{lin} is the linear index of refraction and α is a parameter that controls the strength of the nonlinearity. However, for a general complex, nonlinear dielectric function, the coupled Eqs. (2) and (4) must be solved numerically. Also, to solve for boundary conditions at both film surfaces becomes more complicated because the energy flow through the film is not constant.

At normal incidence the electric field is entirely tangential to the interface, regardless of the polarization. The boundary conditions reduce to continuity of the electric field and its normal derivative at both $z = 0$ and $z = d$ (these equations are given in Ref. 7). We obtain a conservation of energy relationship between these boundary conditions that accounts for the energy lost to absorption. This relationship is derived by equating the Poynting flux across the boundaries and is given by

$$|T|^2 + |R|^2 + (1/k_0)[W(0) - W(d)] = 1, \tag{6}$$

where the third term is the ratio of energy lost to absorption with respect to the incident flux. By writing $|R|^2$ in terms of $A(z)$ and its normal derivative at $z = 0$, Eq. (6) translates into the following relationship between the boundary conditions at $z = 0$, necessarily satisfied for a physically valid solution[10]:

$$A^2(0) - A(0)\left[4 - \frac{1}{k_0^2}\left(\left.\frac{dA}{dz}\right|_{z=0}\right)^2\right]^{1/2} + \frac{1}{k_0}W(0) = 0. \tag{7}$$

The solution proceeds like an initial value problem. A trial (initial) value of the transmissivity $|T|^2$ is chosen, which determines $A(d)$. The two other initial values required for the solution are extracted from the boundary conditions at $z = d$ and are $A'(d) = 0$ and $\phi'(d) = k_0$,[10] where the primes denote derivatives with respect to z. By using Eq. (5), the value of $W(z)$ at $z = d$ is then given by $W(d) = k_0A^2(d)$. With these initial values, Eqs. (2) and (4) are numerically integrated backward to the input side of the film. A valid solution is one that meets the conservation of energy requirement of Eq. (7).

We use this numerical technique to solve for the transmissivity of a thin film having a dielectric function given by $n(I) = n_{\text{lin}}^2(1 + \alpha I + i\beta)$. This dielectric model is similar to the one used by Chen and Mills[7] but is altered to include a small imaginary term, β. We vary β as a parameter to study the role of absorption in this model and to compare our results with those of Chen and Mills.[7] This nonlinear refraction model with a linear absorption term is a Kerr-type nonlinearity and is indeed observed for small-gap semiconductors such as InSb.[11] Since the amplitudes $A(z)$, R, and T are measured in terms of the incident field E_0, we set $E_0 = 1$ and calculate the transmissivity as a function of the nonlinearity parameter $\alpha' = \alpha|E_0|^2$. Note that a plot of transmissivity versus α' is equivalent to a plot of transmissivity versus the incident laser intensity. Multivalued solutions of transmissivity are shown in Fig. 2 for various values of the imaginary term (β) in the dielectric function. The film has a linear index of refraction $n_{\text{lin}} = 8$ and a thickness $d = 0.4\lambda_f$, where $\lambda_f = 2\pi c/\omega n_{\text{lin}}$ is the approximate optical wavelength inside the film. For $\beta = 0$, our results agree with those of Chen and Mills.[7] As β increases, the region of bistability becomes narrower and larger values of incident intensity are required to switch the system from a low-output state to a high-output state and vice versa. Also, peak values of transmissivity are reduced as β increases.

The numerical solution is also applied to a model dielectric function used for CuCl,[5]

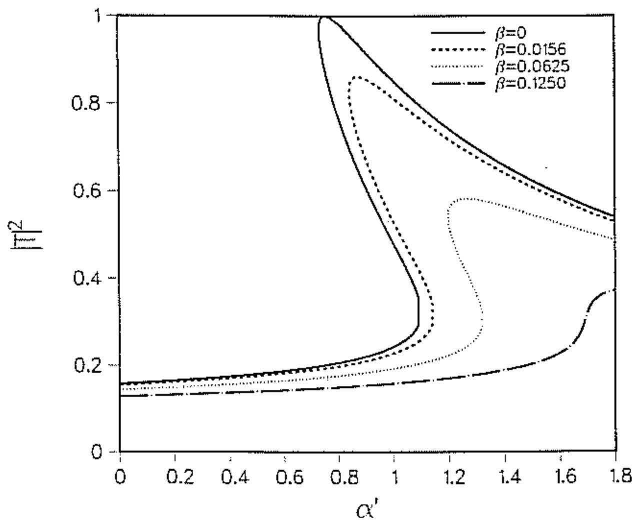

Fig. 2. Transmissivity of a nonlinear thin film as a function of the parameter α' (which is proportional to the incident laser intensity) for various values of the imaginary term β. The linear index of refraction $n_{\text{lin}} = 8$, and the film thickness $d = 0.4\lambda_f$.

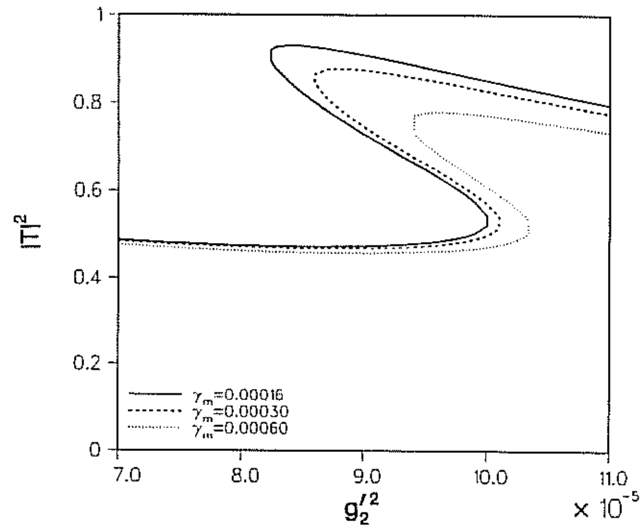

Fig. 3. Transmissivity of a nonlinear thin film, using the CuCl dielectric model, as a function of the parameter $g_2'^2$, which is proportional to the incident laser intensity. The values of the biexcitonic damping constant γ_m are given in electron volts. The incident frequency $\omega = 3.177$ eV, and the film thickness $d = 5$ μm.

$$\epsilon(I) = \epsilon_\infty + \frac{4\pi g_1^2}{\delta - g_2^2 I/\Delta}, \quad (8)$$

where $\delta = \omega_x - \omega - i\gamma_x$ and $\Delta = \omega_m - 2\omega - i\gamma_m$. The values used for the material constants in Eq. (8) are $\epsilon_\infty = 5$, $\omega_x = 3.2027$ eV, $\omega_m = 6.3725$ eV, $4\pi g_1^2 = 0.0275$ eV, and $\gamma_x = 0$. Sung *et al.*[5] point out that various values of the biexcitonic damping constant γ_m are given in the literature. We have varied γ_m from 0.00016 to 0.0006 eV, where $\gamma_m = 0.00016$ eV corresponds to the natural line width and the larger values take into account broadening of the biexcitonic transition.[5,12] Figure 3 shows transmissivity as a function of the nonlinearity parameter $g_2'^2 = g_2^2|E_0|^2$ given an input frequency $\omega = 3.177$ eV and a film thickness $d = 5$ μm. The region of bistability starts at a value of $g_2'^2 = 8.2 \times 10^{-5}$ for $\gamma_m = 0.00016$ eV. With a value of $g_2^2 = 1.57 \times 10^{-22}$ eV^2 cm$^3/\omega$ taken from Ref. 5, this corresponds to an input intensity of 317 MW/cm^2. The switching characteristics as a function of γ_m for the CuCl model exhibit behavior similar to those of the model discussed above when the imaginary term is increased.

We have investigated mirrorless optical bistability in an absorbing, nonlinear thin film by solving the wave equation numerically. The flexibility of this technique was demonstrated by applying it to two different model complex dielectric functions. Our numerical results agreed with the analytical results of Chen and Mills[7] using their nonabsorbing model nonlinear dielectric. Furthermore, we were able to study the effect of absorption on bistability in nonlinear films. Mirrorless optical bistability was predicted for a model semiconductor (CuCl) thin film.

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