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Difference between the second-Brewster and pseudo-Brewster angles when polarized light is reflected at a dielectric–conductor interface

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For a given pseudo-Brewster angle ϕ_{pB} of minimum reflectance $|r_p|$ of p-polarized light at a dielectric-conductor interface, the second-Brewster angle ϕ_{2B} of minimum reflectance ratio $|\rho|=|r_p|/|r_s|$ of the p and s polarizations is determined for all possible values of the complex relative dielectric function ϵ that lead to the same ϕ_{pB}. The difference $\phi_{2B}-\phi_{pB}$ is considered as a function of ϕ_{pB} and $\theta=\arg(\epsilon)$. For any given ϕ_{pB}, the difference $\phi_{2B}-\phi_{pB}=0$ at $\theta=0(\epsilon_r>0,\epsilon_i=0)$ increases monotonically as a function of θ and reaches maximum value $\{\phi_{2B}-\phi_{pB}\}_{\max}$ in the limit as $\theta\to 180°$ ($\epsilon_r<0,\epsilon_i=0$). This maximum difference $\{\phi_{2B}-\phi_{pB}\}_{\max}$ has an upper limit of 15.701° when $\phi_{pB}=28.195°$. © 2010 Optical Society of America

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1. INTRODUCTION

The reflection of monochromatic p- and s-polarized light at an angle ϕ by the planar interface between a transparent medium of incidence of refractive index n_0 and an absorbing medium of refraction of complex refractive index $N_1=n_1-jk_1$ is governed by the well-known complex-amplitude Fresnel reflection coefficients [1–3]:

$$r_p=\frac{\epsilon\cos\phi-(\epsilon-\sin^2\phi)^{1/2}}{\epsilon\cos\phi+(\epsilon-\sin^2\phi)^{1/2}}, \tag{1}$$

$$r_s=\frac{\cos\phi-(\epsilon-\sin^2\phi)^{1/2}}{\cos\phi+(\epsilon-\sin^2\phi)^{1/2}}, \tag{2}$$

$$\epsilon={N_1}^2/{n_0}^2=(n-jk)^2=\epsilon_r-j\epsilon_i. \tag{3}$$

The ratio of complex p and s reflection coefficients, also known as the ellipsometric function ρ [2], is obtained from Eqs. (1) and (2) as

$$\rho=r_p/r_s=\frac{\sin\phi\tan\phi-(\epsilon-\sin^2\phi)^{1/2}}{\sin\phi\tan\phi+(\epsilon-\sin^2\phi)^{1/2}}. \tag{4}$$

For a given value of the complex relative dielectric function ϵ, which is characteristic of a given interface at a given wavelength, $|\rho|$ reaches a minimum at the second-Brewster angle ϕ_{2B} [4–6]. This angle, at which incident unpolarized light is reflected with the maximum degree of polarization, differs from the pseudo-Brewster angle ϕ_{pB}, at which $|r_p|$ is minimum [5,7]. In Fig. 1 $|\rho|$, $|r_p|$ and $|r_s|$ are plotted as functions of ϕ for $\epsilon=-0.5183-j0.2992$; the large difference between $\phi_{pB}=30°$ and $\phi_{2B}=44.9°$ is apparent.

In this paper the difference $\phi_{2B}-\phi_{pB}$ between the second-Brewster and pseudo-Brewster angles is thoroughly investigated as a function of complex ϵ. In Section 2 all possible values of ϕ_{2B} associated with a given ϕ_{pB} are obtained. In Section 3 the maximum difference $\{\phi_{2B}-\phi_{pB}\}_{\max}$ is calculated for each ϕ_{pB} and the upper bound on that maximum is determined. Finally, Section 4 gives a brief summary of the paper.

2. SECOND-BREWSTER ANGLES FOR GIVEN PSEUDO-BREWSTER ANGLE

All possible values of complex $\epsilon=(\epsilon_r,\epsilon_i)$ for which ϕ_{pB} is one and the same angle are obtained as follows [7]:

$$\begin{gathered}\epsilon_r=|\epsilon|\cos\theta,\quad \epsilon_i=|\epsilon|\sin\theta,\\ |\epsilon|=\ell\cos(\zeta/3),\\ \ell=2u\left(1-\tfrac{2}{3}u\right)^{1/2}\Big/(1-u),\\ \zeta=\cos^{-1}\left[-(1-u)\cos\theta\Big/\left(1-\tfrac{2}{3}u\right)^{3/2}\right],\\ u=\sin^2\phi_{pB},\\ 0\leq\theta\leq 180°.\end{gathered} \tag{5}$$

For a specific ϕ_{pB}, θ is increased from 0 to 180° in equal steps and the corresponding values of complex ϵ that share the same ϕ_{pB} are obtained from Eqs. (5). For example, at $\phi_{pB}=30°$, ϵ is calculated for θ values from 0° to 180° in increments of 10°, an $|\rho|$-versus-ϕ curve is generated for each complex ϵ, and the resulting family of curves is plotted in Fig. 2. The bottom curve for $\theta=0°$ ($\epsilon_r>0,\epsilon_i=0$) in Fig. 2 exhibits an exact Brewster angle ($|r_p|=|\rho|=0,\phi_{pB}=\phi_B=30°$); the topmost curve for $\theta=180°$

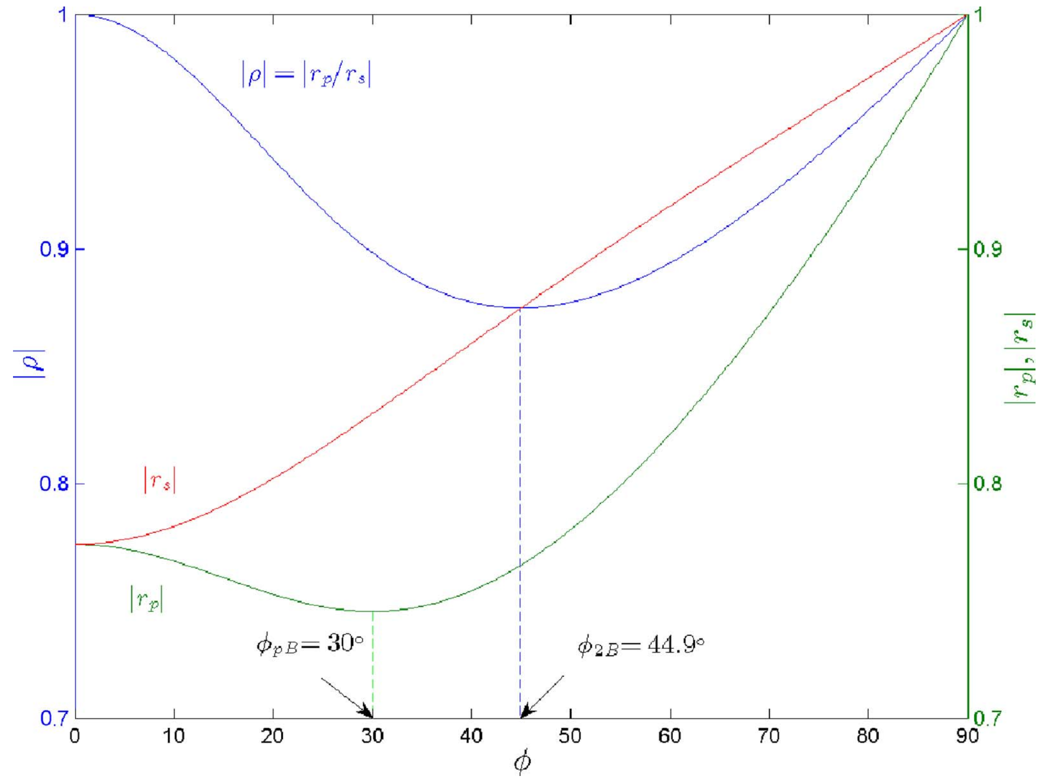

Fig. 1. (Color online) $|\rho|$, $|r_p|$, and $|r_s|$ plotted as functions of the angle of incidence ϕ in degrees for $\epsilon = -0.5183 - j0.2992$. The pseudo-Brewster angle of minimum $|r_p|$ ($\phi_{pB} = 30°$) and the second-Brewster angle of minimum $|\rho|$ ($\phi_{2B} = 44.9°$) are indicated.

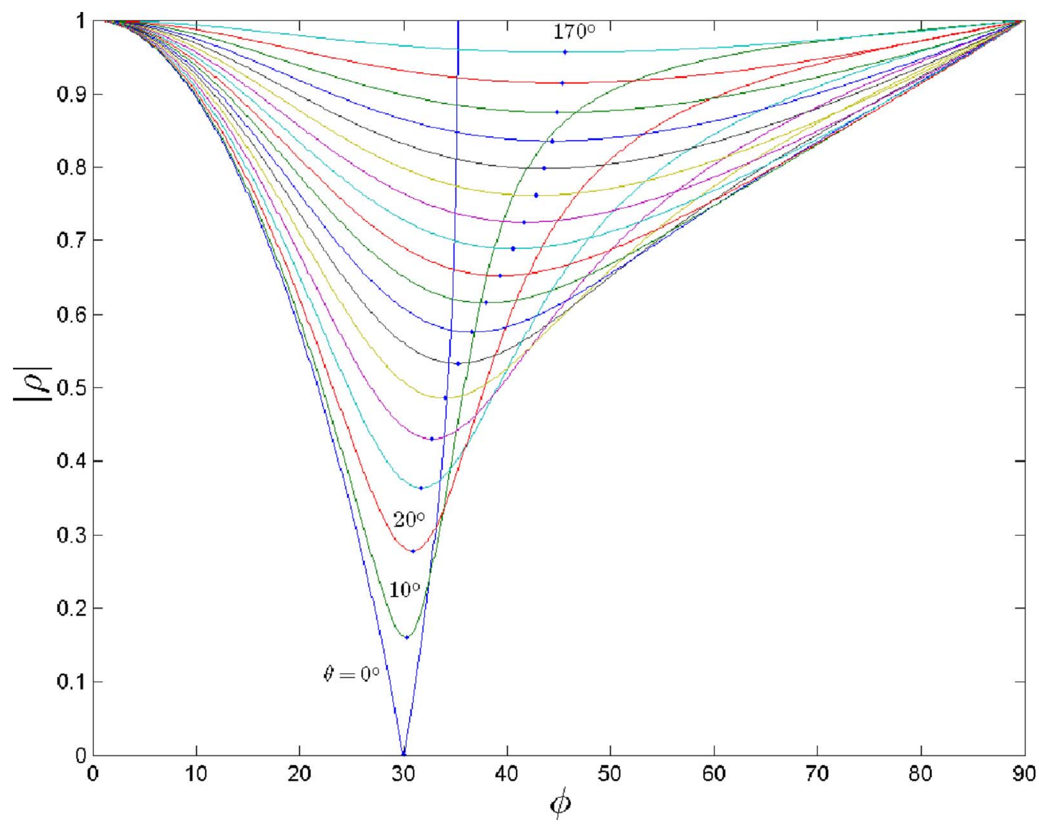

Fig. 2. (Color online) $|\rho|$ as a function of the angle of incidence ϕ in degrees for different values of complex ϵ that are calculated for θ values from 0° to 180° in increments of 10° using Eqs. (5), for pseudo-Brewster angle $\phi_{pB} = 30°$.

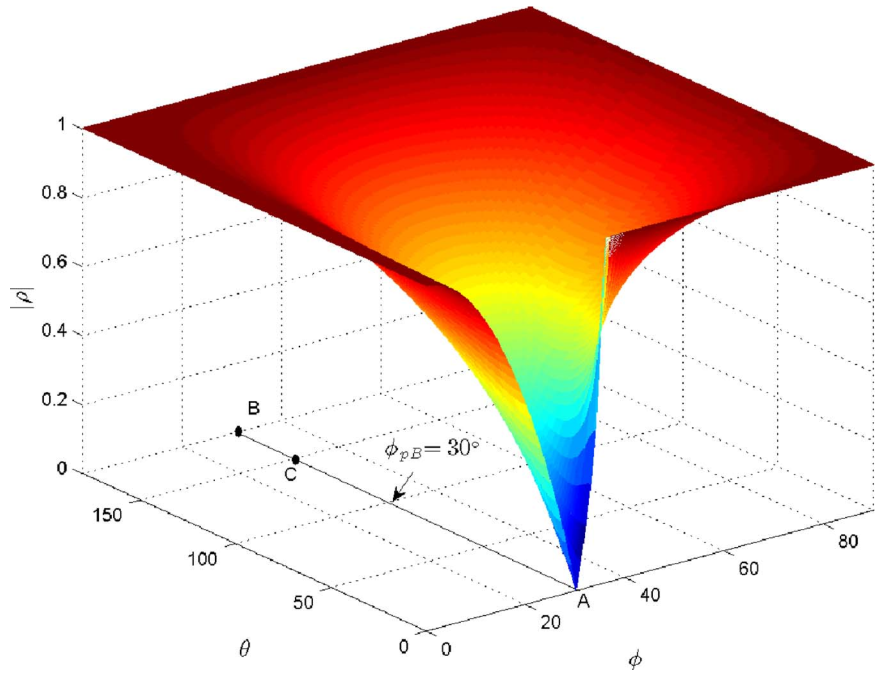

Fig. 3. (Color online) 3-D rendering of $|\rho|$ as a function of ϕ and θ in degrees at constant pseudo-Brewster angle $\phi_{pB}=30°$.

($\epsilon_r<0,\epsilon_i=0$) is the flat line $|\rho|=1$, which represents total reflection of the p and s polarizations at an ideal dielectric–electron-plasma interface. The minimum of each curve in Fig. 2 is highlighted by a dot, and each dot locates ϕ_{2B} for that curve. Notice that the minimum (zero) and maximum differences $\phi_{2B}-\phi_{pB}$ occur when $\theta=0°$ and in the limit as $\theta\rightarrow 180°$, respectively.

A 3-D representation of Fig. 2 is shown in Fig. 3 for $\phi_{pB}=30°$ and with θ assigned values from 0° to 180° in 1° steps. Point A represents a dielectric–dielectric interface for which $|\rho|=0$ at $\theta=0°$ and $\phi_{2B}=\phi_{pB}=\phi_B=30°$. At point B, $\theta=180°$ and $|\rho|=1$; and at point C, $\theta=150°$ and $\epsilon=-0.5183-j0.2992$, which is the value of ϵ used to generate Fig. 1.

3. DIFFERENCE BETWEEN THE SECOND-BREWSTER AND PSEUDO-BREWSTER ANGLES

For a given ϵ, ϕ_{2B} is determined, as shown in [6], by finding the proper root of the following equation:

$$\mathrm{Im}\left[\frac{(u-\epsilon)\left(u-\dfrac{\epsilon}{\epsilon+1}\right)^2}{\left(u-\dfrac{2\epsilon}{\epsilon+1}\right)^2}\right]=0,$$

$$\phi_{2B}=\arcsin\sqrt{u},$$

$$0\leqslant u\leqslant 1. \tag{6}$$

Alternatively [6], u can be explicitly and non-iteratively obtained by solving the equivalent quartic equation:

$$a_4u^4+a_3u^3+a_2u^2+a_1u+a_0=0,$$

$$a_0=\beta_{0r}\gamma_{0i}-\beta_{0i}\gamma_{0r},$$

$$a_1=\beta_{0r}\gamma_{1i}+\beta_{1r}\gamma_{0i}-\beta_{0i}\gamma_{1r}-\beta_{1i}\gamma_{0r},$$

$$a_2=\beta_{2r}\gamma_{0i}+\beta_{1r}\gamma_{1i}-\beta_{0i}-\beta_{1i}\gamma_{1r}-\beta_{2i}\gamma_{0r},$$

$$a_3=\beta_{2r}\gamma_{1i}+\gamma_{0i}-\beta_{1i}-\beta_{2i}\gamma_{1r},$$

$$a_4=\gamma_{1i}-\beta_{2i}; \tag{7}$$

$$\beta_0=-\epsilon(\bar{\epsilon})^2,\quad \beta_1=(\bar{\epsilon})^2+2\epsilon(\bar{\epsilon})^2,\quad \beta_2=-\epsilon-2(\bar{\epsilon}),$$

$$\gamma_0=4(\bar{\epsilon})^2,\quad \gamma_1=-4(\bar{\epsilon}),$$

$$(\bar{\epsilon})=\epsilon/(\epsilon+1),$$

$$\beta_k=\beta_{kr}+j\beta_{ki},\quad \gamma_k=\gamma_{kr}+j\gamma_{ki},\quad k=0,1,2. \tag{8}$$

In external reflection $|\epsilon|>1$ and only one acceptable root ($0\leqslant u\leqslant 1$) of Eq. (7) exists. However, in internal reflection ($|\epsilon|<1$) two additional roots ($0\leqslant u\leqslant 1$) of Eq. (7) appear that represent extrema not of $|\rho|$ but of the associated differential reflection phase shift (or ellipsometric) angle $\Delta=\arg(\rho)$ [8]. The angles of incidence that locate the two extrema of differential phase shift are $>\phi_{2B}$.

Based on the above formulation, the difference $\phi_{2B}-\phi_{pB}$ is first calculated at equi-spaced values of ϕ_{pB} from 2.5° to 27.5° in increments of 2.5°. For each ϕ_{pB}, θ is increased from 0° to 180° in 1° steps, and for each θ the corresponding value of complex ϵ is obtained from Eqs. (5). Equation (7) is solved for $\phi_{2B}=\arcsin\sqrt{u}$ for each complex ϵ, and the difference $\phi_{2B}-\phi_{pB}$ is plotted as a function of θ in Fig. 4(a). In Fig. 4(a) note that $\phi_{2B}-\phi_{pB}$ generally increases as ϕ_{pB} increases from 2.5° to 27.5°. However, for $\phi_{pB}\geqslant 30°$ the difference $\phi_{2B}-\phi_{pB}$ drops as ϕ_{pB} increases,

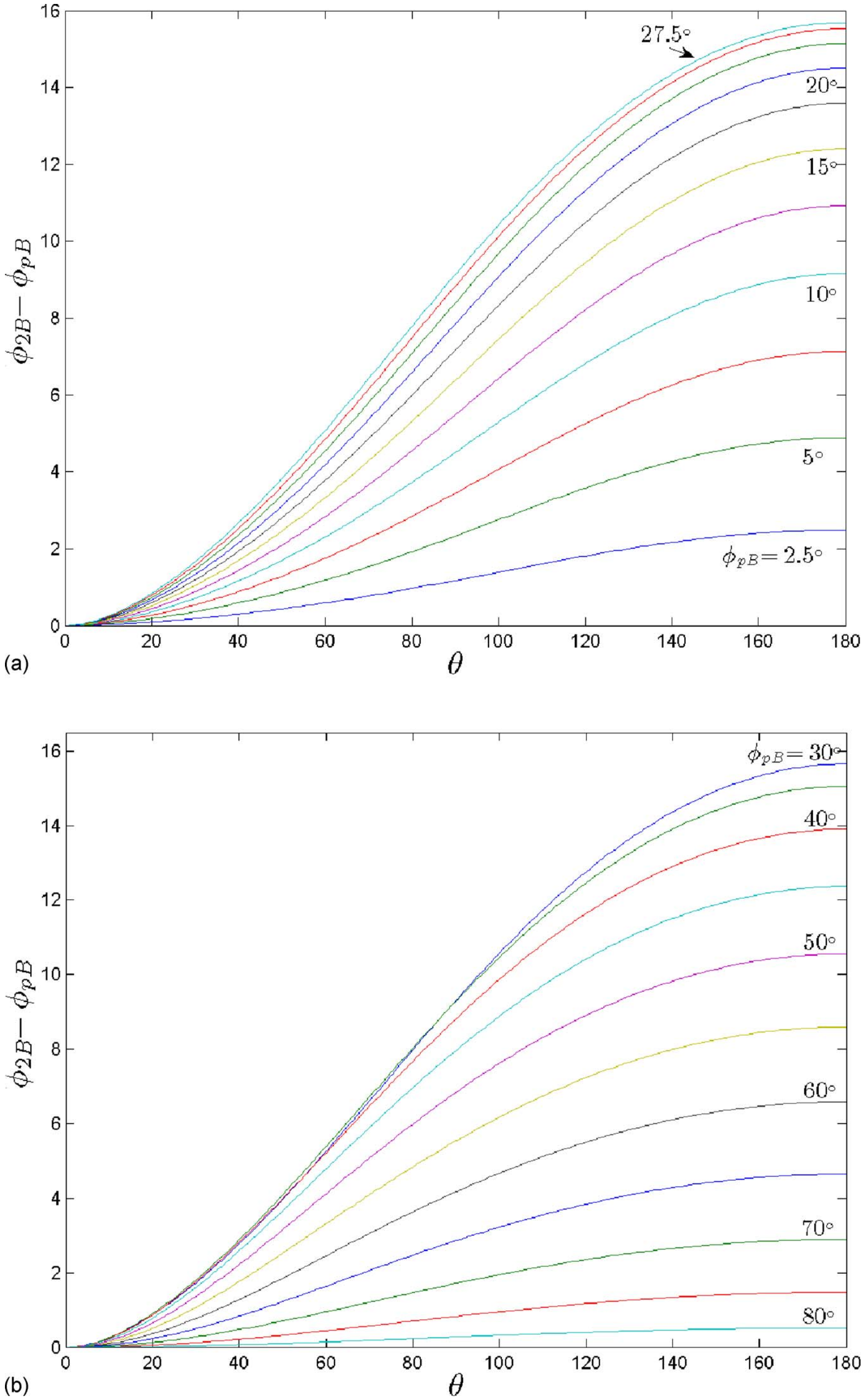

Fig. 4. (Color online) Difference $\phi_{2B}-\phi_{pB}$ plotted as a function of θ (all angles in degrees) for different values of the pseudo-Brewster angle ϕ_{pB}: (a) ϕ_{pB} assumes values from 2.5° to 27.5° in equal increments of 2.5°, and (b) ϕ_{pB} takes values from 30° to 80° in equal steps of 5°.

as shown in Fig. 4(b). Figure 4(b) is a continuation of Fig. 4(a) for $30° \leqslant \phi_{pB} \leqslant 80°$ in equal steps of 5°; it clearly shows that the difference $\phi_{2B}-\phi_{pB}\to 0$ as $\phi_{pB}\to 90°$, as expected in the case of high-reflectance metals in the IR.

For further illustration, Fig. 5 presents a combined 3-D plot of $\phi_{2B}-\phi_{pB}$ as a function of ϕ_{pB} and θ.

Finally, the maximum difference $\{\phi_{2B}-\phi_{pB}\}_{\max}$ is plotted in Fig. 6 as a function of ϕ_{pB}. The maximum difference $\{\phi_{2B}-\phi_{pB}\}_{\max}$ reaches its highest level of 15.701° when $\phi_{pB}=28.195°$.

For reference, Table 1 also lists values of $\{\phi_{2B}-\phi_{pB}\}_{\max}$ at specific values of ϕ_{pB}.

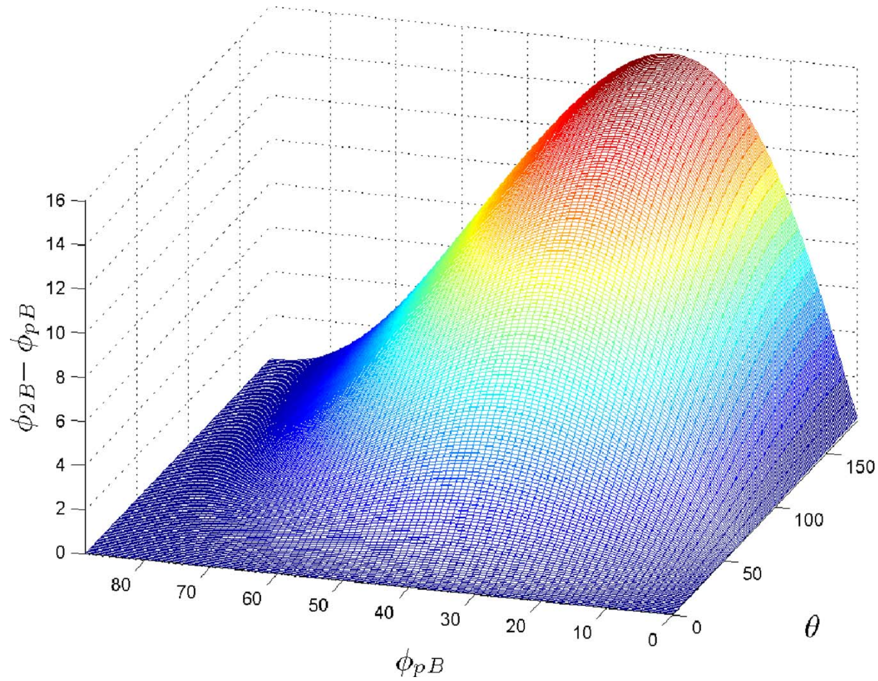

Fig. 5. (Color online) 3-D plot of $\phi_{2B}-\phi_{pB}$ as a function of ϕ_{pB} and θ. All angles are in degrees.

4. SUMMARY

For a given pseudo-Brewster angle ϕ_{pB}, a set of values of the complex relative dielectric function ϵ that share the same ϕ_{pB} is generated by Eqs. (5). Next, for each complex ϵ the second-Brewster angle ϕ_{2B} is obtained from the proper root of Eq. (7). The difference $\phi_{2B}-\phi_{pB}$ is plotted in Figs. 4 and 5. The difference $\phi_{2B}-\phi_{pB}$ reaches an absolute maximum value of 15.701° when $\phi_{pB}=28.195°$ and approaches 0 as $\phi_{pB}\rightarrow 90°$, which corresponds to high-reflectance metals in the IR.

This paper complements earlier work on the plurality of principal angles for a given pseudo-Brewster angle

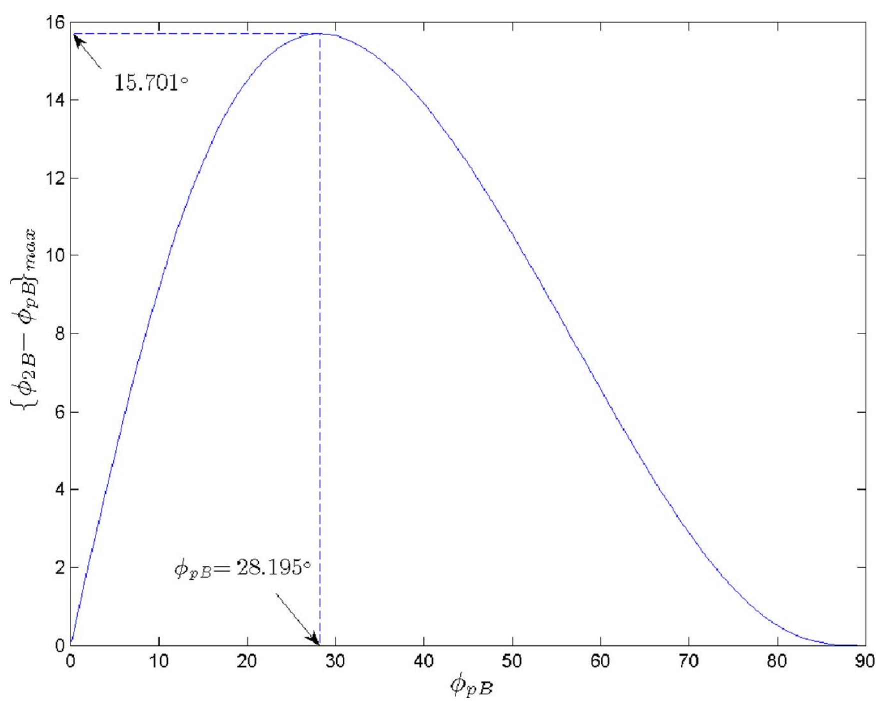

Fig. 6. (Color online) Maximum difference $\{\phi_{2B}-\phi_{pB}\}_{\max}$ is plotted as a function of ϕ_{pB}, with all angles in degrees. The maximum difference reaches an upper limit of 15.701° at $\phi_{pB}=28.195°$.

Table 1. Maximum Difference between the Second-Brewster Angle ϕ_{2B} and Pseudo-Brewster Angle ϕ_{pB} for Selected Values of ϕ_{pB} [a]

ϕ_{pB}	$\{\phi_{2B}-\phi_{pB}\}_{\max}$
15	12.4061
20	14.5064
25	15.5323
28.1951	15.7010
30	15.6513
35	15.0512
40	13.9055
45	12.3641
50	10.5554
55	8.5935
60	6.5869
65	4.6474
70	2.8984
75	1.4776
80	0.5171
85	0.0729

[a] All angles are in degrees.

when polarized light is reflected at a dielectric–conductor interface [9]. Furthermore, the results presented here have immediate application to the determination of complex ϵ from measurements of the two angles ϕ_{pB} and ϕ_{2B}.

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