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Efficiency of linear-to-circular polarization conversion for light reflection at the principal angle by a dielectric–conductor interface

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The efficiency η_{LC} of linear-to-circular polarization conversion when light is reflected at a dielectric–conductor interface is determined as a function of the principal angle $\bar{\phi}$ and principal azimuth $\bar{\psi}$. Constant-η_{LC} contours are presented in the $\bar{\phi}$, $\bar{\psi}$ plane for values of η_{LC} from 0.5 to 1.0 in steps of 0.05, and the corresponding contours in the complex plane of the relative dielectric function ϵ are also determined. As specific examples, efficiencies $\geqslant$88% are obtained for light reflection by a Ag mirror in the visible and near-IR (400–1200 nm) spectral range, and $\geq$40% for the reflection of extreme ultraviolet (EUV) and soft x-ray radiation by a SiC mirror in the 60–120 nm wavelength range. © 2008 Optical Society of America

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1. INTRODUCTION

When linearly polarized light (LPL) is incident at the planar interface between a transparent medium of incidence (medium 0) and an absorbing medium of refraction (medium 1), the reflected light becomes circularly polarized (CP) at a special angle of incidence, called the principal angle $\bar{\phi}$, and for a specific orientation of the incident electric field vector relative to the plane of incidence, called the principal azimuth $\bar{\psi}$, as shown in Fig. 1 [1–3]. In Fig. 1, p and s represent the orthogonal linear polarization directions parallel and perpendicular to the plane of incidence, respectively. If the azimuth of incident LPL is changed from $+\bar{\psi}$ to $-\bar{\psi}$, the handedness of the reflected circular polarization is switched.

The angles $\bar{\phi}$ and $\bar{\psi}$ are determined by the complex relative dielectric function,

$$\epsilon = \epsilon_1/\epsilon_0 = \epsilon_r - j\epsilon_i, \tag{1}$$

of the two media, which are assumed to be linear, homogeneous, and optically isotropic. In particular, $\bar{\phi}$ is obtained by solving a cubic equation in $\sin^2 \bar{\phi}$ whose coefficients are determined by complex ϵ [3]. Conversely, ϵ_r and ϵ_i are derived from the measured angles $\bar{\phi}$ and $\bar{\psi}$ by

$$\epsilon_r = \sin^2 \bar{\phi} + \sin^2 \bar{\phi} \tan^2 \bar{\phi} \cos(4\bar{\psi}),$$

$$\epsilon_i = \sin^2 \bar{\phi} \tan^2 \bar{\phi} \sin(4\bar{\psi}). \tag{2}$$

This is the basis of an experimental technique that was first used by O'Bryan [4] to determine the optical constants of optically thick metal films deposited in vacuum using a return-path ellipsometer [4,5]. Contours of constant $\bar{\phi}$ and constant $\bar{\psi}$ in the complex ϵ plane and the domain of fractional optical constants in which multiple principal angles exist were presented in [3].

The primary objective of this paper is to determine the efficiency of linear-to-circular polarization conversion when LPL is incident at the principal angle and principal azimuth. In Section 2 an expression for this conversion efficiency η_{LC} is derived as a function of $\bar{\phi}$, $\bar{\psi}$ and conditions for achieving high efficiency ($\eta_{LC} > 50\%$) are clarified. The properties of this function are discussed in Section 3. As an application, $\bar{\phi}$, $\bar{\psi}$, and η_{LC} are calculated in Section 4 for light reflection by a Ag mirror in the visible and near IR, and also for the reflection of extreme ultraviolet (EUV) or soft x-ray radiation by a SiC mirror. Section 5 is a brief summary of the paper.

2. EFFICIENCY OF LINEAR-TO-CIRCULAR POLARIZATION CONVERSION AS A FUNCTION OF PRINCIPAL ANGLE AND PRINCIPAL AZIMUTH

At the principal angle, the complex-amplitude Fresnel reflection coefficients r_p and r_s for the p and s polarizations are related by [3]

$$r_p = (j \tan \bar{\psi}) r_s. \tag{3}$$

At any angle of incidence ϕ, r_s is given by

$$r_s = [\cos \phi - (\epsilon - \sin^2 \phi)^{1/2}]/[\cos \phi + (\epsilon - \sin^2 \phi)^{1/2}]. \tag{4}$$

At the principal angle ($\phi = \bar{\phi}$) ϵ in Eq. (4) can be replaced by its equivalent expression in terms of $\bar{\phi}$, $\bar{\psi}$ given by Eqs.

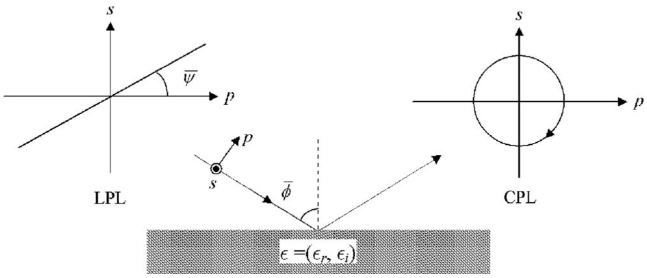

Fig. 1. Incident linerarly polarized light (LPL) is reflected circularly polarized (CPL) at the principal angle $\bar{\phi}$ of a dielectric–conductor interface: $\bar{\psi}$ is the principal azimuth, and p and s represent the orthogonal linear polarization directions parallel and perpendicular to the plane of incidence, respectively.

(2). Use of some trigonometric manipulations transforms Eq. (4) to

$$r_s = (\cos 2\bar{\phi} + j \tan \bar{\psi})/(1 + j \cos 2\bar{\phi} \tan \bar{\psi}). \qquad (5)$$

At $\phi = \bar{\phi}$, the linear-to-circular polarization conversion efficiency η_{LC} is defined as the ratio of the intensity of the reflected circularly polarized light (CPL) to that of the incident LPL and is given by

$$\eta_{LC} = \cos^2 \bar{\psi}(r_p r_p^*) + \sin^2 \bar{\psi}(r_s r_s^*). \qquad (6)$$

Substitution of r_p from Eq. (3) in Eq. (6) gives

$$\eta_{LC} = 2 \sin^2 \bar{\psi}(r_s r_s^*). \qquad (7)$$

From Eqs. (5) and (7) we finally obtain

$$\eta_{LC} = 2 \sin^2 \bar{\psi}\left[\frac{1 - \sin^2 2\bar{\phi} \cos^2 \bar{\psi}}{1 - \sin^2 2\bar{\phi} \sin^2 \bar{\psi}}\right]. \qquad (8)$$

The function $\eta_{LC}(\bar{\psi}, \bar{\phi})$ of Eq. (8) represents the main result of this paper and its properties are examined in Section 3.

3. PROPERTIES OF THE FUNCTION $\eta_{LC}(\bar{\psi}, \bar{\phi})$

Based on Eq. (8), one can readily reach the following conclusions:

(1) $\eta_{LC}=0$ if $\bar{\psi}=0$. This corresponds to $r_p=0$ [Eq. (3)] and light reflection at the Brewster angle of a dielectric–dielectric interface.

(2) $\eta_{LC}=1$ if $\bar{\psi}=45°$. This corresponds to the total internal reflection of the p and s polarizations at the principal angle of a dielectric–dielectric interface ($\epsilon_i=0$) and requires that ϵ_r be in the range $0 < \epsilon_r \leqslant 3 - 2\sqrt{2} = 0.1716$ [3]. It also corresponds to the total reflection at an ideal dielectric–plasma interface with $\epsilon_i = 0, \epsilon_r < 0$.

(3) For any given $\bar{\psi}$

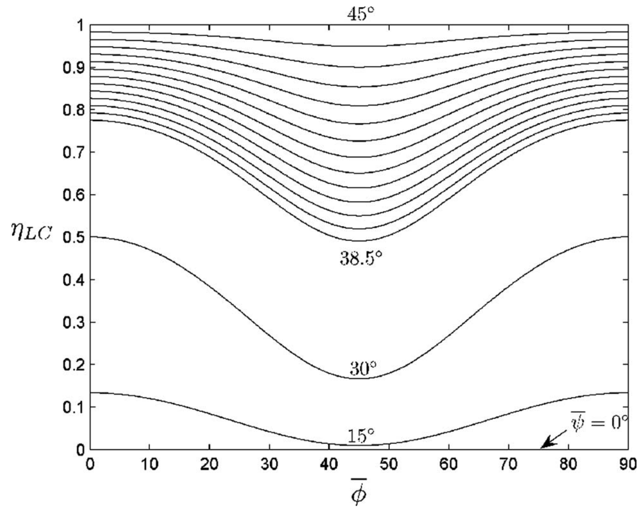

Fig. 2. Efficiency η_{LC} of linear-to-circular polarization conversion as a function of principal angle $\bar{\phi}$ for constant values of the principal azimuth $\bar{\psi}=15°$, 30°, and 38.5° to 45° in steps of 0.5°.

$$\eta_{LC}(\bar{\psi}, \bar{\phi}) = \eta_{LC}(\bar{\psi}, 90° - \bar{\phi}). \qquad (9)$$

Therefore, for a given principal azimuth $\bar{\psi}$, η_{LC} is the same at two principal angles that are equally above and below $\bar{\phi}=45°$.

(4) When $\bar{\phi}=45°$, Eq. (8) becomes

$$\eta_{LC} = 2 \sin^2 \bar{\psi} \tan^2 \bar{\psi}. \qquad (10)$$

From Eq. (10), $\eta_{LC}=1/2$ (50% efficiency) at $\bar{\phi}=45°$ if $\sin^2 \bar{\psi} = (\sqrt{17}-1)/8 = 0.390388$ and $\bar{\psi}=38.6683°$.

(5) In the limits of normal and grazing incidence, i.e., as $\bar{\phi} \to 0, 90°$, Eq. (8) simplifies to

$$\eta_{LC} = 2 \sin^2 \bar{\psi}. \qquad (11)$$

Figure 2 presents a family of curves of η_{LC} as a function

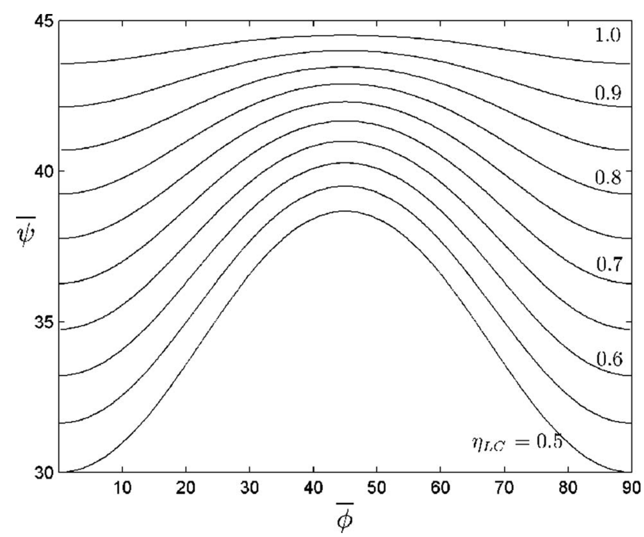

Fig. 3. Constant-η_{LC} contours in the ($\bar{\phi}$, $\bar{\psi}$) plane for $\eta_{LC}=0.5$ to 1.0 in steps of 0.05.

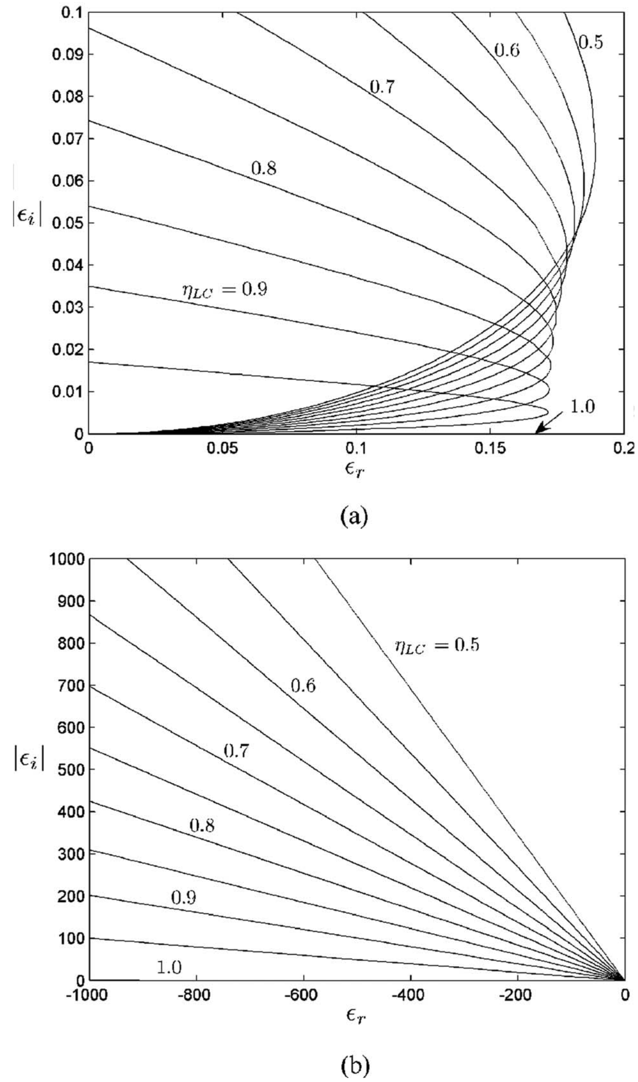

Fig. 4. (a) Family of constant-η_{LC} contours, for $\eta_{LC}=0.5$ to 1.0 in steps of 0.05, in the domain of fractional optical constants of the complex ϵ plane. (b) Continuation of the contours in (a) for large values of (ϵ_r, ϵ_i).

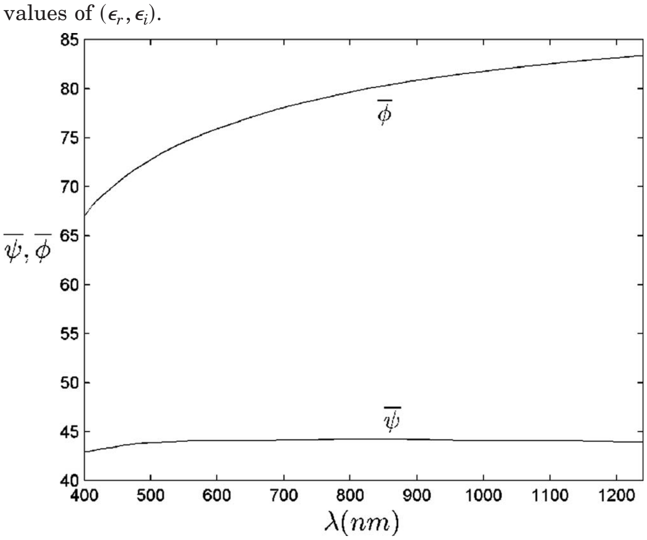

Fig. 5. Principal angle $\bar{\phi}$ and principal azimuth $\bar{\psi}$ for light reflection by a Ag mirror in the visible and near-IR spectral range, $400 \leqslant \lambda \leqslant 1200$ nm. The optical constants of Ag are obtained from [6].

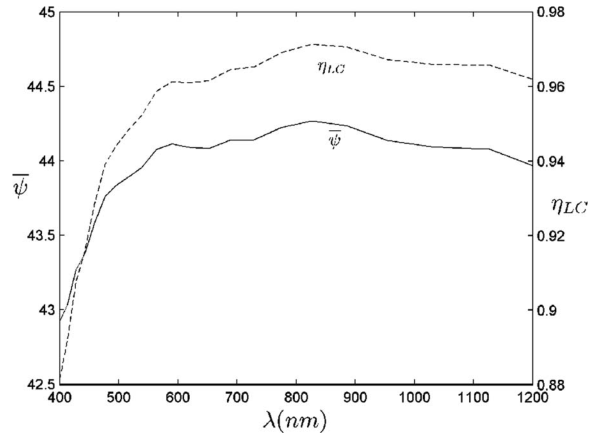

Fig. 6. Linear-to-circular polarization conversion efficiency η_{LC} (dashed curve) and principal azimuth $\bar{\psi}$ (expanded scale) as functions of wavelength λ for light reflection by a Ag mirror in the visible and near-IR spectral range, $400 \leqslant \lambda \leqslant 1200$ nm. The optical constants of Ag are obtained from [6].

of $\bar{\phi}$ for constant values of $\bar{\psi}=15°$, 30°, and 38.5° to 45° in steps of 0.5°. For a given $\bar{\psi}$, Fig. 2 indicates that η_{LC} is maximum at $\bar{\phi}=0$, 90° and is minimum at $\bar{\phi}=45°$.

In Fig. 3 constant-η_{LC} contours are plotted in the $(\bar{\phi}, \bar{\psi})$ plane for values of η_{LC} from 0.5 to 1.0 in steps of 0.05.

It is also of interest to specify the region of the complex ϵ plane in which $\eta_{LC} \geqslant 0.5$. This is done by mapping the family of constant-η_{LC} contours of Fig. 3 to the complex (ϵ_r, ϵ_i) plane by using Eqs. (2). Figure 4(a) shows the corresponding family of constant-η_{LC} contours in the domain of fractional optical constants, and Fig. 4(b) shows the continuation of those contours for large values of (ϵ_r, ϵ_i).

4. APPLICATION TO Ag AND SiC MIRRORS

The principal angle $\bar{\phi}$, principal azimuth $\bar{\psi}$, and conversion efficiency η_{LC} are calculated for light reflection by a Ag mirror (in vacuum or inert ambient) in the visible and near-IR spectrum, $400 \leqslant \lambda \leqslant 1200$ nm. The optical constants of Ag are obtained from Palik [6]. In Fig. 5, $\bar{\phi}$ and $\bar{\psi}$

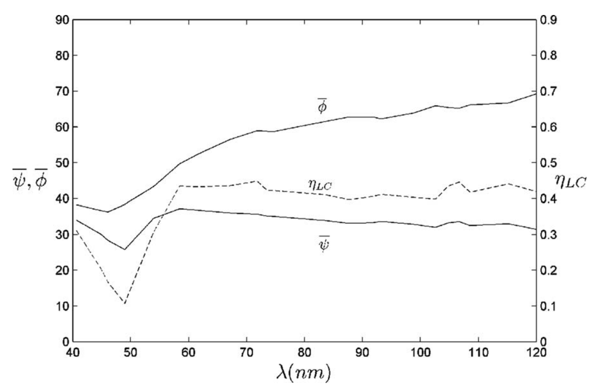

Fig. 7. Principal angle $\bar{\phi}$, principal azimuth $\bar{\psi}$, and linear-to-circular polarization conversion efficiency η_{LC} (dashed curve) as functions of wavelength λ for the reflection of EUV and soft x-ray radiation by a SiC mirror in the $40 \leqslant \lambda \leqslant 120$ nm spectral range. The optical constants of SiC are those published in [7].

are plotted as functions of wavelength λ. Figure 6 shows the correlation between η_{LC} (dashed curve) and $\bar{\psi}$ (on an expanded scale) versus λ. For Ag, $\eta_{LC} \geqslant 88\%$ over the $400 \leqslant \lambda \leqslant 1200$ nm spectral range.

As another example, $\bar{\phi}$, $\bar{\psi}$, and η_{LC} are calculated for the reflection of EUV and soft x-ray radiation by a SiC mirror, and the results are presented in Fig. 7 over the $40 \leqslant \lambda \leqslant 120$ nm spectral range. The optical constants of SiC are those published by Windt *et al.* [7]. Conversion efficiencies $\eta_{LC} \geqslant 40\%$ are achieved at wavelengths of $60 \leqslant \lambda \leqslant 120$ nm.

5. SUMMARY

A detailed analysis of the efficiency of linear-to-circular polarization conversion when light is reflected at a principal angle of a dielectric–conductor interface has been presented. The constraint on the principal angle and principal azimuth for achieving a given efficiency (e.g., ⩾50%) and the corresponding domain in the complex ϵ plane are determined. As examples, efficiencies ⩾88% are obtained for light reflection by a Ag mirror in the visible and near-IR (400–1200 nm) spectral range and ⩾40% for the reflection of EUV and soft x-ray radiation by a SiC mirror in the 60–120 nm wavelength range.

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