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Phase shifts in frustrated total internal reflection and optical tunneling by an embedded low-index thin film

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Simple and explicit expressions for the phase shifts that p- and s-polarized light experience in frustrated total internal reflection (FTIR) and optical tunneling by an embedded low-index thin film are obtained. The differential phase shifts in reflection and transmission Δ_r, Δ_t are found to be identical, and the associated ellipsometric parameters ψ_r, ψ_t are governed by a simple relation, independent of film thickness. When the Fresnel interface reflection phase shifts for the p and s polarizations or their average are quarter-wave, the corresponding overall reflection phase shifts introduced by the embedded layer are also quarter-wave for all values of film thickness. In the limit of zero film thickness (i.e., for an ultrathin embedded layer), the reflection phase shifts are also quarter-wave independent of polarization (p or s) or angle of incidence (except at grazing incidence). Finally, variable-angle FTIR ellipsometry is shown to be a sensitive technique for measuring the thickness of thin uniform air gaps between transparent bulk media.

OCIS codes: 240.0310, 240.7040, 260.2130, 260.5430, 260.6970, 310.6860.

1. INTRODUCTION

Frustrated total internal reflection (FTIR) and optical tunneling have been studied extensively in the past and have found many important applications. (See, for example, the concise review by Zhu *et al.*[1] and references cited therein.) However, most previous studies of FTIR have been limited to considerations of the power fractions that appear in reflection and transmission. A notable exception is the work of Carneglia and Mandel,[2,3] who measured the phase shifts associated with evanescent waves in FTIR using a holographic technique.

In this paper a detailed analysis is provided of the phase shifts that accompany FTIR and optical tunneling by a low-index thin film that is embedded in a high-index medium, and numerous interesting new results are obtained. This extends previous work that dealt with total internal reflection phase shifts at a single interface between two transparent media.[4] Differential reflection and transmission phase shifts are readily measurable by ellipsometry.[5]

In Section 2 simple expressions are derived for the overall reflection and transmission phase shifts introduced by an embedded layer in terms of the Fresnel interface reflection phase shifts and a normalized film thickness. Section 3 considers the ratios of complex-amplitude reflection and transmission coefficients for the p and s polarizations for a layer of any thickness and in the limit of zero and infinite thickness. In Section 4 the results of Sections 2 and 3 are applied to a uniform air gap between two glass prisms. The special case of light reflection and transmission at the critical angle is considered in Section 5. Section 6 gives a brief summary of the paper. Finally, in Appendix A, a new expression for the ratio of slopes of the p and s interface reflection phase shifts as functions of angle of incidence is derived.

2. REFLECTION AND TRANSMISSION PHASE SHIFTS

Consider a uniform layer (medium 1) of thickness d and low refractive index n_1 that is embedded in a bulk medium (0) of high refractive index n_0 (Fig. 1). If monochromatic light of wavelength λ is incident on the layer from medium 0 at an angle ϕ greater than the critical angle, $\phi_c = \arcsin(n_1/n_0)$, FTIR takes place and some of the light tunnels across the thin film as a transmitted wave. Total internal reflection is restored if $d \gg \lambda$.

The overall complex-amplitude reflection and transmission coefficients of the embedded layer for p- and s-polarized incident light are given by[5]

$$R_\nu = |R_\nu| \exp(j\Delta_{r\nu}) = r_{01\nu}(1-X)/(1-r_{01\nu}{}^2 X),$$

$$T_\nu = |T_\nu| \exp(j\Delta_{t\nu}) = (1-r_{01\nu}{}^2)X^{1/2}/(1-r_{01\nu}{}^2 X),$$

$$\nu = p, s, \qquad (1)$$

where $r_{01\nu}$ is the Fresnel reflection coefficient of the 01 interface for the ν polarization ($\nu = p, s$). Above the critical angle, $r_{01\nu}$ is a pure phase factor,[4]

$$r_{01\nu} = \exp(j\delta_\nu), \qquad (2)$$

and the interface reflection phase shifts $\delta_\nu (\nu = p, s)$ are given by[4]

$$\delta_p = 2 \arctan(NU/\cos\phi),$$

$$\delta_s = 2 \arctan(U/N \cos\phi), \qquad (3)$$

in which

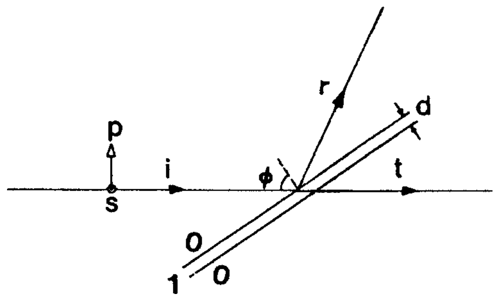

Fig. 1. Reflection and transmission of p- and s-polarized light at an angle of incidence ϕ by a uniform layer (medium 1) of thickness d and refractive index n_1 that is embedded in a bulk medium (0) of refractive index n_0.

$$N = n_0/n_1 > 1, \quad U = (N^2 \sin^2 \phi - 1)^{1/2}. \tag{4}$$

In Eqs. (1) X is an exponential function of film thickness,

$$X = \exp(-2x), \tag{5}$$

$$x = 2\pi n_1 (d/\lambda) U. \tag{6}$$

The overall phase shifts in reflection and transmission are determined by

$$\Delta_{r\nu} = \arg(R_\nu),$$

$$\Delta_{t\nu} = \arg(T_\nu). \tag{7}$$

From Eqs. (1), (2), (5), and (7) we obtain

$$\tan \Delta_{r\nu} = \tan \delta_\nu / \tanh x, \tag{8}$$

$$\tan \Delta_{t\nu} = -\tanh x / \tan \delta_\nu. \tag{9}$$

Equations (8) and (9) are concise and elegant expressions for the overall reflection and transmission phase shifts for the ν polarization in terms of the Fresnel interface reflection phase shift δ_ν and the normalized film thickness x [Eq. (6)].

From Eqs. (8) and (9) it immediately follows that

$$\tan \Delta_{r\nu} \tan \Delta_{t\nu} = -1, \tag{10}$$

$$\Delta_{r\nu} - \Delta_{t\nu} = \pm \pi/2. \tag{11}$$

Equation (11) indicates that the overall reflection and transmission phase shifts for the ν polarization ($\nu = p, s$) differ by a quarter-wave, in agreement with a recent result obtained by Efimov *et al.*[6]

The (ellipsometric) differential reflection and transmission phase shifts are given by

$$\Delta_r = \Delta_{rp} - \Delta_{rs},$$

$$\Delta_t = \Delta_{tp} - \Delta_{ts}. \tag{12}$$

From the trigonometric identity for the tangent of the difference of two angles and Eqs. (8), (9), and (12), we obtain

$$\tan \Delta_r = \tan \Delta_t = \tanh x (\tan \delta_p - \tan \delta_s)/(\tanh^2 x + \tan \delta_p \tan \delta_s), \tag{13}$$

$$\Delta_r = \Delta_t. \tag{14}$$

Equation (14) also follows readily from Eqs. (11) and (12).

The average phase shift on reflection for the p and s polarizations is given by

$$\Delta_{ra} = (\Delta_{rp} + \Delta_{rs})/2. \tag{15}$$

From Eqs. (8), (9), and (15) and the trigonometric identity for the tangent of the sum of two angles, we obtain

$$\tan(2\Delta_{ra}) = \tanh x (\tan \delta_p + \tan \delta_s)/(\tanh^2 x - \tan \delta_p \tan \delta_s). \tag{16}$$

3. RATIOS OF COMPLEX REFLECTION AND TRANSMISSION COEFFICIENTS FOR THE *p* AND *s* POLARIZATIONS

The ellipsometric functions in reflection and transmission are defined by[5]

$$\rho_r = R_p/R_s = \tan \psi_r \exp(j\Delta_r),$$

$$\rho_t = T_p/T_s = \tan \psi_t \exp(j\Delta_t). \tag{17}$$

By substitution of Eqs. (1) in Eqs. (17), we obtain

$$\rho_r = (r_{01p}/r_{01s})[(1 - r_{01s}{}^2 X)/(1 - r_{01p}{}^2 X)],$$

$$\rho_t = [(1 - r_{01p}{}^2)/(1 - r_{01s}{}^2)][(1 - r_{01s}{}^2 X)/(1 - r_{01p}{}^2 X)]. \tag{18}$$

In the limit of zero layer thickness, $d=0$ and $X=1$, Eqs. (18) become

$$\rho_r(X=1) = (r_{01p}/r_{01s})[(1 - r_{01s}{}^2)/(1 - r_{01p}{}^2)],$$

$$\rho_t(X=1) = 1. \tag{19}$$

And in the limit of infinite layer thickness, $d=\infty$ and $X=0$, Eqs. (18) reduce to

$$\rho_r(X=0) = (r_{01p}/r_{01s}),$$

$$\rho_t(X=0) = (1 - r_{01p}{}^2)/(1 - r_{01s}{}^2). \tag{20}$$

Whereas the second of Eqs. (19) and the first of Eqs. (20) are intuitively apparent, the remaining two equations are not.

For a layer of any thickness d, the ratio

$$\Gamma = \rho_r/\rho_t = (\tan \psi_r / \tan \psi_t) \exp[j(\Delta_r - \Delta_t)]$$

$$= (r_{01s} - r_{01s}{}^{-1})/(r_{01p} - r_{01p}{}^{-1}), \tag{21}$$

which is independent of film thickness. Under FTIR conditions, substitution of the interface Fresnel reflection coefficients from Eq. (2) into Eq. (21) gives the surprisingly simple result

$$\Gamma = \rho_r/\rho_t = \sin \delta_s / \sin \delta_p. \tag{22}$$

Between the critical angle and grazing incidence, $\phi_c < \phi < 90°$, $\sin \delta_p, \sin \delta_s > 0$, and the right-hand side of Eq. (22) is a positive real number. It follows from Eqs. (21) and (22) that

$$\Delta_r - \Delta_t = 0, \tag{23}$$

$$\tan \psi_r / \tan \psi_t = \sin \delta_s / \sin \delta_p. \tag{24}$$

Equation (23) is the same as Eq. (14). By use of Eqs. (3) and some trigonometric manipulations, Eq. (24) can be transformed to

$$\tan \psi_r / \tan \psi_t = \sin \delta_s / \sin \delta_p = (N^2 + 1)\sin^2 \phi - 1. \tag{25}$$

In Appendix A we also show that

$$\sin \delta_p / \sin \delta_s = \delta_p' / \delta_s', \tag{26}$$

where δ_p' and δ_s' are the derivatives (slopes) of the interface reflection phase shifts with respect to the angle of incidence ϕ. Equation (26) represents a new interesting relation between the total internal reflection phase shifts for the p and s polarizations at a dielectric–dielectric interface.[4]

4. FRUSTRATED TOTAL INTERNAL REFLECTION PHASE SHIFTS FOR AN AIR GAP BETWEEN TWO GLASS PRISMS

As a specific example, we consider FTIR and optical tunneling by a uniform air gap ($n_0 = 1$) between two glass prisms ($n_1 = 1.5$).

Figure 2 shows the reflection phase shift for the p polarization Δ_{rp} as a function of the angle of incidence ϕ for $d/\lambda = 0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. Note that ϕ covers the full range from 0 to 90°. This range includes both FTIR ($\phi > \phi_c = 41.181°$) and partial internal reflection ($\phi < \phi_c$). The curve of Δ_{rp} versus ϕ for $d/\lambda = 10$ exhibits many oscillations below the critical angle that are not shown in Fig. 2.

A striking feature of Fig. 2 is that all curves pass through a common point A at which $\Delta_{rp} = 90°$. This occurs at the angle of incidence ϕ_p at which the interface reflection phase shift $\delta_p = 90°$, as can be seen from Eq. (8). According to Ref. 4, ϕ_p is determined by

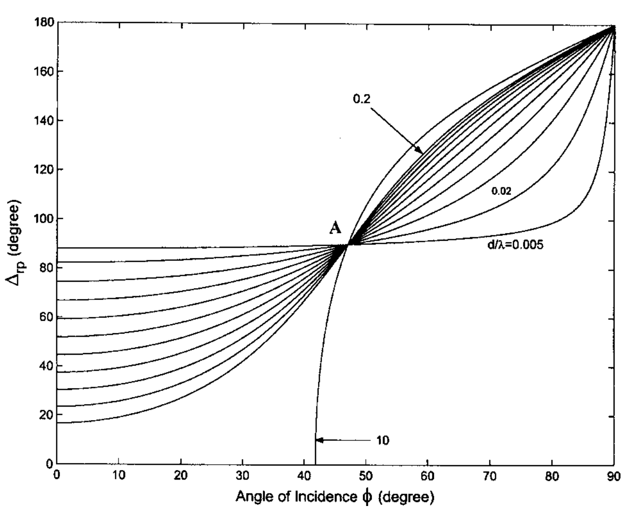

Fig. 2. Reflection phase shift for the p polarization Δ_{rp} as a function of the angle of incidence ϕ for $d/\lambda = 0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. All curves pass through a common point A at which $\Delta_{rp} = 90°$. These results are calculated for an air gap of thickness d between two glass prisms ($N = 1.5$).

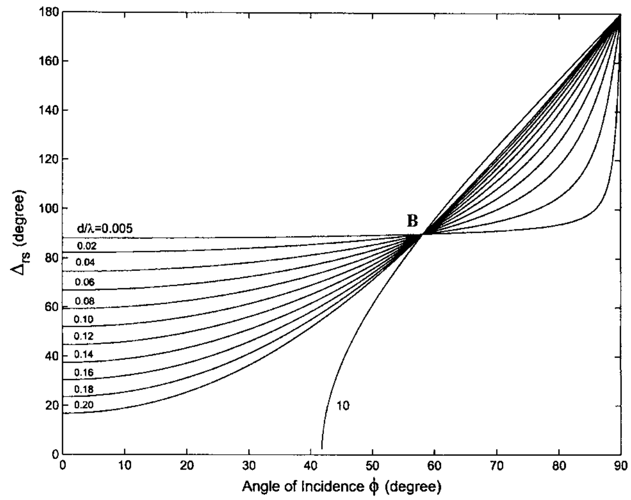

Fig. 3. Reflection phase shift for the s polarization Δ_{rs} as a function of the angle of incidence ϕ for $d/\lambda = 0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. All curves pass through a common point B at which $\Delta_{rs} = 90°$. These results are calculated for an air gap of thickness d between two glass prisms ($N = 1.5$).

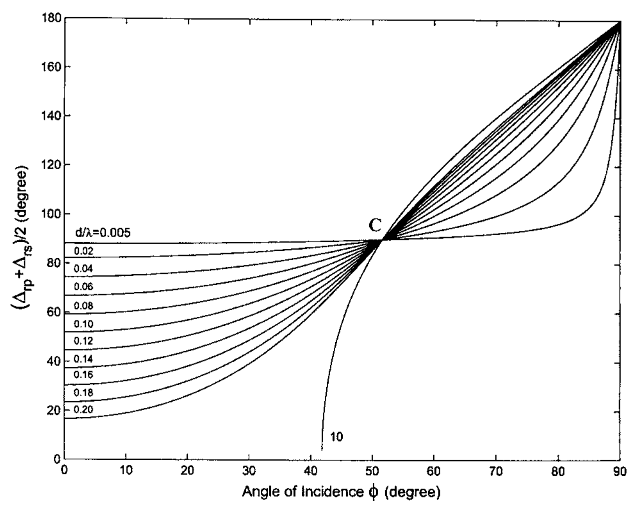

Fig. 4. Average reflection phase shift $\Delta_{ra} = (\Delta_{rp} + \Delta_{rs})/2$ as a function of the angle of incidence ϕ for $d/\lambda = 0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. As in Figs. 2 and 3, all curves pass through a common point C at which $\Delta_{ra} = 90°$. These results are calculated for an air gap of thickness d between two glass prisms ($N = 1.5$).

$$\sin^2 \phi_p = (N^2 + 1)/(N^4 + 1), \tag{27}$$

which gives $\phi_p = 53.248°$ for $N = 1.5$. Therefore the FTIR phase shift for the p polarization is constant at quarter-wave ($\Delta_{rp} = 90°$), independent of film thickness, when light is incident at the special angle ϕ_p. At the same angle of incidence, the corresponding transmission phase shift is zero, $\Delta_{tp} = 0$, independent of film thickness, as can be inferred from Eq. (9).

Similar results are obtained for the s polarization as shown in Fig. 3. Again, all curves pass through a common point B at which $\Delta_{rs} = 90°$. This occurs at the angle of incidence ϕ_s at which the interface reflection phase shift $\delta_s = 90°$, as one expects from Eq. (8). According to Ref. 4, ϕ_s is determined by

$$\sin^2 \phi_s = (N^2+1)/(2N^2), \tag{28}$$

which gives $\phi_s = 58.194°$ for $N=1.5$. Therefore the s-polarization FTIR phase shift is constant at quarter-wave ($\Delta_{rs}=90°$), independent of film thickness, when light is incident at the special angle ϕ_s. At the same angle, the corresponding transmission phase shift is zero, $\Delta_{ts}=0$, independent of film thickness, as can be seen from Eq. (9).

Another important result can be inferred from Figs. 2 and 3. In the limit of zero film thickness (i.e., for an ultrathin embedded layer), the reflection phase shift is quarter-wave independent of polarization (p or s) or angle of incidence (except at grazing incidence). This is also predicted by Eq. (8), which shows that as x (and d) go to zero, $\tanh x$ goes to zero, and $\Delta_{r\nu}=90°$ for all values $\delta_\nu \neq 0, \pi$.

The results for the average phase shift $\Delta_{ra}=(\Delta_{rp}+\Delta_{rs})/2$ as a function of the angle of incidence ϕ are

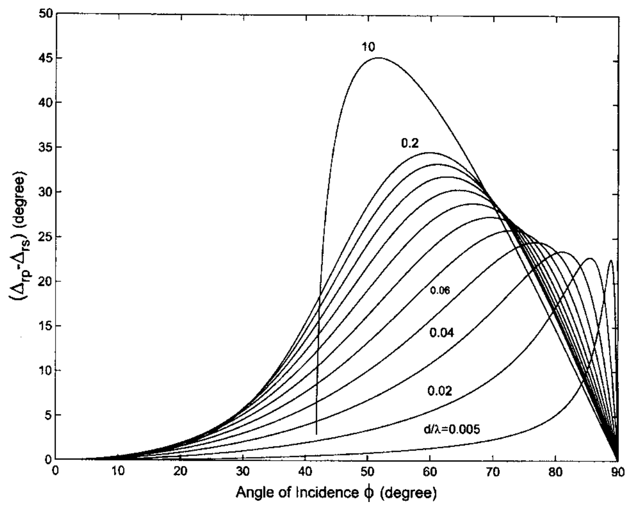

Fig. 5. Differential reflection phase shift $\Delta_r=(\Delta_{rp}-\Delta_{rs})$ as a function of the angle of incidence ϕ for $d/\lambda=0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. These results are calculated for an air gap of thickness d between two glass prisms ($N=1.5$).

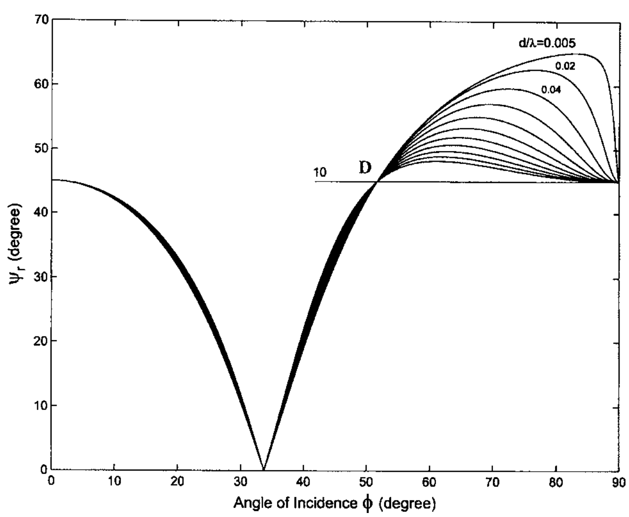

Fig. 6. Reflection ellipsometric parameters ψ_r as a function of the angle of incidence ϕ for $d/\lambda=0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. These results are calculated for an air gap of thickness d between two glass prisms ($N=1.5$).

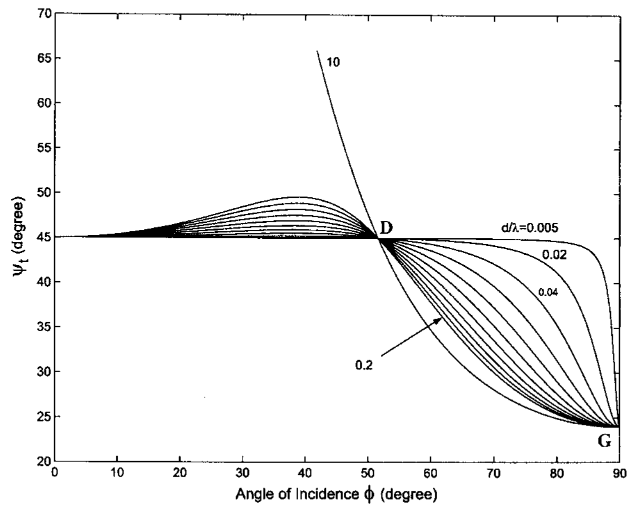

Fig. 7. Transmission ellipsometric parameters ψ_t as a function of the angle of incidence ϕ for $d/\lambda=0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. These results are calculated for an air gap of thickness d between two glass prisms ($N=1.5$).

shown in Fig. 4. As in Figs. 2 and 3, all curves in Fig. 4 pass through a common point C at which $\Delta_{ra}=90°$. This occurs at the angle of incidence ϕ_a at which the average interface reflection phase shift is quarter-wave, $\delta_a=(\delta_p+\delta_s)/2=90°$. Under this condition, $(\tan\delta_p+\tan\delta_s)=0$, and $\Delta_{ra}=90°$, according to Eq. (16). From Ref. 4, ϕ_a is determined by

$$\sin^2 \phi_a = 2/(N^2+1), \tag{29}$$

which gives $\phi_a=52.671°$ for $N=1.5$. Therefore the average of the FTIR phase shifts for the p and s polarizations remains constant at quarter-wave ($\Delta_{ra}=90°$), independent of film thickness, when light is incident at the angle ϕ_a. At the same angle, the corresponding average transmission phase shift is zero, $\Delta_{ta}=0$, independent of film thickness, as can be inferred from Eq. (11).

These results add new insight as to the significance of the special angles[4] ϕ_p, ϕ_s, and ϕ_a in the present context of FTIR by an embedded low-index film.

Figure 5 shows the differential reflection phase shift $\Delta_r=(\Delta_{rp}-\Delta_{rs})$ as a function of the angle of incidence ϕ for $d/\lambda=0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. It is apparent that the angular response of this differential phase is a sensitive function of d/λ, which suggests that ellipsometry[5] would be an excellent technique for measuring air-gap thickness. A salient feature of the Δ_r-versus-ϕ curve is the magnitude and the location of its peak. As d/λ decreases, the maximum value of $\Delta_r=(\Delta_{rp}-\Delta_{rs})$ decreases and its location shifts toward higher angles.

Finally, Figs. 6 and 7 show the ellipsometric parameters ψ_r and ψ_t as functions of the angle of incidence ϕ for $d/\lambda=0.005$, 0.02 to 0.20 in equal steps of 0.02, and 10. In Fig. 6, notice that $\psi_r=0$, independent of d/λ, at the Brewster angle of internal reflection of the glass–air interface, $\phi_B=\arctan(1/1.5)=33.690°$. Also, all curves in Figs. 6 and 7 pass through a common point D at which $\psi_r=\psi_t=45°$, independent of d/λ. The common angle of incidence at point D in Figs. 6 and 7 is given by Eq. (29). If Eq. (29) is substituted into Eq. (25), we obtain $\tan\psi_r=\tan\psi_t=1$. This confirms that ϕ_a is the angle of equal reflection and equal tunneling for the p and s polarizations.[7,8] Note that ϕ_a also happens to be the angle of maximum differential reflection phase shift[4] Δ_r in the limit of $d/\lambda \gg 1$.

At grazing incidence $(\phi = 90°)$, $\psi_{rG} = 45°$, $\tan \psi_{rG} = 1$, and Eq. (25) gives

$$\psi_{tG} = \arctan(1/N^2), \tag{30}$$

independent of film thickness. For $N = 1.5$, Eq. (30) gives $\psi_{tG} = 23.962°$, which corresponds to point G in Fig. 7.

5. REFLECTION AND TRANSMISSION PHASE SHIFTS AT THE CRITICAL ANGLE

It is interesting to consider the reflection and transmission phase shifts by an embedded low-index layer at the critical angle, $\phi_c = \arcsin(1/N)$. At this angle, Eqs. (4) show that $U = 0$. Substitution of $U = 0$ in Eqs. (3) and (6) yields

$$\delta_\nu = \tan \delta_\nu = 0,$$

$$x = \tanh x = 0. \tag{31}$$

If Eqs. (31) are used in Eqs. (8) and (9), the indeterminate forms

$$\tan \Delta_{r\nu} = 0/0, \tag{32}$$

$$\tan \Delta_{t\nu} = 0/0 \tag{33}$$

are obtained. By applying L'Hôpital's rule[9] to Eq. (8), we obtain

$$\Delta_{r\nu}(\phi_c) = \arctan(\delta_\nu'/x')_{\phi_c}, \tag{34}$$

where δ_ν', x' are the angle-of-incidence derivatives of δ_ν, x. Evaluation of these derivatives (which are skipped here to save space) at the critical angle and substitution of the results in Eq. (34) give

$$\Delta_{rp}(\phi_c) = \arctan[\pi^{-1}(d/\lambda)^{-1}N^2(N^2 - 1)^{-1/2}], \tag{35}$$

$$\Delta_{rs}(\phi_c) = \arctan\{[\pi^{-1}(d/\lambda)^{-1}(N^2 - 1)^{-1/2}]\} \tag{36}$$

for the p and s polarizations, respectively. Numerical values obtained from Eqs. (35) and (36) for $N = 1.5$ agree with the results shown in Figs. 2 and 3, respectively.

At the critical angle, the corresponding transmission phase shifts differ from the reflection phase shifts of Eqs. (35) and (36) by $\pi/2$ according to Eq. (11).

6. SUMMARY

Explicit and elegant expressions [Eqs. (8), (9), (13), and (16)] have been obtained for the phase shifts that p- and s-polarized light experience in FTIR and optical tunneling by an embedded low-index thin film. The differential phase shifts in reflection and transmission Δ_r, Δ_t are identical [Eqs. (14) and (23)], so that incident linearly polarized light is reflected and transmitted as elliptically polarized light of the same handedness. The associated ellipsometric parameters ψ_r, ψ_t are governed by a simple relation [Eq. (25)], which is independent of film thickness.

At the special angles of incidence at which the Fresnel interface reflection phase shifts for the p and s polarizations and their average are quarter-wave[4] [Eqs. (27)–(29)], the corresponding overall reflection phase shifts introduced by the embedded layer are also quarter-wave for all values of film thickness. Furthermore, in the limit of zero film thickness (i.e., for an ultrathin embedded layer), the reflection phase shifts are quarter-wave independent of polarization (p or s) or angle of incidence (except at grazing incidence).

Finally, it has been noted that variable-angle FTIR ellipsometry is particularly suited for measuring thin uniform air gaps between bulk dielectric prisms (Fig. 5).

APPENDIX A

The phase shifts that accompany total internal reflection at a dielectric–dielectric interface were considered in Ref. 4. The reflection phase shifts for p and s polarizations δ_p and δ_s [Eqs. (3)] increase monontonically from 0 at the critical angle to π at grazing incidence (see Fig. 1, Ref. 4). In this appendix, we obtain a simple expression for the ratio of the derivatives (slopes) of the δ_p- and δ_s- versus -ϕ curves.

We start with Eq. (8) of Ref. 4, which is repeated here:

$$\tan(\delta_p/2) = N^2 \tan(\delta_s/2). \tag{A1}$$

By taking the derivative of the natural logarithm of both sides of Eq. (A1) with respect to ϕ and applying some trigonometric identities, we obtain

$$\delta_p'/\delta_s' = \sin \delta_p / \sin \delta_s, \tag{A2}$$

where δ_p', δ_s' are the derivatives of the interface reflection phase shifts with respect to ϕ.

At the critical angle, both sides of Eq. (A2) are indeterminate $(\infty/\infty = 0/0)$. However, by applying L'Hôpital's rule,[9] we obtain

$$(\delta_p'/\delta_s')_{\phi_c} = N^2. \tag{A3}$$

At grazing incidence, the right-hand side of Eq. (A2) is again indeterminate. By applying L'Hôpital's rule[9] once more, we obtain

$$(\delta_p'/\delta_s')_{90°} = 1/N^2. \tag{A4}$$

Equations (A3) and (A4) indicate that the ratio of slopes is reversed between the critical angle and grazing incidence. Finally, we note that

$$(\delta_p'/\delta_s')_{\phi_a} = 1, \tag{A5}$$

where ϕ_a is given by Eq. (29). Equation (A5) is equivalent to $(\delta_p - \delta_s)' = 0$, so that ϕ_a is also the angle at which the interface differential reflection phase shift is maximum, as was noted in Ref. 4.

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