A Quadratic, Time-Domain Strip Theory Method for Predicting Global Ship Structure Response in Waves

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A QUADRATIC, TIME-DOMAIN STRIP THEORY METHOD FOR PREDICTING GLOBAL
SHIP STRUCTURE RESPONSE IN WAVES

An Honors Thesis

Presented to

the School of Naval Architecture and Marine Engineering

of the University of New Orleans

In Partial Fulfillment

Of the Requirements for the Degree of

Bachelor of Science

with Honors in Naval Architecture and Marine Engineering

By

Kyle Elias Marlantes

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Acknowledgements

Nearly four years ago, I put forward to my family my intention to move, from our rainy Pacific Northwestern corner of the country, to the city of New Orleans to pursue undergraduate study in Naval Architecture and Marine Engineering at the University of New Orleans. I can say with certainty that they all thought the decision was an odd one, but their support and encouragement throughout the process has been steadfast. I don’t think a simple thank you captures my gratitude toward them, but alas, it is the best I’ve got. Thank you.

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Abstract

This paper outlines the theory, development, validation, and some results of a quadratic strip theory method to predict the global structural response of the KRISO hull geometry due to regular waves in the time-domain. The method attempts to capture nonlinear effects of the dynamic problem due to time-varying underwater hull volume imparted by waves and vessel motions. These effects are formulated by drawing a relationship between the coefficients, $A_{33}$, $B_{33}$, and $C_{33}$ and the sectional draft, $T_s$. Additional nonlinearities are introduced by allowing for a flexible hull girder, and the inclusion of structural damping. Validation is facilitated by running test computations and comparing the linear and nonlinear results to segmented model test data. It is found that the predicted results are validated by the model data, and that nonlinear effects account for a significant increase in predicted bending moment.

Keywords: seakeeping, longitudinal strength, bending moment, ship structure, naval architecture
Section 1 – Introduction

The purpose of this paper is to provide a summary of the theoretical formulation of a computational method for predicting global hull structure response due to regular waves in the time-domain. In addition, some results from the method will be presented and compared to model test data obtained at the University of New Orleans towing tank. These results serve to validate the method, as well as to show some of its capabilities.

This type of analysis is of significant importance for large, ocean-going ships with slender hull forms. The recent trend to capitalize on economies of scale by designing and building ever larger ships makes the question of global hull response even more pertinent. Catastrophic accidents such as the MOL Comfort bring into question the adequacy of existing design methods for evaluating global structural response for these contemporary ship designs.

Established traditional design methods are often founded on statistical approaches, i.e. long-term distributions. One primary example of the use of such methods is in the evaluation criteria published by regulatory and classification societies. In these methods, a long-term sample of data from many ships in a specific, usually severe, seaway (i.e. the North Atlantic) is analyzed and compiled into a generalized bending moment distribution. The distribution is based on the probability of an occurrence of a limiting seaway event that may cause structural failure. The distribution is then scaled based on the principle dimensions of the vessel and the vessel’s still-water bending moment distribution, which is easily computed. Due to the quantity of data considered, these methods provide good results when compared to conventional vessels. However, for unique or otherwise sparsely represented hull forms, these methods may not encompass all possible variations.
Analytical approaches such as linear strip theory are also widely known, however not as common in the design process. These well-established linear strip theories have been shown to provide reasonably good approximations of a ship’s motions in heave and pitch. However, these methods do not consider the hogging and sagging conditions of a hull separately, yielding simplified dynamic bending moment predictions that are often unsatisfactory. Physical measurements show that sagging moments in certain ships can be significantly larger than predicted by these linear theories (Jensen 2001). Furthermore, linear theories assume the underwater volume of the hull is constant over time, regardless of the position of the wave crest or the vessel’s motions, thereby ignoring the time-dependent variation of the hydrostatic and hydrodynamic coefficients and excitation forces. Generally, these methods also consider the ship as a rigid body, and do not consider the flexibility of the hull structure or the effects of structural damping. These physical realities impart complex non-linearity to the hull-girder response and may result in other severe responses, such as springing.

Early work by Jacobs noted that dynamic bending moment response was predominately a second-order effect (1958). Furthermore, the bending moments were found to be “sensitive” to the hydrodynamic added mass and damping terms. His experimental and analytical work concluded that linear predictions were not sufficient in capturing the important “small local load variations”, and the resulting discrepancies were assumed to be attributed partly to the linear coefficients.

More recent experimental studies of a S-175 containership model in linear waves confirmed that wave loading on hulls with small $C_B$ is highly nonlinear (Fonseca 2004). The work concluded that the nonlinear effects are much more pronounced in the internal reactions than for the hull motions, further indicating that these reactions are predominately a higher-order response.
The observation that dynamic peak sagging moments are much larger than peak hogging moments was also reiterated.

Building on these findings, some work has been done to correct these shortcomings in the analytical prediction of dynamic bending moments. Jensen proposed a quadratic strip theory in which the sectional coefficients are a linear function of the local draft, which must be solved in the time-domain (2001). The method proved to predict some of the nonlinearities associated with internal bending moments and was in agreement with model tests on an S175 containership hull. Jensen also extended this work to provide design values based on the results of the nonlinear theory (2001).

The method presented here attempts to extend the linear strip theory to capture these nonlinear effects. In the current formulation, the ship is assumed to be in head seas ($\mu = 180^\circ$) and moving with zero forward speed ($U = 0$ ft/s). The hydrostatic and hydrodynamic coefficients are assumed to be a function of the sectional draft along the hull geometry, as proposed by Jensen. The sectional drafts are computed continuously through time, with consideration given to the vessel’s instantaneous vertical position, and the position of the wave along the hull. External excitation forcing is computed as a combination of simplified diffraction and Froude-Krylov forces using the strip theory equations given by Lewis, with nonlinearities further introduced by the dependence upon the hydrostatic stiffness and hydrodynamic damping coefficients (1989). Incident waves are considered linear for simplicity, although it would be advantageous to improve the method in the future by considering wave spectra and/or nonlinear waves.

The dynamic hull girder structure is modeled using beam elements which are free in vertical translation and rotation at each node. The model is shown by Figure 1.
Computationally, hydrostatic stiffness in heave and pitch is modeled using translational and rotational spring elements, respectively. Structural stiffness is imparted by the sectional moment of inertia and modulus of elasticity in the beam element as is typical in finite-element methods. Only vertical hydrodynamic damping is considered, with structural damping included in the formulation. Mass moment of inertia is computed at each node, with the influence of both physical mass and hydrodynamic added mass considered.

As ship designs continue to evolve, the need for more accurate and adaptable methods for predicting dynamic hull girder bending moments becomes ever more apparent. The methods and results presented in the following sections attempt to build upon existing methods and to show their efficacy in predicting global ship structure response in waves.
Section 2 – Theoretical Formulation

2.1 Equation of Motion

In this section the theory and assumptions behind the method will be discussed. As discussed earlier, Figure 1 illustrates the theoretical dynamic model underlying the method. The foundation of the method, then, is the assembly and solution of the equation of motion describing this model, where the general form is shown by (1).

\[(M + A)\ddot{\eta}(t, x) + (B_{\text{struct}} + B)\dot{\eta}(t, x) + (K + C)\eta(t, x) = F(t, x)\] (1)

Defined at a single node, \(M\) is the physical mass, \(A\) is the hydrodynamic added mass, \(B_{\text{struct}}\) is the structural damping, \(B\) is the hydrodynamic damping, \(K\) is the structural stiffness, and \(C\) is the hydrostatic stiffness. \(F(t, x)\) is the external excitation force due to waves and is a function of time and varies with longitudinal position. \(\ddot{\eta}(t), \dot{\eta}(t),\) and \(\eta(t)\) are the vertical translation and pitch acceleration, velocity, and position of the body in motion.

![Figure 2 Discretized Hull](image)

Equation (1), however, is the scalar form of the equation of motion, thereby describing only a single node in the model shown by Figure 1. To apply the theoretical model to a hull geometry, as shown by Figure 2, the hull is discretized into \(N_s\) stations or nodes. We denote a
particular node by \( n \), where the forward-perpendicular is denoted by \( n = 1 \), and the after-perpendicular is denoted \( n = N_s \). Hull elements are denoted by \( s \). Each hull element is theoretically equivalent to a beam element in Figure 1.

Equation (1) is then defined for each node in Figure 2. The resulting system of equations simplify to the matrix form of the equation of motion (2), where the coefficient matrices are of size \((2N_s, 2N_s)\) and \( \mathbf{F}(t) \) is the external force vector of length \((2N_s)\). One will also observe that the solution and its derivatives must also be vectors of length \((2N_s)\). Parallel to (1), \([M]\) and \([A]\) are the physical and hydrodynamic added mass matrices, respectively. \([B_{struct}]\) is the structural damping matrix, and \([B]\) is the hydrodynamic damping matrix. \([K]\) and \([C]\) are the structural stiffness and hydrostatic stiffness matrices.

\[
([M] + [A])\ddot{\mathbf{u}}(t) + ([B_{struct}] + [B])\dot{\mathbf{u}}(t) + ([K] + [C])\mathbf{u}(t) = \mathbf{F}(t) \quad (2)
\]

The size of the global coefficient matrices in (2) is driven by the number of degrees of freedom of the element that is used to represent a section of the hull. Here we utilize a 4-DOF beam element; i.e. the hull element is free to move in vertical translation (heave) and rotation about the transverse axis (pitch) at each node. This also means that \( \mathbf{F}(t) \) is representative of the external forces and moments, or the heave and pitch wave excitation forces (the latter of which are zero). Computationally, we discretize time by \( \Delta t \) which follows that (2) must be solved at every time step, but these computational details will be outlined later.

The solution of (2) is the global displacement and rotation at each node along the length of the hull girder. Because we are most interested in the structural response of the vessel, we must return to the elemental level to compute the internal reactions according to equation (3), where the subscript \( s \) denotes the elemental matrix of size \((4,4)\).
Equilibrium dictates that the internal moments at nodes shared by two elements will sum to zero. The resulting moment is taken as the average magnitude of these two values. In practice, it was found that the moments are not identical due to numerical limitations. However, a simple average presents no loss in accuracy since the difference is small compared to the magnitude of the values.

2.2 Sectional Coefficients

The sectional heave hydrodynamic coefficients $a_{33}$ and $b_{33}$ are computed using the two-dimensional strip method proposed by Bertram (2000). A potential flow field due to a linear distribution of point sources at the free-surface is developed to satisfy the physical boundary conditions at each section. Once the potential is known, the fluid velocity is therefore also known, and the added mass and damping coefficients are then computed by numerically integrating and decomposing the resulting pressure over the body contour specified by the hull geometry. The resulting coefficients, $a_{33}$ and $b_{33}$, are the hydrodynamic added mass per length, and hydrodynamic damping per length, respectively.

The sectional hydrostatic stiffness per unit length, $c_{33}$, is computed according to equation (4), where $B(x)$ is the sectional beam and $\rho$ and $g$ are the density of water and gravitational acceleration, respectively.

$$c_{33} = \rho g B(x)$$  \hspace{1cm} (4)
For geometry with sections of constant length, the sectional coefficients may be scaled according to equations (5) through (7) to yield the nodal coefficients, $A_{33}$, $B_{33}$, and $C_{33}$. The rotational nodal coefficients, $A_{55}$ and $C_{55}$, are computed according to equations (8) through (10) and are derived from the heave coefficients as shown. Here, $B_{55}$ is considered to be zero. General formulas for $A_{55}$, $B_{55}$, and $C_{55}$ will include a second term that depends upon $U$ and $\omega_e$. Recall that the method presented here assumes a vessel with zero forward speed ($U = 0$) and therefore these additional terms vanish.

\[
A_{33} = a_{33}l_s \quad (5)
\]
\[
B_{33} = b_{33}l_s \quad (6)
\]
\[
C_{33} = c_{33}l_s \quad (7)
\]
\[
A_{55} = \frac{1}{3} A_{33}l_s^2 \quad (8)
\]
\[
B_{55} = 0 \quad (9)
\]
\[
C_{55} = \frac{1}{3} C_{33}l_s^2 \quad (10)
\]

### 2.3 External Force Vector

The complex sectional external heave excitation force per unit length is computed according to equation (11). This equation is derived from the summation of the sectional Froude-Krylov and sectional diffraction exciting forces. Assumptions include exclusively heave-pitch forcing and zero forward speed. A more comprehensive discussion of the derivation of this equation may be found in Lewis (1989).

\[
f_3 = \zeta_a e^{ikwT_x} e^{-k_wT^*(x)} e^{i\omega_e t} \left[ c_{33}(x) - \omega_0 [\omega_e a_{33}(x) - ib_{33}(x)] \right] \quad (11)
\]
Here, $T^*(x)$ is the mean draft of the section at longitudinal position $x$, $t$ is time, and $i$ denotes the imaginary number. $T^*(x)$ is computed according to (12), where $S(x)$ is the sectional area and $B(x)$ is the waterline beam at the section.

$$T^*(x) = \frac{S(x)}{B(x)} \tag{12}$$

The real-valued sectional heave excitation force is then computed by scaling the real part of (11) by the sectional length according to equation (13). Note that this represents the external forces in the model shown by Figure 1.

$$F_3(t) = \text{Re}(f_3)l_s \tag{13}$$

The external force vector is then assembled accordingly.

$$\mathbf{F}(t) = \begin{bmatrix} F_{3_1}(t) & 0 & F_{3_2}(t) & 0 & \cdots & F_{3_{N_s}}(t) & 0 \end{bmatrix}^T \tag{14}$$

2.4 Linear Formulation

At this point, equation (2) is a system of second-order, linear, nonhomogeneous, ordinary differential equations with constant coefficients. The coefficient matrices are, under the assumptions presented up to this point, independent of the solution and are constant with time. With these distinctions, we will formulate the coefficient matrices in (2) to show the fully-linear form of the equation of motion.

2.4.1 Linear Mass Matrices

The global mass matrices are diagonal matrices of size $(2N_s, 2N_s)$. The global matrix is simply the superposition of the elemental mass matrix for each hull element. Equation (2) separates the physical mass matrix $[M]$, from the hydrodynamic added mass matrix $[A]$. In this section, we
will formulate the elemental physical and added mass matrices $[M]_s$ and $[A]_s$ which can be readily extended to the global system.

The elemental physical mass matrix is shown by equation (15), where $M$ is the physical mass of the element at the node, and $I_M$ is the physical mass moment of inertia of the element about the node. In general, $M$ and $I_M$ are known. In this formulation, $M$ is assumed to be a constant distribution over an element length (although not necessarily even over the length of the hull girder), resulting in zero off-diagonal terms.

$$[M]_s = \begin{bmatrix} M_1 & 0 & 0 & 0 \\ 0 & I_{M_1} & 0 & 0 \\ 0 & 0 & M_2 & 0 \\ 0 & 0 & 0 & I_{M_2} \end{bmatrix}$$

(15)

The hydrodynamic added mass matrix is given by equation (16). $A_{33}$ and $A_{55}$ are the hydrodynamic added mass and hydrodynamic mass moment of inertia, respectively. $A_{33}$ is computed at each node according to equation (5) in Section 2.2.

$$[A]_s = \begin{bmatrix} A_{331} & 0 & 0 & 0 \\ 0 & A_{551} & 0 & 0 \\ 0 & 0 & A_{332} & 0 \\ 0 & 0 & 0 & A_{552} \end{bmatrix}_s$$

(16)

In equation (2), the summation of matrices $[M]$ and $[A]$ form the “total mass” matrix coefficient. Note that this means the resulting inertia terms are also the summation of the physical and added mass moment of inertia. This has a significant effect on the magnitude of the resulting internal moments and must not be neglected.

To assemble the global mass matrices, $[M]_s$ and $[A]_s$ for $s \in [1, N_s - 1]$ are superimposed along the diagonal to yield $[M]$ and $[A]$. Expanding $([M] + [A])\ddot{\mathbf{q}}(t)$ from equation (2), one will see that the resulting expression indeed represents a force and moment vector.
2.4.2 Linear Stiffness Matrices

Stiffness in the system is attributed to two physical sources: structural stiffness, $K$, and hydrostatic stiffness, $C$. The elemental structural stiffness matrix, $[K]_s$, is entirely equivalent to the structural stiffness that is used in the formulation of finite-element methods, so it will not be discussed in detail here. $[K]_s$ is therefore given by equation (17). To maintain a general approach, it is not assumed that the element length, $l_s$, or sectional moment of inertia, $I_{yy_s}$, are constant along the length of the hull, and are therefore defined element-wise. However, $E$ is assumed constant for simplicity.

$$[K]_s = \frac{E I_{yy_s}}{l_s^3} \begin{bmatrix} 12 & 6l_s & -12 & 6l_s \\ 6l_s & 4l_s^2 & -6l_s & 2l_s^2 \\ -12 & -6l_s & 12 & -6l_s \\ 6l_s & 2l_s^2 & -6l_s & 4l_s^2 \end{bmatrix}_s$$

(17)

The linear elemental hydrostatic stiffness matrix, $[C]_s$, is shown by equation (18), where $C_{33}$ is the hydrostatic stiffness in heave and $C_{55}$ is the hydrostatic stiffness in pitch, as discussed earlier.

$$[C]_s = \begin{bmatrix} C_{33} & 0 & 0 & 0 \\ 0 & C_{55} & 0 & 0 \\ 0 & 0 & C_{33} & 0 \\ 0 & 0 & 0 & C_{55} \end{bmatrix}_s$$

(18)

The linear global stiffness matrices, $[K]$ and $[C]$, are then constructed by superimposing $[K]_s$ and $[C]_s$ in a diagonal manner as is customary in finite-element formulations.

2.4.3 Linear Damping Matrices

The system damping is composed of two quantities: the structural damping and the hydrodynamic damping. Structural damping, $[B_{struct}]_s$, is computed according to equation (19),
where \( \tan(\delta) \) is the structural damping coefficient and \( [K]_s \) is the elemental structural stiffness matrix from Section 2.4.2. For steel, \( \tan(\delta) \) is taken as 0.001 to 0.005.

\[
[B_{\text{struct}}]_s = \tan(\delta) \ast [K]_s
\]  

(19)

The elemental hydrodynamic damping matrix is then computed by (20) as shown.

\[
[B]_s = \begin{bmatrix}
B_{331} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & B_{332} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}_s
\]  

(20)

The linear global damping matrices, \([B_{\text{struct}}]\) and \([B]\), are then assembled by superimposing \([B_{\text{struct}}]\) and \([B]_s\) along the diagonal.

### 2.5 Nonlinear Formulation

In the linear regime, equation (2), \([A]\), \([B]\), and \([C]\) are assumed constant with time and independent of the solution, \( \eta \). Although this may be a valid simplification in certain instances, this is not at all true when considering the real-world physics. For example, as the wave passes the hull, the wetted surface area, sectional draft, sectional area, and sectional beam at waterline are all changing. Moreover, the vessel is no longer at an even trim, as the hull is in motion due to the excitation of the waves, leading to additional accelerations, velocity, and position changes. All of these factors have an effect on the coefficients, \( A_{33}, B_{33}, \) and \( C_{33} \), which translates into non-constant coefficient matrices. Recall that \( A_{55}, B_{55}, \) and \( C_{55} \) are computed using \( A_{33}, B_{33}, \) and \( C_{33} \) as outlined in Section 2.2. These effects also introduce additional time-variation in the wave excitation forcing, as (11) depends on the unscaled coefficients. From this, it can be deduced that \( A_{33}, B_{33}, \) and \( C_{33} \) are more accurately interpreted as functions of \( \eta \), or more specifically, \( z \). This yields (21), the nonlinear form of equation (2), where the coefficient terms are no longer constant.
with time or position, and the equation contains products of the solution and its derivatives with functions of the solution.

\[
([M] + [A(z)])\ddot{\eta}(t) + ([B_{\text{struct}}] + [B(z)])\dot{\eta}(t) + ([K] + [C(z)])\eta(t) = F(t)
\]  

(21)

These nonlinear terms present obvious problems: one cannot solve (21) unless the solution is already known. To move forward, it helps to consider \( A_{33}, B_{33}, \) and \( C_{33} \) as functions of section draft, \( T_s(t), \) where \( T_s(t) \) depends partly upon \( z(t). \) This distinction, although more of an intermediary than a change of physical dependence, allows the separation of \( A_{33}, B_{33}, \) and \( C_{33} \) from \( z \) in an advantageous manner. The coefficients are computed, at the most basic level, according to underwater geometry and wave characteristics, the former of which is directly dependent upon draft, making this a reasonable substitution. The sectional draft can be computed according to equation (22), where \( T \) is the stillwater draft, \( k_w \) is the wave number, \( x \) is the longitudinal position of the section node, \( \omega_e \) is the wave encounter frequency, and \( t \) is time.

\[
T_s(t) = T + \zeta_a \cos(k_w x - \omega_e t) + z(t)
\]  

(22)

One will notice that as \( T_s(t) \) is still dependent upon \( z(t) \), we have seemingly only complicated the problem. However, if we invoke a small time-step assumption, it is possible to approximate \( T_s(t) \) by replacing \( z(t) \) with \( z(t - \Delta t) \). If \( \Delta t \) is sufficiently small, the difference \( z(t) - z(t - \Delta t) \) will be sufficiently close to zero. The coefficients can now be computed without trouble as \( z(t - \Delta t) \) is known. For long wave periods, it is not difficult to reach a suitable small value for \( \Delta t \), but it is an important consideration when assessing short wave periods, and a worthy consideration otherwise.

Now it is perfectly valid to compute the sectional coefficients at each discrete time-step by interpolating the submerged geometry of the hull using \( T_s \), and this direct method does have several
advantages. However, to simplify the problem, we use a different approach. Prior to solving equation (21), the coefficients are computed at two different stillwater drafts: $T^\pm = T \pm \Delta T$, where $\Delta T$ has been successfully set equal to $0.1 \zeta_a$. Using these two sets of coefficients, it is possible to construct a linear relationship for each coefficient as a function of draft, as shown by equations (23) – (25), where the second term on the right-hand-side is the perturbation term. If the local draft is equal to the design draft, the second term vanishes and we are left with the coefficient at $T$. Note that the difference $|T_s(t) - T|$ will be in some ways limited by the wave amplitude and the vertical position of the node. In resonance, these terms will be constructive and may lead to large perturbation terms.

\[
A_{33}(T_s) = A_{33}(T) + \left. \frac{\partial A_{33}}{\partial T_s} \right|_{T_s=T} (T_s(t) - T) \tag{23}
\]

\[
B_{33}(T_s) = B_{33}(T) + \left. \frac{\partial B_{33}}{\partial T_s} \right|_{T_s=T} (T_s(t) - T) \tag{24}
\]

\[
C_{33}(T_s) = C_{33}(T) + \left. \frac{\partial C_{33}}{\partial T_s} \right|_{T_s=T} (T_s(t) - T) \tag{25}
\]

\[
A_{55}(T_s) = \frac{1}{3} A_{33}(T_s) l_s^2 \tag{26}
\]

\[
B_{55}(T_s) = 0 \tag{27}
\]

\[
C_{55}(T_s) = \frac{1}{3} C_{33}(T_s) l_s^2 \tag{28}
\]

Equations (23) through (25) may be visualized as a linear relationship between the coefficients and sectional draft as shown by Figure 3. To make this simplification valid, the exact function of the hydrodynamic coefficient (indicated arbitrarily by a dashed line) must be close to linear within the draft range of interest. This draft range is difficult to quantify without first estimating the vessel’s motion response due to the wave. Because the heave response of the vessel
may not be symmetric about \( T \), the coefficient also may not oscillate between \( T \pm \zeta_a \). Changes in hull geometry near the waterline should also be considered, as any effects on the coefficients will not be captured unless \( T \pm \Delta T \) includes the desired hull variation. Abrupt changes in hull geometry near the waterline will not be accurately represented by this approximation, and would require a more direct method.

![Figure 3 Linear Approximation of Change in Coefficients](image)

The linear assumption mentioned above is an important consideration when selecting a suitable \( \Delta T \) for a particular wave, and has a significant influence on the validity at certain wave amplitudes. If, for example, the wave amplitude is very large, it may be that the hydrodynamic coefficients are not reasonably linear in the sectional draft range, and therefore this approximation may not be valid. One might compute multiple points within the expected draft range to check the validity of the assumption, and this method may be extended beyond a simple linear relationship, although this has not been evaluated in the work presented here.
2.5.1 Nonlinear Coefficient Matrices

In the non-linear case, the only coefficient matrices that change are \([A], [B], \text{ and } [C]\) to include the perturbation terms. The physical mass matrix, \([M]\), structural damping matrix, \([B_{\text{struct}}]\), and structural stiffness matrix, \([K]\), remain identical between the linear and nonlinear formulations. Here we will present the nonlinear elemental matrices \([A(z)], [B(z)], \text{ and } [C(z)]\) to complete the nonlinear formulation of the equation of motion (21).

The nonlinear elemental hydrodynamic added mass matrix, hydrodynamic damping matrix, and hydrostatic stiffness matrix are given by equations (29), (30), and (31), respectively. Here, the diagonal terms are defined by equations (23) through (28).

\[
[A(z)]_s = \begin{bmatrix}
A_{33}(T_s)_1 & 0 & 0 & 0 \\
0 & A_{55}(T_s)_1 & 0 & 0 \\
0 & 0 & A_{33}(T_s)_2 & 0 \\
0 & 0 & 0 & A_{55}(T_s)_2
\end{bmatrix}_s
\] (29)

\[
[C(z)]_s = \begin{bmatrix}
C_{33}(T_s)_1 & 0 & 0 & 0 \\
0 & C_{55}(T_s)_1 & 0 & 0 \\
0 & 0 & C_{33}(T_s)_2 & 0 \\
0 & 0 & 0 & C_{55}(T_s)_2
\end{bmatrix}_s
\] (30)

\[
[B(z)]_s = \begin{bmatrix}
B_{33}(T_s)_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & B_{33}(T_s)_2 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}_s
\] (31)

2.6 Solution of the Equation of Motion

With the equation of motion fully defined in both the linear and nonlinear formulation, the next logical step is to determine the solution. The system is solved using the Newmark-\(\beta\) direct-integration scheme. With the careful selection of the coefficients, \(\gamma = \frac{1}{2}\) and \(\beta = \frac{1}{4}\), the scheme offers good numerical stability, does not introduce numerical damping, and yields second order...
accuracy. The general method is given by equations (32) through (34). The constants $a_0$ through $a_7$ are also defined below.

\[
[K] \mathbf{\ddot{u}}(t + \Delta t) = F_3(t) + [M](a_0 \mathbf{u}(t) + a_2 \mathbf{\dot{u}}(t) + a_3 \mathbf{\ddot{u}}(t)) + [B](a_1 \mathbf{u}(t) + \\
\cdots a_4 \mathbf{\dot{u}}(t) + a_5 \mathbf{\ddot{u}}(t))
\]

(32)

\[
\mathbf{\ddot{u}}(t + \Delta t) = a_0(\mathbf{u}(t + \Delta t) - \mathbf{u}(t)) - a_2 \mathbf{\dot{u}}(t) - a_3 \mathbf{\ddot{u}}(t)
\]

(33)

\[
\mathbf{\ddot{u}}(t + \Delta t) = \mathbf{\ddot{u}}(t) + a_6 \mathbf{\dot{u}}(t) + a_7 \mathbf{\ddot{u}}(t + \Delta t)
\]

(34)

\[
a_0 = \frac{1}{\beta \Delta t^2}, a_1 = \frac{\gamma}{\beta \Delta t}, a_2 = \frac{1}{\beta \Delta t}, a_3 = \frac{1}{2\beta} - 1, a_4 = \frac{\gamma}{\beta} - 1
\]

(35)

\[
a_5 = \frac{\Delta t}{2} \left( \frac{\gamma}{\beta} - 2 \right), a_6 = \Delta t (1 - \gamma), a_7 = \gamma \Delta t
\]

$[K]$ is called the *structural impedance*, and is given by (36). As implied by equation (32), the structural impedance must be inverted at every time-step to solve for $\mathbf{u}(t + \Delta t)$. Fortunately, $[K]$ is nonsingular and can be inverted relatively easily using a decomposition routine.

\[
[K] = \frac{1}{\beta \Delta t^2} [M] + \frac{\gamma}{\beta \Delta t} [B] + [K]
\]

(36)

The velocity and acceleration are then computed using equations (33) and (34), respectively. It is important to recall that the $[M]$, $[B]$, and $[K]$ matrices in equation (36), as well as $F_3$ in equation (32), are computed at every time-step in the nonlinear formulation and represent the *total* mass, *total* damping, and *total* stiffness and external excitation forcing.
Section 3 – Results

3.1 Experimental Validation

Recent work by Givan, et. al. summarized the midship heave, pitch, and bending moment response of a segmented KRISO container ship model at the University of New Orleans towing tank (2011). The study considered five (5) different wave frequencies at wave amplitudes of 1in and 1.5in. The model was constructed by Single in 2010 at the UNO model shop to perform a similar analysis. The general particulars of the full-scale hull are given by Table 1 (NMRI Japan, KCS). The UNO model was built to a 1:100 scale.

Table 1 Full Scale KRISO Geometry

<table>
<thead>
<tr>
<th>Main Particulars</th>
<th>Full Scale</th>
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<tbody>
<tr>
<td>Length between perpendiculares</td>
<td>$L_{BP}$ (ft)</td>
</tr>
<tr>
<td>Length of waterline</td>
<td>$L_{WL}$ (ft)</td>
</tr>
<tr>
<td>Maximum beam of waterline</td>
<td>$B_{WL}$ (ft)</td>
</tr>
<tr>
<td>Depth</td>
<td>$D$ (ft)</td>
</tr>
<tr>
<td>Draft</td>
<td>$T$ (ft)</td>
</tr>
<tr>
<td>Displacement</td>
<td>$\Delta$ (ft$^3$)</td>
</tr>
<tr>
<td>Block coefficient</td>
<td>$C_B$ (-)</td>
</tr>
<tr>
<td>Longitudinal center of buoyancy</td>
<td>$LCB$ (%$L_{BP}$), fwd+</td>
</tr>
</tbody>
</table>

The method described in this paper was implemented in the FORTRAN programming language, under the name MOMUNO. Using a full-scale computer model of the KRISO hull, a geometry file was prepared for input into the FORTRAN code. A body plan of the geometry is shown by Figure 4. The theoretical weight distribution specified by Givan during his research was also specified in the input file and is given by Figure 5. At the design draft of 35.4ft, results were
generated for ten (10) different wave frequencies at wave amplitudes of 1ft and 2ft. Both linear and nonlinear results were generated for comparison. In the nonlinear case, $\Delta T$ was taken as $0.1\zeta_a$, or 0.1ft for the 1ft wave, and 0.2ft for the 2ft wave. $E$ was taken as $4.176e9$ psf. All simulations generated 300 seconds of data at a 0.1s time-step.

**Figure 4** Full Scale KRISO Body Plan

**Figure 5** Theoretical Weight Distribution
In all cases, the sectional moment of inertia, $I_{yy_s}$, was set to a very large value (10,000,000,000 ft$^4$) to simulate rigid body motion ($I_{yy} = \infty$) and avoid effects of hull flexibility in the initial results. This translates to a constant sectional inertia distribution along the length of the hull girder. Structural damping was also ignored in both the linear and nonlinear results ($\tan(\delta) = 0$). Therefore, the results in this section present nonlinear effects due only to the nonlinear coefficients, $A_{33}$, $B_{33}$, and $C_{33}$.

The midship heave and pitch RAOs for the linear and nonlinear maximum and minimum values were then plotted against the scaled model test measurements obtained by Givan (2011). Although the focus of MOMUNO is the global structural response, the heave and pitch response is the first step in achieving this result. A comparison of the computational motion results to the model test data is important to validate the numerical solution before considering the internal reactions directly. To this end, the heave and pitch results are shown by Figures 6 and 7, respectively.

**Figure 6** Rigid-Body Midship ($n=11$) Heave RAO, $T=35.4$ ft, $\Delta T=0.1\zeta_a$, $dt=0.1$ s
One will immediately notice the relative agreement between the model test data and the results obtained using MOMUNO. As expected, the heave response, $\eta_{33}$, is equivalent to the wave amplitude, $\zeta_a$, at low encounter frequencies, resulting in an RAO of unity. Conversely, the heave response decays to near zero as $\omega_e$ exceeds 1.0 rad·s$^{-1}$. The pitch response appears to be excited at $\omega_e$ near 0.45 rad·s$^{-1}$. Pitch response also decays at low and high encounter frequencies, as expected. In general, the nonlinear predicted heave and pitch response do not differ significantly from the linear formulation. If anything, the nonlinear pitch response may be slightly more pronounced near the natural frequency as well as the dip near 0.63 rad·s$^{-1}$. It does not appear that the nonlinear effects yield any appreciable difference at encounter frequencies less than about 0.4 rad·s$^{-1}$. This makes physical sense, as the lower frequency is paired with a longer wavelength, thereby reducing the change in $T_s$ as the long wave period appears more or less like a still waterline. This convergence is also observed in the heave response at high frequencies. Although minimal, any
deviation between linear and nonlinear heave and pitch response appears to be observed at or above 0.4 rad·s⁻¹.

The rigid-body midship dynamic bending moment amplitude function, $\Phi_M(\omega_e)$, is plotted against the model test data in Figure 8. Again, there appears to be a general agreement in terms of magnitude and shape between the computational and experimental results, with a maximum bending moment occurring between 0.55 rad·s⁻¹ and 0.60 rad·s⁻¹. Contrary to the rigid-body heave and pitch response, nonlinear coefficients impart significant variation in the predicted moment response when compared to the linear formulation. It is evident that the sagging moment is markedly larger than the other predictions at wave frequencies between about 0.53 rad·s⁻¹ and 0.67 rad·s⁻¹. Unfortunately, the available model test data lie on either side of this critical range, making it difficult to conclude that the nonlinear prediction provides a better peak bending moment prediction.

![Figure 8](image-url)

**Figure 8** Rigid-Body Midship ($n=11$) Bending Moment Amplitude Function, $T=35.4$ft, $\Delta T=0.1\zeta$, $dt=0.1s$
The rigid-body longitudinal dynamic bending moment distribution for the linear and nonlinear hogging and sagging results at an encounter frequency of 0.6 rad·s⁻¹ is given by Figure 9. At this wave frequency, the sagging moment is the governing condition, with the hogging moment at midship being about 66% of the corresponding sagging moment in terms of magnitude. Here it is most evident that the linear method may under-predict the midship bending moment by about 20%. More generally, the linear prediction significantly under-predicts the maximum bending moment along the middle 50% of the hull girder.

![Rigid-Body Longitudinal Bending Moment Distribution](image)

**Figure 9** Rigid-Body Longitudinal Bending Moment Distribution, $T=35.4ft, \Delta T=0.1\zeta_a, dt=0.1s$, $\omega_e=0.6$ rad·s⁻¹

### 3.2 Flexible Hull Effects

Flexibility of the hull girder is introduced by reducing the sectional moment of inertia, $I_{yy_s}$, to values similar to what might be specified for an actual structure. In general, $I_{yy_s}$ also varies across the length of the hull girder. In this analysis, the sectional inertia of the model was scaled and applied to the full-scale simulation for consistency. This resulted in a constant $I_{yy_s}$ distribution
of 36,285 ft$^4$. The flexible hull was then subjected to encounter frequencies from 0.1 rad·s$^{-1}$ to 3.0 rad·s$^{-1}$, or a waterline-length to wave-length ratio ($L_{WL}/\lambda$) of approximately 0.04 to 33.93.

Similar to the rigid-body heave and pitch motions, it was found that including hull flexibility resulted in no appreciable influence on the vessel motions. The dynamic bending moment response, however, exhibits significant deviations from the rigid-body results. Figure 10 shows the bending moment amplitude function for both flexible and rigid-body results at midship. In general, flexible hull effects are most apparent for the KRISO hull at $\omega_e$ above 1.5 rad·s$^{-1}$, or $L_{WL}/\lambda = 8.48$. Some difficulties with attaining numerical stability at lower frequencies were also realized.

![Graph showing midship bending moment amplitude function](image)

**Figure 10** Midship Bending Moment Amplitude Function, $T=35.4$ft, $\Delta T=0.1\zeta_a$, $dt=0.1$s,

$I_{yy_s} = 36,285$ ft$^4$

Figure 10 shows a prominent resonance in the nonlinear flexible midship bending moment at an encounter frequency of 2.6 rad/s, $L_{WL}/\lambda = 25.5$. This peak is believed to be the result of springing, a high-frequency wave induced structural vibration that has been observed in container
ships with low stiffness (Jensen 1981). One will notice that the rigid-body linear and nonlinear results do not capture the resonance. To further investigate this result, Figure 11 shows the nonlinear time-domain midship bending moment, heave, and pitch response for the rigid and flexible KRISO hull at the peak encounter frequency of $2.6 \text{ rad}\cdot\text{s}^{-1}$, $L_{WL}/\lambda = 25.5$, for the final 25 seconds of the time-domain results.

![Figure 11](image-url)

**Figure 11** Time-Domain Midship Response, $\omega_e = 2.6 \text{ rad}\cdot\text{s}^{-1}$, $L_{WL}/\lambda = 25.5$

The time-domain bending moment plot shows what appears as high-frequency noise. These sharp variations are typical of a springing response (ABS 2014). Furthermore, the time-domain pitch response exhibits similar high-frequency oscillations.
Springing behavior can be a major contributor to hull girder fatigue. Research by Jensen on a containership hull concluded that the natural frequency of the flexible hull occurred at $L_{WL}/\lambda$ of approximately 10 (2001). For the KRISO hull, this natural resonance is observed at $L_{WL}/\lambda = 25.5$. Although the results presented here are consistent with the expectations of a springing response, further testing of MOMUNO on low-flexibility hulls should be performed to better understand the high-frequency capabilities that the method offers.
Section 4 – Conclusion

The results of the computational method are validated by the model test data. For a rigid hull, the nonlinear effects do not appear to have significant influence on vessel motions in heave and pitch, although some small variations are observed at higher wave frequencies. Dynamic midship bending moments were found to be highly sensitive to nonlinear effects, yielding notable differences between the hogging and sagging condition. It was found that the peak bending moment was in the sagging condition, confirming the experimental and full-scale observations from Jacobs (1958) and Fonseca (2004), among others. Peak nonlinear sagging moments exceeded linear predictions by about 20%. The sparsity of the available model data did not allow conclusive evidence that the nonlinear effects were more accurate, however, the agreement with nonlinear results from Jensen is encouraging.

Hull flexibility was found to impart notable variations in the nonlinear hogging and sagging dynamic bending moment at higher encounter frequencies ($\omega_e \geq 1.5 \text{ rad/s}$). The flexible hull also exhibited a resonance at $\omega_e = 2.6 \text{ rad/s}$, $L_{WL}/\lambda = 25.5$. This resonance is believed to be an indication of hull-girder springing. Steady-state time-domain plots of the midship bending moment, heave and pitch response, also show high-frequency oscillations in the bending moment and pitch results. These findings are in line with findings by Jensen (2001) and springing guidance by the American Bureau of Shipping (2014). Although the results are promising, further testing and validation of MOMUNO is necessary to better understand the capabilities offered by the code when evaluating springing.

The effects of varying $\Delta t$ and $\Delta T$ must also be reiterated. In the nonlinear formulation, these variables may have a significant effect on the stability and accuracy of the solution. This is
especially true when analyzing a flexible hull or evaluating an encounter frequency near resonance. Quantifying these effects would be beneficial to improving the usability of the code, and increasing confidence in the results.

To conclude, the results presented in this paper show that the computational method may offer a viable means of evaluating dynamic global structural response of a ship due to waves. However, further testing and improvements are required before the code might be suitable for use in design analysis. A more robust set of model test data would allow a more comprehensive validation of the computational results. The capability to specify non-zero vessel speed, analyze large-amplitude waves, or input a wave spectra to better represent a realistic sea-state, are possible additions.
References


Taravella, Brandon M. *Offshore Structure and Ship Dynamics*. NAME 3160 Course Notes, University of New Orleans. 2016.


# Appendix A: MOMUNO User Manual

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Contact Information

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Overview

The MOMUNO program was developed as an in-house seakeeping/dynamic structural response code for use by the School of Naval Architecture and Marine Engineering at the University of New Orleans. The first version of the code was developed with future improvements and additions in mind.

The program is a FORTRAN implementation of a time-domain, strip theory method for predicting dynamic bending moments and shear forces in a hull girder due to regular waves. The code offers users the option to investigate the influence of nonlinear sectional coefficients, $a_{33}$, $b_{33}$, and $c_{33}$ by specifying some or all of the coefficients as linear or nonlinear. Nonlinear, time-varying coefficients are approximated using a quadratic strip-theory (Jensen 2001). Some details and assumptions behind this nonlinear theory are found in Section 1 of this manual.

The code can produce results for nearly any hull geometry in any linear wave signal. The hull geometry is specified by longitudinal, transverse, and vertical coordinates in a geometry input file. This file is formatted similarly to a GHS geometry file. However, some modifications and additions must be included for the file to work correctly. Details regarding input files and preparing geometry files are included in Section 3.1 of this manual.

The code can produce a variety of output. However, the current version focuses on generating the time-domain heave, pitch, and dynamic bending moment and shear signals over a period of time specified by the user. An additional post-processing code converts this time-domain data into longitudinal heave, pitch, and dynamic bending moment and shear force distributions along the length of the hull girder. These distributions can then be used to construct response amplitude function plots of the hull. Details regarding output files, and post-processing, are included in Sections 3.3 and 3.4 of this manual.
1 Theory and Assumptions

This section will provide a brief overview of the theory and assumptions behind the method implemented by MOMUNO. Users are encouraged to read the thesis titled, “A Quadratic, Time-Domain Strip Theory Method for Predicting Global Ship Structure Response in Waves” by Kyle E. Marlantes available on http://scholarworks.uno.edu/.

The code utilizes a strip theory approach to computing the external excitation forcing due to waves and a dynamic finite element approach to solving the hull-girder response problem. Discretization of the hull is defined by the input geometry. If the geometry file contains offset data for 21 stations, the program will define 21 nodes and 20 beam elements. Each hull section is modeled as a 4-DOF beam element. Hydrodynamic and hydrostatic terms are modeled as spring and damping elements at each node.

An equation of motion is defined at each node. The resulting system of equations may be written in matrix form as shown by equation (1).

\[
([M] + [A(z)])\ddot{\mathbf{u}}(t) + ([B_{\text{struct}}] + [B(z)])\dot{\mathbf{u}}(t) + ([K] + [C(z)])\mathbf{u}(t) = \mathbf{F}(t)
\] (1)

MOMUNO utilizes this nonlinear form of the equation and toggles the nonlinearity of the coefficient matrices, \([A(z)], [B(z)],\) and \([C(z)],\) to switch between the linear and nonlinear form. In brief, the nonlinearity of the coefficient matrices is introduced by developing a linear relationship between the coefficients, \(a_{33}, b_{33},\) and \(c_{33},\) and the sectional draft, \(T_s,\) at each node. This linear interpolation requires that the wave amplitude be small, and the geometry near the waterline to be without abrupt changes. Because the sectional draft depends partly on the solution, \(z,\) and the wave elevation through time, this captures some of the nonlinearities due to the influence of local variations along the hull. In the nonlinear case, the slope of this linear relationship is non-zero and the intercept is determined from the coefficients. In the linear case, the slope is zero and the intercept is equal to the coefficient value at the stillwater draft. The user is encouraged to read some of the many published works by Jensen, some of which are cited at the end of this manual.

The hydrodynamic coefficients are computed at each node using a two-dimensional panel method by Bertram (2000). The hydrostatic coefficient is computed from the sectional beam. These terms are then assembled into the coefficient matrices accordingly.

The external excitation force in heave is computed at each node by assuming a constant force over a section length centered about the node. In other words, the force is scaled using \(1/2\) the section length on either side of the node. These forces are computed by scaling the real part of equation (2) over this length.

\[
f_3 = \zeta_3 e^{ik_wx}e^{-kwT_s(x)}e^{i\omega t}\left[ c_{33}(x) - \omega_0[\omega a_{33}(x) - ib_{33}(x)] \right]
\] (2)

Assumptions in equation (2) include exclusively heave-pitch forcing, zero forward speed \((U=0)\) and deep water, linear waves. The equation is derived from the summation of the sectional Froude-Krylov and sectional diffraction exciting forces, as outlined in Lewis (1989). The mean sectional draft \(T^*(x)\) is computed numerically as the quotient of the submerged sectional area, \(S(x),\) and the sectional beam,
$B(x)$. MOMUNO uses trapezoidal numerical integration to compute $S(x)$ at each node from the user specified draft.

The solution to equation (1) is found using the Newmark-beta direct integration scheme. The resulting displacements are then used to compute the internal reactions (force/moment) at each node.

**Important Assumptions**

- Linear, deep-water waves
- Exclusively heave-pitch forcing
- Small wave amplitude
- Zero forward speed
2  Code Commentary

In this section, we will give an overview of the program structure. The program is divided into a main program and a number of subroutines. A summary of the main program and a description of each subroutine is provided. It is expected that the reader of this section has access to a copy of the code to follow along with the descriptions provided. This is not intended to be a standalone line-by-line commentary, but rather as a complement to the code.

2.1  Main Program

The main program part of the code is composed of six (6) steps:

Step 1: Read user input from input.txt

User input is simply read from the input file input.txt. This defines many of the variables, such as stillwater draft, time-step, draft increment, and others. See Section 3.1 for more information.

Step 2: Read wave data from wave.txt

The wave characteristics are read from the input file wave.txt using the subroutine readwave(...). See Section 2.2 for more on the subroutine readwave(...).

Step 3: Compute slope and intercepts for the sectional coefficients; a33, b33, c33.

Nonlinearity of the coefficient matrices is introduced by drawing a relationship between the sectional coefficients and the sectional draft, as proposed by Jensen (2001). MOMUNO builds this linear relationship by computing the sectional coefficients at two drafts, one below the user defined stillwater draft, and one above the stillwater draft. The resulting relationship is defined by the coefficient slopes and intercepts, which defines the linear function.

Depending on whether the variables nla, nlb, and/or nlc are defined as TRUE or FALSE in the input.txt file, the code computes the sectional coefficient slopes and intercepts as follows:

If TRUE, we have a nonlinear coefficient. The coefficient is computed at a lower draft, Tin=T-deltaT, and an upper draft, Ts=T+deltaT, using the geom(...) and hmasse(...) subroutines, see Section 2.2 (Bertram 2000). These computations yield the vectors of coefficient values for each node, a33s and a33in, for the upper and lower drafts, respectively. The resulting slope and intercept is defined as shown. The process is identical for the other coefficients.

\[
a33\text{slope} = \frac{(a33s-a33in)}{(Ts-Tin)}
\]
\[
a33\int = a33in-a33\text{slope}*Tin
\]

If FALSE, we have a linear coefficient. The coefficient is computed at the user defined stillwater draft, using the geom(...) and hmasse(...) subroutines, see Section 2.2 (Bertram 2000). This computation yields the vector of coefficient values for each node, a33. The resulting slope is set to zero, and the intercept is set to a33. This results in a constant coefficient over the entire draft range, as is customary in the linear formulation.
Step 4: Read geometry at user defined draft from gfile

After the coefficient slopes and intercepts are computed, the code must once again read the geometry at the stillwater draft, T. This yields the geometric data necessary to compute the excitation forces later in the code. This is again accomplished using the subroutine geom(...).

Step 5: Compute the position, velocity, acceleration, external forcing, and internal reactions using NB(...)

This is where the real bulk of the computation takes place. However, the entire process is contained within the subroutine NB(...). At this step in the main program, we simply call NB(...) and supply all of the necessary information that was computed or read at earlier steps. The resulting position, velocity, acceleration, external forcing, and internal reactions are returned as the arrays sol, sold, soldd, F, and inF. These are the time-domain results.

Step 6: Write results to files

Here we write the results from Step 5 to file. See Section 3.3 on Output Files for more information.

2.2 Subroutines

Outside of the main program, all computations are collected into subroutines. A summary of each subroutine is provided in this section, including the subroutines input and output arguments.

NB(...)

Input: Nx, Ny, x, y, z, B, Lc, rho, gravity, wavnum, omegao, omegae, zeta, T, a33slope, b33slope, c33slope, a33int, b33int, c33int, E, sd, Iyy, mass, time, dt

Output: tv, sol, sold, soldd, F, inF

This subroutine is based around an implementation of the Newmark-β direct integration scheme. The result is the position, velocity, acceleration, external forcing, and internal reactions at each node over time. The subroutine is organized into six (6) tasks:

Task 1: Define Newmark-β constants

In the current version of the code, the Newmark-β constants gam and beta are not specified by the user. These values are internally defined as 0.5 and 0.25. For a helpful discussion as to why these values are selected, see the lecture videos by William Anderson at as available on YouTube at the following link https://www.youtube.com/watch?v=9Pam51Yqg_c&t=1171s (works as of April 2017). The constants a0 through a7 are also computed according to the standard Newmark-β formulation.
**Task 2: Define initial conditions**

Time is initialized at 0, as is the initial position, velocity, and acceleration at each node. These initial values are stored as the first elements in the solution arrays: tv, sol, sold, and soldd.

**Task 3: Compute initial coefficient values**

The initial coefficients $a_{33}$, $b_{33}$, and $c_{33}$ are computed according to the following equation and stored as the first value in the arrays $a_{33 \text{time}}, b_{33 \text{time}},$ and $c_{33 \text{time}}$. These arrays represent the variation in each coefficient over time. One will notice that the coefficients are initialized at the stillwater draft, $T$.

\[
\begin{align*}
a_{33 \text{time}}(:,1) &= a_{33 \text{slope}}T + a_{33 \text{int}} \\
b_{33 \text{time}}(:,1) &= b_{33 \text{slope}}T + b_{33 \text{int}} \\
c_{33 \text{time}}(:,1) &= c_{33 \text{slope}}T + c_{33 \text{int}}
\end{align*}
\]

**Task 4: Compute initial external forcing**

The initial external forcing is computed by calling the subroutine `forceheave(...)` using the geometry data at the stillwater draft, the wave data, and the initial values of $a_{33}, b_{33},$ and $c_{33},$ and time. The result is stored as the first column in the array $F$.

**Task 5: Compute structural stiffness matrix**

Here we must compute the structural stiffness matrix by setting the hydrostatic stiffness terms in the elemental stiffness matrix to 0. This is necessary to compute the internal reactions, where the hydrostatic stiffness does not directly contribute to the internal reactions. This is accomplished by calling the subroutine `element_stiffness(...)` and defining $c_{33}$ as 0 temporarily.

**Task 6: Perform Newmark-$\beta$ method in a time-step loop**

This is where we solve for the response. Within a loop over the number of time-steps, an additional seven (7) steps must be performed at each time-step:

1. Compute the sectional drafts at each node.
2. Compute the new $a_{33}, b_{33},$ and $c_{33}$ vectors using the linear relationship.
3. Construct the elemental and global mass, damping, and stiffness matrices using subroutines `element_mass(...), element_stiffness(...), element_damping(...), and assemble_matrix(...).`
4. Construct the new structural impedance and its inverse, $k_{ha}$ and $k_{hatinv}$, using the subroutine `LUinv(...).`
5. Compute the external forcing at the new time-step using `forceheave(...).`
6. Use the Newmark-beta equations to compute the solution and its derivatives, sol, sold, soldd, at the new time-step.
7. Compute internal reactions for the new time-step using the solution and the subroutine `internal_fm(...). The internal reactions are then stored in the array inF.`
The subroutine reads the input geometry file and assigns its contents to variables. The subroutine references the user specified draft against the geometry data and collects the geometry that represents the underwater volume. Points at the waterline are interpolated linearly. All points are translated by the input draft to realign the geometry with the origin at the waterline as needed by the hmass(...) subroutine.

The number of stations is stored as \(\text{Nx}\), the longitudinal location of each station is stored in the vector \(\text{x}\), the transverse and vertical offsets for each point at each station are stored in the arrays \(\text{y}\) and \(\text{z}\).

After reading the geometry data, the subroutine also computes the mass distribution \(\text{mass}\) from the weight distribution \(\text{wgt}\), and computes the length of each section (distance between nodes) along the length of the hull and stores this value in \(\text{Lc}\).

This subroutine simply reads the wave data from the input file wave.txt. The wave number, wave frequency, wave amplitude, wave heading, and vessel speed are stored in the variables \(\text{wavnum}, \text{omegao}, \text{omegae}, \text{zeta}, \text{mu}, \text{U}\), respectively. The subroutine also computes the encounter frequency, \(\text{omegae}\).

This subroutine computes the sectional hydrostatic stiffness coefficient (per length) using the following equation:

\[
\text{c33}(i) = B(i) \times \rho \times \text{gravity}
\]

The resulting coefficients are stored in the vector \(\text{c33}\).
**hmasse(…)**

**Input:** gravity, yk, zk, nk, ome, ns

**Output:** addedm

This subroutine is a direct use of the `hmasse(…)` subroutine provided by Bertram in his book *Practical Ship Hydrodynamics* (2000). A more comprehensive discussion of the function behind this subroutine may be found in his book. For completeness, however, we offer a brief description here.

The subroutine is an implementation of a two-dimensional panel method and computes the complex hydrodynamic mass for symmetric cross sections in deep water (Bertram 2000). The result is the complex number `addedm`, where the real part may be scaled by density and encounter frequency to yield the added mass per unit length for the section. The hydrodynamic damping per length is computed by scaling the negative of the imaginary part of `addedm` by `rho` and `omegae`.

The subroutine accepts `yk` and `zk` arrays with at most 120 elements. Similarly, the number of points defined at each section may not exceed 50. This is an important consideration when preparing the geometry file. See Section QQQ.

The subroutine also internally calls `SIMQCD(…)` which is an implementation of the Gauss algorithm to solve a system of complex linear equations. This subroutine also comes from Bertram (2000).

**LUinv(…)**

**Input:** mat

**Output:** matinv

This subroutine is an implementation of the LU decomposition method using Gauss elimination to compute the inverse of a square matrix. The only input is the matrix that should be inverted, and the result is the inverse. Note that this subroutine does not perform automatic checks to determine if an inverse exists and it is up to the user to be aware of this limitation.
**forceheave(...)**

**Input:** Nx, Ny, x, y, z, B, Lc, wavnum, omegao, omegae, zeta, a33, b33, c33, t

**Output:** F3

This subroutine computes the external excitation force in heave according to the equation given in the PNA Vol. III (1989), see Section 1.

The subroutine first computes the mean draft, meanT, at each node by using trapezoidal integration to compute the sectional area defined by the input geometry, and the section beam. Note that the geometry data passed to the `forceheave(...)` subroutine must represent the submerged geometry.

The external force vector, F3, is then assembled using a loop over all nodes. In each loop, the external excitation force in heave is defined for the odd vector indices, and the external excitation force in pitch is set to zero.

The external excitation force in heave is computed by scaling the real part of the complex excitation force by $\frac{1}{2}$ the section length on either side of the node. Forces on end nodes are scaled by only a single adjacent $\frac{1}{2}$ section length as there is only a single adjacent hull element.

The resulting F3 vector is then the external excitation force vector at a particular point in time.

---

**element_stiffness(...)**

**Input:** Nx, Iyy, Lc, B, rho, gravity, E, c33

**Output:** elstiff

This subroutine assembled the elemental total stiffness matrix for each element. Both the structural stiffness and hydrostatic stiffness are included in the matrix. The structural stiffness matrix is simply a 4-DOF beam element. The hydrostatic stiffness terms, are computed along the diagonal terms according to the following:

\[
\begin{align*}
\text{elstiff}(1,1,k) &= \ldots + c33(k)\times Lc(k)/2. \\
\ldots \\
\text{elstiff}(2,2,k) &= \ldots + \rho \times \text{gravity} \times (1/3.) \times ((Lc(k)/2)^3) \times B(k) \\
\ldots \\
\text{elstiff}(3,3,k) &= \ldots + c33(k+1)\times Lc(k)/2 \\
\ldots \\
\text{elstiff}(4,4,k) &= \ldots + \rho \times \text{gravity} \times (1/3.) \times ((Lc(k)/2)^3) \times B(k+1)
\end{align*}
\]

The resulting elstiff array is a 3-dimensional array, where k indexes the element along the length of the hull girder.
**element_damping(...)**

**Input:** \( \text{Nx, Iyy, Lc, E, b33, sd} \)

**Output:** \( \text{eldamp} \)

This subroutine assembled the total elemental damping matrix. The structural damping as well as the hydrodynamic damping is included in the formulation. The structural damping is computed as the structural damping coefficient, \( \text{sd} \), multiplied by the structural stiffness matrix. The hydrodynamic damping is defined along the diagonal as:

\[
\begin{align*}
\text{eldamp}(1,1,k) &= ... + b33(k) \times Lc(k)/2. \\
... \\
\text{eldamp}(3,3,k) &= ... + b33(k+1) \times Lc(k)/2. \\
...
\end{align*}
\]

The resulting \( \text{eldamp} \) array is a 3-dimensional array, where \( k \) indexes the element along the length of the hull girder.

**element_mass(...)**

**Input:** \( \text{Nx, mass, Lc, a33} \)

**Output:** \( \text{elmass} \)

This subroutine assembles the total elemental mass matrix. The physical mass as well as the hydrodynamic added mass are included in the formulation. The diagonal elements are computed as shown below.

\[
\begin{align*}
\text{elmass}(1,1,k) &= \text{mass}(k)/2. + a33(k) \times Lc(k)/2. \\
... \\
\text{elmass}(2,2,k) &= (1/3.) \times (\text{mass}(k) + ((a33(k)+a33(k+1)) \times Lc(k)/2.)) \times Lc(k)^2 \\
... \\
\text{elmass}(3,3,k) &= \text{mass}(k)/2. + a33(k+1) \times Lc(k)/2. \\
... \\
\text{elmass}(4,4,k) &= (1/3.) \times (\text{mass}(k) + ((a33(k)+a33(k+1)) \times Lc(k)/2.)) \times Lc(k)^2
\end{align*}
\]

The resulting \( \text{elmass} \) array is a 3-dimensional array, where \( k \) indexes the element along the length of the hull girder.
assemble_matrix(…)

Input: elmat

Output: matrix

This subroutine takes any input 3D elemental matrix array and assembles the individual elemental matrices (indexed by $k$) into the global matrix by superimposing the elements along the diagonal.

The input matrix, elmat, is just an internal variable that may be passed to any external elemental matrix array. The output array, matrix, is the resulting global matrix of size($2N_x,2N_x$).

internal_fm(…)

Input: elmass, eldamp, elstiff, sol, sold, soldd

Output: inF_elmat

Description

This subroutine computes the internal reactions for each hull element at a single time-step. The internal reactions are computed for each element by summing the products of sol and elstiff, sold and eldamp, and soldd and elmass. The reactions for each element are then stored in the output array inF_elmat, where each column represents a different element.
3 Using the Code

This section will focus on how to use MOMUNO to generate output. It will describe the three required input files, their format, and how to set them up. Guidance will be given as to how a user should prepare to run the code. Also, a discussion of the output files and post-processing options is included.

3.1 Input Files

Three input files must be set up to use MOMUNO. One file must define the geometry and the name of this file may be specified by the user. The second file specifies certain parameters needed by the program. This input parameter file must be named input.txt. The third file defines the wave characteristics and must be named wave.txt. All files must be in the same directory as the MOMUNO executable.

A note about units: MOMUNO does not account for different unit systems. It is up to the user to maintain consistent input units in all input files!

Input Parameter File

The input parameter file is called input.txt and is also an ASCII text file. The file is organized as shown by the example below. The first line specifies the name of the geometry file, including extension. The second line specifies gravitational acceleration, shown here as 32.2 ft/s. The third line is water density, in this case 2.0 slugs/ft³. The fourth line specifies the modulus of elasticity, shown here as 4.176e9 psf. Line five specifies the structural damping coefficient, here being zero. The sixth line defines the stillwater draft, or 35.433 ft in this example. The seventh line specifies the draft increment, deltaT, which is often set to 0.1’/a. In this case, a 1.0 ft wave amplitude was used, so a draft increment of 0.1 ft is specified. The next two lines define the total amount of time and the time-step, or 300 seconds and 0.1 seconds, respectively. The last three lines are logical which specify whether a33, b33, and/or c33 are nonlinear or linear. The first logical corresponds to a33, the second to b33, and the third to c33. Here, .TRUE. for each line means all coefficients are nonlinear.

<table>
<thead>
<tr>
<th>KRISO_A.txt</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>32.2</td>
<td>2.0</td>
<td>4.176E9</td>
<td>0.0</td>
<td>35.433</td>
<td>0.1</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>.TRUE.</td>
<td>.TRUE.</td>
<td>.TRUE.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1 Example input.txt File
Geometry File

The geometry file is an ASCII text file with a name specified by the user. The filename may not have more than 12 characters, including the file extension. Figure 2 shows a shortened example geometry file.

```
21
0,37
0,1.824
0.233,1.899
0.451,1.998
0.847,2.251
1.197,2.563
1.513,2.913
2.343,4.068
4.055,6.893
5.349,9.413
6.309,11.695
7.061,14.084
...
38.714,34
0,0
0.283,0
0.447,0.123
0.894,0.533
1.409,1.147
2.969,3.487
4.982,6.944
6.624,10.357
8.717,17.323
8.866,19.56
...
10000000000,1456000
10000000000,3248000
10000000000,5600000
10000000000,6496000
10000000000,6608000
10000000000,6608000
10000000000,6608000
10000000000,6608000
10000000000,6608000
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10000000000,6608000
10000000000,6608000
10000000000,6608000
10000000000,6608000
10000000000,6608000
10000000000,6608000
10000000000,7280000
10000000000,7280000
10000000000,7280000
10000000000,4704000
10000000000,1568000
```

Figure 2 Example Geometry File
The file should follow the format of the example file shown on the previous page. Transverse symmetry of the hull geometry is assumed, therefore the file only defines the starboard portion of the hull. The origin is defined as the intersection of the forward perpendicular, baseline, and centerline. Aft, starboard, and vertical are positive. All geometry files must conform to this coordinate reference.

The value on the first line indicates the total number of stations, or nodes, on the geometry file. This also specifies how many sections define the geometry. In this example the leading value specifies 21 stations. After this leading value, blocks of offsets are then listed to define each section. There should be 21 of these blocks in the example file shown, although most are not shown below. Each block is has a leading pair of numbers, the first giving the longitudinal location of the section from the origin, and the second defining the number of points used to define the section. In this example, two blocks are shown. The first is at a longitudinal position of 0 ft, with a total number of points equal to 37. The second block describes a section at a longitudinal position of 38.714 ft, with a total number of points equal to 34.

Following the first values in each block, points are defined to describe the section. The points are organized such that the first values is the transverse offset from midship and the second value is the vertical offset above baseline. In the first block, the first point is defined as 0 ft transverse by 1.824 ft above baseline (0,1.824). The code requires that each block contain no more than 50 points, and successive points may not have identical transverse offset (vertical line).

After all station contours have been defined, the final block of values is used to define the sectional inertia and weight distribution over the length of the hull. These values are defined for each hull element. Therefore, since the example has 21 stations, it also has 20 elements and 20 inertia-weight pairs. Here, the first element is specified to have an inertia of 10,000,000,000 ft$^4$ and a weight of 1,456,000 lbs.

Because this format closely follows the GHS geometry file format. It is advantageous to use GHS to develop the majority of the geometry file, and simply modify the text file to suit the need of MOMUNO.

Wave File

| 0.01118 |
| 0.6   |
| 3.1415 |
| 0     |
| 1.0   |

**Figure 3 Example wave.txt File**

The file that defines the wave characteristics in called wave.txt and is also an ASCII text file. An example wave file is shown above. The first value defines the wave number, defined as 0.01118 rad/ft. The second line specifies the wave frequency, or 0.6 rad/s in this example. The third line is the wave heading, in radians, here defined as $\pi$. The fourth line is the forward speed of the vessel. The current version of the code assumes this value to be 0 ft/s, therefore it must be set to zero. The variable is included to allow for future development of the code. The final line of the file specifies the wave amplitude, shown here as 1.0 ft.
3.2 Preparing a Run

When preparing to use MOMUNO to generate output, it is recommended that the user devise a folder structure that allows them to store output for various input configurations without over-writing output. Because the output files are written to the same directory as the MOMUNO executable file, it is advised that the folder structure have a program folder that contains \texttt{MOMUNO.exe} and input files are copied into the folder, the program run, and then the output files copied out to their own folder.

To prepare for a run, simply place the input files in the same directory as the executable file and double click the executable to run the code. The command window will appear and the program will provide information to the user about the input parameters and progress of the run. When the run is complete, the window will close automatically and the output files will be found in the same directory.
### 3.3 Output Files

With few modifications, a user may modify the code to print any variable to a text file for use. However, the current version of the code outputs only the solution files, sol.txt, sold.txt, and soldd.txt, as well as the external forcing file, F.txt, the internal reactions file, inF.txt, and a33time.txt, b33time.txt, and c33time.txt, which are the coefficients over time.

<table>
<thead>
<tr>
<th>File</th>
<th>Col. 1</th>
<th>Col. 2</th>
<th>Col. 3</th>
<th>Col. 4</th>
<th>Col. 5</th>
<th>Col. 6</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>sol.txt</td>
<td>time-step</td>
<td>Heave displ., node 1</td>
<td>Pitch displ., node 1</td>
<td>Heave displ., node 2</td>
<td>Heave displ., node 3</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>sold.txt</td>
<td>time-step</td>
<td>Heave vel., node 1</td>
<td>Pitch vel., node 1</td>
<td>Heave vel., node 2</td>
<td>Heave vel., node 3</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>soldd.txt</td>
<td>time-step</td>
<td>Heave accel., node 1</td>
<td>Pitch accel., node 1</td>
<td>Heave accel., node 2</td>
<td>Heave accel., node 3</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>F.txt</td>
<td>time-step</td>
<td>Heave force, node 1</td>
<td>Pitch moment, node 1</td>
<td>Heave force, node 2</td>
<td>Pitch moment, node 2</td>
<td>Heave force, node 3</td>
<td>...</td>
</tr>
<tr>
<td>inF.txt</td>
<td>Block format, each block is a different time-step</td>
<td>node 1, LHS shear</td>
<td>node 2, LHS shear</td>
<td>node 3, LHS shear</td>
<td>node 4, LHS shear</td>
<td>node 5, LHS shear</td>
<td>node 6, LHS shear</td>
</tr>
<tr>
<td></td>
<td></td>
<td>node 1, LHS mom.</td>
<td>node 2, LHS mom.</td>
<td>node 3, LHS mom.</td>
<td>node 4, LHS mom.</td>
<td>node 5, LHS mom.</td>
<td>node 6, LHS mom.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>node 1, RHS shear</td>
<td>node 2, RHS shear</td>
<td>node 3, RHS shear</td>
<td>node 4, RHS shear</td>
<td>node 5, RHS shear</td>
<td>node 6, RHS shear</td>
</tr>
<tr>
<td></td>
<td></td>
<td>node 1, RHS mom.</td>
<td>node 2, RHS mom.</td>
<td>node 3, RHS mom.</td>
<td>node 4, RHS mom.</td>
<td>node 5, RHS mom.</td>
<td>node 6, RHS mom.</td>
</tr>
<tr>
<td>a33time.txt</td>
<td>a33, node 1</td>
<td>a33, node 2</td>
<td>a33, node 3</td>
<td>a33, node 4</td>
<td>a33, node 5</td>
<td>a33, node 6</td>
<td>...</td>
</tr>
<tr>
<td>b33time.txt</td>
<td>b33, node 1</td>
<td>b33, node 2</td>
<td>b33, node 3</td>
<td>b33, node 4</td>
<td>b33, node 5</td>
<td>b33, node 6</td>
<td>...</td>
</tr>
<tr>
<td>c33time.txt</td>
<td>c33, node 1</td>
<td>c33, node 2</td>
<td>c33, node 3</td>
<td>c33, node 4</td>
<td>c33, node 5</td>
<td>c33, node 6</td>
<td>...</td>
</tr>
</tbody>
</table>

*Table 1* MOMUNO.exe Output File Layout

The solution files contain, as implied, the position, velocity, and acceleration for both heave and pitch for each node for each time-step over the time interval specified by the user. Also included in the first column is the time vector, tv, to allow for easy plotting using a plotting engine such as GNUplot. For example, in the file sol.txt, the first column is the time-step, the second column is the vertical displacement for node 1, the third column is the pitch displacement for node 1, the fourth column is the vertical displacement for node 2, and so forth or all nodes. Table 1 gives the layout of each output file for reference.
3.4 Post-Processing

The output files discussed in Section 3.3 are well suited to developing time-domain plots. However, if you wish to develop longitudinal bending moment or shear distributions, or RAO plots of the data, some post-processing of the output files must be performed.

An additional code called *post_MOMUNO.exe* is used to perform these post-processing efforts. The code simply requires the user to specify the range of time from the MOMUNO results to consider, the time-step that the results were generated at, and the number of nodes. These values are set within the code itself. Once these parameters are specified and the code is compiled, post-processing may be performed by placing the MOMUNO output files (*sol.txt* and *inf.txt*) in the same directory as *post_MOMUNO.exe* and running the executable. The resulting files will be named *mom_dist.txt*, *moment.txt*, and *sol_dist.txt*. These files correspond to the longitudinal moment distribution in sag and hog, the time-domain moments at each node, and the longitudinal heave and pitch distributions, respectively. Table 2 below outlines how these output files are organized. **Pitch is always returned in radians.**

| File          | Col. 1     | Col. 2          | Col. 3          | Col. 4 | Col. 5 | Col. 6 | ...
|---------------|------------|-----------------|-----------------|--------|--------|--------|--------
| mom_dist.txt  | node       | hogging moment  | sagging moment  | none   | none   | none   | ...
| moment.txt    | node 1 moment | node 2 moment | node 3 moment | node 4 moment | node 5 moment | node 6 moment | ...
| sol_dist.txt  | node       | heave maximum   | heave minimum   | pitch maximum | pitch minimum | none   | ...

*Table 2* post_MOMUNO.exe Output File Layout
References


Taravella, Brandon M. *Offshore Structure and Ship Dynamics*. NAME 3160 Course Notes, University of New Orleans. 2016.

