10-1982

Explicit determination of the complex refractive index of an absorbing medium from reflectance measurements at and near normal incidence

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Received May 21, 1982

Measurement of reflectance at normal incidence $\mathcal{R}$ and its fractional change $\Delta\mathcal{R}/\mathcal{R}$ caused by a change of the angle of incidence from 0 to a small angle ϕ ($\phi \lesssim 20°$) permits explicit determination of both the refractive index n and extinction coefficient k of an isotropic absorbing medium. The medium of incidence (ambient) is assumed to have a known refractive index (e.g., =1 for vacuum or air), and the incident light is either p or s linearly polarized.

A variety of reflectance methods[1–12] is available to determine the complex refractive index $N = n - jk$ of an absorbing medium. Few of those methods[5,6,9,10] provide explicit solutions for n and k in terms of the measured reflectances. In this Letter we propose a new method that provides simple, direct, and explicit determination of n and k in terms of the intensity reflectance measured at normal incidence $\mathcal{R}$ and the fractional change of such reflectance $\Delta\mathcal{R}/\mathcal{R}$ that results from a given change of angle of incidence from 0 to ϕ, where ϕ is a small angle ($\lesssim 20°$).

Figure 1 shows the normal-incidence reflection of light by the planar interface between a transparent ambient of known refractive index N_0 and an absorbing substrate (mirror) of complex refractive index N_1 to be determined. Both media are assumed to be homogeneous and isotropic. The mirror can be rotated about an axis z in its surface through the point of reflection by a known angle ϕ. The incident light is linearly polarized with its electric vector vibrating either parallel or perpendicular to the rotation axis. (These are the conventional s and p polarizations, respectively.) A complex reciprocal relative refractive index defined by

$$N_r = N_0/N_1 = n_r + jk_r \tag{1}$$

is more readily determined first by using the proposed method. Of course, once N_r is found, N_1 is given by

$$N_1 = N_0/N_r = n_1 - jk_1. \tag{2}$$

The signs of the imaginary parts in Eqs. (1) and (2) are consistent with the Nebraska (Muller) conventions.[13]

The first equation to be used is[14]

$$\frac{\Delta r_\nu}{r_\nu} = \pm N_r \phi^2, \tag{3}$$

which gives the fractional change of Fresnel's complex interface reflection coefficient for the ν polarization (+ for $\nu = s$ and − for $\nu = p$) that results from changing the angle of incidence from 0 to ϕ, where ϕ is a given small angle. The measurable fractional change of intensity reflectance, $\Delta\mathcal{R}_\nu/\mathcal{R}_\nu$, is related to $\Delta r_\nu/r_\nu$ by

$$\Delta\mathcal{R}_\nu/\mathcal{R}_\nu = 2\,\mathrm{Re}(\Delta r_\nu/r_\nu), \tag{4}$$

where Re means the real part of. Substitution of Eq. (3) into Eq. (4) and use of Eq. (1) give

$$|\Delta\mathcal{R}_\nu/\mathcal{R}_\nu| = 2n_r\phi^2. \tag{5}$$

Equation (5) readily determines n_r:

$$n_r = |\Delta\mathcal{R}_\nu/\mathcal{R}_\nu|/2\phi^2. \tag{6}$$

(If $|\Delta\mathcal{R}_\nu/\mathcal{R}_\nu|$, measured at various values of small ϕ, is plotted versus $2\phi^2$, the slope of the resulting straight line gives a precise estimate of n_r.) Subsequently, k_r is found from the normal-incidence reflectance

$$\mathcal{R}_\nu = |(1 - N_r)/(1 + N_r)|^2 \tag{7}$$

or

$$\mathcal{R}_\nu = [(1 - n_r)^2 + k_r{}^2]/[(1 + n_r)^2 + k_r{}^2]. \tag{8}$$

Equation (8) gives

$$k_r = \left[2n_r\left(\frac{1 + \mathcal{R}_\nu}{1 - \mathcal{R}_\nu}\right) - (n_r{}^2 + 1)\right]^{1/2}. \tag{9}$$

Equations (6) and (9) show explicitly how the complex reciprocal relative refractive index, $N_r = n_r + jk_r$, is determined from the normal-incidence reflectance $\mathcal{R}$ and its fractional change $\Delta\mathcal{R}/\mathcal{R}$ caused by changing incidence from normal to a small angle ϕ. The substrate complex refractive index N_1 is calculated from N_r and the known refractive index N_0 of the ambient (usually air, $N_0 = 1$) by using Eq. (2).

The incident light is assumed to be either p or s polarized. The method in its present form would not work if the incident light were unpolarized. This is because $\Delta\mathcal{R}_p/\mathcal{R}_p = -\Delta\mathcal{R}_s/\mathcal{R}_s$, so that

$$\Delta\mathcal{R}_u/\mathcal{R}_u = 0 \tag{10}$$

to second order in ϕ (the subscript u denotes unpolarized).

$\Delta\mathcal{R}/\mathcal{R}$ can be accurately determined by a lock-in technique if the mirror is periodically oscillated (e.g., sinusoidally, so that $\phi = \hat{\phi}\sin\omega t$, where $\hat{\phi}$ is a small-amplitude angular excursion) and the modulation of the reflected light intensity $\Delta I/I$ is

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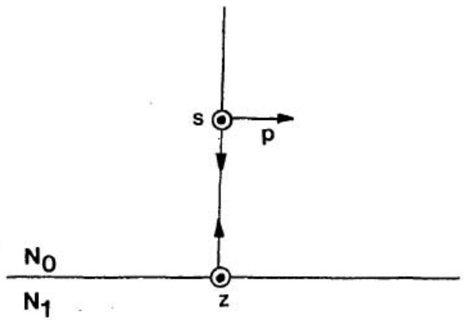

Fig. 1. Normal-incidence reflection of light by the interface between a transparent ambient of known refractive index N_0 and an absorbing substrate (mirror) with unknown complex refractive index N_1. The mirror can be rotated around an axis z in its surface through the point of reflection. p and s are linear-polarization directions perpendicular and parallel to the rotation axis, respectively.

determined. With the intensity of the incident light constant, it is easy to show that

$$\Delta\mathcal{R}/\mathcal{R} = \Delta I/I. \tag{11}$$

The method can be considered as an interesting special case of angle-of-incidence derivative ellipsometry and reflectometry[15,16] and, more closely, of a method previously described by Hunderi.[17]

I am pleased to acknowledge support by the National Science Foundation under grant no. DMR-8018417.

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