Reflection of an electromagnetic plane wave with 0 or $\pi$ phase shift at the surface of an absorbing medium

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Reflection of an electromagnetic plane wave with 0 or \(\pi\) phase shift at the surface of an absorbing medium

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An electromagnetic plane wave incident obliquely from a transparent medium onto the surface of an absorbing medium can be reflected with 0 or \(\pi\) phase shift if (i) the wave is \(p\) (TM) polarized, and (ii) the complex relative dielectric function \(\varepsilon\) is such that 0 < \(|\varepsilon|^2/2\text{Re}(\varepsilon)\) < 1. Furthermore, the locus of \(\varepsilon\) such that the reflection coefficient for the \(p\) polarization is real at the same angle of incidence, is a circle, and that of \(\varepsilon^{1/2}\) (the complex relative refractive index) is Bernoulli's lemniscate.

Except under conditions of total internal reflection, a plane electromagnetic wave experiences a “trivial” phase shift of either 0 or \(\pi\) upon reflection at the interface between two transparent (homogeneous and isotropic) media at all angles of incidence and for both the \(s\) (TE) and \(p\) (TM) polarizations. However, when the medium of refraction (or incidence) becomes absorbing, reflection induces a phase shift \(\delta\) that is generally neither 0 or \(\pi\). In fact, if surface layers (films) can be ruled out, \(\delta \neq 0\) or \(\pi\) may be considered as a manifestation of the existence of absorption. The question arises: Can \(\delta\) revert to one of its trivial values of 0 or \(\pi\) even though absorption is present? The answer is yes. In this Letter we determine when this happens.

The interface Fresnel reflection coefficient for the \(p\) polarization (with electric vibration parallel to the plane of incidence) can be written as:

\[
\begin{align*}
  r &= \frac{(1 - x)/(1 + x)}{1}, \\
  x &= (\varepsilon - \sin^2\phi)^{1/2}/\varepsilon \cos\phi,
\end{align*}
\]

where \(\phi\) is the angle of incidence and \(\varepsilon\) is the relative dielectric function of the two media, that is, the ratio of the dielectric
function of the medium of refraction to that of the medium of incidence. For $r$ to become real, Eq. (1) shows that $x = n$ must be real. Consequently, $x^2$ must be real and positive, or, from Eq. (2),

$$
\epsilon = \frac{\epsilon - \sin^2 \phi}{\cos^2 \phi} = \eta,
$$

where $\eta$ is a positive real number. Equation (3) can be rewritten as a quadratic in $\epsilon$ whose roots are

$$
\epsilon = [1 \pm (1 - \eta \sin^2 2\phi)^{1/2}] / 2\eta \cos^2 \phi.
$$

In Eq. (4) when $\eta \sin^2 2\phi \leq 1$, $\epsilon$ is a positive real number. This is the case of interfaces between transparent media. More interesting is the case when $\eta \sin^2 2\phi > 1$. Equation (4) then indicates that $\epsilon$ becomes complex, $\epsilon = (\epsilon_r, \epsilon_i)$, with real and imaginary parts given by

$$
\epsilon_r = 1/2, \eta \cos^2 \phi,
$$

and

$$
\epsilon_i = \pm (-1 + \eta \sin^2 2\phi)^{1/2} / 2\eta \sin 2\phi.
$$

Elimination of $\eta$ between Eqs. (5a) and (5b) gives

$$
\epsilon_r^2 + \epsilon_i^2 - 2\epsilon_r \sin^2 \phi = 0.
$$

Equation (6) indicates that the locus of $\epsilon = (\epsilon_r, \epsilon_i)$, such that the reflection coefficient for the $p$ polarization at a given angle of incidence $\phi$ is real, is a circle in the complex $\epsilon$ plane with center on the real axis, $\epsilon = (\sin^2 \phi, 0)$, and radius of $\sin^2 \phi$. Fig. 1(a). Varying $\phi$ generates coaxial circles that touch the imaginary axis at the origin, with centers on the segment of the real axis $0 \leq \epsilon \leq 1$. $\phi = 0$ (normal incidence) gives a null circle coincident with the origin ($\epsilon = 0$), while $\phi = 90^\circ$ (grazing incidence) yields a circle with unit radius with center at (1,0).

The interior and boundary of the latter circle define the domain of the $\epsilon$ plane where the reflection coefficient for the $p$ polarization is real at a specific angle of incidence $\phi$ for each $\epsilon$. It is often convenient, particularly in the optical region of the electromagnetic spectrum, to work with refractive indices as well as dielectric functions. If we denote the relative refractive index by $\nu = (n, k)$, then $\epsilon = \nu^2$ and

$$
\epsilon_r = n^2 - k^2,
$$

$\epsilon_i = 2nk$.

If we substitute Eqs. (7) into Eq. (6), we obtain

$$
(n^2 + k^2)^2 = 2 \sin^2 \phi (n^2 - k^2),
$$

which, interestingly enough, is the standard equation of the lemniscate of Bernoulli in the $\nu$ plane, Fig. 1(b). Varying $\phi$ between 0 and $90^\circ$ generates a family of these curves whose symmetry axes are coincident with the $n$ and $k$ coordinate axes and with maximum dimension, $n(k = 0) = \sqrt{2} \sin \phi$, that increases from 0 to $\sqrt{2}$, respectively. The interior and boundary of the largest lemniscate (that corresponds to $\phi = 90^\circ$) defines the domain of the $\nu$ plane in which $r_p$ can become real. Notice that in this domain $n \leq \sqrt{2}$ and $|k| < n$.

So far we have considered the $p$ polarization only. For the $s$ polarization, the reflection coefficient $r_s$ is also given by Eq. (1) but with $x = (\epsilon - \sin^2 \phi)^{1/2} / \cos \phi$. In this case $r_s$ is real when $x = 1$, which, in turn, is satisfied only if $\epsilon$ is real. This is the uninteresting case of interfaces between transparent media.

To conclude, when the relative dielectric function $\epsilon$, or relative refractive index $\nu$, is complex, only the reflection coefficient for the $p$ polarization ($r_p$) can become real. If $\xi$ is defined by

$$
\xi = (\epsilon_r^2 + \epsilon_i^2) / 2\epsilon_r = (n^2 + k^2)^2 / 2(n^2 - k^2),
$$

then $r_p$ is real when

$$
0 \leq \xi \leq 1,
$$

and the angle of incidence at which this happens is given by

$$
\phi = \sin^{-1} \xi^{1/2}.
$$

For a given $\phi$, the locus of $\epsilon = (\epsilon_r, \epsilon_i)$ is a circle, and that of $\nu = (n, k)$ is the lemniscate of Bernoulli.


2For passive or absorbing media, $\epsilon$ (and the relative refractive index $\nu$) is limited to the first or fourth quadrant of the complex plane depending upon whether the $e^{-i\omega t}$ or $e^{i\omega t}$ time dependence of the harmonic fields is selected, respectively. By allowing $\epsilon$ (and $\nu$) to assume values in the entire right-half plane, both choices of the time dependence are simultaneously represented.

3The reflection phase shift $\delta$ ($\arg r$) is either 0 or $\pi$ dependent upon the sign conventions used. If the $e^{-i\omega t}$ time dependence is chosen, and the $p$ directions in the incident and reflected beams are selected such that they are antiparallel at normal incidence (in other words, if we assume the Nebraska (Muller) conventions), then we have $\delta = \pi$.