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Generalized ellipsometry based on azimuth measurements alone

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Generalized ellipsometry based on azimuth measurements alone

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We investigate the azimuth response function, $\theta_o = f(\theta_i)$, of a linear nondepolarizing optical system S, where θ_i and θ_o are the azimuths (orientations) of the generally elliptic vibrations of totally polarized light at the input and output of S. We find that the azimuth response function depends on five of the six parameters that specify the normalized circular Jones matrix of the optical system. Thus the entire *polarization* response of an optical system can be *nearly* completely reconstructed from its *azimuth* response alone. Five input-output azimuth measurements $(\theta_{ik}, \theta_{ok})$, $k = 1, 2, \ldots, 5$ are sufficient to fix the ARF. The procedure for such determination is considerably facilitated if the average of θ_o, when θ_i sweeps a full rang of π, is measured. The general design and automation of an instrument for performing azimuth measurements are discussed and the use of such measurements to determine the optical parameters of elliptic retarders is given as an application.

I. INTRODUCTION

It has been shown[1,2] that measurements of the azimuths (orientations) of the elliptic vibrations of light reflected from an optically isotropic surface, for two given directions of the linear vibration of incident linearly polarized light, enable the determination of the ratio of the complex reflection coefficients of the surface for the p and s polarizations ($\tan\psi e^{j\Delta}$) at any angle of incidence. No ellipticity measurements are involved in this technique so that an instrument based on it need not include a phase-retarding element or compensator. This accounts for the simplicity and higher accuracy of this interesting version of ellipsometry.

In this paper we generalize ellipsometry based on azimuth measurements alone so that it may apply to *any* linear nondepolarizing optical system (including anisotropic surfaces).[3] In Sec. III we show that five of the (maximum total) of six real parameters that completely specify the polarization response of an optical system can be found by azimuth measurements. This follows from an investigation of the azimuth response function of an optical system in Sec. II. Instrumentation for this technique is outlined in Sec. IV and an application of its use to determine the optical parameters of elliptic retarders is given in Sec. V as an example.

II. AZIMUTH RESPONSE FUNCTION (ARF) OF AN OPTICAL SYSTEM

Let θ_i and θ_o denote the azimuths of the electric (generally elliptic) vibrations $\mathbf{E}_i$ and $\mathbf{E}_o$ of totally polarized light at the input and output of a linear nondepolarizing optical system S.[4] If we resolve these vibrations into their right (r) and left (l) circularly polarized components, we can write

$$\begin{pmatrix} E_{ol} \\ E_{or} \end{pmatrix} = \begin{pmatrix} T^c_{11} & T^c_{12} \\ T^c_{21} & T^c_{22} \end{pmatrix} \begin{pmatrix} E_{il} \\ E_{ir} \end{pmatrix}, \quad (1)$$

where $\mathbf{T}^c = (T^c_{ij})$ is the circular Jones matrix[5] of S. The input and output polarization states are uniquely specified by the complex numbers[6]

$$\chi_i = E_{ir}/E_{il}, \quad \chi_o = E_{or}/E_{ol}, \quad (2)$$

which, from Eq. (1), are inter-related by the bilinear transformation[7]

$$\chi_o = A[(B + \chi_i)/(C + \chi_i)], \quad (3)$$

where

$$A = T^c_{22}/T^c_{12}, \quad B = T^c_{21}/T^c_{22}, \quad C = T^c_{11}/T^c_{12}. \quad (4)$$

The *circular* complex polarization number χ that describes

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an elliptic vibration of azimuth θ and ellipticity angle ϵ is given by[6]

$$\chi = \tan[\epsilon + (\pi/4)]e^{-j2\theta}. \quad (5)$$

If we substitute Eq. (5) into Eq. (3) and take the argument of both sides of the resulting equation, we readily obtain the ARF of S as

$$-2\theta_o = \arg(A) + \arg\{B + \tan[\epsilon_i + (\pi/4)]e^{-j2\theta_i}\} - \arg\{C + \tan[\epsilon_i + (\pi/4)]e^{-j2\theta_i}\}. \quad (6)$$

Equation (6) shows that the ARF $\theta_o = f(\theta_i)$ is parametrically dependent on the ellipticity of the incident light ϵ_i. We are specifically interested in the case when the incident light is linearly polarized, $\epsilon_i = 0$, and its vibration direction θ_i is varied. If we set $\epsilon_i = 0$ in Eq. (6), we get

$$-2\theta_o = \arg(A) + \arg(B + e^{-j2\theta_i}) - \arg(C + e^{-j2\theta_i}), \quad (7)$$

which may be called the zero-ellipticity ARF (ZE-ARF).[8] If we make the following change of variable,

$$z = -2\theta, \quad (8)$$

and substitute

$$A = ae^{j\alpha}, \quad B = be^{j\beta}, \quad C = ce^{j\gamma}, \quad (9)$$

into Eq. (7), we obtain

$$z_o = \bar{z}_o + \arg[b + e^{j(z_i-\beta)}] - \arg[c + e^{j(z_i-\gamma)}], \quad (10)$$

where

$$\bar{z}_o = \alpha + \beta - \gamma. \quad (11)$$

If we let z_i sweep the full range of 2π (which occurs when θ_i sweeps the full range of π) and take the average (over 2π) of Eq. (10), we get

$$\langle z_o \rangle = \frac{1}{2\pi}\int_q^{q+2\pi} z_o \, dz_i = \bar{z}_o, \quad (12)$$

because the average value of each of the second and third terms in Eq. (10), when z_i sweeps 2π, is identically zero. Equation (12) shows that $\bar{z}_o$ $(= -2\bar{\theta}_o)$ determines the average value $\bar{\theta}_o$ of the azimuth of the output elliptic vibration. The "variable part" of the ZE-ARF $z_o - \bar{z}_o$ has the geometrical interpretation shown in Fig. 1.

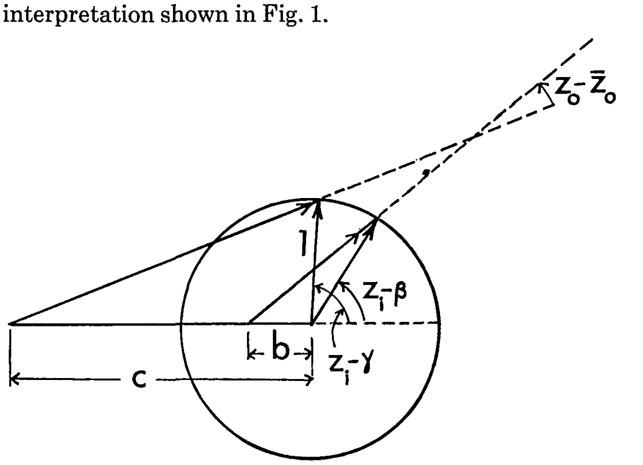

FIG. 1. The azimuth θ_o $[= -(1/2)z_o]$ of the *elliptic* vibration of light leaving an optical system is related to the azimuth θ_i $[= -(1/2)z_i]$ of the *linear* vibration of the incident light via this geometrical construction. $\bar{z}_o$, b, β, c, and γ are five parameters, characteristic of the given system, that completely determine its azimuth response.

If we define the set of functions

$$y(p,x) = \tan^{-1}\frac{\sin x}{p + \cos x}, \quad 0 \leqslant p \leqslant \infty \quad (13)$$

we can rewrite Eq. (10) as

$$z_o = \bar{z}_o + y(b, z_i - \beta) - y(c, z_i - \gamma). \quad (14)$$

Figure 2 shows the set of *periodic* functions $y(p,x)$ vs x with p as a parameter. The ZE-ARF of any linear nondepolarizing optical system can be synthesized using this set of periodic waveforms via the following sequence of steps: (i) select two waveforms corresponding to two different values of the parameter p ($p_1 = b$ and $p_2 = c$); (ii) phase shift (horizontally translate) each of these two waveforms by an appropriate angle (β for p_1 and γ for p_2); (iii) take the difference between the two phase-shifted waveforms; and finally (iv) make a specific vertical translation of the waveform obtained in step iii (by $\bar{z}_o$).

III. NEAR-COMPLETE DETERMINATION OF THE POLARIZATION RESPONSE OF AN OPTICAL SYSTEM BY AZIMUTH MEASUREMENTS ALONE

It is evident from Eq. (10) that the ZE-ARF is completely determined by *five* real parameters, namely α, (b,β), and (c,γ), which are the argument of A, and the absolute value and argument of B and C, respectively, where A, B, and C are ratios of the elements of the circular Jones matrix (CJM) as given by Eqs. (4). In terms of these five parameters, the *normalized* CJM is

$$\mathbf{T}_N^c = \begin{pmatrix} uce^{j(\gamma-\alpha)} & ue^{-j\alpha} \\ be^{j\beta} & 1 \end{pmatrix}, \quad (15)$$

where u is a real parameter $(= 1/a)$ that *cannot* be determined *unless* an ellipticity measurement is made. It follows that the entire *polarization* response of an optical system, which is specified by $\mathbf{T}_N^c$, is *nearly* completely reconstructed from its *azimuth* response alone.

The five parameters α, b, β, c, and γ that completely specify

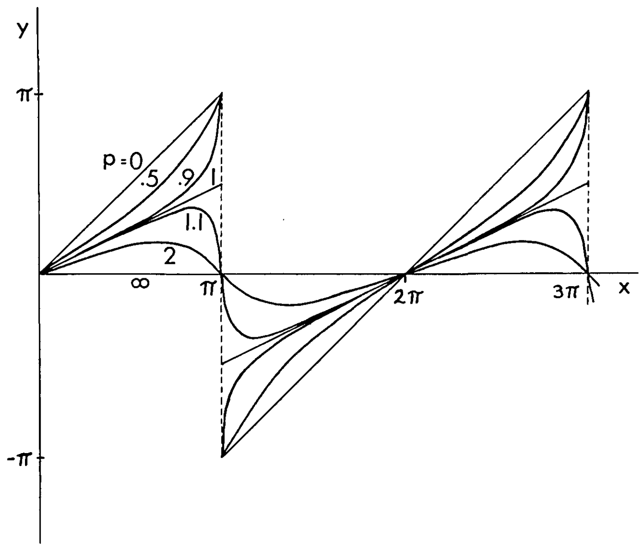

FIG. 2. This set of periodic waveforms, Eq. (13), can be used to synthesize the azimuth response function of *any* linear nondepolarizing optical system.

the ARF can be determined from five input-output azimuth measurements. Let the azimuth θ_i of linearly polarized light incident on the optical system assume *five* different values and the azimuth θ_o of the outgoing (generally) elliptically polarized light be measured for each input azimuth. The five pairs $(\theta_{ik},\theta_{ok})$, $k = 1, \ldots, 5$ of input-output azimuths lead, when substituted in Eq. (14), to five equations in the five unknowns $\bar{z}_o(= \alpha + \beta - \gamma)$, b, β, c, and γ. These equations, however, are nonlinear [see Eqs. (13) and (14)] and cannot be inverted easily to find the five unknowns except numerically by computer fitting.

Significantly, it is possible to *explicitly and analytically* solve for all five parameters *if* the average of the output azimuth θ_o, when the input azimuth θ_i scans a full range of π, is determined. From the average output azimuth $\bar{\theta}_o$ we determine $\bar{z}_o = -2\bar{\theta}_o$. If we take the tangent of $z_o - \bar{z}_o$ in Eq. (14), use Eq. (13), and make some trigonometric and algebraic manipulations, we obtain

$$\tan(z_o - \bar{z}_o) = \frac{\mu_1 + \mu_2 \cos z_i + \mu_3 \sin z_i}{\nu_1 + \nu_2 \cos z_i + \nu_3 \sin z_i}, \qquad (16)$$

where

$$\mu_1 = \sin(\gamma - \beta), \quad \mu_2 = b \sin\gamma - c \sin\beta, \quad \mu_3 = -b \cos\gamma + c \cos\beta;$$

$$\nu_1 = bc + \cos(\gamma - \beta), \quad \nu_2 = b \cos\gamma + c \cos\beta, \quad \nu_3 = b \sin\gamma + c \sin\beta. \qquad (17)$$

Equation (16) is an alternative elegant analytic form of the ZE-ARF. Equation (16) can be readily transformed to a linear form

$$\mu_2' \cos z_i + \mu_3' \sin z_i - \nu_1' \tan(z_o - \bar{z}_o) - \nu_2' \tan(z_o - \bar{z}_o) \cos z_i - \nu_3' \tan(z_o - \bar{z}_o) \sin z_i + 1 = 0, \qquad (18)$$

where

$$\mu_k' = \mu_k/\mu_1 \; (k = 2, 3), \quad \nu_k' = \nu_k/\mu_1 \; (k = 1, 2, 3). \qquad (19)$$

From five input-output azimuth measurements (z_{ik},z_{ok}), $k = 1, \ldots, 5$, Eq. (18) generates five linear equations in the five intermediate unknowns μ_2', μ_3', ν_1', ν_2', and ν_3'. These intermediate parameters determine α, b, β, c, and γ through Eqs. (17) and (19); the results are

$$\tan\beta = (\nu_3' - \mu_2')/(\nu_2' + \mu_3'), \qquad (20a)$$

$$\tan\gamma = (\nu_3' + \mu_2')/(\nu_2' - \mu_3'), \qquad (20b)$$

$$\alpha = \bar{z}_o - \beta + \gamma, \qquad (20c)$$

$$b = \tfrac{1}{2}|\sin(\beta - \gamma)[(\nu_3' + \mu_2')^2 + (\nu_2' - \mu_3')^2]^{1/2}|, \qquad (20d)$$

$$c = \tfrac{1}{2}|\sin(\beta - \gamma)[(\nu_3' - \mu_2')^2 + (\nu_2' + \mu_3')^2]^{1/2}|. \qquad (20e)$$

For higher accuracy in determining the ARF, more than five input-output azimuth measurements can be made and the *overdetermined* set of linear equations generated by Eq. (18) can be solved using standard best-fit least-squares techniques.

IV. INSTRUMENTATION

The optical elements required for an instrument to measure the ARF of an optical system are (i) a linear polarizer that can be rotated in the incident beam to control the input azimuth θ_i, and (ii) an azimuth detector (AD) in the outgoing beam to measure the output azimuth θ_o. A suitable AD may consist of an ac-excited Faraday cell (that produces small-amplitude time-harmonic optical rotation), followed by a linear analyzer and a photodetector. Azimuth detection is achieved by rotating the analyzer to null and fundamental[1] or second-harmonic[9] component of the output signal of the detector. Automation of such an instrument is readily realized by adding motor drive to the polarizer (θ_i scanning) and a servomotor to rotate the analyzer to follow the azimuth of the output vibration (θ_o tracking). The servomotor is under feedback control derived from the ac output signal of the photodetector, the objective of the servo loop being to null the latter signal. On-line data processing to determine the five parameters of the ARF can be carried out if the azimuths of the polarizer and analyzer (obtained by position sensors) are fed to a minicomputer.

V. DETERMINATION OF THE OPTICAL PARAMETERS OF ELLIPTIC RETARDERS: AN APPLICATION

Apart from the requirement that it is linear and nondepolarizing, the optical system under measurement has been assumed up till now to be general so that the results of this paper may be applied to as wide a range of problems as possible. For the sake of illustration by way of a specific example, we now consider the use of azimuth measurements to determine the optical parameters of elliptic retarders. This problem has been the subject of two recent publications.[10,11]

An elliptic retarder is characterized by its orthogonal fast and slow elliptic eigenpolarizations and their associated differential retardation (or phase shift) δ. If

$$\chi_{ef} = \tan[\epsilon_f + (\pi/4)]e^{-j2\theta_f}$$

is the circular complex polarization number that represents the fast eigenpolarization (where θ_f and ϵ_f are the associated major-axis azimuth and ellipticity angle, respectively), the unnormalized CJM of the elliptic retarder (ER) can be written[12]

$$\mathbf{T}_{\mathrm{ER}}^{c} = \begin{pmatrix} P & Q \\ -Q^* & P^* \end{pmatrix}, \qquad (21)$$

where

$$P = e^{j\delta/2} + g^2 e^{-j\delta/2}, \qquad (22a)$$

$$Q = 2g \sin(\delta/2) e^{j[2\theta_f + (\pi/2)]}, \qquad (22b)$$

$$g = \tan[\epsilon_f + (\pi/4)]. \qquad (22c)$$

If we substitute Eq. (21) into Eqs. (4), and make use of Eqs. (22), we get

$$A = ce^{j\alpha}, \quad B = (1/c)e^{j(\gamma+\pi)}, \quad C = ce^{j\gamma}, \qquad (23)$$

where

$$c^2 = \frac{1 + 2g^2 \cos\delta + g^4}{2g^2(1 - \cos\delta)}, \qquad (24a)$$

$$\alpha = -(\delta/2) - [2\theta_f + (\pi/2)] + \arg(1 + g^2 e^{j\delta}), \qquad (24b)$$

$$\gamma = (\delta/2) - [2\theta_f + (\pi/2)] - \arg(1 + g^2 e^{j\delta}). \qquad (24c)$$

In Eqs. (23) only three *real* parameters c, α, γ specify the three *complex* ratios of the elements of the CJM A, B, C that, in turn, determine the ARF. If we substitute Eqs. (23) into Eqs. (17), and use the results in Eq. (16), we obtain the following simple ARF of an elliptic retarder:

$$\tan(z_o - \alpha) = K \tan(z_i - \gamma), \tag{25}$$

where

$$K = (c^2 + 1)/(c^2 - 1). \tag{26}$$

If we first measure the average $\overline{\theta}_o$ of the output azimuth θ_o, when the input azimuth θ_i sweeps π, we can determine α using $\alpha = \overline{z}_o - \pi$, where $\overline{z}_o = -2\overline{\theta}_o$. With α known, Eq. (25) can be linearized by expanding $\tan(z_i - \gamma)$ (as the tangent of the difference between two angles); this gives

$$x_1 \tan z_i + x_2 \tan(z_o - \alpha) + x_3 \tan z_i \tan(z_o - \alpha) + 1 = 0, \tag{27}$$

where

$$x_1 = -\cot\gamma, \quad x_2 = \cot\gamma/K, \quad x_3 = 1/K. \tag{28}$$

If the output azimuth θ_o is measured for three discrete values of the input azimuth θ_i and $(z_i,z_o) = (-2\theta_i, -2\theta_o)$ are substituted into Eq. (27), three linear equations are obtained that can be solved for the three unknowns x_1, x_2, and x_3. From x_1, x_2, and x_3, K and γ are overdetermined using Eqs. (28).

An alternative procedure to determine all of the three independent parameters K, α, γ of the ARF of the elliptic retarder from three discrete input-output azimuth measurements, *without* finding the average of the output azimuth, is derived if Eq. (25) is linearized by expanding both of the two tangent functions that appear in that equation. The result is

$$y_1 \tan z_i + y_2 \tan z_o + y_3 \tan z_i \tan z_o + 1 = 0, \tag{29}$$

where

$$y_1 = \frac{-K - \tan\alpha \tan\gamma}{K \tan\gamma - \tan\alpha}, \quad y_2 = \frac{1 + K \tan\alpha \tan\gamma}{K \tan\gamma - \tan\alpha},$$
$$y_3 = \frac{\tan\gamma - K \tan\alpha}{K \tan\gamma - \tan\alpha}. \tag{30}$$

Three pairs of input-output azimuths generate from Eq. (29) three linear equations that can be solved for the three unknowns y_1, y_2, and y_3. Once y_1, y_2, and y_3 are obtained, Eqs. (30) can be used subsequently to find K, α, and γ.

To determine the three optical characteristics θ_f, ϵ_f and δ of the elliptic retarder from the three parameters K, α, and γ of its ARF (obtained by either one of the two above-mentioned methods), we make use of Eqs. (24). In particular, θ_f is directly obtained by adding Eqs. (24b) and (24c):

$$\theta_f = -(\pi/4) - (1/4)(\alpha + \gamma). \tag{31}$$

If the difference between Eqs. (24b) and (24c) is taken instead, we get

$$\tan(1/2)(\gamma - \alpha) = \frac{\tan(\delta/2)(1 + g^2 \cos\delta) - g^2 \sin\delta}{(1 + g^2 \cos\delta) + g^2 \tan(\delta/2) \sin\delta}. \tag{32}$$

Equation (32) can be solved for g^2 in terms of δ and the result substituted in Eq. (24a) [where $c^2 = (K + 1)/(K - 1)$ from Eq. (26)] so that an equation in δ alone is obtained. Subsequently we find g^2, hence ϵ_f, using Eq. (22c).

VI. CONCLUDING REMARK

Jones has shown[13] that the most general optical anisotropy that any medium can exhibit can be thought of as a superposition of *six* independent basic anisotropic properties, namely, (i) linear birefringence along the coordinate axes, (ii) linear birefringence along the bisectors of the coordinate axes, (iii) circular birefringence, (iv) linear dichroism along the coordinate axes, (v) linear dichroism along the bisectors of the coordinate axes, and (vi) circular dichroism. Of these six properties, only circular dichroism (CD) is *not* amenable to being determined by azimuth measurements because CD induces *ellipticity* changes with no accompanying azimuth changes (i.e., $\theta_i = \theta_o$). On the other hand, any anisotropy which is an admixture of the *first five* properties listed above can be characterized by azimuth measurements alone. This should lend some physical meaning to our finding that azimuth measurements are capable of determining all but one of the six parameters that specify the polarization response of any linear nondepolarizing optical system. The anisotropy possessed by an elliptic retarder (which is the example that we have pursued in Sec. V) can be considered as a superposition of all of the three birefringence properties (i), (ii), and (iii).

APPENDIX: ARF IN TERMS OF THE CARTESIAN JONES MATRIX

In Sec. II we have purposely selected the right and left circular polarizations as basis states because this simplifies the derivation and form of the ARF of an optical system. However, it is also often convenient to work with orthogonal linear polarizations as basis states and the associated *Cartesian* Jones matrix. To be able to use the results of this paper we can, of course, determine the circular Jones matrix that corresponds to a given Cartesian Jones matrix using the known transformation that links the two.[5] Alternatively, we can *directly* derive the ARF when orthogonal linear polarizations are used as basis states. In this case we obtain

$$\tan 2\theta_o = \frac{2ab \cos(\alpha - \beta)}{b^2 - a^2}, \tag{A1}$$

where

$$\begin{aligned} ae^{j\alpha} &= T_{22}\chi_i + T_{21}, \\ be^{j\beta} &= T_{12}\chi_i + T_{11}, \end{aligned} \tag{A2}$$

and

$$\chi_i = \frac{\tan\theta_i + j \tan\epsilon_i}{1 - j \tan\theta_i \tan\epsilon_i}. \tag{A3}$$

T_{ij} are the elements of the Cartesian Jones matrix and χ_i is the Cartesian complex polarization number that describes an input vibration of azimuth θ_i and ellipticity angle ϵ_i [Eq. (1.79), p. 29, Ref. 5].

The ZE-ARF is obtained by setting $\epsilon_i = 0$, which makes $\chi_i = \tan\theta_i$, and Eqs. (A2) are simplified accordingly.

As an example, light reflection from an optically isotropic surface, discussed in Refs. 1 and 2, is represented by a Cartesian Jones matrix with elements $T_{12} = T_{21} = 0$, $T_{11} = \tan\psi e^{j\Delta}$, and $T_{22} = 1$. (ψ and Δ are the usual ellipsometric parameters.) If we substitute these elements in Eqs. (A2), and assume linearly polarized incident light so that $\chi_i = \tan\theta_i$, we obtain from Eq. (A1) the following ARF:

$$\tan 2\theta_o = \frac{2\tan\theta_i \tan\psi \cos\Delta}{\tan^2\psi - \tan^2\theta_i}, \qquad \text{(A4)}$$

which agrees with Eq. (3) of Ref. 1, as expected.

If, in the above example, we try to determine the ARF by first finding the equivalent circular Jones matrix and then using any of the expressions of the ARF in Secs. II or III, we discover that this procedure is indeed considerably more complicated.

Noted added in proof. A recent article[14] provides a significant additional reference for this paper.

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Simulation of mechanical rotation by optical rotation: Application to the design of a new Fourier photopolarimeter

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The *mechanical* rotation of an optical element around the axis of a beam of polarized light can be easily simulated by using the phenomenon of *optical* rotation. Because optical rotation can be magnetically or electrically induced, virtually any kind of mechanical rotation can be mimicked. This interesting principle is applied to the design of a new Fourier photopolarimeter that uses an oscillating-azimuth retarder (OAR). The OAR consists of a quarter-wave plate surrounded by two ac-excited Faraday cells that produce equal and opposite sinusoidal optical rotations. Analysis of the operation of this polarimeter of no moving parts proves its ability to measure all four Stokes parameters of incident partially polarized radiation.

I. INTRODUCTION

In this paper we show that the *mechanical* rotation of an optical element around the axis of a beam of polarized light can be easily simulated by using the phenomenon of *optical* rotation. Because optical rotation can be induced and controlled by magnetic and electric fields (e.g., in Faraday and liquid-crystal cells), it is possible, in principle, to simulate, any type of mechanical rotation by tailoring the required electric or magnetic input. After giving the formal proof of this interesting concept, we apply it to the design of a new (automatic) Fourier photopolarimeter which is capable of measuring the four Stokes parameters of a partially polarized light beam. Applications other than in polarimetry are anticipated, e.g., in the up/down frequency shifter of Sommargren[1] that requires a synchronously rotating half-wave plate.

II. CONTROL OF THE AZIMUTH OF AN OPTICAL ELEMENT BY OPTICAL ROTATION

In Fig. 1, let $T_E^{x,y}$ be the (Mueller or Jones) matrix of an optical element E with respect to a transverse coordinate system xy fixed to the element. We assume that xy is rotated around the beam axis with respect to another reference coordinate system $x'y'$ by the azimuth

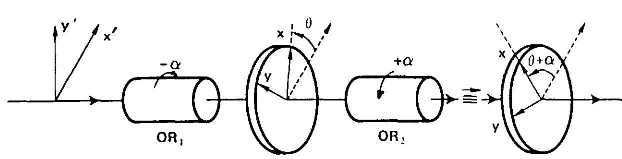

FIG. 1. Optical element E of azimuth θ which is surrounded by two optical rotators OR_1 and OR_2 with equal and opposite rotations $+\alpha$ and $-\alpha$ is equivalent to itself in a new azimuth position $\theta+\alpha$.