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# Design of film-substrate single-reflection linear partial polarizers

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# Design of film–substrate single-reflection linear partial polarizers*

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The results of a preceding paper [J. Opt. Soc. Am. 65, 1464, (1975)] are viewed from a different angle as providing the basis for the design of film–substrate single-reflection linear partial polarizers (LPP), which also operate as reflection optical rotators. The important characteristics of a comprehensive set of discrete designs of $SiO_2$-Si LPP's at $\lambda = 6328$ Å are shown graphically.

Index Headings: Polarization; Reflection; Films; Silicon.

We have determined the conditions under which a film–substrate system preserves the state of linear polarization of light of arbitrary azimuth that is obliquely reflected from such a system.[1] We were interested in the implication of this observation as a basis of a new method of polarizer-surface-analyzer null ellipsometry (PSA-NE). However, the results of Ref. 1 can be considered differently, as providing a design procedure for film–substrate single-reflection linear partial polarizers. We further discuss this point in this paper.

A linear partial polarizer (LPP) differentially attenuates two orthogonally linearly polarized electric-field components of a light wave, without shifting the phase of one component with respect to the other.[2] An example of a transmission LPP is a plane-parallel slab of a linearly dichroic material. In this paper, we consider film–substrate reflection LPP's, in which case the linear polarizations parallel ($p$) and perpendicular ($s$) to the plane of incidence are the components that experience different attenuations. Strictly speaking, the foregoing definition of a LPP includes only film–substrate designs that give a reflection phase difference $\Delta = 0$, but excludes those for which $\Delta = \pi$. A film–substrate system that gives $\Delta = \pi$ acts as a combination of a half-wave retarder (HWR) and a LPP. For simplicity and generality, we will consider both cases $\Delta = 0$ and $\Delta = \pi$ together, keeping in mind the aforementioned distinction. Note that the $p$- and $s$-suppressing film–substrate reflection polarizers represent limiting cases of the LPP, in which one field component ($p$ or $s$, respectively) is entirely extinguished upon reflection.

The LPP ($\Delta = 0$) or the LPP + HWR ($\Delta = \pi$) acts also as a reflection optical rotator, which adds to the practical importance of such a device. Thus, if $\theta_i$ and $\theta_r$ are the azimuth angles of the linearly polarized incident ($i$) and reflected ($r$) light, we can easily prove that

$$\tan \theta_r = \pm \cot\psi \tan \theta_i , \tag{1}$$

where the + and − signs correspond to $\Delta = 0$ and $\Delta = \pi$, respectively. From Eq. (1), it is evident that the optical rotation $(\theta_r - \theta_i)$ is a function of the azimuth of the incident polarization $\theta_i$ and the ellipsometric angles $\psi$ and $\Delta$, where $\Delta$ is either 0 or $\pi$. Figure 1 shows the optical rotation $(\theta_r - \theta_i)$ computed from Eq. (1) as a function of $\psi$ when $\theta_i = 45°$ for both cases of $\Delta = 0$ and $\Delta = \pi$.

The most important parameter that describes the operation of a reflection LPP is the ellipsometric angle $\psi$, the square of whose tangent or cotangent gives the extinction ratio (e.r.) of the device,

$$\begin{aligned} \text{e.r.} &= \tan^2\psi , \quad 0 \le \psi \le 45° , \\ \text{e.r.} &= \cot^2\psi , \quad 45° \le \psi \le 90° . \end{aligned} \tag{2}$$

To design a LPP (or a LPP + HWR) of a given $\psi$ from a film–substrate system with known optical properties

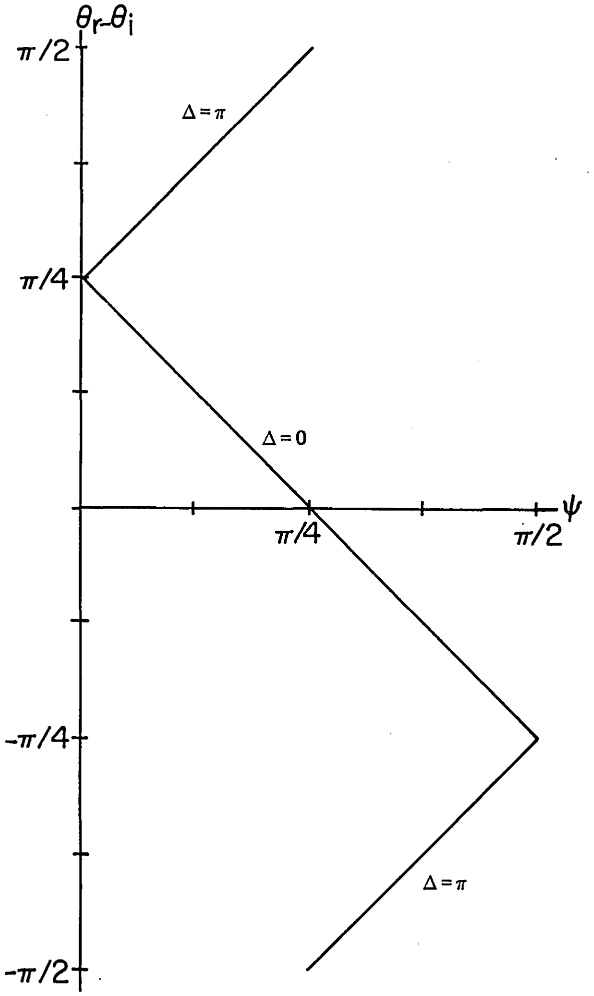


FIG. 1. Rotation $(\theta_r - \theta_i)$ of plane of polarization of light upon reflection from a linear partial polarizer, as a function of $\psi$ for $\Delta = 0$ and $\Delta = \pi$; $\theta_i = \pi/4$.

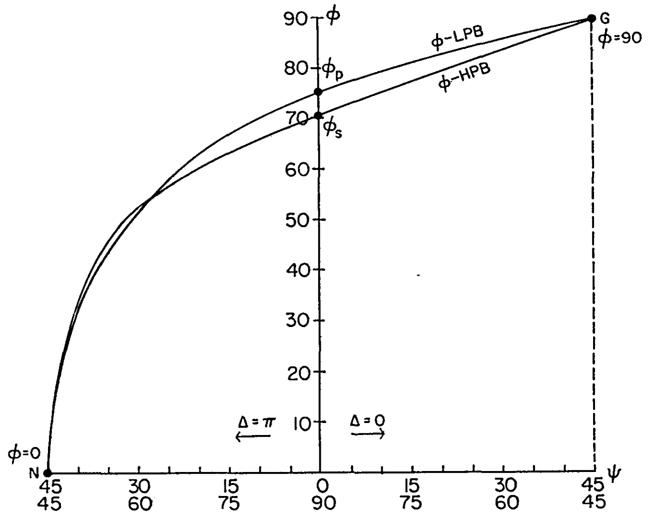


FIG. 2. Angle of incidence $\phi$ at which a $SiO_2$–Si film–substrate system operates as a linear partial polarizer for $\lambda = 6328$ Å, shown as a function of $\psi$ for $\Delta = 0$ and $\Delta = \pi$. $\phi_p$ and $\phi_s$ are the $p$- and $s$-polarizing angles of incidence for total $p$ and $s$ suppression, respectively.

at a given wavelength, we follow the procedure explained in association with Eqs. (25) and (26) of Ref. 3 (see Sec. III of Ref. 1). The same procedure has recently been put in an easy-to-follow step-by-step format in connection with the design of reflection retarders.[4] By use of $\rho = +\tan\psi$ ($\Delta = 0$), or $\rho = -\tan\psi$ ($\Delta = \pi$), steps 2–5 of Ref. 4 can be applied without change for the design

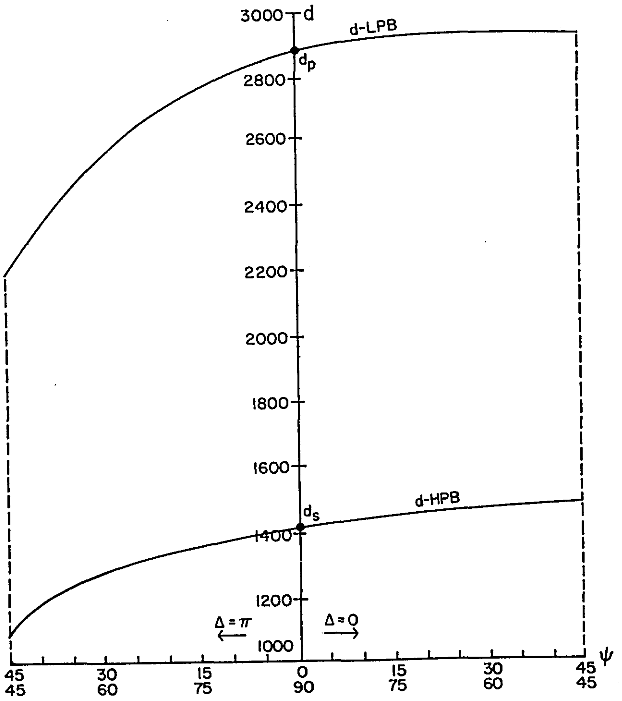


FIG. 3. Least $SiO_2$ film thickness $d$ required to make a $SiO_2$–Si system operate as a linear partial polarizer at $\lambda = 6328$ Å, shown as a function of $\psi$ for $\Delta = 0$ and $\Delta = \pi$. $d_p$ and $d_s$ are the least $p$- and $s$-polarizing film thicknesses for total $p$ and $s$ suppression.

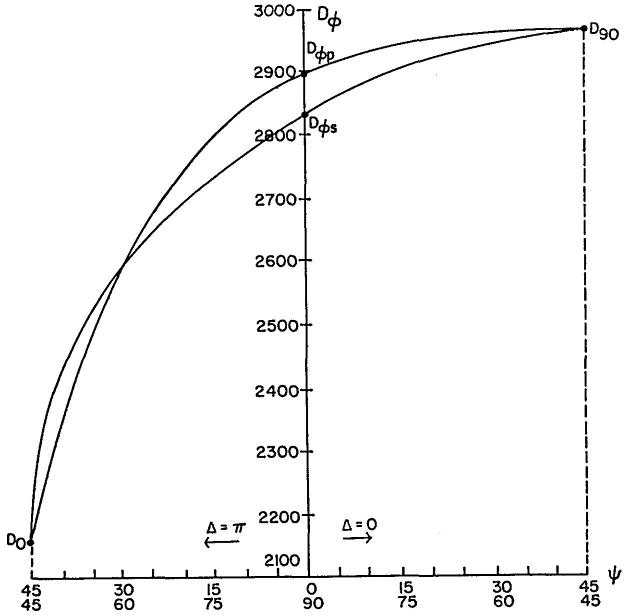


FIG. 4. Film-thickness period $D_\phi$ for the designs of Figs. 2 and 3.

of LPP. The purpose of the design is to determine the angle of incidence $\phi$ and the least film thickness $d$ that make the film–substrate system operate as a LPP, that has the prescribed value of $\rho$ ($= \pm\tan\psi$).

Tables I and II of Ref. 1 document a comprehensive set of discrete designs of $SiO_2$-Si film–substrate single-reflection LPP and LPP + HWR at $\lambda = 6328$ Å. In these tables, the primary design parameter $\psi$ assumes values from 0° to 90° in steps of 5° and $\Delta$ is allowed both values of 0° (LPP) and $\pi$ (LPP + HWR). Figure 5 of Ref. 1 shows the relation between the angle of incidence $\phi$ and least film thickness $d$ for operation of the $SiO_2$-Si film–substrate system as a LPP or a LPP + HWR at $\lambda = 6328$ Å. The relation assumes the form of two sepa-

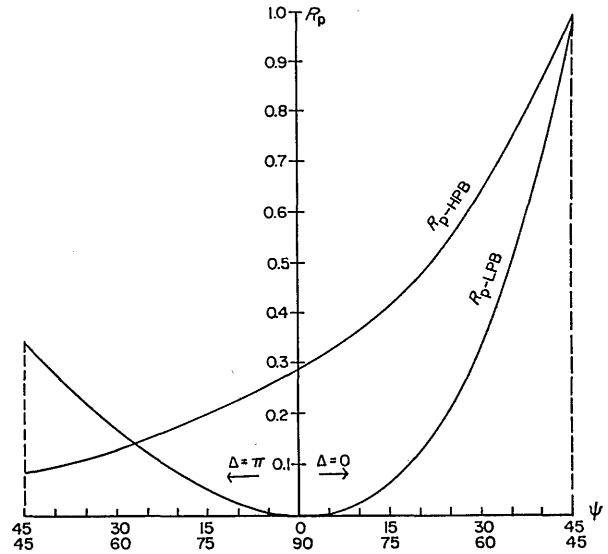


FIG. 5. Parallel reflectance $\mathcal{R}_p$ for the designs of Figs. 2 and 3.

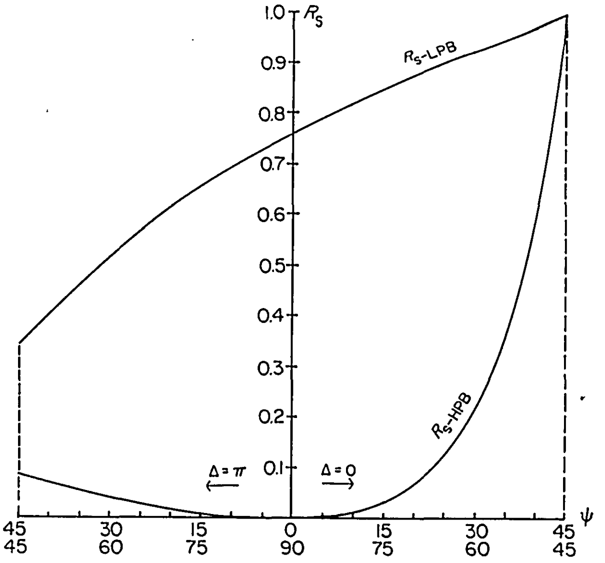


FIG. 6. Perpendicular reflectance $\mathcal{R}_s$ for the designs of Figs. 2 and 3.

rate branches: a high-psi branch (HPB) $d_H$ and a low-psi branch (LPB) $d_L$.

To gain a clear view of the design information given in Tables I and II of Ref. 1, we present it graphically in Figs. 2–6. Figure 2 shows the angle of incidence $\phi$ at which the $SiO_2$-Si film–substrate system acts as a LPP ($\Delta = 0$) or LPP + HWR ($\Delta = \pi$) of a given value of $\psi$ at $\lambda = 6328$ Å. Figures 3 and 4 show the associated least film thickness $d$, and film-thickness period $D_\phi$. Figures 5 and 6 give the parallel ($p$) and perpendicular ($s$) reflectances $\mathcal{R}_p$ and $\mathcal{R}_s$, respectively. The graph for each of the quantities $\phi$, $d$, $D_\phi$, $\mathcal{R}_p$, and $\mathcal{R}_s$ as a function of the device parameter $\psi$ in Figs. 2–6 is split into two branches: one for low $\psi$ ($0 \leq \psi \leq 45°$) and the other for high $\psi$ ($45° \leq \psi \leq 90°$).

The reflectances $\mathcal{R}_p$ and $\mathcal{R}_s$ are important performance parameters; they determine the absolute attenuation of the incident light upon reflection from the device.[5]

The film-thickness period $D_\phi$, or any integral multiple of it, can be added to the least film thickness $d$ to generate a new LPP with the same $\psi$. The higher thicknesses permit additional flexibility when a device is physically realized.

The permissible-thickness bands (PTB) of Fig. 6 of Ref. 1 specify where the $SiO_2$ film thickness should lie for a $SiO_2$-Si system to operate as a LPP or LPP + HWR at $\lambda = 6328$ Å. The graphical construction shown in Fig. 8 of Ref. 1 can be used to determine the angles of incidence at which a film–substrate system with known film thickness can operate as a LPP or LPP + HWR. The concepts and procedures that govern the design of film–substrate linear partial polarizers are similar to those for the design of film–substrate retarders[4], both are special cases of the same general design scheme.[3] In summary, we have shown that the results of Ref. 1 can be profitably considered from a different perspective as applicable to the design of film–substrate single-reflection linear partial polarizers.

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[5]The absolute attenuation (or net reflectance $\mathcal{R}$) is a function of $\mathcal{R}_p$, $\mathcal{R}_s$ and the polarization of the incident light.