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Kenneth V. Cartwright  
*The College of the Bahamas*

Patrick Russell  
*The College of the Bahamas*

Edit J. Kaminsky  
*University of New Orleans, ejbourne@uno.edu*

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# A further look at capacitors in complex arrangements



Kenneth V. Cartwright<sup>1</sup>, Patrick Russell<sup>1</sup> and Edit J. Kaminsky<sup>2</sup>

<sup>1</sup>School of Mathematics, Physics, and Technology, College of The Bahamas, P.O. Box N4912, Nassau, Bahamas.

<sup>2</sup>Department of Electrical Engineering, EN 846 Lakefront Campus, University of New Orleans, New Orleans, LA 70148, USA.

E-mail: kvcartwright@yahoo.com

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## Abstract

Recently, a previous contributor showed how the delta-wye conversion may be applied to find the equivalent capacitance of five capacitors arranged in a bridge configuration. In fact, that contributor gave an algorithm for this purpose which showed that the equivalent capacitance depends upon all five capacitors. However, in this paper, we point out that there is a special case where the equivalent capacitance does not depend upon one of the five capacitors.

**Keywords:** Combination of capacitors, Bridge circuits, Delta-wye and wye-delta conversions.

## Resumen

Un contribuidor anterior recientemente demostró cómo usar la conversión Delta-Estrella para encontrar la capacitancia equivalente de cinco capacitores en una configuración de puente. De hecho, ese contribuidor presentó un algoritmo para este propósito y mostró que la capacitancia equivalente depende de los cinco capacitores. Sin embargo, en este artículo mostramos que existe un caso especial para el cual la capacitancia equivalente no depende de uno de los cinco capacitores.

**Palabras Clave:** Combinaciones de capacitores, Circuitos puente, Conversiones delta-estrella y estrella-delta.

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## I. INTRODUCTION

In [1], Rizhov investigated the problem of finding the equivalent capacitance  $C_0$  (with respect to terminals  $T_3$  and  $T_4$ ) of five capacitors connected in a bridge configuration as shown in Figure 1.

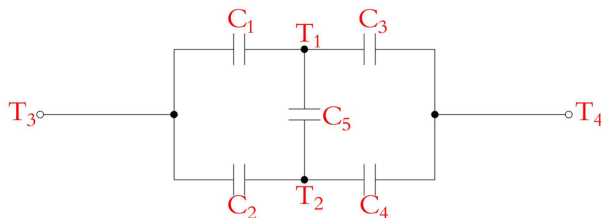


FIGURE 1. Five capacitors connected in a bridge configuration.

His approach was to arrive at a system of seven unknowns, which were solved for one particular case. However, as pointed out by Atkin [2], this method “did not reveal how a general solution could be obtained, giving  $C_0$  as a function of the five capacitances”. On the other hand, Atkin [2] showed how the delta-wye conversion may be applied for this very purpose. (Incidentally, the use of delta-wye and

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wye-delta conversions to solve problems of this sort is a well-known technique described in many electrical circuit analysis textbooks [3, 4]).

Unfortunately, the algorithm given by Atkin in [2] hides the fact that  $C_0$  is independent of the value of  $C_5$  if:

$$\frac{C_3}{C_1} = \frac{C_4}{C_2} = C.$$

In fact, there is no hint at all in [1] or [2] that the equivalent capacitance might not depend upon  $C_5$ . It is the purpose of this paper to demonstrate this fact in two ways: by using i) the algorithm given by Atkin [2] and ii) the concept of a balanced bridge circuit.

## II. REVIEW OF THE METHOD DESCRIBED BY ATKIN

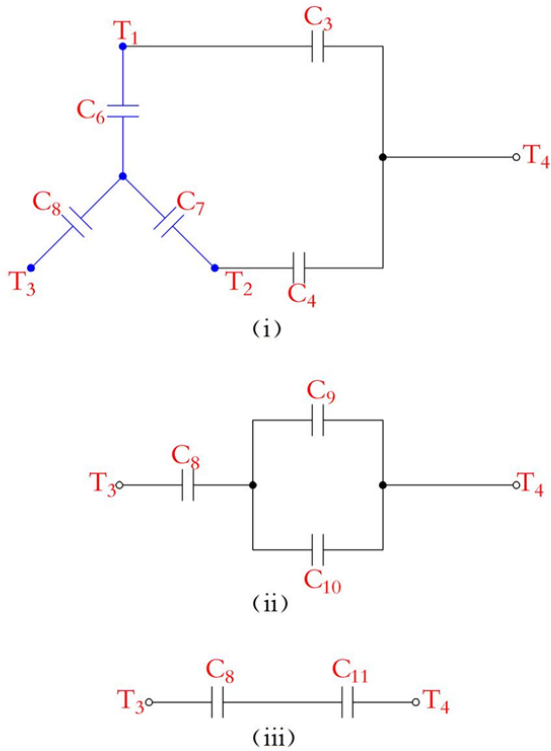
In Atkin’s method [2], the delta configuration of  $C_1$ ,  $C_2$  and  $C_5$  in Figure 1 is first converted to a wye circuit, given by  $C_6$ ,  $C_7$  and  $C_8$  of Figure 2i, where:

$$C_6 = C_1 + C_5 + \frac{C_1 C_5}{C_2}, \quad (1a)$$

$$C_7 = C_2 + C_5 + \frac{C_2 C_5}{C_1}, \quad (1b)$$

and:

$$C_8 = C_1 + C_2 + \frac{C_1 C_2}{C_5}. \quad (1c)$$



**FIGURE 2.** Simplification of Figure 1 using a delta-wye transformation.

Comparing Figure 2i with Figure 2ii shows that  $C_9$  is the series combination of  $C_3$  and  $C_6$  and  $C_{10}$  is the series combination of  $C_7$  and  $C_4$  Hence:

$$C_9 = \frac{C_3 C_6}{C_3 + C_6}, \quad (2a)$$

and:

$$C_{10} = \frac{C_4 C_7}{C_4 + C_7}. \quad (2b)$$

Furthermore, comparing Figure 2ii with Figure 2iii shows that  $C_{11}$  is the parallel combination of  $C_9$  and  $C_{10}$  Hence:

$$C_{11} = C_9 + C_{10}. \quad (3)$$

Finally, from Figure 2iii, it is clear that  $C_0$  is the series combination of  $C_8$  and  $C_{11}$

Hence:

$$C_0 = \frac{C_8 C_{11}}{C_8 + C_{11}}. \quad (4)$$

### III. DETERMINING THE EQUIVALENT CAPACITANCE FOR THE SPECIAL CASE

WHERE  $\frac{C_3}{C_1} = \frac{C_4}{C_2}$

#### A. Using the algorithm given by Atkin

From Equations 2a and 1a:

$$\begin{aligned} C_9 &= \frac{C_3}{1 + \frac{C_3}{C_6}} = \frac{C_3}{1 + \frac{C_3}{C_1 + C_5 + \frac{C_1 C_5}{C_2}}}, \\ &= \frac{C_3}{1 + \frac{C_3 / C_1}{1 + \frac{C_5}{C_1} + \frac{C_5}{C_2}}}, \\ &= \frac{C_3}{1 + \frac{C}{K}}, \\ &= \frac{KC_3}{K+C}, \end{aligned} \quad (5)$$

where  $K = \left( \frac{C_1 + C_2}{C_1 C_2} \right) C_5 + 1$ , and  $C = \frac{C_3}{C_1}$ .

Similarly, from Equations 2b and 1b:

$$\begin{aligned} C_{10} &= \frac{C_4}{1 + \frac{C_4}{C_7}} = \frac{C_4}{1 + \frac{C_4}{C_2 + C_5 + \frac{C_2 C_5}{C_1}}}, \\ &= \frac{C_4}{1 + \frac{C_4 / C_2}{1 + \frac{C_5}{C_2} + \frac{C_5}{C_1}}}, \\ &= \frac{C_4}{1 + \frac{C}{K}}, \\ &= \frac{KC_4}{K+C}, \end{aligned} \quad (6)$$

With:

$$C = \frac{C_4}{C_2}$$

Substituting Equations 5 and 6 into Equation 3 gives:

$$C_{11} = \frac{K}{K+C} (C_3 + C_4) \tag{7}$$

Furthermore, Equation (1c) can be written as:

$$C_8 = \left( \left( \frac{C_1 + C_2}{C_1 C_2} \right) C_5 + 1 \right) \frac{C_1 C_2}{C_5}, \tag{8}$$

$$= K \frac{C_1 C_2}{C_5}$$

Substituting Equation (7) and Equation (8) into Equation (4) gives:

$$C_0 = \frac{K^2 \frac{C_1 C_2}{C_5} \frac{1}{K+C} (C_3 + C_4)}{K \frac{C_1 C_2}{C_5} + \frac{K}{K+C} (C_3 + C_4)}, \tag{9}$$

$$= \frac{K \frac{C_1 C_2}{C_5} \frac{1}{K+C} (C_3 + C_4)}{\frac{C_1 C_2}{C_5} + \frac{1}{K+C} (C_3 + C_4)}$$

Further simplification of Equation (9) gives:

$$C_0 = \frac{\frac{C_1 C_2}{C_5} K (C_3 + C_4)}{\frac{C_1 C_2}{C_5} (K+C) + (C_3 + C_4)}, \tag{10}$$

$$= \frac{K (C_3 + C_4)}{K+C + (C_3 + C_4) \frac{C_5}{C_1 C_2}},$$

$$= \frac{K (C_3 + C_4)}{1+C + (C_1 + C_2 + C_3 + C_4) \frac{C_5}{C_1 C_2}},$$

$$= \frac{K (C_3 + C_4)}{(1+C) \left[ 1 + \left( \frac{C_1 + C_2 + C_3 + C_4}{1+C} \right) \frac{C_5}{C_1 C_2} \right]},$$

$$= \frac{K (C_3 + C_4)}{(1+C) \left[ 1 + \left( \frac{C_1 (1+C) + C_2 (1+C)}{1+C} \right) \frac{C_5}{C_1 C_2} \right]}$$

Hence:

$$C_0 = \frac{K (C_3 + C_4)}{(1+C)K}, \tag{11}$$

$$= \frac{C_3}{1+C} + \frac{C_4}{1+C},$$

$$= \frac{C_3}{1 + \frac{C_3}{C_1}} + \frac{C_4}{1 + \frac{C_4}{C_2}},$$

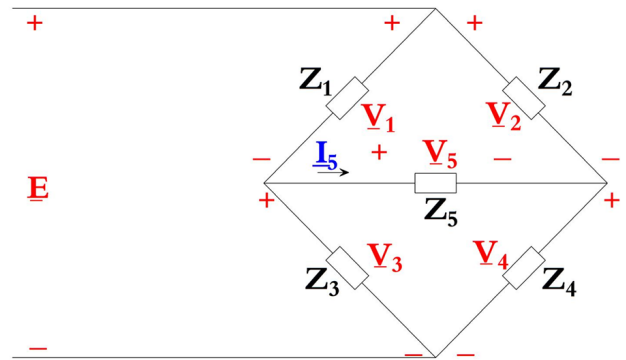
$$= \frac{C_3 C_1}{C_1 + C_3} + \frac{C_4 C_2}{C_2 + C_4}$$

As can be seen from Equation (11), the equivalent capacitance does not depend upon  $C_5$  as claimed. In fact, the value given by Equation (11) is the same value that would be given if  $C_5$  is not connected to the circuit. Hence, if  $\frac{C_3}{C_1} = \frac{C_4}{C_2} = C$ , the bridge circuit behaves the same regardless of the value of  $C_5$ .

**B. Using the concept of the balanced bridge**

As can be seen from the above, it is straightforward but tedious to derive the equivalent capacitance for this special case. Fortunately, there is a well-known alternative way of deriving this same result using the idea of a balanced bridge [3, 4]. For completeness, we will review this method very briefly.

Consider the bridge circuit of Figure 3. If the current through  $Z_5$  is zero, the bridge is said to be balanced and  $Z_5$  will have no effect on the bridge circuit. In fact,  $Z_5$  can now be an open or short circuit.



**FIGURE 3.** General bridge circuit with arbitrary impedances. Note the underscore indicates a phasor voltage or current.

If we replace  $Z_5$  with an open circuit, *i.e.*, remove  $Z_5$  the total impedance will be given by the parallel combination of  $Z_1 + Z_3$  and  $Z_2 + Z_4$  giving rise to Equation 11.

Furthermore, to ensure that the bridge is balanced, it is easily shown that  $\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}$ , or equivalently,  $\frac{C_3}{C_1} = \frac{C_4}{C_2}$ .

To see this, note that the current through  $Z_5$  is zero when the voltage across  $Z_5$  is zero, *i.e.*:

$$\underline{V}_5 = \underline{V}_3 - \underline{V}_4 = 0. \quad (12)$$

Furthermore, when  $\underline{I}_5 = 0$ , we apply the voltage divider rule to obtain:

$$\underline{V}_3 = \frac{Z_3}{Z_1 + Z_3} \underline{E} = \frac{1}{1 + Z_1/Z_3} \underline{E}, \quad (13)$$

and:

$$\underline{V}_4 = \frac{Z_4}{Z_2 + Z_4} \underline{E} = \frac{1}{1 + Z_4/Z_2} \underline{E}. \quad (14)$$

Substituting Equation 13 and 14 into Equation 12 gives the desired result:

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4}.$$

## IV. CONCLUSIONS

We have shown that the equivalent capacitance of five capacitors connected in a bridge configuration does not depend upon one of the capacitors for a special case. We showed this by using the algorithm provided by Atkin and also by using the concept of a balanced bridge.

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