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Non-Linear Electromechanical System Dynamics

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Non-Linear Electromechanical System Dynamics

A Thesis

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Dedicated to

My dearest one

Acknowledgement

I wish to express my heartfelt thanks to all the people who supported me during the composition of this work. I have learned many new and exciting things, and for whichI would like to especially thank Dr Nikolaos Xiros for guiding me throughout the thesis and making me complete by instilling so much confidence. I would like to thank God and my dearest one who stood by me and kept pushing me in every step with lots of cheers.

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Nomenclature

Description	Notations Used	Units, (If applicable)	Values, (if applicable)
Capacitance of Left EM	c_l	F	5.00E-06
Capacitance of Right EM	c_r	F	2.00E-05
Charge Envelope of Left EM	$\tilde{q}_l$	C	
Charge Envelope of Right EM	$\tilde{q}_r$	C	
Current Envelope of left EM	$\dot{\tilde{q}}_l$	A	
Current Envelope of Right EM	$\dot{\tilde{q}}_r$	A	
Damping	b	kg/s	0.25
Displacement	y	cm	
Displacement influencing the inductance, Left EM	ξ_l	cm	
Displacement influencing the inductance, right EM	ξ_r	cm	
Distance between EM's	y_e	cm	10
Disturbance	d	N	
Frequency of Left EM	ω_{el}	rad/s	2000
Frequency of Mechanical system	ω_m	rad/s	10
Frequency of Right EM	ω_{eR}	rad/s	1000
Inductance of Left EM	L_{0l}, L_{1l}	H, H/m	0.05,0.5
Inductance of Right EM	L_{0r}, L_{1r}	H, H/m	0.05,0.5
Mass	m	kg	0.25
Rate of change of charge Envelope of Left EM	$\dot{\tilde{q}}_l$	C/s (A)	
Rate of change of Charge Envelope of Right EM	$\dot{\tilde{q}}_r$	C/s (A)	
Rate of change of current Envelope of Left EM	$\ddot{\tilde{q}}_l$	A/s	
Rate of change of Current Envelope of Right EM	$\ddot{\tilde{q}}_r$	A/s	
Resistance of Left EM	$R_l,$	Ω(ohms)	1

Description	Notations Used	Units, (If applicable)	Values, (if applicable)
Resistance of Right EM	R_r,	Ω(ohms)	1
State space Matrices	$F(x_1), G(x_1)$	1/s, A/Vs (1/ Ωs)	
State space vector of current and charge envelopes	$\tilde{x}_{2l}, \tilde{x}_{2r}$	C, A	
State space vector of displacement and velocity	x_1,	m, m/s	
Stiffness of spring	K	Kg/s^2	0.1
Voltage Envelope of Left EM	$\tilde{u}_l$	V	
Voltage Envelope of Right EM	$\tilde{u}_r$	V	

Abstract

Electromechanical systems dynamics analysis is approached through non-linear differential equations and further creating a state space model for the system. There are three modules analyzed and validated; the first module consists two magnet coupled with a mass spring damper system as a band-pass system; a Low-pass equivalent system and a Low-pass equivalent system through perturbation analysis. Initially Band Pass frameworks for the systems are formulated considering the relation between the mechanical forcing and current. Using Mathematical tools such as the Hilbert Transforms, a Low-Pass equivalent of the systems is derived. The state equations of the systems are then used to design a working model in MATLAB and simulations investigated completely.

Keywords

Electromechanical system, Hilbert Transform, non-linear coupling, perturbation

1 Introduction

1.1 Objective

The Electromechanical system is non-linearly coupled; if analyzed extensively and with the prospective development in their controls will make to be a great tool in the field of mechatronics for a number of applications including vibration analysis, vibration control and energy harvesting.

1.2 Applications Overview

The vast and continuing reduction in the size and power led to focus on development of effective tools for monitoring & control in various fields.

1.2.1 Energy Harvesting

The micro-electromechanical systems (MEMS) have been proven to be an attractive technology for harvesting small magnitudes of energy from ambient vibrations. This technology eliminates the need for replacing chemical batteries or complex wiring in micro-sensors/micro-systems, moving us closer toward battery-less autonomous sensors systems and networks. The recent year development shows that they can provide real time information on consumption of energy. The mechanical component vibrates due to the force induced by the ambient vibration, this movement of the component changes the flux linked in the electric circuit in turn changing the current envelope which could in-turn be used for the components operation.

1.2.2 Vibration Analysis & Vibration Control

Vibrations are inherent part of any system and structure which needs to be attended continuously through-out the span of operation. This can be effectively addressed by coupling an electrical system to the mechanical structure and by controlling the

electromagnetic force created by the electrical system. The displacement of the mechanical system can be monitored and controlled effectively. With recent development in the self-powered MEMS, the analysis and control has improved efficiently with remote and autonomous control and monitoring. The Vibration analysis field is of numerous types, some types of vibration where this work on electromechanical systems is of great use are,

1.2.2.1 Structural Vibrations

A smaller sensor reduces the strain caused by the added sensors on the structure, preserving dynamics of structures. Smaller sensors can also be used to monitor the local strain on structures without averaging the strain at the neighborhood. MEMS/NEMS could be solution for the structural vibration control and monitoring.

1.2.2.2 Vortex Induced Vibrations

In fluid dynamics, vortex-induced vibrations [1] are motions induced on bodies interacting with an external fluid flow, produced by or the motion producing periodical irregularities on this flow. MEMS actuators have recently made their way to the forefront of flow control research. Flow control is most effective when applied near the transition or separation points, i.e., near the critical flow regimes where flow instabilities amplify quickly. This is especially important for micro actuators because these actuators cannot deliver large forces or high power. The matching in length scale between the micro transducers and the structures makes the manipulation of these structures possible. This phenomenon being most common in offshore structures and underwater cables, such extension of the work proposed in this thesis will also address the issue.

1.3 Text Outline

The chapters following from now on will be in the outline as follows, the module one consisting of initial chapters detailing Band-pass systems governing equation derivation and the low-pass equivalent governing equations derived using the Hilbert transform tool and following chapters details the validation of work done by Dr. Xiros, Associate Professor, University of New Orleans and Mr. Psarrou. The later chapters present the two magnet model with the developed concept of single magnet model and validation of this system for various disturbance forms and also for different voltage profiles.

1.4 Context of Thesis

In this thesis, analysis of spectrally decoupled but non-linearly coupled subsystems is carried out using appropriate mathematical tools. The electrical system (RLC circuit) is evidently band-pass operating near resonant frequency while the mechanical system which is a mass, spring & damper system operating at much lower frequency is a low-pass subsystem. When these two systems are coupled and its dynamics analyzed by designing a simulated model, the system has to be run at twice the frequency of the higher frequency. This is overcome by deriving low-pass equivalent of the subsystem describing its essential dynamics. Thus the mechanical subsystem is influenced by the voltage source of the electric (RLC circuit), but the mechanical subsystems position also influences the inductance of the circuit therefore presenting a mutual interaction. If there is external disturbance the subsystems can be suitably designed to cope, thus the objective of the system as a whole is maintained.

Both the systems are validated for different disturbance conditions and different voltage profile. The validation of single magnet model and two magnet models carried out and results compared and effectiveness of both discussed for further extension in the field of analysis and monitoring.

This two magnet system is used for payload positioning. The results are discussed for future work in different applications as afore said. The band-pass system current through resistor will be in phase with the voltage source.

The appendix at end of the report contains the Simulink coding of the systems built for analysis.

1.5 Description of Systems

As put-forth earlier electromechanical systems comprises of two subsystems. Before describing the systems there are few basic definitions required to better understand the dynamics of coupled subsystems.

There are different types of Signals and Systems [2], in this thesis we will be focusing on the linear or Non-linear time invariant systems. A linear system is any system that obeys the properties of scaling (homogeneity) and superposition, while a non-linear system is any system that does not obey at least one of these. The coupled subsystems exhibit similar properties of the Non-linear time invariant systems; this will be evident as the dynamics of the coupled subsystems will be desired.

1.6 Model Description

1.6.1 Single magnet model

The single magnet model [3] comprises of One RLC circuit (Band-Pass subsystem) and a mass-spring- damper arrangement. The mass's spring when at its natural length places the mass at midpoint from the fixed end towards the electromagnet. The fixed end is at y_0=0 and the electromagnet is at distance of y_e. Thus the mass can move from 0 to maximum y_e. The dynamics of the non-linearly coupled subsystems will be analyzed by deriving its equations governing their dynamics.

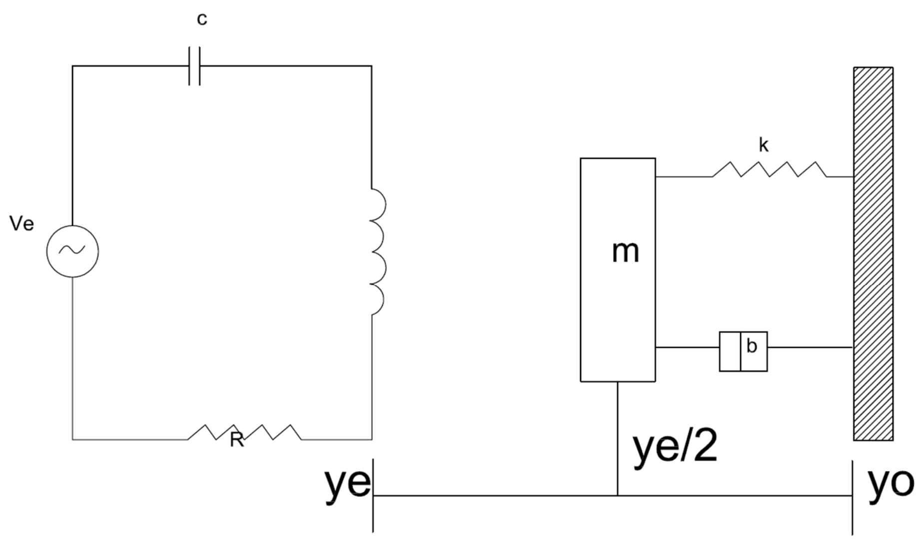
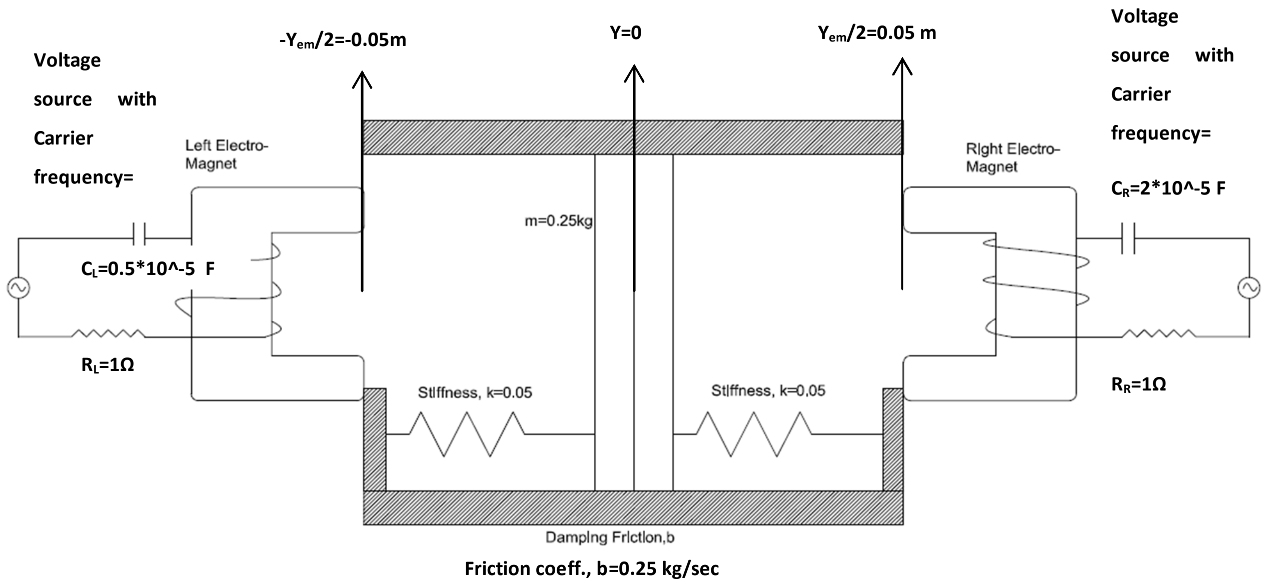


Figure 1- One Magnet Model

1.6.2 Two – Magnet Model

The validations and verification is made using the two magnet model, in the mass's spring when at its natural length is at y_0=0 and the both the electromagnets are at effective distance of y_e/2. Based on these limits and already derived governing equations of single magnet model as shown in (1-10) & (1-12), governing equations depicting this models dynamics is derived.



Inductance of the circuit

$$L_{left} = L_0 + L_1\xi_{left};\ \xi_{left} = -y + y_e/2$$

$$L_{right} = L_0 + L_1\xi_{right};\ \xi_{right} = y + y_e/2$$

Figure 2- two magnet model

1.7 Uncoupled System Dynamics

1.7.1 Mechanical Subsystem

Considering the mechanical subsystem as represented above **Figure 1- One Magnet Model**, the second order force equation of the system can be represented as following

$$m\ddot{y} + b\dot{y} + ky = fem(t) + d \tag{1—1}$$

Where,

m--- Mass of the payload

b--- Damping coefficient

k--- Spring constant

fem (t) --- Electromagnetic force experienced by the mass

d--- Disturbance force

y--- Displacement of mass, and its corresponding velocity and acceleration.

The natural resonant frequency of the mass spring damper system is given by,

$$\omega_m = \sqrt{k/m} \tag{1—2}$$

1.7.2 Electrical system

The second order force equation of the electrical system can be represented as,

$$L\ddot{q} + R\dot{q} + q/C = e(t) \tag{1—3}$$

Where,

L--- Inductance,

R--- Resistance,

C--- Capacitance,

e(t) - Voltage source,

q--- Charge at the capacitor, corresponding current and rate of change of current

The resonant frequency of the RLC circuit is,

$$\omega_e = \sqrt{1/(\mathrm{LC})} \qquad (1—4)$$

Thus equations represent the linear dynamics of both subsystems.

1.8 Coupled system dynamics

In this chapter, an analytical model defining the governing equations of the coupled system will be shown [4]. The subsystem when analyzed for its dynamics separately they are shown above, but these two systems are coupled and both are interacting with each other.

Let's see the electrical subsystem now, the inductance of the electromagnet a function of the displacement of mass because the flux linkage is proportional to inductance of the circuit. It is a clear fact that as the mass is closer to the coil the flux cut by the mass will be more so the inductance will be higher. But as the mass is farther out of the influence of the flux linkage there will be still a self -inductance of the coil existing. Based on this basic electromagnetic theory, the inductance as a function of displacement can be given as,

$$\mathrm{L(y)} = \mathrm{L}_0 + \mathrm{L}_1\mathrm{y}, \text{for } 0 \leq \mathrm{y} \leq \mathrm{y_e} \qquad (1—5)$$

Where,

L_0 – Self-Inductance of the coil,

L_1 – Sensitivity of the coil as influenced by the displacement of the mass

Outside the limits though the flux linkage still exists it is negligible and the systems will be decoupled, so the limits are placed.

Let's consider the mechanical subsystem, the force influenced by the mass is from two sources one will be the electromagnetic force and exogenous disturbance as explained earlier. The electromotive force [5] caused by the electromagnet is given by,

$$\mathrm{fem} = \int \mathrm{edl} \tag{1—6}$$

Where

e—Voltage source

I— Current flowing through the coil,

$$\mathrm{e} = \dot{\varphi}, \text{ rate of change of flux} \tag{1—7}$$

$$\varphi = \mathrm{L(y).\,I}, \text{ flux linkage} \tag{1—8}$$

Combining equation above, it can be easily found that

$$\mathrm{fem} = (\mathrm{I}^2\mathrm{L}_1/2) \tag{1—9}$$

$$\mathrm{m\ddot{y} + b\dot{y} + ky = L_1 \frac{I^2}{2} + d} \tag{1—10}$$

$$\frac{d(L(y)\dot{q})}{dt} + R\dot{q} + \frac{q}{C} = e(t) \qquad (1—11)$$

$$(L_0 + L_1 y)\ddot{q} + (L_1\, \dot{y} + R)\dot{q} + q/C \ = e(t) \qquad (1—12)$$

Thus the above four equations clearly show the non-linearly coupled electromechanical system. The inductance of the coil is the factor that couples the subsystems, if the L_1 tends to becomes "0" the subsystems decouple. This is the governing equation which depicts the dynamics of the systems clearly.

Electrical subsystem is Band-pass system as the natural frequency of subsystem will be much higher than the natural frequency of the mass spring damper system. If seen little clearly, the voltage source will be a combination of low-pass signal that is the signal corresponding to that of the mass and band-pass signal containing the electrical system higher frequency. This is similar to modulated signal used in signal transmission [6]. That is the electrical signal is the carrier signal which is required for the low-pass signal to carry out the positioning of the mass in the field of the magnetic flux.

The variables considered for representing system are displacement and velocity of the mechanical system, while for the electrical system the charge and the current are the variables. Thus those are the states space variables. Hence the governing equations with these variables can be given as following,

$$\ddot{q} = (e(t) - (L_1\dot{y} + R)\dot{q} - q/C)/(L_0 + L_1 y) \qquad (1—13)$$

$$\dot{q} = I \qquad (1—14)$$

$$\ddot{y} = (L_1 \frac{I^2}{2} - d - b\dot{y} - ky)/m \qquad (1—15)$$

$$\dot{y} = \frac{dy}{dt} \qquad (1—16)$$

The above equations are used to build the band-pass model representing the entire system.

1.8.1 Low-pass equivalent system

In this chapter, a detailed description will be of given how a low-pass equivalent set of governing equations will be derived using the Hilbert Transform. The objective is finding an equivalent in low pass is because as “Nyquist” [7] proposed that in order to properly reconstruct a signal, any signal, baseband or band-pass, needs to be sampled by at least two times its highest spectral frequency. The point here is that the mathematical concept will help us get around the signal processing requirements by Nyquist for sampling of band-pass systems.

As earlier notified that band-pass systems signal is a combination of carrier signal and a low-pass signal. Such a signal can be expressed by an analytic signal. Before explaining this part a short description on the mathematical tool ‘Hilbert transform’ will be helpful in further understanding.

1.8.1.1 Hilbert Transform

The Hilbert Transform [8] provides signal analysis with some additional information about amplitude, instantaneous phase, and frequency. The Hilbert Transform is equivalent to linear filter, where all the amplitudes of the spectral components are left unchanged, but their phases are shifted by π/2.

The Hilbert Transform representation of the original function is the convolution integral, written as

$$\hat{u}(t) = u(t)(\pi t)^{-1} \qquad (1—17)$$

Hilbert transform has few important properties which is useful in this thesis is the product property, if there exists a product of low-pass and a band-pass signal as in our

case, then Hilbert transform applies only to the band-pass signal. Consider a signal with low-pass and band-pass components,

$$u(t) = u_1(t)u_2(t) \qquad (1—18)$$

Then the Hilbert transform of the signal is given by,

$$\hat{u}(t) = u_1(t)\hat{u}_2(t) \qquad (1—19)$$

In frequency domain,

$$\widehat{U}(f) = -j\mathrm{sign}(f)U(f) \qquad (1—20)$$

In Digital Signal Processing we often need to look at relationships between real and imaginary parts of a complex signal. These relationships are generally described by Hilbert transforms. Hilbert transform not only helps us relate the real component that is the ‘In-phase’ component and imaginary component that is the ‘Quadrature’ components but it is also used to create a special class of causal signals called analytic which are especially important in simulation.

1.8.1.2 Analytic Signal

The analytic signals help us to represent band-pass signals as complex signals which have especially attractive properties for signal processing [8]. The complex envelope is nothing but the low pass signal. Thus for low pass equivalent derivation a signals split into low pass and carrier signal is all we want. This is accomplished by the analytic signal of a signal.

$$u_+(t) = u(t) + j\hat{u}(t) \qquad (1—21)$$

$$u_+(t) = \tilde{u}(t)e^{j\omega_c t} \qquad (1—22)$$

Where,

$\tilde{u}(t)$ is the complex envelope of the signal of interest.

1.8.1.3 Low-Pass Equivalent governing Equations

The above brief description on the mathematical tool will be used to derive the low-pass equivalent of the electromechanical system. It is clearly understood that the band-pass signal is that of the electrical subsystem and the mechanical subsystems governing equations dynamics are already low pass. So the equations have to be represented in their equivalent forms.

The complex envelope of the current flowing in the circuit is found as following,

$$i_+(t) = \frac{dq}{dt} + j\frac{d\hat{q}(t)}{dt} \tag{1—23}$$

$$i_+(t) = \frac{dq_+}{dt} \tag{1—24}$$

$$q_+(t) = q(t) + j\hat{q}(t) \tag{1—25}$$

Multiplying both sides by $e^{j\omega_e t}$we get the analytic signal represented as complex envelope and solving that we get

$$\dot{\tilde{q}} = j\omega_c\tilde{q} + \dot{\tilde{q}} \tag{1—26}$$

Where,

$$\tilde{q} = q_+ e^{-j\omega_e t} \tag{1—27}$$

$$\dot{\tilde{q}} = [\frac{dq}{dt}]_+ e^{-j\omega_e t} \tag{1—28}$$

This gives the complex envelope of the charge current.

By similar method we can find the other equation’s complex envelope and we get,

$$\frac{d\tilde{\imath}}{dt} = \frac{\left[-\frac{\tilde{q}}{c} - (R + L_1\dot{y})\tilde{\imath} + \tilde{e}\right]}{[L_0 + L_1 y]} - j\omega_e \tilde{\imath} \tag{1—29}$$

The low pass systems equations are,

$$\ddot{y} = \left[\frac{L_1}{2}\frac{|\tilde{\imath}|^2}{2} + d - b\dot{y} - ky\right]/m \tag{1—30}$$

The modulus of complex envelope will be equal to the square root of the sum of squares of the imaginary and real part of the complex current envelope when derived it will be found that it will be twice the analytic signals value hence it has to be halved. Other way to put this point is that to make it low pass the value needs to be replaced by its RMS value.

$$\dot{y} = dy/dt \tag{1—31}$$

These equations represent the low pass equivalent of the coupled system. Since now the low pass signals could be in complex form, for simulation purpose the system has to be split into real and imaginary parts.

$$\mathrm{Real}\frac{dq}{dt} = \omega_e \mathrm{Img}(\tilde{q}) + \mathrm{Real}(\dot{\tilde{q}})$$

$$\mathrm{Img}\frac{d\tilde{q}}{dt} = -\omega_e \mathrm{Real}(\tilde{q}) + \mathrm{Img}(\dot{\tilde{q}})$$

$$\mathrm{Real}\ddot{\tilde{q}} = \frac{\left[\mathrm{real}(\tilde{e}) - \frac{\mathrm{real}(\tilde{q})}{c} - \left(R + (L_1\dot{y})\right)\mathrm{real}\left(\dot{\tilde{q}}\right)\right]}{[L_0 + L_1 y]} + \omega_e \mathrm{img}(\dot{\tilde{q}})$$

$$\mathrm{Img}\ddot{\tilde{q}} = \frac{\left[\mathrm{img}(\tilde{e}) - \frac{\mathrm{img}(\tilde{q})}{c} - \left(R + (L_1\dot{y})\right)\mathrm{img}\left(\dot{\tilde{q}}\right)\right]}{[L_0 + L_1 y]} - \omega_e \mathrm{real}(\dot{\tilde{q}})$$

$$(1—32)$$

The above set of equations gives the complete low pass equivalent set of governing equations.

1.8.2 Single Magnet Description

By setting the initial conditions for the governing conditions as “0” for velocity, acceleration, current and rate of change of current variables at $y=y_e/2$, considering $y_e/2$ as the equilibrium point. This is the point where all forces equalize each other and the mass will remain at equilibrium.

Since the forces will be equal to each other i.e., the magnetic force and the spring force considering no exogenous disturbance, then

$$I = \sqrt{k\frac{y}{L_1}} \tag{1—33}$$

For the parameters used, the values of current required to maintain the mass at equilibrium is 2.236A.

Using this value of current and tending the equation to ‘0’ from initial conditions, we get

$$|\hat{e}| = 79.099\ \mathrm{V} \tag{1—34}$$

Thus, the input voltage signal will be of the form,

$$e\ (t) = (79.099 + A\cos\omega_m\, t)\ \cos\omega_e\, t \qquad (1—35)$$

Where,

A - Amplitude of the signal,

ω_m –Resonant frequency of the mass,

ω_e - Carrier frequency, equal to electrical resonance frequency

The governing equations are incorporated in the MATLAB functions and coupled with integrators. The velocity integrator is of limiter type while the acceleration integrators will be of rising hold type which will be explained briefly in the limiters chapter.

The low pass equivalent input complex envelope is the low pass part of the band-pass input signal,

$$e\ (t) = (79.099 + A\cos\omega_m\, t) \qquad (1—36)$$

The corresponding governing equations for the real and imaginary part of the low-pass equivalent system is incorporated in the MATLAB functions and coupled with integrators to get the displacement and current envelope of the system. The simulation results will be validated in later chapters along with different input conditions.

1.8.3 Limiters for single magnet system

The payload/mass which gets displaced when it gets experienced by the exogenous disturbance and the electromagnetic force, it is necessary to limit the movement of the mass so that after it touches the electromagnet or the fixed end it should be brought back or stopped from moving further. This is done by making the velocity of the payload "zero" and the force influenced is brought to zero.

$$\text{At, y=0; } \dot{y} = 0, \ddot{y} = 0 \ \& \tag{1—37}$$

$$\text{At, y=ye, } \dot{y} = 0, \ddot{y} = 0 \tag{1—38}$$

And since at y=0, the displacement interaction with the inductance of the magnet will also be 0, so only self-inductance exists. Similarly at y=ye, the electromagnetic force is made 0, thereby, there exists only spring force which will pull back the payload avoiding the mass to extend further of electromagnet.

These limiters are incorporated in the embedded MATLAB functions. These limiters are in addition to that of the limits incorporated directly on the integrators.

Validation of single magnet Systems:

The systems simulation is run for three cases, typically depicting

- When the system experiences no exogenous disturbance and influenced only by the DC signal optimum to maintain the mass in equilibrium.
- When the system is experiencing single tone and polychromatic low-pass exogenous disturbances.
- Limiter checked for higher amplitude disturbances.
- For different voltage profile.

This validation is carried out for all the types of model with similar set up.

1.8.3.1 Band-Pass & Low-pass System

As explained earlier the band-pass model will be validated for 4 conditions,

When the system is inputted with DC signal optimum to maintain at midpoint, is the equilibrium point. This voltage has been calculated in the above section. There will be initial slight oscillation of the payload but soon it will stabilize at the equilibrium point in

this case the distance between the minimum and maximum the mass can be moved is 10cm. Thus equilibrium point is 5cm. This will be seen in Figure 3.

The other condition maintained for this validation is that there is no exogenous disturbance on the mass. This is also clearly shown in the scope represented in the Figure 3.

It is clearly visible that the low-pass equivalent and the band-pass settles at the same set equilibrium point proving the point the system can be sampled at just twice the resonant frequency of the mechanical system instead that of the electrical signal which will act as carrier signal in the low-pass equivalent model.

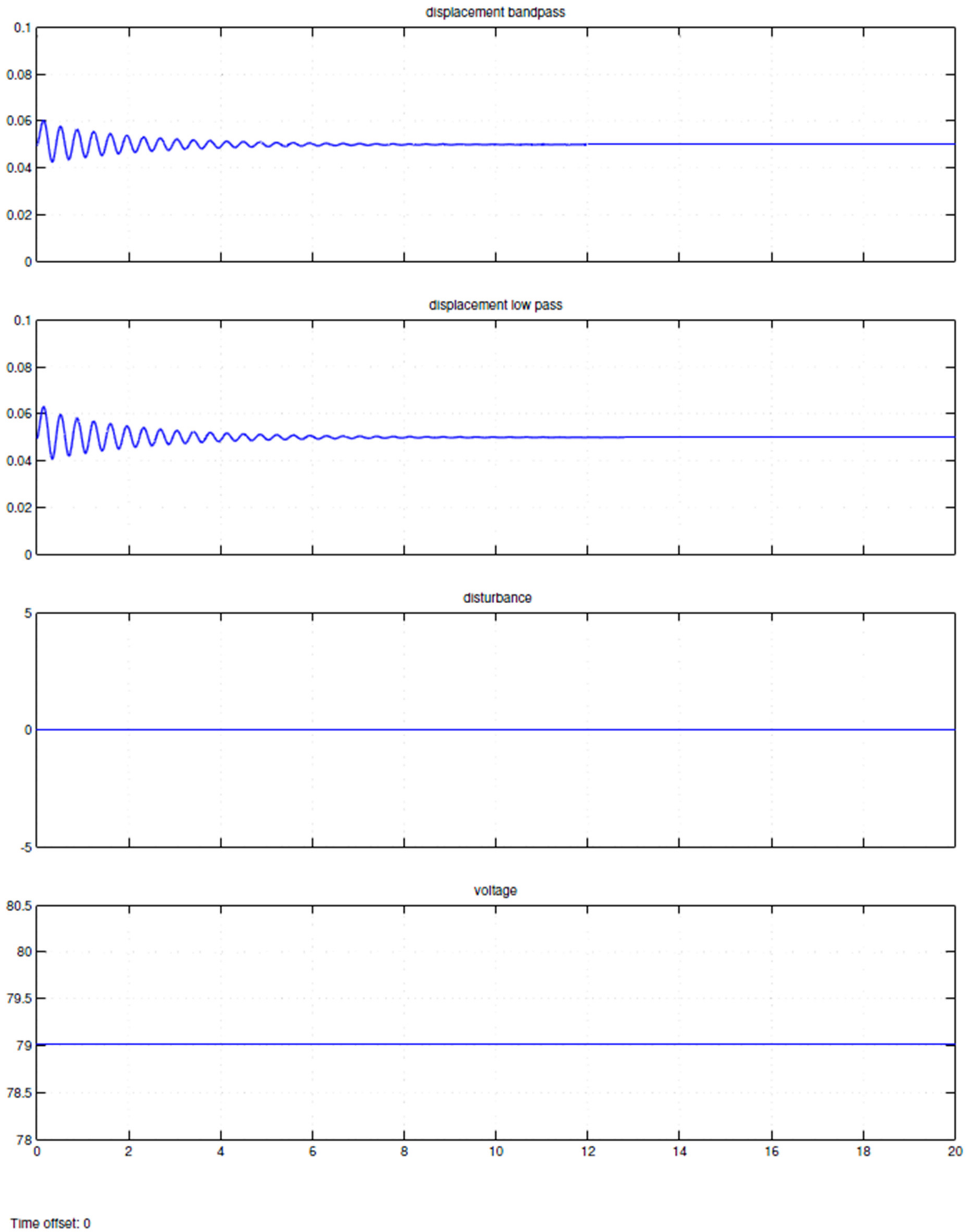


Figure 3- Band-pass & Low-pass Flat Line validations

For the monochromatic disturbance as in Figure 4 is obtained when the system is influenced by single sinusoidal disturbance. And also there is the informational signal on to the mass i.e., the voltage signal with resonant frequency of the mass.

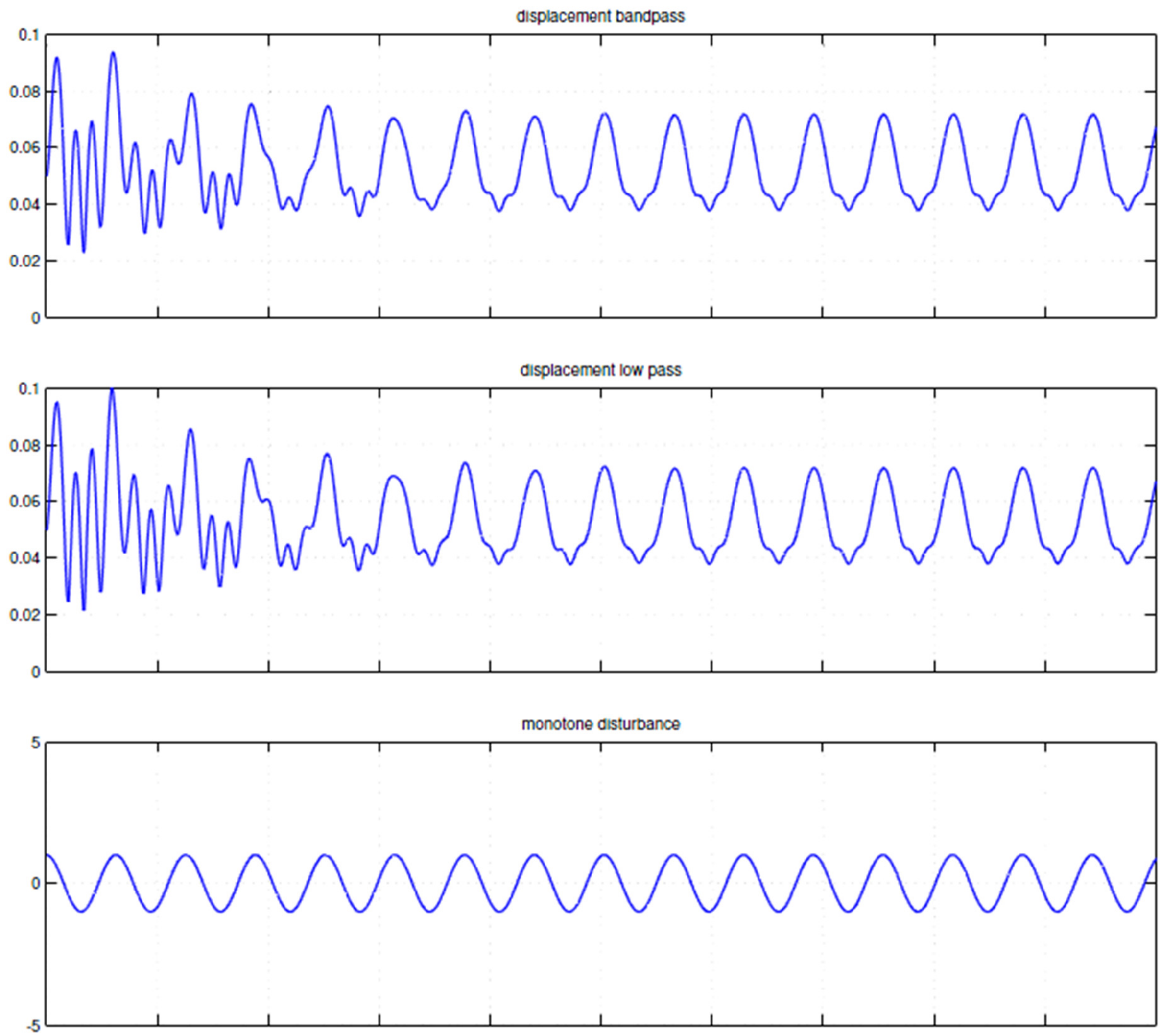


Figure 4- Monochromatic Disturbance- of amplitude 1.0cos5t N

The Figure 5 is obtained for polychromatic disturbance with three disturbances of amplitude and phase as follows, $0.5\cos 5t + 0.3\sin 11t + 0.4\cos 8t$ N, at ω_m. It can be noticed that the movement of the mass is exactly the same as that of the band-pass model displacement.

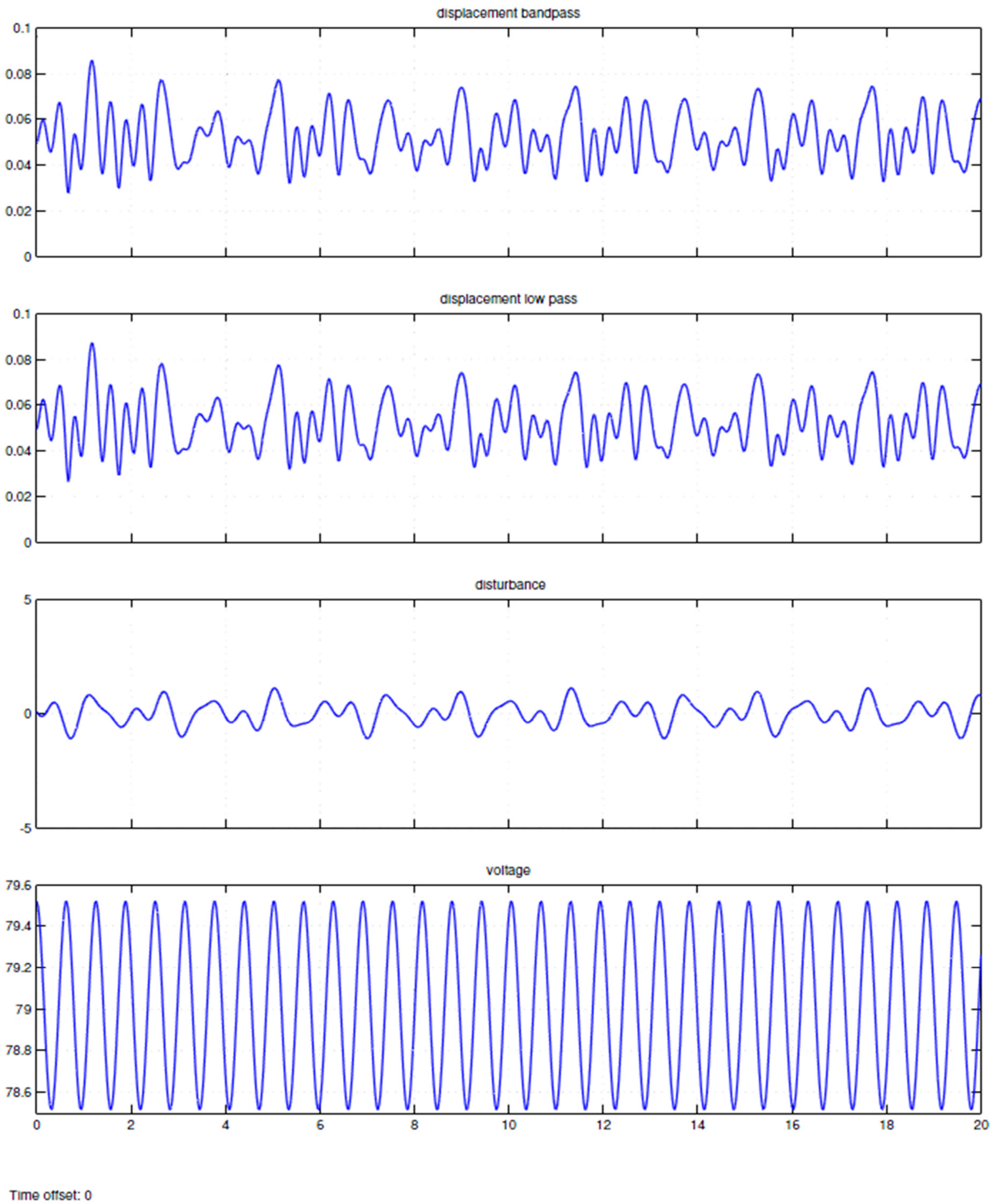


Figure 5- Polychromatic disturbance

Though the displacement of the mass of band-pass system matches almost to that of the low-pass system but at the start of time especially at the peaks (Figure 6) there is a

small difference between the displacements of the low-pass equivalent and that of the band-pass model. This is because of the existence non-linearity in the low-pass equivalent governing equations. This can be overcome in the two magnet model. This will be validated and verified in the later text. But this difference is not the permanent one, the difference voids off as the system stabilizes as the time frame increases.

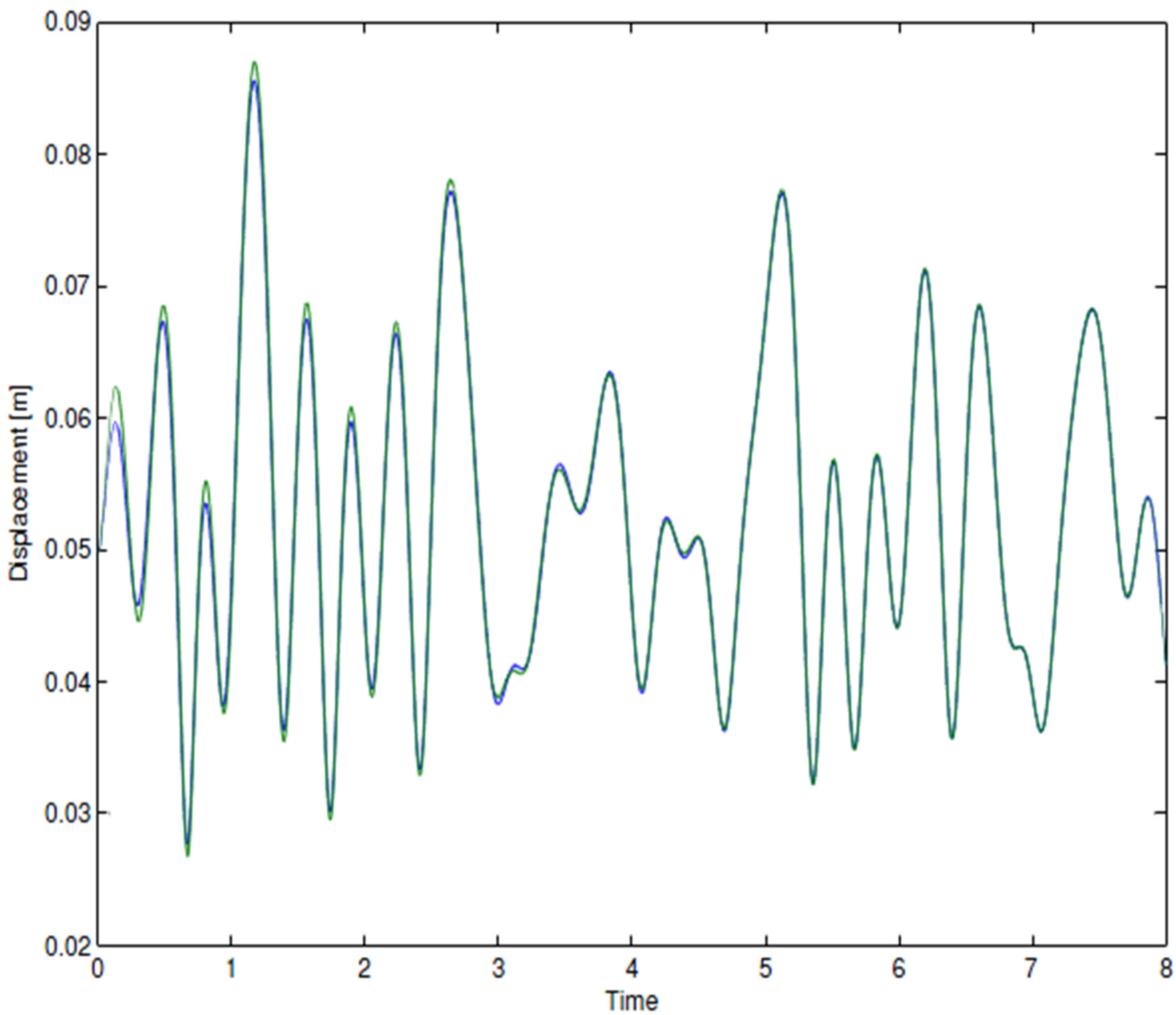


Figure 6- displacement comparison for low-pass (green) and band-pass(blue)

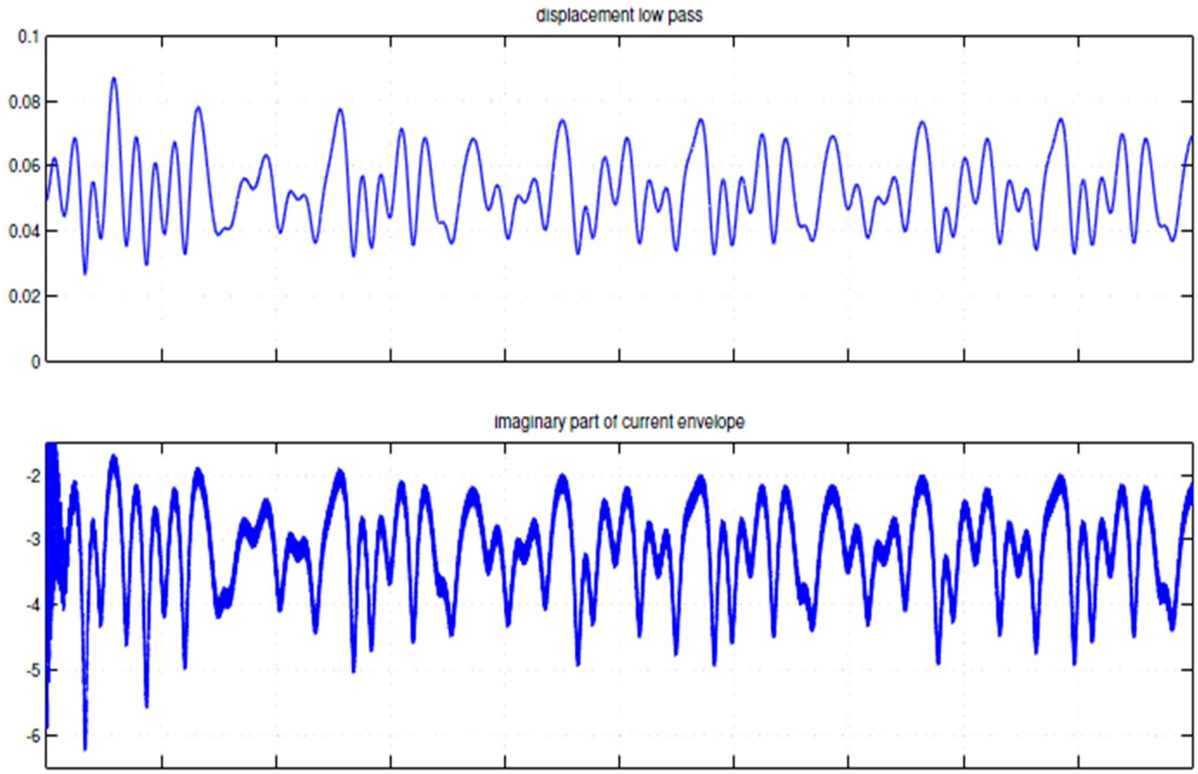


Figure 7- Displacement and imaginary current envelope comparison

Figure 7, the scope shows clearly the low-pass equivalent is that the imaginary part of the complex current envelope is in phase of that displacement of the payload/mass. Though there is a non-linearity difference. But for analysis of the type of vibration or amplitude of vibration or control of vibration this result can be employed. The relation between both the displacement and imaginary part of the current envelope could be found out by calculating the ratio between the mean values of the both the scopes. This will be the factor relating the two.

Figure 8 validates the setup of the limiters. The mass is not allowed to cross the y_e=10cm, as soon as it hits the values y_e, the limiters comes into force setting up the velocity and acceleration 0, there by pulling back the mass.

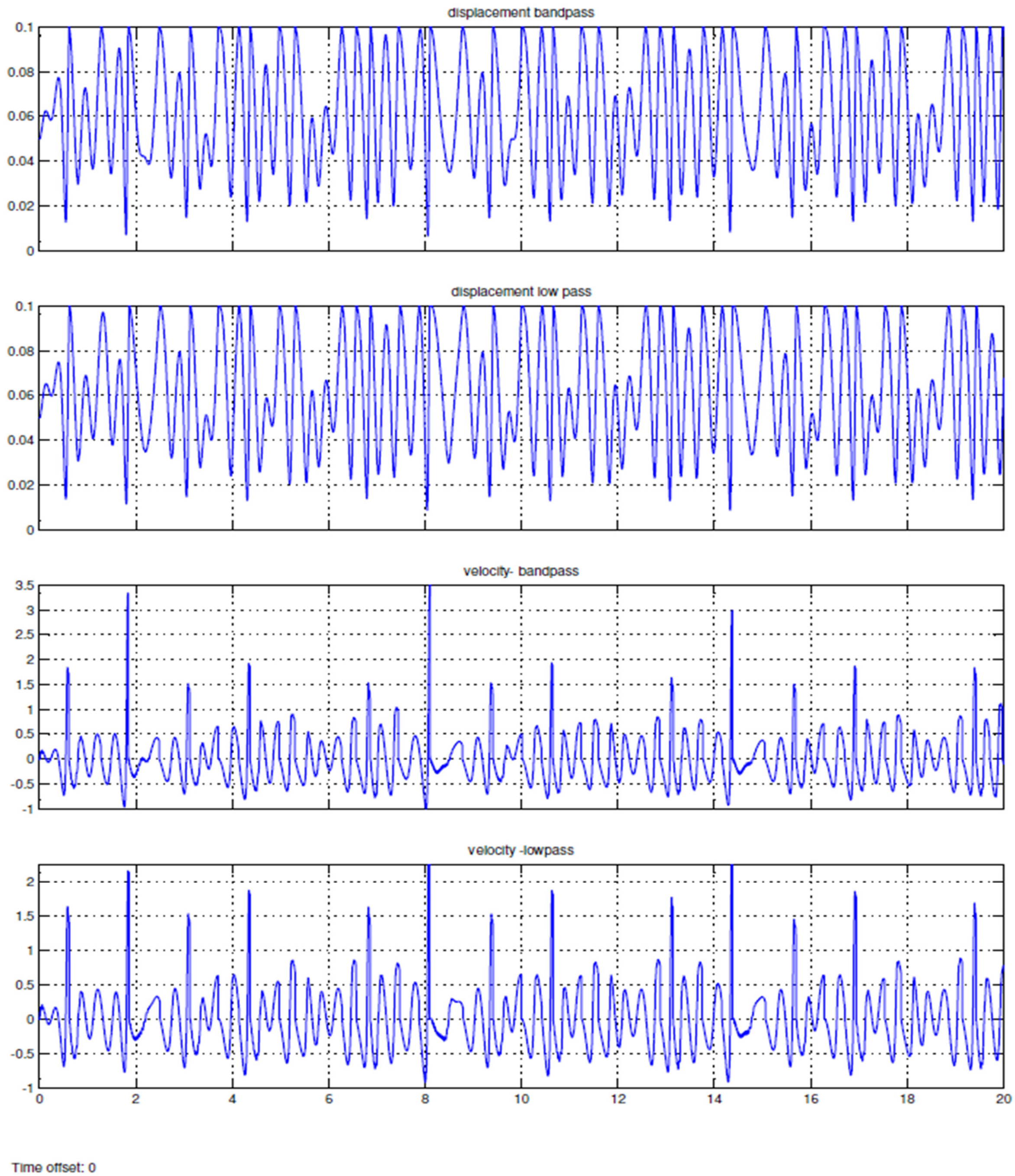


Figure 8- limiter check

2 Two Magnet systems

2.1.1 Two- Magnet model Governing Equations

The basic governing equations are used for both types of model being discussed henceforth. In the two magnets model there is same set of equations but for both magnets individually. Since the mass is placed at y=0, and the electromagnets at $y_e/2$ and $-y_e/2$, there is equivalent displacement introduced such that influence of displacement on the inductance of both the electromagnets could be calculated effectively.

$$\text{For left magnet, } \xi_l = -y + y_e/2, \tag{2—1}$$

$$\text{For right magnet, } \xi_r = y + y_e/2, \tag{2—2}$$

The respective inductances of the magnets and the low pass governing equations will have their corresponding coefficients. The electromagnetic force will be the effective force of both the magnets on the mass. This will be shown on the hence forth results of the simulation run of the model made using these governing equations.

The important aspect of using the two magnet model is being the advantage of using not the predefined voltage source set to bring the mass to equilibrium. By maintaining same current values in both the electromagnets the mass can be maintained in equilibrium and this plays major role in control part in positioning the mass.

Now that the governing equations have been derived for both the cases, henceforth variables will be defined and the set of values used for testing the model designed using these governing equations.

2.1.2 Two Magnet Description

The two magnet model's governing equations are derived similar to that of the single magnet model only thing being the electromagnetic force being the effective force of

the both the magnets on the mass. The current flow and rate of change of current are found for both the magnets with the parameters.

Input DC signals of the electromagnets to maintain the mass at equilibrium at y=0 can be any value with current maintained same in both the magnets. If there is change in current and corresponding difference in voltage signal based on the potential of the electromagnets the mass will be inclined towards that electromagnet.

Using equation for an arbitrary value of voltage current is found for one model and the same current is used for second model to find the corresponding signal voltage.

2.1.3 Limiters for two magnet system

Limiters conditions are of similar to that of the single magnet limiters, difference being the positioning of the electromagnets.

$$\text{At } y=-y_e/2;\ \dot{y}=0, \ddot{y}=0 \qquad \& \tag{2—3}$$

$$\text{At } y=+y_e/2,\ \dot{y}=0, \ddot{y}=0 \tag{2—4}$$

The integrators are incorporated in this too for velocity and acceleration as additional limiters to that of direct in governing equation.

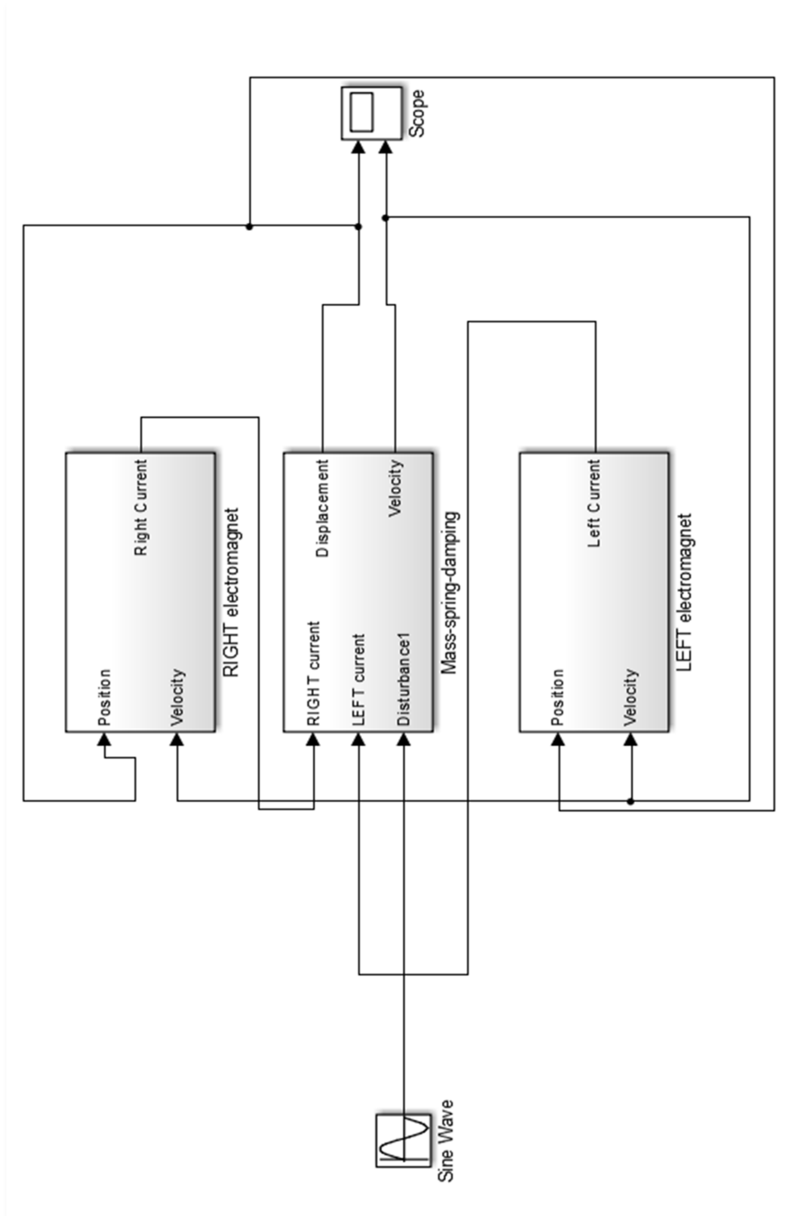


Figure 9- Two Magnet Model

3 Two Magnet, Band-Pass System

3.1 Model Description

The Band-Pass system is developed by using the coupled non-linear equations afore said. The two magnet model or the base model for all the systems is made in the Simulink as following, the mechanical system state space decomposition constitutes a single block which will get the influence of the current vectors from both the magnets. In the similar way the velocity and the displacement of the mechanical system as a result of the mutual effect of the electrical system over the mechanical influences the electrical system.

3.2 System Equations

The system equation for one magnet is derived by splitting system in to state space model, by splitting them into state vectors [10],

$$\mathbf{x_1} = \begin{bmatrix} \mathbf{y} \\ \dot{\mathbf{y}} \end{bmatrix} \tag{3—1}$$

$$\mathbf{x_2} = \begin{bmatrix} \mathrm{q_r} \\ \dot{\mathrm{q}}_\mathrm{r} \end{bmatrix} \tag{3—2}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\mathrm{k/m} & -\mathrm{b/m} \end{bmatrix} \tag{3—3}$$

$$\boldsymbol{\varphi}(\mathbf{x_1}) = \begin{bmatrix} 0 \\ \frac{|\ddot{\tilde{\mathrm{q}}}|^2 \mathrm{L_{1r}}}{2\mathrm{m}} \end{bmatrix} \tag{3—4}$$

$$\mathbf{F}(\mathbf{x_1}) = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\left[\mathrm{C_r}\left[\mathrm{L_{0r}}+\left[\mathrm{y}+\frac{\mathrm{y_e}}{2}\right]\mathrm{L_{1r}}\right]\right]} & -\frac{[\mathrm{R_r}+\mathrm{L_{1r}}\dot{\mathrm{y}}]}{\left[\mathrm{L_{0r}}+\left[\mathrm{y}+\frac{\mathrm{y_e}}{2}\right]\mathrm{L_{1r}}\right]} \end{bmatrix} \tag{3—5}$$

$$\mathbf{G}(\mathbf{x_1}) = \begin{bmatrix} 0 \\ 1/[\mathrm{L_{0r}} + \left[\mathrm{y} + \frac{\mathrm{y_e}}{2}\right]\mathrm{L_{1r}}] \end{bmatrix} \tag{3—6}$$

$$\mathrm{u} = \mathrm{e} \tag{3—7}$$

$$\mathrm{y_2} = \dot{\mathrm{q}} \tag{3—8}$$

By this state- state space decomposition, the following second order final vectors are obtained from the basic equations,

$$\begin{bmatrix} \dot{\mathrm{y}} \\ \ddot{\mathrm{y}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\mathrm{k/m} & -\mathrm{b/m} \end{bmatrix} \begin{bmatrix} \mathrm{y} \\ \dot{\mathrm{y}} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{|\dot{\mathrm{q}}|^2 \mathrm{L_{1r}}}{2\mathrm{m}} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\mathrm{d}}{\mathrm{m}} \end{bmatrix} \tag{3—9}$$

$$\begin{bmatrix} \dot{\mathrm{q}}_\mathrm{r} \\ \ddot{\mathrm{q}}_\mathrm{r} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{\left[\mathrm{C_r}\left[\mathrm{L_{0r}}+\left[\mathrm{y}+\frac{y_e}{2}\right]\mathrm{L_{1r}}\right]\right]} & -\frac{[\mathrm{R_r}+\mathrm{L_{1r}}\dot{\mathrm{y}}]}{\left[\mathrm{L_{0r}}+\left[\mathrm{y}+\frac{y_e}{2}\right]\mathrm{L_{1r}}\right]} \end{bmatrix} \begin{bmatrix} \mathrm{q_r} \\ \dot{\mathrm{q}}_\mathrm{r} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\left(\mathrm{L_{0r}}+\left[\mathrm{y}+\frac{y_e}{2}\right]\mathrm{L_{1r}}\right)} \end{bmatrix} e_r \tag{3—10}$$

The above set of matrices is used in the Simulink model for the mechanical and the electrical system correspondingly. The set of equations derived are for one side of the magnet for the “Right Magnet”. The left magnet has similar set of matrices but the distance of the mass from the magnet changes opposite to that of the right magnet.

3.3 Results of Band- pass system under different conditions

3.3.1 Systems displacement of mass

3.3.1.1 Different voltages to both magnets and a Monochromatic disturbance

The displacement of the mechanical system which includes the influence of the current of both the magnets and effective inductance of the system since the inductance of the magnets is kept same value for testing purpose the influence of magnets different currents is considered important. The effective current of the magnets on the mass and the monochromatic disturbance 0.005 N, play role in positioning the mass since in this particular condition the monochromatic disturbance and voltage value of the magnets are set to a value so that the mass doesn’t reach the limiters for the time series applied.

The maximum displacement the mass can attain is 0.05m on either side of origin. This is explained earlier as in two cases of limits. The displacement (position) of the mass can be effectively controlled using the voltage envelope applied compensating the disturbance there by influencing the current. This part will explain why Low-pass model of the electromechanical system is built and analyzed.

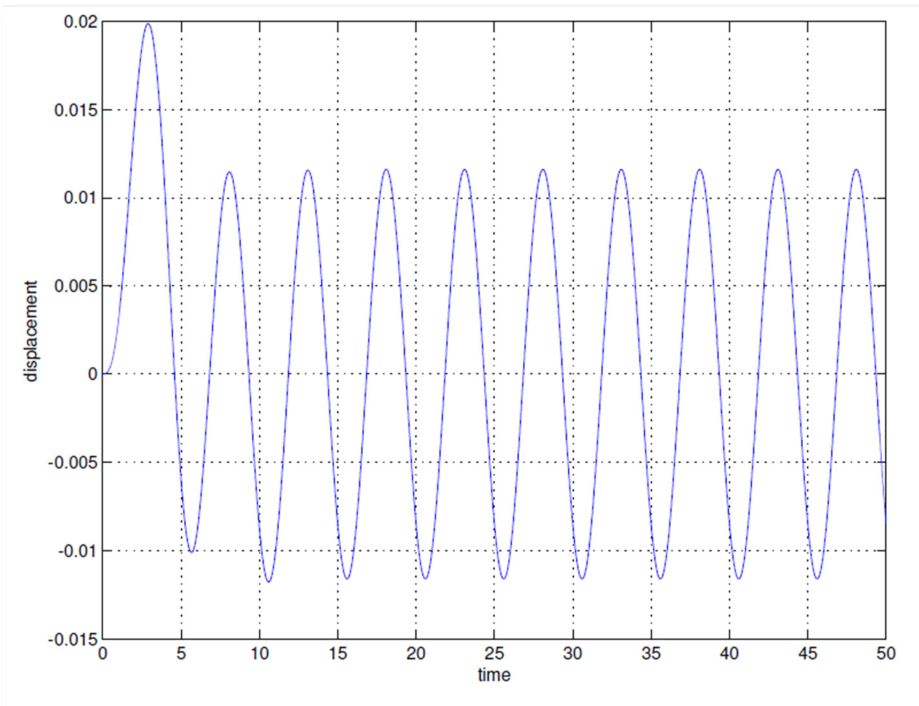


Figure 10- displacement

3.3.2 Systems current

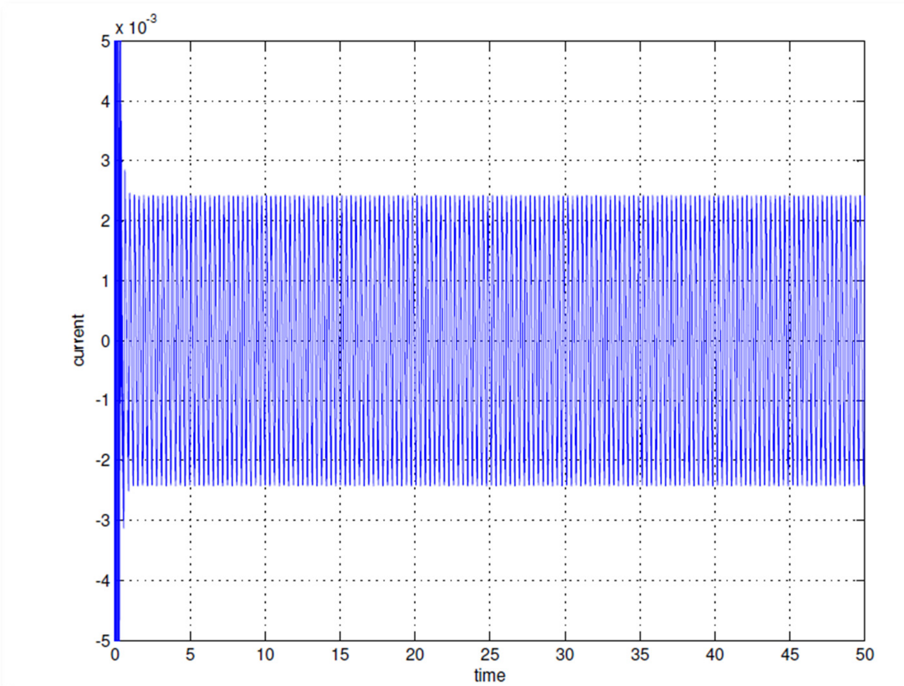


Figure 11-Right Magnet Current

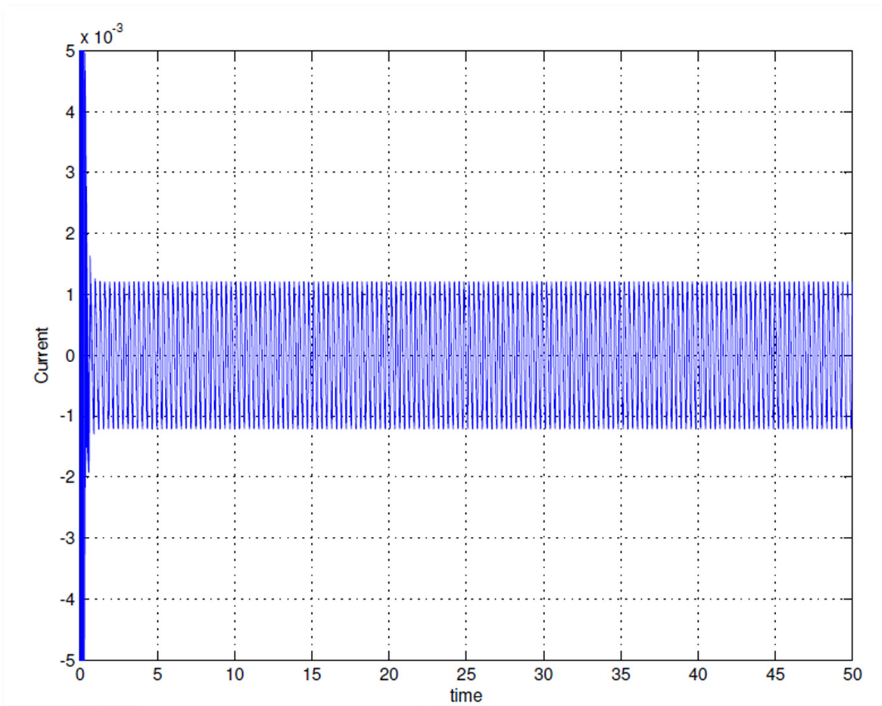


Figure 12-Left Magnet Current

The above results are under the condition of not only a monochromatic disturbance but also excitation voltage supply of 10V to right magnet and 20V to left magnet.

4 Two Magnet, Low-Pass system

4.1 Systems Equations

The system equations of the mechanical system [9] is similar to that of the band-pass system since it is already in linear form so the effective magnitude of the current in the complex envelope form and the analytic form remains the same. Thus the mechanical system follows similar way as that of that of the band-pass system.

The electrical system employs the low-pass equivalent system of equations that is the complex envelope of the charge and envelope which reduces the carrier and the information of the system and at much lower frequency there by the systems response is effective but still maintaining the information that is linked with the mechanical system. This also reduces the effect of non-linearity on to the mechanical systems' displacement because the complex envelope reduces the effect of non-linearity by eliminating the higher order variables and the effect of high frequency on them. This is well explained in the above chapters describing the math tools used in deriving the governing equations of the complex envelope there by deriving the low-pass system equations.

The following equations follow the same principle as that of the band-pass system wherein the variables are split into state space vectors and their corresponding state space decomposition is carried out. But since the vectors of current and charge are of complex variables the time series run results in generating complex dimension.

$$\mathbf{x_1} = \begin{bmatrix} \mathrm{y} \\ \dot{\mathrm{y}} \end{bmatrix} \tag{4—1}$$

$$\mathbf{x_2} = \begin{bmatrix} \tilde{\mathrm{q}}_\mathrm{r} \\ \dot{\tilde{\mathrm{q}}}_\mathrm{r} \end{bmatrix} \tag{4—2}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\mathrm{k/m} & -\mathrm{b/m} \end{bmatrix} \tag{4—3}$$

$$\boldsymbol{\varphi}(\mathbf{x_1}) = \begin{bmatrix} 0 \\ \frac{|\dot{\tilde{\mathrm{q}}}|^2 \mathrm{L_{1r}}}{2\mathrm{m}} \end{bmatrix} \tag{4—4}$$

$$\mathbf{F}(\mathbf{x_1}) = \begin{bmatrix} 0 & 1 \\ -1/[\mathrm{C_r}\left[\mathrm{L_{0r}} + \left[\mathrm{y} + \frac{\mathrm{y_e}}{2}\right]\mathrm{L_{1r}}\right]] & -[\mathrm{R_r} + \mathrm{L_{1r}}\dot{\mathrm{y}}]/\left[\mathrm{L_{0r}} + \left[\mathrm{y} + \frac{\mathrm{y_e}}{2}\right]\mathrm{L_{1r}}\right] \end{bmatrix} \tag{4—5}$$

$$\mathbf{G}(\mathbf{x_1}) = \begin{pmatrix} 0 \\ \frac{1}{\left(\mathrm{L_{0r}} + [\mathrm{y} + \frac{\mathrm{y_e}}{2}]\mathrm{L_{1r}}\right)} \end{pmatrix} \tag{4—6}$$

$$\dot{\mathbf{x}}_\mathbf{1} = \mathbf{A}\mathbf{x_1} + \boldsymbol{\varphi}(\mathbf{x_1}) + \mathrm{d} \tag{4—7}$$

$$\begin{bmatrix} \dot{\mathrm{y}} \\ \ddot{\mathrm{y}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\mathrm{k/m} & -\mathrm{b/m} \end{bmatrix}\begin{bmatrix} \mathrm{y} \\ \dot{\mathrm{y}} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{|\dot{\tilde{\mathrm{q}}}|^2 \mathrm{L_{1r}}}{2\mathrm{m}} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\mathrm{d}}{\mathrm{m}} \end{bmatrix} \tag{4—8}$$

$$\dot{\mathbf{x}}_\mathbf{2} = \mathbf{F}(\mathbf{x_1})\mathbf{x_2} + \mathbf{G}(\mathbf{x_1})\mathrm{e} \tag{4—9}$$

$$\begin{bmatrix} \dot{\tilde{\mathrm{q}}}_\mathrm{r} \\ \ddot{\tilde{\mathrm{q}}}_\mathrm{r} \end{bmatrix} = \begin{bmatrix} -\mathrm{j}\omega_\mathrm{e} & 1 \\ -\frac{1}{\left[\mathrm{C_r}\left[\mathrm{L_{0r}} + \left[\mathrm{y} + \frac{\mathrm{y_e}}{2}\right]\mathrm{L_{1r}}\right]\right]} & -\frac{[\mathrm{R_r} + \mathrm{L_{1r}}\dot{\mathrm{y}}]}{\left[\mathrm{L_{0r}} + \left[\mathrm{y} + \frac{\mathrm{y_e}}{2}\right]\mathrm{L_{1r}}\right]} - \mathrm{j}\omega_\mathrm{e} \end{bmatrix}\begin{bmatrix} \tilde{\mathrm{q}}_\mathrm{r} \\ \dot{\tilde{\mathrm{q}}}_\mathrm{r} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\left(\mathrm{L_{0r}} + \left[\mathrm{y} + \frac{\mathrm{y_e}}{2}\right]\mathrm{L_{1r}}\right)} \end{bmatrix}\tilde{e}_r \tag{4—10}$$

Similar to that in the Band-Pass system the left magnet equations are similar to that of the right magnet varying with the distance of the mass from the position that of the right magnet. The complex envelope of the charge and current are derived by using the ‘Hilbert Transform’, where in the pre envelope is the sum of the input signal vector and Hilbert transform of the input signal vector in the frequency domain as shown in (1-17) to (1-20). When converted to time domain we get real part of the analytic signal gives the input signal vector. This is used in converting the input signal vector to complex envelope where in retaining the signal information but in low-pass state.

4.2 Model testing under same condition as Band-pass

4.2.1 Displacement of the system

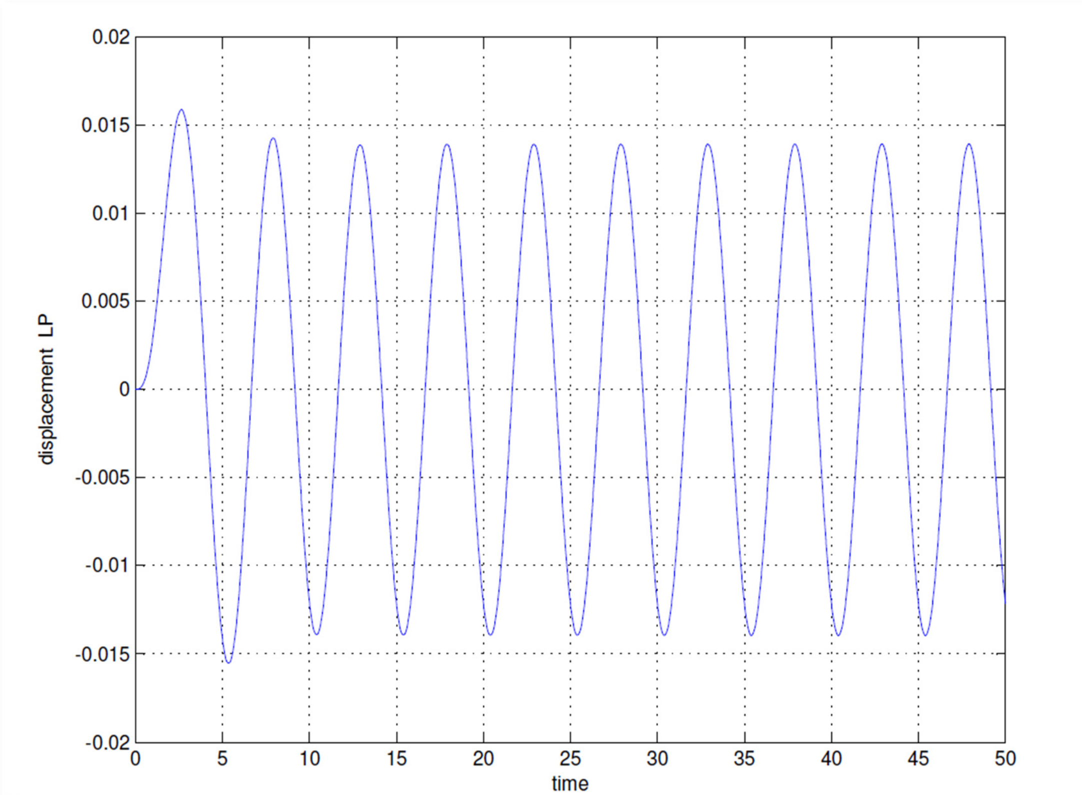


Figure 13 – Displacement LP

The above plot displays the displacement of the mass of the Low pass equivalent system of the electromechanical system because of a monochromatic disturbance of 0.005N and voltage supply of 10V to right magnet and 20V to the left magnet. The below plots are the current envelopes of the magnets with imaginary and real parts. In the future chapters conditions used for this LP equivalent testing will be used for validation with the perturbation model. The current envelope is since proportional to the charge envelope, in effect the plots of the current envelope are shown and charge envelope will be evident.

4.2.2 Current Envelope

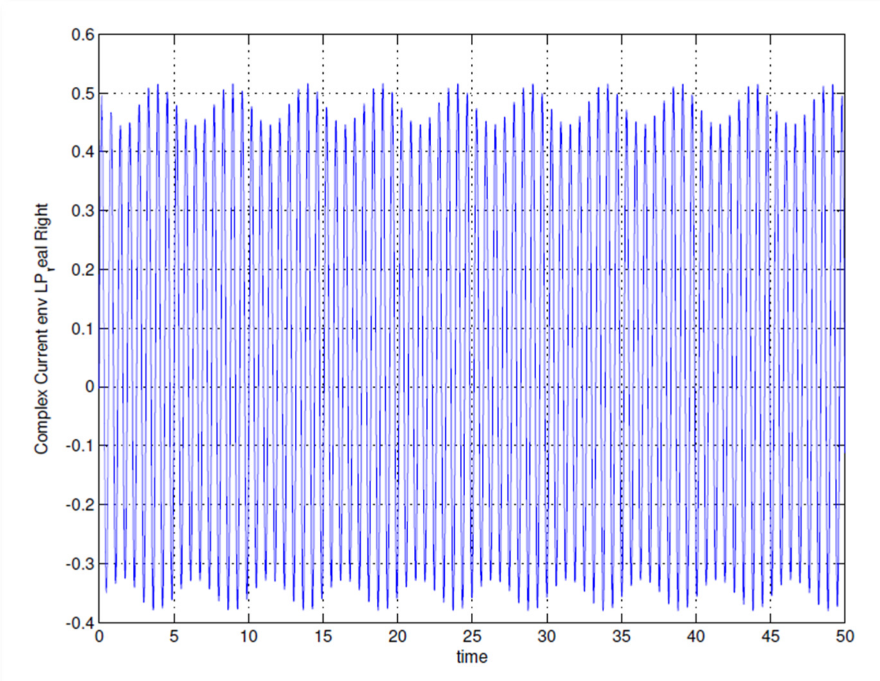


Figure 14- Real Part of right Magnet current envelope

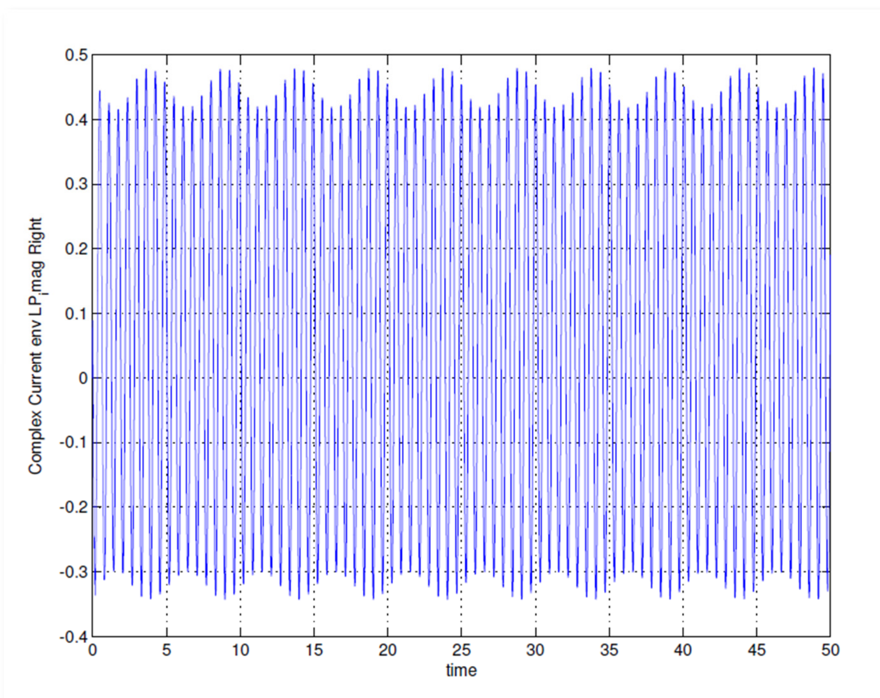


Figure 15- Imaginary Part of right Magnet current envelope

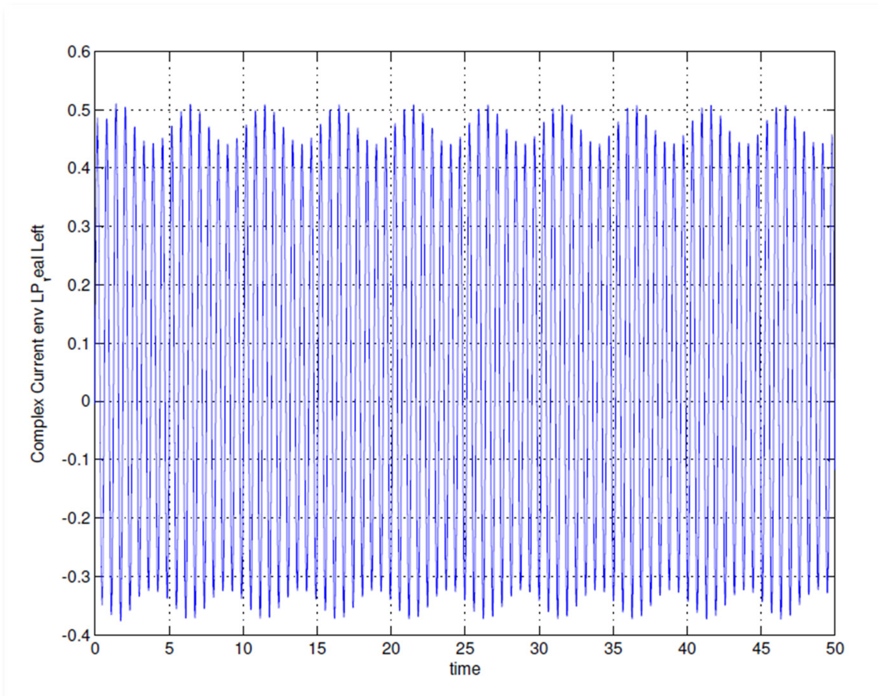


Figure 16- Real Part of left Magnet current envelope

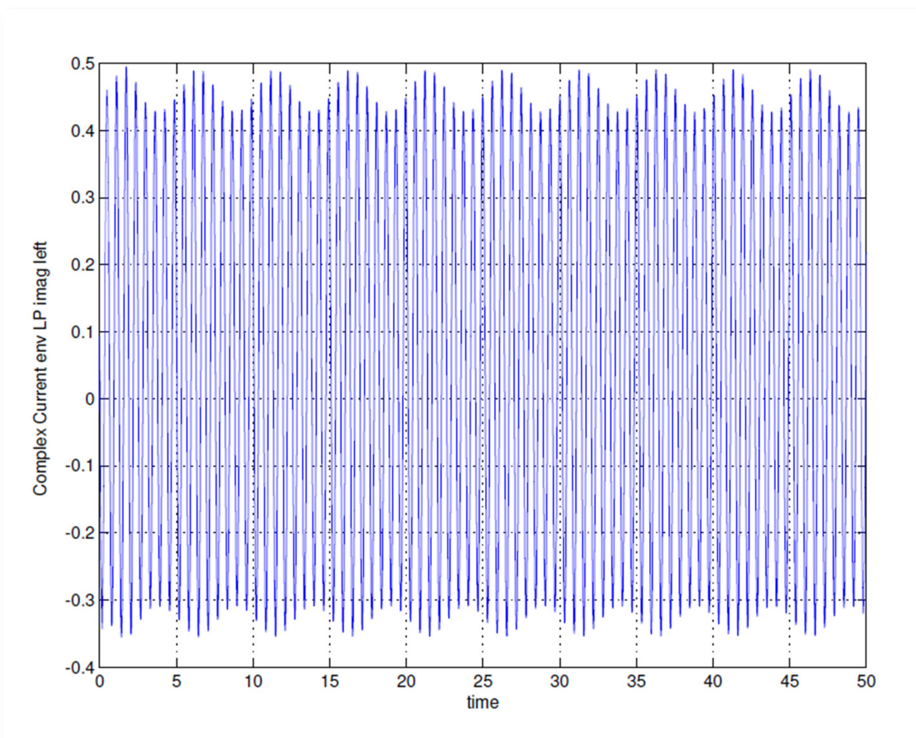


Figure 17- Imaginary Part of left Magnet current envelope

5 Two Magnet, Low-Pass Perturbation System

5.1 Model description

Low-pass equivalent system is validated by linearizing with respect to electrical subsystem variables with parameters as displacement and velocity of the non-linearly coupled mechanical and electrical subsystems. The electrical sub-systems complex envelope of the LP equivalent is considered and using Taylor expansion using the Jacobian of the variables involved the subsystem is expanded and used as the electrical subsystem.

5.2 Governing Equations of Right Magnet

5.2.1 Mechanical Subsystem

The state space variables of the subsystems is,

$$\mathbf{x_1} = \begin{bmatrix} \mathrm{y} \\ \dot{\mathrm{y}} \end{bmatrix} \tag{5—1}$$

$$\mathbf{x_2} = \begin{bmatrix} \tilde{\mathrm{q}}_\mathrm{r} \\ \dot{\tilde{\mathrm{q}}}_\mathrm{r} \end{bmatrix} \tag{5—2}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\mathrm{k/m} & -\mathrm{b/m} \end{bmatrix} \tag{5—3}$$

$$\boldsymbol{\varphi}(\mathbf{x_1}) = \begin{bmatrix} 0 \\ \frac{|\tilde{\mathrm{q}}|^2 \mathrm{L_{1r}}}{2\mathrm{m}} \end{bmatrix} \tag{5—4}$$

$$\dot{\mathbf{x}}_\mathbf{1} = \mathbf{A}\mathbf{x_1} + \boldsymbol{\varphi}(\mathbf{x_1}) + \mathrm{d} \tag{5—5}$$

$$\begin{bmatrix} \dot{\mathrm{y}} \\ \ddot{\mathrm{y}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\mathrm{k/m} & -\mathrm{b/m} \end{bmatrix} \begin{bmatrix} \mathrm{y} \\ \dot{\mathrm{y}} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{|\tilde{\mathrm{q}}|^2 \mathrm{L_{1r}}}{2\mathrm{m}} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\mathrm{d}}{\mathrm{m}} \end{bmatrix} \tag{5—6}$$

The above set of state space vectors constitutes the mechanical subsystem.

5.2.2 Electrical Subsystem

The electrical subsystem state space vectors,

$$\mathbf{F}(\mathbf{x_1}) = \begin{bmatrix} 0 & 1 \\ -1/[C_r\left[L_{0r} + \left[y + \frac{y_e}{2}\right]L_{1r}\right]] & -[R_r + L_{1r}\dot{y}]/\left[L_{0r} + \left[y + \frac{y_e}{2}\right]L_{1r}\right] \end{bmatrix} \quad (5\text{—}7)$$

$$\mathbf{G}(\mathbf{x_1}) = \begin{pmatrix} 0 \\ \frac{1}{\left(L_{0r}+[y+\frac{y_e}{2}]L_{1r}\right)} \end{pmatrix} \quad (5\text{—}8)$$

$$\dot{\tilde{\mathbf{x}}}_{\mathbf{2r}} = F(x_1)\tilde{x}_2 + G(x_1)\tilde{e}_r \quad (5\text{—}9)$$

The above equation is the governing equation for the electrical subsystem, which is expanded using the Taylor's expansion and neglecting the higher order functions and considering the steady state is also included. But since it is of negligible value when the model is tested in Simulink the steady state component is not considered. The function to the order of only first order is only considered [11, 12].

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_{\mathbf{2<\#>}} + \dot{\tilde{\mathbf{x}}}_{\mathbf{2<0>}} = {} & \left[[\mathbf{F}(\mathbf{x_{1\#}})]\tilde{\mathbf{x}}_{\mathbf{2<0>}} + \mathbf{G}(\mathbf{x_{1\#}})\tilde{\mathbf{u}}_{\mathbf{<0>}}\right] + \\ & \left[[\mathbf{F}(\mathbf{0}) - \mathbf{j}\boldsymbol{\omega}_{\mathbf{e}}\mathbf{I}]\tilde{\mathbf{x}}_{\mathbf{2<\#>}}\right] + \\ & [\mathbf{G}(\mathbf{0})\tilde{\mathbf{u}}_{\mathbf{<\#>}}] + \\ & \left[[\mathbf{F}(\mathbf{0}) - \mathbf{j}\boldsymbol{\omega}_{\mathbf{e}}\mathbf{I}]\tilde{\mathbf{x}}_{\mathbf{2<0>}}\right] + \\ & [\mathbf{G}(\mathbf{0})\tilde{\mathbf{u}}_{\mathbf{<0>}}] \end{aligned} \quad (5\text{—}10)$$

The state space matrix is differentiated and variables influencing the electrical vectors are the velocity and displacement which is termed up to "0".

$$\mathbf{F}(\mathbf{0}) = \begin{bmatrix} 0 & 1 \\ -1/(C_r[L_{0r} + \frac{y_e}{2}L_{1r}]) & -R_r/[L_{0r} + \frac{y_e}{2}L_{1r}] \end{bmatrix} \quad (5\text{—}11)$$

$$\left|\frac{\partial \mathbf{F}}{\partial \mathbf{x_1}}_{\mathbf{x1=0}}\right| = \begin{bmatrix} 0 & 0 \\ -\frac{L_{1r}y}{C_r\left(L_{0r}+\frac{y_e}{2}L_{1r}\right)^2} & -\frac{L_{1r}\left[\dot{y}\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]-R_r y\right]}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]^2} \end{bmatrix} \tag{5—12}$$

$$\mathbf{G}(\mathbf{0}) = \begin{pmatrix} 0 \\ 1/[L_{0r}+\frac{y_e}{2}L_{1r}] \end{pmatrix} \tag{5—13}$$

$$\left|\frac{\partial \mathbf{G}}{\partial \mathbf{x_1}}_{\mathbf{x1=0}}\right| = \begin{pmatrix} 0 \\ \frac{-L_{1r}y}{\left(L_{0r}+\frac{y_e}{2}L_{1r}\right)^2} \end{pmatrix} \tag{5—14}$$

$$\tilde{\mathbf{x}}_{\mathbf{2<0>}} = \begin{pmatrix} \tilde{q}_{r<0>} \\ \dot{\tilde{q}}_{r<0>} \end{pmatrix} \tag{5—15}$$

$$\tilde{\mathbf{x}}_{\mathbf{2<\#>}} = \begin{pmatrix} \tilde{q}_{r<\#>} \\ \dot{\tilde{q}}_{r<\#>} \end{pmatrix} \tag{5—16}$$

$$\tilde{\mathbf{u}}_{<0>} = \mathbf{e_r} \tag{5—17}$$

$$\mathbf{x}_{\mathbf{1}<\#>} = \begin{pmatrix} y_{<\#>} \\ \dot{y}_{<\#>} \end{pmatrix} \tag{5—18}$$

$$\mathbf{F}(\mathbf{x_{1\#}}) = \begin{bmatrix} 0 & 0 \\ -\frac{L_{1r}y}{C_r\left(L_{0r}+\frac{y_e}{2}L_{1r}\right)^2} & -\frac{L_{1r}[\dot{y}[L_{0r}+\frac{y_e}{2}L_{1r}]-R_r y]}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]^2} \end{bmatrix} \tag{5—19}$$

$$\mathbf{F}(\mathbf{x_{1\#}})\tilde{\mathbf{x}}_{\mathbf{2<0>}} = \begin{bmatrix} 0 & 0 \\ -\frac{L_{1r}y}{C_r\left(L_{0r}+\frac{y_e}{2}L_{1r}\right)^2} & -\frac{L_{1r}\left[\dot{y}[L_{0r}+\frac{y_e}{2}L_{1r}]-R_r y\right]}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]^2} \end{bmatrix} \begin{bmatrix} \tilde{q}_{r<0>} \\ \dot{\tilde{q}}_{r<0>} \end{bmatrix} \tag{5—20}$$

$$\mathbf{F}(\mathbf{x_{1\#}})\tilde{\mathbf{x}}_{\mathbf{2<0>}} = \begin{bmatrix} 0 & 0 \\ -\frac{L_{1r}y}{C_r\left(L_{0r}+\frac{y_e}{2}L_{1r}\right)^2} & -\frac{L_{1r}\left[\dot{y}\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]-R_r y\right]}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]^2} \end{bmatrix} \begin{bmatrix} \tilde{q}_{rc<0>} + j\tilde{q}_{rs<0>} \\ \dot{\tilde{q}}_{rc<0>} + j\dot{\tilde{q}}_{rs<0>} \end{bmatrix} \tag{5—21}$$

$$\mathbf{G}(\mathbf{x_{1\#}}) = \begin{pmatrix} 0 \\ \frac{-L_{1r}y}{\left(L_{0r}+\frac{y_e}{2}L_{1r}\right)^2} \end{pmatrix} \tag{5—22}$$

$$\left[\mathbf{F}(\mathbf{0})-[j\omega_e\mathbf{I}]\right]\tilde{\mathbf{x}}_{\mathbf{2}<\#>}=\left[\begin{bmatrix}0 & 1\\ -\frac{1}{C_r[L_{0r}+\frac{y_e}{2}L_{1r}]} & -\frac{R_r}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]}\end{bmatrix}-[j\omega_e I]\right]\begin{bmatrix}\tilde{q}_{rc<\#>}+j\tilde{q}_{rs<\#>}\\ \dot{\tilde{q}}_{rc<\#>}+j\dot{\tilde{q}}_{rs<\#>}\end{bmatrix}$$

(5—23)

$$\mathbf{F}(\mathbf{x}_{\mathbf{1}\#})\tilde{\mathbf{x}}_{\mathbf{2}<0>}=\begin{bmatrix}0\\ -\frac{\tilde{q}_{rc<0>}L_{1r}y}{C_r\left(L_{0r}+\frac{y_e}{2}L_{1r}\right)^2}-\frac{L_{1r}\left[\dot{y}\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]-R_r y\right]\dot{\tilde{q}}_{rc<0>}}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]^2}\end{bmatrix}$$
$$+j\begin{bmatrix}0\\ -\frac{\tilde{q}_{rs<0>}L_{1r}y}{C_r\left(L_{0r}+\frac{y_e}{2}L_{1r}\right)^2}-\frac{L_{1r}\left[\dot{y}[L_{0r}+\frac{y_e}{2}L_{1r}]-R_r y\right]\dot{\tilde{q}}_{rs<0>}}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]^2}\end{bmatrix}$$

(5—24)

$$\left[\mathbf{F}(\mathbf{0})-[j\omega_e\mathbf{I}]\right]\tilde{\mathbf{x}}_{\mathbf{2}<\#>}$$
$$=\begin{bmatrix}\omega_e\tilde{q}_{rs<\#>}+\dot{\tilde{q}}_{rc<\#>}\\ -\frac{\tilde{q}_{rc<\#>}}{C_r\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]}-\frac{\dot{\tilde{q}}_{rc<\#>}R_r}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]}+\dot{\tilde{q}}_{rs<\#>}\omega_e\end{bmatrix}$$
$$+j\begin{bmatrix}-\omega_e\tilde{q}_{rc<\#>}+\dot{\tilde{q}}_{rs<\#>}\\ -\frac{\tilde{q}_{rs<\#>}}{C_r\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]}-\frac{\dot{\tilde{q}}_{rs<\#>}R_r}{\left[L_{0r}+\frac{y_e}{2}L_{1r}\right]}-\dot{\tilde{q}}_{rs<\#>}\omega_e\end{bmatrix}$$

5—25

$$[\mathbf{G}(\mathbf{0})]\tilde{\mathbf{u}}_{<\#>}=\begin{bmatrix}0\\ \frac{1}{\left[\left[L_{0r}+\frac{y_e}{2}*L_{1r}\right]\right]}\end{bmatrix}[\tilde{u}_{rc<\#>}+j\tilde{u}_{rs<\#>}]\qquad 5—26$$

$$[\mathbf{F}(\mathbf{0}) - [j\omega_e\mathbf{I}]]\tilde{\mathbf{x}}_{\mathbf{2}<0>}$$
$$= \left[\begin{array}{c} \omega_e \tilde{q}_{rs<0>} + \dot{\tilde{q}}_{rc<0>} \\ -\dfrac{\tilde{q}_{rc<0>}}{C_r\left[L_{0r} + \dfrac{y_e}{2}L_{1r}\right]} - \dfrac{\dot{\tilde{q}}_{rc<0>}R_r}{\left[L_{0r} + \dfrac{y_e}{2}L_{1r}\right]} + \dot{\tilde{q}}_{rs<0>}\omega_e \end{array} \right]$$
$$+ j \left[\begin{array}{c} -\omega_e \tilde{q}_{rc<0>} + \dot{\tilde{q}}_{rs<0>} \\ -\dfrac{\tilde{q}_{rs<0>}}{C_r\left[L_{0r} + \dfrac{y_e}{2}L_{1r}\right]} - \dfrac{\dot{\tilde{q}}_{rs<0>}R_r}{\left[L_{0r} + \dfrac{y_e}{2}L_{1r}\right]} - \dot{\tilde{q}}_{rs<0>}\omega_e \end{array} \right]$$

(5—27)

$$[\mathbf{G}(\mathbf{0})]\tilde{\mathbf{u}}_{<0>} = \left[\begin{array}{c} 0 \\ \frac{1}{\left[L_{0r} + \frac{y_e}{2}L_{1r}\right]} \end{array} \right] [\tilde{u}_{rc<0>} + j\tilde{u}_{rs<0>}] \qquad (5—28)$$

$$\mathbf{G}(\mathbf{x}_{\mathbf{1}\#})\tilde{\mathbf{u}}_{\mathbf{r}<0>} = \left[\begin{array}{c} 0 \\ \frac{-L_{1r}y}{\left(L_{0r} + \frac{y_e}{2}L_{1r}\right)^2} \end{array} \right] [\tilde{u}_{rc<0>} + j\tilde{u}_{rs<0>}] \qquad (5—29)$$

The set of equations are derived for the right magnet are derived, the left magnet is self- evident with only the displacement varying. The final vectors since in the complex dimension the imaginary and real parts of the charge and current envelope are shown in the following equations.

$$\dot{\tilde{q}}_{rr} = \omega_e \tilde{q}_{rs<\#>} + \dot{\tilde{q}}_{rc<\#>} + \omega_e \tilde{q}_{rs<0>} + \dot{\tilde{q}}_{rc<0>} \qquad 5—30$$

$$
\begin{aligned}
\ddot{\tilde{q}}_{rr} = & -\frac{\tilde{q}_{rc<0>} L_{1r} y}{C_r \left(L_{0r} + \frac{y_e}{2} L_{1r}\right)^2} - \frac{L_{1r}\left[\dot{y}\left[L_{0r} + \frac{y_e}{2} L_{1r}\right] - R_r y\right] \dot{\tilde{q}}_{rc<0>}}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]^2} - \frac{\tilde{q}_{rc<\#>}}{C_r \left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} \\
& - \frac{\dot{\tilde{q}}_{rc<\#>} R_r}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} + \dot{\tilde{q}}_{rs<\#>} \omega_e + - \frac{\tilde{q}_{rc<0>}}{C_r \left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} - \frac{\dot{\tilde{q}}_{rc<0>} R_r}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} \\
& + \dot{\tilde{q}}_{rs<0>} \omega_e + \frac{1}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} \tilde{u}_{rc<0>} + \frac{1}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} \tilde{u}_{rc<\#>} \\
& + \frac{-L_{1r} y}{\left(L_{0r} + \frac{y_e}{2} L_{1r}\right)^2} \tilde{u}_{rc<0>}
\end{aligned}
\tag{5—31}
$$

$$
\dot{\tilde{q}}_{ri} = -\omega_e \tilde{q}_{rc<\#>} + \dot{\tilde{q}}_{rs<\#>} - \omega_e \tilde{q}_{rc<0>} + \dot{\tilde{q}}_{rs<0>} \tag{5—32}
$$

$$
\begin{aligned}
\ddot{\tilde{q}}_{ri} = & -\frac{\tilde{q}_{rs<0>} L_{1r} y}{C_r \left(L_{0r} + \frac{y_e}{2} L_{1r}\right)^2} - \frac{L_{1r}\left[\dot{y}\left[L_{0r} + \frac{y_e}{2} L_{1r}\right] - R_r y\right] \dot{\tilde{q}}_{rs<0>}}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]^2} - \frac{\tilde{q}_{rs<\#>}}{C_r \left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} \\
& - \frac{\dot{\tilde{q}}_{rs<\#>} R_r}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} - \dot{\tilde{q}}_{rs<\#>} \omega_e - \frac{\tilde{q}_{rs<0>}}{C_r \left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} - \frac{\dot{\tilde{q}}_{rs<0>} R_r}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} \\
& - \dot{\tilde{q}}_{rs<0>} \omega_e + \frac{1}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} \tilde{u}_{rs<\#>} + \frac{1}{\left[L_{0r} + \frac{y_e}{2} L_{1r}\right]} \tilde{u}_{rs<0>} \\
& + \frac{-L_{1r} y}{\left(L_{0r} + \frac{y_e}{2} L_{1r}\right)^2} \tilde{u}_{rs<0>}
\end{aligned}
\tag{5—33}
$$

5.3 Model Testing

5.3.1 Displacement of the mechanical subsystem:

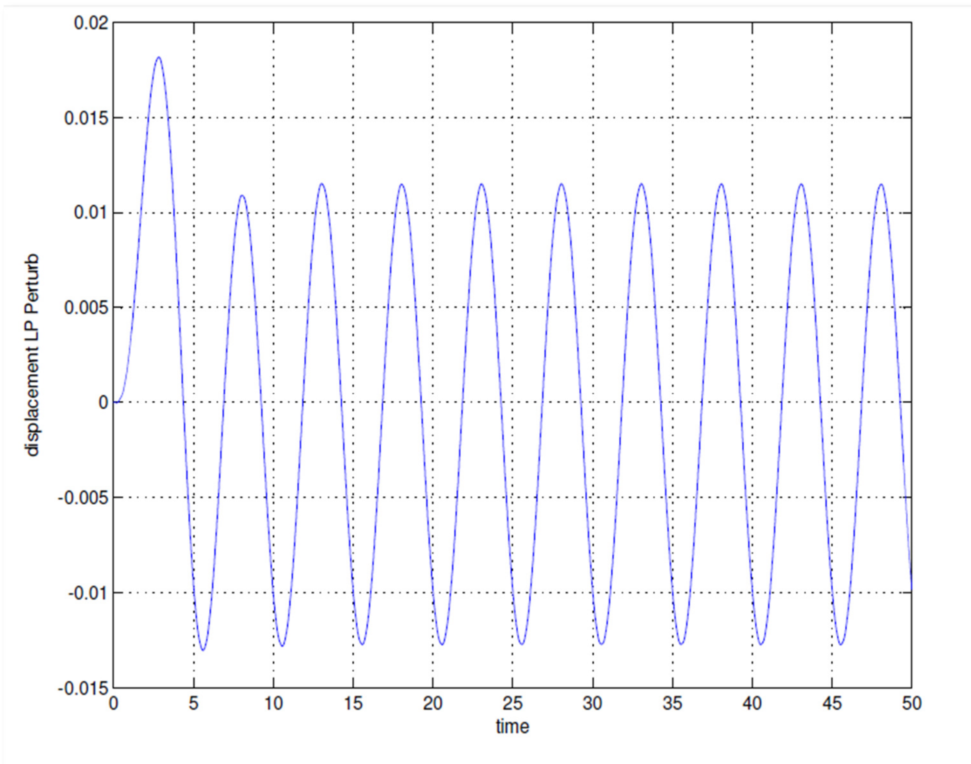


Figure 18- Displacement –LP Perturb

This particular test for the perturbation analysis is carried out with the same conditions as of the other two earlier models. Value of the disturbance is of $0.005 \sin 2\pi/6$ N. The excitation voltage supply of the right magnet is of 10V and the left magnet is supplied with 20V. But the model will be run for other different conditions and will be compared with the LP- equivalent. The current envelope plots are shown in the following pages.

5.3.2 Current Envelope

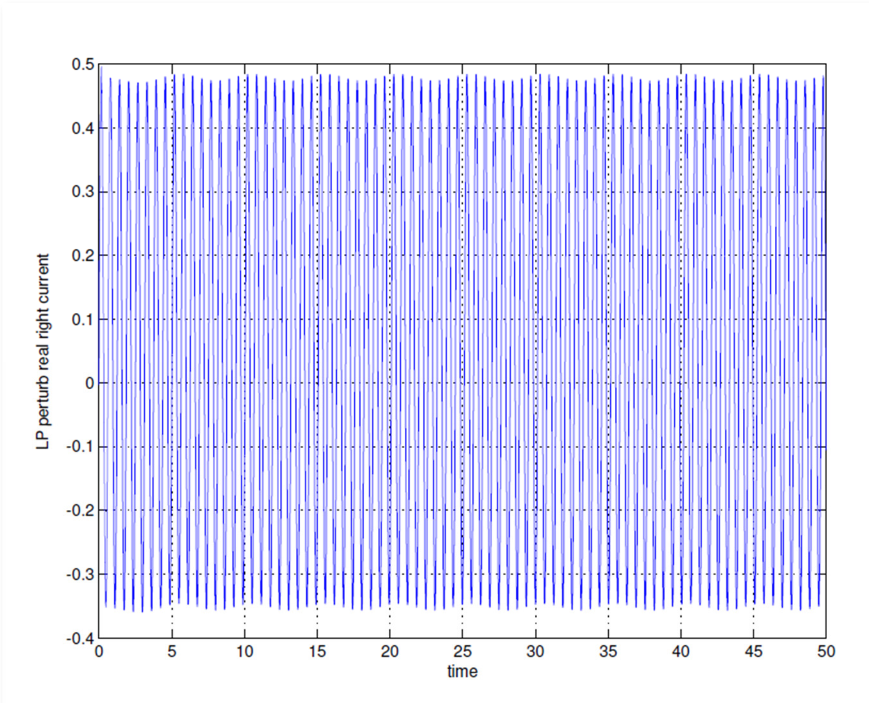


Figure 19- Real Part of right Magnet Current Envelope

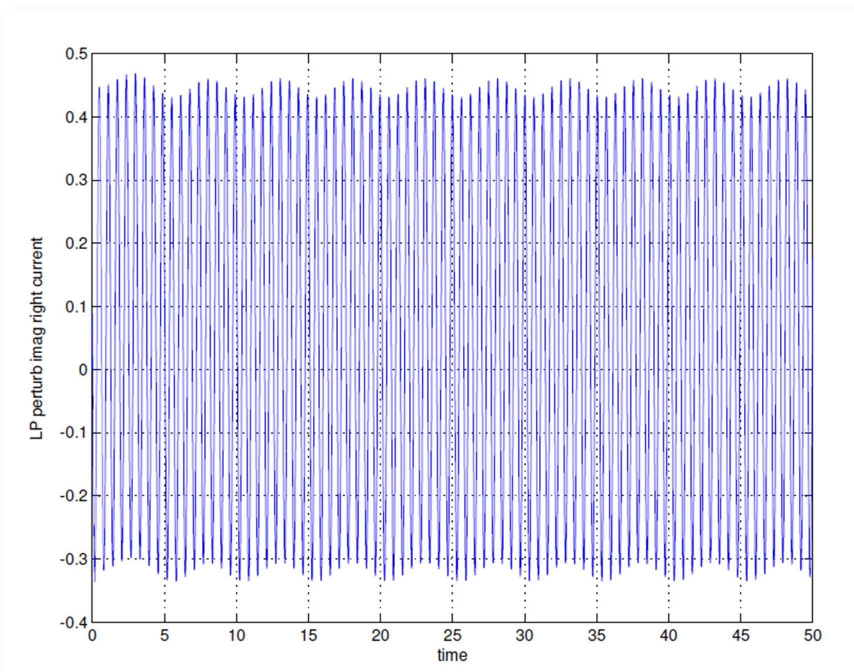


Figure 20- Imaginary part of Right Current Envelope

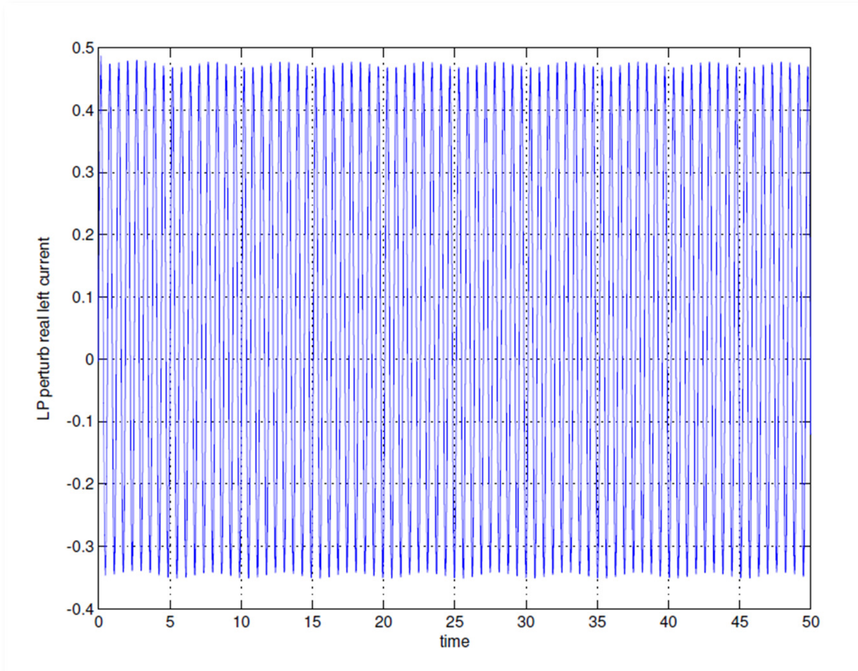


Figure 21- Real part of Left Current Envelope

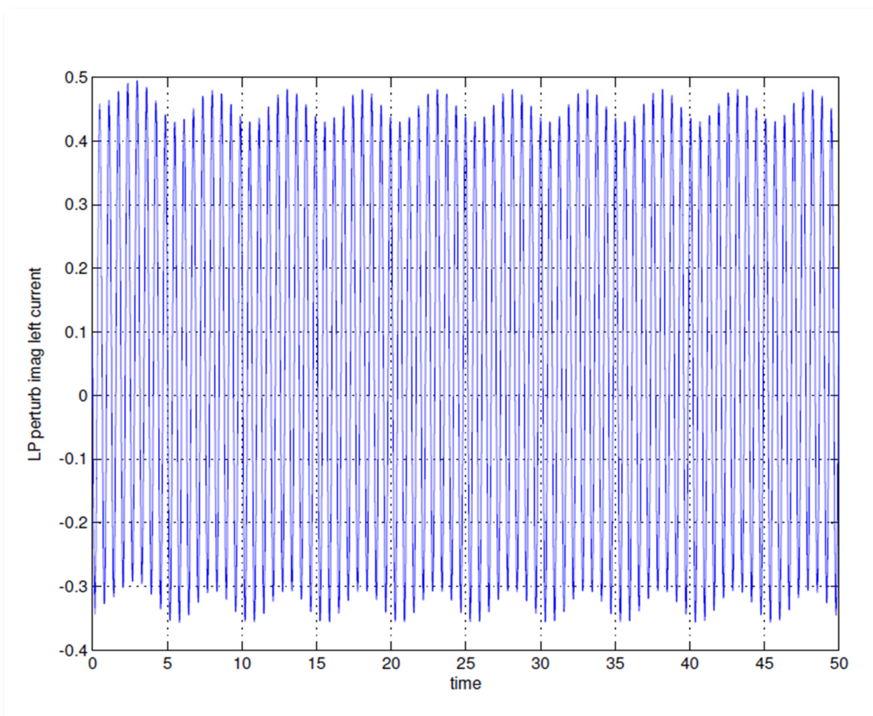


Figure 22- Imaginary part of Left Current Envelope

6 Validation of Low-Pass Equivalent & Low-pass Perturb Models

6.1 Description

The LP- equivalent model and LP-perturbation model are compared in the following sections to show the LP-Equivalent is not by neglecting the related variables and conditions. The above methodology for the derivation of LP- equivalent of the electromechanical systems dynamics proves the point that voltage excitation of the electric circuit needs to be BP if it is around resonance frequency of the circuit then only its complex envelope is needed in order to perform calculations for the output system. If the disturbance is LP then it has some dynamic effect on the system but it will be greatly attenuated due to the LP characteristic of the electromechanical subsystems.

Both models are combined on a single time series with same disturbance and excitation voltages for magnets of both models are maintained the same. Three conditions in the following pages are considered to show the comparison between the systems. The current envelope is only shown in the plots comparison because the displacement and velocity is coupled with the electrical system and comparison becomes self-evident for the other variables.

6.2 Condition 1

The conditions employed for the comparison,

- External disturbance maintained zero for both systems.
- Excitation voltages of both the systems are maintained zero.
- Only the initial conditions for the steady state purpose are employed.

6.2.1 Right Magnet Real Parts

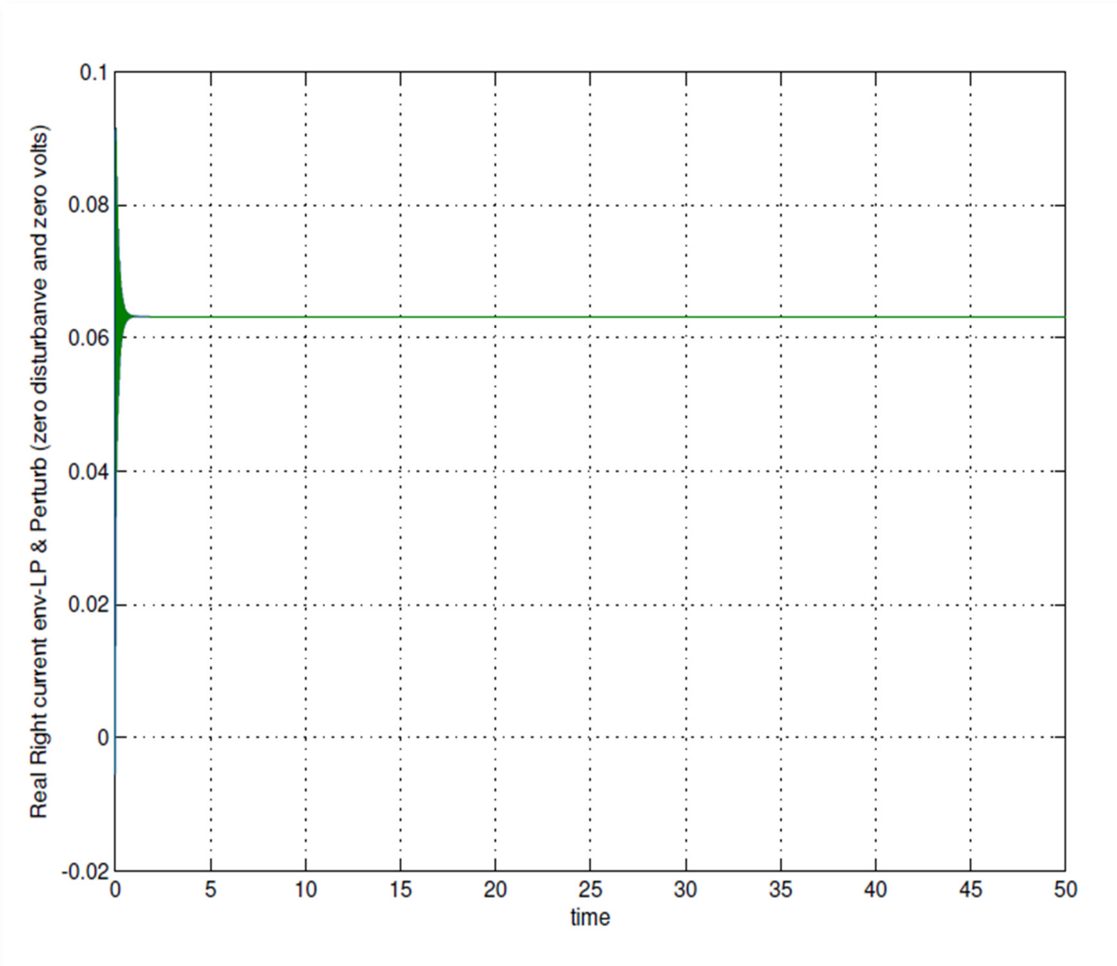


Figure 23-Zero disturbance & Zero excitation voltage

The above plot of the current envelopes of both the systems with the conditions explained shows clearly after initial fluctuation for the system to get stablised and it settles at the initial condition. And it is clearly evident that the LP equivalent when compared with LP perturbation model stabilises at the same value indicating the LP equivalents dynamics follows as such of linearized system. The above and below plots are for the real and imaginary parts of the current envelope of the right magnet of both systems.

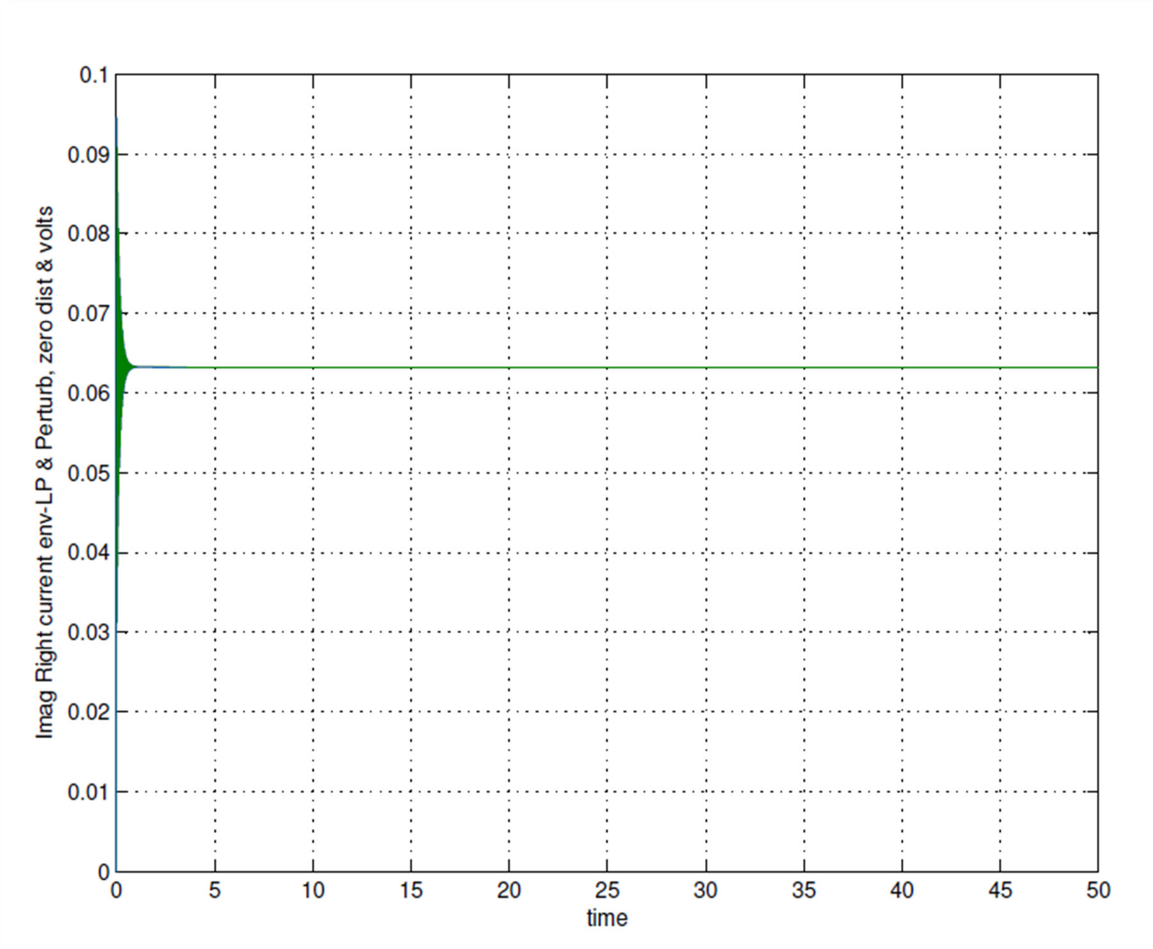


Figure 24-Zero disturbance & Zero excitation voltage

6.3 Condition 2

The conditions employed for the comparison are:

- External disturbance maintained zero for both systems.
- Excitation voltages of both the systems are maintained 10V for both magnets.
- The initial conditions for steady state.

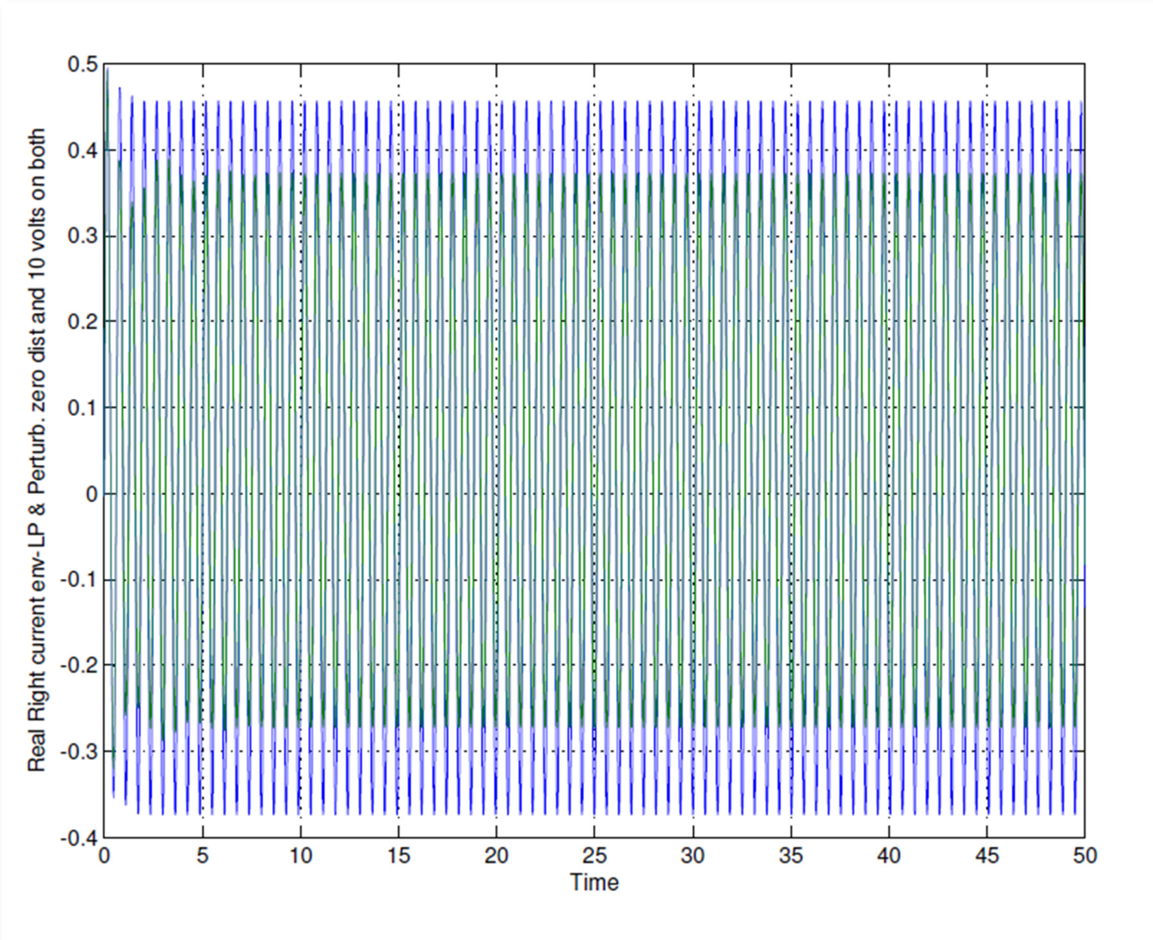


Figure 25- Zero disturbance & 10V for both the magnets

The above plot shows the similarity in the behavior of both systems. With LP equivalent varying slightly less than that of the LP linearized perturbation model. This exists because of the non-linearity of the electromechanical systems. But the phase of the system is well preserved.

6.4 Condition 3

The conditions employed for the comparison,

- External disturbance maintained 0.005N and monochromatic sine-wave for both systems.
- Excitation voltages of both systems are maintained 10V for right magnet and 20V for the left magnet.
- The initial conditions for steady state.

The above condition is to show complete dynamics comparison of the system reflecting the scope. The LP disturbance is greatly attenuated due to the mechanical subsystems LP characteristic. The difference in voltages corresponding to the difference in the frequencies of the magnets stabilizes the systems too. This is very much followed by the LP equivalent system. The linearized LP equivalent by perturbation analysis results match perfectly to that of the LP equivalent system with very or negligible difference.

The dynamics of the subsystems established by the LP equivalent system using the mathematical tools showing that the electromechanical systems information can be established using only the complex envelope of the input signal which carries the information and influenced by low frequency of the mechanical subsystem and superficially by the electrical subsystems high frequency.

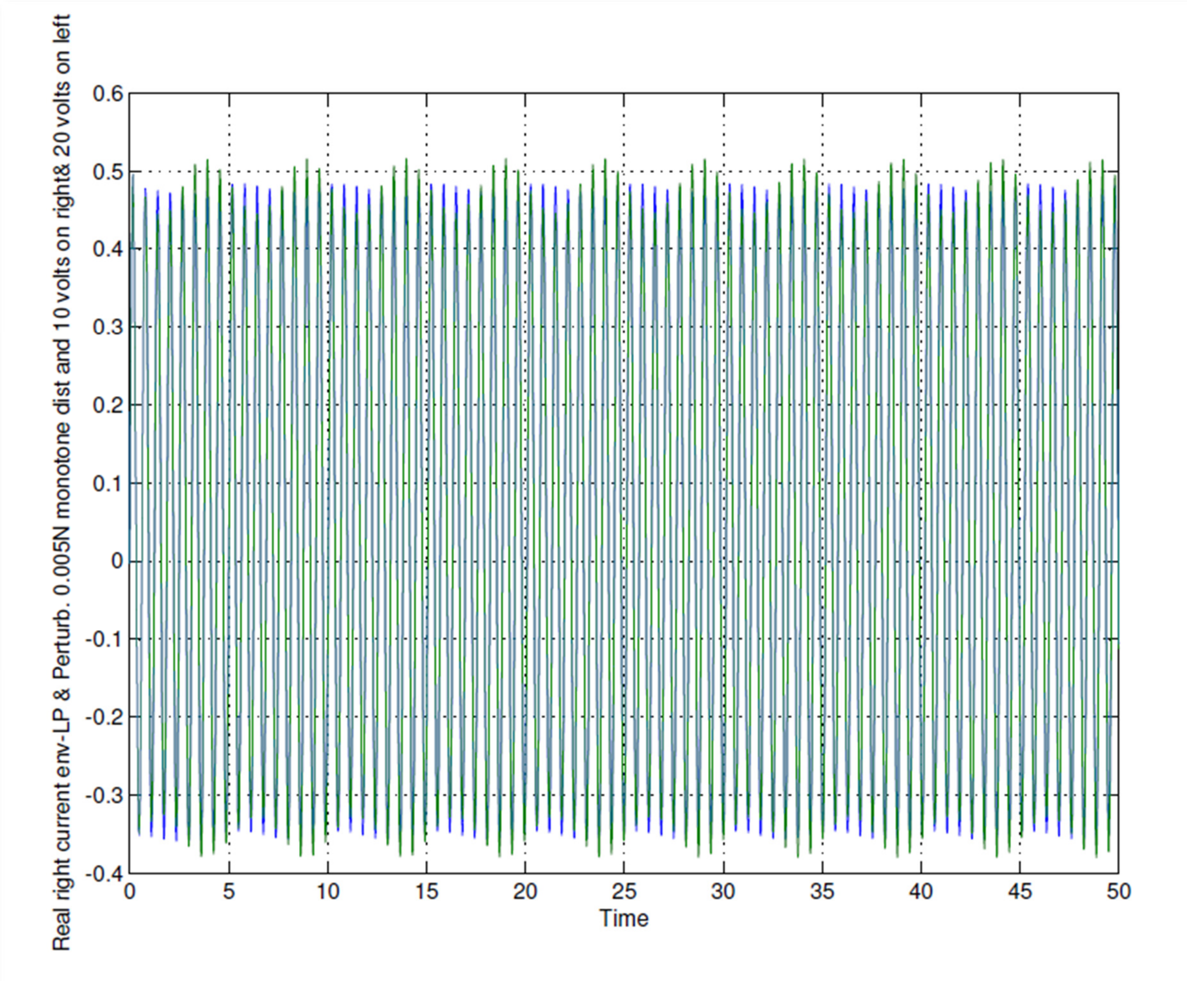


Figure 26- Real right current envelope at 0.005N Monochromatic disturbance and 10V on right magnet & 20V on left magnet.

The above plots and below plots shows the comparison between the real and imaginary parts of the right magnet's current envelope. It is clearly visible that the phase, magnitude of both the models is in agreement with each other. The negligible variations are because of the linearization of the LP- Perturbation model.

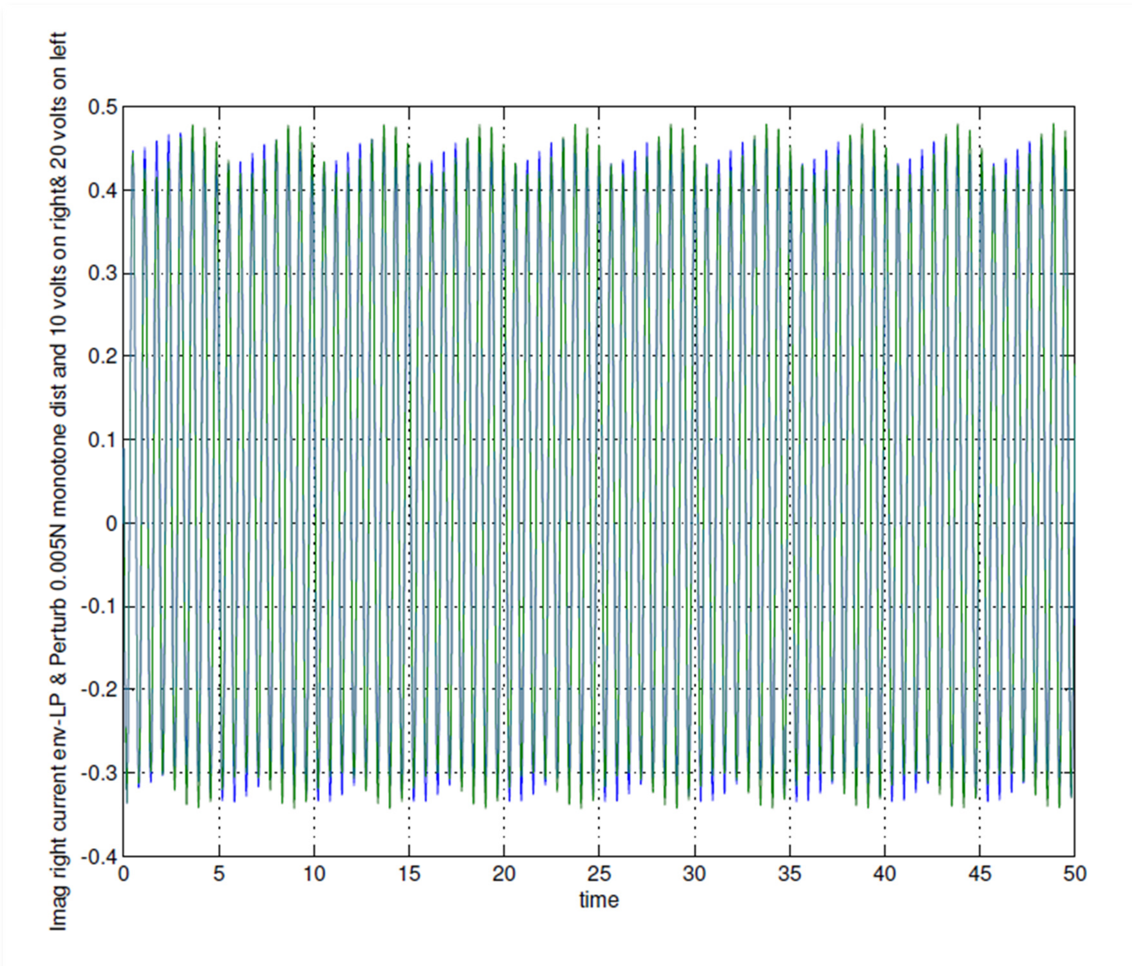


Figure 27- Imaginary right current envelope at 0.005N Monochromatic disturbance and 10V on right magnet & 20V on left magnet.

7 Conclusion

The analysis of the dynamics of an electromechanical system considering the non-linearity arising when the systems are coupled gives great information which could be used for control and analysis of vibrations. The non-linearity arises because of the mutual interaction of the magnetic force on to the mechanical system and the interaction of the mechanical system on to the inductance of the coil.

As seen, the resonant frequency of the electrical signal is much higher than that of the resonant frequency of the mechanical system, so the information voltage signal with very high frequency which controls the displacement of mass other than the exogenous disturbance is modulated. In effect the information signal is close to the resonant frequency of the mechanical system but also has the electrical signal's frequency as the carrier frequency. To sample this signal the sampling rate has to be twice the highest frequency. This increases considerably the simulation time.

Considering this the low-pass equivalent system takes the low-pass part of the band-pass signal, that is the complex envelope, and this information signal is close to the frequency of the mechanical system. Thus sampling rate will be just little higher than (double) the natural frequency of the mechanical system but representing the dynamics of the system perfectly, and reducing the simulation time considerably.

As validation and comparison between the single magnet model and the two magnet model is made the agreement is shown clearly, however the sensitivity is higher for the two magnet model; even slightest disturbance affects the displacement of the mass. But with proper analysis of the current envelope on both the magnets, and the displacement of the magnet can be stabilized faster. The measurement of the complex current envelope directly relates with the displacement profile, with just a factor

involved, the latter could be found by the finding peak values of both and finding the ratio.

It is also a known fact that “A charged particle cannot be held [statically] in a stable equilibrium by electrostatic forces alone.” This calls for a feedback control for the system.

An effective control will be of great use in the various fields where vibration has to be analyzed or controlled. In the future works, the scope of development of MEMS is unlimited for the time being because it’s just in the initial phase.

The validation of the LP equivalent system using the linearized LP-Perturbation model projects the scope of developing a control system effectively to position the mass at desired location by varying the excitation voltage compensating any external disturbance.

The recent application which is under development of an Active Hydromagnetic Journal Bearing [13], this employs the same principle as applying a direct electromagnetic force on arising the shaft and levitating the shaft there by removing the surface contact as in the case of traditional bearings. This analysis will help in the effective control design of the system for appropriate sensing and actuation.

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Appendix A

Mechanical subsystem

```
m = .25;

b = .25;

k = 0.1;

yem = .1;

L1 = .5;

y = u(1);

y_dot = u(2);

i_Right = u(3);

i_Left = u(4);

d = u(5);

Trig = -1;

Dy = y_dot;

if y <= (-yem/2);

    Dy_dot = ( d - (k * y) - (b * y_dot) ) /m;

    %dx22= (e-(x21/C)- (R*x22))/ L0 ;

    if Dy_dot < 0

        Dy_dot = 0;

    end;

    if Dy < 0

        Trig = 1;

    end;
```

```
    elseif y >= (yem/2)

    Dy_dot = ( d - (k * y) - (b * y_dot) ) /m;

    if Dy_dot > 0

        Dy_dot = 0;

    end;

    %dx22= (e-(x21/C)- (R*x22))/ (L0+(L1*yem)) ;

       if Dy > 0

        Trig = 1;

    end;

         else

    Dy_dot = ( (L1/4) * ( (i_Right)^2 - (i_Left)^2 ) + d -
(k * y) - (b * y_dot) ) /m;

    %dx22= (e-(x21/C)- (R+ ( L1*x12))* x22)/ (L0+(L1*x11))
```

Electrical subsystem-LP

```
L0 = .05; L1 = .5;

yem = .1;

R = 1;

omega_em_L = 2000;

C_L=.5e-5;

x = u(1:2);

y = u(3); y_dot = u(4);

e = u(5);

if (y <= -yem/2)

    Xi_L = yem;

elseif (y >= (yem/2))
```

```
    Xi_L = 0;
else
    Xi_L = -y+yem/2;
end;
A = [0, 1; -1/(L0+L1*Xi_L)/C_L, -(R-L1*y_dot)/(L0+L1*Xi_L)];
B = [0; 1/(L0+L1*Xi_L)];
Dx = (A-j*omega_em_L*eye(2))*x+B*e;
```

Electrical Subsystem-BP

```
L0 = .05; L1 = .5;
yem = .1;
R = 1;
omega_em_L = 2000;
C_L=.5e-5;
x = u(1:2);
y = u(3); y_dot = u(4);
e = u(5);
if (y <= -yem/2)
    Xi_L = yem;
elseif (y >= (yem/2))
    Xi_L = 0;
else
    Xi_L = -y+yem/2;
end;
```

```
A = [0, 1; -1/(L0+L1*Xi_L)/C_L, -(R-L1*y_dot)/(L0+L1*Xi_L)];

B = [0; 1/(L0+L1*Xi_L)];

Dx = A*x+B*e;
```

Vita

Shathiyakkumar Ganapathy Annadurai, is a Masters student of Naval Architecture & Marine Engineering, at University of New Orleans. His career started as Marine Engineer. And he worked with different ships which includes Oil Tankers, Pure Car-Carriers and Bulk Carriers. He holds a Bachelor’s degree in Marine Engineering and Certificate of Competency for Marine Engineering Officers. His research interests are signal, data analysis and control systems.