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Distributed Emitter Detector Design under Imperfect Communication Channel

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Distributed Emitter Detector Design under Imperfect Communication Channel

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Dedicated to my family
for their constant support and encouragement
through thick and thin

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You can't connect the dots looking forward; you can only connect them looking backwards. - Steve Jobs

Looking back at the last three years spent at the University of New Orleans I see numerous learning experiences, some of them beyond the realms of academia. I take this opportunity to thank all the people that have played a crucial role in their own right to shape my learning experience.

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Abstract

We consider the distributed detection of an emitter using multiple sensors deployed at deterministic locations. The signal from the emitter follows a signal attenuation model dependent on the distance between the sensor and the emitter. The sensors transmit their decisions to the fusion center through a parallel access Binary Symmetric Channel (BSC) with a cross-over probability. We seek to optimize the detection performance under a prescribed false alarm at the sensor level and at the system level. We consider the triangular topology structure and using the least favorable emitter range study the impact of the BSC on the system level detection fusion rules. The MAJORITY fusion rule is found to be optimal under certain conditions.

Chapter 1

Introduction

With the confluence of revolutionized internet, information and communications technology coupled with advances in electronics engineering, wireless sensor networks have come a long way from traditional sensor-actuator networks with wired communication. The area of detection and decision making with networks of sensors has been researched for over three decades and has well established results with regards to a broad spectrum of applications, both civilian and military. To name a few, sensor networks are useful in detecting topological events such as forest fires, floods and earthquakes. Also, they are applicable in industrial automation, military surveillance, national security and emergency health care. Wireless sensors have small size, low power capacity, limited communication capabilities and processing power. They may measure distance, speed, humidity, temperature and various other parameters.

With respect to the data available for processing, wireless sensor networks are classified as centralized and decentralized detection networks. In centralized networks all the information measured by the sensors are availble for processing at the sink node or central processor. With increased communication and bandwidth requirements distributed decision making has replaced centralized detection in a number of areas. In distributed sensor networks the sensors deployed to monitor a surveillance area are capable of processing the observed data and generating decisions which are then communicated to the fusion center for processing and eventually taking the global decision. The information from sensors can be sent through parallel access channels (PAC) or a multiple access channel (MAC). In a decentralized setting, various interrelated issues have to be dealt with by the system designer.

They include optimal processing of the sensor measurements, optimal mixing of the sensor decisions at the fusion center, communication channel limitations on efficient communication of the local sensor level decisions and the sensor deployment strategies to obtain maximum coverage. A lot of research has been conducted assuming conditional independence of sensors observations. While conditional independence aids in achieving optimal solutions for both the local sensor level as well as the system level fusion rules, in practical scenarios this assumption does not hold true. In detection of a low level point radiation source such as a radioactive emitter the sensor measurements are conditionally dependent. The sensor observations are typically dependent on the distance or range between the sensors and the emitter. This results in a composite hypothesis testing problem. One of the ways to deal with it is to consider least favorable locations of the emitter within the area of monitoring [1]. The other methods for composite detection include the Generalized Likelihood Ratio Test (GLRT). The GLRT, because of its ease of use finds widespread implementations, however optimality is hard to establish. On the other hand, Bayesian approach is a good replacement but is optimal only when the prior probabilities and the prior probability density functions of the system parameters are true. Other approaches to tackle this problem include estimating the range between the sensors and the emitter (target) under the assumption that the emitter is present with probability 1. In this case, the detection performance critically depends on the estimation of the emitter location which might not be feasible under low signal to noise ratio.

Furthermore, the communication from the sensors to the fusion center can be corrupted by channel imperfections. Significant portion of research has been devoted towards distributed detection considering Rayleigh fading, Rician fading and Binary Symmetric or Erasure Channels. In addition, the channel aware methods have been explored with sub-optimal fusion strategies at the local sensors [2]. One of the possible channel imperfection is a Binary Symmetric Channel in which the transmitted bit gets flipped with a probability that is referred to as the channel cross-over probability. In some applications not all the links connecting the sensors to the fusion center are binary symmetric for which selective exclusion of sensor nodes help achieve close to desired performance. Detection performance limitations posed by channel imperfection is an important communication aspect that cannot be ignored in sensor network design.

Chapter 2

Preliminaries and Motivation

2.1 Research Background

2.1.1 Decision Fusion in a Wireless Sensor Network

A distributed wireless sensor network is a collection of intelligent sensors capable of obtaining observations from the environment, processing relevant information and taking decisions either by sending them to a fusion center or in collaboration [3]. The observations have statistical distribution based on the phenomenon that could be described by M-ary hypotheses. The problem of single stationary emitter detection in a region of interest is a binary classification problem. The observations received at the sensors could be assumed to be conditionally independent. One of the advantages of this assumption is the formulation of the sensor decision rules as threshold type tests based solely on the likelihood ratio of the senor observations. However, in practical scenarios the sensor observations are correlated. This arises due to a random signal in noise or if the noise in the observations are correlated when detecting a deterministic signal.

A wireless sensor network could centralized or decentralized. In a centralized detection system the individual sensors observe a global phenomenon characterized by hypotheses H_0 and H_1 and send the received measurements to the processing center that processes the data and reaches a global decision. The biggest advantage of the centralized system over the distributed system is that the central processing center has complete access to the measurements received by the sensors. In other words, the information profile at the processing center is more complete. Centralized detection network is also referred to as *Measurement Fusion* or *Data Fusion* for emitter detection. On the other hand, in decentralized or distributed setting each node has intelligence to process the measurements based on optimal sensor level rules and take local decisions.

The local decisions are sent to the fusion center where the information from the sensors are processed based on an optimal fusion rule and a global decision is taken thereafter. The centralized system is more restrictive and can be considered inadvisable in many practical scenarios. In those cases where it is required to completely cover the region of interest and not all the sensor decision regions overlap, the signal is not received by all the sensors. As a result, the sensor measurements fail to transmit accurate messages to the processing center. In such cases, it is required that the local sensors process the observations received and take decisions based on local decision rules. Some of the advantages include reduced bandwidth requirement, reduced cost and increased reliability. Unlike the central processing center, the fusion center has only partial information. This results in a degradation of the system level performance. The performance loss can be reduced by implementing optimal tests at the sensors. The challenges faced in designing a distributed detection system are sensor deployment(choice of topology) ability to readjust in case of sensor/link failures, the communication channel imperfection between sensors and the fusion center.

2.1.2 Conditional Independence of Sensor Observations

The observations at the sensors are said to be conditionally independent when the joint density of the observations given the hypothesis can be expressed as the product of the marginal densities. If $p(x_{ij} \mid H_l)$, for $i = 1, \ldots, N$, $j = 1, \ldots, M$ and $l = 0, 1$ are the marginal densities of the observations with respect to hypothesis H_1 or H_0 then conditional assumption leads to the following formulation for the joint density:

$$p(x_{i1}, \ldots, x_{iM} \mid H_l) = p(\boldsymbol{x_i}^M \mid H_l) = \prod_{j=1}^{M} p(x_{ij} \mid H_l) \tag{2.1}$$

where $i = 1, \ldots, N$ are the number of sensors, $j = 1, \ldots, M$ and $l = 0, 1$ are the binary hypothesis index values. Because of the conditional independence assumption the decision rules at the sensors and the fusion center are threshold rules based on appropriate likelihood ratios under the Neyman-Pearson (NP) criterion. With the conditional independence assumption the test statistic $T(\boldsymbol{X})$ for each sensor is a likelihood ratio test (LRT). For $i = 1, \ldots, N$,

$$T(\boldsymbol{x}_i^M) = \frac{P(\boldsymbol{x}_i^M \mid H_1)}{P(\boldsymbol{x}_i^M \mid H_0)} = \prod_{j=1}^{M} \frac{P(x_{ij} \mid H_1)}{P(x_{ij} \mid H_0)} \tag{2.2}$$

2.1.3 Parallel Configuration of Sensors

It is assumed that there is no mutual communication between the sensors and there is no feedback from the fusion center to the sensors. The sensors $\{s_i\}_{i=1}^N$ employ decision rules $\{\delta_i\}_{i=1}^N$ and send the decision bits $\{I_i\}_{i=1}^N$ to the fusion center. Based on the received information the fusion center employs a decision rule δ_F that favors either H_1 or H_0. In the serial configuration the $(i-1)$th sensor sends its decision to the ith sensor that generates its own decision based on the observation it receives from the surrounding and the decision received from the preceding sensor. Only the first sensor in the network uses its own observations to generate its own local decision. Usually the last sensor in the network has the responsibility of classifying the observations corresponding to the two possible hypotheses(for binary hypothesis problem). Unlike the serial configuration, in the parallel configuration the sensors transmit the decisions at the same time to the fusion center. The transmitted decision bits $\{I_1, \ldots, I_N\}$ are mixed at the fusion center based on an optimal strategy that satisfies the false alarm constraint and maximizes the detection probability (Neyman-Pearson criterion)
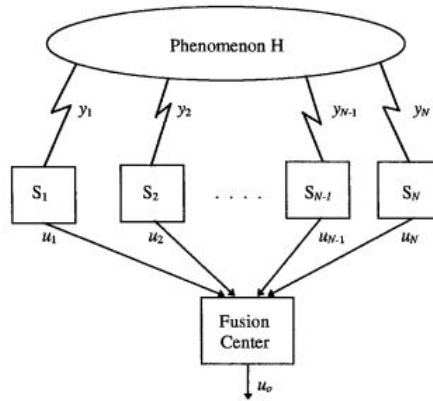
Figure 2.1: Parallel Configuration of sensors with a Fusion Center.

In addition, the parallel configuration of the sensor network can be parallel access or multiple access. In parallel access communication each sensor has its own communication channel with the fusion center. The sensors send the decision bits at the same time but through dedicated channels. As against this, in multiple access communication all sensors transmit the decision bits through a single channel linking to the fusion center.

2.1.4 Neyman-Pearson Lemma

We assume that the observations at the individual sensors either correspond to the hypothesis H_1 or H_0. The Neyman-Pearson formulation can be stated as finding optimum local $(\{\delta_i\}_{i=1}^N)$ and global (δ_F) decision rules that maximize the global probability of detection P_D^F or minimize the global probability of miss P_M^F subject to a prescribed bound on the global probability of false alarm P_{FA}^F. Consider the likelihood ratio test

$$T\left(\boldsymbol{x}_i^M\right) = \frac{p(\boldsymbol{x}_i^M \mid H_1)}{p(\boldsymbol{x}_i^M \mid H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \tau_i$$

where τ_i is chosen so that $P\left(T\left(\boldsymbol{x}_i^M\right) > \tau_i \mid H_0\right) = \alpha_i$. α_i is a fixed false alarm value for s_i. The Likelihood Ratio Test is uniformly most powerful with the probability of false alarm less than or equal to α_i. In other words, it gives the maximum detection probability among all possible tests with false alarm less than or equal to α_i.

There are variations of the NP formulation that include optimizing the fusion rule for a given set of local decision rules or optimizing the local decision rules for a fixed fusion rule. Also, the solution to the problem depends on whether the sensor observations are conditionally independent or conditionally dependent.

One of the limitations of the Neyman-Pearson Test is that it is not uniformly most powerful (UMP) for composite hypothesis testing. A composite hypothesis is a hypothesis that is not Simple. And a Simple hypothesis is one that uniquely specifies the distribution of the population from which the observations (samples) are taken. If the alternative hypothesis depends on a quantity that itself is unknown or varying then a UMP test cannot be established using the Neyman-Pearson lemma. However, there are methods that can be implemented to convert the composite hypothesis into a simple hypothesis.

2.1.5 Sensor Deployment in a Region of Interest

The placement or deployment of the sensor nodes in the area of surveillance is a major contributing factor determining the coverage, connectivity and the lifetime of the wireless sensor network (WSN). In many situations the sensors have to be strategically placed because the sensors are not capable of distant monitoring due to limited energy capacities.

Because of limited sensing range the sensors have to be placed at a suitable distance from the probable location of the target. The deployment of the sensors could be random or deterministic depending on the accessibility of the region and the application. Although random deployment finds widespread use in the theoretical analysis of coverage, connectivity and evaluation of various algorithms, it is comparatively more expensive than deterministic deployment. Also existing research corroborates that the node density required in placing the sensors strategically at fixed locations is lower than deploying them randomly using aerial vehicle.

Having said that, its important to consider such applications where the region of interest or the surveillance area under consideration is inaccessible due to harsh climate, rough terrain or high adversarial monitoring.

The optimization criteria and the role of the individual nodes in the network influence the deployment strategies. Whenever the application requires massive number of sensors in potential target areas and the cost of the sensors deployed are of lesser significance, non-deterministic strategies are more practical for use. However, for small scale coverage and monitoring the deterministic strategies outperform the non-deterministic random counterparts.

Figure 2.2 depicts different sensor deployment strategies. Possible classification are square, triangular, hexagonal or random coverage.
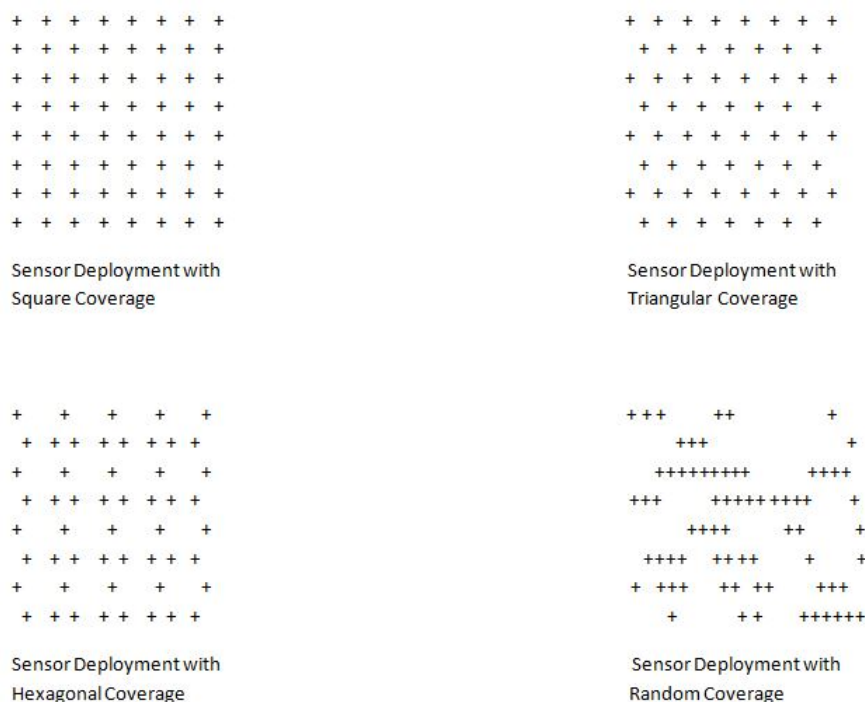
Figure 2.2: Various Sensor Deployment Strategies

2.1.6 Communication between the Sensors and the Fusion Center

In distributed detection systems the optimal information processing at the local sensors is of prime importance in order to optimize the system level performance. However, one is confronted with the transmission channel constraints while communicating the decision to the fusion center. The conventional approach to distributed detection ignores the unreliable nature of the transmission channels and hence considerable focus is on the first source of uncertainty, i.e. the sensor measurements. However, in practical settings the errors induced in communication due to the transmission channel imperfection cannot be ignored. As such, a prevalent approach to accommodate the channel imperfection involves two-stage solution: First, a communication block at the fusion center recovers the transmitted bits from the sensors. Second, a signal processing block combines the transmitted bits (decisions) according to a fusion rule to take the final decision.
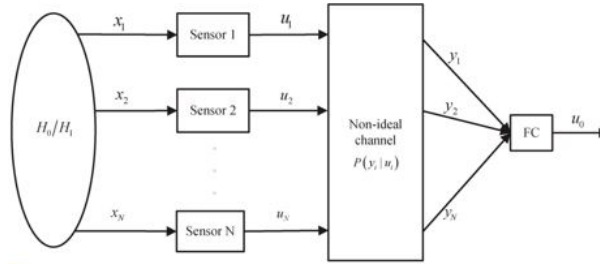
Figure 2.3: Parallel Configuration Distributed Detection System with Non-Ideal Channel.

The imperfection in the transmission channel between the sensors and the fusion center could be classified as: Shadowing, Fading, Interference between the observation and transmission channel, Binary Erasure and Binary Symmetric Channel.

Fading channels could be further categorized as Rayleigh fading or Rician fading, depending on the distribution of the amplitude gain of the signal transmitted. Rayleigh fading is characterized by the absence of a line of sight signal and is a special case of the more general concept of Rician fading in which a line of sight is much stronger than others.

A binary erasure channel is one which a transmitted bit can get scrambled with a certain probability so that the fusion center has no idea what the transmitted bit was. On the other hand, in a Binary Symmetric Channel a bit transmitted has a flipping probability with which it flips into a 0 or 1.

2.2   Literature Review

Significant amount of research has been dedicated towards the conventional distributed detection (decision fusion) under the conditional independence assumption [4], [5], [6], [7], [8]. In practical settings the sensor measurements are correlated to one another because of correlated measurement noise. Also, in order achieve satisfactory coverage the sensor deployment in a region of interest may be spatially dense. As a result, multiple sensors record information about a single event which in turn results in the correlation of observations. The degree of correlation increases and the inter-node separation between the sensors decreases. Correlated observations require different strategies for distributed detection of a target [9], [10], [11], [12]. With the sensor observations correlated, finding optimal local sensor rule and the system fusion rule becomes a sophisticated problem even for detecting a mean shift under dependent Gaussian noise [9].

Unlike centralized detection where the sensors communicate data to a central processing unit [13], in distributed detection the sensor measurements need to be mapped to decisions according to sensor decision rules. The decision bits are transmitted to a fusion center that takes the final decision based on an optimal fusion rule. The optimization of sensor level rules can be done by maximizing the detection performance under a prescribed false alarm constraint which is known as the Neyman-Pearson criterion [14], [15], [16]. Also, optimal rules at the sensor ad the system level can be found by minimizing a Risk associated with error in decision making (Bayesian criterion) [17]. In [18] the authors talk about randomized mixing of the sensor performance models at the fusion center using Dynamic Programming under the assumption that the fusion center has full knowledge of the performance of the individual sensors. Furthermore, with the assumption of similar performance model of the individual sensors the problem can be simplified. [19] talks about implementing dynamic sensor thresholds to achieve improved performance. The intuition behind employing different thresholds is that the distance between the sensors and the emitter is not known which results in unequal signal -to-noise ratios (SNRs) for different sensors.

The requirement of improved performance calls upon more than satisfactory coverage of the area of surveillance. This in turn depends on the deployment of the sensors. The sensors can be deployed at deterministic locations or can be randomly deployed from the sky using an aerial vehicle or robot [20].

Random deployment is most suitable for areas that are difficult to access on account of adverse climate, hostile regions etc.. However, it has been shown in a number of works that the deterministic deployment leads to achievement of better coverage than mere random deployment. In [21] authors have shown results that override a previous conclusion that deploying sensors at fixed locations in a grid fashion requires higher node density than random node distribution. In the study the authors explore the problem of determining the critical node density required to maintain k-coverage in a square region of interest. The two random strategies (Poisson Point Process and Uniform Distribution) have identical density requirement for k-coverage and the deterministic grid deployment requires lesser node density for the same problem. [22] confirms the fact that the choice of sensor placement strategy has a big role to play in the network performance. [23] states that a factor of $log(n)$ additional sensors are required in random deployment as compared to optimal deterministic deployment if the number of sensors required in random deployment is n. The authors explore the effects of placement errors and random failures on the density needed to achieve full coverage when sensors are deployed randomly versus deterministically.

A separate but pertinent issue in wireless sensor networks is the communication of data from the sensors to the fusion center. The transmission path through which the sensors communicate their decisions cannot be guaranteed to be perfect. Various imperfections include fading such as Rayleigh fading, Rician fading, erasure of decisions in the transmission path (Erasure Channel) or Binary Symmetric Channel having a certain bit-flipping probability. [24], [2] incorporated the Rayleigh fading channel layer in the parallel structure of the distributed detection system. Considering fixed locations for the local decision devices [2] computed likelihood ratio based rule as the decision fusion rule. The optimal fusion rule, however requires perfect knowledge of the local decision performance indices as well as the fading channel and claims that the channel aware fusion strategies are more energy efficient than considering the two stage approach of fusion and communication separately. In a distributed setting the sensors deployed in the region of interest are usually low-end devices with limited communication capacities. As such, communicating the instantaneous channel information is a task beyond the communication capability of the sensors. [25] considers the knowledge of Rician fading channel statistics instead of instantaneous channel information in deriving a likelihood ratio (LR) based optimal fusion rule. The channel statistics based fusion rule approaches the optimal LR based fusion rule under high signal-to-noise ratio.

2.3 Motivation behind the thesis

The motivation behind this thesis is the problem of distributed emitter detection with deterministic sensor deployment under a multiple access binary symmetric channel. Distributed detection is a widely studied and well researched area with a considerable amount of work towards emitter detection. However, channel imperfection arising out of the presence of a binary symmetric channel is an important issue considered in this work. [26] explores the emitter detection system design with sensor observations dependent on the range between the fixed sensor locations and the least favorable emitter location in a region of interest. Among all the decision fusion rules characterized as the random mixture of decision trees the optimal fusion policy turned out to be the OR rule with each sensor applying identical decision rule to generate the decision bit. However, the presence of an imperfect channel linking the sensors and the fusion center influences the optimal system level decision rule.

The sensors deployed in the region of interest at deterministic locations generate local decisions using the least favorable sensor-emitter distance using identical likelihood ratio tests. The performance model of each sensor and the channel cross-over probability is known to the fusion center (FC). The sensors send information to the fusion center about the presence or absence of the emitter. The decision bit transmitted to the FC has a probability p of getting flipped. This implies that if a sensor sends decision corresponding to a false declaration of emitter presence and if the decision gets flipped, then the resulting error is in favor of the detector system. Similarly,if the sensor sends a decision corresponding to the false declaration of emitter absence and if the decision gets flipped, then the resulting error is in favor of the detector system. As a result of this influence of the binary symmetric channel under certain conditions the optimal fusion rule is no longer the OR rule.

Chapter 3

Problem Formulation

Consider a region of interest (ROI) having an area S_e that is suspected to have a low-level point radiation source. We have a distributed emitter detection system having N sensors located at fixed locations $\{l_i\}_{i=1}^N$. Denote by l_e the actual emitter location within the region of interest with area S_e. Under the distributed detection setting, the sensors $\{s_i\}_{i=1}^N$ deployed uniformly at fixed locations $\{l_i\}_{i=1}^N$, take respective localized decisions I_i employing localized decision rules δ_i. The local decisions are transmitted to the fusion center (FC) that takes the global decision regarding the presence or absence of the emitter by employing a global decision rule δ_F. The observations at the local sensors depend on the signal strength from the emitter which in turn depends on the range between the sensors and the emitter. As a result, the conditional independence assumption on the observations at the sensors is inapplicable in this scenario.

Furthermore, the local decisions from the individual sensors are sent to the fusion center through imperfect transmission channel. The transmission channel is considered to be a parallel multiple access binary symmetric channel having a cross over probability p.

This chapter considers the formulation of the problem of obtaining optimal local decision rules and the corresponding fusion rule in the presence of channel cross over probability.

3.1   Statistical Distribution of the Sensor Measurements

Under the emitter absent hypothesis, i.e. the hypothesis H_0, each sensor has an observation model given by:

$$X \sim f_0(X) \tag{3.1.1}$$

Typically f_0 does not depend on the sensor index i or its location l_i. Under the signal present hypothesis, i.. hypothesis H_1, the observation model of senor s_i is given by:

$$X_i \sim f_1\left(X \mid \xi\left(d_i\right)\right), \qquad i = 1, .., N \tag{3.1.2}$$

In the above notation for the statistical distribution of the sensor observations $d_i = \|l_i - l_e\|$ is the range between the sensors and the emitter and $\xi\left(\cdot\right)$ is a known function depending on the signal propagation model of the emitter.

We denote by $\boldsymbol{x}_i^M$ the M independent observations $\{x_{i_1}, ..., x_{i_M}\}$ made by sensor s_i. Denote by $\boldsymbol{X}^M$ the collection of observations $\{\boldsymbol{x_1}^M, ..\boldsymbol{x_N}^M\}$ from all the N sensors.

In centralized detection setting, the sensors collect observations and transmit them to the fusion center that can utilize the measurements from all sensors by employing a decision rule δ that computes a test statistic $T\left(\boldsymbol{X}^M\right)$ and compares it with a threshold to decide whether an emitter is present.

However, in the distributed setting each local sensor s_i utilizes a local decision rule δ_i that computes a test statistic $T_i\left(\boldsymbol{x_i}^M\right)$ and compares it with a threshold to decide whether an emitter is present. Sensor s_i then sends its local decision I_i to the fusion center that applies a fusion rule $\delta\left(I_1, .., I_N\right)$ using all the local decisions of the sensors to decide upon the presence of an emitter within the region of interest. In this work, we have considered the transmission of the individual decisions from the sensors to the fusion center via communication channel having a cross over probability p. It means that a bit $(say1)$ has a probability p of getting flipped (0) during its transmission to the fusion center.

The goal is to optimize the sensor locations and the decision rules under the distributed setting in the presence of an imperfect transmission channel to satisfy the desired false alarm and detection probability constraints for any possible emitter locations in the area covered by the region of interest with the minimal number of observations M.

3.2 Decision Fusion with Least Favorable Range between the Sensor and the Emitter

We consider sensor s_i to have a false alarm $p_{FAi} = \alpha_{(\delta)_i}$ under a certain decision rule (δ_i). Let f_e be the distribution of the emitter location.

Also, let $p_{Di} = \beta_{(\delta)_i}(f_e)$ be the detection probability of sensor s_i under decision rule δ and the emitter location distribution f_e. We call $f_e^\star$ the least favorable distribution of the emitter for a given decision rule δ when $f_e^\star$ satisfies the following:

$$\beta_{(\delta)}(f^\star) \leq \beta_{(\delta)}(f) \tag{3.2.1}$$

for any proper distribution f_e. Note that $f_e^\star$ may not be unique.

We assume that the hypotheses H_0 and H_1 become harder to distinguish as the distance between the emitter and the sensor increases. We assume that the relative entropy $D(f_1(d_i)//f_0)$ is strictly decreasing in d_i where

$$D(f_1//f_0) = \int_X f_1(x) \log \frac{f_1(x)}{f_0(x)} dx \tag{3.2.2}$$

for continuous density functions f_1 and f_0.

The decision rule δ_i for the sensor s_i to declare the presence of emitter ($H_1 hypothesis$) is

$$\delta_i\left(\boldsymbol{x_i^M}\right) = \mathbb{1}\left(T_i\left(\boldsymbol{x_i^M}\right) > \tau_i\right) \tag{3.2.3}$$

where

$$T_i\left(\boldsymbol{x_i^M}\right) = \frac{1}{M} \sum_{j=1}^{M} \log\left(\frac{f_1(x_{ij} \mid \xi(d_i^\star))}{f_0(x_{ij})}\right) \tag{3.2.4}$$

is the test statistic which is the log-likelihood ratio of the observation under hypotheses H_1 and H_0. τ_i is some threshold to achieve the desirable false alarm probability $p_{FAi} = \alpha_i$. If $d_i^\star$ is the least favorable range between the emitter and the sensor such that $S_e^\star = \{l_e : \|l_e - l_i\| = d_i^\star\}$, then the coverage area of each senor s_i will be $S_{ei} = \{l_e : \|l_e - l_i\| \leq d_i^\star\}$. Each sensor achieves the detection probability at least $\beta_i(d_i^\star)$ under the false alarm constraint no greater than α_i for an emitter located in the region $S_{ei} = \{l_e : \|l_e - l_i\| \leq d_i^\star\}$. We call $d_i^\star$ the coverage radius of sensor s_i. with false alarm probability α_i. and detection probability β_i under the least favorable emitter range from the sensor. Using N sensors to cover the region of interest having area $_e$ with each sensor having an area S_{ei} for $i = 1, .., N$, we want to jointly optimize the sensor deployment $\{l_i\}|i = 1^N$ and the local decision rule $\{\delta_i\}_{i=1}^{N}$ as well as the decision fusion rule δ_F so that each sensor s_i satisfy the sensor coverage constraint, i.e. the area under the region of interest is the proper subset of the union of the coverage areas of the individual sensors.

In other words, the sensors should completely cover the region of interest.

$$S_e \quad \subseteq \quad \bigcup_{i=1}^{N} S_{ei} \qquad (3.2.5)$$

It should be noted that if the emitter is within the coverage area S_{ei} of the sensor, then the sensor detection probability $\beta_i(d_i)$ under a false alarm constraint α_i is at least the detection probability $\beta_i(d_i^\star)$ (considering the least favorable range between the sensor and the emitter). The decision rule δ_i is optimal under the Neyman-Pearson criterion when the range between the sensor and the emitter is $d_i^\star$. It is also optimal under Bayesian framework with suitable choice of the threshold τ_i.

3.3 Transmission of Local Decisions in the presence of Channel Crossover Probability

In practical scenarios the transmission channel linking the sensors and the fusion center are never ideal. Channel imperfection is an important aspect in the design of signal detection systems that cannot be ignored. Here we consider a Binary Symmetric Channel with a cross-over or bit flipping probability p. The decision in favor of emitter present H_1 hypothesis is denoted as $I_i = 1$ and the decision in favor of emitter absent H_0 i denoted as $I_i = 0$. During transmission through a binary symmetric channel (BSC), there is a probability p with which the decision received by the fusion center I_i^{rec} is equal to the flipped decision $1 - I_i$. The probability of receiving a $'1'$ when H_0 is the true hypothesis is different as seen from the fusion center than as seen by the sensor. If $Pr\{I_i = 1 \mid H_0\} = p_{FAi}(\delta_i) = \alpha_{\delta_i}$ is the false alarm at the sensor level, then the false alarm as seen from the fusion center must take into consideration the bit flipping probability p, because the decision sent to the fusion center might be erroneous.

The false alarm corresponding to sensor s_i as seen from the fusion center is as follows:

$$p_{FAi}(\delta_i) = Pr\{I_i^{rec} = 1 \mid H_0\} = Pr\{I_i^{rec} = 1 \mid I_i = 1\}Pr\{I_i = 1 \mid H_0\} \quad +$$
$$Pr\{I_i^{rec} = 1 \mid I_i = 0\}Pr\{I_i = 0 \mid H_0\}$$

The probabilities $Pr\{I_i^{rec} = 1 \mid I_i = 1\}$ and $Pr\{I_i^{rec} = 1 \mid I_i = 0\}$ represent the probabilities of correct and incorrect transmission and is equal to $1 - p$ and p respectively.

So the expression for the false alarm corresponding to sensor s_i as seen from the fusion center is:

$$p_{FAi}^{F}(\delta_i) = (1 - p)\, p_{FAi}(\delta_i) + p\,(1 - p_{FAi}(\delta_i)) \tag{3.3.1}$$

where $Pr\{I_i = 1 \mid H_0\} = p_{FAi}(\delta_i)$ is the false alarm probability of the sensor s_i.

Similarly, the detection probability corresponding to the sensor s_i as seen from the sensor is

$$p_{Di}^{F}(\delta_i) = (1 - p)\, p_{Di}(\delta_i) + p\,(1 - p_{Di}(\delta_i)) \tag{3.3.2}$$

Note that the above two expressions (3.3.1) and (3.3.2) are the false alarm and the detection probability including the bit flipping probability p.

So the sensor level false alarm and detection probabilities can be expressed as a function of the false alarm and detection probabilities as seen from the fusion center. They are as shown below:

$$
\begin{aligned}
p_{FAi}(\delta_i) &= \frac{p_{FAi}^{F}(\delta_i) - p}{1 - 2p} \\
p_{Di}(\delta_i) &= \frac{p_{Di}^{F}(\delta_i) - p}{1 - 2p}
\end{aligned}
\tag{3.3.4}
$$

So we can see from the above equation that the channel cross over probability p can not exceed the false alarm and the detection probability as seen from the fusion center, i.e. $p_{FA_i}^{F}(\delta_i)$ and $p_{D_i}^{F}(\delta_i)$ respectively for $p < 0.5$ and $p_{FA_i}^{F}(\delta_i)$ and $p_{D_i}^{F}(\delta_i)$ greater than p. Because $p_{FA_i}(\delta_i)$ and $p_{D_i}(\delta_i)$ are probabilities and cannot have negative values.

3.4   Problem Statement

With the assumption that the fusion center has knowledge of the performance model of each sensor s_i i.e., $\left(\alpha_i^{F}, \beta_i^{F}\right)$, the least favorable distance of the emitter from each sensor $d_i^{\star}$ and the channel cross over probability of the binary symmetric channel, p, the optimal fusion rule is in general a random mixture of the local decisions (decision rules to be specific). The optimal fusion policy is a solution to the following constrained optimization problem.

$$
\begin{aligned}
\text{maximize} \quad & P_D^{F}\left(\delta_F \mid \delta_1, \delta_2, \delta_3, \ldots, \delta_N\right) \\
\text{subject to} \quad & P_{FA}^{F}\left(\delta_F \mid \delta_1, \delta_2, \delta_3, \ldots, \delta_N\right) \le \alpha_F
\end{aligned}
$$

The resulting optimal fusion rule is a randomized mixture of decision trees. Subsequently, we optimize the sensor-level decision rule and the sensor deployment (that satisfies the coverage constraint of $Eq.3.2.5$) using the likelihood ratio test as the desirable performance model.

$$\text{maximize} \quad P_{D_i}\left(\delta_i \mid d_i^\star\right)$$

$$\text{subject to} \quad P_{FA_i}\left(\delta_i^\star\right) \leq \alpha_i$$

This in turn amounts to finding the optimal threshold $\tau_i^\star$ such that $P_{FA_i}\left(\tau_i^\star\right) = \alpha_i$. By employing identical performance model for the individual sensors, both constrained optimization problems can be simplified. The optimization problem resulting in the optimal fusion rule is limited to a k/N rule, wherein we can use fusion rules such as OR rule, ALL rule or the MAJORITY rule. Likewise, the optimization of the local sensor performance is limited to finding an optimal threshold $\tau_i^\star = \tau^\star$ as mentioned above. We also take notice of the fact that in such a case all the sensors employ the same coverage radius $d_i^\star$.

Chapter 4

System Design and Performance Analysis

In the previous chapter we saw the statistical distribution of the sensor measurements in the presence and absence of a radioactive emitter at an unknown location in a region of interest. And because of the unknown distance between the sensors and the emitter, the least favorable emitter location was incorporated into the formulation of the localized decision rule at the individual sensors. The localized fusion strategy at each sensor is a likelihood ratio test that takes in a collection of M observations and takes local decision based on a suitably chosen threshold. We also saw the different sensor deployment strategies and the system level and sensor level optimization to optimize the detection performance.

In this chapter, we look at a specific model of the distributed emitter detection system. In order to derive insights into the problem of emitter detection with an imperfect channel linking the sensors and the fusion center, we consider a simplified model for a channel imperfection. The channel is modeled as a binary symmetric channel with channel cross-over probability p. In other words, the transmitted bit has a probability p of being "flipped". It is further assumed that this probability is known to the fusion center. However, in reality the probability p apart from being unknown can be dependent on specific characteristics of the transmission link (such as modulation format, channel coding, fading, detection strategy at the fusion center, etc.).

4.1 A 3-sensor example

Consider a network of three sensors covering a region of interest with area S_e. The three sensors are arranged as shown in Figure 2. The locations of the sensors are $(0,0)$, $(b,0)$ and $(b/2, 1.732b/2)$. The network is in accordance with the parallel topology wherein the sensors detect a global phenomenon characterized by a null or alternate hypothesis and transmit their local decisions to the fusion center at the same time. We also assume that the channel imperfection modeled as a binary symmetric channel (with flipping probability p) is also a multiple access channel. In other words, the flipping probability is the same for all the sensor-fusion center transmission path. The measurement noise at each sensor is modeled to have Gaussian distribution having zero mean and variance σ^2. Also they are independent and identically distributed. For local sensor s_i the binary hypothesis testing problem is defined as follows:

$$
\begin{aligned}
\mathcal{H}_1: \quad & X_{i_j} = A_i + n_{i_j} \\
\mathcal{H}_0: \quad & X_{i_j} = n_{i_j} \\
& i = 1, \ldots, 3 \\
& j = 1, \ldots, M
\end{aligned}
$$
(4.1.1)

$n_{i_j} \sim N\left(0, \sigma^2\right)$. A_i is a known function depending on the propagation model of the emitter. It is assumed that the radioactive emitter source has an isotropic signal attenuation model defined as:

$$
A_i^2 = \frac{P_0}{1 + a d_i^n}
$$
(4.1.2)

In the above equation n is the signal decay exponent and can take values in between 2 and 3. a is an adjustable constant. This model is applicable with unknown distance between sensors and the emitter being 0. Also, it can be extended to the three dimensional case.

From equations (4.1.1) and (4.1.2) we see that the binary hypothesis problem is a composite hypothesis problem. The sensor observations depend on the amplitude A_i which in turn depends on the distance between the sensors and the emitter (an unknown quantity). As a result, the sensor s_i employs a likelihood ratio test δ_i assuming least favorable distance $d_i^\star$.

So the test statistic at each sensor is given as:

$$T_i\left(\boldsymbol{x_i}^M\right) = \frac{1}{M} \sum_{j=1}^{M} \log \left\{ \frac{\frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ \frac{-1}{2\sigma_2}\left(x_{ij} - A_i\right)^2 \right\}}{\frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ \frac{-1}{2\sigma_2}\left(x_{ij}\right)^2 \right\}} \right\}$$

This further simplifies to

$$T_i\left(\boldsymbol{x_i^M}\right) = \frac{1}{M} \sum_{j=1}^{M} \left(x_{ij} A_i - \frac{A_i^2}{2} \right) \tag{4.1.3}$$

The sensor decides in favor of hypothesis H_1 if the test statistic is above a threshold $\tau_i^\star$ (to achieve the desired false alarm rate α_i).

$$\frac{1}{M} \sum_{j=1}^{M} \left(x_{ij} A_i - \frac{A_i^2}{2} \right) > \tau_i^\star$$

$$\frac{1}{M} \sum_{j=1}^{M} \left(x_{ij} \sqrt{\frac{P_0}{1 + a d_i^\star}} - \frac{1}{2}\left(\frac{P0}{1 + a d_i^\star} \right) \right) > \tau_i^\star$$

The test statistic is the sample mean of M observations at each sensor. However, we notice that the expression on the right hand side of the inequality below is a dependent quantity which becomes a constant only under the assumption that $d_i = d_i^\star$.

$$\frac{1}{M} \sum_{j=1}^{M} x_{ij} > \tau_i^\star \sqrt{\frac{1 + a\left(d_i^\star\right)^n}{P_0}} + \frac{1}{2}\sqrt{\frac{P_0}{1 + a\left(d_i^\star\right)^n}}$$

The above inequality is also expressed as:

$$\bar{X}_i > \frac{\tau_i^\star}{A_i} + \frac{A_i}{2} \tag{4.1.4}$$

4.2 Performance Analysis

The sample mean is the sufficient statistic for an individual sensor for the optimality test against the least favorable emitter location. With the sensors employing identical coverage radius equal to the least favorable distance between the sensors and the emitter the composite hypothesis testing problem is now a simple hypothesis. Also note that the observations at the sensors are *iid*.

This culminates in the following statistical distribution for the sample mean for hypothesis H_1 and H_0.

$$\mathcal{H}_1: \quad \bar{X}_i \sim \mathcal{N}\left(A_i\left(d_i^\star\right), \frac{\sigma^2}{M}\right)$$

$$\mathcal{H}_0: \quad \bar{X}_i \sim \mathcal{N}\left(0, \frac{\sigma^2}{M}\right)$$

(4.2.1)

In the previous section we noticed that the threshold for each sensor depends on the distance $d_i^\star$. Hence, we shall denote it as $\tau_i\left(d_i^\star\right) = \tau_i'$.

The detection probability and the false alarm probability for each sensor is given as follows:

$$p_{Di}\left(\delta_i \mid d_i^\star\right) = \frac{\sqrt{M}}{\sqrt{2\pi}\sigma} \int_{\tau_i'}^\infty exp\left\{\frac{-M}{2\sigma^2}\left(t - A_i\left(d_i^\star\right)\right)^2\right\} = Q\left(\frac{\tau_i' - A_i\left(d_i^\star\right)}{\frac{\sigma}{\sqrt{M}}}\right)$$

(4.2.2)

$$p_{FAi}\left(\delta_i\right) = \frac{\sqrt{M}}{\sqrt{2\pi}\sigma} \int_{\tau_i'}^\infty exp\left\{\frac{-M}{2\sigma^2}t^2\right\} = Q\left(\frac{\tau_i'}{\frac{\sigma}{\sqrt{M}}}\right)$$

(4.2.3)

Therefore, as mentioned in the previous chapter the optimal test for the sensors against the least favorable emitter location boils to finding the optimal $\tau_i\left(d_i^\star\right) = \tau_i'$ so that $p_{FAi}\left(\delta_i\right) = \alpha_i$. So, if α_i is a false alarm probability value for a decision rule δ_i then the threshold can be expressed as:

$$\tau_i' = Q^{-1}\left(\alpha_i\right)\frac{\sigma}{\sqrt{M}}$$

(4.2.4)

Consequently, the corresponding detection probability is given as

$$\beta_i\left(\delta_i \mid d_i^\star\right) = Q\left(Q^{-1}\left(\alpha_i\right) - \frac{A_i\left(d_i^\star\right)\sqrt{M}}{\sigma}\right)$$

Here we notice that the false alarm probability at the sensor and the false alarm probability of the sensor as seen from the fusion center is related by equation (3.3.4). Hence the detection probability becomes

$$\beta_i\left(\delta_i \mid d_i^\star\right) = Q\left(Q^{-1}\left(\frac{\alpha_i^F - p}{1 - 2p}\right) - \frac{A_i\left(d_i^\star\right)\sqrt{M}}{\sigma}\right)$$

(4.2.5)

α_i^F is the false alarm probability for sensor s_i as seen from the fusion center.
This difference is a result of the binary symmetric channel between the sensors and the fusion center.

The detection probability and false alarm as seen from the fusion center is:

$$\beta_i^F \left(\delta_i \mid d_i^\star \right) = (1 - 2p) \, \beta_i \left(\delta_i \mid d_i^\star \right) + p$$

$$\alpha_i^F \left(\delta_i \right) = (1 - 2p) \, \alpha_i \left(\delta_i \right) + p$$

The performance model of the sensors that the fusion center utilizes to take the global decision is $\left(\alpha_i^F, \beta_i^F \right)$. It is important to note that in this case, since the flipping probability p and the performance model "at the individual sensor" i.e. (α_i, β_i) is known to the fusion center, it might appear that it doesn't make any significant difference if $\left(\alpha_i^F, \beta_i^F \right)$ or (α_i, β_i) is used. In actuality, the fusion center has to take the flipping probability into account and incorporate it in its knowledge of the performance model of each sensor. In this work, the imperfect channel is modeled as a binary symmetric channel whose probability is a known constant. However, there are more practical situations in which the flipping probability depends on the decision strategy being used at the fusion center or other channel characteristics such as presence of fading or modulation.

4.3   Sensor Deployment and Coverage

The deployment strategy employed by the emitter detection network is the triangular sensor constellation. In this arrangement the sensors are placed in what can be interpreted as triangular cells. Equilateral triangular arrangement is considered here. An emitter could be within any triangular cell. The signal from the emitter follows a signal propagation that depends on the unknown distance between the sensor location and the emitter location. Out approach against this constraint lies in optimizing the deployment of sensors, the local decision rules and the global fusion rule with respect to the least favorable location of the emitter.

When the region of interest is large and sensors are densely arranged in congruent circles (two dimensional space) or spheres (three dimensional space), the optimization of the local decision rules δ_i of the sensors and the system level fusion rule δ_F is treated with regards to a circle (sphere) packing problem. We consider three or four nearest range sensors of primary interest to the least favorable emitter location. We solve a constrained optimization problem involving these primary neighboring sensors to the least favorable emitter location to optimize the local decision rules for s_i.

The figure below shows the sensor deployment for a triangular cell in the triangular arrangement of sensors. The coverage of the sensors using least favorable emitter distance for OR and ALL fusion rule are shown respectively.
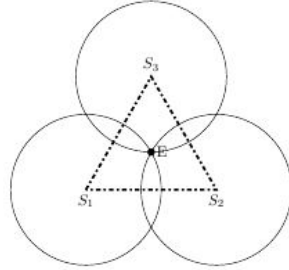
(a) Sensor Coverage for OR rule
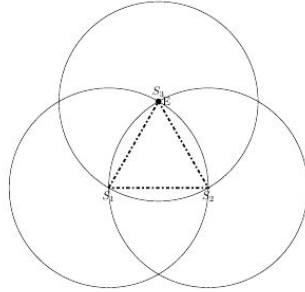
(b) Sensor Coverage for ALL rule

Figure 4.1: The sensor coverage for two different fusion rules, the OR rule and the ALL rule. The circles represent the sensing area of the three sensors. The intersection of the circles within the triangle(ROI) is the overlapped decision regions of the sensors. The intersection of the triangle and an individual circle represent the decision region of an individual sensor.

The OR rule is characterized by at least one sensor in favor of the emitter present leading to the global decision in favor of emitter present hypothesis. The ALL rule on the other hand requires all the sensors to be in favor of emitter present in order for the global decision to be in favor of emitter present hypothesis. Note that, we only consider the sensors on the immediate neighborhood of the emitter as the ones of primary interest. For the OR rule, the point E (the centroid of the triangle) is the least favorable emitter location since the emitter tends to be equidistant from all the sensors. Also, notice that the emitter places itself at the boundary of the sensing radii of the individual sensors when Or rule is used as the optimal policy. However, for the ALL rule the emitter tends to be very close to any one sensor (in this example s_2). The figure above shows that when ALL rule is adopted as the fusion policy the emitter places itself at the boundary of the sensing radii of the other two sensors.

4.4   Extension to N sensors

The sensors implement similar decision rules over the observations. As represented by equations (4.2.2) and (4.2.3) the detection probability depends on the threshold τ'_i and the least favorable emitter-sensor distance $d^\star_i$. Note that the distance $d^\star_i$ is also the coverage radius of each sensor. Including identical coverage radius and threshold at the sensors to optimize the decision rule (with deployment limited to the coverage problem) leads to k out of N mixing with the false alarm less than or equal to α_F. With the system false alarm tolerance set to $\alpha_F = 0.05$ (say), the objective is to maximize the system detection probability. The k out of N mixing with k $= 0,..,N$ involves $2^N = \binom{N}{0} + \binom{N}{1} + \cdots + \binom{N}{N}$ combinations. Out of the total combinations, based on the number of sensors in favor the presence of the emitter the fusion policies can be characterized into the OR, MAJORITY or ALL rule. If number of sensors in favor of the presence of emitter is at least 1 i.e. $k \geq 1$ and the fusion center declares H_1 then the fusion policy is referred to as the OR rule. Likewise for $k = N$ it is AND. For the majority rule we have to cases. When the number of sensors N is odd, it is $k \geq \lceil \frac{N}{2} \rceil$. And when N is even, it is $k \geq \frac{N}{2} + 1$. The probability of false alarm for each sensor (as seen from the fusion center) is related to the system level false alarm tolerance α_F upon implementation of the OR rule as the following:

$$P^F_{FA} = 1 - \left(1 - \alpha^F_{i(OR)}\right)^N \tag{4.4.1}$$

The value of α_F can be fixed beforehand to a desired value within which we want the system level false alarm to be restricted. Note that, the false alarm probability for each sensor is taken as $\alpha^F_{i(OR)}$ and not $\alpha_{i(OR)}$ because the performance model used by the fusion center has to take into consideration the imperfection in the channel modeled as binary symmetric channel.

Similarly, the detection probability of the system P^F_D is related to the detection probability for each sensor $\beta^F_{i(OR)}(d^\star_i)$ for OR rule as:

$$P^F_D = 1 - \left(1 - \beta^F_{i(OR)}(d^\star_i)\right)^N \tag{4.4.2}$$

Likewise, when the optimal policy used at the fusion center is the ALL rule (k=N), the false alarm probability is as follows:

$$P^F_{FA} = \left(\alpha^F_{i(ALL)}\right)^N \tag{4.4.3}$$

The probability of detection for the ALL rule is:

$$P_D^F = \left(\beta_{i(ALL)}^F \left(d_i^\star \right) \right)^N \tag{4.4.4}$$

The system performance for MAJORITY rule is as shown below.

a) When N is odd:

Denoting $\beta_{i(MAJ)}^F \left(d_i^\star \right) = \beta$, the system level probability of detection is:

$$P_D^F = \binom{N}{\lceil \frac{N}{2} \rceil} (\beta)^{\lceil \frac{N}{2} \rceil} (1 - \beta)^{N - \lceil \frac{N}{2} \rceil} + \binom{N}{\lceil \frac{N}{2} \rceil + 1} (\beta)^{\lceil \frac{N}{2} \rceil + 1} (1 - \beta)^{N - \lceil \frac{N}{2} \rceil - 1} + \ldots + (\beta)^N \tag{4.4.5}$$

Denoting $\alpha_{i(MAJ)}^F = \alpha$, the system level false alarm is:

$$P_{FA}^F = \binom{N}{\lceil \frac{N}{2} \rceil} (\alpha)^{\lceil \frac{N}{2} \rceil} (1 - \alpha)^{N - \lceil \frac{N}{2} \rceil} + \binom{N}{\lceil \frac{N}{2} \rceil + 1} (\alpha)^{\lceil \frac{N}{2} \rceil + 1} (1 - \alpha)^{N - \lceil \frac{N}{2} \rceil - 1} + \ldots + (\alpha)^N \tag{4.4.6}$$

b) When N is even:

The probability of system detection and false alarm for $\beta_{i(MAJ)}^F \left(d_i^\star \right) = \beta$ and $\alpha_{i(MAJ)}^F = \alpha$ is:

$$P_D^F = \binom{N}{\frac{N}{2} + 1} (\beta)^{\frac{N}{2} + 1} (1 - \beta)^{\frac{N}{2} - 1} + \binom{N}{\frac{N}{2} + 2} (\beta)^{\frac{N}{2} + 2} (1 - \beta)^{\frac{N}{2} - 2} + \ldots + (\beta)^N \tag{4.4.7}$$

$$P_{FA}^F = \binom{N}{\frac{N}{2} + 1} (\alpha)^{\frac{N}{2} + 1} (1 - \alpha)^{\frac{N}{2} - 1} + \binom{N}{\frac{N}{2} + 2} (\alpha)^{\frac{N}{2} + 2} (1 - \alpha)^{\frac{N}{2} - 2} + \ldots + (\alpha)^N \tag{4.4.8}$$

So we looked at a specific problem wherein a system of three sensors arranged in an equilateral triangle monitor an area of interest that contains an emitter at an unknown location. The individual sensors collect observations that are corrupted by Gaussian noise having zero mean and variance σ^2. Also, it was found that the signal propagation model of the radioactive emitter depends on the sensor-to-emitter distance rendering the binary hypothesis problem as a composite hypothesis problem. To overcome this constraint, we considered the least favorable range between the sensor and the emitter. Next, a performance analysis of the optimization of the individual sensors given the least favorable distance was carried out. The optimal sensor level detection boiled down to finding a suitable threshold satisfying the false alarm constraint of the sensors. Also, by considering identical performance model and coverage radius of the sensors the constrained optimization of the system level performance was reduced to a k/N policy with false alarm fixed to predetermined value. In the next chapter, we look at numerical results to analyze and gain insights into detection performance of the system.

Chapter 5

Comparative Study of Detection Performance

In the previous chapter we looked at a specific three sensor emitter detection system and the mathematical expressions of the sensor performance model. Also, we looked at the system level performance under a set of practical assumptions.

In this chapter, we further our analysis with numerical examples and graphical plots. As discussed in section (4.1) we consider three sensors placed at (0,0), (b,0) and (b/2, 1.732b/2,). Let us assume $b=\sqrt{3}$. So, the sensors are at locations $(0,0), (\sqrt{3}, 0), \left(\frac{\sqrt{3}}{2}, 1.5\right)$. The sensor level and system level detection performance is dealt with taking into consideration the fact that the number of sensors nearest to the least favorable emitter location is three. This approach can be extended to a large region of interest covered with sensors in a triangular arrangement. The emitter could be located at an unknown location within any of the triangular cells. Note that the emitter could also be located on the line joining two adjacent sensors. The detection of an event triangle (i.e., a triangular cell within which an emitter is present) is a separate problem. However, we are interested in the performance optimization of the sensors and the detection system within a sub-region of interest monitored by the three nearest sensors to the emitter location.

The signal propagation model is as shown in equation (4.1.2). The model is applicable even when the distance between the emitter and the sensor is equal to 0. The figure on the following page shows the variation of the amplitude of the emitter radiation with distance. A low level point radiation source is considered with P_0 equal to 5 dB, a is an adjustable constant equal to 100 and n is the signal decay exponent that can take values between 2 and 3. Here, we consider n equal to 2.5.

The signal propagation model depends on the unknown distance between the sensor and the emitter as shown below:
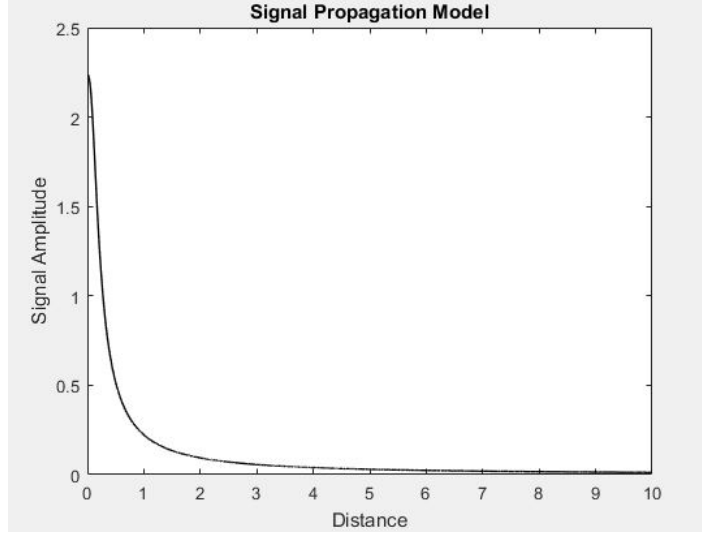

Figure 5.1: The variation of Signal Amplitude with distance.

The measurement noise at each individual sensor is considered to be *iid* Gaussian with zero mean and variance $\sigma^2 = 1$. As discussed in section (4.2) under the sensor performance analysis, the observation samples at each sensor are $\mathcal{N}\left(A_i\left(d_i^\star\right), \frac{1}{M}\right)$ under hypothesis H_1 and $\mathcal{N}\left(0, \frac{1}{M}\right)$ under hypothesis H_0. Since $A_i\left(d_i^\star\right)$ is dependent on the distance $d_i^\star$ it is a composite binary hypothesis problem. The value of $d_i^\star$ depends on the fusion rule used at the fusion center. Upon employing the OR rule the emitter places itself equidistant from all the sensors (which is the centroid of the triangle in this case) and for the ALL fusion rule the emitter places itself very close to or at any one of the sensors. However, it is not not known to the fusion center at which sensor the emitter places itself. As a result, the emitter treats all the sensors equally and considers equal chance of the emitter to be located at any of the three sensors.

5.1 Probability of False Alarm at each sensor

The performance model of each sensor (α_i, β_i) is considered identical. And as the cross-over probability p linking the sensors and the fusion center is assumed to be known to the fusion center (p=0.01), the sensor performance as seen from the fusion center $\left(\alpha_i^F, \beta_i^F\right)$ is also identical. Since the fusion center has to take the channel probability into consideration it mixes $\left(\alpha_i^F, \beta_i^F\right)$ according to the k/N policy. It is desired that the system level false alarm be within five per cent, i.e. $P_{FA}^F \leq 0.05$.

For the OR fusion rule α_i^F can be found out using equation (4.3.1). $\alpha_i^F = 0.017$ for $N = 3$. And the false alarm for each sensor as seen by itself is $\alpha_i = \frac{\alpha_i^F - p}{1 - 2p} = 0.00714$ for $p = 0.01$.

So, $\alpha_{i(OR)}^F = 0.017$ and $\alpha_{i(OR)} = 0.00714$.

Likewise, for $N = 3$ sensors the individual sensor false alarm a seen by itself and the fusion center can be found out for the ALL rule using equation (4.3.3). With P_{FA}^F fixed at 0.05, $\alpha_i^F = 0.3684$ and $\alpha_i = \frac{\alpha_i^F - p}{1 - 2p} = 0.3657$ for $p = 0.01$.

So, $\alpha_{i(ALL)}^F = 0.3684$ and $\alpha_{i(ALL)} = 0.3657$.

5.2 Sensor Performance with Increasing Number of Observations

As mentioned in the preceding sections, the emitter places itself equidistant from the sensors when the optimal fusion rule is the OR rule. Whereas, it places itself close to any of three sensors when the optimal fusion rule is the ALL rule. With the given arrangement the values of $d_i^\star$ is equal to 1 and $\sqrt{3}$ for the OR and ALL fusion rule respectively. The figure below shows a comparison of the performance of each sensor (as seen from the fusion center) with increasing number of observations while implementing the OR and the ALL rule as the optimal rule by the fusion center.
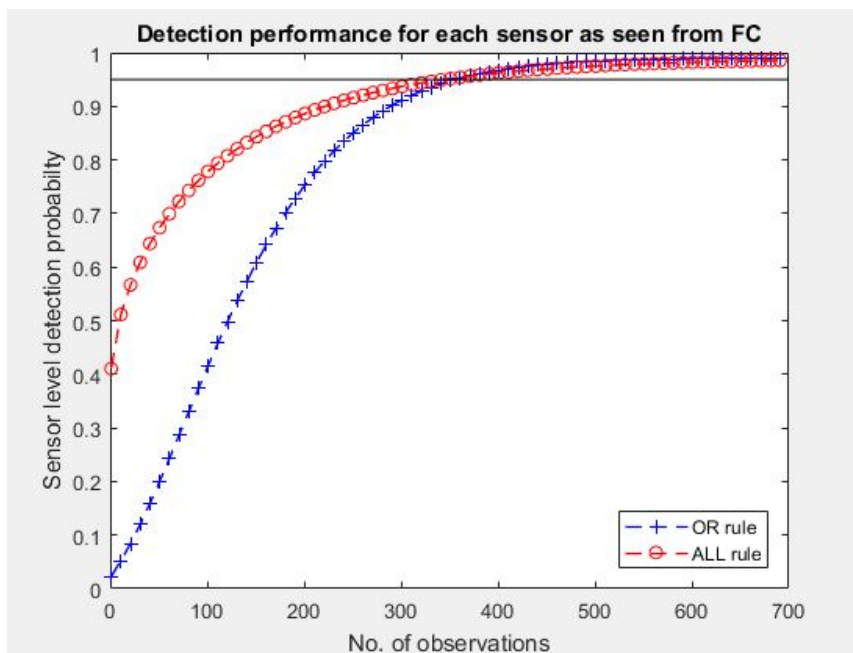

Figure 5.2: Sensor level detection performance with increasing number of observations. OR rule: $d_i^\star = 1$, ALL rule: $d_i^\star = \sqrt{3}$.

With the specific values taken for the different parameters, equation (4.2.5) for OR and ALL is given as follows:

$$\beta_{i(OR)}\left(\delta_i \mid d_i^\star\right) = Q\left(Q^{-1}\left(0.00714\right) - \sqrt{\frac{5M}{1 + 100\left(1\right)^{2.5}}}\right)$$

$Q^{-1}\left(0.00714\right) = 2.4501.$

$$\beta_{i(ALL)}\left(\delta_i \mid d_i^\star\right) = Q\left(Q^{-1}\left(0.3657\right) - \sqrt{\frac{5M}{1 + 100\left(\sqrt{3}\right)^{2.5}}}\right)$$

$Q^{-1}\left(0.3657\right) = 0.3433.$ Comparing the two expressions it can be seen that although both $\beta_{i(OR)}\left(\delta_i \mid d_i^\star\right)$ and $\beta_{i(ALL)}\left(\delta_i \mid d_i^\star\right)$ increase with increasing number of observations, however the detection probability for each sensor is more towards the upper left corner for the ALL rule. This is evident from the fact that the probability of false alarm at each sensor for the ALL rule is much higher than for the OR rule.

Analytically, it is evident from equation (4.2.5) that increasing the argument of the $Q\left(.\right)$ function leads to an increase in detection probability. The false alarm at each sensor for ALL is higher than that for OR. Also, $d_i^\star = \sqrt{3}$ is greater than $d_i^\star = 1$. Furthermore, $Q_1\left(\alpha_{i(ALL)}\right)$ ($= Q^{-1}\left(0.3657\right) = 0.3433$) is significantly lower than $Q^{-1}\left(\alpha_{i(OR)}\right)$ ($= Q^{-1}\left(0.0.00714\right) = 2.4501$). So, to start with $\beta_{i(ALL)}\left(\delta_i \mid d_i^\star\right)$ is greater than $\beta_{i(OR)}\left(\delta_i \mid d_i^\star\right)$ and it increases in a similar fashion for every collected observation until the 95 per cent probability mark. The figure below shows the variation of the sensor threshold for a fixed false alarm (for both OR and ALL rule) with increasing number of observations.
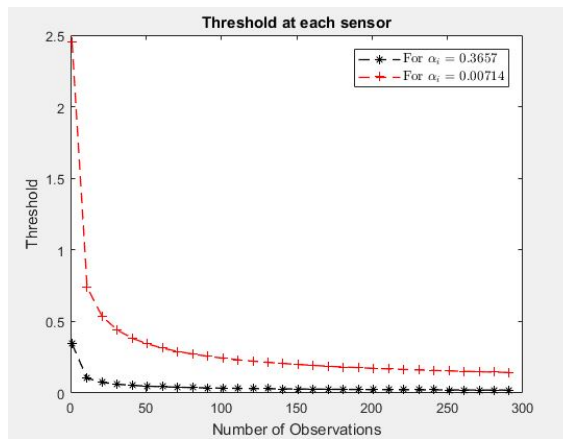

Figure 5.3: Plot showing the decrease of sensor threshold with an increase in observations for a fixed α_i=0.3657 and α_i=0.00714.

5.2.1 Sensor level Threshold

Going back to chapter 3, section (3.4) where the constrained optimization of the sensor level decision rule was discussed, we saw that the problem boils down to finding a suitable threshold that satisfies the false alarm constraint. $P_{D_i}(\delta_i \mid d_i^\star) = Q\left(\tau_i'\sqrt{M} - \frac{1}{\sigma}\sqrt{\frac{P_0 M}{1 + a(d_i^\star)^n}}\right)$. Substituting the values for p, σ, a and n we obtain the following:

$$\text{maximize} \quad Q\left(\tau_i'\sqrt{M} - \sqrt{\frac{5M}{1 + 100\,(d_i^\star)^{2.5}}}\right)$$

$$\text{subject to} \quad Q\left(\tau_i'\sqrt{M}\right) \le \frac{\alpha_i^F - p}{1 - 2p}$$

For a fixed $\alpha_i^F = 0.3684$ and $M = 30$ (say), the value of threshold is $\tau_i' = 0.0627$. However, it is important to note that this threshold varies with M. And as shown in figure 6 it decreases with increasing M. Also, the sensor optimizes its decision rule without considering the other sensors. So, the optimization turns out to be a calculation of $\tau_i' = Q^{-1}\left(\frac{\alpha_i^F - p}{1 - 2p}\right)\frac{1}{\sqrt{M}}$ as the sensor s_i gradually collects observations M. For a fixed α_i^F the detection probability is given as $Q\left(Q^{-1}\left(\frac{\alpha_i^F - p}{1 - 2p}\right) - \sqrt{\frac{5M}{1 + 100(d_i^\star)^{2.5}}}\right)$.

5.3 System Performance

5.3.1 Ideal Channel

For the ideal channel, the channel probability is $p = 0$. Among all the decision fusion rules computed from the constrained optimization (in section 3.4) the OR rule is the optimal rule. It takes 130 observations for the system to reach 95 per cent detection probability. However, with the ALL rule the number of observations required goes up to 480 to reach the same performance.

5.3.2 Non-Ideal Channel

For the non-ideal channel it can be seen that the number of observations required for achieving 95 per cent detection probability while implementing the OR rule is 160. For ALL rule, it takes 600 observations for the system to achieve the same performance.

Plots showing the detection performance for the ideal channel and the non-ideal channel are shown in the following page.
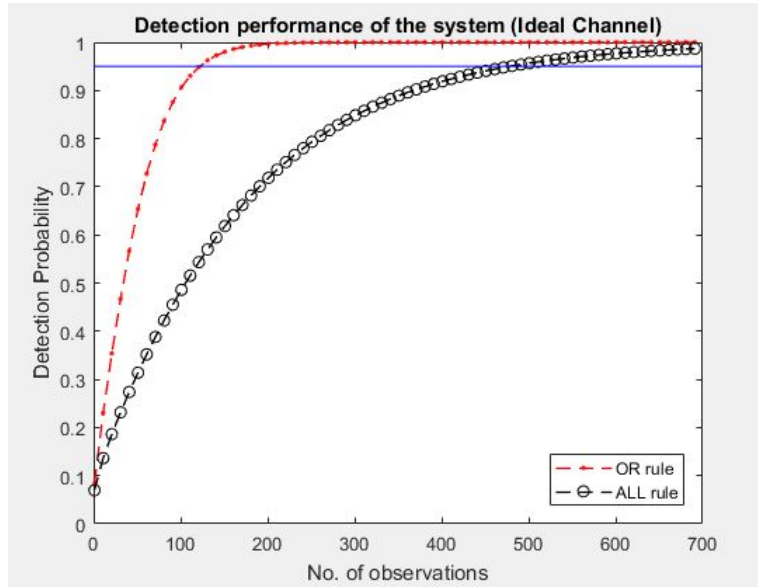
Figure 5.4: Detection Performance of the system when the channel probability $p = 0$ (Ideal Channel).
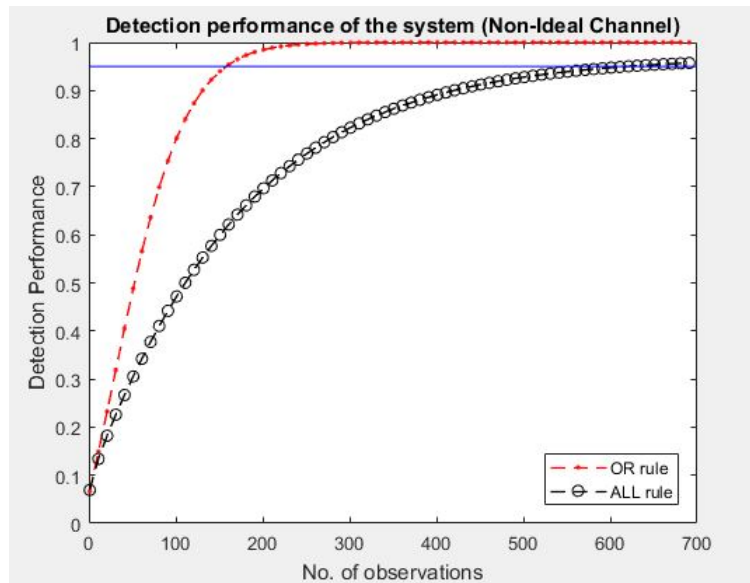

Figure 5.5: Detection performance of the system for channel probability $p = 0.01$ (Non-Ideal Channel).

It can be seen that there is an increase in the number of observations required to reach the same performance level $(P_D^F = 0.95)$ from the ideal situation to the non-ideal situation. There is about 23% for thee OR rule and about 25% increase for the ALL rule. Also, the increase in number of observations from OR to ALL for the ideal channel is about 269% and for the non-ideal channel it is 275%. Thus, we see that the OR rule outperforms the ALL rule by a large margin for both situations when channel probability $p = 0$ and $p = 0.01$.

5.3.3 What if Channel Probability is greater than 0.017 ?

So far it is evident that the OR rule is the optimal detection rule. But, for $p < 0.5$ and assuming that probability of detection at sensor is greater than the probability of false alarm at the sensor, $p_{FA_i}(\delta_i) \in [p, 1-p]$ and $p_{D_i}(\delta_i \mid d_i^\star) \in [p, 1-p]$. For $N = 3$, and the system level false alarm constraint fixed at 0.05, $\alpha_i^F = 0.017$. Also, the false alarm as seen by the sensor and the false alarm of the sensor as seen by the fusion center are related as $\alpha_i = \frac{\alpha_i^F - p}{1 - 2p}$, which implies that the value of p can not exceed 0.017, otherwise α_i will be negative which is probabilistically not consistent. So, if p takes values such as 0.02 or higher then the ALL rule seems to be applicable. It could be argued that even for ALL rule the relation between α_i and α_i^F holds and hence there is a similar limitation. However, it is important to note that although theoretically the value of p could be greater than $\alpha_{i(ALL)}^F = 0.3684$ (for the three sensor system), in practical scenarios such a value is far from realization.

However, if the cross-over probability reaches larger values then such a situation would call for a separation approach for the distributed detection. The communication schemes have to be separately dealt with from the signal processing algorithms involved in the computation of the decision rules.

5.4 The MAJORITY Rule

Given a system level false alarm the false alarm for each sensor s_i under the MAJORITY rule can be calculated from equation (4.3.8).

$$\binom{3}{2} \left(\alpha_{i(MAJ)}^F\right)^2 \left(1 - \alpha_{i(MAJ)}^F\right) + \left(\alpha_{i(MAJ)}^F\right)^3 = 0.05$$

From the above equation we get three values for $\alpha_{i(MAJ)}^F$ (0.13535, -0.12407, 1.48872). $\alpha_{i(MAJ)}^F$ being a probability can not take a negative value or a value greater than 1. So $\alpha_{i(MAJ)}^F = 0.13535$. And $\alpha_{i(MAJ)} = \frac{0.13535 - 0.01}{1 - 2(0.01)} = 0.128$. The optimization of sensor level decision rule and deployment is confined to the following optimization problem:

$$\text{maximize} \quad Q\left(\tau_i'\sqrt{M} - \sqrt{\frac{5M}{1 + 100\left(d_i^\star\right)^{2.5}}}\right)$$
$$\text{subject to} \quad Q\left(\tau_i'\sqrt{M}\right) \leq \frac{0.13535 - 0.01}{1 - 2(0.01)}$$

In implementing the MAJORITY rule a question arises as to where the emitter would place itself in the region of interest. In other words, what should be the least favorable distance between the sensor and the emitter. The three-sensor system can be thought of as a sub-region of interest which is part of a bigger region of interest deployed with sensors in triangular arrangement. For the OR rule the least favorable location of the emitter was equidistant from the all the three sensors (centroid). For the ALL rule the least favorable location would be very close to any one of the three sensors. However, for the MAJORITY rule it is difficult to decide what the appropriate least favorable location of the emitter would be. The sensors are arranged in a triangular (equilateral) fashion so that they form triangular cells. The figure below shows a similar arrangement.

The scope of $d_i^\star$ in the three sensor system is $[1, \sqrt{3}]$.

The least favorable distance cannot be less than 1 if the adjacent distance between the sensors is $\sqrt{3}$. Likewise, $d_i^\star$ cannot be greater than $\sqrt{3}$. If $d_i^\star$ is less than 1, the triangular region of interest is not fully covered as shown in the figure below. And if $d_i^\star$ is greater than $\sqrt{3}$, it implies that the emitter is outside the triangular region which is an infeasible case. However, if we consider the entire region deployed with sensors in the triangular arrangement then it is not infeasible. Because, in that case the emitter falls in another triangular three-sensor arrangement of sensors.
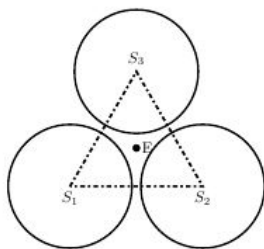

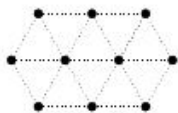**Figure 5.6:** The Region of Interest is not covered for $d_i^\star < 1$.

Figure 5.7: Triangular Arrangement. An emitter is assumed to be within one triangular region of interest at a time. It ca be seen that if $d_i^\star$ is greater than $\sqrt{3}$ then it falls in a different triangular region.

The least favorable emitter location may not be unique, and hence the least favorable distance $d_i^\star$ is not a unique value within the region of interest. It can take values in $[1, \sqrt{3}]$. Its important to remember that the very reason of having a composite binary hypothesis testing problem is the presence of the unknown distance d_i in the sensor measurements (under hypothesis H_1). In actuality the distance d_i varies from sensor to sensor based on the location of the emitter. However, in order to convert the composite hypothesis to a simple hypothesis problem we consider the least favorable distance $d_i^\star$ and it is further assumed that all three sensors use the same $d_i^\star$ to optimize the sensor decision rule δ_i. This explains why we consider $d_i^\star = \sqrt{3}$ for all the sensors in case of ALL rule when in reality the emitter could be close to only one of the three sensors.

5.4.1 System Performance under the MAJORITY rule

Since the sensors apply identical likelihood ratio test with identical $d_i^\star$, $d_i^\star$ could be thought of as the coverage radius or the sensing radius of each sensor.
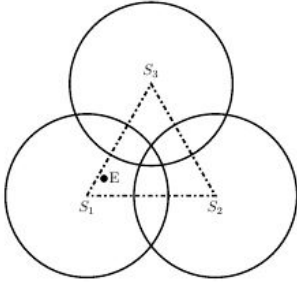

Figure 5.8: Overlap of decision regions for $d_i^\star = 1.25$. Emitter E lies in a region covered only by s_1.
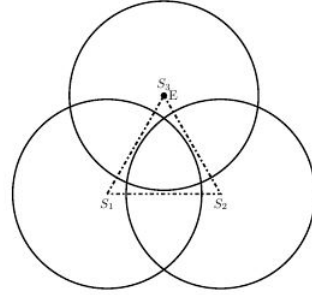
Figure 5.9: Overlap of decision regions for $d_i^\star = 1.65$. Emitter E lies in a region covered only by s_3.

The decision region overlap depends on the choice of $d_i^\star$. The scope of $d_i^\star$ is $[1, \sqrt{3}]$. Let D_i be the decision region of s_i corresponding to $d_i^\star$. Since the sensors use identical $d_i^\star$ the area of D_i is same. Now for $d_i^\star = \sqrt{3}$ the entire triangular region of interest is fully covered by $D_1 \cap D_2 \cap D_3$. However for $1 < d_i^\star < \sqrt{3}$ there are some non-overlapped decision regions as shown in figures 5.8 and 5.9 respectively. For the majority rule the H_1 hypothesis is declared only when the emitter is detected by at least two out of three sensors. In other words, the emitter has to be in the decision region of two or more sensors, i.e it has to be located in the overlap of at least two decision regions.

As a result, $d_i^\star = \sqrt{3}$ is the ideal choice. Also, it follows analytically that for the MAJORITY rule the emitter would distance itself as far as possible from at least two sensors. This can be achieved only if the emitter is very close to any of the three sensors.

However, one might argue that in the presence of binary symmetric channel even if the emitter is beyond the decision region of a sensor there is a probability of the decision to be flipped in favor of H_1 although the sensor correctly decided in favor of H_0. This situation in fact is in favor of the system performance.

Figure 5.10 shows the system performance under MAJORITY rule for $d_i^\star = 1$ and $d_i^\star = \sqrt{3}$. It can be inferred that for all the values of $d_i^\star \in (1, \sqrt{3})$ the performance curve is in between the curves for $d_i^\star = 1$ and $d_i^\star = \sqrt{3}$.
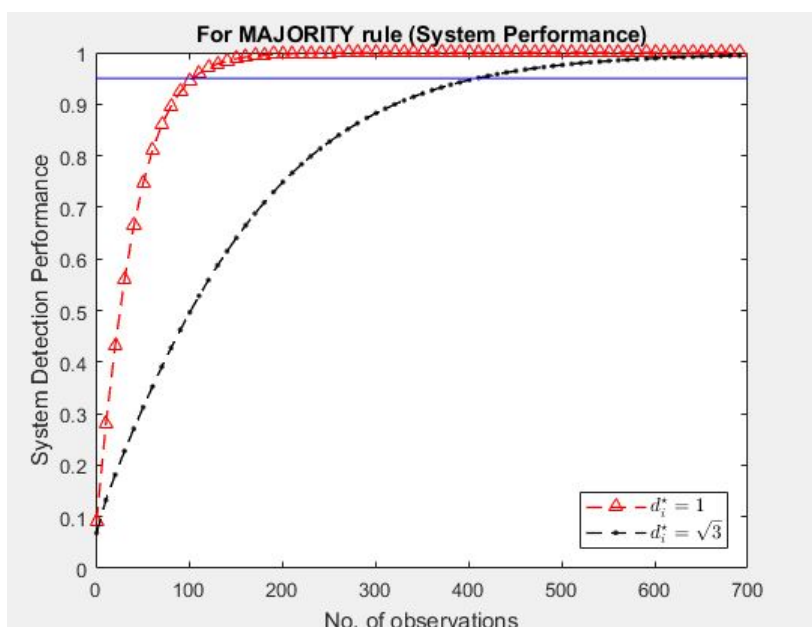

Figure 5.10: The detection performance of the system under the MAJORITY rule for $d_i^\star = 1$ and $d_i^\star = \sqrt{3}$. Note that for all values of $d_i^\star \in (1, \sqrt{3})$ the detection performance curve is between the ones for $d_i^\star = 1$ and $d_i^\star = \sqrt{3}$.

Figure 5.11 shows a comparison of the AND rule and the MAJORITY rule for p = 0.02 (knowing the fact that OR rule is infeasibe for $p > 0.017$) at $d_i^\star = \sqrt{3}$. The number of observations required to reach the desired detection performance of 95 per cent for the MAJORITY rule is 425 whereas for the ALL rule the performance does not reach the desired level after 700 observations.
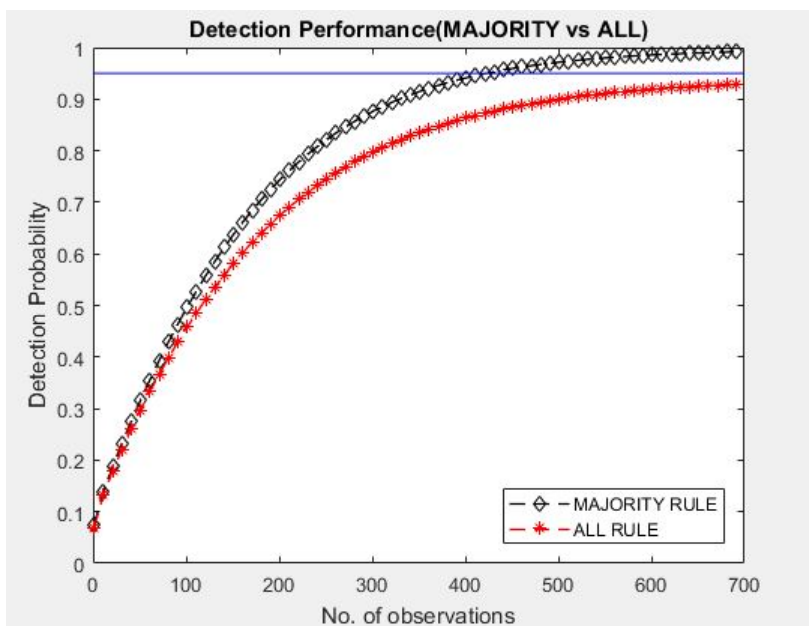
Figure 5.11: Comparison of detection performance of the ALL rule and the MAJORITY rule with the least favorable emitter to sensor distance set at $d_i^\star = \sqrt{3}$. Note that there is significant difference in the performance of the two fusion rules.

Thus, we see that the OR rule is the optimal fusion rule under the assumption that the transmission of decisions from the sensors to the fusion center is through ideal channels. Under the assumption that the channel linking the sensors and fusion center is binary symmetric with cross-over probability p, the OR rule is still optimal. However, for value of p greater than 0.017 the rule is not optimal. This calls for the sensors to lower their decision thresholds. For ALL rule, the threshold is lowered significantly resulting in a much reduced false alarm and an eventual reduction in sensor detection probability leading to the significant increase in the number of observations to reach the desired performance. Among the fusion rules classified as OR, ALL and MAJORITY, the MAJORITY rule was found to be optimal.

Chapter 6

Conclusion

The focus of this study was to explore the emitter detector system issues of finding optimal local sensor rules and system fusion rule with deterministic sensor deployment. The channel linking the sensors and the fusion center was considered to be a multiple access binary symmetric channel. The communication of the sensors with the fusion center was in accordance with the parallel configuration of distributed detection. The results obtained were different from the ideal channel assumption of [26] in which the OR rule was declared to be the optimal fusion rule. This research shows that the possibility of bit-flipping upon transmission influences the choice of optimal decision fusion rule. The optimal local sensor decision rules are likelihood ratio test based with each sensor utilizing identical sensing radius equal to the least favorable sensor-emitter distance. The optimal sensor rule boiled down to suitable threshold computation satisfying a false alarm constraint. With the fusion center having complete knowledge of the performance model of the sensors and the channel cross-over probability, the optimal fusion rule was the solution of a constrained problem in the Neyman-Pearson sense. Among all fusion rules classified as the OR, ALL and MAJORITY, the MAJORITY rule was found to be optimal given that the channel cross-over probability p is greater than 0.017 and less than 0.5.

There are a few limitations of this research. First, the methodology in this work is applicable for cross-over probability p less than 0.5. In addition, the threshold for each sensor is assumed to be identical on account of employing identical sensing radius for each sensor. The sensing radius is equal to the least favorable distance of the emitter from the sensor. In practice, however, the sensor-emitter distance is different for different sensors. Hence, there is a need to adopt collaborative detection among the sensors. Also, for larger values of p the sensors will have to transmit a stream of bits instead of single-bit decision, thus requiring sensors with increased processing and communication capacities.

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Vita

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