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Pricing of Idiosyncratic Risk in an Intermediary Asset Pricing Model

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Abstract

Standard asset pricing theories suggest that only systematic risk is priced. Empirical studies report a relationship between idiosyncratic volatility or risk (IVOL) and asset price. The most common explanation for this anomaly is that households under-diversify creating a Bad Model problem. This paper uses an Intermediary Asset Pricing Model (IAPM) as a way to control for under-diversification in evaluating the relationship between IVOL and asset price. We find that IVOL premia is lower in an IAPM. Our findings indicate that under-diversification can explain the anomaly partially.

Keywords: idiosyncratic risk, asset pricing, intermediary asset pricing model.

1. Introduction

Asset pricing theories suggests a relation between systematic risk and return and no relation between idiosyncratic risk and return because investors diversify away this risk. However, empirical studies on idiosyncratic risk or volatility (IVOL) are at odds with the standard asset pricing predictions that find a relation between IVOL and asset prices.

The empirical connection between IVOL and asset prices may be a “Bad Model” problem, as discussed by (Fama 1998), resulting in mispricing. The mispriced component is part of the IVOL because it is measured by the variance of residuals from a particular asset-pricing model. Adrian, Etula and Muir (AEM, 2014) list the strong assumptions of the standard asset pricing models found in the literature. Models require: a) participation of all households, b) no transaction costs, c) widespread knowledge of complicated trading strategies, d) knowing the moments of asset returns, and e) continuously optimizing agents. Violation of any of these assumptions produces frictions and causes mispricing. AEM (2014) offer an Intermediary Asset Pricing Model (IAPM) that does not suffer from shortcomings of these representative consumers. Their model is based on Broker-dealer behavior rather than the behavior of households.

Most asset pricing studies employ a stochastic discount factor (SDF) variant. A SDF is thought of as the intertemporal marginal rate of substitution (IMRS) of aggregate household consumption or wealth. SDF based models determine asset prices by premia that arise through their covariance to the SDF. Idiosyncratic volatility produces no premia in this framework because it is orthogonal to the SDF by construction. An incorrect SDF results in an incorrect orthogonal projection that causes mispricing. The IAPM uses an SDF linked to the marginal value of aggregate wealth of financial intermediaries to correct the SDF for shortcomings of its standard form.

The IAPM SDF is a direct function of Broker-Dealers (BDs) leverage which a ratio of financial assets-to-liabilities. Leverage as an empirical proxy for the marginal value of aggregate wealth of financial intermediaries. When credit is tight funding constraints for intermediaries become binding which can force them to deleverage (sell assets), leading to higher marginal value of wealth and visa-versa in when credit is plentiful. This SDF has the advantage of not making assumptions about the behavior of households.

Long before IAPM, Levy (1978) in a theoretical study draws attention to the assumptions of perfect indivisibility of an investment and no transaction costs infering that individuals hold market portfolio. He concluded that beta would explain price behavior of widely held assets. But most stocks are not widely held, and individual stock volatility is better at explaining price behavior of corresponding stock. Similarly, Merton (1987) proposes an extension to CAPM drawing attention to assumptions about frictionless market, complete information, and rational and optimizing agents. He argues that investors care about total risk because they do not hold the market portfolio. Thus, they require compensation for holding high IVOL stocks. Both extensions propose a beta on market-wide IVOL along with market beta in the pricing equation.

Tinic and West (1986), and Malkiel and Xu (1997, 2002) provide empirical support for IVOL in explaining the cross-section of expected stock returns. All these studies draw attention to the fact that investors do not hold perfectly diversified portfolios as an explanation for IVOL premia. Transaction costs and employee stock options can expose investors to concentrated ownership.

Benartzi (2001), Benartzi and Thaler (2001), and Huberman (2001) find evidence that investors willingly ignore opportunities to diversify because of private information and familiarity of firms they invest in. Bonaparte and Cooper (2001) report that on annual basis less than 71% of stockholders adjust their portfolios[1]

Perfect diversification requires continuously optimizing households that face no transaction costs. This is not as much of a concern in the IAPM. Financial intermediaries trade in much broader set of assets than average households, face lower transaction costs, and specialize the analysis of securities.

Goyal and Santa-Clara (2003) argue that background risk from non-traded assets[2] of investors can explain the positive IVOL premia. If the risk of non-traded assets in an investor's portfolio increases, investors will require higher expected return for their traded assets. Their finding support pricing models based on investor heterogeneity.

There are some other explanations that an IAPM cannot address. Ang, Hodrick, Xing, and Zhang (AHXZ, 2006) argue that higher exposure of high IVOL stocks to aggregate volatility risk can partially explain negative IVOL premia. They leave the rest as a puzzle. Bali and Cakici (2008) adjust data frequency, weighting scheme, breakpoints, and use screening for size, liquidity, and price. They argue that these adjustments and screening can explain the IVOL premia.

[1] From averages of question X7193 from Survey of consumer Finances on purchase or selling stocks or securities through a broker. This rate is an average of the 1995, 1998, 2001 and 2005 cross-sections.

[2] Example: (i) human capital, such as labor income, and (ii) private business, such as private equity capital.

While IAPM cannot address all the justification for IVOL premia, it can address mispricing from under-diversification, optimizing agents, and transaction costs. So, this study re-examines the relationship of idiosyncratic volatility and expected stock return using a financial intermediary SDF. We test whether pricing of idiosyncratic risk is arising from these friction present in pricing models based on marginal aggregate household wealth.

First, we take a single stock approach. We test whether the measure of IVOL are significantly different based on three different factor sets: (i) intermediary leverage mimicking factor (LMF), (ii) LMF and market return, (iii) Fama French three factors. We find that the IVOL measures are very similar with different factors. The similarity continues to use of no pricing model in IVOL measures. A possible reason could be the pricing models fail to separate systematic risk from total risk and defining total risk as idiosyncratic risk.

We continue the analysis with portfolio approach. We use the LMF as a measure of IVOL and form quintile IVOL portfolios. We estimate alphas of these portfolios using aforementioned factor sets. Here, we find positive alphas for lower IVOL portfolios and negative alphas for the highest IVOL portfolio. Alphas generated by LMF are higher than alphas generated by the other two factor sets. While the intermediary asset pricing model reduces some mispricing for high IVOL stock, it cannot entirely remove IVOL premia.

The rest of the paper is organized as follows: section 2 discusses related literature, section 3 defines data and methodology, section 4 provides results, section 5 concludes.

2. Literature

Earlier theoretical studies by Levy (1978) and Merton (1987) particularly focus on imperfect diversification by investors. They argue that assumptions about frictionless market or no transaction costs, complete information, and optimizing agents do not hold, leading to empirical anomalies. They propose IVOL as a pricing factor for an extension to CAPM. In empirical studies on similar thread, Tinic and West (1986) analyze 20 market beta portfolios and provide evidence that portfolios with higher IVOL generate higher return. Malkiel and Xu (1997) form 100 market beta and size portfolios to control for size and reach same conclusion. These studies do not look at IVOL on firm level.

Lehmann (1990) analyzes firm level IVOL and return and finds positive relationship between return and IVOL. The relationship can change sign in different econometric specification. Malkiel and Xu (2002) form 200 market beta and size portfolios and use IVOL of a single stock in each portfolio as the IVOL of corresponding portfolio. They find a positive IVOL premia.

The positive IVOL premia is justified by the fact that households cannot achieve perfect diversification. Barber and Odean (2000) collect data on 66,465 household from a discount broker during 1991 to 1996 and find that households invest in 4.3 stocks on average and the median is 2.61 stocks. Imperfect diversification happens for a number of reasons. Bonaparte and Cooper (2009) argue that portfolio optimization is costly. They draw attention to large fixed and quadratic cost of adjustment. Such large adjustment costs lead households not to optimize their portfolios dynamically.

In some cases, investors choose to invest in familiar stocks and keep their portfolios under-diversified. Huberman (2001) provides evidence that investors are more likely to invest in companies operating in their regions. He argues that such geographic bias often leads households to ignore the principles of portfolio theory and keep their portfolios concentrated. Benartzi and Thaler (2001) analyze how individuals allocate assets in defined contribution saving plans and report that individuals place a disproportionate amount of funds in companies they work. Benartzi (2001) provide additional evidence that employees allocate 20-30% of their discretionary funds in companies they work.

Theories considering under-diversification, report a positive relationship between idiosyncratic risk and expected stock return[3]. Goyal and Santa-Clara (2003) suggest that investors' non-traded assets add "background risk" in their portfolio decisions. The relationship of such risks with total risks of individual stocks leads to a tradeoff between market return and average stock risk. Various studies show that investors hold non-traded undiversifiable assets. Heaton and Lucas (2000) argue that human capital and private businesses constitute a large portion of an individual's portfolio and these are undiversifiable assets. Storesletten, Telmer and Yaron (2001) provide evidence that IVOL in labor income is persistent and can explain equity returns in their study on income and consumption inequality. Moskowitz and Vissing-Jorgensen (2002) report that private equity is consistently dominant over public equity in the US. Investors hold a significant amount of private equity in their highly concentrated portfolios. Goetzmann and

[3] See Lintner (1965), Douglas (1968), Lehmann (1990), Xu and Malkiel (2002), Goyal and Santa-Clara (2003), Ghysels, Santa-Clara and Valkanov (2004), Fu (2005), and Fu (2009).

Kumar (2008) argue that majority of household investors hold very few stocks and fails to form a well-diversified portfolio.

There also exist studies that come to different conclusions about the sign of IVOL premia. Baker and Wurgler (2006) conclude that conditional on investor sentiment idiosyncratic risk can be positively or negatively correlated with the expected returns. When sentiment is low (high) high IVOL stocks earn high (low) returns.

Guo and Savickas (2006) and AHXZ (2006) find a negative relationship between returns and idiosyncratic risk. Guo and Savickas (2006) use value-weighted IVOL and aggregate market volatility jointly and find that IVOL is negatively related to future stock market returns. They argue that IVOL is a pervasive macro variable and it's forecasting power is similar to consumption-wealth ratio.

AHXZ (2006) estimate IVOL from individual stocks based on residuals from three-factor Fama-French (1993) model. Then, they sort portfolios ranked on IVOL and estimate the difference in average returns between high and low IVOL portfolios. They find strong negative IVOL premia. AHXZ (2009) later do the same analysis on international stocks and report similar results. Spiegel and Wang (2005) report similar finding in a related research. They argue that idiosyncratic risk and liquidity are negatively correlated and studies interaction between these factors. They conclude that idiosyncratic risk has much stronger explanatory power than liquidity and idiosyncratic risk eliminates the explanatory power of liquidity in some cases.

Bali, Cakici, Yan, and Zhang (2005), and Bali and Cakici (2008) argue that idiosyncratic risk is not priced in the market. The pricing effects reported in the earlier studies are mere measurement error. AHXZ (2006) finding is biased by return reversal of a subset of small stocks with very high idiosyncratic risks. Fu (2009) proposes improvement in idiosyncratic volatility measurement. He uses exponential GARCH model to measure idiosyncratic risk and reports a significantly positive relationship with expected stock return.

A study by Bartram, Brown and Stulz (2018) argue that there has been a dramatic change in number and composition of listed firms since 1990s. Listed firms are now larger and older, which have lower idiosyncratic risks. This change has reduced average IVOL in recent years and contributes to high market model R-squareds.

3. Methodology and Data

3.1. Leverage mimicking factor

We employ standard factor based idiosyncratic volatility measures AHXZ (2006) Fama-French factor variants and new measure based on broker-dealer leverage as proposed by AEM (2014). Broker-dealer leverage as defined by AEM (2014) is,

$$leverage^{BD} = \frac{Total\ Financial\ Assets}{Total\ Financial\ Assets - Total\ Liabilities}, \tag{1}$$

from the Federal Reserve System flow of funds, (Financial Accounts of the United States-Z.1) accounts table L.130 published quarterly.[4] The broker-dealer factor is estimated as the

[4] $Total\ Financial\ Assets = Line\ 1 - Line\ 3; Total\ Liabilities = Line\ 14 - Line\ 3 + Line\ 22$

seasonally adjusted log changes in broker dealer leverage by taking the residual from regressing changes in log leverage on quarterly dummy variables,

$$LevFac = \Delta \ln(leverage^{BD}) - \gamma_q d_q. \tag{2}$$

Estimated $Levfac$ is presented in figure 1. The measure is 97% correlated with the measure used by AEM (2014)[5].

The quarterly leverage factor is converted to daily values needed for our idiosyncratic volatility measures through the use of factor mimicking portfolios. The leverage-mimicking factor LMF is from a moving regression of 20 quarterly excess returns r on 20 quarters of leverage,

$$r = a + b\, LevFac + \epsilon, \tag{3}$$

to obtain an N-dimensional vector of factor betas b for each quarter. The betas for each quarter are placed into deciles determining a securities portfolio location of the following quarter. Daily and monthly averages of security returns comprise the returns of each of the 10 either daily or monthly factor portfolios. The leverage-mimicking factor is an arbitrage portfolio that is long in high beta securities and short in low beta securities (1 minus 10 portfolio). Figure 2 reports quarterly returns and monthly returns from the LMF. LMF and $LevFac$ have an 80% correlation and a simple time-series relation,

[5] Tyler Muir made the factor available at http://faculty.som.yale.edu/tylermuir/LEVERAGEFACTORDATA_001.txt. The correlation is not one because of FED data revisions.

$$LMF = \underset{(.04)}{0.0005} + \underset{(17.49)}{0.0109}\, LevFac + \epsilon; \quad R^2 = 64\%. \tag{4}$$

where t-stats are in parentheses.

3.2. Idiosyncratic Volatility in Cross-Section:

For each firm in each month a regression of daily excess returns, r, is estimated

$$r = a_k + \mathrm{B} f_k + \varepsilon_k, \tag{5}$$

with four factor sets to obtain $IVOL$ estimates, $\sqrt{T}\sigma_{\varepsilon_k}$, which is the residual standard deviation scaled by the number of non-missing return days in each month. Firms must have at least 15 days of non-missing returns for the month to be included in a portfolio for the following month. The sets of IVOL estimates from the prior month place firms into a particular quintile portfolio for the following month. The factor sets are, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3], [\text{FF3 then LMF}]\}$, where LMF is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$ and the last factor set estimates $IVOL$ by regressing the errors from FF3 on LMP. We rank stocks based on previous month's IVOL estimation and form IVOL quintile portfolios for further analysis.

Then IVOL is defined as the standard deviation of residuals, multiplied with square-root of number of observations available in each month.

We are using CRSP monthly and daily return data for the period 1962 through 2016. The sample consists of NYSE, AMEX, and NASD stocks. The Fama-French factor data are obtained from the website of Kenneth R. French[6]. Risk-free rate is defined as one-month Treasury bill rate at the beginning of each month.

4. Results

4.1. Return and betas of IVOL portfolios

Portfolio mean returns increase first and then decrease as idiosyncratic volatility increase. The returns are at their lowest for the highest and lowest IVOL portfolios as reported in Table 1.[7] An equal weighted arbitrage portfolio formed by taking long position in high IVOL quintile stocks and short position in low IVOL quintile stocks generate insignificant returns, but a value weighted one generates negative returns. One possible reason is higher valued stocks in high IVOL quintile generate much lower returns than the low valued stocks.

Portfolio betas can shed some light on the trend of increasing and decreasing return behavior. Betas of IVOL portfolios are reported in Table 2. Betas provide a measure of portfolio volatility relative to the market. Beta coefficients steadily increase as IVOL of stocks in quintile portfolios increase. However, returns start decreasing in quintile 4 and altogether vanishes in highest quintile value-weighted portfolios. While portfolio betas can justify the increases in return across portfolios, it cannot do so for the decreases.

The arbitrage portfolios display less volatility than the market. Also, portfolios formed on different IVOL estimates do not generate much different returns or betas.

[6] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html#Benchmarks
[7] The return trend is very similar to the findings in literature, e.g. AHXZ (2006), Bali and Cakici (2008).

4.2.1. Distribution of IVOL portfolios

Strong similarity of portfolios across different IVOL measures indicates that firms are assigned the same ranks by different IVOL estimation techniques. We test whether different IVOL estimation methods have any effect on stock assignment in different portfolios. For this test we generate another set of IVOL quintile portfolios. Here, we refrain from using any factor model and define IVOL estimate as $\sqrt{T}\sigma_r$.

We check how the stocks are distributed in quintile portfolios using these five measures. Table 3 shows the probability distribution and cumulative probability distribution of number of matches across the five measures. Perfect matches in ranks across all five techniques are achieved 66.30% of times, whereas an independent distribution would achieve such match in 0.16% of times.

A two-sample Kolmogorov-Smirnov (K-S) test is used to test whether the observed distribution is different from an independent distribution. The K-S statistic is defined as

$$D_{n,m} = \underset{x}{Sup}\left|F_{i,n}(x) - F_{o,m}(x)\right|, \tag{6}$$

where sup is the supremum function, $F_{i,n}(x)$ is the cumulative distribution function of an independent distribution for n number of observations, and $F_{o,m}(x)$ is that of the observed distribution of for n number of stock-months. We generate the independent distribution using 2112286 randomly generated ranks, which is the same as the number of stock-month observations. The critical value is defined as

$$D - crit = \sqrt{-\frac{1}{2}\ln\alpha\left(\frac{n+m}{nm}\right)}. \tag{7}$$

The K-S test generates a $D - stat$ of 0.7427, where $D - crit$ at $\alpha = .01$ is .0015. $D - stat > D - crit$, rejecting null hypothesis that both independent ranks and observed ranks come from the same distribution.

We also check pairwise correlation of IVOL ranking among these five measures. The correlation matrix is reported in Table 4. Each pair of rankings is more than 90% correlated, the smallest one is 91.99% for FF-LMF and $\sqrt{T}\sigma_r$, and the largest one is 98% for LMF and $\sqrt{T}\sigma_r$.

IVOL portfolios estimated from different factor sets give very similar rankings. Control for systematic risk is ineffective in separating idiosyncratic risk from total risk in individual stocks. Thus, choice of a particular factor set (or none) does not influence how IVOL quintile portfolios are formed. It becomes arbitrary. We are going to use IVOL quintiles generated using LMF in further analyses.[8]

4.2.2. Distribution controlling for market capitalization

We further check whether the strong similarity among stock rankings are size specific. For this purpose, stocks are first distributed in size quintile portfolios. Then, we compare distribution of stocks with an independent distribution for each size quintiles. These are provided in Table 3. Percentages of perfect matches steadily increase from 52% for the largest size quintiles to 73%

[8] Of the three major candidates, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, FF3 achieves the highest average R^2 at 24.19% and LMF the lowest at 5.54%. We should not draw any conclusion from there because we are looking at regressions of 15 to 23 observations, where $FF3$ uses three independent variables and LMF uses one.

for the smallest size quintiles. A possible reason is that the smallest and lowest priced stocks add noise to IVOL measures. A minimum tick of $1/8 greatly affects the returns of these firms.

Two sample K-S test reject null hypothesis that both independent ranks and observed ranks come from the same distribution for all size quintiles. The test statistics and critical values at $\alpha = .01$ are also provided in Table 3.

Table 4 presents the correlation matrices of IVOL rankings within each size quintiles. Pairwise correlation among these five measures range from 88% to 99%. Correlations are slightly lower for larger size quintiles, and vice versa.

4.3.1. Alphas of IVOL portfolios

We estimate portfolio alphas as a measure of abnormal return from IVOL quintile portfolios. Time series alphas of equal & value weighted IVOL quintile portfolios are estimated by regressing excess return on three different factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$. The results are reported in Table 5.

Portfolio alphas show somewhat similar trend as returns. They increase as we move from the lowest IVOL quintile to the higher ones, and then they decrease. Equal weighted portfolios generate positive time-series alphas for all three factor sets. Value-weighted portfolios starts at positive (or statistically insignificant) for lower IVOL portfolios. The alphas stay very close for three quintiles. Then, they drop and get negative. Particularly, the high IVOL portfolio generates very large negative alpha.

An arbitrage portfolio formed by buying high IVOL stocks and selling low IVOL ones generate negative alphas in all cases, except for equal weighted portfolio regressed on single LMF factor. Alphas from $FF3$ regressions are somewhat similar to those from LMF and r_{MKT} regressions in value-weighted portfolios.

LMF alphas are higher than $FF3$ and $LMF - mkt$ in all cases. They are statistically insignificant for higher IVOL portfolios, whereas $FF3$ and $LMF - mkt$ alphas are insignificant for lower IVOL portfolios.

We run a two-pass regression of excess return and the factor sets to see whether the differences in alphas can be attributed to some weakness in the model. First, we regress monthly excess returns on the factor sets for each portfolio. Then, we regress time-series mean excess returns of these six portfolios on factor coefficients from the first step. The results of the second step regressions are reported in Table 6. LMF is a statistically significant factor, when the r_{MKT} factor is included in the regression.

We also check how close the predicted and actual returns are in Figure 3. Realized mean return is higher than predicted by LMF and lower than predicted by FF3 and LMF-mkt. The predicted return of the hedge portfolio by LMF is very close to the realized return.

4.3.2. Alphas controlling for market capitalization

Bali and Cakici (2008) report negative correlation between firm size and idiosyncratic volatility. There is a possibility that firm sizes are contributing to the trend present in IVOL alphas. Small firms are highly concentrated in the higher IVOL portfolios. So, we estimate time-series alphas for double sorted portfolios. Every month, five portfolios are formed on size. Then, IVOL quintile portfolios are formed within each size quintile based on previous month's stock IVOL. Alphas are estimated by regressing portfolio excess return on the factor sets used earlier. The results are presented in Table 7.

Similar to IVOL quintile portfolios, alphas are lower for the highest and lowest size quintiles. Within each size quintile alphas are also lower for the highest and lowest IVOL quintiles. LMF alphas are statistically insignificant for the highest IVOL portfolios, but positive and significant for other portfolios. Adding the market factor to LMF decreases alphas for all portfolios, leading to smaller alphas for lower IVOL portfolios, but large negative alphas for the highest IVOL portfolios.

Regression with $FF3$ generates alphas very similar to those of $LMF - mkt$ for the highest IVOL portfolios. Alphas for other portfolios are even smaller and statistically insignificant in some cases. Arbitrage portfolios, formed by taking a long position in the lowest size-highest IVOL portfolio and a short position in the highest size-lowest IVOL portfolio, are used to assess return differential between high and low IVOL stocks. An equal weighted arbitrage portfolio generate positive and very similar alphas with the three factor sets. The value-weighted portfolio does not generate any significant alpha.

A two-pass regression displayed in Table 8 shows that LMF is a statistically significant priced risk factor. Mean-absolute pricing error of the single factor model is also very close to those obtained by, $f_k = \{[LMF, r_{MKT}], [FF3]\}$. Figure 4 shows that the predicted returns by a single LMF factor is closer to realized mean returns than the two factor or three factor predicted returns.

4.3.3. Alphas screening for price and market capitalization

As a more direct way of handling added IVOL noise from smallest and lowest priced stocks, we exclude stocks valued less than $5 at 1982-84 dollars and stocks with market capitalization less than $5 million. Estimated alphas from IVOL quintile portfolios formed from this sample are in Table 9. While the trend is similar to what we found for the whole sample, alphas for the highest IVOL portfolio and the arbitrage portfolio are much smaller in size. LMF factor generate statistically insignificant alpha for the equal weighted arbitrage portfolio, but a negative one for the value weighted one. However, these alphas are smaller in magnitude than those generated by $f_k = \{[LMF, r_{MKT}], [FF3]\}$.

4.4. Robustness checks

This section confirms that stock distribution among different IVOL quintile portfolios using all five measures are persistent through time. We also show that IVOL risk premia holds across different subsamples.

4.4.1. Stock distribution among IVOL quintile portfolios across time

There is a possibility that the observed similarity in stock distribution among IVOL quintile portfolios by the five measures are not persistent over time. We estimate the percentage of stocks assignment in quintile rank matches by the five measures. The results are provided in Figure 5. The figure shows that around 70% stocks get perfect matches in quintile IVOL ranks over time. Perfect matches are slightly lower after 2000, but it is still very large.

4.4.2. Alphas in time subsamples

Since there is a shift in quintile rank matches in stock distribution after 2000, we estimate time series alphas in two subsamples, one from October 1972 through December 2000, and the other from January 2001 through December 2016. The results are provided in Table 10. While the negative alphas of highest IVOL portfolio and the arbitrage portfolio are present in both subsamples, differences in magnitude is noticeable. During the time of lower perfect matches in quintile rank matches, the alphas are lower.

The reduction in magnitude of alphas align with findings of Bartram, Brown and Stulz (2018). They argue that average idiosyncratic risk has been lower in recent years. They also argue that higher number of large and old listed firms are contributing to lower risk and better pricing model fits.

4.4.3. Alphas in portfolio match subsamples

We estimate time series alphas for subsamples of stocks based on quintile portfolio matches using different measures. The subsamples range from stocks achieving the same quintile rank by all five measures to stocks achieving unique quintile ranks. Table 11 provides a set of subsamples based on portfolio rank matches. Panel A provides IVOL quintile alphas from portfolios formed from subsample of stocks receiving the same IVOL quintile rank by all five measures. Portfolios in Panel B is formed with the rest of the stocks. Although the negative alphas persist, they are higher in magnitude for the perfect match subsample. Panel C to panel E divide panel B subsample by 4 matches to 2 matches by five IVOL measures. All these subsamples produce negative alphas. While the alphas rise in magnitude as number of matches increase, subsample of match=4 generate larger alphas than the perfect match, the rest of the subsamples shows diminishing trend in magnitude as we move from higher matches to lower matches.

We also test whether negative risk premia of IVOL are model specific in IVOL measures. So, we prepare a subsample of stocks based on IVOL ranking mismatch by $FF3$ and LMF. We form portfolios based on $FF3$ IVOL and LMF IVOL separately and estimate alphas. the results are provided in Table 12. We find that the alphas are still negative and very similar in both cases. However, they are smaller in magnitude than in the case of whole sample.

5. Conclusion

Idiosyncratic risk premia is a puzzle in empirical asset pricing research. Standard asset pricing theories suggest that only systematic risk is priced. But empirical findings indicate the presence of an IVOL premia. The most common explanation for the IVOL premia is under-diversification by households. Under-diversification can arise from lack of participation, dynamic optimization, knowledge of trading knowledge, and transaction costs. An IAPM uses an SDF linked to the marginal value of aggregate wealth of financial intermediaries to correct the SDF for such shortcomings of its standard form.

This study uses an IAPM proposed by AEM (2014) in estimation of IVOL and IVOL premia to test whether an under-diversification corrected SDF reduces IVOL premia. First, we test whether the IVOL rankings are model dependent. We compare IVOL ranks based on different factor sets. We find that the ranks are very similar regardless of chosen factor sets or simple standard deviation of daily returns. Then we estimate alphas of quintile portfolios ranked on previously estimated IVOL measures. While the size of high IVOL portfolio alphas are smaller in an IAPM, they persist. One possible reason is that while IAMP can account for under-diversification, it cannot solve the mispricing arising from background risks. We further control for size and price and reach the same conclusion.

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Appendix: Figures and Tables

Figure 1: Quarterly Leverage Factor (LevFac)

This figure shows the values of leverage factor ($LevFac$) from 1968 through 2016. These values are quarterly. $LevFac$ is estimated as seasonally adjusted log changes in broker-dealer leverage, $LevFac = \Delta \ln(leverage^{BD}) - \gamma_q d_q$, where $leverage^{BD} = \frac{Total\ Financial\ Assets}{Total\ Financial\ Assets - Total\ Liabilities}$ and d_q is quarterly dummy variable. Data are collected from U.S. Flow of Funds Table L.130.

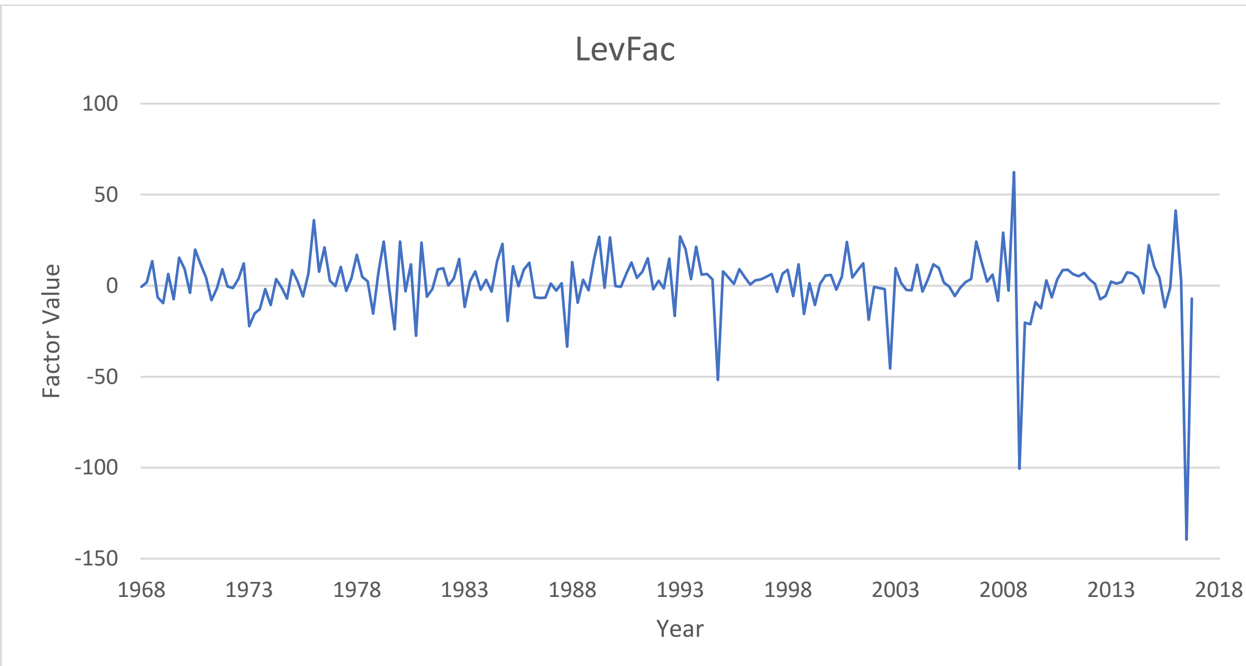
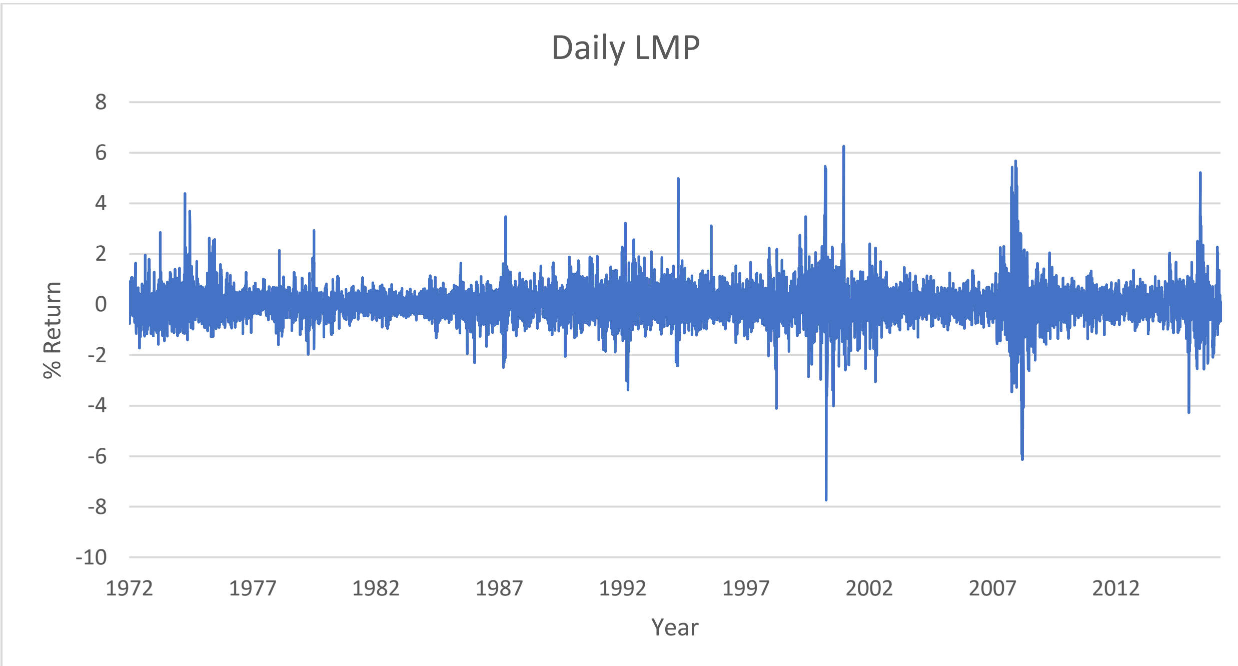


Figure 2: Leverage factor-mimicking portfolio (LMP) performance

These figures depict daily and monthly returns of Leverage factor-mimicking portfolios from October 1972 through December 2016. For each firm in each quarter a 20-quarter rolling window regression of past quarterly excess returns, r, is estimated, $r = a + b\ LevFac + \epsilon$. The coefficient from this regression places a firm to a decile portfolio for the next quarter. Then, a hedge portfolio is formed by buying the equally weighted high decile stocks and selling the low decile ones at the beginning of each quarter.



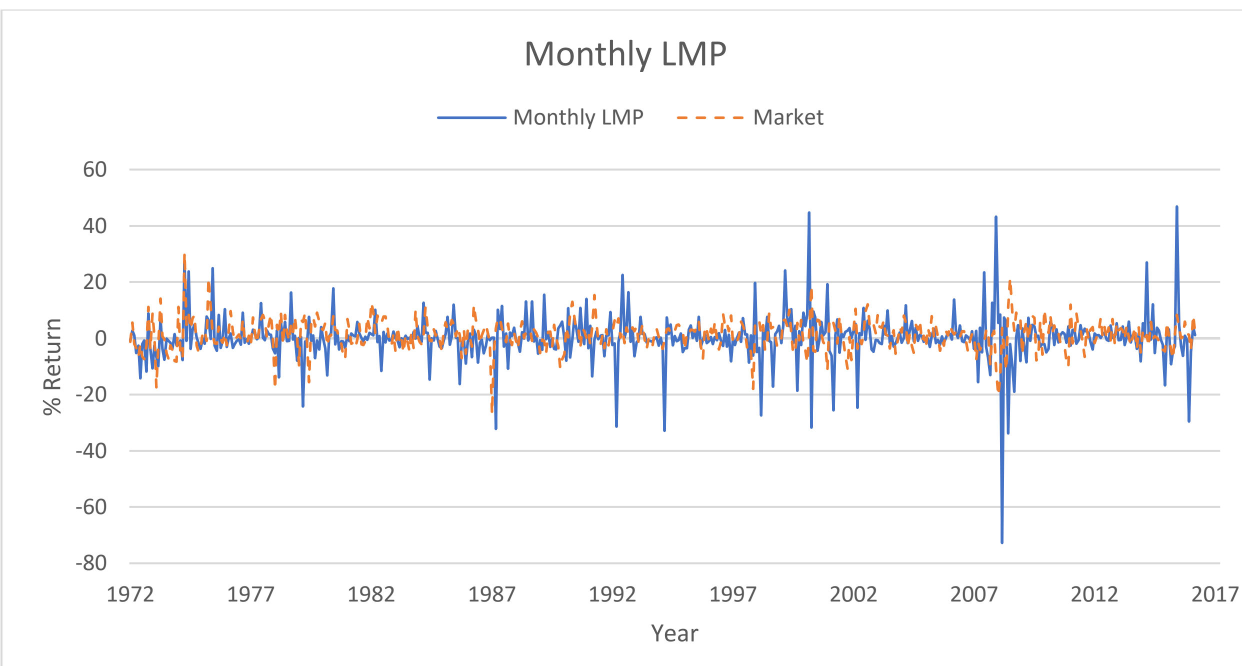


Figure 3: Predicted and realized mean returns using different factor sets

This figure pairs realized time-series mean returns of IVOL portfolios and expected returns from October 1972 through December 2016. X axis is realized mean return and y axis is predicted expected return. Expected returns are estimated using three factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return defined in section 3.1, r_{MKT} is the CRSP value weighted return, and FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$.

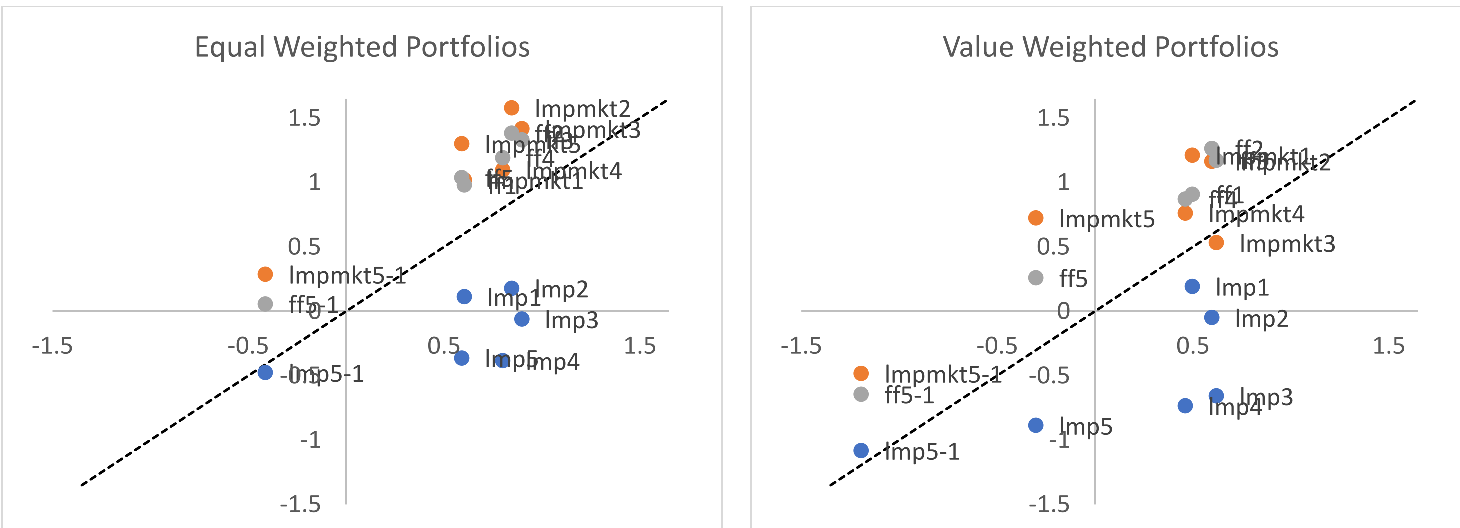
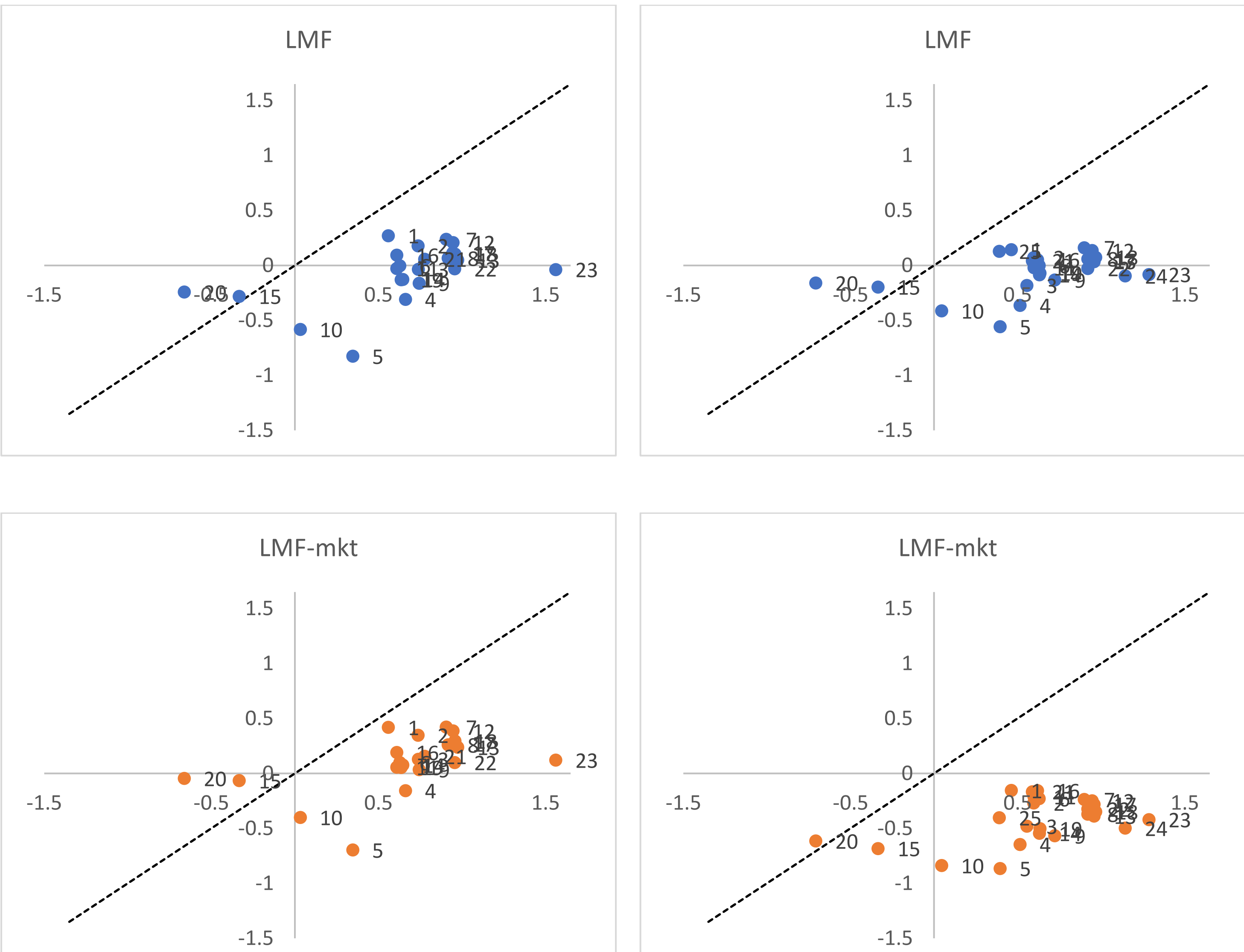


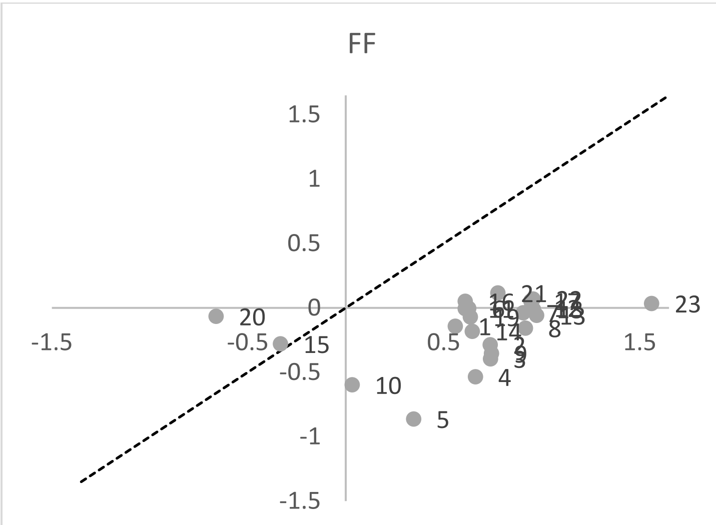
Figure 4: Predicted and realized mean returns of size-IVOL portfolios

This figure pairs realized time-series mean returns of size-IVOL portfolios and expected returns from October 1972 through December 2016. X axis is realized mean return and y axis is predicted expected return. Expected returns are estimated using three factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return defined in section 3.1, r_{MKT} is the CRSP value weighted return, and FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$.

(a) Equal Weighted Portfolios

(b) Value Weighted portfolios

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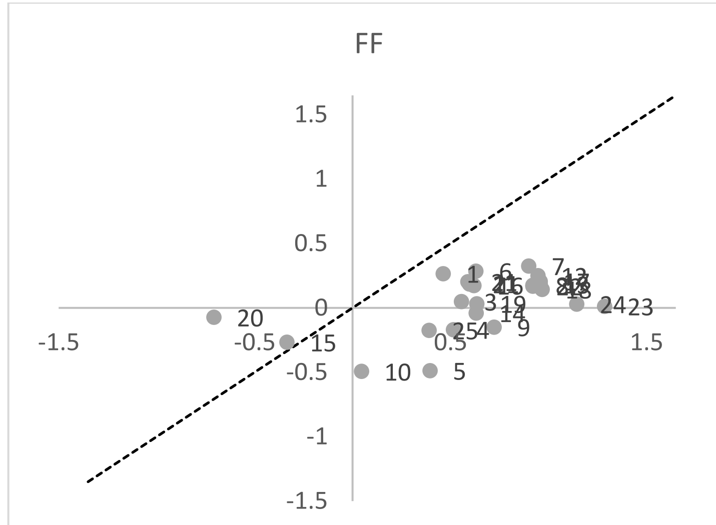
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Figure 5: Quintile portfolio distribution across time using different IVOL estimation models

This figure depicts percentage of individual assets achieving the same quintile rank using the four different idiosyncratic volatility estimation discussed section 3.2 and a fifth IVOL estimate defined as $\sqrt{T}\sigma_r$ for each stock over time. No match is where five measures assign a stock in five different quintiles and perfect match is where five measures assign a stock in the same quintile.

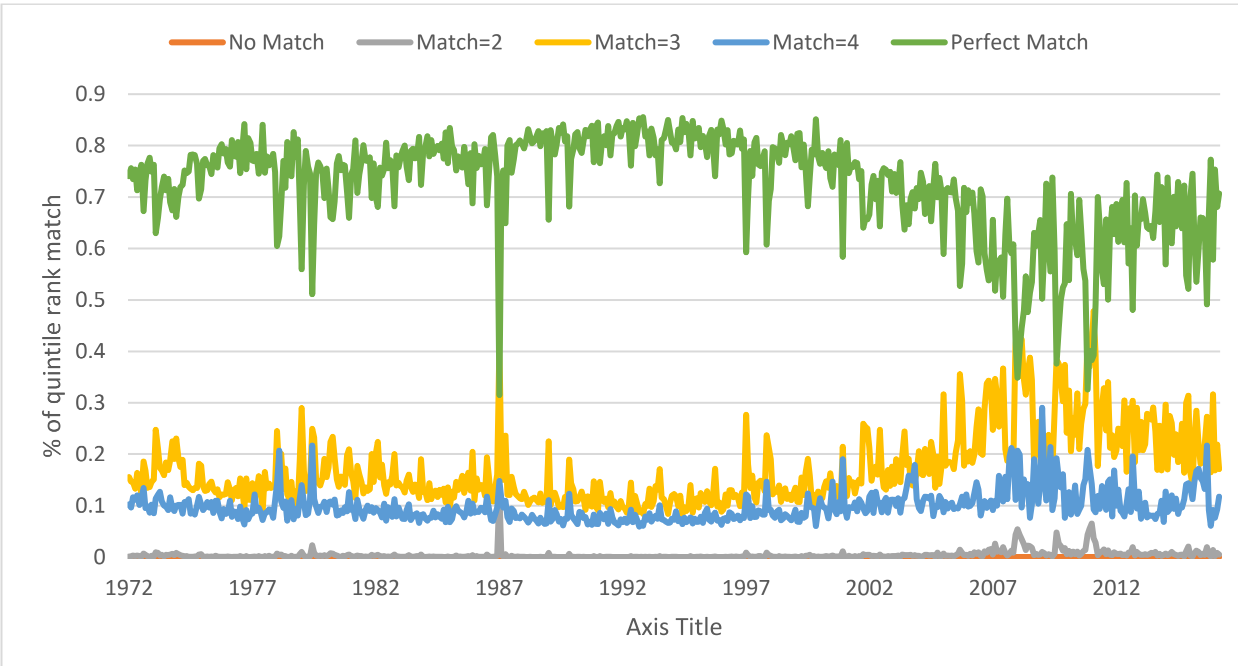

Table 1: Mean Return of Idiosyncratic Volatility Portfolios

Table depicts mean percent monthly return of equal & value weighted quintile portfolios formed on idiosyncratic volatility ($IVOL$) from October 1972 through December 2016. For each firm in each month a regression of daily excess returns, r, is estimated, $r = a_k + \mathrm{B} f_k + \varepsilon_k$,with four factor sets to obtain $IVOL$ estimates, $\sqrt{T}\sigma_{\varepsilon_k}$, where T is the number of non-missing return days in each month. Firms must have at least 15 days of non-missing returns for the month to be included in a portfolio for the following month. The sets of IVOL estimates from the prior month place firms into a particular quintile portfolio for the following month. The factor sets are, $f_k = \{[LMP], [LMP, r_{MKT}], [FF3], [\text{FF3 then LMF}]\}$, where LMP is the leverage factor mimicking return defined in section 3.1, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$ and the last factor set collects $IVOL$ by regressing the errors from FF3 on LMP.

Values are in percentages per month. Values in parentheses are Newey-West standard errors. * is significance at 10%, ** is at 5%, and *** is at 1%.

	Equal Weighted Portfolios				**Value Weighted Portfolios**			
	LMF	LMF-mkt	FF	FF-LMF	LMF	LMF-mkt	FF	FF-LMF
Low	1.00***	1.00***	1.00***	1.00***	0.89***	0.95***	0.94***	0.94***
	[0.14]	[0.14]	[0.14]	[0.14]	[0.15]	[0.16]	[0.16]	[0.16]
2	1.24***	1.23***	1.23***	1.24***	0.99***	0.97***	0.98***	0.98***
	[0.20]	[0.20]	[0.20]	[0.20]	[0.19]	[0.20]	[0.20]	[0.20]
3	1.29***	1.29***	1.28***	1.28***	1.01***	1.03***	1.05***	1.03***
	[0.24]	[0.24]	[0.24]	[0.24]	[0.24]	[0.25]	[0.25]	[0.25]
4	1.19***	1.17***	1.17***	1.17***	0.86***	0.78**	0.73**	0.75**
	[0.30]	[0.30]	[0.30]	[0.30]	[0.32]	[0.31]	[0.31]	[0.31]
Hi	0.99**	1.01***	1.01***	1.03***	0.09	0.10	0.09	0.11
	[0.39]	[0.39]	[0.39]	[0.39]	[0.39]	[0.38]	[0.38]	[0.37]
Hi-Low	-0.02	0.01	0.01	0.03	-0.80**	-0.85***	-0.85***	-0.83***
	[0.31]	[0.30]	[0.30]	[0.30]	[0.33]	[0.31]	[0.30]	[0.30]

Table 2: Beta of Idiosyncratic Volatility Portfolios

Betas of equal & value weighted quintile portfolios formed on idiosyncratic volatility (IVOL) from October 1972 through December 2016. Four different factor sets, $f_k = \{[LMP], [LMP, r_{MKT}], [FF3], [\text{FF3 then LMF}]\}$, are used to form IVOL portfolios, where LMP is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$ and the last factor set collects $IVOL$ by regressing the errors from FF3 on LMP. Monthly returns of these portfolios are regressed on CRSP value weighted monthly returns to get beta estimates.

Values in parentheses are Newey-West standard errors. * is significance at 10%, ** is at 5%, and *** is at 1%.

	Equal Weighted Portfolios				**Value Weighted Portfolios**			
	LMF	LMF-mkt	FF	FF-LMF	LMF	LMF-mkt	FF	FF-LMF
Low	0.58***	0.62***	0.62***	0.62***	0.67***	0.77***	0.77***	0.78***
	[0.03]	[0.03]	[0.03]	[0.03]	[0.02]	[0.02]	[0.02]	[0.02]
2	0.89***	0.89***	0.90***	0.90***	0.93***	0.99***	0.99***	0.99***
	[0.03]	[0.03]	[0.03]	[0.03]	[0.01]	[0.01]	[0.01]	[0.01]
3	1.09***	1.09***	1.08***	1.08***	1.19***	1.22***	1.21***	1.21***
	[0.03]	[0.03]	[0.03]	[0.03]	[0.02]	[0.02]	[0.02]	[0.02]
4	1.29***	1.27***	1.27***	1.26***	1.45***	1.39***	1.40***	1.40***
	[0.04]	[0.05]	[0.05]	[0.05]	[0.04]	[0.04]	[0.04]	[0.04]
Hi	1.40***	1.37***	1.37***	1.37***	1.61***	1.56***	1.53***	1.53***
	[0.07]	[0.07]	[0.07]	[0.07]	[0.06]	[0.07]	[0.06]	[0.06]
Hi-Low	0.83***	0.76***	0.76***	0.76***	0.94***	0.80***	0.76***	0.75***
	[0.07]	[0.07]	[0.07]	[0.07]	[0.08]	[0.08]	[0.08]	[0.07]

Table 3: Quintile portfolio distribution using different IVOL estimation models

The table depicts percentage of individual assets achieving the same quintile rank using the four different idiosyncratic volatility estimation discussed section 3.2 and a fifth IVOL estimate defined as $\sqrt{T}\sigma_r$ for each stock, 1 being no match across the five measure, and 5 being the perfect match. First, percentage of matches across size quintiles are reported. Then, matches for the whole sample and independent draws are reported. The last two rows provide the two=sample Kolmogorov-Smirnov (K-S) test statistic and critical values. The K-S test statistic is defined as, $D_{n,m} = \underset{x}{Sup}\left|F_{i,n}(x) - F_{o,m}(x)\right|$, and critical value is defined as, $D - crit = \sqrt{-\frac{1}{2}\ln\alpha\left(\frac{n+m}{nm}\right)}$, where sup is the supremum function, $F_{i,n}(x)$ is the cumulative distribution function of an independent distribution for n number of observations, and $F_{o,m}(x)$ is that of the observed distribution for n number of stock-months.

* is significance at 10%, ** is at 5%, and *** is at 1%.

	Size					Whole Sample	Random Draw
	Large	2	3	4	Small		
No Match	0.00	0.00	0.00	0.00	0.00	0.00	0.04
2	0.02	0.01	0.01	0.01	0.01	0.01	0.67
3	0.30	0.24	0.21	0.19	0.17	0.21	0.26
4	0.15	0.13	0.12	0.10	0.08	0.11	0.03
Perfect Match	0.52	0.62	0.67	0.71	0.73	0.66	0.00
$D_{n,m}$	0.6868 ***	0.7143 ***	0.7510 ***	0.7751 ***	0.7830***	0.7427***	
$D - crit$	0.0026	0.0026	0.0026	0.0026	0.0026	0.0015	

Table 4: Correlation Matrix of Portfolio Ranks across different Measures

Pairwise correlation matrix of portfolio ranks assigned by IVOL estimates from four different factor sets, $f_k = \{[LMP], [LMP, r_{MKT}], [FF3], [\text{FF3 then LMF}]\}$ and a no factor estimate defined as $\sqrt{T}\sigma_r$ for each stock. Panel A reports pairwise correlations for the whole sample. Panel B to F reports correlations for five size quintiles where (B) is the biggest size quintile and (F) is the smallest size quintile.

Values in parentheses are p-values. * is significance at 10%, ** is at 5%, and *** is at 1%.

Panel A: Whole Sample

	LMF	LMF-mkt	FF	FF-LMF	$\sqrt{T}\sigma_r$
LMF	1				
LMF-mkt	0.94 ***	1			
	[0.00]				
FF	0.93 ***	0.97 ***	1		
	[0.00]	[0.00]			
FF-LMF	0.93 ***	0.97 ***	0.99 ***	1	
	[0.00]	[0.00]	[0.00]		
$\sqrt{T}\sigma_r$	0.98 ***	0.93 ***	0.92 ***	0.92 ***	1
	[0.00]	[0.00]	[0.00]	[0.00]	

Panel B: Size-Large

	LMF	LMF-mkt	FF	FF-LMF	$\sqrt{T}\sigma_r$
LMF	1				
LMF-mkt	0.90 ***	1			
	[0.00]				
FF	0.88 ***	0.95 ***	1		
	[0.00]	[0.00]			
FF-LMF	0.89 ***	0.96 ***	0.98 ***	1	
	[0.00]	[0.00]	[0.00]		
$\sqrt{T}\sigma_r$	0.97 ***	0.89 ***	0.88 ***	0.88 ***	1
	[0.00]	[0.00]	[0.00]	[0.00]	

Panel C: Size-2

	LMF	LMF-mkt	FF	FF-LMF	$\sqrt{T}\sigma_r$
LMF	1				
LMF-mkt	0.94 ***	1			
	[0.00]				

FF	0.92 ***	0.96 ***	1		
	[0.00]	[0.00]			
FF-LMF	0.92 ***	0.96 ***	0.99 ***	1	
	[0.00]	[0.00]	[0.00]		
$\sqrt{T}\sigma_r$	0.97 ***	0.92 ***	0.91 ***	0.91 ***	1
	[0.00]	[0.00]	[0.00]	[0.00]	

Panel D: Size-3

	LMF	LMF-mkt	FF	FF-LMF	$\sqrt{T}\sigma_r$
LMF	1				
LMF-mkt	0.95 ***	1			
	[0.00]				
FF	0.93 ***	0.96 ***	1		
	[0.00]	[0.00]			
FF-LMF	0.93 ***	0.96 ***	0.99 ***	1	
	[0.00]	[0.00]	[0.00]		
$\sqrt{T}\sigma_r$	0.98 ***	0.94 ***	0.93 ***	0.92 ***	1
	[0.00]	[0.00]	[0.00]	[0.00]	

Panel E: Size-4

	LMF	LMF-mkt	FF	FF-LMF	$\sqrt{T}\sigma_r$
LMF	1				
LMF-mkt	0.96 ***	1			
	[0.00]				
FF	0.94 ***	0.97 ***	1		
	[0.00]	[0.00]			
FF-LMF	0.94 ***	0.97 ***	0.99 ***	1	
	[0.00]	[0.00]	[0.00]		
$\sqrt{T}\sigma_r$	0.98 ***	0.95 ***	0.94 ***	0.93 ***	1
	[0.00]	[0.00]	[0.00]	[0.00]	

Panel F: Size-Small

	LMF	LMF-mkt	FF	FF-LMF	$\sqrt{T}\sigma_r$
LMF	1				
LMF-mkt	0.95 ***	1			
	[0.00]				
FF	0.94 ***	0.97 ***	1		

	[0.00]	[0.00]			
FF-LMF	0.94 ***	0.97 ***	0.99 ***	1	
	[0.00]	[0.00]	[0.00]		
$\sqrt{T}\sigma_r$	0.98 ***	0.94 ***	0.93 ***	0.93 ***	1
	[0.00]	[0.00]	[0.00]	[0.00]	

Table 5: Alphas of Idiosyncratic Volatility Portfolios

Time series alphas of equal & value weighted quintile portfolios formed on idiosyncratic volatility (IVOL) from October 1972 through December 2016. Monthly returns of quintile portfolios formed on IVOL estimated using LMF in section 3.1 are regressed on three different factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$.

Values are in percentages per month. Values in parentheses are Newey-West standard errors. * is significance at 10%, ** is at 5%, and *** is at 1%.

	Equal Weighted Portfolios			**Value Weighted Portfolios**		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.60***	0.29***	0.14*	0.50***	0.14*	0.06
	[0.14]	[0.08]	[0.08]	[0.15]	[0.07]	[0.07]
2	0.84***	0.37***	0.18***	0.60***	0.10**	0.06
	[0.20]	[0.09]	[0.07]	[0.19]	[0.05]	[0.04]
3	0.90***	0.31***	0.13**	0.63**	-0.01	0.00
	[0.24]	[0.11]	[0.07]	[0.24]	[0.07]	[0.07]
4	0.80***	0.11	-0.04	0.47	-0.31**	-0.27**
	[0.30]	[0.16]	[0.09]	[0.32]	[0.13]	[0.11]
Hi	0.59	-0.16	-0.36*	-0.29	-1.15***	-1.16***
	[0.39]	[0.27]	[0.20]	[0.39]	[0.22]	[0.17]
Hi-Low	-0.41	-0.86***	-0.89***	-1.18***	-1.69***	-1.61***
	[0.31]	[0.26]	[0.21]	[0.33]	[0.27]	[0.21]

Table 6: Two-pass test of pricing models used to estimate alphas

Time series regression of monthly excess return is estimated, $r = a_k + \mathrm{B}f_k + \varepsilon_k$ using three factor sets to obtain factor coefficients from October 1972 through December 2016. Then, the mean returns of each time-series are regressed on the factor coefficients. The factor sets are, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return defined in section 0, r_{MKT} is the CRSP value weighted return, and FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$.

Values in parentheses are Newey-West standard errors. * is significance at 10%, ** is at 5%, and *** is at 1%.

	Equal Weighted Portfolios			**Value Weighted Portfolios**		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Alpha	0.72***	-0.56	-0.44*	0.65*	-0.54	-0.52
	[0.12]	[0.35]	[0.12]	[0.24]	[0.64]	[0.19]
LMF	20.93	38.92**		15.93	25.72**	
	[15.96]	[8.40]		[7.91]	[4.68]	
MKT		1.01*	1.47*		1.08	1.22**
		[0.35]	[0.34]		[0.68]	[0.18]
SMB			-0.56			-1.59**
			[0.19]			[0.29]
HML			1.09			-1.16
			[0.53]			[0.89]
R^2	15.59	77.32	96.51	33.82	52.84	95.35
MAPE	0.72	0.56	0.44	0.69	0.57	0.52

Table 7: Alphas of equal & value weighted quintile portfolios formed on Size and idiosyncratic volatility (IVOL)

Time series alphas of equal & value weighted quintile portfolios formed on size and idiosyncratic volatility (IVOL) from October 1972 through December 2016. First, quintile portfolios are formed on size. Then IVOL quintile portfolios are formed for each of the size quintiles. Monthly returns of resulting 25 portfolios are regressed on three different factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMP is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$. Alphas estimated from a single leverage factor ($f = LMF$) are reported in Panel A, a leverage factor with a market factor ($f = [LMF, r_{MKT}]$) are in Panel B, Fama-French model ($f = [MKT, HML, SMB]$) are in Panel C. Panel D reports alphas estimated from regressing excess return of an arbitrage portfolio, formed by buying small-size-high-IVOL portfolio and selling large-size-low-IVOL portfolio, on each of the three different factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$.

Values are in percentages per month. * is significance at 10%, ** is at 5%, and *** is at 1%.

Panel A: LMF factor

	Equal Weighted Portfolios					Value Weighted Portfolios				
Idio/Size	Large	2	3	4	Small	Large	2	3	4	Small
Low	0.55***	0.63***	0.61***	0.61***	0.77***	0.46***	0.63***	0.60***	0.62***	0.59***
	[0.15]	[0.15]	[0.14]	[0.15]	[0.17]	[0.15]	[0.15]	[0.14]	[0.15]	[0.16]
2	0.73***	0.90***	0.94***	0.94***	0.96***	0.59***	0.89***	0.94***	0.96***	0.92***
	[0.18]	[0.20]	[0.21]	[0.22]	[0.24]	[0.18]	[0.20]	[0.21]	[0.22]	[0.24]
3	0.74***	0.92***	0.97***	0.96***	1.56***	0.56***	0.92***	0.96***	0.97***	1.29***
	[0.21]	[0.25]	[0.26]	[0.27]	[0.31]	[0.21]	[0.25]	[0.26]	[0.27]	[0.32]
4	0.67***	0.75**	0.65**	0.64*	1.79***	0.52**	0.73**	0.63**	0.64*	1.15***
	[0.25]	[0.30]	[0.32]	[0.33]	[0.38]	[0.25]	[0.30]	[0.32]	[0.33]	[0.38]
High	0.36	0.04	-0.33	-0.66	2.12***	0.41	0.05	-0.33	-0.70*	0.39
	[0.35]	[0.39]	[0.39]	[0.40]	[0.49]	[0.33]	[0.39]	[0.39]	[0.40]	[0.45]

Panel B: LMF & MKT factors

	Equal Weighted Portfolios					Value Weighted Portfolios				
Idio/Size	Large	2	3	4	Small	Large	2	3	4	Small
Low	0.21***	0.30***	0.32***	0.34***	0.49***	0.11	0.29***	0.31***	0.34***	0.31**
	[0.08]	[0.09]	[0.10]	[0.11]	[0.14]	[0.08]	[0.09]	[0.10]	[0.11]	[0.13]
2	0.27***	0.42***	0.46***	0.50***	0.52***	0.14**	0.41***	0.46***	0.50***	0.47***
	[0.07]	[0.10]	[0.12]	[0.14]	[0.18]	[0.06]	[0.10]	[0.12]	[0.14]	[0.18]
3	0.18***	0.32***	0.38***	0.39**	1.03***	0.01	0.32***	0.36**	0.39**	0.74***
	[0.06]	[0.11]	[0.14]	[0.17]	[0.25]	[0.05]	[0.11]	[0.14]	[0.17]	[0.24]
4	0.01	0.03	-0.07	-0.03	1.17***	-0.12	0.00	-0.10	-0.05	0.49*
	[0.08]	[0.14]	[0.17]	[0.22]	[0.31]	[0.08]	[0.14]	[0.18]	[0.22]	[0.29]
High	-0.47***	-0.84***	-1.17***	-1.42***	1.42***	-0.39**	-0.83***	-1.17***	-1.47***	-0.32
	[0.16]	[0.21]	[0.25]	[0.29]	[0.42]	[0.15]	[0.21]	[0.25]	[0.29]	[0.37]

Panel C: FF factors

	Equal Weighted Portfolios					Value Weighted Portfolios				
Idio/Size	Large	2	3	4	Small	Large	2	3	4	Small
Low	0.06	0.11	0.17**	0.18*	0.29**	0.04	0.10	0.16*	0.19**	0.12
	[0.07]	[0.07]	[0.08]	[0.09]	[0.12]	[0.07]	[0.07]	[0.08]	[0.09]	[0.11]
2	0.14**	0.18***	0.23***	0.26**	0.28*	0.11**	0.18***	0.23***	0.27***	0.24
	[0.06]	[0.07]	[0.08]	[0.11]	[0.15]	[0.05]	[0.07]	[0.08]	[0.10]	[0.15]
3	0.07	0.11*	0.13	0.15	0.80***	0.01	0.12*	0.11	0.15	0.52**
	[0.05]	[0.06]	[0.08]	[0.12]	[0.20]	[0.05]	[0.06]	[0.08]	[0.12]	[0.20]
4	-0.03	-0.09	-0.28***	-0.27*	0.91***	-0.07	-0.11*	-0.30***	-0.29*	0.22
	[0.06]	[0.06]	[0.09]	[0.16]	[0.26]	[0.07]	[0.06]	[0.09]	[0.16]	[0.24]

High	-0.37***	-0.87***	-1.34***	-1.68***	1.15***	-0.27**	-0.85***	-1.33***	-1.74***	-0.55*
	[0.12]	[0.13]	[0.16]	[0.21]	[0.36]	[0.13]	[0.13]	[0.16]	[0.21]	[0.31]

Panel D: Arbitrage portfolio alphas

	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Alpha	1.17***	0.81*	0.69*	-0.47	-0.82**	-0.98***
	[0.45]	[0.44]	[0.39]	[0.43]	[0.41]	[0.34]

Table 8: Two-pass test for portfolios formed on size & IVOL

Time series regression of monthly excess return is estimated, $r = a_k + \mathrm{B}f_k + \varepsilon_k$ using three factor sets to obtain factor coefficients from October 1972 through December 2016 for 25 portfolios formed on size and IVOL. Then, the mean returns of each time-series are regressed on the factor coefficients. The factor sets are, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return defined in section 3.1, r_{MKT} is the CRSP value weighted return, and FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$.

Values in parentheses are t-stats. * is significance at 10%, ** is at 5%, and *** is at 1%.

	Equal Weighted Portfolios			**Value Weighted Portfolios**		
	LMP	LMP-mkt	FF	LMP	LMP-mkt	FF
Alpha	0.81***	0.64	0.90	0.67***	1.03***	0.57
	[0.11]	[0.47]	[126.80]	[0.09]	[0.33]	[110.93]
LMF	11.41**	13.13*		7.46**	4.28	
	[4.83]	[6.86]		[3.44]	[4.41]	
MKT		0.03	-0.58		-0.41	0.02
		[0.41]	[81.89]		[0.30]	[3.42]
SMB			0.29			-0.24
			[0.32]			[0.21]
HML			0.85			1.06
			[0.87]			[0.63]
R^2	16%	12.7%	6.48%	13.36%	14.45%	16.04%
MAPE	0.85	0.71	0.95	0.72	1.04	0.63

Table 9: Alphas of IVOL Portfolios after screening for price and market capitalization

Time series alphas of equal & value weighted quintile portfolios formed on idiosyncratic volatility (IVOL) from October 1972 through December 2016. Stocks priced at less than $5 in 1982-1984 dollars and less than $5 million in market capitalization are excluded. Monthly returns of quintile portfolios formed on IVOL estimated using LMF in section 3.1 are regressed on three different factor sets, $f_k = \{[LMP], [LMP, r_{MKT}], [FF3]\}$, where LMP is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$.

Values are in percentages per month. * is significance at 10%, ** is at 5%, and *** is at 1%.

	Equal Weighted Portfolios			**Value Weighted Portfolios**		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.59***	0.27***	0.14*	0.48***	0.12	0.04
	[0.14]	[0.08]	[0.08]	[0.15]	[0.07]	[0.07]
2	0.85***	0.39***	0.22***	0.58***	0.11*	0.07
	[0.19]	[0.08]	[0.07]	[0.18]	[0.06]	[0.05]
3	0.95***	0.39***	0.22***	0.60***	0.01	-0.01
	[0.22]	[0.09]	[0.06]	[0.22]	[0.06]	[0.06]
4	0.92***	0.26**	0.17***	0.49*	-0.21**	-0.15*
	[0.27]	[0.12]	[0.05]	[0.28]	[0.10]	[0.09]
Hi	0.60*	-0.16	-0.20*	0.05	-0.78***	-0.71***
	[0.34]	[0.19]	[0.10]	[0.36]	[0.18]	[0.15]
Hi-Low	-0.39	-0.83***	-0.74***	-0.82***	-1.30***	-1.14***
	[0.26]	[0.20]	[0.15]	[0.29]	[0.23]	[0.19]

Table 10: Alphas of portfolios formed on IVOL across time subsamples

Time series alphas of equal & value weighted quintile portfolios formed on idiosyncratic volatility (IVOL) from October 1972 through December 2016. Monthly returns of quintile portfolios formed on IVOL estimated using LMF in section 3.1 are regressed on three different factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$. Panel A reports alphas of portfolios from October 1972 to December 2000 and panel B is from January 2001 to December 2016.

Values are in percentages per month. Values in parentheses are Newey-West standard errors. * is significance at 10%, ** is at 5%, and *** is at 1%.

Panel A: Oct 1972 to Dec 2000

	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.56***	0.21*	-0.06	0.48**	0.08	-0.08
	[0.19]	[0.11]	[0.08]	[0.20]	[0.09]	[0.08]
2	0.80***	0.30**	0.03	0.61**	0.08	0.00
	[0.25]	[0.12]	[0.09]	[0.25]	[0.06]	[0.06]
3	0.86***	0.26*	0.06	0.72**	0.07	0.12
	[0.31]	[0.15]	[0.09]	[0.31]	[0.09]	[0.08]
4	0.66*	-0.03	-0.13	0.51	-0.26	-0.08
	[0.38]	[0.22]	[0.10]	[0.39]	[0.17]	[0.12]
Hi	0.36	-0.35	-0.48**	-0.57	-1.37***	-1.22***
	[0.47]	[0.34]	[0.22]	[0.46]	[0.28]	[0.19]
Hi-Low	-0.75**	-1.13***	-0.99***	-1.60***	-2.01***	-1.69***
	[0.36]	[0.32]	[0.24]	[0.39]	[0.34]	[0.24]

Panel B: Jan 2001 to Dec 2016

	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.67***	0.43***	0.35***	0.51**	0.23**	0.23**
	[0.19]	[0.12]	[0.11]	[0.21]	[0.11]	[0.10]
2	0.89***	0.48***	0.34***	0.55*	0.13*	0.12*
	[0.29]	[0.12]	[0.10]	[0.29]	[0.08]	[0.07]
3	0.94**	0.41***	0.21**	0.44	-0.15	-0.19*
	[0.38]	[0.15]	[0.10]	[0.40]	[0.11]	[0.10]

4	1.01**	0.36	0.11	0.37	-0.39*	-0.47**
	[0.49]	[0.22]	[0.16]	[0.54]	[0.21]	[0.19]
Hi	0.95	0.17	-0.13	0.16	-0.77**	-0.85***
	[0.67]	[0.42]	[0.34]	[0.69]	[0.34]	[0.30]
Hi-Low	0.17	-0.38	-0.59*	-0.47	-1.12***	-1.20***
	[0.55]	[0.42]	[0.34]	[0.58]	[0.41]	[0.36]

Table 11: Alphas of portfolios formed on different stock matches

Time series alphas of equal & value weighted quintile portfolios formed on idiosyncratic volatility (IVOL) from October 1972 through December 2016. Monthly returns of quintile portfolios formed on IVOL estimated using LMF in section 3.1 are regressed on three different factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$.

Panel A reports alphas of portfolios formed from stocks that achieved the exact same quintile ranks using four different idiosyncratic volatility estimation discussed section 3.2 and a fifth IVOL estimate defined as $\sqrt{T}\sigma_r$ for each stock. Alphas of the rest of the stocks are reported in panel B. Panel C to E report alphas of portfolios formed from stocks receiving 4, 3, and 2 matches in portfolio ranks respectively.

Values are in percentages per month. Values in parentheses are Newey-West standard errors. * is significance at 10%, ** is at 5%, and *** is at 1%.

Panel A: Stock assignment to portfolio matches=5						
	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.61***	0.37***	0.24***	0.53***	0.24***	0.16**
	[0.12]	[0.08]	[0.07]	[0.13]	[0.08]	[0.08]
2	0.87***	0.42***	0.22***	0.59***	0.15**	0.06
	[0.18]	[0.09]	[0.07]	[0.18]	[0.07]	[0.07]
3	1.05***	0.46***	0.23***	0.60**	-0.02	-0.11
	[0.25]	[0.13]	[0.09]	[0.24]	[0.09]	[0.09]
4	1.13***	0.41**	0.21	0.36	-0.43**	-0.50***
	[0.34]	[0.20]	[0.14]	[0.34]	[0.17]	[0.13]
Hi	1.06**	0.27	0.04	-0.23	-1.10***	-1.16***
	[0.44]	[0.32]	[0.26]	[0.42]	[0.27]	[0.23]
Hi-Low	0.05	-0.50	-0.60**	-1.15***	-1.73***	-1.71***
	[0.37]	[0.31]	[0.26]	[0.39]	[0.32]	[0.26]

Panel B: Stock assignment to portfolio matches<5						
	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.72***	0.28***	0.14	0.60***	0.13	0.11
	[0.19]	[0.09]	[0.09]	[0.20]	[0.10]	[0.10]

2	0.79***	0.26***	0.12	0.52**	-0.06	0.01
	[0.22]	[0.09]	[0.08]	[0.24]	[0.10]	[0.10]
3	0.61**	-0.02	-0.10	0.81***	0.13	0.25**
	[0.25]	[0.10]	[0.07]	[0.28]	[0.13]	[0.13]
4	0.37	-0.30**	-0.36***	0.74**	-0.02	0.11
	[0.28]	[0.14]	[0.09]	[0.33]	[0.17]	[0.15]
Hi	-0.48	-1.20***	-1.28***	0.23	-0.66***	-0.55**
	[0.32]	[0.17]	[0.12]	[0.40]	[0.24]	[0.23]
Hi-Low	-1.60***	-1.88***	-1.81***	-0.76**	-1.18***	-1.05***
	[0.21]	[0.18]	[0.16]	[0.32]	[0.29]	[0.27]

Panel C: Stock assignment to portfolio matches=4

	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.86***	0.41***	0.23*	1.12***	0.66***	0.57***
	[0.22]	[0.15]	[0.14]	[0.26]	[0.20]	[0.20]
2	0.77***	0.20	0.03	0.64**	0.02	-0.03
	[0.26]	[0.15]	[0.13]	[0.31]	[0.21]	[0.22]
3	0.36	-0.26	-0.34**	0.54*	-0.11	-0.10
	[0.28]	[0.16]	[0.14]	[0.33]	[0.23]	[0.22]
4	0.21	-0.42**	-0.53***	0.74*	-0.02	0.05
	[0.31]	[0.20]	[0.18]	[0.42]	[0.30]	[0.30]
Hi	-0.76**	-1.41***	-1.58***	-0.06	-0.88**	-0.90**
	[0.36]	[0.26]	[0.24]	[0.49]	[0.37]	[0.37]
Hi-Low	-2.01***	-2.22***	-2.21***	-1.57***	-1.93***	-1.87***
	[0.30]	[0.28]	[0.28]	[0.46]	[0.42]	[0.43]

Panel D: Stock assignment to portfolio matches=3

	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.69***	0.23**	0.13	0.66***	0.19	0.18
	[0.20]	[0.10]	[0.11]	[0.22]	[0.13]	[0.13]
2	0.82***	0.30***	0.18**	0.57**	-0.02	0.04

	[0.22]	[0.10]	[0.09]	[0.25]	[0.13]	[0.13]
3	0.66**	0.03	-0.06	1.03***	0.33*	0.45**
	[0.26]	[0.12]	[0.09]	[0.32]	[0.18]	[0.20]
4	0.47	-0.22	-0.24**	0.64*	-0.13	-0.03
	[0.30]	[0.16]	[0.12]	[0.35]	[0.21]	[0.20]
Hi	-0.44	-1.19***	-1.24***	0.19	-0.67**	-0.55**
	[0.34]	[0.20]	[0.15]	[0.42]	[0.28]	[0.26]
Hi-Low	-1.52***	-1.82***	-1.76***	-0.86**	-1.26***	-1.12***
	[0.25]	[0.22]	[0.19]	[0.37]	[0.34]	[0.32]

Panel E: Stock assignment to portfolio matches=2

	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	-1.11	-0.90	-0.86	-0.50***	-0.59***	-0.67***
	[1.22]	[0.98]	[0.98]	[0.16]	[0.17]	[0.19]
2	-0.43	0.47	0.35	-0.53**	-0.68***	-0.72***
	[1.12]	[1.01]	[0.95]	[0.22]	[0.20]	[0.23]
3	-2.61***	-2.31***	-2.47***	-0.94***	-1.08***	-1.13***
	[0.97]	[0.79]	[0.79]	[0.21]	[0.22]	[0.24]
4	0.32	-0.33	-0.29	-0.49**	-0.60***	-0.89***
	[0.87]	[0.74]	[0.65]	[0.21]	[0.21]	[0.22]
Hi	-1.44*	-1.57**	-1.56**	-0.62***	-0.78***	-1.02***
	[0.78]	[0.66]	[0.65]	[0.22]	[0.22]	[0.21]
Hi-Low	-3.32**	-3.57***	-3.64***	-0.51**	-0.59**	-0.75***
	[1.31]	[1.21]	[1.20]	[0.24]	[0.25]	[0.25]

Table 12: Alphas of portfolios formed on stocks where FF3 and LMF IVOL rankings mismatch

Time series alphas of equal & value weighted quintile portfolios formed on idiosyncratic volatility (IVOL) from October 1972 through December 2016. Only stocks with mismatched quintile rankings assigned by FF3 and LMF estimates are included. Monthly returns of quintile portfolios formed on IVOL estimated using LMF in section 3.1 are regressed on three different factor sets, $f_k = \{[LMF], [LMF, r_{MKT}], [FF3]\}$, where LMF is the leverage factor mimicking return, r_{MKT} is the CRSP value weighted return, FF3 are the Fama-French 3 factors $[MKT, HML, SMB]$. Panel A reports alphas of quintile portfolios created by FF3 IVOL estimates. Panel B reports alphas of quintile portfolios created by LMF IVOL estimates.

Values are in percentages per month. Values in parentheses are Newey-West standard errors. * is significance at 10%, ** is at 5%, and *** is at 1%.

Panel A: FF3 sort						
	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.74***	0.22***	0.11	0.58**	0.04	0.06
	[0.21]	[0.08]	[0.07]	[0.22]	[0.11]	[0.10]
2	0.74***	0.15	0.08	0.80***	0.14	0.20
	[0.24]	[0.10]	[0.09]	[0.28]	[0.15]	[0.15]
3	0.70***	0.11	0.04	0.79***	0.10	0.25
	[0.25]	[0.12]	[0.09]	[0.30]	[0.17]	[0.16]
4	0.45	-0.19	-0.31***	0.89**	0.13	0.21
	[0.28]	[0.16]	[0.11]	[0.36]	[0.23]	[0.20]
Hi	-0.40	-1.08***	-1.13***	0.24	-0.61**	-0.50*
	[0.32]	[0.20]	[0.15]	[0.42]	[0.28]	[0.27]
Hi-Low	-1.53***	-1.69***	-1.64***	-0.74**	-1.05***	-0.95***
	[0.21]	[0.20]	[0.16]	[0.34]	[0.32]	[0.30]

Panel B: LMF sort						
	Equal Weighted Portfolios			Value Weighted Portfolios		
	LMF	LMF-mkt	FF	LMF	LMF-mkt	FF
Low	0.67***	0.22**	0.11	0.57***	0.11	0.09
	[0.19]	[0.10]	[0.10]	[0.21]	[0.12]	[0.12]
2	0.78***	0.26***	0.13	0.44*	-0.13	-0.07
	[0.22]	[0.10]	[0.09]	[0.24]	[0.13]	[0.12]
3	0.70***	0.08	0.00	0.85***	0.17	0.29*
	[0.26]	[0.11]	[0.09]	[0.29]	[0.16]	[0.16]

4	0.43	-0.25*	-0.30***	0.69**	-0.07	0.02
	[0.30]	[0.15]	[0.11]	[0.34]	[0.20]	[0.19]
Hi	-0.34	-1.07***	-1.12***	0.26	-0.61**	-0.53**
	[0.33]	[0.19]	[0.14]	[0.42]	[0.27]	[0.27]
Hi-Low	-1.40***	-1.68***	-1.62***	-0.71*	-1.12***	-1.01***
	[0.23]	[0.21]	[0.18]	[0.36]	[0.33]	[0.31]

VITA

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