Parallel-slab polarizing beam splitter and photopolarimeter

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Parallel-slab polarizing beam splitter and photopolarimeter

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A dielectric-slab polarizing beam splitter (PBS) is described that generates two parallel beams of orthogonal \( p \) and \( s \) linear polarizations in reflection and functions as a diattenuator in transmission. The plane-parallel slab, which is set at Brewster’s angle, is uncoated on one side and has an \( s \)-polarization antireflection coating (\( s \)-ARC) on the other side. Analytical results are presented for a PBS that uses a high-index slab coated with a low-index single-layer \( s \)-ARC, which is particularly suited for the IR. A novel multistage photopolarimeter that uses two such PBSs in series is described as being capable of sequential and simultaneous measurement of all four Stokes parameters of light. © 2007 Optical Society of America

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1. Introduction

Conventional polarizing beam splitters (PBSs) use crystal optics\(^1\) or multilayer interference coatings\(^2\) to divide an incident light beam into two (\( p \) and \( s \)) orthogonally linearly polarized beams that travel in different directions. In contrast with these one-to-two PBSs, a one-to-three, dielectric-slab BS is presented that produces two parallel beams of orthogonal \( p \) and \( s \) linear polarizations in reflection and a third beam in transmission. A multistage complete photopolarimeter that uses two such PBSs in a series is described for the sequential and simultaneous measurements of the first, second, and third normalized Stokes parameters of light using three pairs of detection channels. Previous designs of division-of-amplitude photopolarimeters are briefly reviewed elsewhere.\(^3\)

2. Parallel-Slab Polarizing Beam Splitter

Figure 1 shows the PBS as a plane-parallel dielectric slab of thickness \( d_2 \) and refractive index \( n_2 \) whose front surface is uncoated and reflects incident light in air at the Brewster angle, \( \phi_B = \arctan(n_2) \). Therefore the first reflected beam (beam 1) from the front surface of the slab is purely \( s \) polarized. The back surface has an antireflection coating for the \( s \) polarization (\( s \)-ARC) at the Brewster angle. Consequently, the light beam reflected from the backside of the slab is purely \( p \) polarized and is totally refracted as it exits the slab to air (beam 2) in a direction parallel to the first-reflected beam. There are no higher-order reflected beams. The only transmitted beam (beam 3) has both \( p \)- and \( s \)-polarized components that have experienced different attenuations in propagating through the slab.

From basic geometrical optics and the Brewster condition, the lateral separation \( D \) between the two parallel, orthogonally polarized, reflected beams is given by

\[
D = \frac{2d_2}{n_2(n_2^2 + 1)^{1/2}}.
\]  

(1)

The simplest \( s \)-ARC at the Brewster angle is a transparent single layer of refractive index \( n_1 \) and metric thickness \( d_1 \) given by

\[
n_1 = \sqrt{2n_2/(n_2^2 + 1)^{1/2}},
\]

(2)

\[
d_1 = 0.3536(\lambda/n_1),
\]

(3)

where \( \lambda \) is the vacuum wavelength of light. The thin-film \( s \)-ARC specified by Eqs. (2) and (3) was first proposed in Ref. 4, and applied to selected areas on the same (front) side of a dielectric substrate to produce any desired two-dimensional spatial binary polarization patterns in reflected light. A reflected beam with periodic temporal binary polarization modulation (between the \( p \) and \( s \) states) is also obtained when the same coating (with thickness that alternates between 0 and \( d_1 \)) is applied to the front surface of a synchronously rotating disk.\(^5\) It is apparent that a multilayer \( s \)-ARC can be applied on the backside of the slab, but this is not considered here.
At the Brewster angle, the intensity reflectances of the front and back surfaces of the slab for s- and p-polarized light, respectively, are given by:

\[ R_{fs} = \cos^2(2\phi_B) = \left[ \left( n_2^2 - 1 \right) / \left( n_2^2 + 1 \right) \right]^2, \quad \text{(4)} \]

\[ R_{bp} = \left[ R_{fs} / (2 - R_{fs}) \right]^2. \quad \text{(5)} \]

The corresponding intensity transmittances of the transparent slab for the p and s polarizations are given by:

\[ T_p = 1 - R_{bp}, \quad T_s = 1 - R_{fs}. \quad \text{(6)} \]

The average reflectance of the slab for incident light with equal p and s components (i.e., light whose first Stokes parameter \( s_1 = 0 \)) is given by

\[ R_{av} = (R_{fs} + R_{bp}) / 2. \quad \text{(7)} \]

From Eqs. (5) and (7), we obtain

\[ R_{av} = (R_{fs} / 2)(R_{fs}^2 - R_{fs} + 4) / (R_{fs}^2 - 4R_{fs} + 4). \quad \text{(8)} \]

For incident linearly polarized light of azimuth angle \( \alpha \) from the plane of incidence (i.e., from the p direction), the two reflected beams have equal power when

\[ \tan \alpha = (R_{bp} / R_{fs})^{1/2} = \left( n_2^4 - 1 \right) / \left( n_2^4 + 6n_2^2 + 1 \right). \quad \text{(9)} \]

The overall reflectance of the slab under the condition given by Eq. (9) is

\[ R = 2R_{fs}R_{bp} / (R_{fs} + R_{bp}), \]

\[ R^{-1} = \left( 1/2 \right)(R_{fs}^{-1} + R_{bp}^{-1}). \quad \text{(10)} \]

As a specific example, for a Ge slab with refractive index \( n_2 = 4 \) in the IR, we obtain

\[ \phi_B = 75.964^\circ, \quad D = 0.1213d_2, \]

\[ n_1 = 1.372, \quad d_1 = 0.2577\lambda, \]

\[ R_{fs} = 77.855\%, \quad R_{bp} = 40.627\%, \]

\[ R_{av} = 59.241\%, \quad R = 53.392\%, \]

\[ T_s = 22.145\%, \quad T_p = 59.373\%, \]

\[ \alpha = 35.844^\circ. \quad \text{(11)} \]

The film refractive index \( n_1 = 1.372 \) is close to that of Si over a broad visible range. \( n_2 = 3.383 \) is also the refractive index of GaP at \( \lambda = 0.580 \mu\text{m} \) in the visible. For an average reflectance \( R_{av} = \frac{1}{2} \), a different cubic equation,

\[ 3R_{fs}^3 - 11R_{fs}^2 + 20R_{fs} - 8 = 0, \quad \text{(15)} \]

is obtained that yields \( R_{fs} = 0.533990 \) and \( n_2 = 2.53634 \). The latter refractive index is that of ZnS at the short wavelength end of the visible spectrum.

4. Maximum Difference between \( R_{fs} \) and \( R_{bp} \)

It is interesting to consider the difference between \( R_{fs} \) and \( R_{bp} \). For simplicity, \( R_{fs} \) is denoted by \( x \). It follows from Eq. (5) that

\[ \Delta R = R_{fs} - R_{bp} = (x^3 - 5x^2 + 4x)/(2 - x^2). \quad \text{(16)} \]

Equation (16) shows that \( \Delta R = 0 \) in the limiting cases of \( x = 0 \) and \( x = 1 \), hence \( \Delta R \) must reach a maximum at some value of \( x \) between 0 and 1. By setting the derivative of Eq. (16) equal to 0, we obtain yet another cubic equation,

\[ x^3 - 6x^2 + 16x - 8 = 0. \quad \text{(17)} \]

![Fig. 1. Dielectric-slab PBS. A light beam incident on the slab at the Brewster angle \( \phi_B \) is split into two reflected beams 1 and 2 that are purely s and p polarized, respectively, and a transmitted beam 3 that has both p- and s-polarized components. The front surface of the slab is uncoated and the back surface has an s-ARC. \( d_2 \) is the thickness of the slab, and \( D \) is the separation of the parallel reflected beams with orthogonal polarizations.](image-url)
Equation (17) has one acceptable solution, 
\[ x = R_{ls} = 0.635344. \]  
(18)

The corresponding slab refractive index, calculated from Eq. (4), is 
\[ n_2 = 3.9844. \]  
(19)

This index is essentially the same as that of Ge in the IR. The maximum reflectance difference is given by 
\[ \Delta R_{\text{max}} = 0.373354. \]  
(20)

5. Photopolarimeter Using Two Parallel-Slab Polarizing Beam Splitters

Whereas a conventional PBS splits an incoming light beam into two beams of orthogonal linear polarizations, the parallel-slab PBS shown in Fig. 1 does the same in reflection, and provides a third beam in transmission. This makes this PBS particularly suited for Stokes-parameter photopolarimetry.

Figure 2 shows a photopolarimeter that employs two parallel-slab PBSs (PBS1 and PBS2) with a 45° optical rotator (OR) in the middle. (Because of the diattenuation introduced by the slab in transmission, a rotation other than 45° may be optimum.) PBS1 generates reflected beams 1 and 2, and PBS2 produces reflected beams 3 and 4. The 45° optical rotator OR can be a quartz plate, whose optic axis is perpendicular to its faces and parallel to the beam, a twisted-nematic liquid-crystal cell, or a magneto-optic Faraday rotator. Alternatively, one can do without this OR by rotating the plane of incidence for light reflection at PBS2 by 45° with respect to the plane of incidence for light reflection at PBS1. By use of the Mueller calculus,\textsuperscript{11} it can readily be shown that detection of light beams 1 and 2 can be dedicated (and calibrated) to determining the first normalized Stokes parameter \( s_1 \). Likewise, detection of light beams 3 and 4 can be dedicated to determining the second normalized Stokes parameter \( s_2 \).

If the incoming light is totally polarized (which is often the case in ellipsometry\textsuperscript{11}), the remaining third normalized Stokes parameter \( s_3 \) is obtained by 
\[ s_3 = \pm (1 - s_1^2 - s_2^2)^{1/2}. \]  
(21)

Therefore operation of this photopolarimeter is similar to that of the widely used rotating-analyzer ellipsometer but with no moving parts.

To measure the third normalized Stokes parameter \( s_3 \) independently (which is essential if the input light is generally partially polarized), a third stage is added to the photopolarimeter as shown in Fig. 2. It consists of a quarter-wave retarder (QWR) followed by a conventional PBS PBS3. Detection of light beams 5 and 6 enables the measurement of \( s_3 \), given that \( s_1 \) and \( s_2 \) are already determined by the first two stages of the polarimeter. To the best of my knowledge, this is the only division-of-amplitude photopolarimeter in which the first, second, and third Stokes parameters are determined separately and simultaneously.

6. Summary

A novel parallel-slab polarizing beam splitter is described that splits an incoming light beam into two reflected beams of orthogonal \( p \) and \( s \) linear polarizations and a third transmitted beam that retains both the \( p \) and \( s \) components. A detailed analysis of the essential features of this design is presented. A novel photopolarimeter that consists of two such beam splitters in succession, plus a circular-polarization detector, is realized in which the first, second, and third normalized Stokes parameters of input light are measured separately and simultaneously by three dual channels of orthogonal polarizations.

References

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