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Efficiency of linear-to-circular polarization conversion for light reflection at the principal angle by a dielectric-conductor interface

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1. INTRODUCTION

When linearly polarized light (LPL) is incident at the planar interface between a transparent medium of incidence (medium 0) and an absorbing medium of refraction (medium 1), the reflected light becomes circularly polarized (CP) at a special angle of incidence, called the principal angle, and for a specific orientation of the incident electric field vector relative to the plane of incidence, called the principal azimuth, as shown in Fig. 1 [1–3]. In Fig. 1, \( p \) and \( s \) represent the orthogonal linear polarization directions parallel and perpendicular to the plane of incidence, respectively. If the azimuth of incident LPL is changed from \( +\psi \) to \( -\psi \), the handedness of the reflected circular polarization is switched.

The angles \( \phi \) and \( \psi \) are determined by the complex relative dielectric function,

\[
\epsilon = \epsilon_r/e_0 = \epsilon_r - j\epsilon_i, \tag{1}
\]

of the two media, which are assumed to be linear, homogeneous, and optically isotropic. In particular, \( \phi \) is obtained by solving a cubic equation in \( \sin^2 \phi \) whose coefficients are determined by complex \( \epsilon \) [3]. Conversely, \( \epsilon_r \) and \( \epsilon_i \) are derived from the measured angles \( \phi \) and \( \psi \) by

\[
\epsilon_r = \sin^2 \phi + \sin^2 \phi \tan^2 \phi \cos(4\psi),
\]

\[
\epsilon_i = \sin^2 \phi \tan^2 \phi \sin(4\psi). \tag{2}
\]

This is the basis of an experimental technique that was first used by O’Bryan [4] to determine the optical constants of optically thick metal films deposited in vacuum using a return-path ellipsometer [4,5]. Contours of constant \( \phi \) and constant \( \psi \) in the complex \( \epsilon \) plane and the domain of fractional optical constants in which multiple principal angles exist were presented in [3].

The primary objective of this paper is to determine the efficiency of linear-to-circular polarization conversion when LPL is incident at the principal angle and principal azimuth. In Section 2 an expression for this conversion efficiency \( \eta_{LC} \) is derived as a function of \( \phi \), \( \psi \) and \( \epsilon \) for achieving high efficiency (\( \eta_{LC} \geq 50\%) \) are clarified. The properties of this function are discussed in Section 3.

As an application, \( \phi \), \( \psi \), and \( \eta_{LC} \) are calculated in Section 4 for light reflection by a Ag mirror in the visible and near-IR, and also for the reflection of extreme ultraviolet (EUV) or soft x-ray radiation by a SiC mirror. Section 5 is a brief summary of the paper.

2. EFFICIENCY OF LINEAR-TO-CIRCULAR POLARIZATION CONVERSION AS A FUNCTION OF PRINCIPAL ANGLE AND PRINCIPAL AZIMUTH

At the principal angle, the complex-amplitude Fresnel reflection coefficients \( r_p \) and \( r_s \) for the \( p \) and \( s \) polarizations are related by [3]

\[
r_p = (j\tan \psi)r_s. \tag{3}
\]

At any angle of incidence \( \phi \), \( r_s \) is given by

\[
r_s = [\cos \phi - (\epsilon - \sin^2 \phi)^{1/2}] [\cos \phi + (\epsilon - \sin^2 \phi)^{1/2}]. \tag{4}
\]

At the principal angle \( (\phi = \bar{\phi}) \) \( \epsilon \) in Eq. (4) can be replaced by its equivalent expression in terms of \( \psi \) \( \psi \) given by Eqs.
(2). Use of some trigonometric manipulations transforms Eq. (4) to

\[ r_s = \frac{\cos 2\phi \tan \psi + j \tan \psi}{1 + j \cos 2\phi \tan \psi} \]  

At \( \phi = \tilde{\phi} \), the linear-to-circular polarization conversion efficiency \( \eta_{LC} \) is defined as the ratio of the intensity of the reflected circularly polarized light (CPL) to that of the incident LPL and is given by

\[ \eta_{LC} = \cos^2 \psi(r_p r_p^*) + \sin^2 \psi(r_s r_s^*) \]  

Substitution of \( r_p \) from Eq. (3) in Eq. (6) gives

\[ \eta_{LC} = 2 \sin^2 \tilde{\psi}(r_s r_s^*) \]  

From Eqs. (5) and (7) we finally obtain

\[ \eta_{LC} = 2 \sin^2 \tilde{\psi} \left[ \frac{1 - \sin^2 2\phi \cos^2 \tilde{\psi}}{1 - \sin^2 2\phi \sin^2 \tilde{\psi}} \right] \]  

The function \( \eta_{LC}(\tilde{\psi}, \tilde{\phi}) \) of Eq. (8) represents the main result of this paper and its properties are examined in Section 3.

3. PROPERTIES OF THE FUNCTION \( \eta_{LC}(\tilde{\psi}, \tilde{\phi}) \)

Based on Eq. (8), one can readily reach the following conclusions:

1. \( \eta_{LC} = 0 \) if \( \tilde{\psi} = 0 \). This corresponds to \( r_s = 0 \) [Eq. (3)] and light reflection at the Brewster angle of a dielectric–dielectric interface.

2. \( \eta_{LC} = 1 \) if \( \tilde{\phi} = 45^\circ \). This corresponds to the total internal reflection of the \( p \) and \( s \) polarizations at the principal angle of a dielectric–dielectric interface (\( \epsilon_r = 0 \)) and requires that \( \epsilon_p \) be in the range \( 0 < \epsilon_p < 3 - 2\sqrt{2} = 0.1716 \) [3]. It also corresponds to the total reflection at an ideal dielectric–plasma interface with \( \epsilon_r = 0, \epsilon_p < 0 \).

3. For any given \( \tilde{\psi} \)
of \( \phi \) for constant values of \( \psi = 15^\circ, 30^\circ, \) and 38.5° to 45° in steps of 0.5°. For a given \( \psi \), Fig. 2 indicates that \( \eta_{LC} \) is maximum at \( \phi = 0, 90^\circ \) and is minimum at \( \phi = 45^\circ \).

In Fig. 3 constant-\( \eta_{LC} \) contours are plotted in the \((\phi, \psi)\) plane for values of \( \eta_{LC} \) from 0.5 to 1.0 in steps of 0.05.

It is also of interest to specify the region of the complex \( \epsilon \) plane in which \( \eta_{LC} > 0.5. \) This is done by mapping the family of constant-\( \eta_{LC} \) contours of Fig. 3 to the complex \((\epsilon_r, \epsilon_i)\) plane by using Eqs. (2). Figure 4(a) shows the corresponding family of constant-\( \eta_{LC} \) contours in the domain of fractional optical constants, and Fig. 4(b) shows the continuation of those contours for large values of \((\epsilon_r, \epsilon_i)\).

4. APPLICATION TO Ag AND SiC MIRRORS

The principal angle \( \phi \), principal azimuth \( \psi \), and conversion efficiency \( \eta_{LC} \) are calculated for light reflection by a Ag mirror (in vacuum or inert ambient) in the visible and near-IR spectral range, 400 \( \leq \lambda \leq 1200 \) nm. The optical constants of Ag are obtained from Palik [6]. In Fig. 5, \( \phi \) and \( \psi \)

Fig. 6. Linear-to-circular polarization conversion efficiency \( \eta_{LC} \) (dashed curve) and principal azimuth \( \psi \) (expanded scale) as functions of wavelength \( \lambda \) for light reflection by a Ag mirror in the visible and near-IR spectral range, 400 \( \leq \lambda \leq 1200 \) nm. The optical constants of Ag are obtained from [6].

Fig. 7. Principal angle \( \phi \), principal azimuth \( \psi \), and linear-to-circular polarization conversion efficiency \( \eta_{LC} \) (dashed curve) as functions of wavelength \( \lambda \) for the reflection of EUV and soft x-ray radiation by a SiC mirror in the 40 \( \leq \lambda \leq 120 \) nm spectral range. The optical constants of SiC are those published in [7].
are plotted as functions of wavelength $\lambda$. Figure 6 shows the correlation between $\eta_{LC}$ (dashed curve) and $\tilde{\phi}$ (on an expanded scale) versus $\lambda$. For Ag, $\eta_{LC} \approx 88\%$ over the $400 \leq \lambda \leq 1200$ nm spectral range.

As another example, $\tilde{\phi}$, $\tilde{\psi}$, and $\eta_{LC}$ are calculated for the reflection of EUV and soft x-ray radiation by a SiC mirror, and the results are presented in Fig. 7 over the $40 \leq \lambda \leq 120$ nm spectral range. The optical constants of SiC are those published by Windt et al. [7]. Conversion efficiencies $\eta_{LC} \approx 40\%$ are achieved at wavelengths of $60 \leq \lambda \leq 120$ nm.

5. SUMMARY
A detailed analysis of the efficiency of linear-to-circular polarization conversion when light is reflected at a principal angle of a dielectric–conductor interface has been presented. The constraint on the principal angle and principal azimuth for achieving a given efficiency (e.g., $\approx 50\%$) and the corresponding domain in the complex $\varepsilon$ plane are determined. As examples, efficiencies $\approx 88\%$ are obtained for light reflection by a Ag mirror in the visible and near-IR ($400–1200$ nm) spectral range and $\approx 40\%$ for the reflection of EUV and soft x-ray radiation by a SiC mirror in the 60–120 nm wavelength range.

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REFERENCES