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Division-of-wave-front thin-film beam splitter for generating binary patterns of orthogonal elliptical polarization states

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A division-of-wave-front thin-film beam splitter is described that reflects monochromatic light at oblique incidence with orthogonal elliptical polarization states. It consists of a metallic substrate partially covered with a transparent thin film that inverts the ratio ρ of the complex p and s reflection coefficients at the principal angle of the metal. Any pattern of coated and uncoated areas of the substrate is imprinted upon the reflected wave front as a corresponding two-dimensional spatial binary polarization pattern. A specific design is given that uses a Au substrate at a wavelength of 632.8 nm. The effects of small errors in the film refractive index, the film thickness, and the angle of incidence are discussed. It is noted that a layer that inverts ρ at a certain (especially high) angle of incidence is an effective ρ-inverting layer at all angles.

1. INTRODUCTION

Recently[1,2] I described a new optical device that consists of a dielectric or absorbing (semiconductor or metallic) substrate coated by a transparent thin film of a certain refractive index and of a thickness that alternates between two specific values over adjacent areas of the substrate. When a monochromatic plane wave of light is incident upon the device at a certain angle (such that the wave is refracted in the film at 45°), the two-dimensional (2-D) binary thickness pattern (obtained, e.g., by lithographic techniques) is imprinted upon the reflected wave front as a 2-D pattern of orthogonal linear (p and s) polarization states. Subsequently[3] this concept was extended to the generation of 2-D spatial binary patterns of orthogonal, right- and left-handed, circular polarizations by using an optically dense transparent substrate (e.g., Ge in the IR) on which +90° and −90° thin-film reflection quarter-wave retarders are integrated (i.e., reside side by side) and the incident light is linearly polarized at a 45° azimuth from the plane of incidence.

In this paper such a reflective division-of-wave-front thin-film beam splitter (DOW TF BS) is generalized further to produce orthogonal elliptical polarization states.

2. DIVISION-OF-WAVE-FRONT THIN-FILM BEAM SPLITTER FOR ORTHOGONAL ELLIPTICAL POLARIZATION STATES

Operation of the DOW TF BS is explained with reference to Fig. 1. A monochromatic plane wave of light is incident from a transparent ambient of refractive index N_0 (usually air, for which $N_0 = 1$) onto the planar surface of an absorbing substrate of complex refractive N_2, which is covered partially by a transparent thin film of refractive index N_1 and uniform thickness d. All media are assumed to be linear, homogeneous, isotropic, and nonmagnetic. The angle of incidence is chosen to be the principal angle ϕ of the substrate. In terms of the complex relative dielectric constant $\epsilon = (N_2/N_0)^2 = (\epsilon_r, \epsilon_i)$, ϕ is obtained by solving the cubic equation[4]

$$a_3u^3 + a_2u^2 + a_1u + a_0 = 0, \tag{1}$$

where

$$\begin{aligned} a_0 &= \epsilon_r^2 + \epsilon_i^2, \\ a_1 &= -2(\epsilon_r^2 + \epsilon_i^2) - 2\epsilon_r, \\ a_2 &= (\epsilon_r^2 + \epsilon_i^2) + 4\epsilon_r + 1, \\ a_3 &= -2\epsilon_r - 2, \end{aligned} \tag{2}$$

and $u = (\sin\phi)^2$.

Let $\rho = r_p/r_s$ be the ratio of complex p and s reflection coefficients. At ϕ, the ρ value of the uncoated substrate becomes pure imaginary,

$$\rho = \rho_u = j\tan\psi, \tag{3}$$

where ψ is called the principal azimuth.

The refractive index and the thickness (N_1 and d) of the film are chosen to invert ρ[5,6] so that, for the coated substrate,

$$\rho = \rho_c = -j\cot\psi. \tag{4}$$

It is convenient to represent the state of polarization of light by the complex number[7]

$$\chi = E_p/E_s, \tag{5}$$

which is the ratio of the phasor components of the electric vector that are parallel (E_p) and perpendicular (E_s) to the plane of incidence. It follows immediately that, for the optically isotropic system under consideration, the reflected and incident polarizations are related by

$$\chi_r = \rho\chi_i. \tag{6}$$

We assume that the incident light is linearly polarized at a 45° azimuth from the plane of incidence, so that $\chi_i = 1$. From Eqs. (3), (4), and (6), the polarization states reflected by the uncoated and coated parts of the substrate are given by

$$\chi_{ru} = j\tan\psi, \qquad \chi_{rc} = -j\cot\psi. \tag{7}$$

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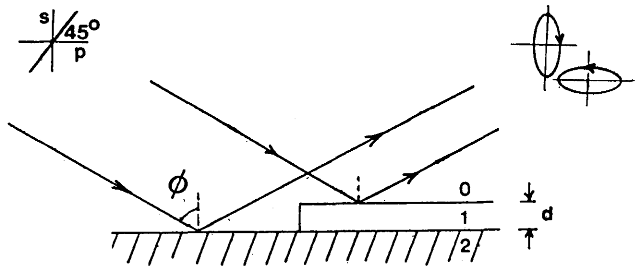

Fig. 1. DOW TF BS for generating orthogonal elliptical polarization states. Linearly polarized light at a 45° azimuth from the plane of incidence is incident upon a metal substrate at the principal angle. The metal is partially coated by a transparent ρ-inverting layer.

Two polarization states χ_1 and χ_2 are orthogonal if they satisfy the condition[7]

$$\chi_1\chi_2^* = -1, \tag{8}$$

where the asterisk denotes the complex conjugate. The two reflected polarizations given by Eqs. (7) do satisfy the orthogonality condition of Eq. (8). χ_{ru} and χ_{rc} represent right- and left-handed elliptical polarization states of the same axial ratio whose major axes are aligned perpendicular and parallel to the plane of incidence, respectively.

It is also of interest to calculate the intensity reflectances of the uncoated and coated areas of the substrate for the incident linearly polarized light at a 45° azimuth, given by

$$R_k = (r_{pk}r_{pk}^* + r_{sk}r_{sk}^*)/2, \tag{9}$$

where the subscripts $k = u$ and c refer to the uncoated and coated areas, respectively.

In summary, a DOW TF BS can be designed to produce orthogonal elliptical polarizations by illuminating an absorbing substrate that is partially covered by a ρ-inverting layer with linearly polarized light that is incident at the principal angle and that has a vibration direction 45° from the plane of incidence.

3. SPECIFIC EXAMPLE

As a concrete example of a DOW TF BS for the generation of orthogonal elliptical polarization states, we take a Au substrate and assume incident light of wavelength $\lambda = 632.8$ nm from a He–Ne laser. The complex refractive index of Au at this wavelength[8] is $0.20 - j3.71$. Solving Eq. (1) gives the principal angle $\phi = 75.8266°$. At ϕ, the reflectances of the Au substrate for p-polarized, s-polarized, and 45° linearly polarized light are 0.8925, 0.9873, and 0.9399, respectively, and the principal azimuth $\psi = \arctan(R_p/R_s)^{1/2} = 43.5553°$.

By a procedure already described in Refs. 5 and 6, the refractive index and the thickness of the transparent ρ-inverting layer at ϕ are calculated to be $N_1 = 1.9475$ and $d = 81.85$ nm. A suitable thin-film coating material is silicon nitride, the stoichiometry of which can be controlled[9] to achieve this desired value of N_1. It should be noted that d is the least film thickness; thicker ρ-inverting layers are obtained by adding any integral multiple of the film thickness period $D_\phi = 187.33$ nm. The reflectances of the Au coated with the ρ-inverting layer for p-polarized, s-polarized, and 45° linearly polarized light are 0.9399, 0.8497, and 0.8948, respectively. The associated principal azimuth is 46.4447°, which is equal to $90° - \psi$, as anticipated. Also, it can be verified by direct calculation that the differential reflection phase shift, $\Delta = \arg \rho$, equals $-90.0000°$ and $+90.0000°$, respectively, for the uncoated and coated parts of the substrate, as required by design.

The reflectances, for incident linearly polarized light at a 45° azimuth, of the uncoated and coated areas of the substrate are sufficiently high (93.99 and 89.48%, respectively) to make this device practical. Their difference of 4.5% produces a weak spatial-intensity modulation of the reflected wave front, which is superimposed upon, and registered with, the much more significant binary polarization modulation between orthogonal elliptical states.

For convenience, the important device characteristics are summarized in Table 1. For comparison, Table 1 also lists data obtained at another wavelength, $\lambda = 488.0$ nm, of the Ar^+-ion laser. Because of its diminished reflectance, Au is not a particularly good substrate at this shorter wavelength. Also, the reflectance difference between the coated and uncoated areas is relatively large and would cause appreciable intensity modulation of the reflected wave front.

4. ERROR ANALYSIS

In this section we examine the effects of small errors in the film refractive index N_1, the film thickness d, and the angle of incidence ϕ on the ρ-inversion condition ($\rho_c\rho_u = 1$), which is key to the operation of the DOW TF BS. The deviation of each of these parameters from its design value at $\lambda = 632.8$ nm (listed in Table 1) is considered individually. Each parameter error causes a magnitude error (ME) and a phase error (PE) that are defined by

$$\mathrm{ME} = |\rho_c\rho_u| - 1, \qquad \mathrm{PE} = \arg(\rho_c\rho_u), \tag{10}$$

where ρ_c is a function of (N_1, d, ϕ) and ρ_u is a function of ϕ.

Figure 2 shows the ME and PE caused by shifting N_1 by ± 0.05 around the design value of 1.9475. To keep ME < 1% and PE < 3°, the deviation ΔN_1 should not exceed ± 0.01.

Figure 3 shows the ME and PE caused by thickness changes of ± 5 nm around the design value of 81.85 nm. For ME < 1% and PE < 3°, d should be controlled to within ± 1

Table 1. Characteristics of DOW TF BS's Using a Au Substrate at Two Laser Wavelengths[a]

λ (nm)	n_2	k_2	ϕ (°)	N_1	d (nm)	ψ (°)	R_u	R_c
632.8	0.20	3.71	75.8266	1.9475	81.85	43.5553	0.9399	0.8948
448.0	0.94	1.70	66.1446	2.5394	27.96	32.9133	0.5198	0.2070

[a] λ is the wavelength of light, $n_2 - jk_2$ is the Au substrate complex refractive index from Ref. 8, ϕ is the substrate principal angle from Eq. (1), N_1 and d are the refractive index and the thickness of the ρ-inverting layer, ψ is the substrate principal azimuth, and R_u and R_c are the reflectances for incident linearly polarized light at a 45° azimuth of the uncoated and coated parts of the Au substrate.

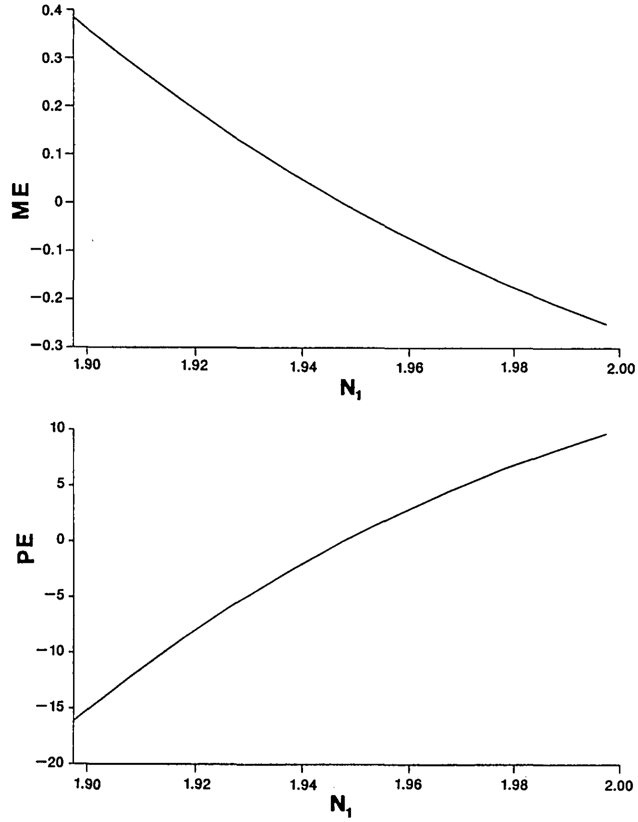

Fig. 2. ME and PE caused by shifting the refractive index of a transparent film on Au from the value (1.9475) required for ρ inversion at the principal angle (75.8266°) and at wavelength of 632.8 nm.

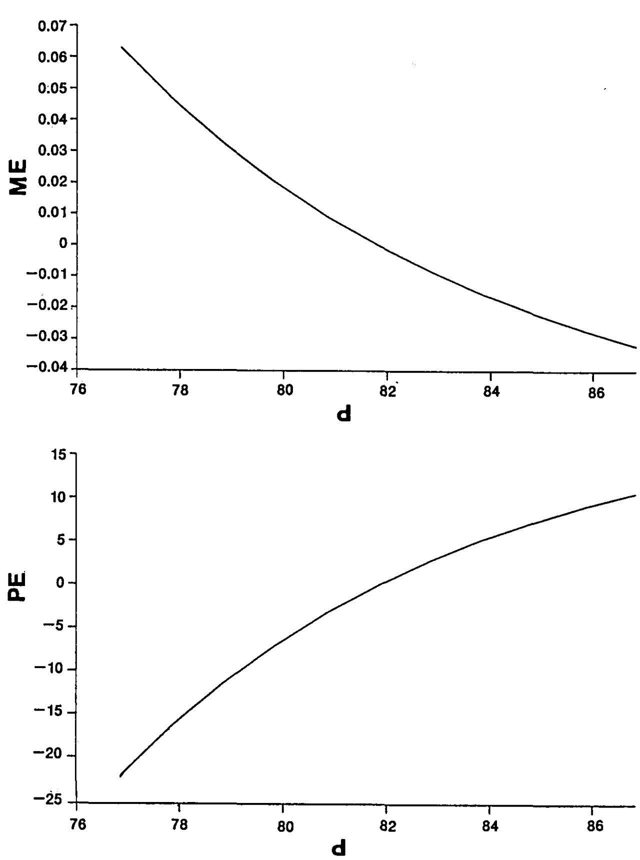

Fig. 3. Same as in Fig. 2, except that here the film thickness is shifted around the design value of 81.85 nm.

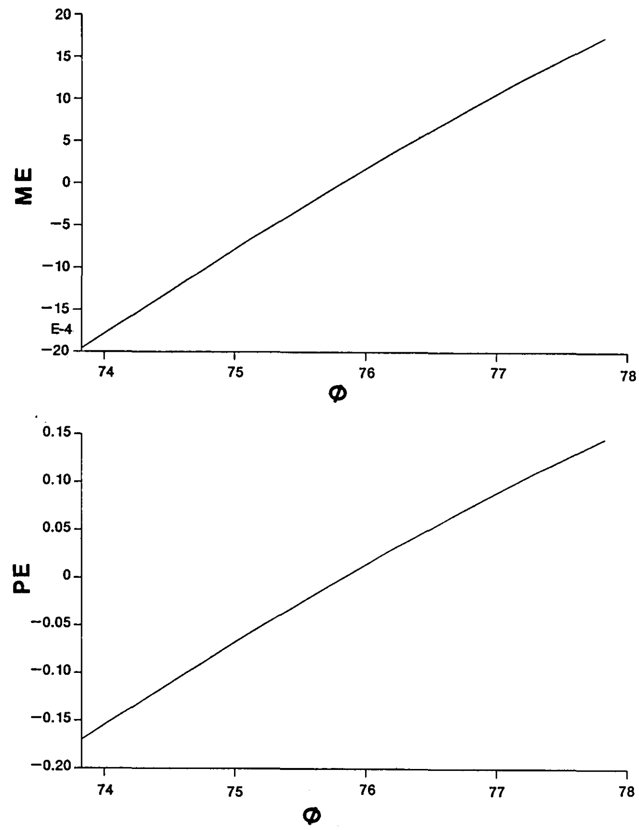

Fig. 4. Same as in Fig. 2, except that here the angle of incidence is shifted around the principal angle of 75.8266°.

nm. These tolerances on the refractive index and the thickness of the film are achievable with modern film-deposition technologies in conjunction with advanced ellipsometric monitoring.

Finally, Fig. 4 shows the ME and PE that result from shifting the angle of incidence by ±2° around the principal angle $\phi = 75.8266°$. It is remarkable that ME $< 2 \times 10^{-3}$ and PE $< 0.2°$, so that appreciable angular errors do not substantially affect the ρ-inversion condition.

In Appendix A it is verified that ρ inversion is maintained over nearly the entire range of ϕ from normal to grazing incidence.[10]

5. CONCLUSION

The DOW TF BS is a new optical device that embosses a binary pattern of orthogonal polarization states onto an optical wave front. This pattern corresponds to a binary pattern of thickness of a transparent thin film coating upon a transparent or absorbing substrate. In this paper previous work[1–3] that dealt with the generation of orthogonal linear and circular states is extended to the generation of orthogonal elliptical states. This is accomplished by reflecting light from a metallic (or semiconductor) substrate at the principal angle and using a ρ-inverting layer for the coating. A specific device based on a Au substrate is described for the common 632.8-nm He–Ne laser radiation, and the effects of small errors in the film refractive index, the film thickness, and the angle of incidence are considered.

The DOW TF BS can serve as a useful half-shade device.[11] More importantly, it provides a new means of spatial binary polarization modulation for a new class of 2-D optical information processors that operate on polarization instead of the scalar complex field amplitude or intensity. It should also be noted that, if a spatially homogeneous analyzer is placed in the reflected wave front, a controllable binary intensity pattern can be obtained. For example, if the analyzer is crossed with one of the orthogonal reflected states, the binary levels of transmitted intensities will be zero (or a minimum) and a maximum.

For a review of other single-layer-coated optical devices for polarized light, the reader may consult Ref. 12. Other useful reviews that discuss applications of single-layer and multilayer thin films are given in Refs. 11 and 13.

APPENDIX A

In Section 4 and in Fig. 4 it is shown that ρ inversion is affected negligibly by an angular swing of $\pm 2°$ around ϕ. This suggests that a layer that inverts ρ at one angle of incidence may act as an effective ρ-inverting layer over a broad range of incidence angles. Figure 5 confirms this. It shows ME and PE versus ϕ over the entire range $0 \leq \phi \leq 90°$ for the same transparent film on a Au substrate that inverts ρ at the principal angle, with $\lambda = 632.8$ nm. The maximum ME and PE values are 1.6% and 2°, respectively.

To determine whether this is perhaps a peculiarity of ρ inversion at the principal angle only, we repeat the calculations for layers that invert ρ at angles of incidence from 20 to 80° in steps of 10°. N_1 and d values of these layers are listed in Table 2. Again, we assume a Au substrate ($N_2 = 0.20 - j3.71$) at $\lambda = 632.8$ nm. Figures 6 and 7 show the ME and PE

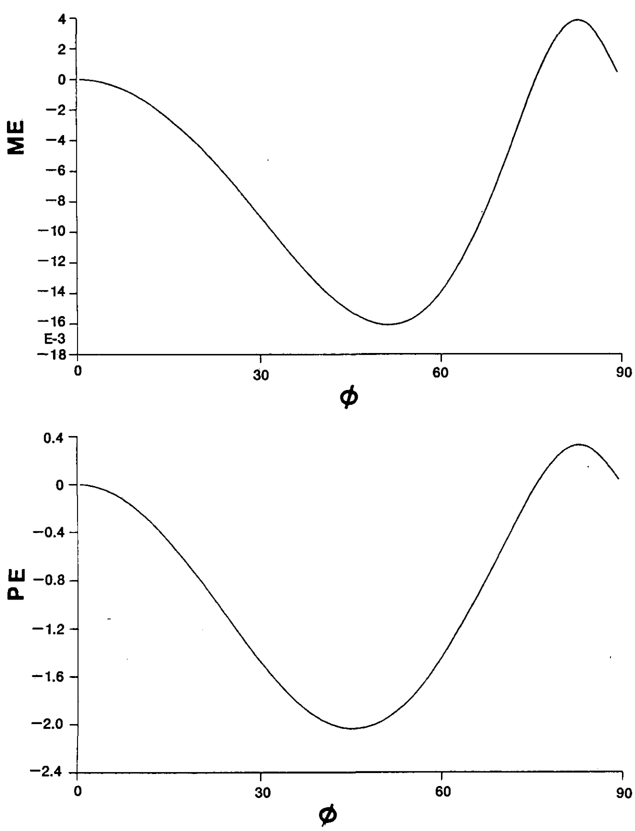

Fig. 5. Same as in Fig. 4, except that the entire range of incidence angles from 0 to 90° is covered.

Table 2. Refractive Index N_1 and Thickness d of a ρ-Inverting Layer on Au at Several Angles of Incidence ϕ^a

ϕ (°)	N_1	d (nm)
20	3.0195	38.26
30	2.8295	42.60
40	2.6116	48.69
50	2.3925	56.50
60	2.1914	65.88
70	2.0226	76.22
80	1.9064	85.27

[a] We assume a wavelength of 632.8 nm and a corresponding complex refractive index of Au of $0.20 - j3.71$ (from Ref. 8).

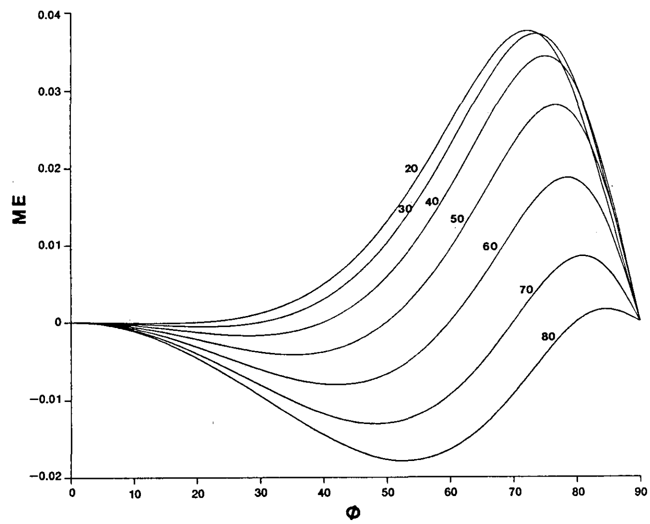

Fig. 6. ME as a function of the angle of incidence ϕ for different ρ-inverting layers on Au at a wavelength of 632.8 nm. Each layer, whose refractive index and thickness are given in Table 2, inverts ρ exactly at the angle of incidence marked by each curve.

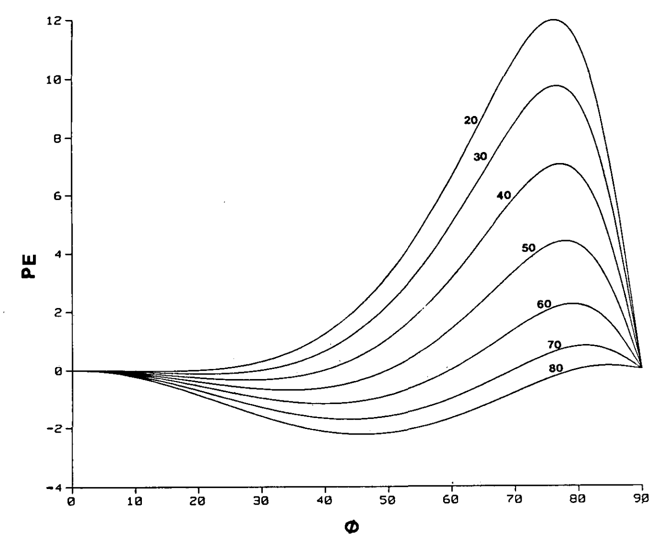

Fig. 7. Same as in Fig. 6, except that here PE is plotted versus ϕ.

values, respectively. It is apparent that the ρ-inversion condition is maintained reasonably well, to a good approximation, over the entire range of ϕ. This is so especially for layers that invert ρ at high angles, e.g., at 60°, 70°, and 80°.

The reason for this broad angular insensitivity is that ρ inversion is invariably satisfied at $\phi = 0$ and 90° as well as at the design angle of oblique incidence. This establishes three nodes of zero error in the ME-versus-ϕ and PE-versus-ϕ curves, and the excursion of these errors becomes constrained between nodes.

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