

5-22-2006

50%-50% Beam Splitters Using Transparent Substrates Coated by Single- or Double-Layer Quarter-Wave Thin Films

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50%-50% BEAM SPLITTERS USING TRANSPARENT SUBSTRATES COATED BY
SINGLE- OR DOUBLE-LAYER QUARTER-WAVE THIN FILMS

A Thesis

Submitted to the Graduate Faculty of the
University of New Orleans
in partial fulfillment of the
requirements for the degree of

Master of Science
in
Engineering

by

Faisal Sudradjat

B.S. University of New Orleans, 2004

May, 2006

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Dedication

This thesis is dedicated to my parents and to my family.

Acknowledgments

I would like to express my sincere gratitude and humble appreciation toward Dr. Rasheed Azzam who has been my research advisor, teacher, and mentor for the past three years. I will forever be indebted for his guidance and for his teachings. I would also like to thank Dr. Alsamman and Prof. Jovanovich for serving in my thesis committee. Many thanks go to the whole faculty of the Department of Electrical Engineering for their academic contributions to me. Finally, I would like to thank my parents, without whom I will not be typing this thesis, my dearest siblings, A' Deni, Teh Eva, Teh Ade, Teh Ayi, Teh Novy, and Teh Fitri for everything.

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Abstract

A pair of light beams that have orthogonal polarizations and equal intensity can be generated through reflection and refraction of a monochromatic light at a dielectric surface. Sytematic procedures to design beam splitters which can produce such output light beams are described in this thesis. Two designs that are of particular interest are prismatic substrates coated by a single layer and a double layer of thin films. Specific examples of each beam splitter in the visible and infrared are included. The performance of each beam splitter as a function of incidence angle, film thickness, and wavelength is also discussed.

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Chapter 1

Introduction

The state of polarization of totally polarized light can be described by the the following complex parameter [1]:

$$\chi(\theta, \epsilon) = \frac{\tan \theta + j \tan \epsilon}{1 - \tan \theta \tan \epsilon}, \quad (1.1)$$

where θ is the azimuth angle and ϵ is the ellipticity angle of the particular polarization state. Figure (1.1) shows the parameters θ and ϵ on the Cartesian coordinate plane. $\vec{E}(t)$, which is the instantaneous electric vector at time t , is also shown.

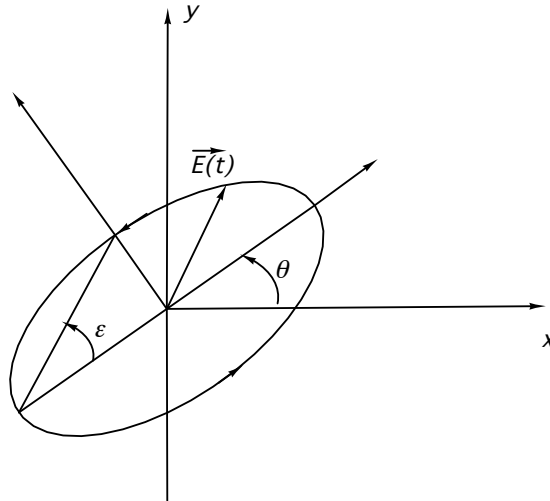


Figure 1.1: Elliptical Polarization of Light

Let a beam of light be incident on a dielectric surface, whose refractive index is n , at an angle ϕ as shown in Figure (1.2). The electric field of the incident light is decomposed into two components: the p component, for the electric field that is parallel to the plane of incidence, and the s component, for the electric field that is perpendicular to the plane of incidence.

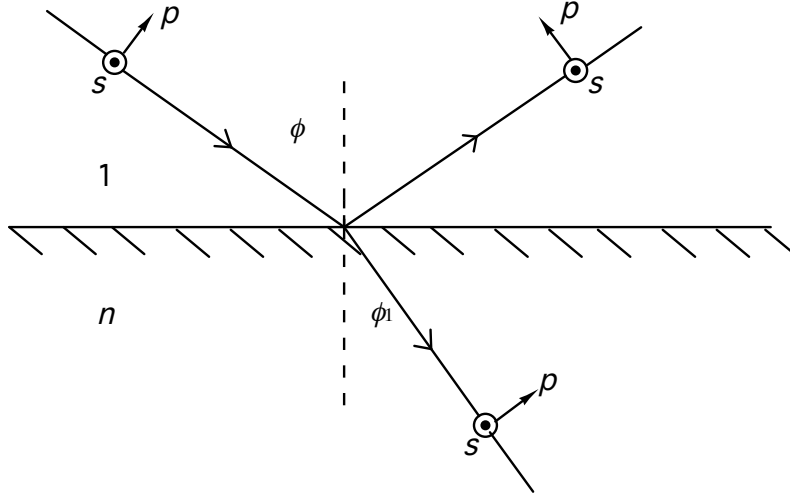


Figure 1.2: Reflection and Refraction of Light at a Dielectric Surface

The reflected and refracted light can be decomposed into their p and s components as well. Let R_ν and T_ν be the complex-amplitude reflection and transmission coefficients for the ν polarization, respectively, where $\nu = p, s$. Then, the following ellipsometric parameters can be defined [1]:

$$\rho_r = \frac{R_p}{R_s} = \tan \psi_r e^{j\Delta_r} \quad (1.2)$$

$$\rho_t = \frac{T_p}{T_s} = \tan \psi_t e^{j\Delta_t} \quad (1.3)$$

In addition, we have the following relationships [1]

$$\rho_r = \frac{\chi_i}{\chi_r} \quad (1.4)$$

$$\rho_t = \frac{\chi_i}{\chi_t}, \quad (1.5)$$

where χ_r and χ_t are complex parameters that describe the polarization states of the reflected and the refracted light, respectively. χ_i is the polarization state of the input light. The polarization states of the reflected and the refracted light are orthogonal if the following condition is satisfied [1]:

$$\chi_r \chi_t^* = -1. \quad (1.6)$$

A well-known method to produce orthogonal polarizations is to use double refraction by birefringent crystals [2]. In addition, recent researches have suggested the use of diffractive gratings [3, 4], negative refraction of photonic crystals [5], and embedded multiple thin-film structures [6] to generate beams of orthogonal polarizations.

However, Azzam proposed that beams of equal power and orthogonal polarizations can be generated using a relatively simple optical structure, such as an uncoated prism or a prism coated by a quarter-wave layer of thin film [7]. The proposed beam splitters have the following properties:

$$\Delta_r = \pi, \quad \Delta_t = 0 \quad (1.7)$$

$$\mathcal{R}_u = \tau_u = 0.5, \quad (1.8)$$

where \mathcal{R}_u and τ_u are the average reflectance and transmittance, respectively.

An interesting result that comes as a consequence of Eq. (1.8) is [8] :

$$\psi_r + \psi_t = \pi/2 \quad (1.9)$$

Substituting Eqs. (1.7) into Eqs. (1.2) and (1.3), we have

$$\rho_r = -\tan \psi_r \quad (1.10)$$

$$\rho_t = \tan \psi_t. \quad (1.11)$$

After substituting Eqs. (1.10) and (1.11) into Eqs. (1.4) and (1.5), respectively, we obtain the following relationships

$$\chi_r = \chi_i / (-\tan \psi_r) \quad (1.12)$$

$$\chi_t = \chi_i / (\tan \psi_t). \quad (1.13)$$

Taking the tangent of both sides of Eq. (1.9) and using the angle sum identity, we have:

$$\tan(\psi_r + \psi_t) = \tan \frac{\pi}{2} = \infty = \frac{\tan \psi_r + \tan \psi_t}{1 - \tan \psi_r \tan \psi_t}. \quad (1.14)$$

Equation (1.14) is satisfied if

$$\tan \psi_r \tan \psi_t = 1. \quad (1.15)$$

Multiplying Eq. (1.12) and the conjugate of Eq. (1.13) and using Eq. (1.15), we obtain

$$\chi_r \chi_t^* = -\chi_i \chi_i^* \quad (1.16)$$

If the polarization state of the input light is selected such that [7]:

$$\chi_i \chi_i^* = 1, \quad (1.17)$$

then the orthogonality condition of the reflected and refracted light as stated in Eq. (1.6) is satisfied.

Equation (1.17) restricts the polarization state of the incoming light to the unit circle in the Cartesian χ plane. This amounts to restricting the azimuth angle θ of the input polarization to $\pm 45^\circ$. Obviously, an infinite number of input polarization states can satisfy Eq. (1.17) and as a result, an infinite number of pairs of output orthogonal polarization states can be generated.

As examples [7], Fig. 1.3 shows splitting an input light, which is polarized at 45° azimuth,

into the reflected and refracted light whose polarizations are linear and orthogonal. Figure 1.4 shows the generation of reflected and refracted beams of light whose polarizations are orthogonal elliptical when the input light is right-handed circularly polarized.

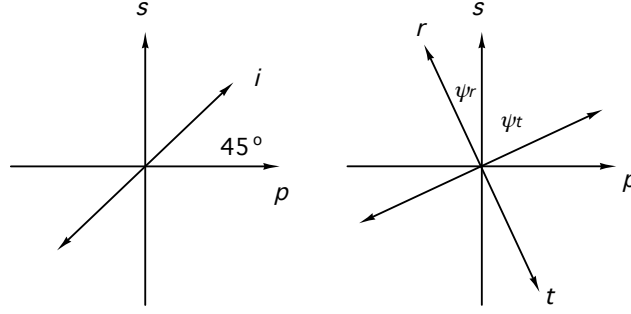


Figure 1.3: Splitting an Incident Light that is Linearly Polarized at 45° into Two Orthogonal Linear Polarizations

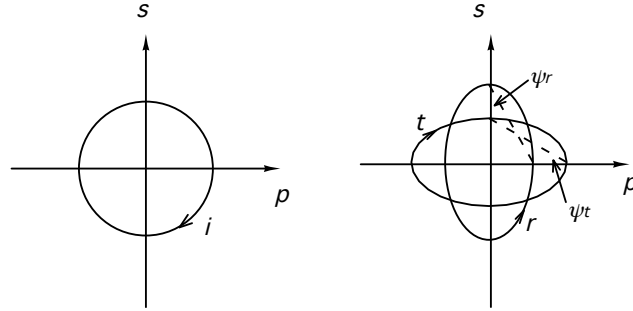


Figure 1.4: Splitting an Incident Light that is Right-Handed Circularly Polarized into Two Orthogonal Elliptical Polarizations

This thesis will expand on the concepts introduced in [7] and will describe the design procedures of beam splitters, which consist of a prismatic substrate which is coated by quarter-wave single- and double-layer thin films. Chapter 2 and 3 describe the design procedure and the specific examples of single- and double-layer-coated beam splitter, respectively. Chapter 4 gives a brief summary.

Chapter 2

Design of Single-Layer-Coated Beam Splitter

The simplest beam splitter which can generate beams of equal power and orthogonal polarizations consists of an uncoated PbTe prism [7]. However, this beam splitter utilizes an exotic material and requires a high angle of incidence. This limitation can be reduced by depositing a quarter-wave thick thin film whose refractive index is high on a substrate whose refractive index is relatively low.

Figure (2.1) shows the design of single-layer-coated beam splitter. The beam splitter consists of a dielectric substrate of refractive index n_2 , which is coated by a transparent thin film of refractive index n_1 and thickness d_1 . A monochromatic light beam of wavelength λ , whose polarization satisfies Eq. (1.17), is incident on the entrance face of the prism at an angle ϕ . The prism angle α is equal to the angle of refraction in the prism, such that the refracted light is incident normally to the exitance plane. The exitance face is antireflection coated.

The complex-amplitude reflection coefficients of the coated surface are given by [1]:

$$R_\nu = \frac{r_{01\nu} + r_{12\nu}X_1}{1 + r_{01\nu}r_{12\nu}X_1}, \quad (2.1)$$

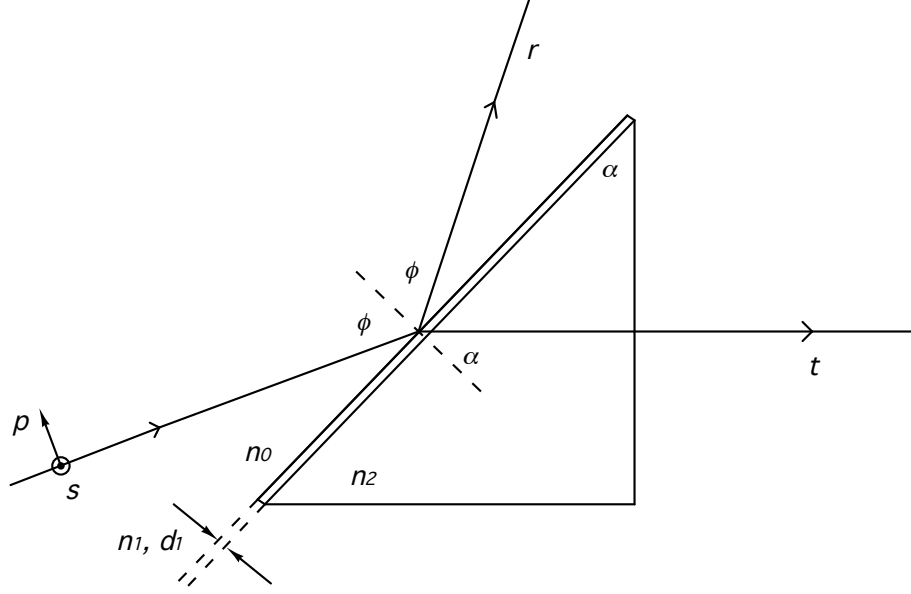


Figure 2.1: Single-Layer-Coated Beam Splitter

where $r_{ij\nu}$ is the Fresnel reflection coefficient of the ij interface for the ν polarization ($\nu = p, s$) and

$$X_1 = e^{-j2\pi\zeta_1} \quad (2.2)$$

$$\zeta_1 = d_1/D_1 \quad (2.3)$$

$$D_1 = (\lambda/2)/(n_1^2 - \sin^2 \phi)^{1/2} \quad (2.4)$$

In Eqs. (2.2) - (2.4), ζ_1 is the normalized film thickness, d_1 is the metric thickness of the thin film, D_1 is the film-thickness period, and λ is the wavelength of light. All materials are assumed to be transparent and $n_0 = 1$.

The Fresnel reflection coefficients in Eq. (2.1) can be written as follows:

$$r_{01p} = (n_1^2 S_0 - S_1)/(n_1^2 S_0 + S_1), \quad (2.5)$$

$$r_{12p} = (n_2^2 S_1 - n_1^2 S_2)/(n_2^2 S_1 + n_1^2 S_2),$$

$$r_{01s} = (S_0 - S_1)/(S_0 + S_1), \quad (2.6)$$

$$r_{12s} = (S_1 - S_2)/(S_1 + S_2),$$

where

$$\begin{aligned} S_0 &= \cos \phi \\ S_1 &= (n_1^2 - \sin^2 \phi)^{1/2} \\ S_2 &= (n_2^2 - \sin^2 \phi)^{1/2}. \end{aligned} \tag{2.7}$$

If the thin film has a quarter-wave optical thickness, then

$$d_1 = D_1/2 = \frac{\lambda}{4}(n_1^2 - \sin^2 \phi)^{-1/2} \tag{2.8}$$

$$X_1 = -1. \tag{2.9}$$

With a quarter-wave coating [Eqs. (2.8), (2.9)], the complex-amplitude reflection coefficients of the coated surface [Eq (2.1)] become real and are given by

$$R_p = \frac{n_1^4 p - n_2^2}{n_1^4 p + n_2^2} \tag{2.10}$$

$$R_s = \frac{p - 1}{p + 1}, \tag{2.11}$$

where

$$p = S_0 S_2 / S_1^2. \tag{2.12}$$

For all values of ϕ , we find that the sign R_s is negative. However, this is not the case for R_p ; which is required to satisfy Eq. (1.7) for orthogonal polarizations.

In order to achieve the equal intensity split of Eq. (1.8), the following condition must hold:

$$\mathcal{R}_u = \frac{\mathcal{R}_p + \mathcal{R}_s}{2} = \frac{1}{2}, \tag{2.13}$$

where $\mathcal{R}_\nu = |R_\nu|^2$ for the $\nu = p, s$ polarization. Substituting Eqs. (2.10) and (2.11) in Eq. (2.13), we obtain a quartic equation in p given by:

$$a_4 p^4 + a_3 p^3 + a_2 p^2 + a_1 p + a_0 = 0 \tag{2.14}$$

where

$$\begin{aligned}
a_4 &= n_1^8 \\
a_3 &= -2n_1^4(n_1^4 + n_2^2) \\
a_2 &= n_1^4(n_1^4 - 12n_2^2) + n_2^4 \\
a_1 &= -2n_2^2(n_1^4 + n_2^2) \\
a_0 &= n_2^4.
\end{aligned} \tag{2.15}$$

If Eq. (2.14) is solved for $p \in \Re, 0 < p < 1$, then, the operating angle of incidence ϕ at which Eq. (2.13) is satisfied can be determined. Squaring Eq. (2.12) and using Eqs. (2.7), we obtain:

$$p^2 = \frac{\cos^2 \phi (n_2^2 - \sin^2 \phi)}{(n_1^2 - \sin^2 \phi)^2}. \tag{2.16}$$

Let

$$\sin^2 \phi = u. \tag{2.17}$$

Substituting Eq. (2.17) in Eq. (2.16) and using the trigonometric identity $\cos^2 \phi + \sin^2 \phi = 1$, we obtain a quadratic equation in u

$$b_2 u^2 + b_1 u + b_0 = 0 \tag{2.18}$$

where

$$\begin{aligned}
b_2 &= (p^2 - 1) \\
b_1 &= [(n_2^2 + 1) - 2n_1^2 p^2] \\
b_0 &= (p^2 n_1^4 - n_2^2).
\end{aligned} \tag{2.19}$$

Equation (2.18) has a valid solution for the operating angle of incidence ϕ only if $u \in \Re, 0 < u < 1$.

The procedure to design a single-layer-coated beam splitter, which can generate beams of equal power and orthogonal polarizations, is summarized below:

1. Choose an appropriate refractive index of the substrate material n_2 .

2. For $1 \leq n_1 \leq 6$, solve Eq. (2.14) for p , where $p \in \Re, 0 < p < 1$.
3. Substitute p to find u in Eq. (2.18), where $u \in \Re, 0 < u < 1$.
4. Recover ϕ from u using Eq. (2.17).
5. Substitute ϕ in Eqs. (2.10) and (2.10) to calculate R_p and R_s .
6. Plot R_p and R_s as a function of n_1 . The values of ϕ at which R_p and R_s have opposite sign will guarantee that the reflected light and the refracted light have orthogonal polarizations.

Figure (2.2) shows the angle of incidence ϕ as a function of n_1 for the special case when $n_2 = 1$. As can be seen, two solution branches exist. The first solution with solid line (this will be called Solution 1 from now on), gives the value of ϕ for which equal intensity is achieved for the reflected and refracted light and $\Delta_r = 0$. The polarization of the reflected and refracted light are not orthogonal. The second solution represented by the dashed lines (this will be called Solution 2 from now on), gives the value of ϕ at which the reflected and refracted light have equal intensity and $\Delta_r = \pi$. In this case, the polarizations of the reflected and refracted light are orthogonal, i.e., Eq. (1.7) is satisfied.

As examples, Figures (2.3) and (2.5) show the amplitude reflectances for the p and s polarizations for Solutions 1 and 2 when $n_2 = 1$. Figures (2.4) and (2.6) show the plots of intensity reflectances for the p and s polarizations for Solutions 1 and 2. Notice that as n_1 increases, \mathcal{R}_p diminishes. The average intensity reflectance, \mathcal{R}_u , always takes the value of 0.5 for both Solutions 1 and 2, which is required by the design.

Figure (2.7) shows the angle of incidence ϕ as a function of n_1 when $n_2 = 1.2$. Similar to the case where $n_2 = 1$, there are two solution branches for ϕ . Figures (2.8) and (2.10) show the amplitude reflectances for the p and s polarizations for Solutions 1 and 2 when $n_2 = 1.2$. Figures (2.9) and (2.11) show the plots of intensity reflectances for the p and s polarizations for Solutions 1 and 2.

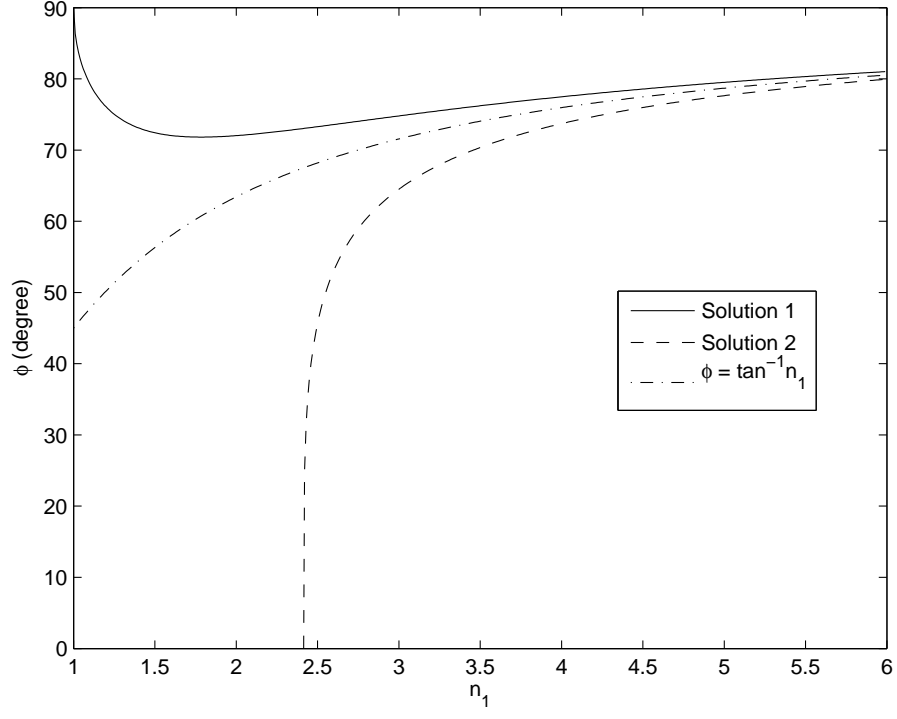


Figure 2.2: ϕ as a function of n_1 when $n_2 = 1$

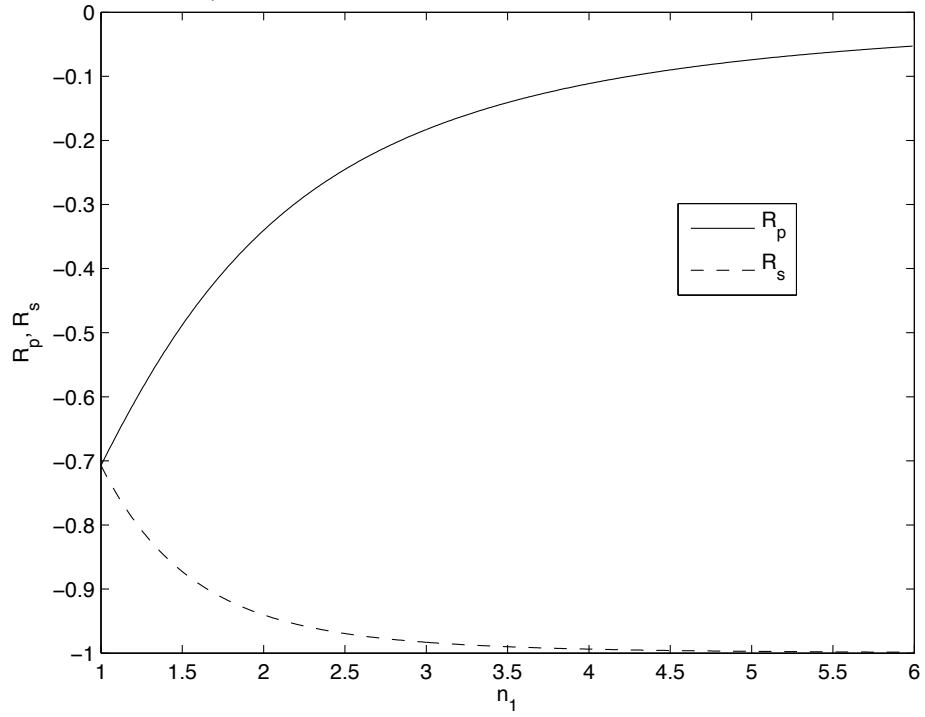


Figure 2.3: R_p, R_s as functions of n_1 when $n_2 = 1$, Solution 1 ($\Delta_r = 0$)

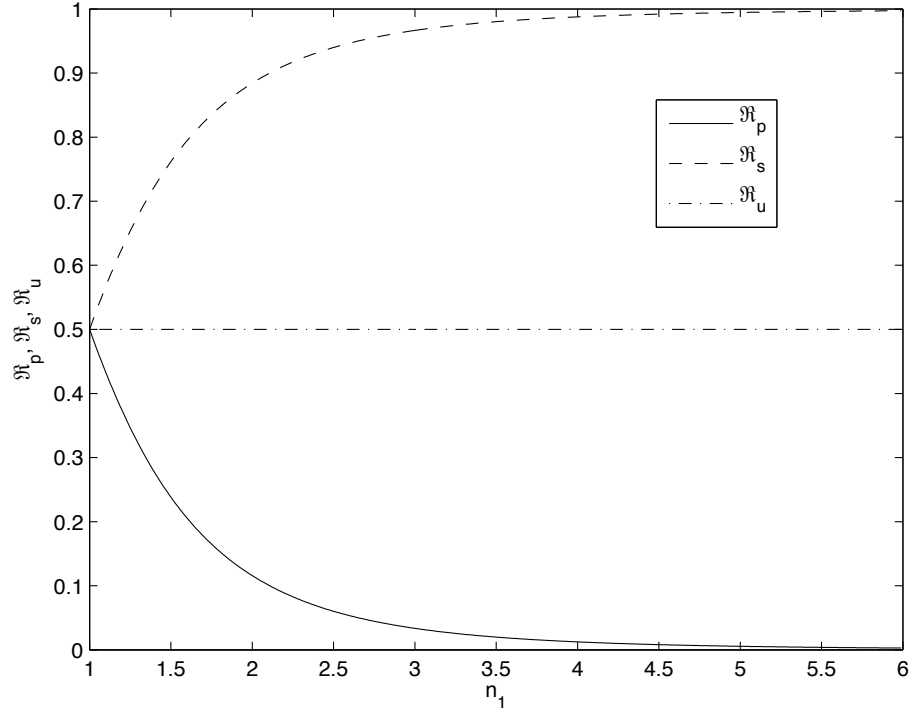


Figure 2.4: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1$, Solution 1 ($\Delta_r = 0$)

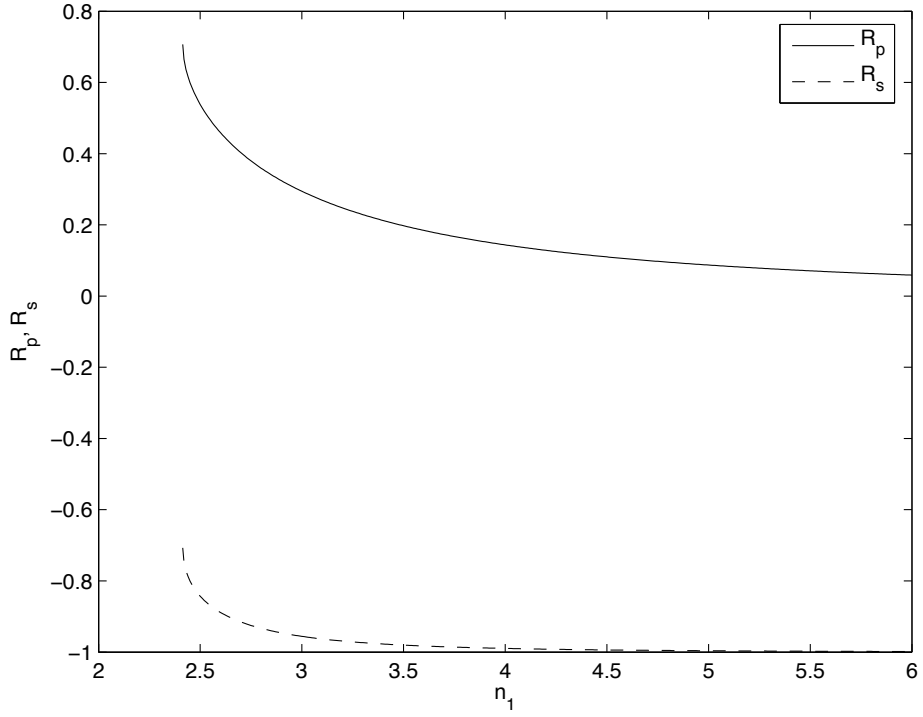


Figure 2.5: R_p, R_s as functions of n_1 when $n_2 = 1$, Solution 2 ($\Delta_r = \pi$)

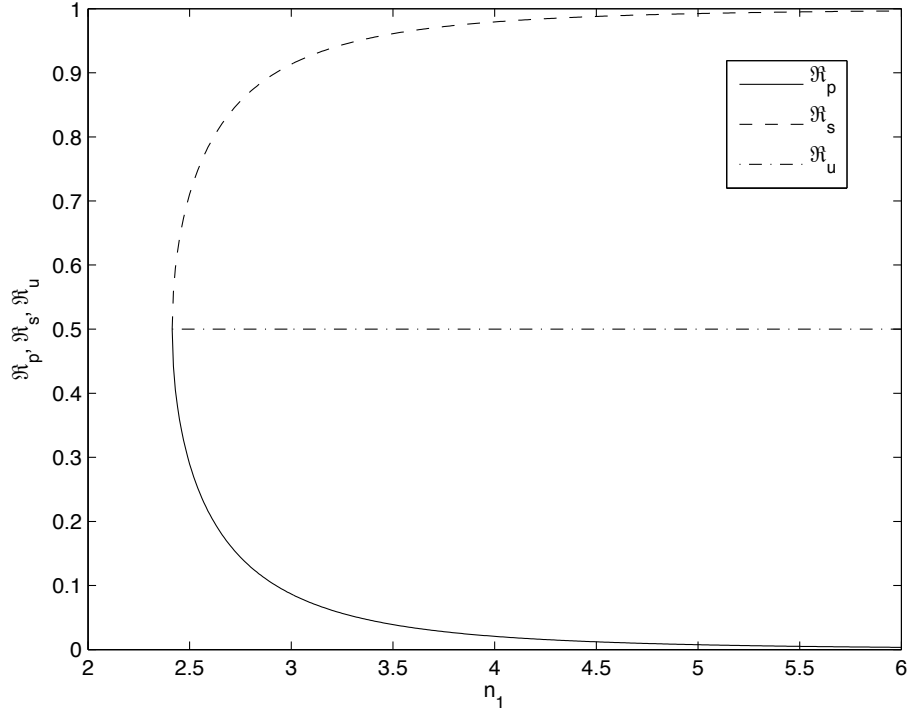


Figure 2.6: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1$, Solution 2 ($\Delta_r = \pi$)

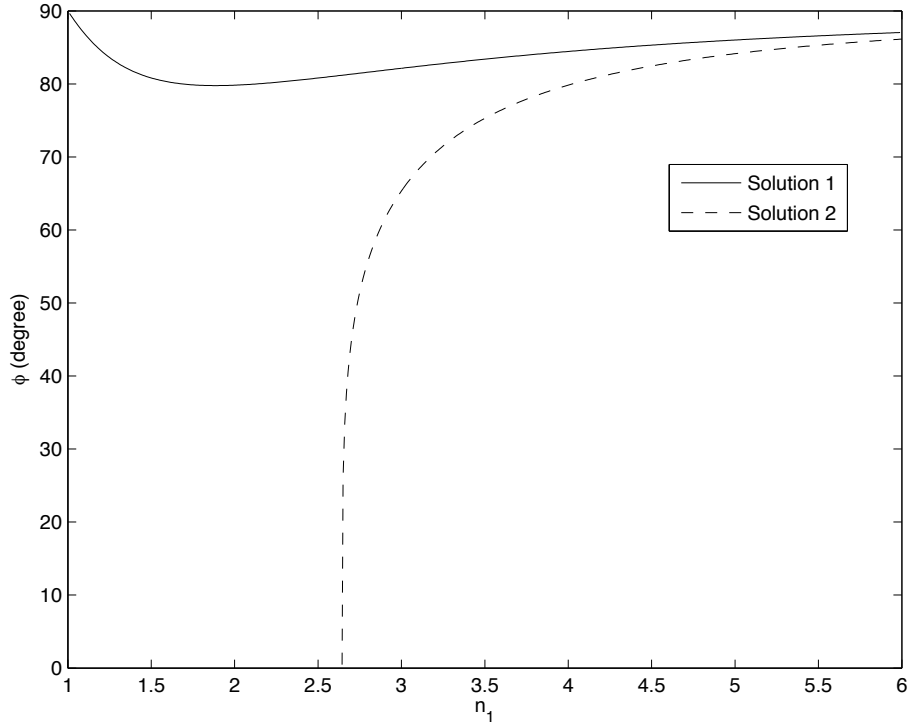


Figure 2.7: ϕ as a function of n_1 when $n_2 = 1.2$

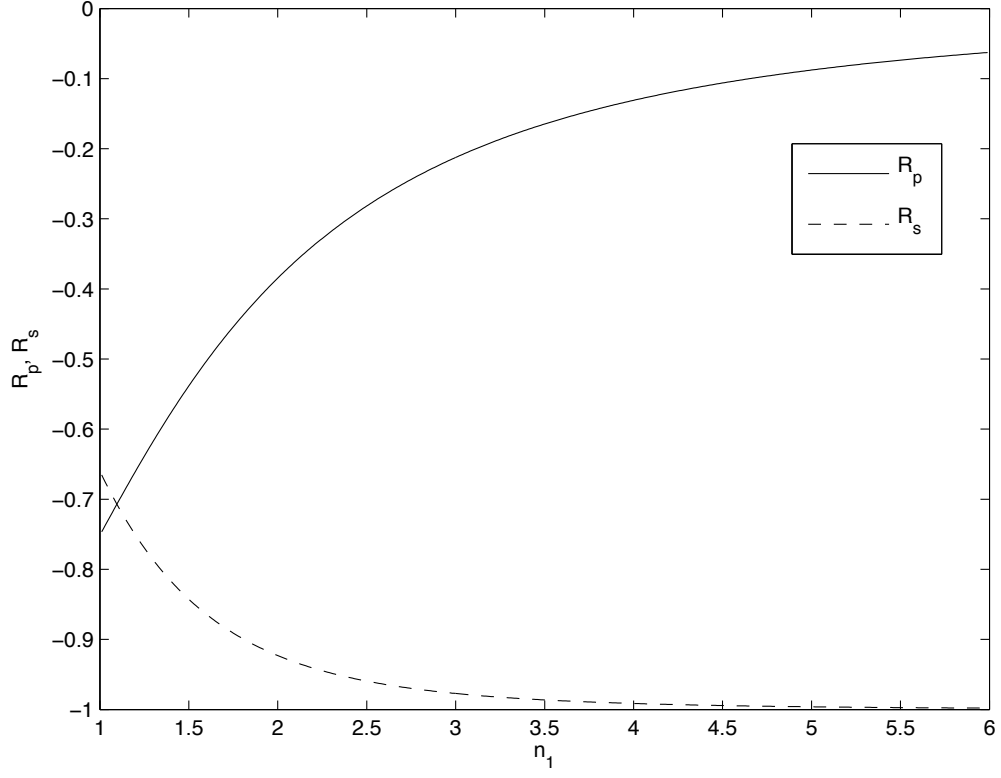


Figure 2.8: R_p, R_s as functions of n_1 when $n_2 = 1.2$, Solution 1 ($\Delta_r = 0$)

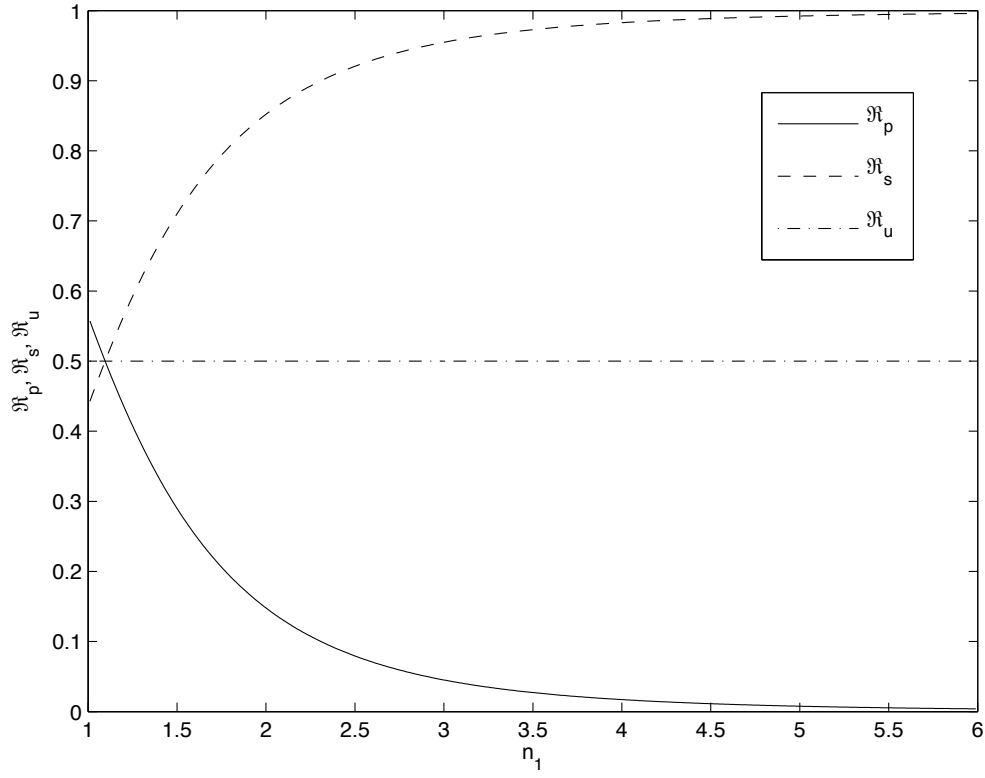


Figure 2.9: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1.2$, Solution 1 ($\Delta_r = 0$)

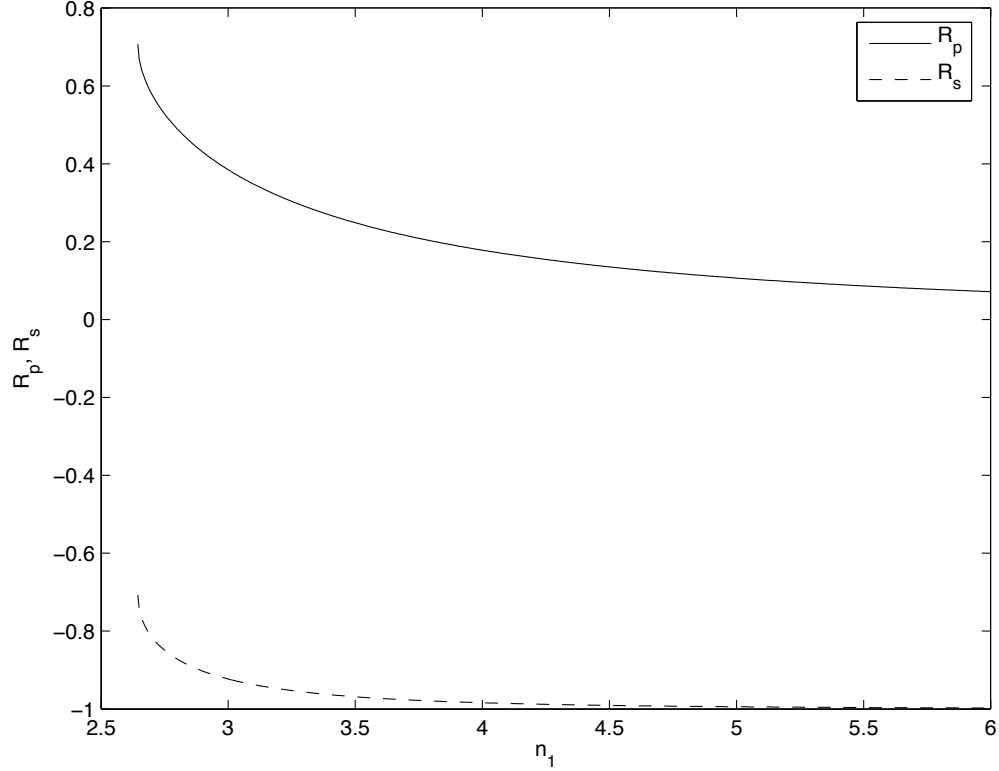


Figure 2.10: R_p, R_s as functions of n_1 when $n_2 = 1.2$, Solution 2 ($\Delta_r = \pi$)

Figure (2.12) shows the two branches of solution of ϕ as a function of n_1 when $n_2 = 1.5$. Figures (2.13) and (2.15) show the amplitude reflectances for the p and s polarizations for Solutions 1 and 2 when $n_2 = 1.5$. Figures (2.14) and (2.16) show the intensity reflectances for the p and s polarizations for Solutions 1 and 2.

From Figs. (2.2), (2.7), and (2.12), it is seen that that the refractive index of the substrate n_2 is increased, the Solution 2 curve for ϕ tends to shift to right. This means that thin films of higher refractive index n_1 are required. In order to design a beam splitter that operates at relatively low angle of incidence ϕ , there should be a high enough difference between n_1 and n_2 , where $n_1 > n_2$.

The value of n_1 for which beams of equal intensity and orthogonal polarization are generated at normal incidence can be found. At normal incidence, the complex-amplitude for

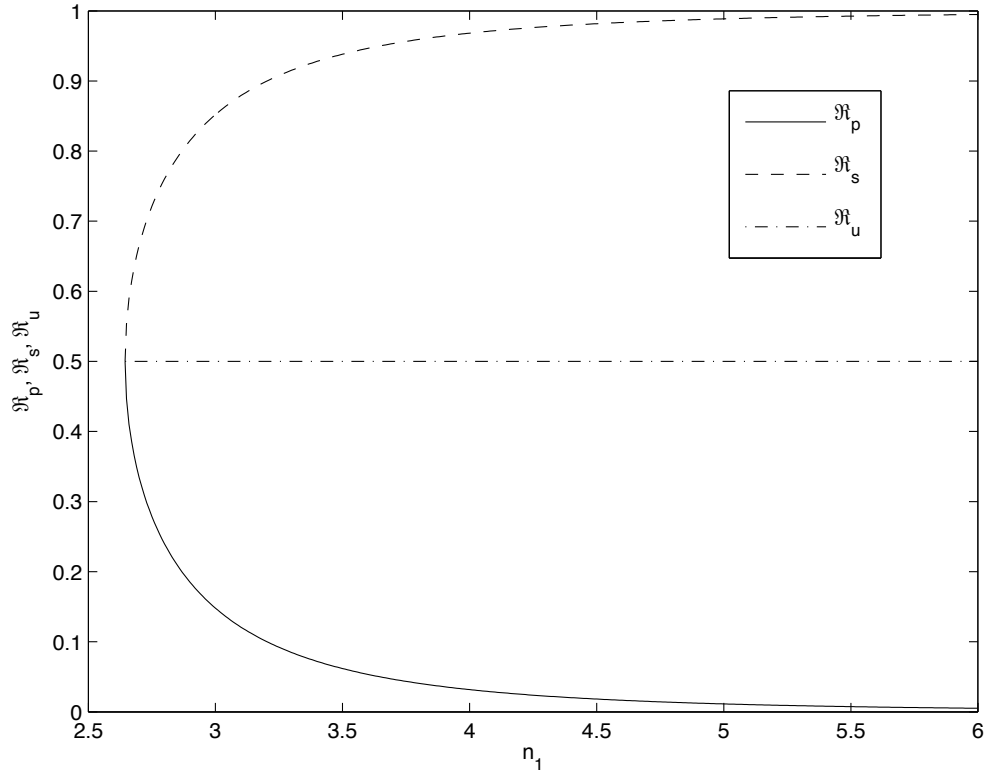


Figure 2.11: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1.2$, Solution 2 ($\Delta_r = \pi$)

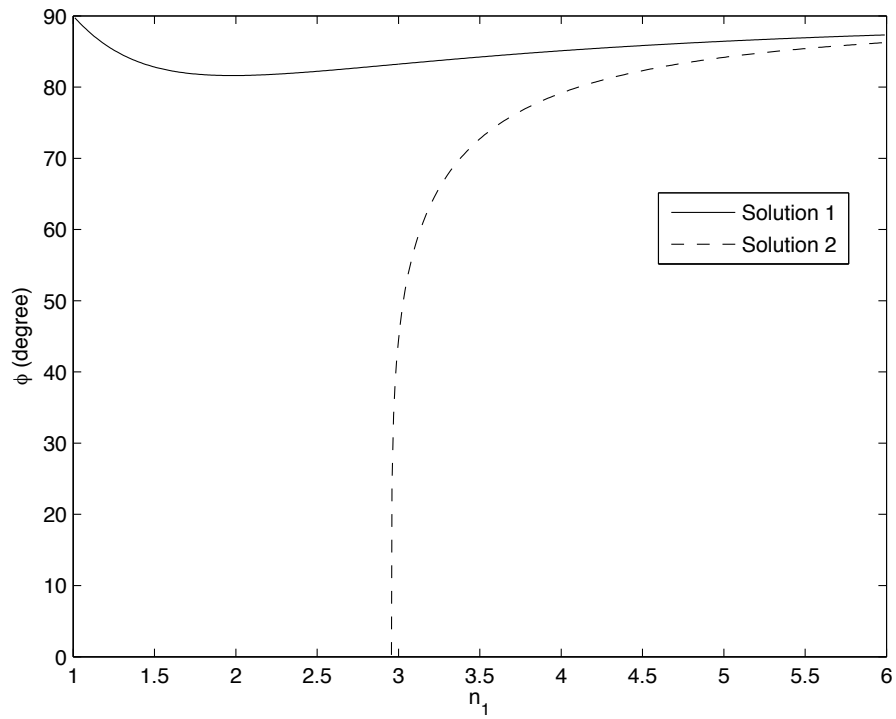


Figure 2.12: ϕ as a function of n_1 when $n_2 = 1.5$

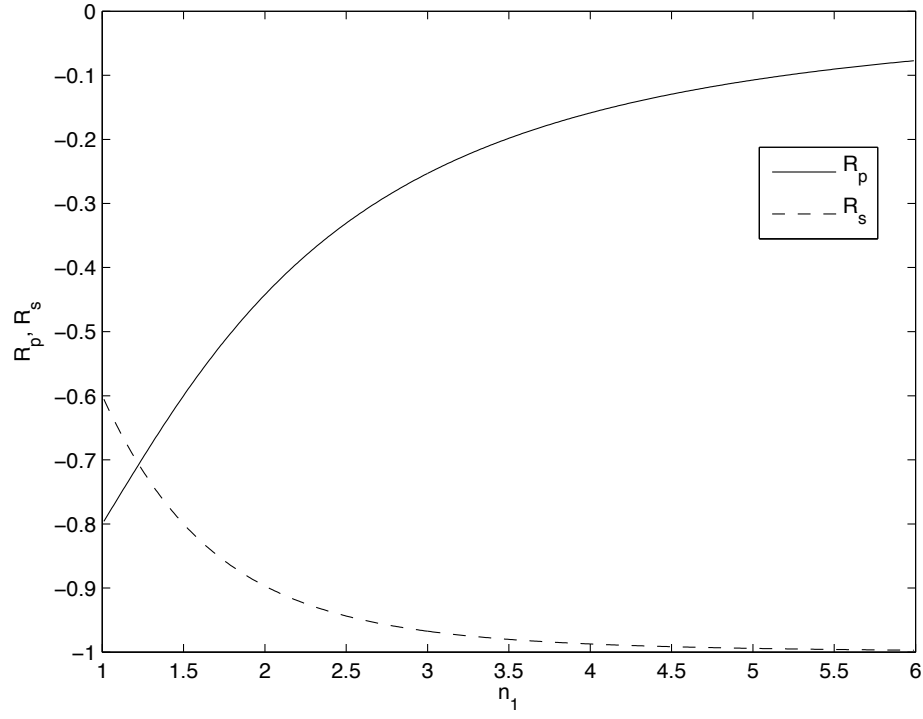


Figure 2.13: R_p, R_s as functions of n_1 when $n_2 = 1.5$, Solution 1 ($\Delta_r = 0$)

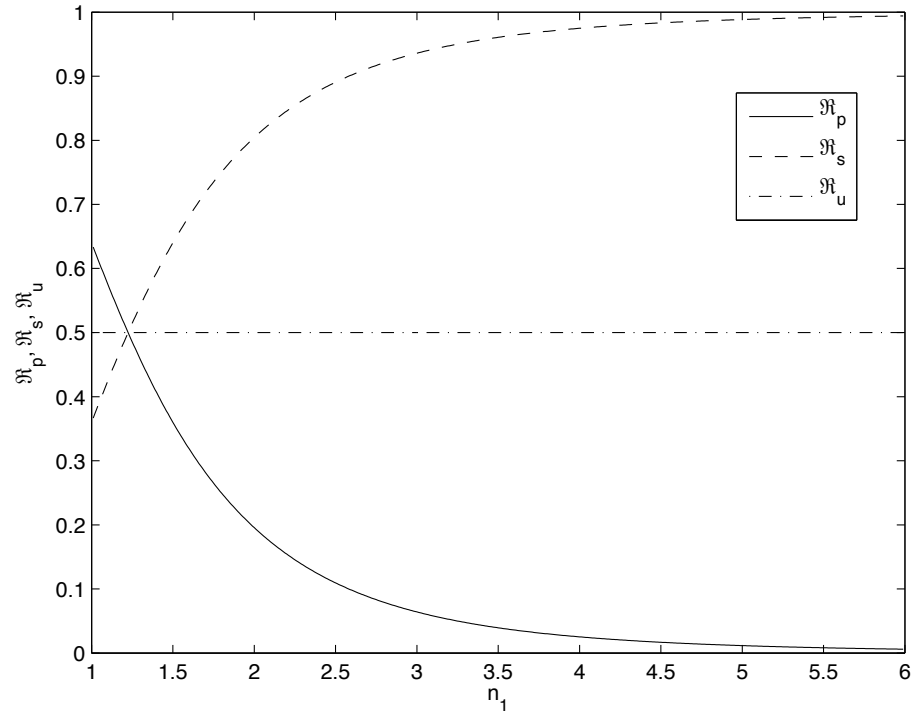


Figure 2.14: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1.5$, Solution 1 ($\Delta_r = 0$)

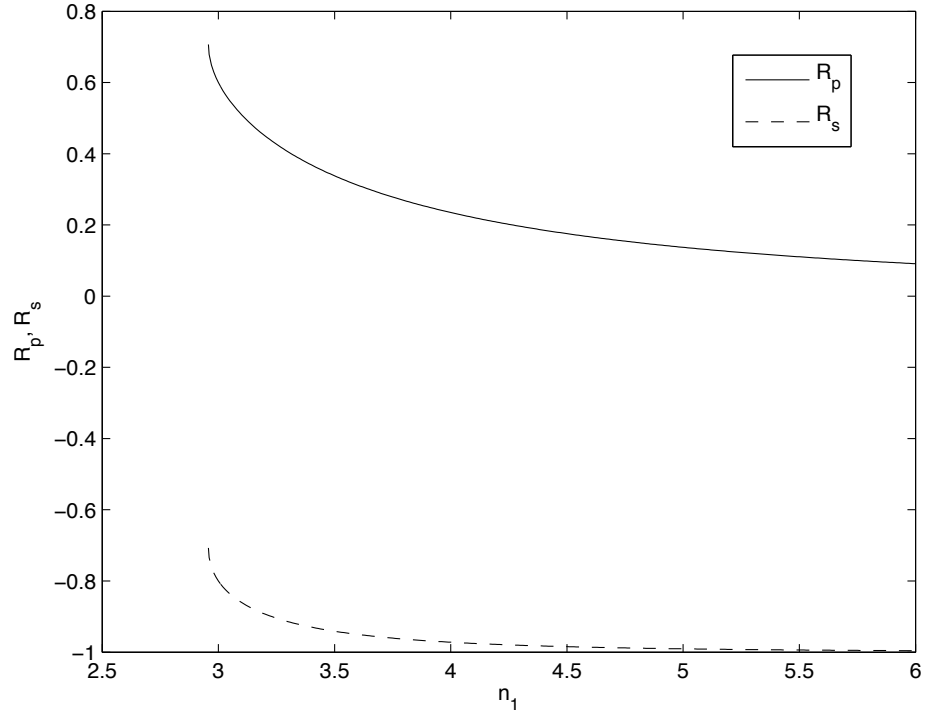


Figure 2.15: R_p, R_s as functions of n_1 when $n_2 = 1.5$, Solution 2 ($\Delta_r = \pi$)

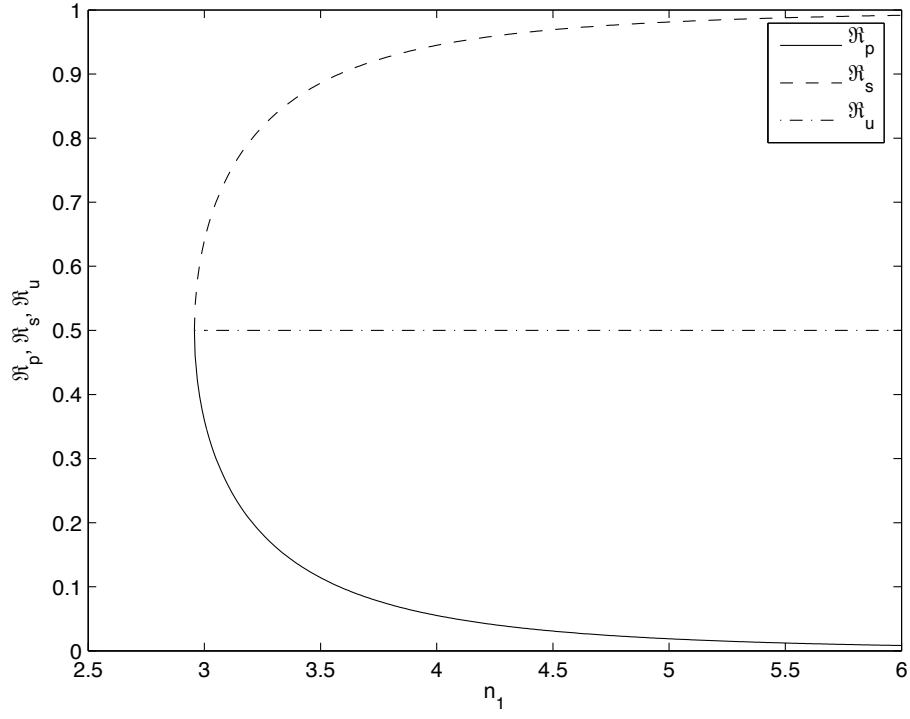


Figure 2.16: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1.5$, Solution 2 ($\Delta_r = \pi$)

reflection coefficients are:

$$R_p = (n_2 - n_1^2)/(n_2 + n_1^2) \quad (2.20)$$

$$R_s = (n_1^2 - n_2)/(n_1^2 + n_2) \quad (2.21)$$

Substituting Eqs. (2.20) and (2.21) in Eq. (1.8), we obtain a quadratic equation in n_1^2

$$(n_1^2)^4 - (6n_2)n_1^2 + n_2^2 = 0. \quad (2.22)$$

Solving the above equation for n_1 , we obtain

$$n_1 = \sqrt{(3 + \sqrt{2})n_2}, \quad (2.23)$$

the solution $\sqrt{(3 - \sqrt{2})n_2} < 1$ is discarded since it lacks physical meaning.

From Figs. 2.8 and 2.13, there exists a point where $R_p = R_s$. At this point, the beam splitter acts as a polarization-independent beam splitter (PIBS) [9], where the polarization state of the input light is reflected and refracted unchanged.

Now that we have presented the mathematical background for the design of single-layer-coated beam splitters that can generate beams of equal intensity and orthogonal polarizations, we proceed with specific physical designs of such beam splitters. Two designs at a visible wavelength and two designs at an infrared wavelength are presented in the following sections. In addition, it is also important to analyze the properties of the reflected and the refracted beams should the beam splitters operate at wavelengths, angles of incidence, and film thicknesses *other* than the designed values. Illustrations on how \mathcal{R}_u , ψ_r , ψ_t , and Δ_r , Δ_t vary as functions of wavelength, angle of incidence, and film thickness are included.

Recall that \mathcal{R}_u indicates the power of the reflected beam. ψ_r and ψ_t are ellipsometric parameters whose sum need to be 90° if $\mathcal{R}_u = 0.5$. Lastly, Δ_r and Δ_t must satisfy Eqs. (1.7) so that the reflected beam and the refracted beam have orthogonal polarizations.

2.1 Design at Visible Wavelengths

2.1.1 Fused Silica Prism Coated by a GaP Thin Film

The first design of single-layer-coated beam splitter uses a fused silica prism which is coated by a GaP thin film. The operating wavelength for this beam splitter is $\lambda = 0.633 \text{ } \mu\text{m}$. The refractive indices [10] of GaP and fused silica (SiO_2) are 3.3077 and 1.4570, respectively. The following data gives the operating design parameters of the beam splitter:

$$\begin{aligned}\phi &= 68.9876^\circ \\ \alpha &= 39.8438^\circ \\ \zeta_1 &= 0.5000 \\ d_1 &= 99.7416 \text{ nm} \\ D_1 &= 49.8708 \text{ nm}\end{aligned}\tag{2.24}$$

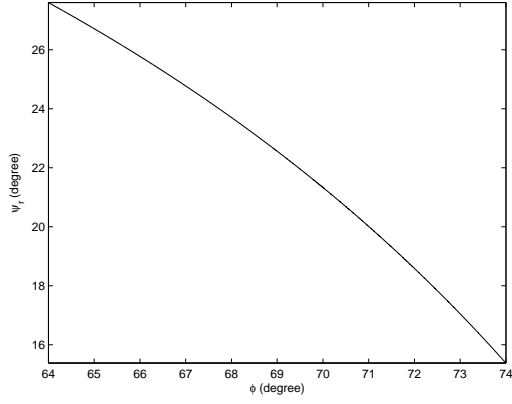
Figures (2.17) (a)-(f) show the performance of the beam splitter when the angle of incidence is in the range of $\pm 5^\circ$ of the design angle and all other parameters are fixed.

Figures (2.18) (a)-(f) show the performance of the beam splitter when a shift of $\pm 5\%$ is introduced in the wavelength λ and all other parameters are fixed.

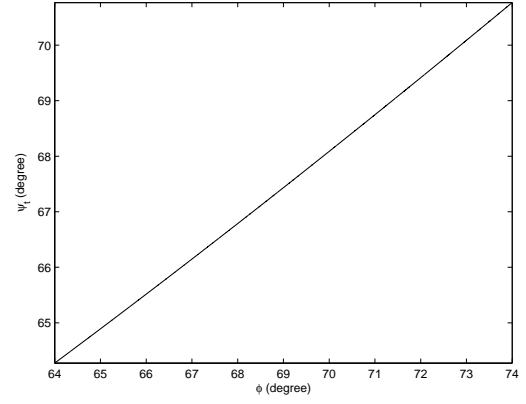
Figures (2.19) (a)-(f) show the performance of the beam splitter when an error of $\pm 5\%$ is introduced in the thickness d_1 of the thin film and all other parameters are fixed.

2.1.2 MgF_2 Prism Coated by a TiO_2 Thin Film

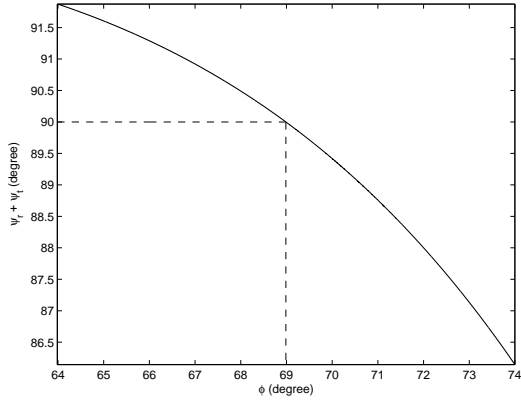
The operating wavelength for this beam splitter is $\lambda = 0.488 \text{ } \mu\text{m}$. The refractive indices [10] of TiO_2 and MgF_2 are 2.895 and 1.3856, respectively. The following data gives the operating



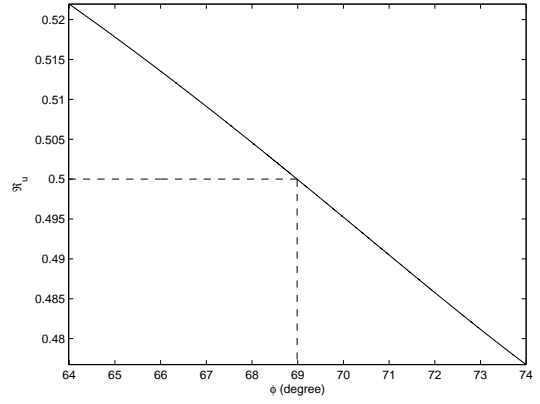
(a)



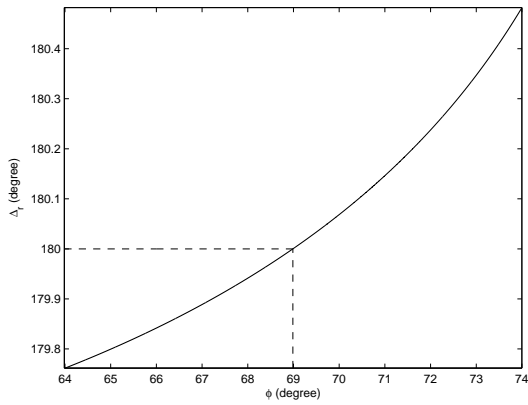
(b)



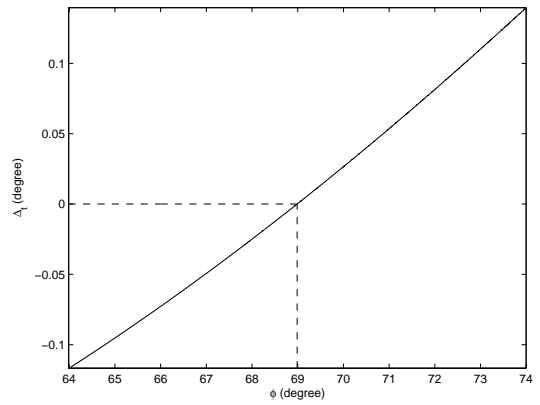
(c)



(d)

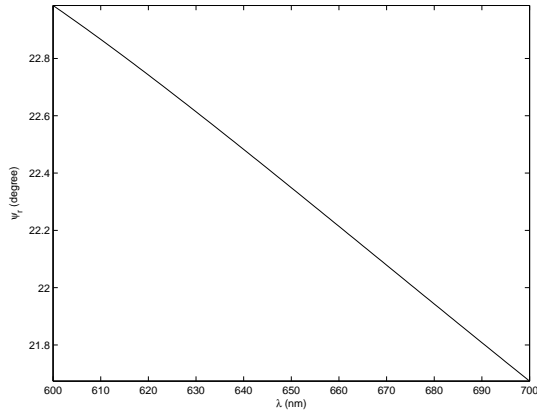


(e)

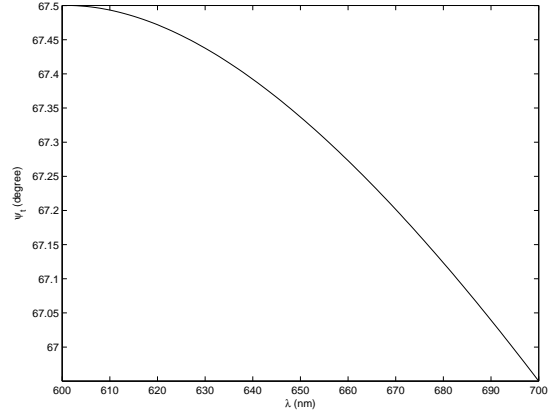


(f)

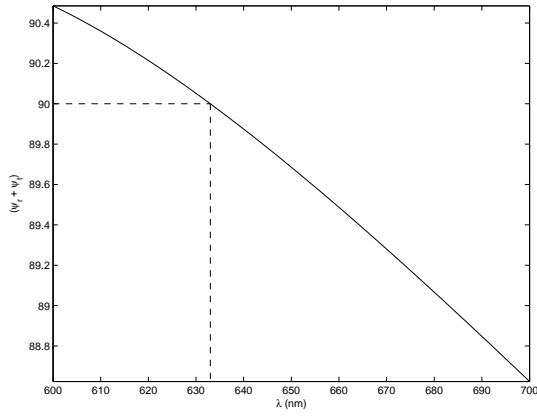
Figure 2.17: Angular Response of Beam Splitter Using Fused Silica Substrate Coated by a GaP Thin Film



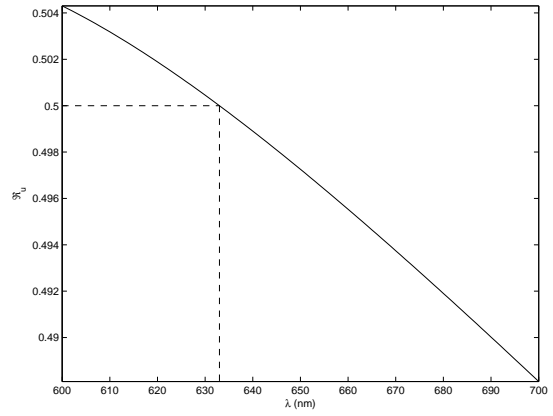
(a)



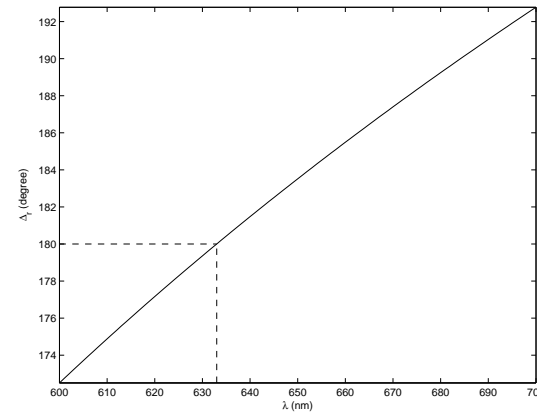
(b)



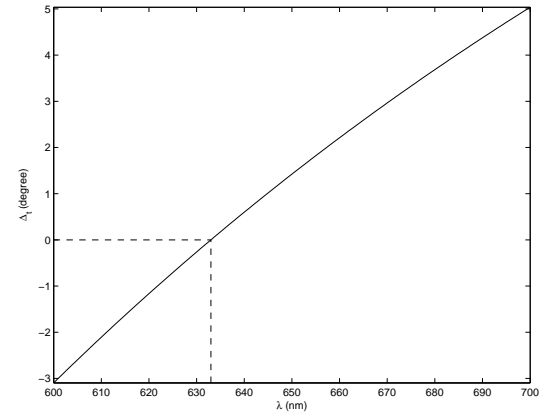
(c)



(d)



(e)



(f)

Figure 2.18: Spectral Response of Beam Splitter Using Fused Silica Substrate Coated by a GaP Thin Film

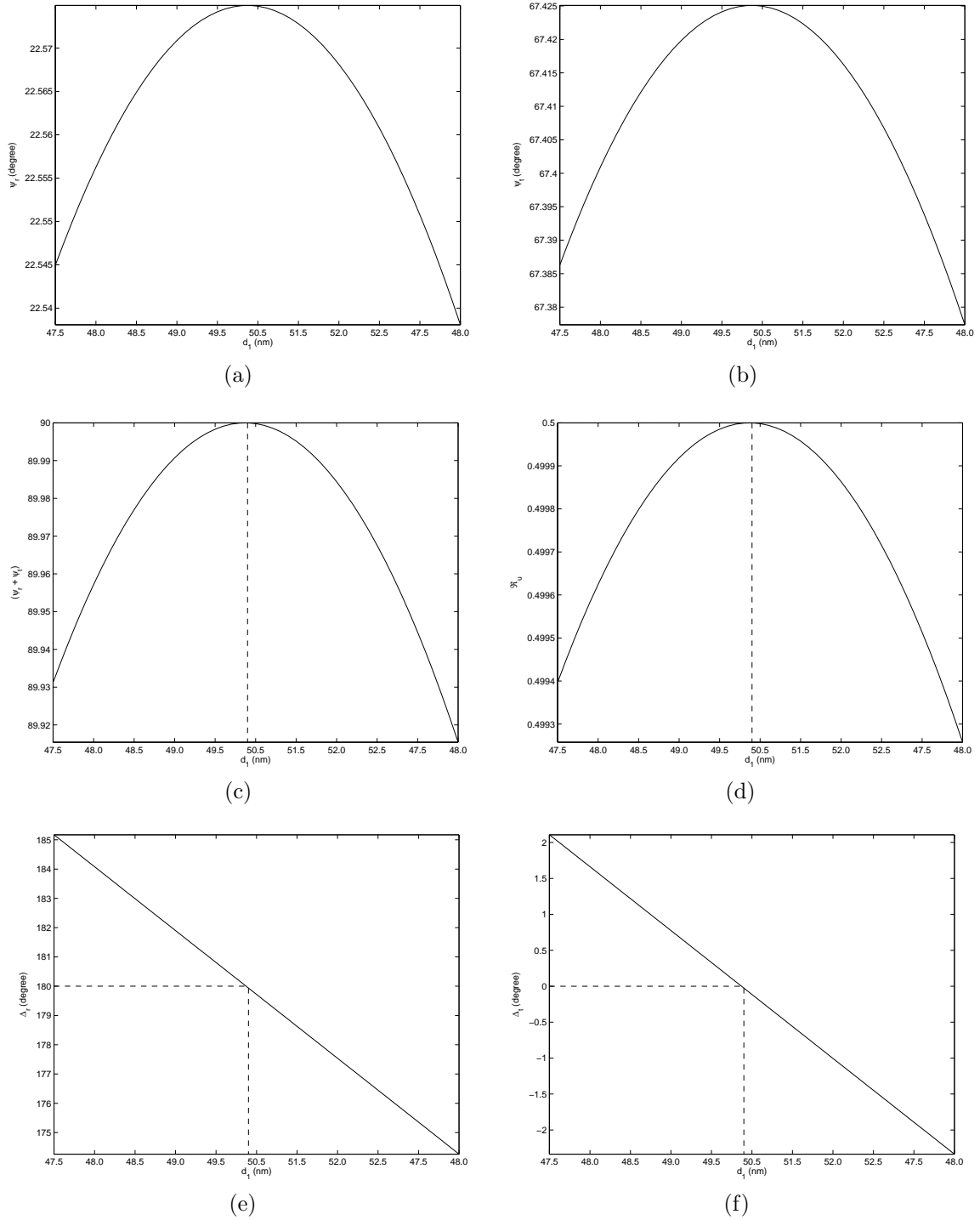


Figure 2.19: Film Thickness Response of Beam Splitter Using Fused Silica Substrate Coated by a GaP Thin Film

design parameters of the beam splitter:

$$\begin{aligned}
\phi &= 46.0367^\circ \\
\alpha &= 31.2961^\circ \\
\zeta_1 &= 0.5000 \\
d_1 &= 43.5077 \text{ nm} \\
D_1 &= 87.0155 \text{ nm}
\end{aligned} \tag{2.25}$$

Figures (2.20) (a)-(f) show the performance of the beam splitter when the angle of incidence is in the range of $\pm 5^\circ$ of the design angle and all other parameters are fixed.

Figures (2.21) (a)-(f) show the performance of the beam splitter when a shift of $\pm 5\%$ is introduced in the wavelength λ and all other parameters are fixed.

Figures (2.22) (a)-(f) show the performance of the beam splitter when an error of $\pm 5\%$ is introduced in the thickness d_1 of the thin film and all other parameters are fixed.

2.2 Design at an Infrared Wavelength

2.2.1 Irtran 3 Prism Coated by a GaP Thin Film

The operating wavelength for this beam splitter is $\lambda = 10.6 \text{ } \mu\text{m}$. The refractive index of GaP is 2.9560 [10]. Using a polynomial curve fit [11] of the 3^{rd} order, the refractive index of Irtran 3 is found to be 1.2820. The following data gives the operating design parameters of the beam splitter:

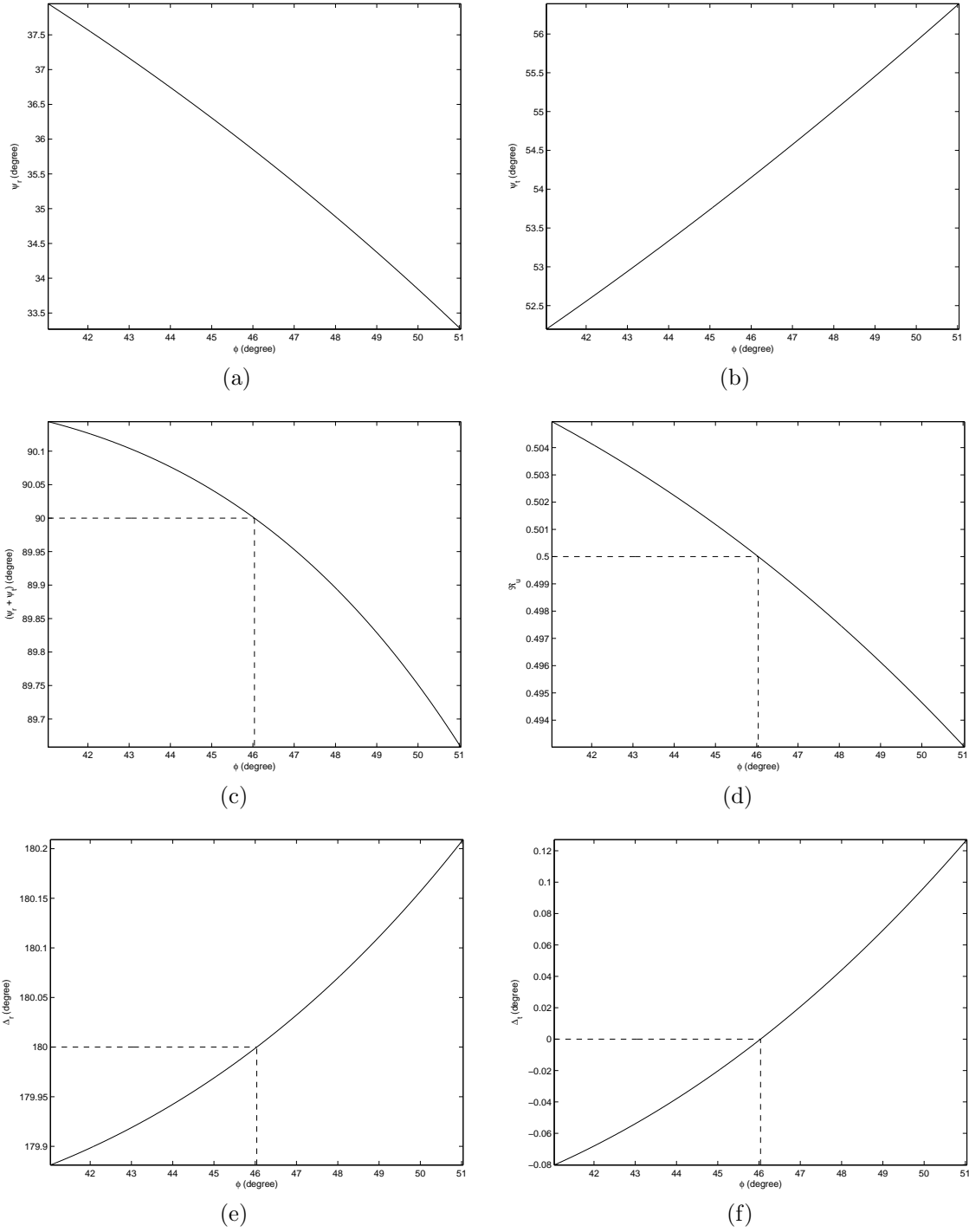


Figure 2.20: Angular Response of Beam Splitter Using MgF_2 Substrate Coated by a TiO_2 Thin Film

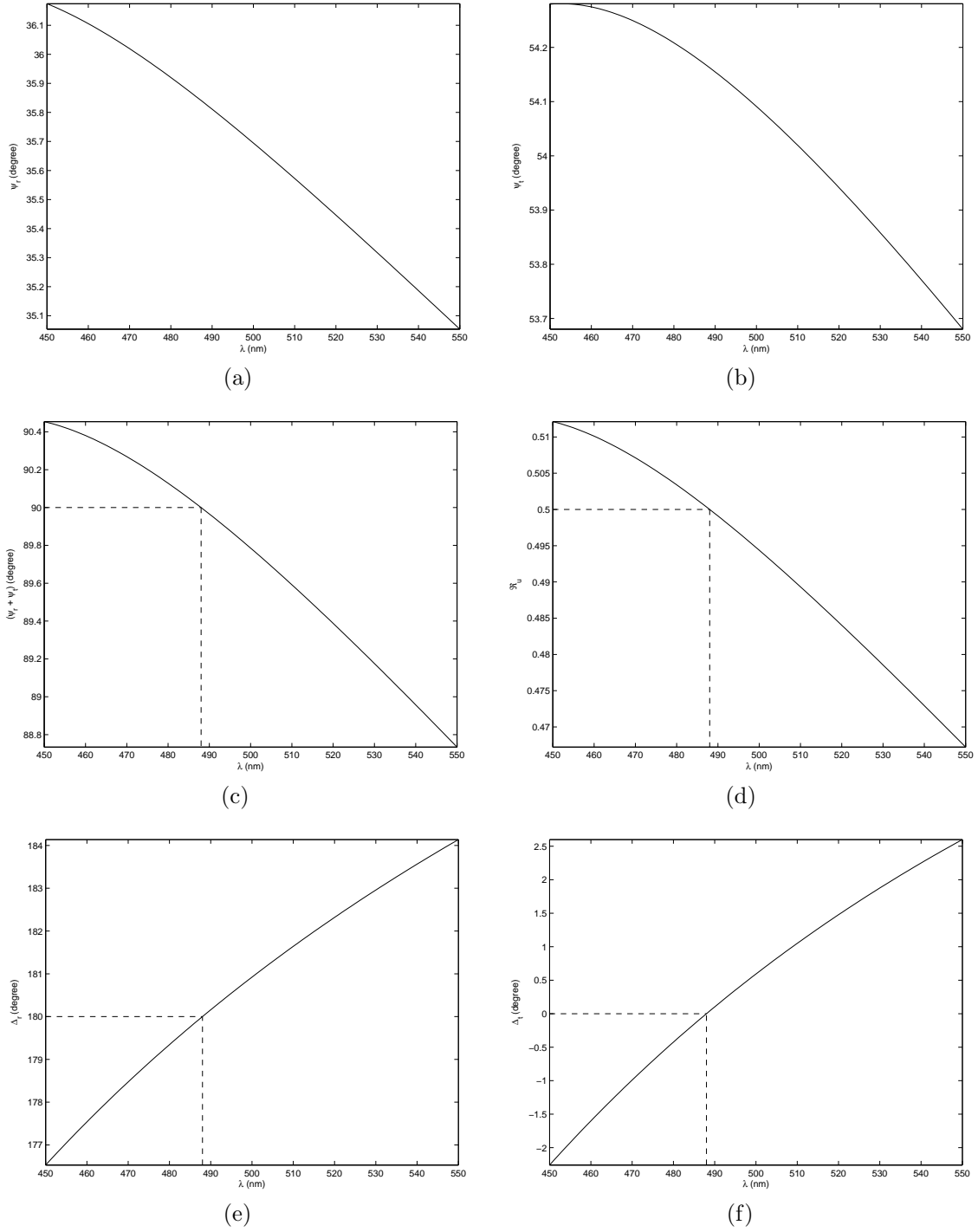


Figure 2.21: Spectral Response of Beam Splitter Using MgF_2 Substrate Coated by a TiO_2 Thin Film

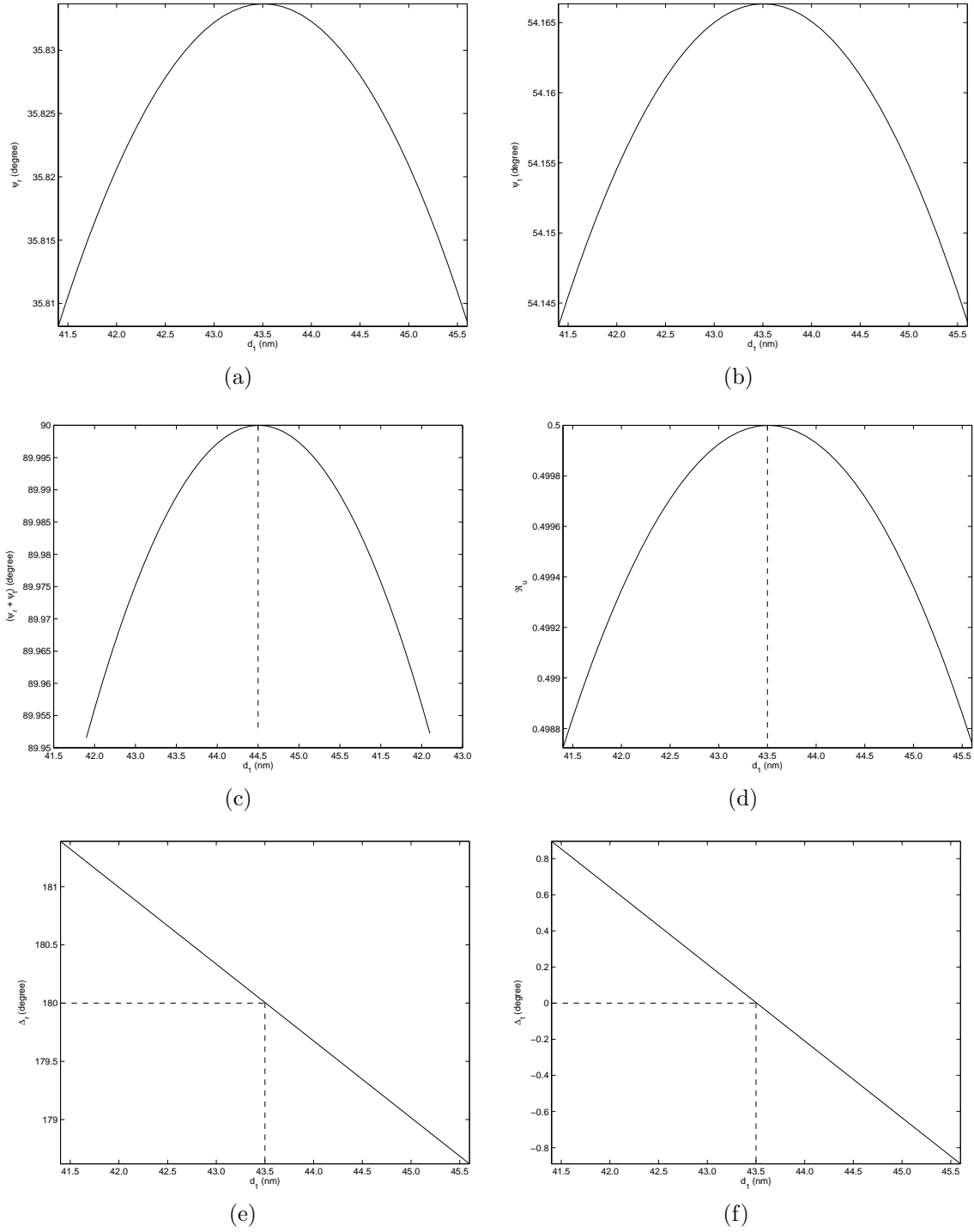


Figure 2.22: Film Thickness Response of Beam Splitter Using MgF_2 Substrate Coated by a TiO_2 Thin Film

$$\begin{aligned}
\phi &= 61.0003^\circ \\
\alpha &= 43.01957^\circ \\
\zeta_1 &= 0.5000 \\
d_1 &= 93.8503 \text{ nm} \\
D_1 &= 187.7006 \text{ nm}
\end{aligned} \tag{2.26}$$

Figures (2.23) (a)-(f) show the performance of the beam splitter when the angle of incidence is in the range of $\pm 5^\circ$ of the design angle and all other parameters are fixed.

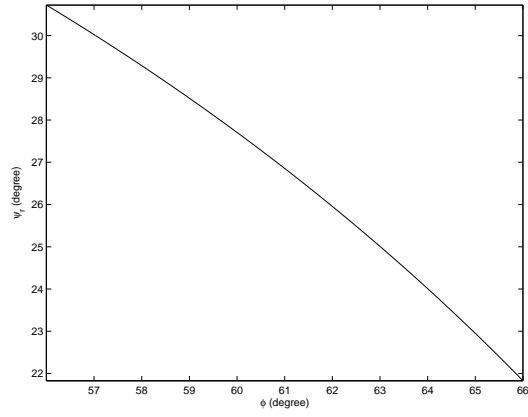
Figures (2.24) (a)-(f) show the performance of the beam splitter when a shift of $\pm 5\%$ is introduced in the wavelength λ and all other parameters are fixed.

Figures (2.25) (a)-(f) show the performance of the beam splitter when an error of $\pm 5\%$ is introduced in the thickness d_1 of the thin film and all other parameters are fixed.

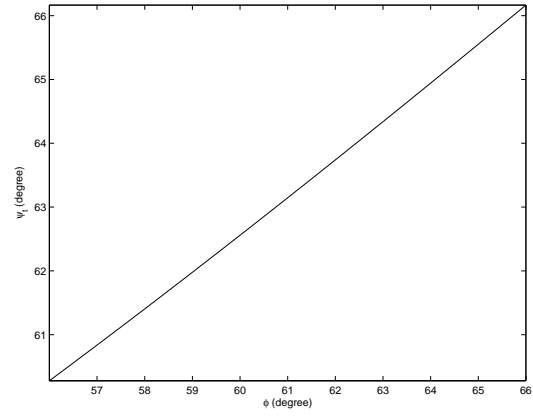
2.2.2 Irtran 3 Prism Coated by a Si Thin Film

The operating wavelength for this beam splitter is $\lambda = 10.6 \text{ } \mu\text{m}$. The refractive index of Si is 3.4176 [10]. Using a polynomial curve fit [11] of the 3^{rd} order, the refractive index of Irtran 3 is found to be 1.2820. The following data gives the operating design parameters of the beam splitter:

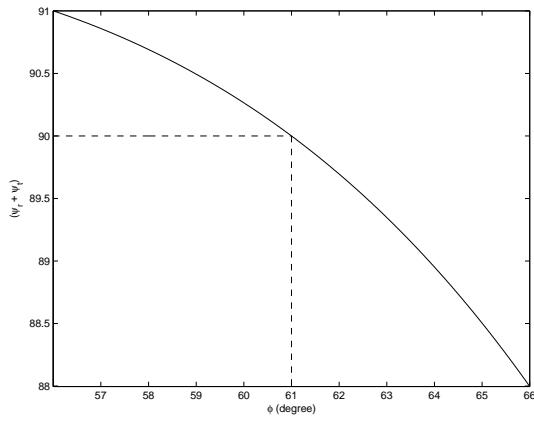
$$\begin{aligned}
\phi &= 73.9162^\circ \\
\alpha &= 48.5485^\circ \\
\zeta_1 &= 0.5000 \\
d_1 &= 80.7989 \text{ nm} \\
D_1 &= 161.5977 \text{ nm}
\end{aligned} \tag{2.27}$$



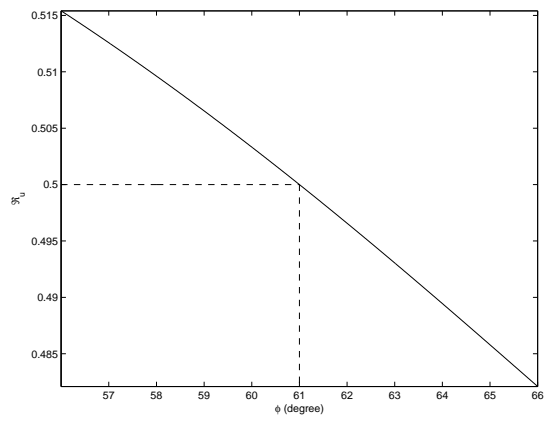
(a)



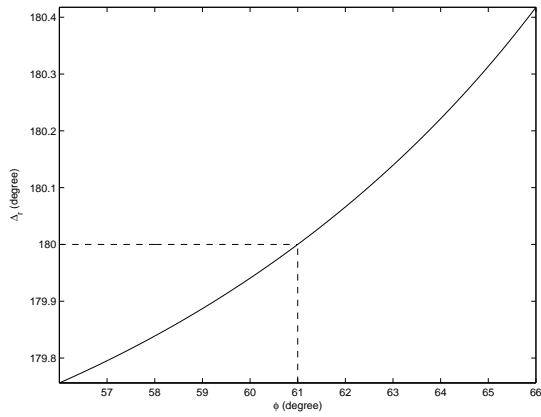
(b)



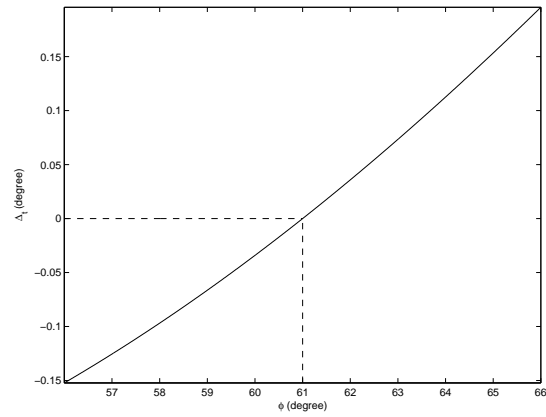
(c)



(d)



(e)



(f)

Figure 2.23: Angular Response of Beam Splitter Using Irtran 3 Substrate Coated by a GaP Thin Film

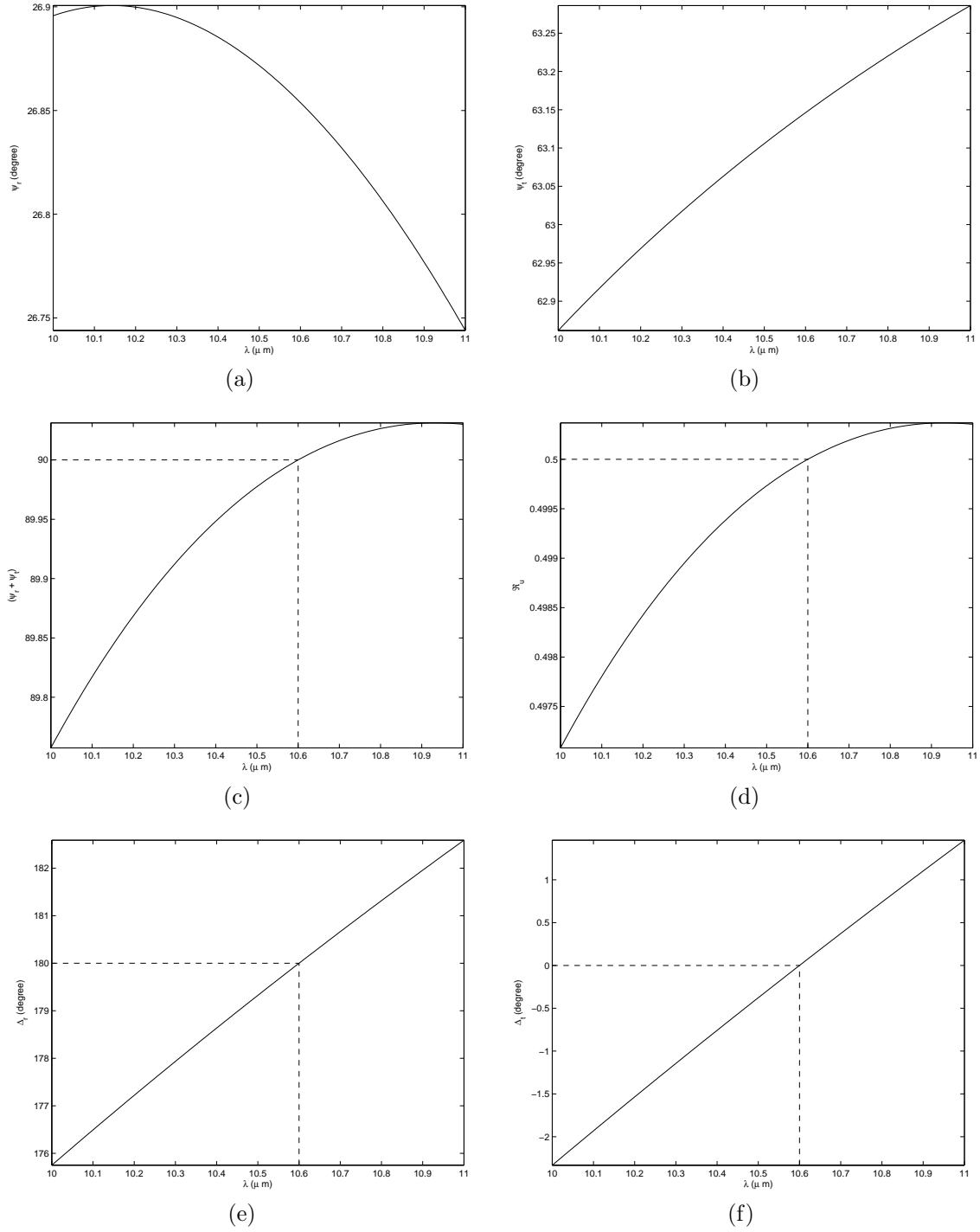


Figure 2.24: Spectral Response of Beam Splitter Using Irtran 3 Substrate Coated by a GaP Thin Film

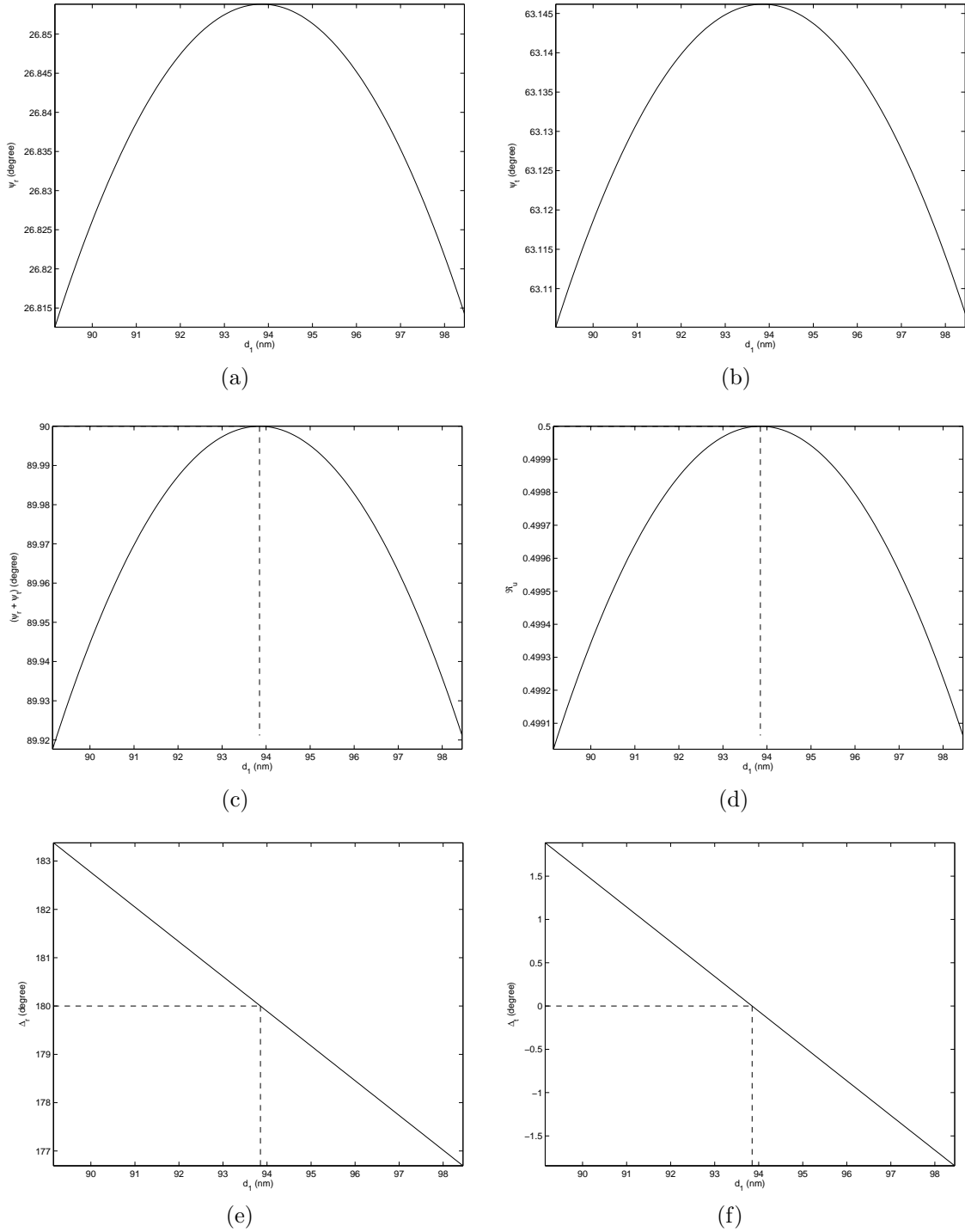
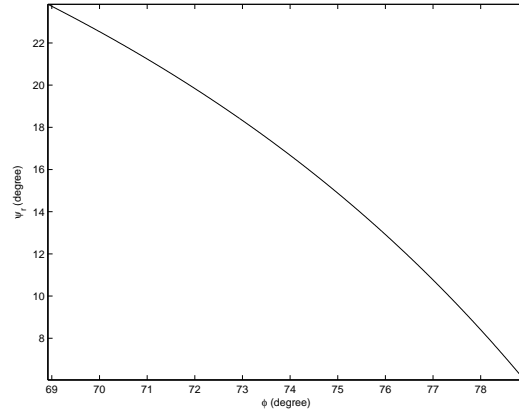


Figure 2.25: Film Thickness Response of Beam Splitter Using Itrane 3 Substrate Coated by a GaP Thin Film

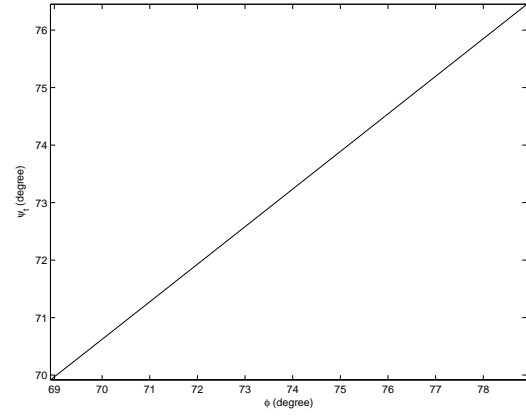
Figures (2.26) (a)-(f) show the performance of the beam splitter when the angle of incidence is in the range of $\pm 5^\circ$ of the design angle and all other parameters are fixed.

Figures (2.27) (a)-(f) show the performance of the beam splitter when a shift of $\pm 5\%$ is introduced in the wavelength λ and all other parameters are fixed.

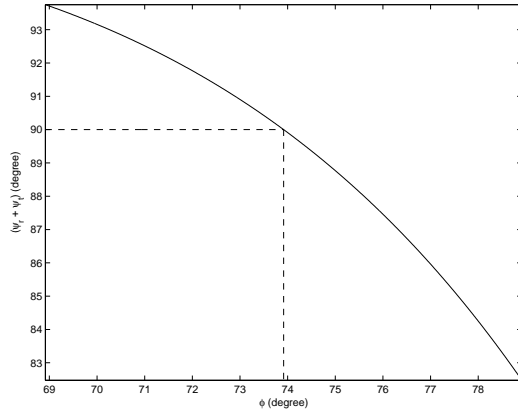
Figures (2.28) (a)-(f) show the performance of the beam splitter when an error of $\pm 5\%$ is introduced in the thickness d_1 of the thin film and all other parameters are fixed.



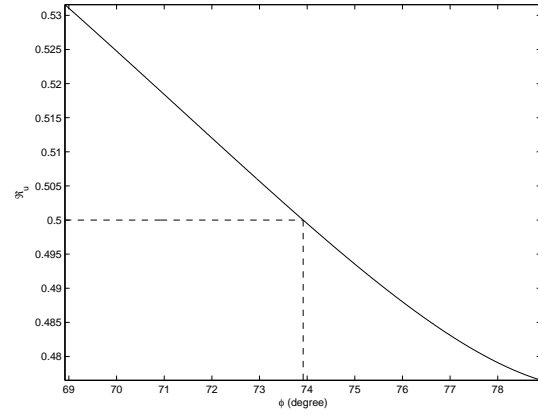
(a)



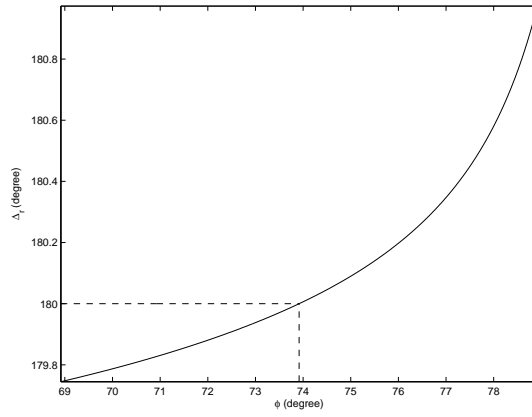
(b)



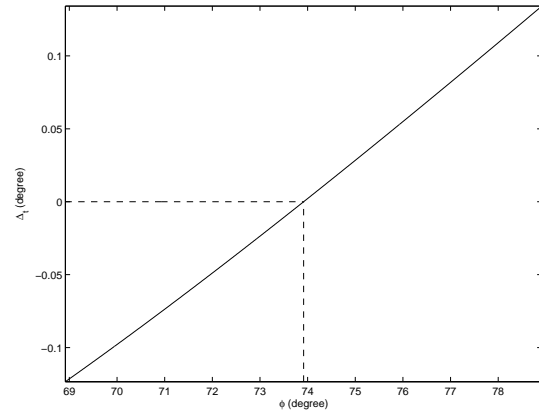
(c)



(d)



(e)



(f)

Figure 2.26: Angular Response of Beam Splitter Using Irtran 3 Substrate Coated by a Si Thin Film

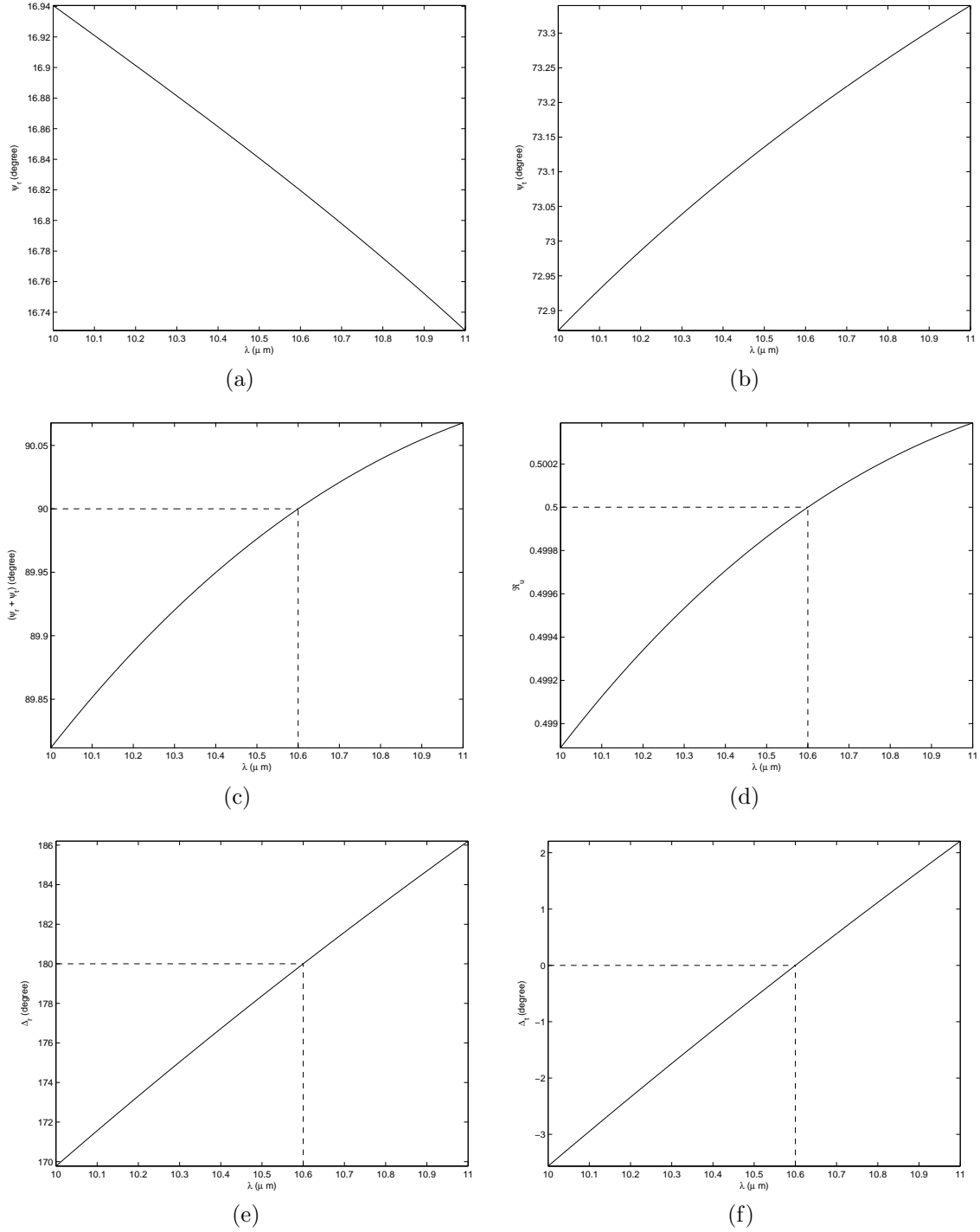


Figure 2.27: Spectral Response of Beam Splitter Using Irtran 3 Substrate Coated by a Si Thin Film

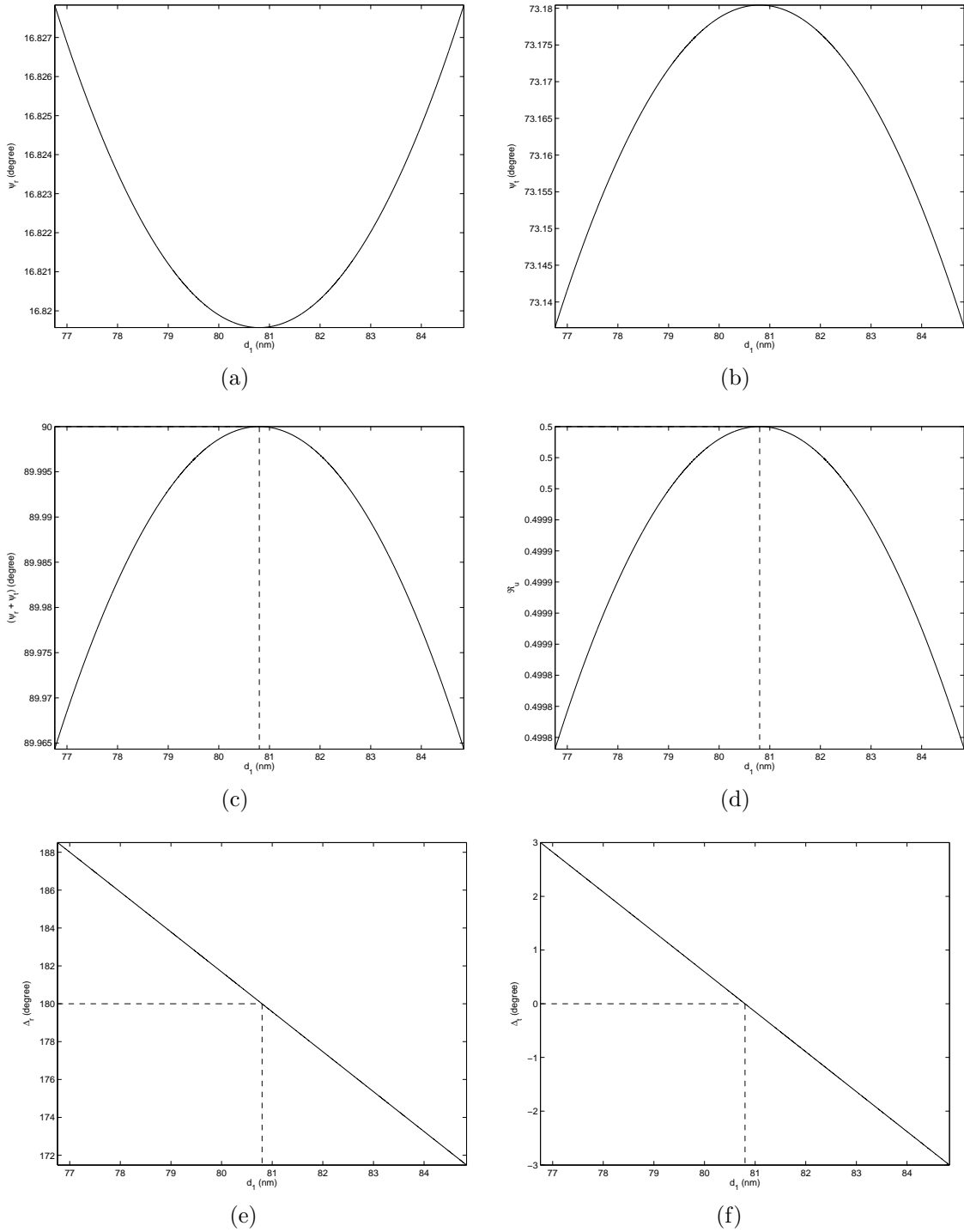


Figure 2.28: Film Thickness Response of Beam Splitter Using Itrtran 3 Substrate Coated by a Si Thin Film

Chapter 3

Design of Double-Layer-Coated Beam Splitter

The single-layer-coated prism beam splitter designs described in Chapter 2 satisfy all the conditions given by Eqs. (1.7)-(1.9). It is of interest to find out what happens when a prism is coated by two thin films each of which is quarter-wave thick. Figure 3.1 shows the proposed beam splitter design.

The beam splitter consists of a prism of refractive index n_3 . The prism is coated by two thin films whose refractive indices are n_1 and n_2 . It is assumed that all materials are transparent. The prism angle α is equal to the angle of refraction in the prism, such that the refracted light is incident normally to the exitance plane. A monochromatic beam of light of wavelength λ is incident on the structure at an angle ϕ .

The complex-amplitude reflection coefficients of the double-layer coated surface are given by [1]:

$$R_\nu = \frac{r_{01\nu} + r_{12\nu}X_1 + r_{01\nu}r_{12\nu}r_{23\nu}X_2 + r_{23\nu}X_1X_2}{1 + r_{01\nu}r_{12\nu}X_1 + r_{12\nu}r_{23\nu}X_2 + r_{01\nu}r_{23\nu}X_1X_2} \quad (3.1)$$

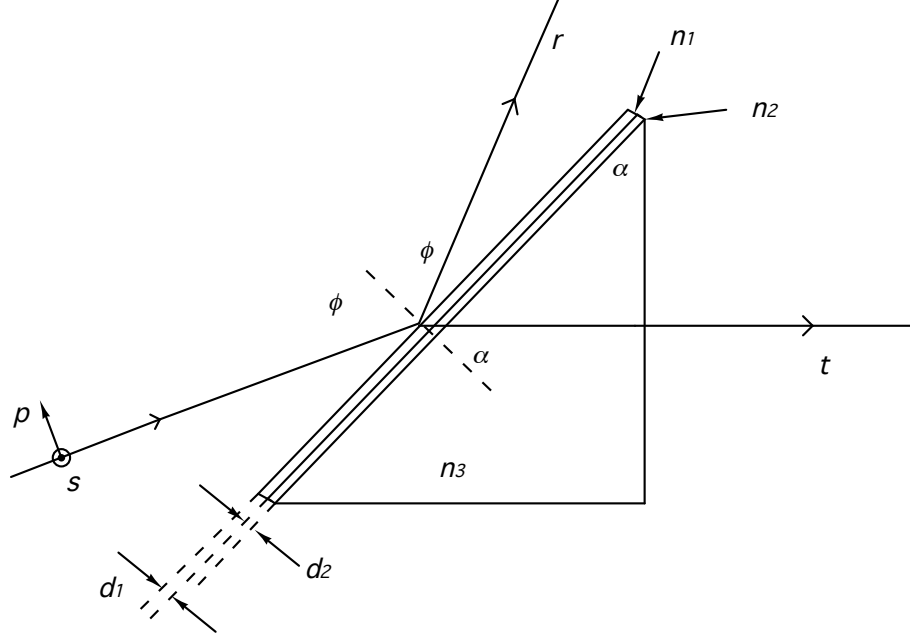


Figure 3.1: Double-Layer-Coated Beam Splitter

where $r_{ij\nu}$ is the Fresnel reflection coefficient of the ij interface for the ν polarization ($\nu = p, s$) and

$$X_1 = e^{-j2\pi\zeta_1} \quad (3.2)$$

$$X_2 = e^{-j2\pi\zeta_2} \quad (3.3)$$

$$\zeta_1 = d_1/D_1 \quad (3.4)$$

$$\zeta_2 = d_2/D_2 \quad (3.5)$$

$$D_1 = (\lambda/2)/(n_1^2 - \sin^2 \phi)^{1/2} \quad (3.6)$$

$$D_2 = (\lambda/2)/(n_2^2 - \sin^2 \phi)^{1/2}. \quad (3.7)$$

In Eqs. (3.2) - (3.7), ζ_1 and ζ_2 are the normalized film thicknesses for the first and the second thin film, respectively; D_1 and D_2 are the corresponding film-thickness periods, respectively. d_1 and d_2 are the metric thicknesses of the first and the second thin films. λ is the wavelength of light. Light is assumed to be incident from air, so that $n_0 = 1$.

The Fresnel interface reflection coefficients in Eq. (3.1) can be written in the following form:

$$\begin{aligned} r_{01p} &= (n_1^2 S_0 - S_1)/(n_1^2 S_0 + S_1), \\ r_{12p} &= (n_2^2 S_1 - n_1^2 S_2)/(n_2^2 S_1 + n_1^2 S_2), \\ r_{23p} &= (n_3^2 S_2 - n_2^2 S_3)/(n_3^2 S_2 + n_2^2 S_3), \end{aligned} \quad (3.8)$$

$$\begin{aligned} r_{01s} &= (S_0 - S_1)/(S_0 + S_1), \\ r_{12s} &= (S_1 - S_2)/(S_1 + S_2), \\ r_{23s} &= (S_2 - S_3)/(S_2 + S_3). \end{aligned} \quad (3.9)$$

where

$$\begin{aligned} S_0 &= \cos \phi \\ S_1 &= (n_1^2 - \sin^2 \phi)^{1/2} \\ S_2 &= (n_2^2 - \sin^2 \phi)^{1/2} \\ S_3 &= (n_3^2 - \sin^2 \phi)^{1/2}. \end{aligned} \quad (3.10)$$

Let both thin films have quarter-wave optical thickness, i.e. ,

$$d_1 = D_1/2 = \frac{\lambda}{4}(n_1^2 - \sin^2 \phi)^{-1/2} \quad (3.11)$$

$$X_1 = -1, \quad (3.12)$$

$$d_2 = D_2/2 = \frac{\lambda}{4}(n_2^2 - \sin^2 \phi)^{-1/2} \quad (3.13)$$

$$X_2 = -1. \quad (3.14)$$

If we substitute Eqs. (3.12) and (3.14) into Eq. (3.1), the complex-amplitude reflection coefficients become purely real and simplify to :

$$R_p = \frac{n_1^4 n_3^2 q - n_2^4}{n_1^4 n_3^2 q + n_2^4} \quad (3.15)$$

$$R_s = \frac{q - 1}{q + 1}, \quad (3.16)$$

where

$$q = (S_0 S_2^2)/(S_1^2 S_3). \quad (3.17)$$

In order to achieve the equal intensity split condition of Eq. (1.8), the average intensity reflectance and transmittance for incident unpolarized light must be equal to one half. This condition is set forth in the previous chapter in Eq. (2.13). Substituting Eqs. (3.15) and (3.16) into Eq. (2.13), we obtain a quartic equation in q given by:

$$c_4 q^4 + c_3 q^3 + c_2 q^2 + c_1 q + c_0 = 0 \quad (3.18)$$

where

$$\begin{aligned} c_4 &= n_1^8 n_3^4 \\ c_3 &= -2n_1^4 n_3^2 (n_2^4 + n_1^4 n_3^2) \\ c_2 &= n_1^4 n_3^2 (n_1^4 n_3^2 - 12n_2^4) + n_2^8 \\ c_1 &= -2n_2^4 (n_1^4 n_3^2 + n_2^4) \\ c_0 &= n_2^8. \end{aligned} \quad (3.19)$$

If Eq. (3.18) is solved for $q \in \Re, 0 < q < 1$, then, the operating angle of incidence ϕ for which Eq. (2.13) is satisfied can be determined. Squaring Eq. (3.17) and utilizing Eq. (3.10), we obtain the following result:

$$q^2 = \frac{\cos^2 \phi (n_2^2 - \sin^2 \phi)^2}{(n_1^2 - \sin^2 \phi)^2 (n_3^2 - \sin^2 \phi)}. \quad (3.20)$$

Let

$$\sin^2 \phi = v. \quad (3.21)$$

Substituting Eq. (3.21) in Eq. (3.20) and using the trigonometric identity $\cos^2 \phi + \sin^2 \phi = 1$, we obtain a cubic equation in terms of v

$$d_3 v^3 + d_2 v^2 + d_1 v + d_0 = 0 \quad (3.22)$$

where

$$\begin{aligned}
d_3 &= -q^2 + 1 \\
d_2 &= q^2 n_3^3 + 2n_1^2 q^2 - 1 - 2n_2^2 \\
d_1 &= -2n_1^2 n_3^2 q^2 - n_1^4 q^2 + 2n_2^2 + n_2^4 \\
d_0 &= n_1^4 n_3^2 q^2 - n_2^4
\end{aligned} \tag{3.23}$$

Equation (3.22) has a valid solution for the operating angle of incidence ϕ if $v \in \Re, 0 < v < 1$.

The procedure for the design of double-layer-coated beam splitter, which can generate beams of equal intensity and orthogonal polarizations, are very similar to those given in Chapter 2. They are summarized as follows:

1. Choose an appropriate refractive index n_3 of the the substrate material and the second thin film n_2 .
2. For $n_2 \leq n_1 \leq 6$, solve Eq. (3.18) for q , where $q \in \Re, 0 < q < 1$.
3. Substitute q into Eq. (3.22) and solve for v , where $v \in \Re, 0 < v < 1$.
4. Find ϕ using Eq. (3.21).
5. Substitute ϕ in Eqs. (3.15) and (3.15) to calculate R_p and R_s .
6. Plot R_p and R_s as a function of n_1 .

The values of ϕ , which give opposite signs for R_p and R_s guarantee that the reflected light and the refracted light have orthogonal polarizations.

It is interesting to note that there are also two solutions for ϕ as n_1 is varied. As an example, Figure (3.2) shows the two solution branches when the refractive index of the substrate $n_3 = 1.5$ and the refractive index of the second film, $n_2 = 1.38$. The first solution (Solution 1), which is shown in solid line, gives the value of ϕ for which reflected and refracted beams have equal power. However, their polarization states are not orthogonal to each other. The second solution (Solution 2), which is shown by a dashed line, gives the value of ϕ for

which both equal power and orthogonal polarizations are achieved for the reflected and refracted beams.

Figures (3.3) and (3.5) show the amplitude reflectances for the p and s polarizations for Solutions 1 and 2 when $n_2 = 1.38$, $n_3 = 1.5$.

Figures (3.4) and (3.6) show the plots of intensity reflectances for the p and s polarization for Solutions 1 and 2. Notice that as n_1 increases, \mathcal{R}_p diminishes. The average intensity reflectance, \mathcal{R}_u , always takes the value of 0.5 for both Solutions 1 and 2, which is required by design.

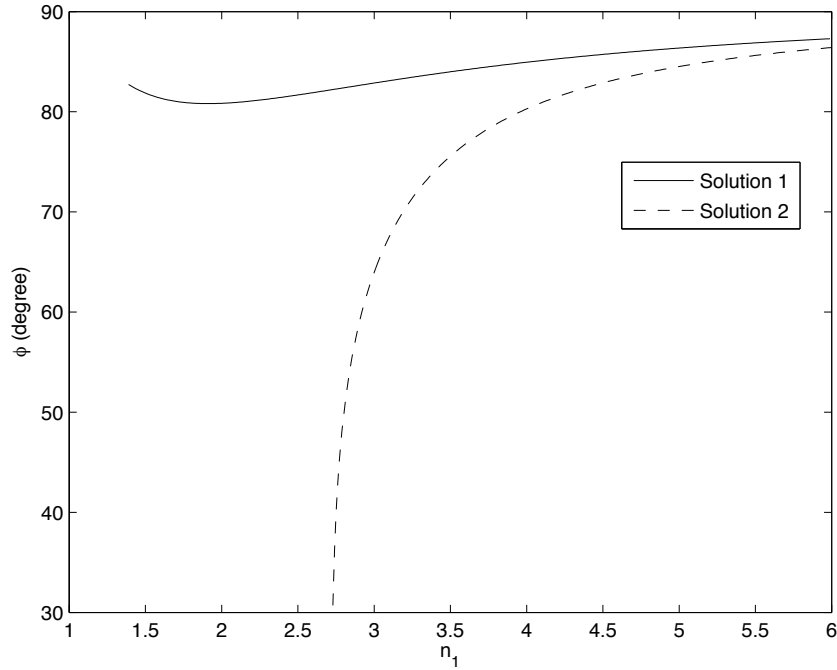


Figure 3.2: ϕ as a function of n_1 when $n_2 = 1.38$ and $n_3 = 1.5$

Figure (3.7) shows the angle of incidence ϕ as a function of n_1 when $n_2 = 1.2$. Note that there are also two solution branches for ϕ . Figures (3.8) and (3.10) show the amplitude reflectances for the p and s polarizations for Solutions 1 and 2 when $n_2 = 1.2$. Figures (3.9) and (3.11) show the intensity reflectances for the p and s polarization for Solutions 1 and 2.

The effect of decreasing the refractive of the second thin film layer is to decrease the operating angle for which equal power and orthogonal polarization can be achieved. Figures

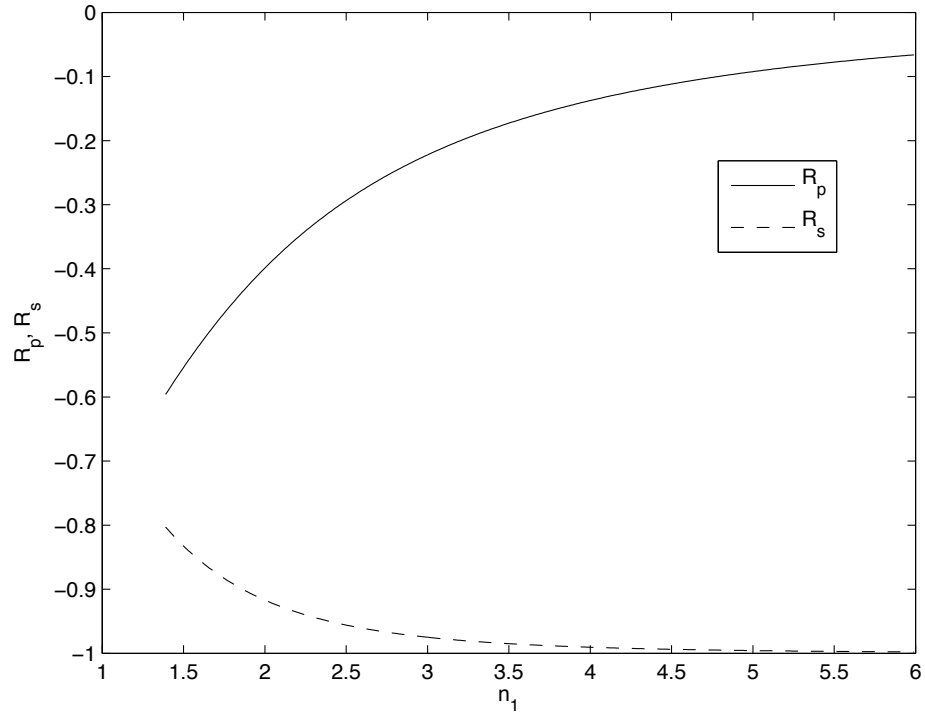


Figure 3.3: R_p, R_s as functions of n_1 when $n_2 = 1.38$ and $n_3 = 1.5$, Solution 1 ($\Delta_r = 0$)

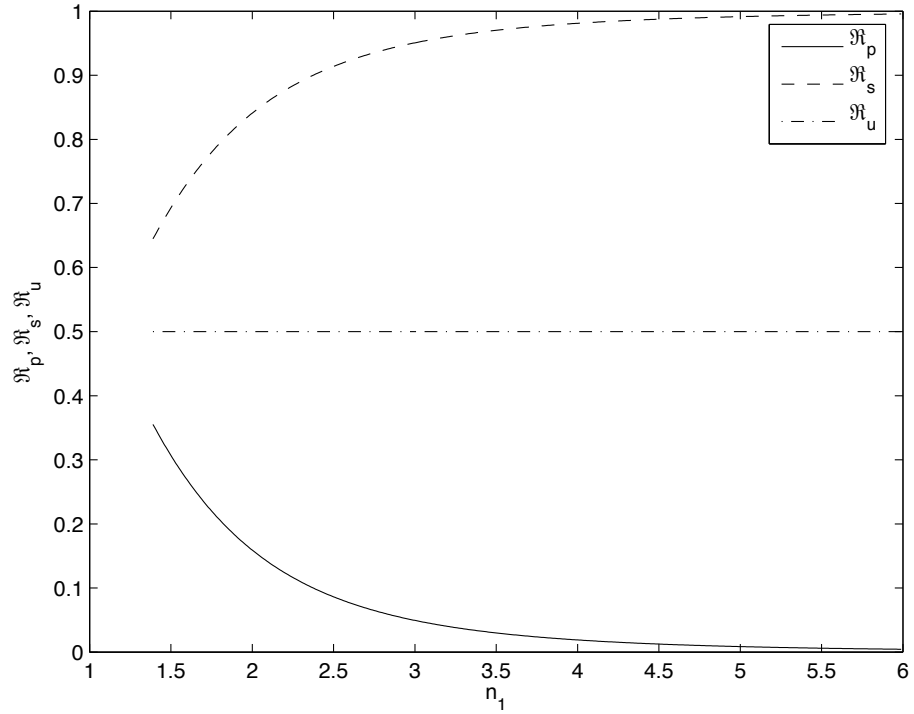


Figure 3.4: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1.38$ and $n_3 = 1.5$, Solution 1 ($\Delta_r = 0$)

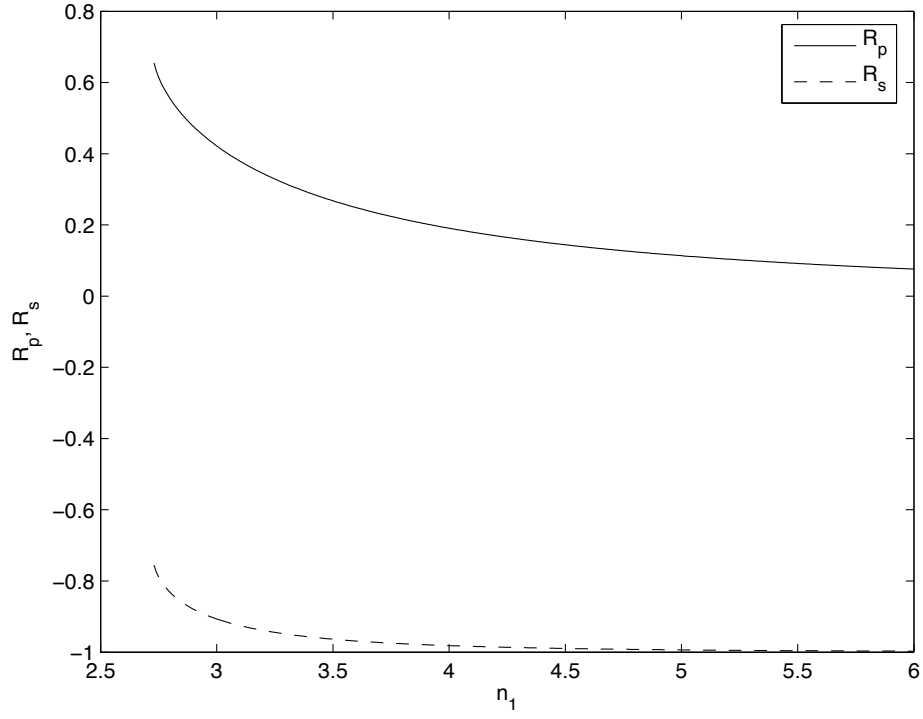


Figure 3.5: R_p, R_s as functions of n_1 when $n_2 = 1.38$ and $n_3 = 1.5$, Solution 2 ($\Delta_r = \pi$)

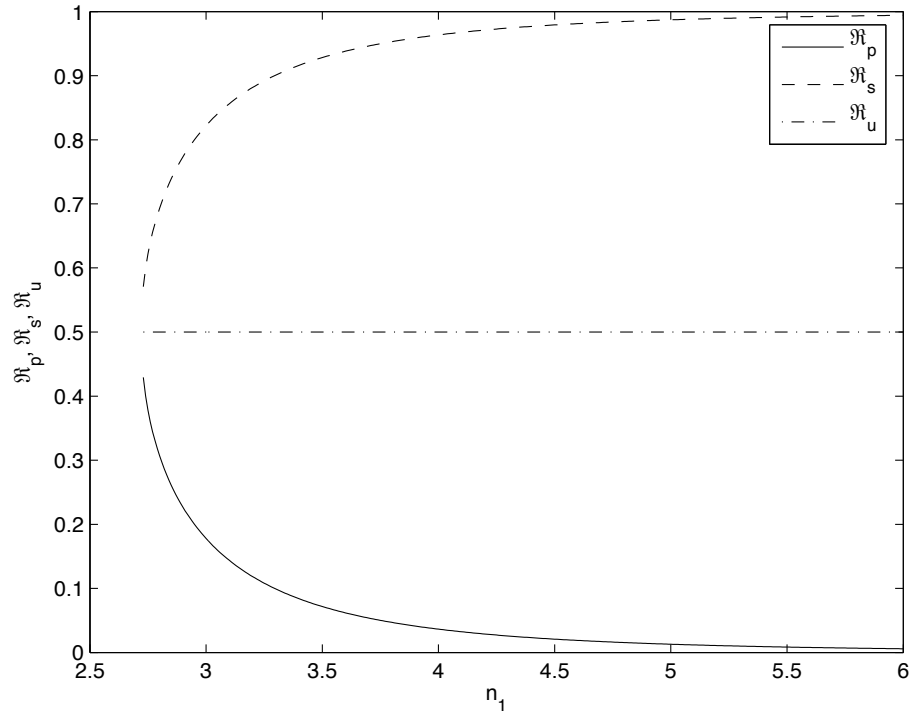


Figure 3.6: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1.38$ and $n_3 = 1.5$, Solution 2 ($\Delta_r = \pi$)

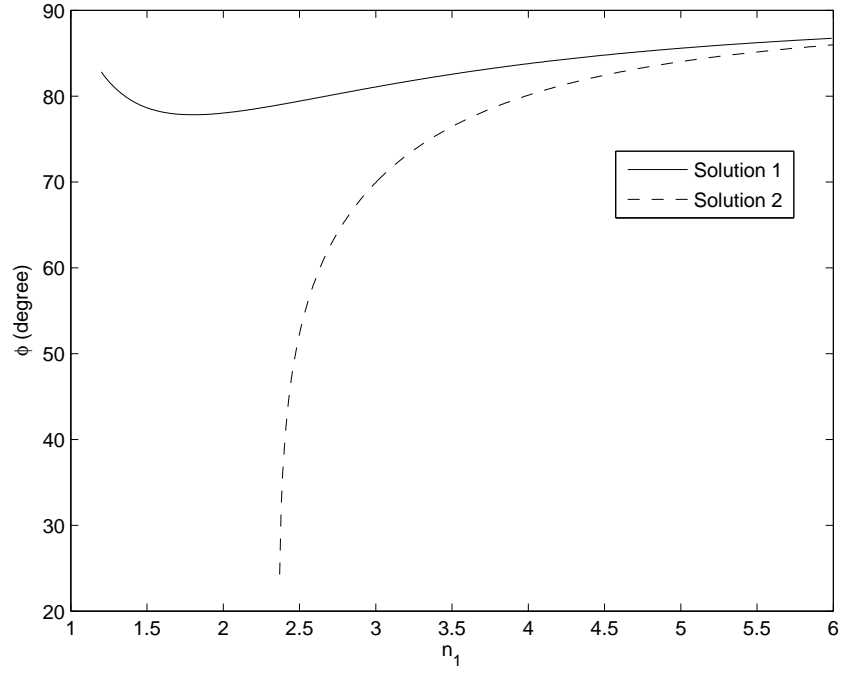


Figure 3.7: ϕ as a function of n_1 when $n_2 = 1.2$ and $n_3 = 1.5$

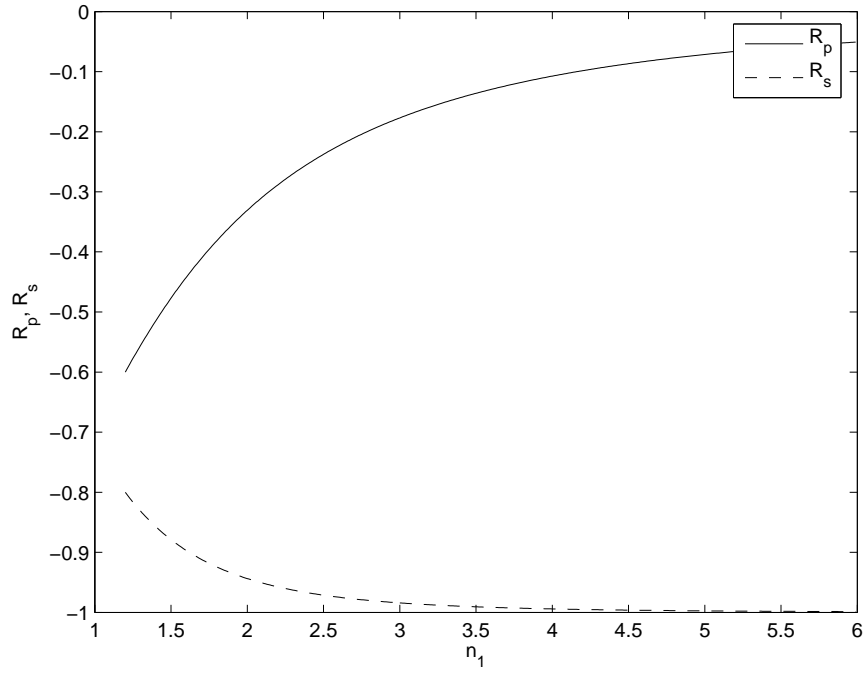


Figure 3.8: R_p, R_s as functions of n_1 when $n_2 = 1.2$ and $n_3 = 1.5$, Solution 1 ($\Delta_r = 0$)

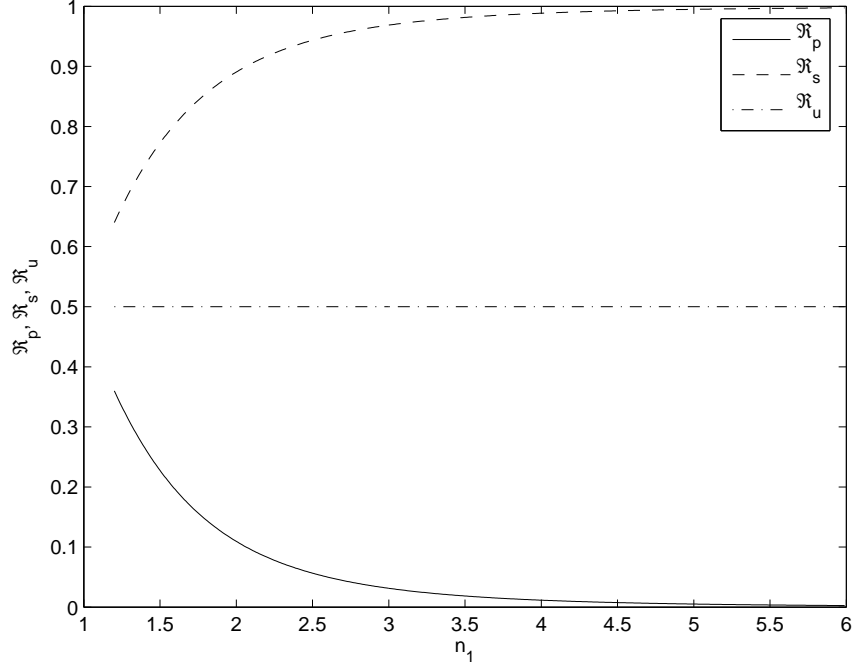


Figure 3.9: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1.2$ and $n_3 = 1.5$, Solution 1 ($\Delta_r = 0$)

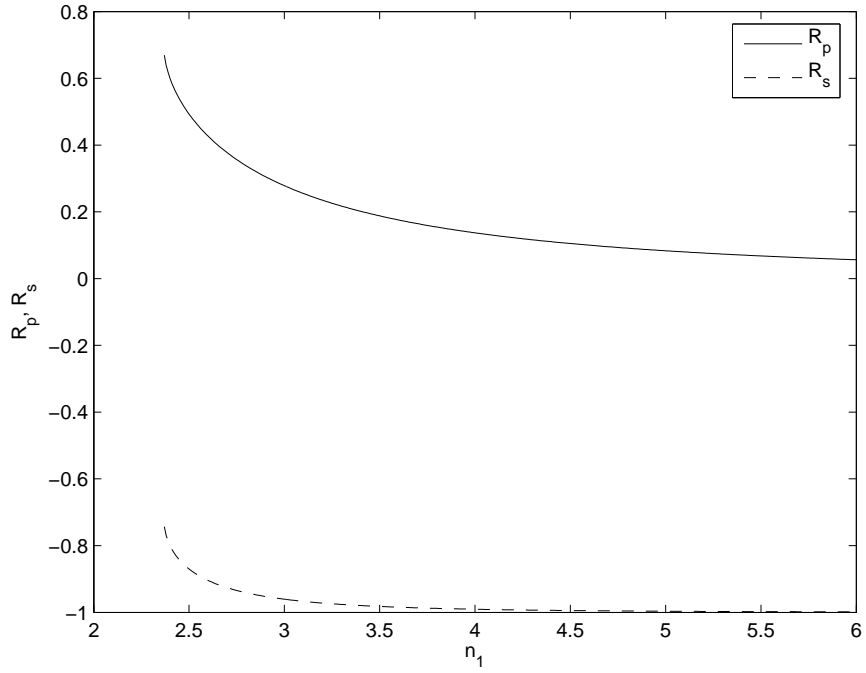


Figure 3.10: R_p, R_s as functions of n_1 when $n_2 = 1.2$ and $n_3 = 1.5$, Solution 2 ($\Delta_r = \pi$)

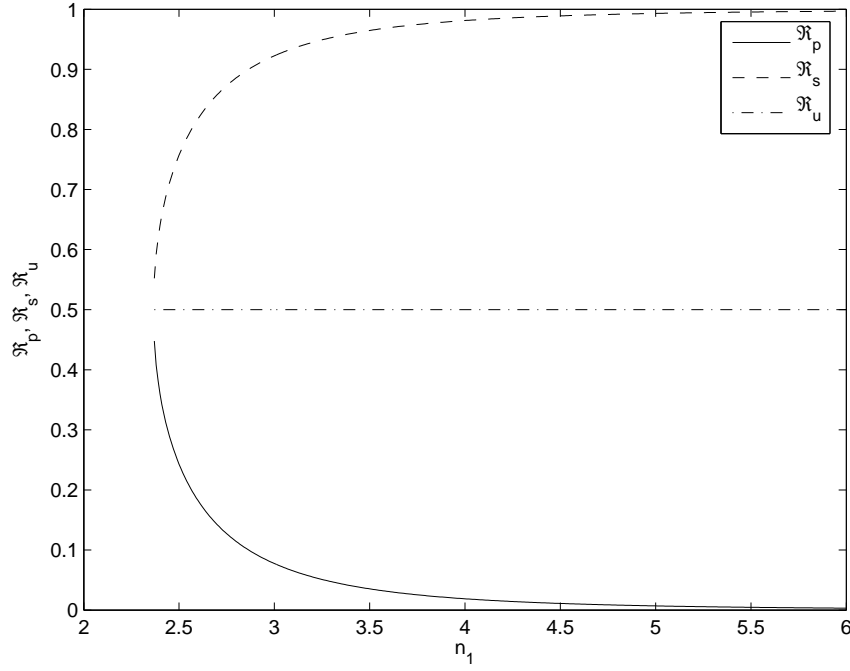


Figure 3.11: $\mathcal{R}_p, \mathcal{R}_s, \mathcal{R}_u$ as functions of n_1 when $n_2 = 1.2$ and $n_3 = 1.5$, Solution 2 ($\Delta_r = \pi$)

(3.2) and (3.7) show that as n_2 is decreased, the curve for Solution 2 shifts to the left. The same effect can also be achieved by increasing the refractive index of the substrate n_3 .

Specific examples of double-layer-coated beam splitters in for visible and infrared wavelengths are described in the following subsections. The performance of these beam splitters over a range of angle of incidence, wavelength, first film thickness, and second film thickness will also be discussed.

3.1 Design at a Visible Wavelength

3.1.1 Fused Silica Substrate Coated by Double Layer of TiO_2 and MgF_2 Thin Films

The operating wavelength for this beam splitter is $\lambda = 0.488 \mu\text{m}$. The refractive indices of TiO_2 , MgF_2 , and fused silica are 2.895, 1.3856, and 1.4630, respectively [10]. The following

data gives the operating design parameters of the beam splitter:

$$\begin{aligned}
\phi &= 55.2219^\circ \\
\alpha &= 34.1541^\circ \\
\zeta_1 &= 0.5000 \\
\zeta_2 &= 0.5000 \\
d_1 &= 43.9474 \text{ nm} \\
d_2 &= 109.3241 \text{ nm} \\
D_1 &= 87.8949 \text{ nm} \\
D_2 &= 218.6484 \text{ nm}
\end{aligned} \tag{3.24}$$

Figures (3.12) (a)-(f) show the performance of the beam splitter when the angle of incidence is in the range of $\pm 5^\circ$ of the design angle and all other parameters are fixed.

Figures (3.13) (a)-(f) show the performance of the beam splitter when a shift of $\pm 5\%$ is introduced in the wavelength λ and all other parameters are fixed.

Figures (3.14) (a)-(f) show the performance of the beam splitter when an error of $\pm 5\%$ is introduced in the thickness d_1 of the first thin film and all other parameters are fixed.

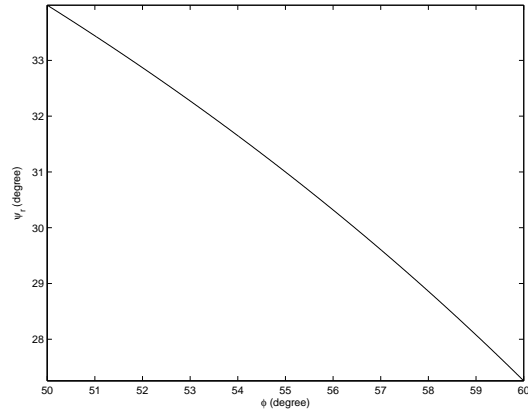
Figures (3.15) (a)-(f) show the performance of beam splitter when an error of $\pm 5\%$ is introduced in the thickness d_2 of the second thin film and all other parameters are fixed.

3.2 Design at an Infrared Wavelength

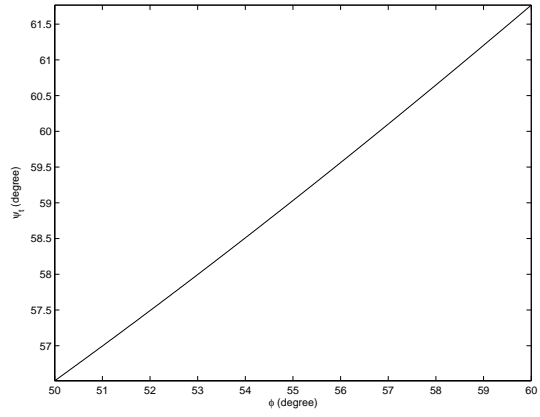
3.2.1 KBr Substrate Coated by Double Layer of CdTe and CdF₂

Thin Films

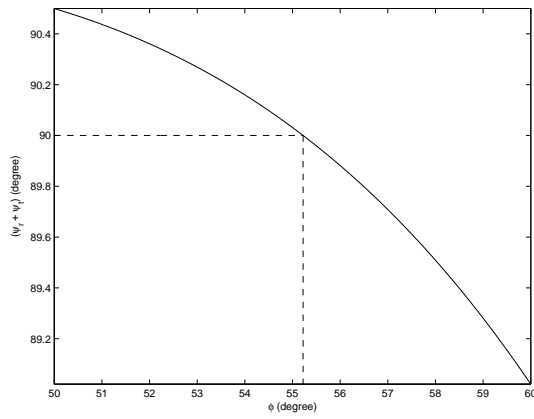
The operating wavelength for this beam splitter is $\lambda = 10.6 \text{ } \mu\text{m}$. The refractive indices of CdTe and KBr are 2.6706 and 1.525, respectively [10] . Using polynomial curve fit [11] of



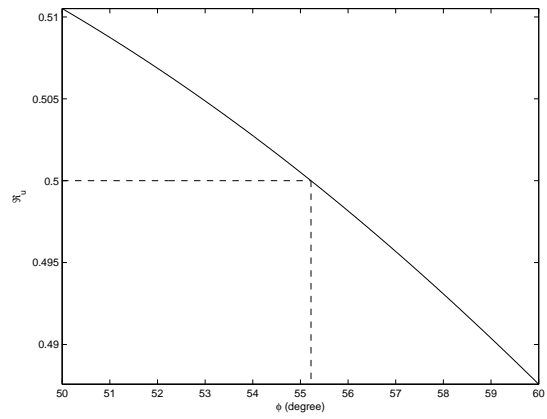
(a)



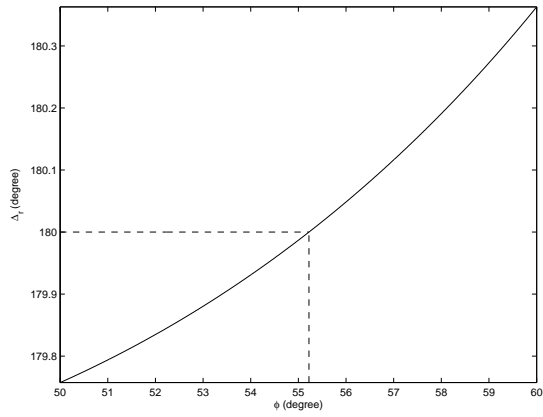
(b)



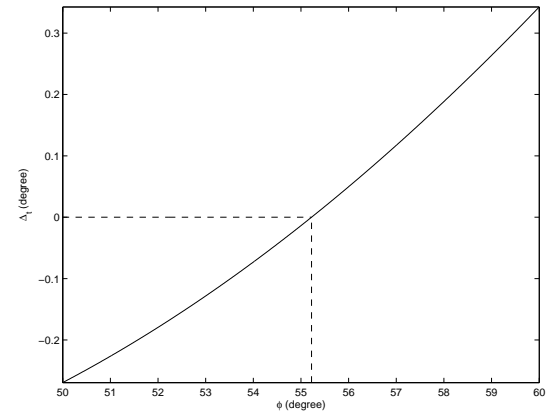
(c)



(d)



(e)



(f)

Figure 3.12: Angular Response of Beam Splitter Using SiO₂ Substrate Coated by TiO₂ and MgF₂ Thin Films

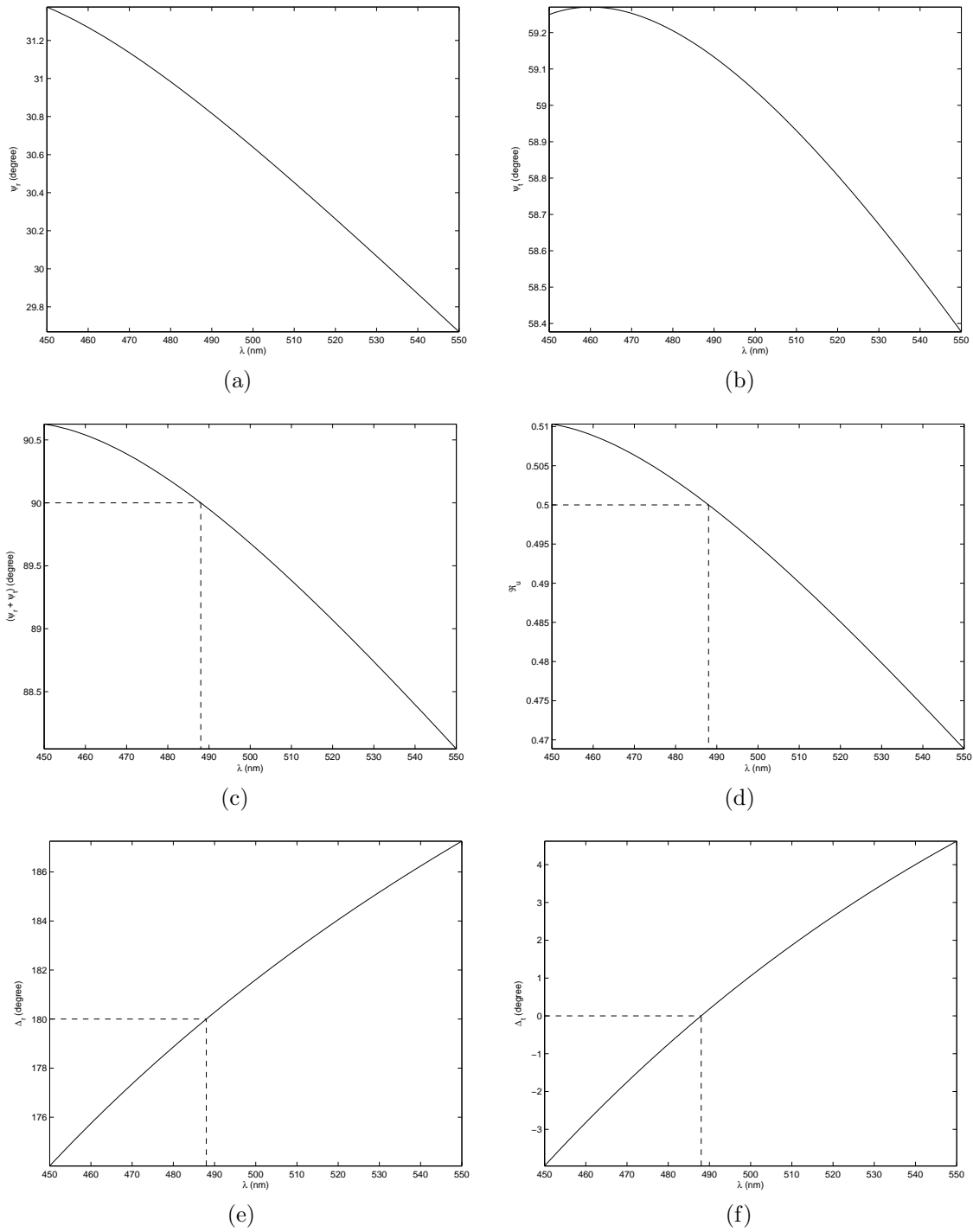
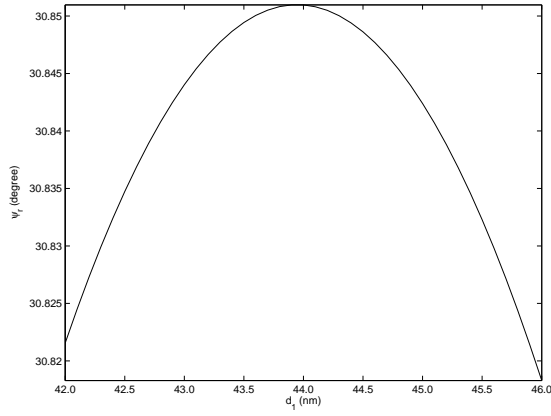
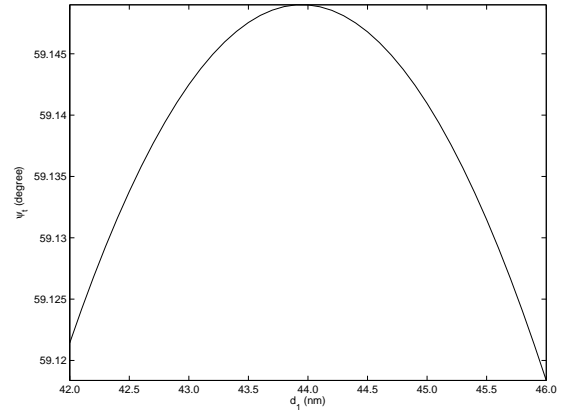


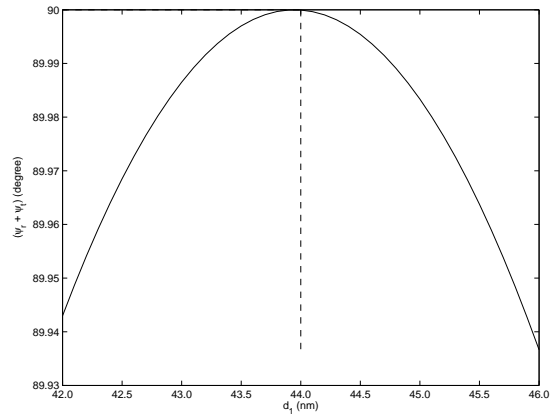
Figure 3.13: Spectral Response of Beam Splitter Using SiO₂ Substrate Coated by TiO₂ and MgF₂ Thin Films



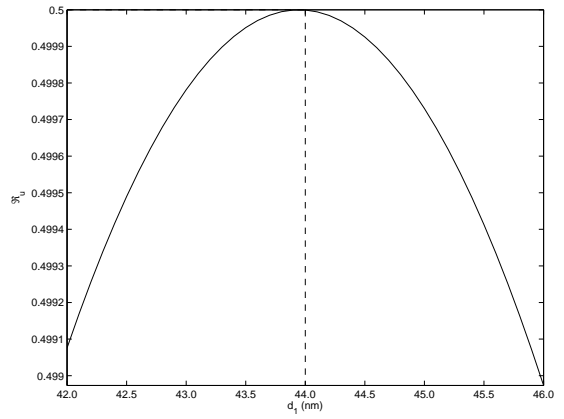
(a)



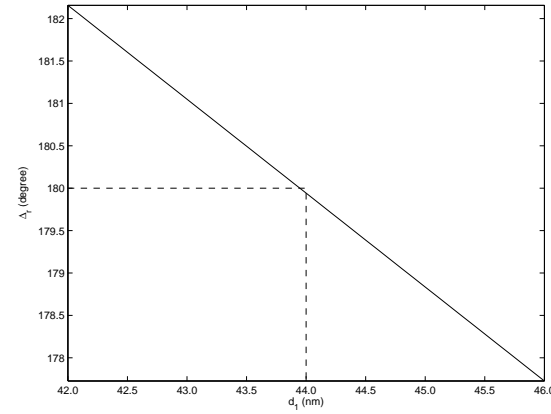
(b)



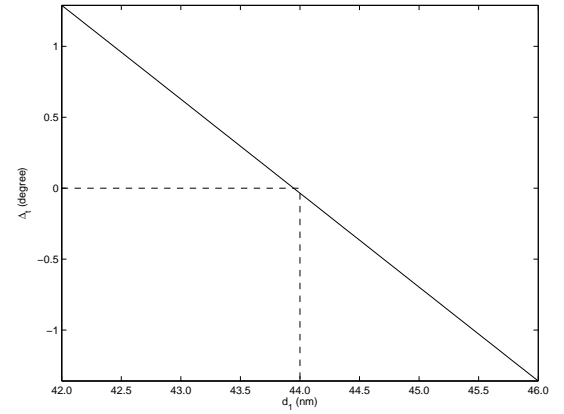
(c)



(d)



(e)



(f)

Figure 3.14: d_1 Response of Beam Splitter Using SiO_2 Substrate Coated by TiO_2 and MgF_2 Thin Films

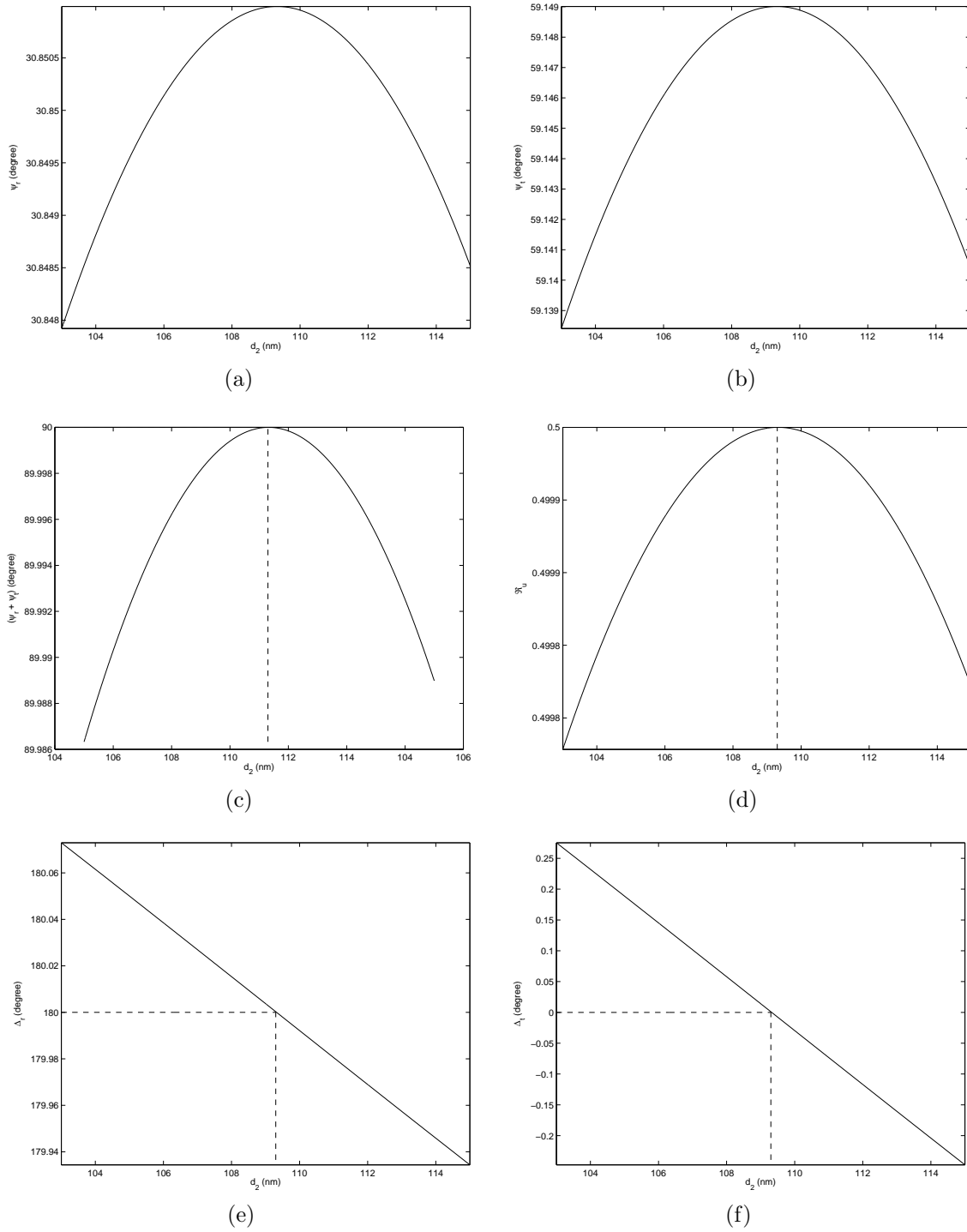


Figure 3.15: d_2 Response of Beam Splitter Using SiO_2 Substrate Coated by TiO_2 and MgF_2 Thin Films

the 3rd order, the refractive index of CdF₂ is found to be 1.2820. The following data gives the operating design parameters of the beam splitter:

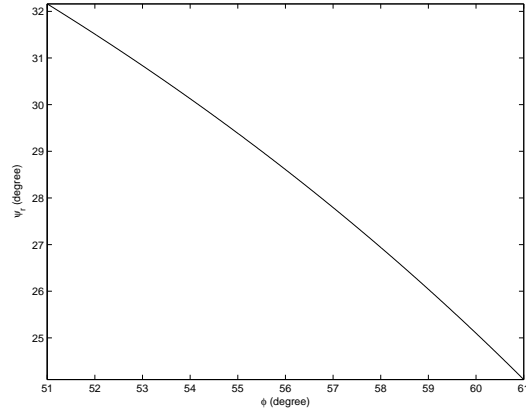
$$\begin{aligned}
\phi &= 56.1390^\circ \\
\alpha &= 32.98851^\circ \\
\zeta_1 &= 0.5000 \\
\zeta_2 &= 0.5000 \\
d_1 &= 1.0440 \text{ } \mu\text{m} \\
d_2 &= 2.7133 \text{ } \mu\text{m} \\
D_1 &= 2.0881 \text{ } \mu\text{m} \\
D_2 &= 5.4266 \text{ } \mu\text{m}
\end{aligned} \tag{3.25}$$

Figures (3.16) (a)-(f) show the performance of the beam splitter when the angle of incidence is in the range of $\pm 5^\circ$ of the design angle and all other parameters are fixed.

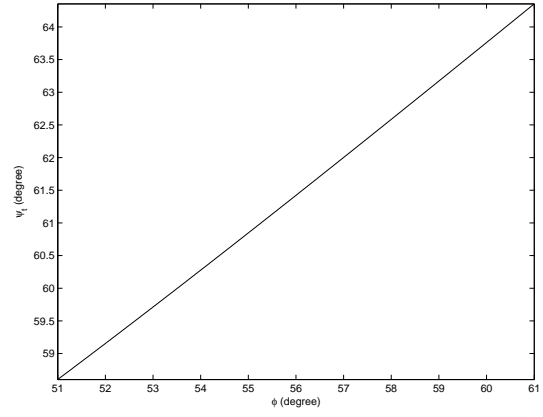
Figures (3.17) (a)-(f) show the performance of the beam splitter when a shift of $\pm 5\%$ is introduced in the wavelength λ and all other parameters are fixed.

Figures (3.18) (a)-(f) show the performance of the beam splitter when an error of $\pm 5\%$ is introduced in the thickness d_1 of the first thin film and all other parameters are fixed.

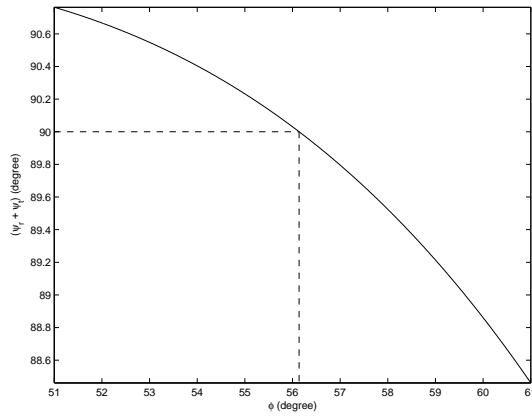
Figures (3.19) (a)-(f) show the performance of the beam splitter when an error of $\pm 5\%$ is introduced in the thickness d_2 of the second thin film and all other parameters are fixed.



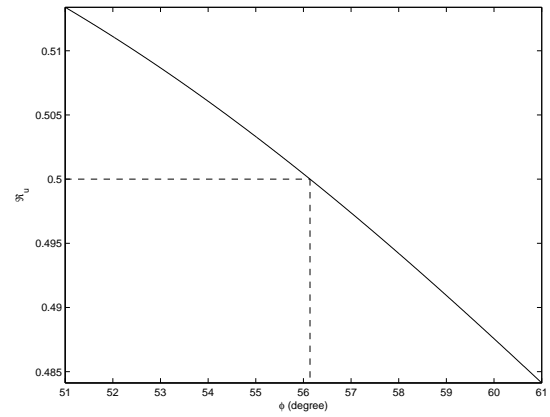
(a)



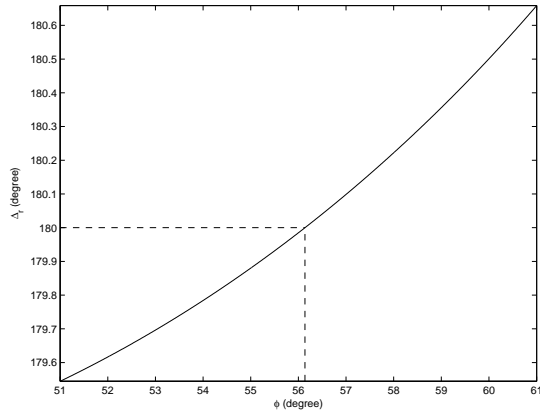
(b)



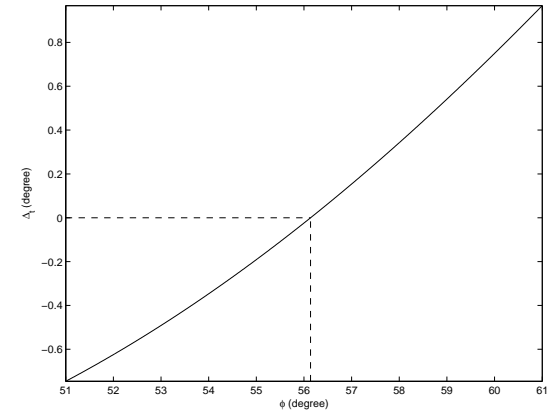
(c)



(d)

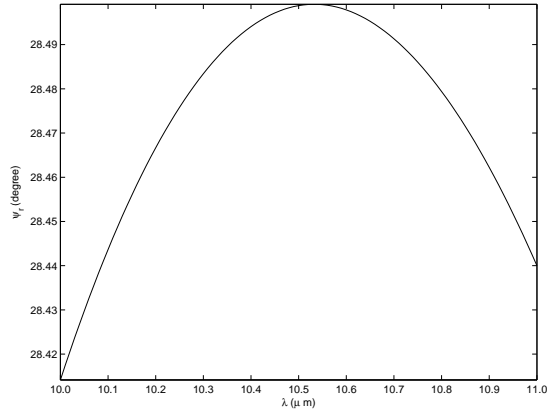


(e)

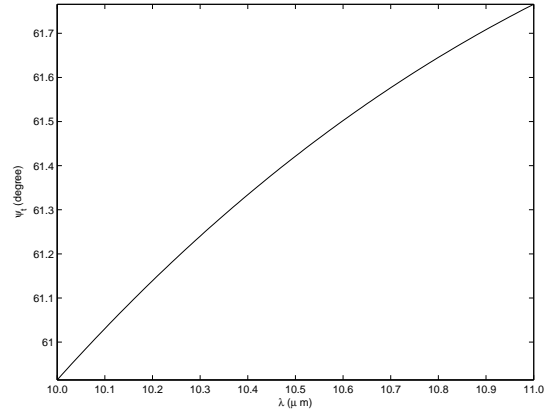


(f)

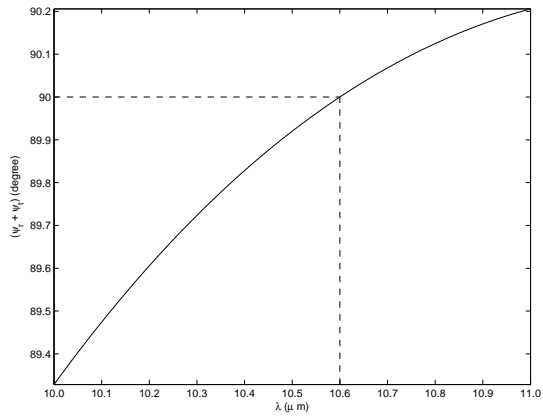
Figure 3.16: Angular Response of Beam Splitter Using KBr Substrate Coated by CdTe and CdF₂ Thin Films



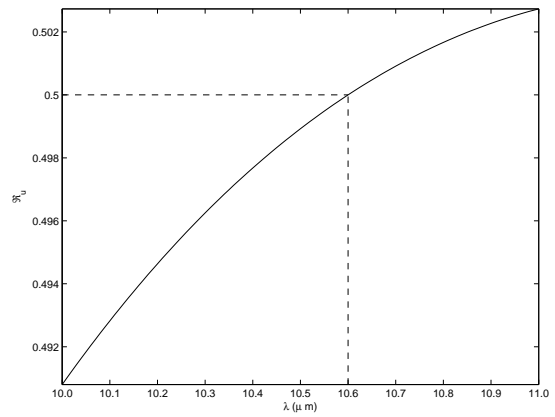
(a)



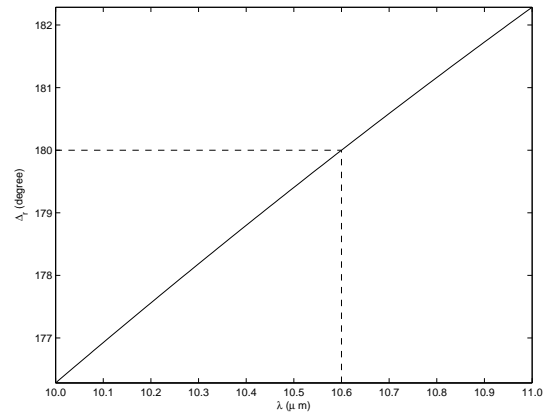
(b)



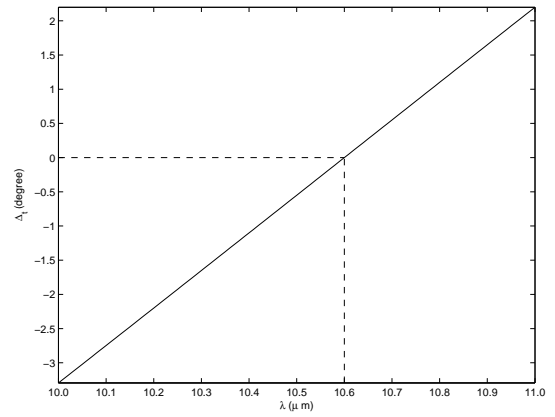
(c)



(d)



(e)



(f)

Figure 3.17: Spectral Response of Beam Splitter Using KBr Substrate Coated by CdTe and CdF₂ Thin Films

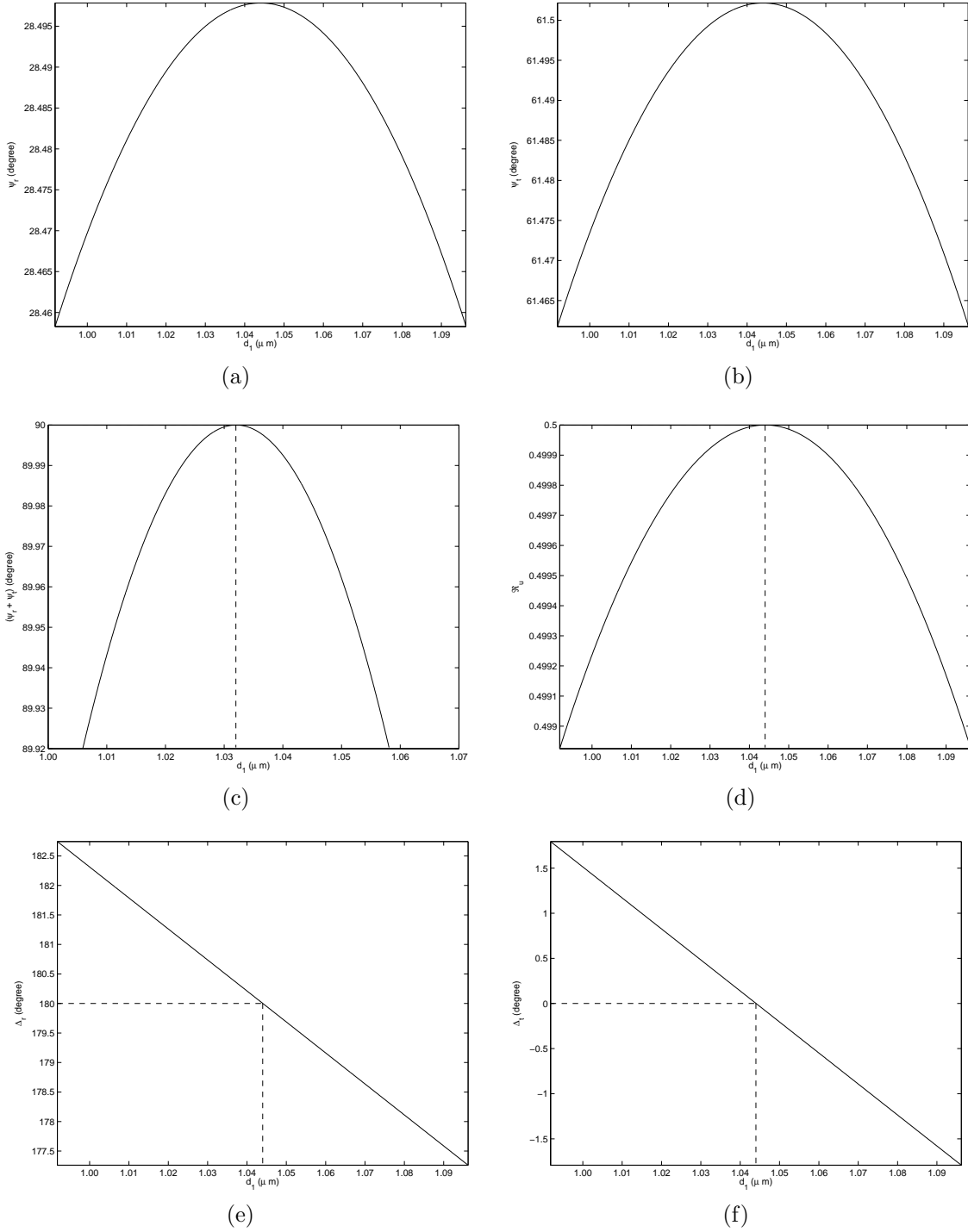


Figure 3.18: d_1 Response of Beam Splitter Using KBr Substrate Coated by CdTe and CdF₂ Thin Films

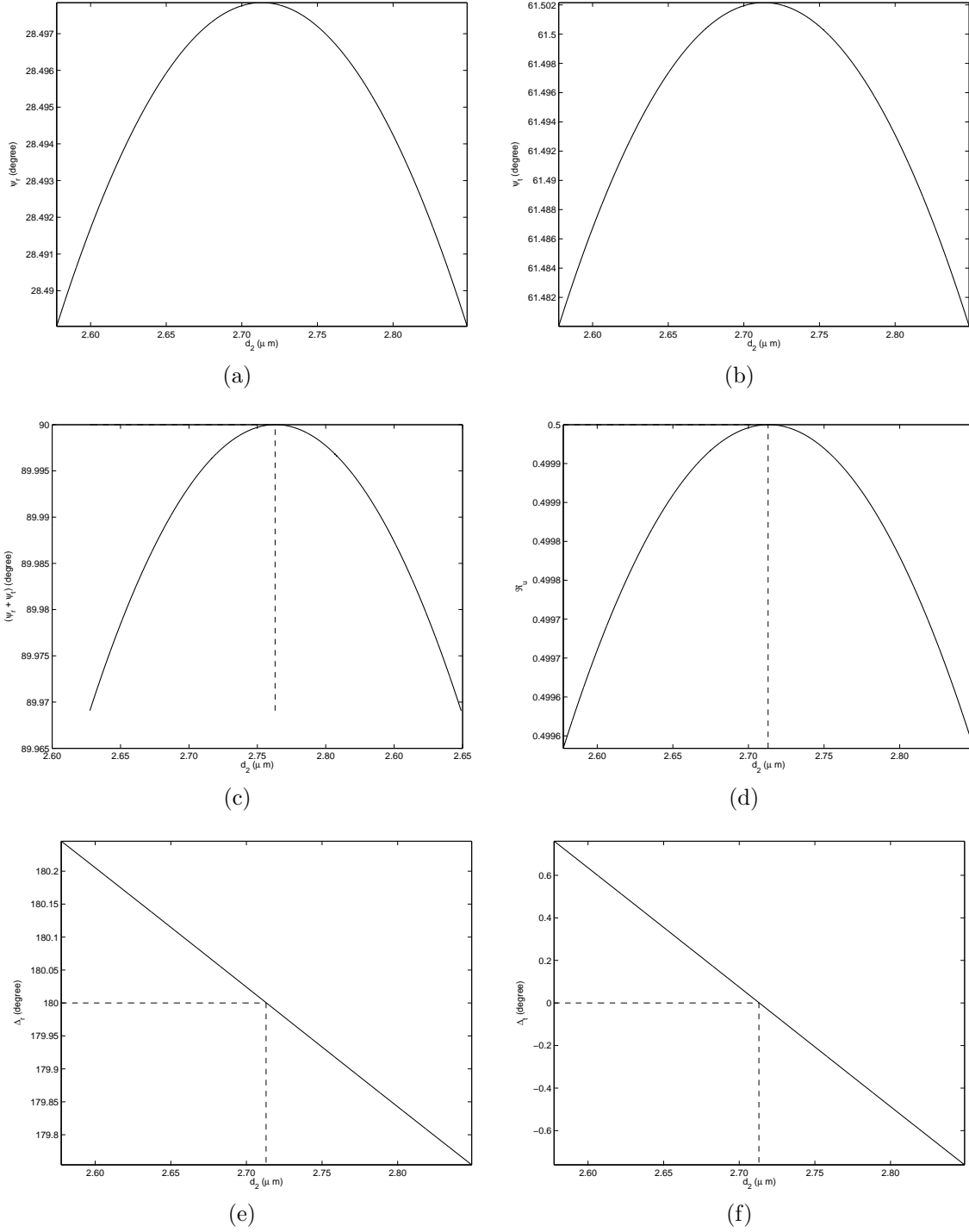


Figure 3.19: d_2 Response of Beam Splitter Using KBr Substrate Coated by CdTe and CdF₂ Thin Films

Chapter 4

Conclusions

Beams splitters that can generate beams of equal intensity and orthogonal polarizations have been presented. These beam splitters consist of prism substrates that are coated by one or two layers that are quarter-wave thick. The simplest design of beam splitter uses a prism substrate that is coated by a single transparent thin film. Examples of single-layer-coated beam splitters in the visible range are fused silica prism that is coated by a GaP thin film and MgF_2 prism that is coated by a TiO_2 thin film. Examples of single-layer-coated beam splitter in the infrared range are Irtran 3 prism that is coated by a GaP and a Si thin film. All of these beam splitters satisfy the equal intensity and the orthogonal polarization conditions. Double-layer coated beam splitters are also presented. Examples of these beam splitters are fused silica substrate that is coated by a bilayer of TiO_2 and MgF_2 in the visible range and KBr substrate that is coated by a bilayer of CdTe and CdF_2 in the infrared range.

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Appendix

This section contains the Matlab codes that are used for calculation and plotting.

```
%QWL for 50% Reflectance for Incident UPL
%Calculate p (by solving a quartic equation given n1 and n2),
%then solve for phi (by solving a quadratic equation given n1, n2, and p).
%SINGLE LAYER BS

clear

clc

%n1 = 2.41421356237310; %sqrt(2) + 1, phi is zero when n2 = 1
%n1 = 2.644638454; %phi is almost zero when n2 = 1.2
%n1 = 2.956795679; %phi is almost zero when n2 = 1.5

%n1 = [1.000001 [1.01:0.01:2.41] 2.41421356237310 [2.42:0.01:6]]; %case 1
%n1 = [[1.01:0.01:2.63] 2.644638454 [2.65:0.01:6.0]]; %case 2
%n1 = [[1.01:0.01:2.95] 2.956795679 [2.96:0.01:6]]; %case 3
n1 = 1.01:0.01:6;

n2 = 1; %Refractive index of substrate
```

```

%*****SOLVE FOR p = S0*S2/S1^2, where
%S0 = cos(phi)
%S1 = (n1^2 - sin(phi)^2)^(1/2)
%S2 = (n2^2 - sin(phi)^2)^(1/2)
%*****

%Calculate the coefficients for the quartic equation
a4 = n1.^8;
a3 = -2.*(n1.^4).*(n1.^4 + n2.^2);
a2 = (n1.^4).*(n1.^4 - 12.*(n2.^2)) + n2.^4;
a1 = -2.*(n2.^2).*(n1.^4 + n2.^2);
a0 = n2.^4;

%Calculate the values of p for different values of n1
%p is going to be a 4*length(n1) x 1 array. Each root of the
%quadratic equation is repeated twice on the same column.
%This is just for convenience reasons.
possRoots = [];
p = [];
pTemp = [];
for i = 1 : length(n1)
    possRoots = roots([a4(i) a3(i) a2(i) a1(i) a0]);
    [r c] = size(possRoots);
    %In case the quartic eq. reduced to a quadratic eq.
    if r ~= 4
        possRoots = [zeros(4-r,1); possRoots];
    end
end

```



```

for j = 1:4
    pTemp = [pTemp; repmat(possRoots(j,:),2,1)];
end

p = [p ; pTemp];
pTemp = [];
end

%Reformat the array n1 to be a 8*length(n1) x 1 array
N1 = [];
for j = 1:length(n1)
    N1 = [N1; repmat(n1(j),8,1)];
end

%*****SOLVE FOR u = sin(phi)^2*****
possRoots = [];
u = [];

%Calculate the coefficients for the quadratic equations
for j = 1:2:8*length(n1)

    b2 = (p(j).^2 - 1);
    b1 = [(n2.^2) + 1) - 2*(N1(j).^2)*p(j).^2];
    b0 = (p(j).^2).*(N1(j).^4) - n2.^2;

    %The solution is stored in an array u, whose size is
    %8*length(n1) x 1

```

```

    u = [u; roots([b2 b1 b0])];
end

%Calculate phi and form a matrix PHI. The elements of PHI are
%the same as phi except that the complex values are eliminated
%because they don't physically make sense.
phi = asin(sqrt(u))*180/pi;
%indexRealPhi = find(abs(imag(phi)) < 1e-7);
%find the index for which phi is real
indexRealPhi = find(imag(phi) == 0);
PHI = zeros(length(phi),1);
PHI(indexRealPhi) = phi(indexRealPhi);

%Calculate Rs and form a matrix RS. The elements of RS are
%the same as Rs except that the corresponding elements whose
%phi's are complex are replaced by zeros.
Rs = (p - 1)./(p + 1);
RS = zeros(length(Rs),1);
RS(indexRealPhi) = Rs(indexRealPhi);

%Calculate Rs and form a matrix RP. The elements of RP are
%the same as Rp except that the corresponding elements whose
%phi's are complex are replaced by zeros.
Rp = ((N1.^4).*p - n2^2)./((N1.^4).*p + n2^2);
RP = zeros(length(Rp),1);
RP(indexRealPhi) = Rp(indexRealPhi);

```

```

scriptRp = RP.^2;
scriptRs = RS.^2;

scriptRu = 0.5*(scriptRp + scriptRs);

%*****CALCULATE Tp and Ts*****

j = sqrt(-1);
S0 = cos(PHI*pi/180);
S1 = sqrt((N1.^2 - (sin(PHI*pi/180).^2)));
S2 = sqrt((n2.^2 - (sin(PHI*pi/180).^2)));

r01s = (S0 - S1)./(S0 + S1);
r12s = (S1 - S2)./(S1 + S2);

r01p = ((N1.^2).*S0 - S1)./((N1.^2).*S0 + S1);
r12p = ((n2.^2).*S1 - (N1.^2).*S2)./((n2.^2).*S1 + (N1.^2).*S2);

t01s = (2*S0)./(S0 + S1);
t12s = (2*S1)./(S1 + S2);

t01p = (2.*N1.*S0)./((N1.^2).*S0 + S1);
t12p = (2.*N1.*n2.*S1)./((n2.^2).*S1 + (N1.^2).*S2);

% Remember that there are prefactor for the transmittances
PHI0=PHI.*pi/180;
PHI1=asin(sin(PHI0)./N1);
PHI2=asin(sin(PHI0)./n2);

```

```

TP = (-j*(t01p.*t12p))./(1 - (r01p.*r12p));
TS = (-j*(t01s.*t12s))./(1 - (r01s.*r12s));

taup = (n2.*cos(PHI2)./cos(PHI0)).*abs(TP).^2;
taus = (n2.*cos(PHI2)./cos(PHI0)).*abs(TS).^2;

tauu = (taup + taus)./2;

%*****DATA ORGANIZATION*****
%plotData = [n1, phi, Rp, Rs scriptRp, scriptRs, scriptRu]
%Only valid data are included, i.e. those whose phi are real and
%0<= phi <= 90, whose p are real and positive, and whose u are real
%and <= u <=1

validIndex = find(PHI > 0 & p > 0); %Find the valid values of phi
validData = [N1(validIndex) PHI(validIndex)...
             RP(validIndex) RS(validIndex)...
             scriptRp(validIndex) scriptRs(validIndex) scriptRu(validIndex)...
             TP(validIndex) TS(validIndex) ...
             taup(validIndex) taus(validIndex) tauu(validIndex)];

%Since there are 2 solution sets for certain n1's consider separating
%the data for each set.
indSolution1 = find(diff(validData(:,1))~=0);
indSolution2 = find(diff(validData(:,1))==0);

```

```

%*****PLOTING STATEMENTS*****

%phi, validData column 2

figure (1)

hold on

box on

plot(validData(indSolution1,1), validData(indSolution1,2), 'k')
plot(validData(indSolution2,1), validData(indSolution2,2), 'k--')
%plot(validData(indSolution1,1), atan(validData(indSolution1,1))*180/pi, 'k-.')
%title(['\phi vs. n_1 for 50% Reflectance with QWL and n_2 = ',num2str(n2)])
%xlabel('n_1'), ylabel('\phi (degree)'), legend('Solution 1',['Solution ' ...
%
%                '2'], '\phi = tan^{-1}n_1')
xlabel('n_1'), ylabel('\phi (degree)'), legend('Solution 1','Solution 2')

hold off


figure(2)

scatter(N1,p,'k.')

% $$$
% $$$ %Rp and Rs, validData column 3 & 4, solution 1
% $$$ figure (2)
% $$$ hold on
% $$$ box on
% $$$ plot(validData(indSolution1,1), validData(indSolution1,3), 'k')
% $$$ plot(validData(indSolution1,1), validData(indSolution1,4), 'k--')
% $$$ title(['R_p, R_s vs. n_1 for 50% Reflectance with QWL and n_2 = ',...
num2str(n2), ', Solution 1'])
% $$$ xlabel('n_1'), ylabel('R_p, R_s'), legend('R_p','R_s')

```

```

% $$$ hold off

% $$$

% $$$ %Rp and Rs, validData column 3 & 4, solution 2

% $$$ figure (3)

% $$$ hold on

% $$$ box on

% $$$ plot(validData(indSolution2,1), validData(indSolution2,3), 'k')

% $$$ plot(validData(indSolution2,1), validData(indSolution2,4), 'k--')

% $$$ title(['R_p, R_s vs. n_1 for 50% Reflectance with QWL and n_2 = ',...
num2str(n2),',', Solution 2'])

% $$$ xlabel('n_1'), ylabel('R_p, R_s'), legend('R_p','R_s')

% $$$ hold off

% $$$

% $$$ %scriptRp, scriptRs, scriptRu validData column 5,6,7 solution 1

% $$$ figure (4)

% $$$ hold on

% $$$ box on

% $$$ plot(validData(indSolution1,1), validData(indSolution1,5), 'k')

% $$$ plot(validData(indSolution1,1), validData(indSolution1,6), 'k--')

% $$$ plot(validData(indSolution1,1), validData(indSolution1,7), 'k-.' )

% $$$ hold off

% $$$ title(['\Re_p, \Re_s, \Re_u vs. n_1 for 50% Reflectance with QWL ...
and n_2 = ',num2str(n2),',', Solution 1'])

% $$$ xlabel('n_1'), ylabel('\Re_p, \Re_s, \Re_u'), legend('\Re_p',...
'\Re_s', '\Re_u')

% $$$

% $$$ %scriptRp, scriptRs, scriptRu validData column 5,6,7 solution 2

```

```

% $$$ figure (5)
% $$$ hold on
% $$$ box on
% $$$ plot(validData(indSolution2,1), validData(indSolution2,5), 'k')
% $$$ plot(validData(indSolution2,1), validData(indSolution2,6), 'k--')
% $$$ plot(validData(indSolution2,1), validData(indSolution2,7), 'k-.')
% $$$ hold off
% $$$ title(['\Re_p, \Re_s, \Re_u vs. n_1 for 50% Reflectance with ...
QWL and n_2 = ',num2str(n2),', Solution 2'])
% $$$ xlabel('n_1'), ylabel('\Re_p, \Re_s, \Re_u'), legend('\Re_p',...
'\Re_s',['\Re_u'])
% $$$
% $$$ %tauu validData column 12 solution 1 and 2
% $$$ figure (6)
% $$$ hold on
% $$$ box on
% $$$ plot(validData(indSolution1,1), validData(indSolution1,12),'k')
% $$$ plot(validData(indSolution2,1), validData(indSolution2,12),'k--')
% $$$ hold off
% $$$ title(['\tau_u vs. n_1 for 50% Reflectance with QWL and n_2 = '...
,num2str(n2)])
% $$$ xlabel('n_1'), ylabel('\tau_u'), legend('Solution 1', 'Solution 2')

%QWL for 50% Reflectance for Incident UPL
%Calculate p (by solving a quartic equation given n1, n2, and n3),
%then solve for phi (by solving a cubic equation given n1, n2, n3, and q).
%DOUBLE LAYER BS

```

```

clear

clc

n1 = 1.38:0.01:6; %Refractive index of the first film
%n2 = 1; %Refractive index of the second film
%n3 = 2; %Refractive index of the substrate

n2 = 1.38;
n3 = 1.5;

%*****SOLVE FOR q = S0*S2^2/(S1^2 * S3), where
%S0 = cos(phi)
%S1 = (n1^2 - sin(phi)^2)^(1/2)
%S2 = (n2^2 - sin(phi)^2)^(1/2)
%S3 = (n3^2 - sin(phi)^2)^(1/2)
%*****

%Calculate the coefficients for the quartic equation
c4 = (n1.^8).*(n3.^4);
c3 = -2*((n1.^4)*(n3.^2)).*(n2.^4 + ((n1.^4)*(n3.^2)));
c2 = ((n1.^4)*(n3.^2)).*(((n1.^4)*(n3.^2)) - 12.*n2.^4) + n2.^8;
c1 = -2.*(n2.^4).*(((n1.^4)*(n3.^2)) + n2.^4);
c0 = n2.^8;

%Calculate the values of q for different values of n1
%p is going to be a 12*length(n1) x 1 array. Each root of the
%quadratic equation is repeated three times on the same column.

```



```

%This is just for convenience reasons.
possRoots = [];
q = [];
qTemp = [];
for i = 1 : length(n1)
    possRoots = roots([c4(i) c3(i) c2(i) c1(i) c0]);
    [r c] = size(possRoots);

    for j = 1:4
        qTemp = [qTemp; repmat(possRoots(j,:),3,1)];
    end

    q = [q ; qTemp];
    qTemp = [];
end

%Reformat the array n1 to be a 12*length(n1) x 1 array
N1 = [];
for j = 1:length(n1)
    N1 = [N1; repmat(n1(j),12,1)];
end

%*****SOLVE FOR v = sin(phi)^2*****
possRoots = [];
v = [];

%Calculate the coefficients for the quadratic equations
for j = 1:3:12*length(n1)

```

```

d3 = -q(j).^2 + 1;
d2 = (q(j).^2).*(n3.^2) + 2.*(N1(j).^2).*(q(j).^2) - 1 - 2.*(n2.^2);
d1 = -2.*(N1(j).^2).*(n3.^2).*(q(j).^2) - (N1(j).^4).*(q(j).^2) + ...
      2.*(n2.^2) + (n2.^4);
d0 = (N1(j).^4).*(n3.^2).*(q(j).^2) - n2.^4;

%The solution is stored in an array v, whose size is
%12*length(n1) x 1
v = [v; roots([d3 d2 d1 d0])];
end

%Calculate phi and form a matrix PHI. The elements of PHI are
%the same as phi except that the complex values are eliminated
%because they don't physically make sense.
phi = asin(sqrt(v))*180/pi;
indexRealPhi = find(imag(phi) == 0);
PHI = zeros(length(phi),1);
PHI(indexRealPhi) = phi(indexRealPhi);

%Calculate Rs and form a matrix RS. The elements of RS are
%the same as Rs except that the corresponding elements whose
%phi's are complex are replaced by zeros.
Rs = (q - 1)./(q + 1);
RS = zeros(length(Rs),1);
RS(indexRealPhi) = Rs(indexRealPhi);

```

```

%Calculate Rs and form a matrix RP. The elements of RP are
%the same as Rp except that the corresponding elements whose
%phi's are complex are replaced by zeros.
Rp = ((N1.^4).*(n3.^2).*q - n2.^4)./((N1.^4).*(n3.^2).*q + n2.^4);
RP = zeros(length(Rp),1);
RP(indexRealPhi) = Rp(indexRealPhi);

scriptRp = RP.^2;
scriptRs = RS.^2;

scriptRu = 0.5*(scriptRp + scriptRs);

%*****CALCULATE Tp and Ts*****
% j = sqrt(-1);
%Calculate corresponding Fresnel coefficients
X1 = -1;
X2 = -1;
PHI0=PHI.*(pi/180);

S0 = cos(PHI0);
S1 = sqrt((N1.^2 - sin(PHI0).^2));
S2 = sqrt((n2^2 - sin(PHI0).^2));
S3 = sqrt((n3^2 - sin(PHI0).^2));

r01s = (S0 - S1)./(S0 + S1);
r12s = (S1 - S2)./(S1 + S2);
r23s = (S2 - S3)./(S2 + S3);

```

```

r01p = ((N1.^2).*S0 - S1)./((N1.^2).*S0 + S1);
r12p = ((n2.^2).*S1 - (N1.^2).*S2)./((n2.^2).*S1 + (N1.^2).*S2);
r23p = ((n3.^2).*S2 - (n2.^2).*S3)./((n3.^2).*S2 + (n2.^2).*S3);

t01s = 2.*S0./(S0 + S1);
t12s = 2.*S1./(S1 + S2);
t23s = 2.*S2./(S2 + S3);

t01p = 2.*N1.*S0./((N1.^2).*S0 + S1);
t12p = 2.*N1.*n2.*S1./((n2.^2).*S1 + (N1.^2).*S2);
t23p = 2.*n2.*n3.*S2./((n3.^2).*S2 + (n2.^2).*S3);

% Remember that there are prefactor for the transmittances
PHI1=asin(sin(PHI0)./N1);
PHI2=asin(sin(PHI0)./n2);
PHI3=asin(sin(PHI0)./n3);

TP = (t01p.*t12p.*t23p.*sqrt(X1.*X2))./...
      (1+(r01p.*r12p.*X1)+(r12p.*r23p.*X2)+(r01p.*r23p.*X1.*X2));
TS = (t01s.*t12s.*t23s.*sqrt(X1.*X2))./...
      (1+(r01s.*r12s.*X1)+(r12s.*r23s.*X2)+(r01s.*r23s.*X1.*X2));

taup = (n3.*cos(PHI3)./cos(PHI0)).*abs(TP).^2;
taus = (n3.*cos(PHI3)./cos(PHI0)).*abs(TS).^2;

tauu = (taup + taus)./2;

```

```

%*****DATA ORGANIZATION*****
%plotData = [n1, phi, Rp, Rs scriptRp, scriptRs, scriptRu]
%Only valid data are included, i.e. those whose phi are real and
%0<= phi <= 90, whose p are real and positive, and whose u are real
%and <= u <=1

validIndex = find(PHI > 0 & q > 0); %Find the valid values of phi

validData = [N1(validIndex) PHI(validIndex)...
             RP(validIndex) RS(validIndex)...
             scriptRp(validIndex) scriptRs(validIndex) scriptRu(validIndex)...
             TP(validIndex) TS(validIndex) ...
             taup(validIndex) taus(validIndex) tauu(validIndex)];

%Since there are 2 solution sets for certain n1's consider separating
%the data for each set.

indSolution1 = find(diff(validData(:,1))~=0);
indSolution2 = find(diff(validData(:,1))==0);

% $$$ figure (2)
% $$$ scatter(N1(validIndex), PHI(validIndex),'k.')
%*****PLOTING STATEMENTS*****

%phi, validData column 2
figure (1)

```

```

hold on

box on

plot(validData(indSolution1,1), validData(indSolution1,2), 'k')
plot(validData(indSolution2,1), validData(indSolution2,2), 'k--')

%title(['\phi vs. n_1 for 50% Reflectance with QWL bilayer BS, n_2 = ' ...
%      ,num2str(n2), ', n_3 = ', num2str(n3)])

xlabel('n_1'), ylabel('\phi (degree)'), legend('Solution 1','Solution 2')

hold off


% $$$ %Rp and Rs, validData column 3 & 4, solution 1
% $$$ figure (2)
% $$$ hold on
% $$$ box on
% $$$ plot(validData(indSolution1,1), validData(indSolution1,3), 'k')
% $$$ plot(validData(indSolution1,1), validData(indSolution1,4), 'k--')
% $$$ %title(['R_p, R_s vs. n_1 for 50% Reflectance with QWL ...
and n_2 = ',num2str(n2), ...
% $$$ %      ', n_3 = ', num2str(n3), ', Solution 1'])
% $$$ xlabel('n_1'), ylabel('R_p, R_s'), legend('R_p','R_s')
% $$$ hold off
% $$$

% $$$ %Rp and Rs, validData column 3 & 4, solution 2
% $$$ figure (3)
% $$$ hold on
% $$$ box on
% $$$ plot(validData(indSolution2,1), validData(indSolution2,3), 'k')
% $$$ plot(validData(indSolution2,1), validData(indSolution2,4), 'k--')

```

```

% $$$ %title(['R_p, R_s vs. n_1 for 50% Reflectance with QWL and ...
            n_2 = ',num2str(n2), ...
% $$$ %            ', n_3 = ', num2str(n3), ', Solution 2'])
% $$$ xlabel('n_1'), ylabel('R_p, R_s'), legend('R_p','R_s')
% $$$ hold off
% $$$
% $$$ %scriptRp, scriptRs, scriptRu validData column 5,6,7 solution 1
% $$$ figure (4)
% $$$ hold on
% $$$ box on
% $$$ plot(validData(indSolution1,1), validData(indSolution1,5), 'k')
% $$$ plot(validData(indSolution1,1), validData(indSolution1,6), 'k--')
% $$$ plot(validData(indSolution1,1), validData(indSolution1,7), 'k-.')
% $$$ %title(['\Re_p, \Re_s, \Re_u vs. n_1 for 50% Reflectance with ...
            %QWL and n_2 ' ...
% $$$ %            '= ',num2str(n2), ', n_3 = ', num2str(n3), ', Solution 1'])
% $$$ xlabel('n_1'), ylabel('\Re_p, \Re_s, \Re_u'), ...
%legend('\Re_p','\Re_s','\Re_u')
% $$$
% $$$ %scriptRp, scriptRs, scriptRu validData column 5,6,7 solution 2
% $$$ figure (5)
% $$$ hold on
% $$$ box on
% $$$ plot(validData(indSolution2,1), validData(indSolution2,5), 'k')
% $$$ plot(validData(indSolution2,1), validData(indSolution2,6), 'k--')
% $$$ plot(validData(indSolution2,1), validData(indSolution2,7), 'k-.')
% $$$ hold off

```

```

% $$$ %title(['\Re_p, \Re_s, \Re_u vs. n_1 for 50% Reflectance ...
%with QWL and n_2 ' ...
% $$$ %      '= ', num2str(n2), ', n_3 = ', num2str(n3), ', Solution 2'])
% $$$ xlabel('n_1'), ylabel('\Re_p, \Re_s, \Re_u'), ...
%legend('\Re_p', '\Re_s', '\Re_u')

%Specific Visible Design of Single Layer Polarizing Beam Splitter
%GaP film on SiO2 (fused silica) substrate

clear

format long %make sure we get enough numbers behind the decimal point.
%Find the exact solution for the specific design at lambda = 0.633 micro
%meter.

lambda = 0.633; %wavelength of light in micrometer.

%Calculate the refractive index of GaP
A = 1.390.*(lambda.^2)./(lambda.^2 - 0.172.^2);
B = 4.131.*(lambda.^2)./(lambda.^2 - 0.234.^2);
C = 2.570.*(lambda.^2)./(lambda.^2 - 0.345.^2);
D = 2.056.*(lambda.^2)./(lambda.^2 - 27.52.^2);
nGaP = sqrt(1 + A + B + C + D);

%Calculate the refractive index of SiO2 (fused silica)
D = 0.6961663.*(lambda.^2)./(lambda.^2 - 0.0684043.^2);
E = 0.4079426.*(lambda.^2)./(lambda.^2 - 0.1162414.^2);
F = 0.8974794.*(lambda.^2)./(lambda.^2 - 9.896161.^2);
nSiO2 = sqrt(1 + D + E + F);

```



```

n1 = nGaP;
n2 = nSiO2;

%Calculate the coefficients for the quartic equation
a4 = n1.^8;
a3 = -2.*(n1.^4).*(n1.^4 + n2.^2);
a2 = (n1.^4).*(n1.^4 - 12.*(n2.^2)) + n2.^4;
a1 = -2.*(n2.^2).*(n1.^4 + n2.^2);
a0 = n2.^4;

%Calculate the values of p for different values of n1
%p is going to be a 4*length(n1) x 1 array. Each root of the
%quadratic equation is repeated twice on the same column.
%This is just for convenience reasons.
possRoots = [];
p = [];
pTemp = [];
for i = 1 : length(n1)
    possRoots = roots([a4(i) a3(i) a2(i) a1(i) a0]);
    [r c] = size(possRoots);
    %In case the quartic eq. reduced to a quadratic eq.
    if r ~= 4
        possRoots = [zeros(4-r,1); possRoots];
    end

    for j = 1:4
        pTemp = [pTemp; repmat(possRoots(j,:),2,1)];
    end
end

```

```

end

p = [p ; pTemp];
pTemp = [];
end

%Reformat the array n1 and n2 to be a 8*length(n1) x 1 array
N1 = [];
N2 = [];
for j = 1:length(n1)
    N1 = [N1; repmat(n1(j),8,1)];
    N2 = [N2; repmat(n2(j),8,1)];
end

%*****SOLVE FOR u = sin(phi)^2*****
possRoots = [];
u = [];
%Calculate the coefficients for the quadratic equations
for j = 1:2:8*length(n1)

    b2 = (p(j).^2 - 1);
    b1 = [((N2(j).^2) + 1) - 2*(N1(j).^2)*p(j).^2];
    b0 = (p(j).^2).*(N1(j).^4) - N2(j).^2;

    %The solution is stored in an array u, whose size is
    %8*length(n1) x 1
    u = [u; roots([b2 b1 b0])];

```

end

%Calculate phi and form a matrix PHI. The elements of PHI are
%the same as phi except that the complex values are eliminated
%because they don't physically make sense.

```
phi = asin(sqrt(u))*180/pi;  
indexRealPhi = find(imag(phi) == 0);  
PHI = zeros(length(phi),1);  
PHI(indexRealPhi) = phi(indexRealPhi);
```

%Calculate Rs and form a matrix RS. The elements of RS are
%the same as Rs except that the corresponding elements whose
%phi's are complex are replaced by zeros.

```
Rs = (p - 1)./(p + 1);  
RS = zeros(length(Rs),1);  
RS(indexRealPhi) = Rs(indexRealPhi);
```

%Calculate Rp and form a matrix RP. The elements of RP are
%the same as Rp except that the corresponding elements whose
%phi's are complex are replaced by zeros.

```
Rp = ((N1.^4).*p - N2.^2)./((N1.^4).*p + N2.^2);  
RP = zeros(length(Rp),1);  
RP(indexRealPhi) = Rp(indexRealPhi);
```

```
scriptRp = RP.^2;
```

```
scriptRs = RS.^2;
```

```

scriptRu = 0.5*(scriptRp + scriptRs);

%*****CALCULATE Tp and Ts*****

j = sqrt(-1);
S0 = cos(PHI*pi/180);
S1 = sqrt((N1.^2 - (sin(PHI*pi/180).^2)));
S2 = sqrt((N2.^2 - (sin(PHI*pi/180).^2)));

r01s = (S0 - S1)/(S0 + S1);
r12s = (S1 - S2)/(S1 + S2);

r01p = ((N1.^2).*S0 - S1)/((N1.^2).*S0 + S1);
r12p = ((N2.^2).*S1 - (N1.^2).*S2)/((N2.^2).*S1 + (N1.^2).*S2);

t01s = (2*S0)/(S0 + S1);
t12s = (2*S1)/(S1 + S2);

t01p = (2.*N1.*S0)/((N1.^2).*S0 + S1);
t12p = (2.*N1.*N2.*S1)/((N2.^2).*S1 + (N1.^2).*S2);

TP = (-j*(t01p.*t12p))/(1 - (r01p.*r12p));
TS = (-j*(t01s.*t12s))/(1 - (r01s.*r12s));

%Find the index for which Rp and Rs have opposite sign. This index
%gives where the operating angle is.
op = find(sign(RP).*sign(RS) == -1);

```

```

%The order of the film thickness, may be > 0 for shorter wavelength.
m = 0;

%Operating conditions, X1 = -1, d1/D1 = 1/2
PHI(op)
%Notice that this thickness satisfies the quarter wave optical thickness
%condition.
zeta1 = 1/2; %always, for quarter wave opt. thick
D1 = (lambda./2)./sqrt(n1.^2 - sin(PHI(op)*pi/180).^2)
d1 = (zeta1 + m).*D1
X1 = exp(-j.*2*pi.*(d1./D1))

%Save variables for use in system response calculation
phi = PHI(op);
save GaPonSiO2Data phi d1 n1 n2

phi1 = asin(sin(phi*pi/180)/n1)*180/pi
phi2 = asin(sin(phi1*pi/180)*(n1/n2))*180/pi
alpha = phi2
format short

%Specific design for IR wavelength (Double Layer BS)

clear
clc

lambda = 10.6;

```

```

%CdTe
nCdTe = sqrt(1 + (6.1977889.*lambda.^2)./(lambda.^2-0.317069^2)...
            + (3.2243821.*lambda^2)./(lambda^2-72.0663^2));

%Irtran 3
p1 = -4.624e-5;
p2 = -0.0006347;
p3 = -0.002136;
p4 = 1.431;
nIrtran3 = p1*lambda^3 + p2*lambda^2 + p3*lambda + p4;

%KBr
nKBr = sqrt(1.39408 + (0.79221.*lambda^2)./(lambda^2 - 0.146.^2) + ...
            (0.01981.*lambda^2)./(lambda^2 -0.173^2) + ...
            (0.15587.*lambda^2)./(lambda^2 - ...
            0.187^2) + (0.17673.*lambda^2)./(lambda^2 -60.61^2)...
            + (2.06217.*lambda^2)./(lambda^2 - 87.72^2));

n1 = nCdTe;
n2 = nIrtran3;
n3 = nKBr;

%*****SOLVE FOR q = S0*S2^2/(S1^2 * S3), where
%S0 = cos(phi)
%S1 = (n1^2 - sin(phi)^2)^(1/2)
%S2 = (n2^2 - sin(phi)^2)^(1/2)

```

```

%S3 = (n3^2 - sin(phi)^2)^(1/2)

%*****

%Calculate the coefficients for the quartic equation
c4 = (n1.^8).*(n3.^4);
c3 = -2*((n1.^4)*(n3.^2)).*(n2.^4 + ((n1.^4)*(n3.^2)));
c2 = ((n1.^4)*(n3.^2)).*(((n1.^4)*(n3.^2)) - 12.*n2.^4) + n2.^8;
c1 = -2.*(n2.^4).*(((n1.^4)*(n3.^2)) + n2.^4);
c0 = n2.^8;

%Calculate the values of q for different values of n1
%p is going to be a 12*length(n1) x 1 array. Each root of the
%quadratic equation is repeated three times on the same column.
%This is just for convenience reasons.
possRoots = [];
q = [];
qTemp = [];
for i = 1 : length(n1)
    possRoots = roots([c4(i) c3(i) c2(i) c1(i) c0]);
    [r c] = size(possRoots);

    for j = 1:4
        qTemp = [qTemp; repmat(possRoots(j,:),3,1)];
    end

    q = [q ; qTemp];
    qTemp = [];

```

```

end

%Reformat the array n1 to be a 12*length(n1) x 1 array
N1 = [];
for j = 1:length(n1)
    N1 = [N1; repmat(n1(j),12,1)];
end

%*****SOLVE FOR v = sin(phi)^2*****
possRoots = [];
v = [];
%Calculate the coefficients for the quadratic equations
for j = 1:3:12*length(n1)

    d3 = -q(j).^2 + 1;
    d2 = (q(j).^2).*(n3.^2) + 2.*(N1(j).^2).*(q(j).^2) - 1 - 2.*(n2.^2);
    d1 = -2.*(N1(j).^2).*(n3.^2).*(q(j).^2) - (N1(j).^4).*(q(j).^2) + ...
        2.*(n2.^2) + (n2.^4);
    d0 = (N1(j).^4).*(n3.^2).*(q(j).^2) - n2.^4;

    %The solution is stored in an array v, whose size is
    %12*length(n1) x 1
    v = [v; roots([d3 d2 d1 d0])];
end

%Calculate phi and form a matrix PHI. The elements of PHI are
%the same as phi except that the complex values are eliminated

```



```

%because they don't physically make sense.
phi = asin(sqrt(v))*180/pi;
indexRealPhi = find(imag(phi) == 0);
PHI = zeros(length(phi),1);
PHI(indexRealPhi) = phi(indexRealPhi);

%Calculate Rs and form a matrix RS. The elements of RS are
%the same as Rs except that the corresponding elements whose
%phi's are complex are replaced by zeros.
Rs = (q - 1)./(q + 1);
RS = zeros(length(Rs),1);
RS(indexRealPhi) = Rs(indexRealPhi);

%Calculate Rp and form a matrix RP. The elements of RP are
%the same as Rp except that the corresponding elements whose
%phi's are complex are replaced by zeros.
Rp = ((n1.^4).*(n3.^2).*q - n2.^4)./((n1.^4).*(n3.^2).*q + n2.^4);
RP = zeros(length(Rp),1);
RP(indexRealPhi) = Rp(indexRealPhi);

scriptRp = RP.^2;
scriptRs = RS.^2;

scriptRu = 0.5*(scriptRp + scriptRs);

%*****CALCULATE Tp and Ts*****
j = sqrt(-1);

```

```

%Calculate corresponding Fresnel coefficients

X1 = -1;
X2 = -1;
PHI0=PHI.*(pi/180);

S0 = cos(PHI0);
S1 = sqrt((N1.^2 - sin(PHI0).^2));
S2 = sqrt((n2^2 - sin(PHI0).^2));
S3 = sqrt((n3^2 - sin(PHI0).^2));

r01s = (S0 - S1)./(S0 + S1);
r12s = (S1 - S2)./(S1 + S2);
r23s = (S2 - S3)./(S2 + S3);

r01p = ((N1.^2).*S0 - S1)./((N1.^2).*S0 + S1);
r12p = ((n2.^2).*S1 - (N1.^2).*S2)./((n2.^2).*S1 + (N1.^2).*S2);
r23p = ((n3.^2).*S2 - (n2.^2).*S3)./((n3.^2).*S2 + (n2.^2).*S3);

t01s = 2.*S0./(S0 + S1);
t12s = 2.*S1./(S1 + S2);
t23s = 2.*S2./(S2 + S3);

t01p = 2.*N1.*S0./((N1.^2).*S0 + S1);
t12p = 2.*N1.*n2.*S1./((n2.^2).*S1 + (N1.^2).*S2);
t23p = 2.*n2.*n3.*S2./((n3.^2).*S2 + (n2.^2).*S3);

% Remember that there are prefactor for the transmittances

```

```

PHI1=asin(sin(PHI0)./N1);
PHI2=asin(sin(PHI0)./n2);
PHI3=asin(sin(PHI0)./n3);

TP = (t01p.*t12p.*t23p.*sqrt(X1.*X2))./...
      (1+(r01p.*r12p.*X1)+(r12p.*r23p.*X2)+(r01p.*r23p.*X1.*X2));
TS = (t01s.*t12s.*t23s.*sqrt(X1.*X2))./...
      (1+(r01s.*r12s.*X1)+(r12s.*r23s.*X2)+(r01s.*r23s.*X1.*X2));

taup = (n3.*cos(PHI3)./cos(PHI0)).*abs(TP).^2;
taus = (n3.*cos(PHI3)./cos(PHI0)).*abs(TS).^2;

tauu = (taup + taus)./2;

%Find the index for which Rp and Rs have opposite sign. This index
%gives where the operating angle is.
op = find(sign(RP).*sign(RS) == -1);

%The order of the film thickness, may be > 0 for shorter wavelength.
m = 0;

%Operating conditions, X1 = -1, d1/D1 = 1/2
PHI(op)

%Notice that this thickness satisfies the quarter wave optical thickness
%condition.
zeta1 = 1/2; %always, for quarter wave opt. thick
zeta2 = 1/2;

```

```

D1 = (lambda./2)./sqrt(n1.^2 - sin(PHI(op)*pi/180).^2)
D2 = (lambda./2)./sqrt(n2.^2 - sin(PHI(op)*pi/180).^2)

d1 = (zeta1 + m).*D1
d2 = (zeta2 + m).*D2

X1 = exp(-j.*2*pi.*(d1./D1))
X2 = exp(-j.*2*pi.*(d2./D2))

%Save variables for use in system response calculation
phi = PHI(op);
save CdTeIrtran3onKBrData phi d1 d2 n1 n2 n3

phi1 = asin(sin(phi*pi/180)/n1)*180/pi
phi2 = asin(sin(phi1*pi/180)*(n1/n2))*180/pi
phi3 = asin(sin(phi2*pi/180)*(n2/n3))*180/pi
alpha = phi3

format short

```

Vita

Faisal Sudradjat was born in Tembagapura, Indonesia, on June 1, 1982. He graduated from Yayasan Pendidikan Jayawijaya (YPJ) Middle School with honors in 1997. The following August, he entered Southwestern Academy High School from which he graduated with honors in the year of 2000. Then, he obtained his B. S. in Electrical Engineering (magna cum laude) from the University of New Orleans in 2004.