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A Study of Conditional Volatilities in Financial Markets using Generalized Conditional Heteroscedasticity Jump Models

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Dedications

“Except the Lord builds the house, they labor in vain that build it: except the Lord keep the city, the watchman waketh but in vain” (Psalm 127:1).

To the glory of Almighty God, my family, and all those who seek a better world for all mankind.

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Abstract

In this manuscript, I investigate the time-varying volatilities and co-volatilities in the fixed income and equities market using jump augmented stochastic volatility models. The results highlights that the fact that jumps are inherent in financial markets and have implications for the dynamics of volatilities and co-volatilities of financial assets over time. Jump augmented models provide a superior description of instantaneous market conditions and a promising avenue for future research in areas of asset pricing, portfolio selection, and risk management.

Chapter 1

Overview of Volatility Models and Research Motivation

Volatility pervades every sphere of human endeavor that involves uncertainty. The financial market is by no means immune from this phenomenon. Since the dawn of modern times, the study of volatility and its impact on asset prices, portfolio management, risk management and public policy analysis continues to remain on the frontier of empirical and theoretical research in finance and economics.

The earliest insight into understanding the dynamics of asset prices in presence of uncertainty was the seminal paper by Bachelier (1900). He argued under the theory of speculation that stock price change is independent of previous price movements, and concludes by proposing a model that in the future be widely characterized as the random walk or Brownian motion theory. This theory set the stage for the first paradigm of how volatility was modeled in empirical research. Motivated by Ito's (1957) advance in stochastic calculus, a spurt of literatures (general termed as stochastic volatility models) that follows the Brownian motion paradigm was the hallmark of financial research in the 70's and 80's. These literatures played a crucial role in modern finance theory by providing the basis for most option pricing, asset allocation and term structure theory currently in use today. In general, a stochastic volatility process models the value of the underlying security and its volatility as a random process, governed by state variables. Such state variables could be the level of the underlying process or the tendencies for the underlying process and its volatility to revert to a long-run mean.

Prominent amongst these class of literatures are the diffusion process in Hall and White's (1987) generalization of the Black-Scholes option pricing model to allow for

stochastic volatility, and Heston's (1993) model. The Heston's model allows volatility; to revert to a long-run mean value; be correlated with the underlying process, and to have its own constant volatility of volatility.

In 1982, Robert Engle, introduced a stochastic volatility model that characterizes the distribution of the stochastic error ε_t to be conditional on the realized values of the set of information $I_{t-1} = \left\{\varepsilon_{t-1}, \ldots\ldots\ldots, \varepsilon_{t-q}\right\}$:

$$\varepsilon_t \mid I_{t-1} \sim N(0, h_t), \quad h_t = a_0 + a_1\varepsilon_{t-1}^2 + \ldots + a_q\varepsilon_{t-q}^2 \tag{1}$$

with $a_0 > 0$ and $a_1 \geq 0$, $i = 1, \ldots, q,$ to ensure that the conditional variance is positive. Bollerslev (1986) extended Engle's (1982) work by proposing a generalization of the conditional variance function (1), which he termed generalized ARCH (GARCH) model. A standard GARCH model simply parametizes volatility as a function of unexpected shocks to the value of the underlying process. In other words, the standard GARCH computes the next period's variance by taking the sum of the square of the current period's innovations, and the current period's variance in an ARMA framework.

The contributions of ARCH and GARCH models to understanding the time-varying dynamics of volatilities are so colossal, that in 2003, Robert Engle, along with Clive Granger (for his own work on cointegration) were awarded the Alfred Nobel's prize in Economics. Today, the literature is awash with numerous refinements to Bollerslev's (1996) approach to modeling conditional volatility. To name a few; the V-GARCH models suggested by Engle and Ng (1993), the threshold GARCH model (Thr.-GARCH) by Zakoian (1994), the GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993), the Exponential GARCH of Nelson (1990), and the NGARCH model of

Engle and Ng (1993) and Duan (1995).

Since financial volatilities tend to move together over time and across markets. It is pertinent that researchers reflect on the time-varying comovement between asset returns, and between financial markets when pricing financial assets, selecting portfolios, and managing risks. To address this challenge, a new class of GARCH model (termed Multivariate generalized ARCH model or MGARCH) which possess the ability to capture the vector of covariance between assets over time was introduced by Bollerslev, Engle and Wooldridge (1988). A MGARCH model is a VARMA generalization of the univariate GARCH models. It models the conditional covariance matrix of m-assets as a function of newly revealed information from last period's joint asset returns. Under the assumption that returns follow a joint normal distribution, the econometrician can then assess the impact of shock in one asset or market on the volatilities and covolatilities of several assets or markets in a tractable framework.

The benefits of multivariate modeling of volatility spurred another growth in the generalization and specification of multivariate GARCH models. Prominent amongst these class of literatures are, Engle, Ng, and Rothchild's (1990) asset pricing relation based on a factor ARCH (FARCH) model of T-bills and the value- weighted NYSE index returns, Engle and Kroner's (1995) proposition of the BEKK model to guarantee positivity of the covariance matrix, Kroner and Ng's (1998) generalization of the VECH, BEKK, FARCH and Bollerslev's (1990) model into the generalized dynamic covariance model, and Engle's (2002) dynamic conditional correlation (DCC-MGARCH) model of various indices.

Although Multivariate GARCH models have come to be the empirical workhorse

in many areas of financial research, they are computationally demanding and often become impracticable to implement as the number of asset in the model grows. Nevertheless they provide a promising avenue for future research into understanding the relationship between financial assets, and markets across time.

One problem that continues to pose a limitation to the efficacy of stochastic volatility models in research is the nature of the financial data itself. Various literatures have pointed to the fact that financial data often have leptokurtic distributions that are characterized by large infrequent moves in either their levels, or their volatilities. Research has responded to this challenge by augmenting stochastic volatility models with jump dynamics based on a Poisson and recently Levy arrival process.

Jump diffusion model was first proposed in Press (1967) study of equity prices. They achieve prominence when Merton (1976) added Poisson jumps to the standard geometric Brownian motion model in his modification of the Black-Scholes option pricing model. Poisson jumps are defined so that during any short interval of time, the evolution of asset Prices and returns can be affected by a discrete number of random shocks, drawn from a normal distribution. The arrival of these shocks within each interval of time is governed by a Poisson distribution.

The success achieved in the integration of jump dynamics into stochastic volatility models engendered a new class of literature widely known as the stochastic-volatility-jump (SVJ) diffusion models. A few of these includes; Bates (1996) work in continuous time on foreign exchange options, Jorion (1988) ARCH/Jump model of foreign exchange and stock market return, and Bates (1991,2000) study of the 1987 stock market crash.

To address the issue of state dependencies or systematic pattern in the arrival discrete shocks mentioned in Bates (1991), recent works in the area of SV-jump-diffusion have incorporated time-varying jump intensities in their specification. Examples of these include Anderson, Benzoni, and Lund (2002), Bates (2000) and Eraker, Johannes and Polson (2003), Chan and Maheu (2002), Bekart and Gray (1998), Maheu and McCurdy (2004) and Duan (2004) work on stocks and stock market returns. And Das (2002), and Anderson, Benzoni and Lund (2004) on short-term interest rate. Jump models have performed creditably well in mitigating the excess kurtosis problem, and in their ability to capture the effect of peculiar shocks on the evolution of volatility and levels of financial data.

A more recent adaptation of jump dynamics is in the areas of multivariate-GARCH modeling of financial time series. MGARCH-Jump models present a new perspective of the comovement of financial assets and markets under normal and unusual news events. By incorporating jumps in a MGARCH framework, the econometrician is able to capture the incidence of leptokurtosis that are driven by local jump shocks, and jump shocks that foreign to the series. First amongst this class of models is Chan (2003) study of the foreign exchange returns. He found that the conditional covariances of returns in the foreign exchange market are driven by both normal innovations and correlated jump shocks. This insight raises the question as to whether the incidence of correlated jump shocks might also exist amongst other financial assets and markets over time.

My research follows the direction of the literatures listed in the last two paragraphs. I seek a better understanding of financial data through the eye of jump-

augmented stochastic volatility models in both a univariate, and a multivariate framework. In my first essay, I examined the ability of a new class of GARCH- jump augmented Cox, Ingersoll and Ross (1985) model to capture the dynamics of the U.S. short-term interest rate. In particular, I study the impact of normal and jump innovations on the intertemporal levels and, the volatility of 3-months U.S. Treasury bill yields. The structure of the model is such that leverage effect and leptokurtosis in interest rate series are accounted for. The goal of this essay is to examine the efficacy of the model in describing the salient characteristics of the short-term fixed income market.

In the second essay, I apply a multivariate-GARCH (MGARCH) jump model to investigate the contemporaneous comovement between equity and bond returns. Next I focus my attention on the issue of flight-to-quality by examining if the time-varying correlation between equities and the bond market is conditional on a set of information variables and market conditions. My interest in this research is the impact of decline or volatility in equity (bond) market on the dynamic correlation between equities and bonds.

In general, I find that in similar fashion to equity returns, yields on short-term default-free bonds responds to good news and bad news of equal magnitude differently. I also find evidence of the state dependency of jump arrival in the volatility of 3-month T-bill yields. Furthermore, I uncover the prevalence of correlated jumps between equity and bond returns and show that the contemporaneous correlation between equity and bond returns is driven by the arrival of independent and correlated jumps in the equity and bond market returns.

From the public policy point of view, studies such as this essay will have implication for a better understanding of how economic shocks is propagated within and

across markets. For example, it will do good to examine what factors drives the conditional comovement of equity and debt securities in both normal and unusual times. An understanding of these factors could provide a meaningful signal and platform for intervention by policy makers when the likelihood or level of uncertainty increases in financial markets.

From an investor's point of view, the findings of this research would provide a more accurate understanding of investors' reaction to a shift in the investment opportunity set due to the arrival of shocks in financial markets. In particular, an understanding of the nature of shock propagation in short-term debt market will be of immense benefits when pricing interest rate derivatives such as swaps and swaptions. Furthermore, the results from my second essay would also have implication for asset allocation, risk management, asset pricing, and cross- market hedging.

Chapter 2

Jumps and Asymmetric Volatility in the Short-Term Interest Rate

2.1 Introduction

Since Vasicek (1977) and Cox, Ingersoll and Ross (1985) (hereafter CIR) many other single, and multi-factor models that include, and do not include stochastic volatility components have been developed and analyzed to understand the dynamics of the short-term interest rate. Financial economists are in particular, interested in modeling the dynamics of the short-term interest rate because of its effect on the term structure of interest rate. More so the short-term interest rate is fundamental to the valuation of securities.

Although a substantial amount of studies exist today on modeling and estimating the dynamics of short rate, the lack of consensus about the appropriateness of any model to accurately capture the features of short-term interest rate necessitate that researchers continue to seek a better understanding of its time-varying characteristics. In this essay, I extend the work of Das (2002) and Bali (2000) by augmenting the Level-GARCH specification to accommodate the asymmetric effects that news have on volatility. Significant evidences exist that dissimilar types of news have different impact on the volatility dynamics of securities. For example, research has shown that bad news have significantly more impact on volatility than good news of equal magnitude (Bali 2000; Engle and Ng 1993; McCurdy and Maheu, 2004). This asymmetric effect provided the motivation for my examination of the suitability of fit of an asymmetric level-GARCH jump specification to the dynamics of the short rate.

This research offers the following contributions to the literature. First, unlike

existing literatures, this essay scrutinizes the evolution of the short rate for leverage effect and state dependency of jumps by the augmenting the CIR model with conditional jump and asymmetric volatility dynamics. I find that the asymmetric CIR-NGARCH-jump model is superior to existing Level-GARCH specification in explaining the dynamics of short-term interest rate. In particular, I find that jumps in short-term interest rate are state dependent in way that has not been highlighted in existing literatures. Das (2002) found evidence suggesting higher probability of jumps on Wednesdays to Fridays and 2-days meetings of Federal Reserve Open Market Committee for the Federal Funds rate. I document evidence of state dependencies in the arrival of jumps in 3-months T-Bill yield from a generalized perspective. In general, jump arrival is positively related to the level of uncertainty in the market and is more likely to arrive if there was a jump in the immediate past period. By employing an autoregressive intensity framework in modeling the arrival of jumps, the CIR-NGARCH-jump model is better equipped to capture instantaneous discrete shocks arrival in the T-bill yield, and thus provide a more accurate description of the instantaneous condition in the fixed income market.

Second, I also find that jumps have implications for conditional volatility. The estimates of the GARCH coefficient obtained from the jump enhanced Level-GARCH model for the 3-months T-bill yields is significantly lower than those obtained in the CIR-GARCH models. The introduction of jumps parameters into the model severely dampens the ARCH effect in the dynamics of the short-rate. Again, by accounting for the impact of large discrete shocks to the short rate, the jump coefficients allow such shocks to quickly dissipate away in the process. There by enabling the GARCH coefficient to focus only on capturing the impact of the smoothly arriving news on the dynamics of the

short rate.

Third, yields on short-term default-free bond respond asymmetrically to news information. In particular, the negative size bias test reveals that the squared standardize residuals obtained from the CIR-NGARCH-jump and two other models estimated in this essay, are sensitive to high levels of negative shocks to the short rate. The volatility of the short rate is more sensitive to bad news than good news. Unfortunately all jump enhanced conditional volatility specification estimated in this essay could not adequately account for this asymmetric effect.

Taking all these into account, I surmise that jumps have a significant implication for the evolution of the short-term interest rate. The CIR-NGARCH-Jump framework provides an improved fit for modeling the dynamics of short-term rate over pre-existing models and would be a realistic benchmark for future research into modeling the time-series of short-term interest rate.

2.2 Review of Literature

Following the lead of Vasicek and CIR is class of pure diffusion models where the primary source of uncertainty is the instantaneous interest rate itself. For example, Constantinide (1992) proposed a non-linear generalization of the CIR model, Chan, Karolyi, Longstaff and Sanders (1992) (hereafter CKLS) specified a general framework to estimate and compare a range of single-factor (Levels) models for the short-term interest rate, Heston (1993), and Fong and Vasicek (1991) model the behavior of interest rate and its volatility as the term structure of a driving force, Longstaff and Schwartz (1992) specified a model that allowed the volatility of the short rate to be stochastic in a

two-factor model, and lastly, Ait-Sahalia (1996) proposed a model that associate the volatility of the short rate to its level, along with an accommodation for mean reversion .

Another class of models follows the generalized autoregressive conditional heteroscedasticity (GARCH) specification proposed by Engle, (1982) and extended by Bollerslev (1986). In these models, the volatility of the short rate is conditional on the information arrival process and not the instantaneous rate itself. These literatures include; Cecchetti, Cumby and Figlewski (1988) three equation ARCH model which yields the time varying estimates of the covariance matrix of equity return and interest rate under the assumption of a constant correlation, and Engle, Ng, and Rothschild (1992) FACTOR-ARCH model of the daily Treasury bill yields.

Despite the popularity of single factor and GARCH models, both suffer from several limitations. Single-factor models imply that the instantaneous returns on bonds of all maturities are perfectly correlated[1], and that the volatility process depends strictly on the interest rate itself. As a result, single factor models of the short rate tend to overemphasize the sensitivity of volatility to interest rate level[2]. GARCH only models also suffer from misspecification errors. Brenner, Harjes and Kroner (1996) (hereafter BHK) show that GARCH-only models relies too heavily on the serial correlation in the variances and fail to capture the relationship between volatility and the level of the short rates. Furthermore, GARCH models also imply that current shocks pervade in the volatility process infinitely, and permit negative interest rate.

[1] Since prices at all maturities are driven by a single stochastic factor, therefore the all yields levels irrespective of its maturities are perfectly correlated. Empirical research has shown that though yields are highly correlated, they are not perfectly correlated as claimed by single factor models. See Anderson and Lund (1997) Chen and Scott (1993) Dai and Singleton (1997), Brick and Thompson (1978).

[2] BHK argues that levels models in accurately restrict volatility to be a function of interest rate levels only, because volatility of the short rate increases as levels increase

As an alternative to single factor and GARCH models, BHK introduced a model that incorporates GARCH effect into the dynamics of the CKLS model under a general discrete time specification. This generalization of the CKLS model provides the motivation for a third class of short rate models. Level-GARCH models (as they are popularly referred to) allow volatility to depend on both interest rate levels and the smoothly evolving information arrival process. As a result, they provide a superior fit to the data than the single factor models (such as the CIR and Vasicek models), and GARCH only models. In continuous time, Anderson and Lund (1997) introduced a set of stochastic volatility models of the short rate that nest EGARCH coefficients into a simple diffusion model. They show that this model is also superior to the GARCH only models in responding to interest rate shocks. Cvsa and Ritchken (2001) examines the pricing of interest-rate-contingent claims when the underlying interest rate process is driven by a two-state-variable GARCH process, and conclude that level-GARCH models have a good chance of removing the volatility smiles observed when normal and lognormal process are used in pricing claims such as interest rate swaps and swaptions. While level-GARCH models are generally well accepted in the literature today, they also suffer other limitations along with single-factor and GARCH-only models. For example, level-GARCH models are unable to capture the skewness, excessive kurtosis and pervasive autocorrelation patterns clearly evident in interest rate data[3].

A parallel class of literatures examines the term-structure of interest rates when the underlying state variables follow a jump-diffusion process. Ahn and Thompson (1988) show that traditional expectation hypothesis is inconsistent with the equilibrium

[3] Bollerslev (1987), Nelson (1991) points to the problem of excess kurtosis in interest rate data. In addition the descriptive statistic obtained reveals strong evidence of fat tails in the *dr(t)* (see Table I.)

model, under jump-diffusion framework. Das (1997) extends Vasicek (1977) model by including a normally distributed jump dynamics into the interest rate process. Attari (1999) develops a general methodology for pricing discount bonds and options on bond when the short rate follows a jump diffusion process. Andersen, Benzoni and Lund (hereafter, ABL, 2004) estimated a system that introduces multiple factors along with jumps into the drift and diffusion coefficients. Piazzesi (2005) developed a high frequency policy rule by implying that the Federal Open Market Committee reacts to information contained in the yield curve. This yield curve is derived from rates on LIBOR and swaps contracts, and is assumed to follow a jump diffusion process. In all these literatures, the conclusion is that the introduction of jumps ameliorates excess kurtosis and persistent autocorrelation pattern observed in interest rate series.

The incidence of unanticipated news arrival is crucial to understanding the role of jumps in the interest rate process. Attari (1999) suggest that jumps may reflect economic announcement and/or Federal Reserve activity that deviates from expectation of market participants. Das (2002) provide substantiation to this assertion by showing that two-day meetings by the Federal Open market committee have a substantially greater jump effect on Federal Funds rate than one-day meetings. For the equities market, Maheu and McCurdy (2004) also show that normal news and unanticipated news have different impact on the returns and volatility of returns in the stock markets. Normal news results in the smoothly evolving changes in the conditional variance and levels of return, while unanticipated news often results in large infrequent changes in the levels and the volatility of equity returns.

A more recent strand of literatures examines the performance of single factor

models when augmented with jumps and stochastic volatility components. These jump-enhanced Level-GARCH models are specified in such a way that the stylized behavior of the short rate can be accounted for, from a statistical point of view. The GARCH coefficients in the model will capture the persistent volatility in the data. The levels components to account for mean reversion, which may arise naturally from underlying macroeconomic events or correction on account of bond market overreaction, and lastly the jump coefficient to account for excessive skewness and kurtosis in the data. Das (2002) follow this strategy by augmenting the Vasicek (1977) with GARCH and jump components. He showed that the goodness-of-fit as well as the economic implications of this model far exceed other pre-existing models.

I extend the frontier of level-GARCH modeling by addressing the issue of asymmetry in the response of the short rate to the arrival of news. I test to see if an asymmetric level-GARCH-jump framework will provide a superior fit to the data than symmetric level-GARCH-jump models. Rather than employ the level-GARCH-jump framework of the Vasicek type model that theoretically can permit negative interest rate, I follow the CIR model, which rules out negative interest rate in its continuous time specification.

2.3 Model Specification

a. Background

In a single-good continuous-time production economy in which a representative agent has a logarithmic Von Neumann-Morgenstern utility function, CIR (1985) show that the equilibrium (instantaneous) real interest rate r, follows a “mean reverting

square-root" process specified as:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dz_t\,, \tag{2}$$

where $\kappa > 0$ and $\theta > 0$ are constants and z_t is a standard (zero drift and unit variance) Brownian motion. The parameter κ is the speed of adjustment of the interest rate *r* towards the long-term mean θ. The CIR model has some appealing features, which are unavailable in Vasicek (1977), for example; Negative interest rates are precluded. If the interest rate reaches zero, it can subsequently become positive. In addition, the absolute variance of the interest rate increases when the interest rate itself increases. The explicit dependence of yields on interest rate volatility allows the model to capture many of the observed properties of the term structure such as humps, troughs, the relations between term premia, and interest rate volatility. Lastly this interest rate model has a steady state distribution

The conditional density function for the CIR model, which highlights the probability density of the short rate at time *s* conditional on its value at the current time *t*, is shown below.

$$f\left(r_s, s \mid r_t, t\right) = ce^{-u-v}\left(\frac{\upsilon}{u}\right)^{q/2} I_q\left(2(u\upsilon)^{1/2}\right), \tag{3}$$

Where

$$c \equiv \frac{2\kappa}{\sigma^2\left(1 - e^{-\kappa(s-t)}\right)} \tag{4}$$

$$u \equiv cr(t)e^{-\kappa(s-t)} \tag{5}$$

$$\upsilon \equiv cr(s) \tag{6}$$

$$q \equiv \frac{2\kappa\theta}{\sigma^2} - 1 \tag{7}$$

$(s > t)$ and (I_q) is the modified Besseli function of the first kind of order q. The distribution function is the non-central chi-square, $\chi^2[2cr_s; 2q+2, 2u]$, with 2q+2 degrees of freedom, and parameter of non-centrality 2u, proportional to the current spot rate.

The conditional expectation and variance of the CIR model is also expressed as:

$$E(r_s \mid r_t) = r_t e^{-\kappa(s-t)} + \theta\left(1 - e^{-\kappa(s-t)}\right), \tag{8}$$

$$\operatorname{var}(r_s \mid r_t) = r_t\left(\frac{\sigma^2}{\kappa}\right)\left(e^{-\kappa(s-t)} - e^{-2\kappa(s-t)}\right) + \theta\left(\frac{\sigma^2}{2\kappa}\right)\left(1 - e^{-\kappa(s-t)}\right), \tag{9}$$

ABL (2004) mentions that discretizing the CIR model does not rule out negative interest rates; however, the probability of such arising is minimal at best. In view of this, I present, without loss of generality, the discrete time approximation of the CIR model as:

$$\begin{aligned} &r_{t+s} = r_t + \kappa(\theta - r_t)\Delta t + \varepsilon_{t+s} \\ &\textit{where } E(\varepsilon_{t+s} \mid \Phi_t) = 0, \quad E(\varepsilon_{t+s}^2 \mid \Phi_t) = \sigma_{t+s}^2 = \sigma^2 r_t \Delta z_t, \quad \Delta t = s - t \end{aligned} \tag{10}$$

An ARCH(q) model by Engle (1982) characterize the distribution of the stochastic error ε_t conditional on the realized values of the information set $I_{t-1} = \{\varepsilon_{t-1}, \ldots\ldots\ldots, \varepsilon_{t-q}\}$ as:

$$\varepsilon_t \mid I_{t-1} \sim N(0, h_t), \quad h_t = a_0 + a_1\varepsilon_{t-1}^2 + \ldots + a_q\varepsilon_{t-q}^2 \tag{11}$$

with $a_0 > 0$ and $a_1 \geq 0$, $i = 1, \ldots, q$, to ensure that the conditional variance is positive. Bollerslev (1986) proposed a generalization of the conditional variance function (11), which he termed, generalized ARCH (GARCH). BHK (1996) followed by augmenting

the standard diffusion model of CKLS (1992) which nest the CIR model, with GARCH components as specified below:

$$dr_t = \kappa(\theta - r_t)dt + \sqrt{h_t r_t}\, dz_t \tag{12}$$

where

$$dh_t = k_1(\phi - h_t)dt + \xi dz_{1,t} \tag{13}$$

The constant variance parameter in the CIR model is replaced with an autoregressive volatility process (h_t). The generalization of (8) obtained by discretizing (12) and (13) are also shown below:

$$r_{t+s} - r_t = \kappa\left(\theta - r_t\right)\Delta t + \varepsilon_{t+s} \tag{14}$$

$$E\left(\varepsilon_{t+s} \mid \Phi_t\right) = 0, \qquad E\left(\varepsilon_{t+s}^2 \mid \Phi_t\right) = \sigma_{t+s}^2 = h_{t+s} r_t \Delta z_t{}^4 \tag{15}$$

where, $$h_t = a_0 + \sum_{i=1}^{p} a_i \cdot \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \cdot h_{t-i} \tag{16}$$

Bali (2000) extended BHK's work by proposing a two-factor discrete time volatility model with non-linear asymmetry dynamics in the short-term rate. In addition, he also tested the ability of several well-known symmetric and asymmetric GARCH models to capture the dynamics of the interest rate series. He found that all symmetric mean stochastic volatility models appear to be weak in their forecasting ability of the actual yield changes, and conclude that their predictive power are inferior to asymmetric level-GARCH models. Amongst the 8 models tested in Bali (2000), the non-linear GARCH (NGARCH) specification of Engle and Ng (1993) nested in the CIR framework, provided the best fit to the data. I extend Bali's works by augmenting the CIR-NGARCH

model with jump components and present the model in the next section.

b. Model

I specify a general Level-GARCH-jump model that follows Das (1998) and ABL (2004). Consider an extension of (12) by including an independent Poisson process embodying a random jump $J(\mu_j, \delta_j^2)$ drawn from an independent normal distribution. The arrival of $J(\mu_j, \delta_j^2)$ is governed by a Poisson arrival frequency parameter $n(t) \in \{0,1,2,....\}$ over the interval *(t-1, t).* While it is feasible to allow for feedback from the jump process into the GARCH and vice versa, (See, McCurdy and Maheu, 2004; Daal and Yu, 2005), however, for tractability I assume that the diffusion process embodied in the GARCH coefficients is independent of the jump dynamics. The following equations describe the CIR model with GARCH and jump innovations:

$$dr_t = k(\theta - \frac{\lambda\mu_j}{k} - r_t)dt + \sqrt{h_t} r_t dz(t) + J(\mu_j, \delta_j^2)dn(t) \quad (17)$$

where ,

$$dh_t = \omega(\phi - h_t)dt + \eta dB_{2,t} \quad (18)$$

Estimating continuous-time diffusion models for the interest rate process is a daunting task because few of these processes have a tractable density function. A trendy solution to this problem is to discretize the underlying process. In addition, in the presence of leptokurtosis, overwhelming number of literatures calls into the question the suitability of the normal density function for estimating the parameter of the model. Nevertheless, assuming a normal distribution serves as a good starting point for this

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[4] z(t) is the variance of the standard Brownian motion and is a function of time Δt, it is strictly increasing in time, where Δt=1, Δz(t)=1, so it will always be strictly positive. Since I am only considering the change with a unit time, Δz(t) becomes irrelevant in the model.

investigation. I can discretize (17) and (18) without the loss of generality as shown below.

$$\Delta r_t = k(\theta - \frac{\lambda\mu_j}{k} - r_t)\Delta t + \sqrt{h_t r_t} z_t + \sum_{i=1}^{n(t)} J(\mu_j, \delta_j^2) \tag{19}$$

where

$$h_t = a_0 + \sum_{i=1}^{p} a_i \cdot \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i \cdot h_{t-i} \tag{20}$$

Notice that h_t can be a general outline for nesting various GARCH models in the CIR-GARCH-jump framework. For example, I nest the Non-linear asymmetric GARCH (NGARCH) model of Engle and Ng (1993) within the framework of (19) and (20) by specifying:

$$\begin{gathered} h_t = a_0 + a_1\left(\varepsilon_{t-1} + \varphi\sqrt{h_{t-1}}\right)^2 + \beta_1 h_{t-1} \\ \textit{Where } a_0 > 0,\ 0 \le a_1 < 1, 0 \le a_2 < 1, a_1 + \beta_1 < 1, \text{ and } \psi > 0 \end{gathered} \tag{21}$$

Also, notice that when $\Delta t = 1$, (19) can be reduced to a tractable form given by

$$r_t = r_{t-1} + k(\theta - r_{t-1}) + \varepsilon_{1t} + \varepsilon_{2t} \text{ [5]} \tag{22}$$

where,

$$\begin{aligned} \varepsilon_{1t} &= \sqrt{h_t r_{t-1} \Delta z(t)} \quad , \quad z_t \sim NID(0,1) \\ \varepsilon_{1t} &\sim NID(0, \sigma_t^2)) \quad , \quad \sigma_t^2 = h_t r_{t-1} \end{aligned} \tag{23}$$

$$\varepsilon_{2t} = \sum_{i=1}^{n(t)} J(\mu_j, \delta_j^2) \text{ -}\lambda\mu_j, \qquad J(\mu_j, \delta_j^2) \sim NID(\mu_j, \delta_j^2) \tag{24}$$

The jump intensity $\lambda = \mathrm{E}\left[n_t \mid \Phi_{t-1}\right]$ is the unconditional expected number of

jumps that occurs within the time interval $\Delta t = 1$. Since the Poisson jump arrival frequency parameter $n(t) \in \{0,1,2,....\}$ over the interval $\Delta t = 1$, the conditional density of n_t can be expressed as:

$$P(n(t) = j \mid \Phi_{t-1}) = \frac{\exp(-\lambda)\lambda^j}{j!} \tag{25}$$

In subsequent section of this paper, I relax the restriction on λ by allowing the jump intensity to be state dependent. The conditional moments for the model are[6]:

$$E\left[r_t \mid \Phi_{t-1}\right] = r_{t-1} + k(\theta - r_{t-1}) \tag{26}$$

$$\begin{aligned} Var\left[r_t \mid \Phi_{t-1}\right] &= Var\left[\varepsilon_{1t} \mid \Phi_{t-1}\right] + Var\left[\varepsilon_{2t} \mid \Phi_{t-1}\right] \\ &= \sigma_t^2 + \lambda\left[\mu_j^2 + \delta_j^2\right] \end{aligned} \tag{27}$$

$$Sk\left[r_t \mid \Phi_{t-1}\right] = \frac{\lambda\left(\mu_j^3 + 3\mu_j\delta^2\right)}{\sigma_t^2 + \lambda\left[\mu_j^2 + \delta_j^2\right]} \tag{28}$$

$$Ku\left[r_{t|} \mid \Phi_{t-1}\right] = 3 + \frac{\lambda\left(\mu_j^4 + 6\mu_j^2\delta_j^2 + 3\delta_j^4\right)}{\left(\sigma_t^2 + \lambda\left[\mu_j^2 + \delta_j^2\right]\right)^2} \tag{29}$$

When $\lambda = 0$, (i.e. no jumps) then $P(n(t) = j \mid \Phi_{t-1}) = 0$, the skewness is zero and the Kurtosis is 3, which is the normal level. Thus, the sign of the conditional skewness depends on the sign of the mean jump size μ_j.

The unconditional variance for the short rate model specified above is also shown as.

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[5] This model can also be specified as $\Delta r_t = \alpha + \beta r_{t-1} + \varepsilon_{1t} + \varepsilon_{2t}$ where $\alpha = k\theta$ and $\beta = k$ .The probability densities for both specifications are essentially the same. For more intuitive economic implications, it is more efficient to estimate the parameters separately.

[6] For derivations of the conditional moments, see Das and Sundaram (1997)

$$Var(r_t) = \frac{\theta a_0}{1 - a_1\varphi - \beta - a_1} + \lambda\left[\mu_j^2 + \delta_j^2\right] \tag{30}$$

$$Where\ a_0 > 0,\ 0 \le a_1 < 1, 0 \le a_2 < 1, a_1 + \beta_1 < 1, \text{ and } \psi > 0$$

Broze, Scaillet, and Zakoian (1993) revealed that the GMM estimators of level models are inconsistent when the power of the r_t in the error term is greater than one. Duffie (2002) compare benefits of quasi-maximum Likelihood with the Efficient Methods of Moment technique of GMM and argued that there is positive probability that quasi-maximum likelihood estimated model can generate the actual distributional properties of the observed term structure series. Thus, I apply quasi-maximum likelihood approach in lieu of the GMM method.

Two popular methods of estimating jump diffusion models using the maximum likelihood approach are; (1) use a mixture of Poisson-normal probability densities, and (2) use a mixture of a discrete sum of two normal probability densities using the Bernoulli distribution as an approximation of the Poisson distribution (Das 2002). The Bernoulli approximation is easier and quicker to estimate at the expense of accuracy. On the other hand, the Poisson-normal distribution, which is more appropriate, is more exigent to estimate and often requires careful selection of the point of truncation for the estimation to be feasible. For the sake of accuracy, I employ the Poisson-normal density function for the estimation of (19) and construct a log-likelihood function defined as expressed below:

$$L(\Theta; r_1, \ldots r_T) = \sum_{t=2}^{T} \ln\left[f(r_t \mid \Phi_{t-1})\right] \tag{31}$$

Where

$$f(r_t \mid \Phi_{t-1}) = \sum_{j=0}^{\infty} P(n(t) = j \mid \Phi_{t-1}) f(r_t \mid n(t) = j, \Phi_{t-1}) \tag{32}$$

$$= \sum_{j=0}^{\infty} \frac{\exp(-\lambda)\lambda^j}{j!} \cdot \frac{1}{\sqrt{2\pi\left(\sigma_t^2 + j\delta_j^2\right)}} \exp\left(-\frac{\left(r_t - r_{t-1} - k(\theta - r_{t-1}) - \lambda\mu_j + j\mu_j\right)^2}{2\left(\sigma_t^2 + j\delta_j^2\right)} \right) \quad (33)$$

2.4 Data Description

I analyzed two sets of data for this study. The first set consists of daily 3-months nominal Federal constant maturity T-bill yield from 01/04/1982 to 12/15/2005. The second set of data, which I use as a robustness check during the model selection stage, consist of daily observations of the 6-months nominal Federal constant maturity T-bill yield from 01/04/1982 to 12/15/2005. These are market yields on U.S. Treasury securities and are quoted on investment basis. The series are published by the United States Treasury Department and are available in the Federal Reserve H.15 database.

Table 1

Summary Statistics

Moments

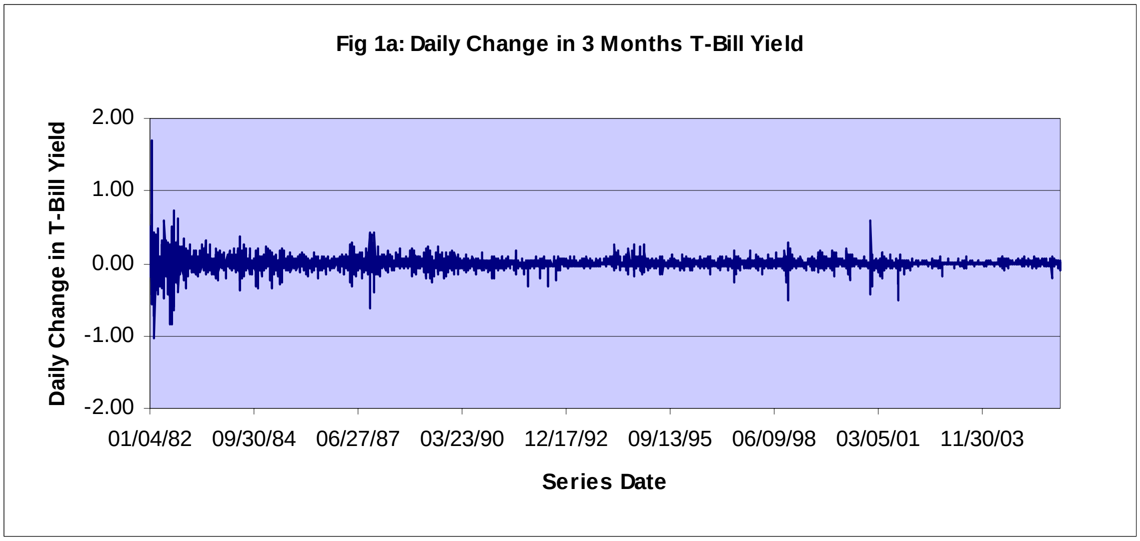
Panel A:	N	Mean	Std Deviation	Skewness	Variance	Kurtosis	Std Error Mean
3 Months	5988	5.491	2.655	0.438	7.051	0.313	0.034
6 Months	5988	5.719	2.762	0.494	7.626	0.397	0.036
Panel B: Interest Rate Change							
3 Months	5987	-0.001	0.083	0.502	0.007	48.616	0.001
6 Months	5987	-0.001	0.079	0.297	0.006	37.399	0.001

The daily data set is obtained from the Federal Reserve H.15 database and consists of the three and six months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms.

Table 1 supplies the summary statistics of the level and the first difference of the two series. The unconditional mean level of the three- month T-bill yield is 5.49%, with a

standard deviation of 2.65%. For the 6 months T-bill yield, the unconditional mean level is 5.72% with a standard deviation of 2.76%. The average daily change for the 3-, and the 6-months data is -0.001%. Fig 1a and Fig 1b graphs the daily change in the yield for the 3-, and the 6-months series respectively. The largest single day change (not reported in the table) in the yields are 1.68% (13% change), and 1.39% (9.76% change) for the 3-months and the 6-months data, respectively.

Figure 1: Times Series of T-Bill Yields. Time series plot of the daily change in the 3-, and 6-months T-bill yields.



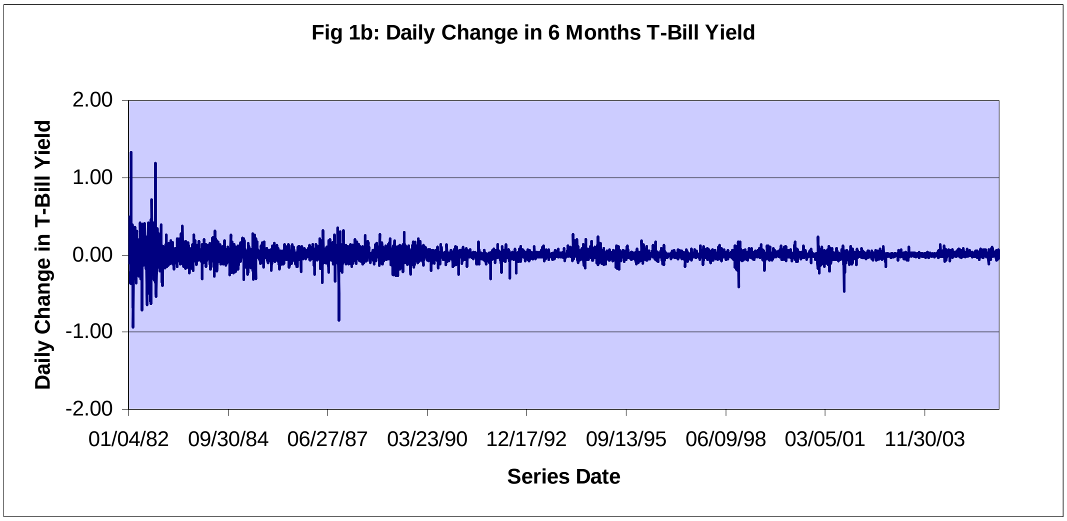


Table 2

Q and LM Test for Arch Disturbance

Panel A:					**Panel B:**				
3 Months					**6 Months**				
Q and LM Tests for ARCH Disturbances					**Q and LM Tests for ARCH Disturbances**				
Order	**Q**	**Pr > Q**	**LM**	**Pr > LM**	**Order**	**Q**	**Pr > Q**	**LM**	**Pr > LM**
1	5944.9668	<.0001	5937.6309	<.0001	**1**	5949.2855	<.0001	5940.9105	<.0001
2	11839.8352	<.0001	5937.8107	<.0001	**2**	11853.5529	<.0001	5941.0468	<.0001
3	17681.504	<.0001	5937.8610	<.0001	**3**	17710.9582	<.0001	5941.0693	<.0001
4	23474.938	<.0001	5937.9803	<.0001	**4**	23521.5416	<.0001	5941.0694	<.0001
5	29228.5752	<.0001	5938.3027	<.0001	**5**	29292.0853	<.0001	5941.3062	<.0001
6	34923.3519	<.0001	5940.7263	<.0001	**6**	35014.887	<.0001	5941.8132	<.0001
7	40568.3378	<.0001	5941.3667	<.0001	**7**	40689.2861	<.0001	5941.8134	<.0001
8	46155.3215	<.0001	5941.7472	<.0001	**8**	46311.0812	<.0001	5941.935	<.0001
9	51685.1968	<.0001	5941.7524	<.0001	**9**	51882.2495	<.0001	5941.9769	<.0001
10	57160.3667	<.0001	5941.7548	<.0001	**10**	57398.8795	<.0001	5942.1868	<.0001
11	62564.7421	<.0001	5942.7452	<.0001	**11**	62855.8918	<.0001	5942.3000	<.0001
12	67914.5765	<.0001	5943.8348	<.0001	**12**	68257.1341	<.0001	5942.3587	<.0001

The daily data set is obtained from the Federal Reserve H.15 database and consists of the three and six months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms. The rates are expressed in annualized percentage terms. The Lagrange Multiplier and Portmanteau Q Test for the presence of ARCH are specified below:

$$LM(q) = \left[\frac{(\text{NW'Z }(\text{Z'Z})^{-1}\text{Z 'W})}{(\text{W'W})}\right] \text{ and } Q(q) = N(N+2)\sum_{1}^{q}\frac{r(i;\hat{v}_t^2)}{(N-i)}$$

Notice that the magnitude and the frequency of extreme change in the evolution of the yields appears to diminish as we progress from the 1980's into the 1990's. In October, 1979, the Federal Reserve made an abrupt shift from interest rate oriented monetary measures to reserved oriented measures. The result was an unprecedented level of uncertainty in the short-term fixed income market as regard the evolution of the short rate. Towards the end of 1982, the Feds realizing that financial innovations have weakened the historical link between monetary base and the economic goal of monetary policy responded by making more flexible decision about money market conditions[7]. The Feds then shifted focus to using a broader assortment of monetary and financial variable

[7] More information is available in the Federal Reserve Bulleting for November 1997

to gauge the necessity for an adjustment of the short rate. Beginning in 1994, the Feds started publicizing changes in its policy objective, and in 1995 embarked on policy of explicitly stating its target level for the Federal Funds rate. A plausible consequential implication as shown in Fig 1a and Fig 1b is a significant reduction in the frequency and magnitude of extremities in the time series of the short rates.

I also report the skewness and excess kurtosis statistics for test of the distributional assumption of normality. Panel B shows that distribution of the short rate process have extremely large fat tails compared to a normal distribution. In support of the existing volume of literature, both the Lagrange multiplier test and the Portmanteau Q-test of conditional heteroscedasticity in the two series, presented in Table 2 reveals strong evidence of heteroscedasticity up to the 12th lag.

2.5 Results

First, I present the parameter estimates of the pure diffusion models in continuous and discrete time in Table 3. I then compare the results of the estimation of the CIR model following the discrete time and the continuous time specification. Next, I present the estimates of the Level-GARCH models in Table 4 and discuss their gains over the pure-diffusion CIR model. In Table 5, I compare the performance of various CIR-GARCH specifications using the results of the likelihood ratio statistics. In Table 6, I highlight the suitability of fit of the CIR-GARCH specifications and suggest a candidate GARCH dynamics for the CIR-GARCH-jump model. The results of the estimation of this jump model along with the Vasicek-GARCH-jump model are then presented in Table 7. Next, I present in Tables 8 and 9, comparison tests of the appropriateness of fit, of the

different level-GARCH-jump specifications highlighted in Table 7, using the likelihood ratio test and the information criteria statistic. In Table 10, I present the results of the CIR-NGARCH with autoregressive jump intensity model. Lastly, in Table 11 and 12, I highlight some specifications issues of interest in the jump augmented models.

a. Level Models

Table 3 shows the result of the continuous time and the discrete-time quasi-maximum likelihood estimates of the CIR model of 3-, and 6-months T-bill yields. The discrete-time maximum likelihood exploits the normal density function, while the continuous time estimation is based on the non-central Chi-square distribution proposed in the CIR (1985) paper. Discrete time maximum likelihood parameter estimates were derived using the Marquardt-Levenberg numerical minimization algorithm, while the continuous time model estimation utilized the Quasi-Newton numerical minimization method. In both estimations I employed a variety of starting values to ensure robustness of the estimates of the parameters. From each model, I extract the maximized log-likelihood, the parameter estimates and the standard errors. The standard errors are the square root of the diagonal elements of corresponding asymptotic covariance matrix derived from the inverse of the Hessian matrix. The Hessian matrix consists of the second-order partial derivatives of the log-likelihood functions with respect to the parameters.

The estimates for the continuous time specification are reported in Panel A of Table 3, while the estimates reported in Panel B are for the discrete time specification. A cursory look at the log-likelihood and the parameter estimates obtained from both

specifications appears to suggest that there is no significant benefit to be obtained from employing the continuous time over the discrete time specification. The coefficients obtained from both models are not significantly different and both lied within reason bound of existing literatures. Furthermore, Brenner, Harjes and Kroner (1996), Bali (2000), Das (2002) to name a few all utilized the discrete-time specification of the modified CKLS (1992) model. Thus, I also employed the discrete-time specification in my subsequent augmentation and estimation of the CIR model.

b. Levels-GARCH Models

In Table 4, I present the discrete time quasi-maximum likelihood parameter estimates, asymptotic *p*-value and the log-likelihood of the CIR-GARCH, CIR-AGARCH, and CIR-NGARCH, and CIR-QGARCH, models for both the 3-months and the 6-months T-bill yields. The specification for each model is shown in the table. As in the previous estimation a variety of starting values were employed to ensure the robustness and convergence of the model. In light of the leptokurtic distribution of the series, it would be beneficial to assume distribution such as the Student-*t* distribution or the generalized error distribution (See, Daal and Yu (2005) and, Bollerslev (1987), and Nelson (1991)). However, in order to make the estimation more tractable, I assume that the error terms are drawn from a normal distribution (Anderson and Lund, 1997 shows that there is only a minute gain in accuracy from changing the distribution assumption from normal to Student-*t* for the interest rate process.).

Table 3

Continuous Time Maximum Likelihood Estimate of CIR Model					Discrete Time Maximum Likelihood Estimate of CIR Model					
	Continuous Time					Discrete Time				
Panel A:					Panel B:					
	3 Months	k	θ	σ	Log L	3 Months	k	θ	σ	Log L
Estimates		0.0006	3.3963	0.0305	7756.22	Estimates	0.0006	3.1331	0.0305	7761.00
SE		0.0003	1.2215	0.0003		SE	0.0003	1.3981	0.0003	
T- Stats		2.1097	2.7805	103.73		T- Stats	1.8900	2.2400	109.42	
P-Value		[0.0175]	[0.0027]	[0.0000]		P-Value	[0.0593	[0.0251]	[0.0000]	
	6 Months					6 Months				
Estimates		0.0007	3.4821	0.0290	7925.70	Estimates	0.0006	3.2887	0.0290	7929.00
SE		0.0003	1.1540	0.0003		SE	0.0003	1.2655	0.0003	
T-Stats		2.3227	3.0173	103.71		T-Stats	2.1400	2.6000	109.43	
P-Value		[0.0101]	[0.0013]	[0.0000]		P-Value	[0.0325]	[0.0094]	[0.0000]	

This table displays the maximum likelihood estimates of the Continuous time and the Discrete time CIR model, The parameter estimates with asymptotic t-statistics in parentheses are presented for each model. The maximized log likelihoods (Log-L) for the models are shown to compare the explanatory power of these models. The daily data set is obtained from the Federal Reserve H.15 database and consists of the three and six months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms.

Table 4

Maximum Likelihood Estimates of the CIR-GARCH Models.

Models	κ	θ	a_0	a_1	β_1	φ	**Log-L**
Panel A **3 months**							
Vasicek	0.0009	4.0909					6420.00
	[0.0026]	[0.0423]					
CIR-GARCH	0.0001	10.5573	0.0003	0.0492	0.0459		8791.30
	[0.6354]	[0.5168]	[0.0000]	[0.0000]	[0.0000]		
CIR-AGARCH	0.0004	4.6271	0.0003	0.0490	0.0466	0.0137	8801.80
	[0.1161]	[0.0064]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	
CIR-NGARCH	0.0004	4.4838	0.0003	0.0479	0.0446	-0.2237	8802.30
	[0.0889]	[0.0030]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	
CIR-QGARCH	0.0004	4.5741	0.0003	0.0490	0.0467	-0.0014	8802.20
	[0.1090]	[0.0054]	[0.0000]	[0.0000]	[0.0000]	[0.0000]	
Panel B: **6 months**							
Vasicek	0.0010	4.2353					6688.00
	[0.0011]	[0.0200]					
CIR-GARCH	0.0001	13.9067	0.0003	0.0364	0.0497		8689.30
	[0.8102]	[0.7560]	[0.0000]	[0.0000]	[0.0000]		
CIR-AGARCH	0.0002	5.2387	0.0003	0.0359	0.0500	0.0128	8695
	[0.3700]	[0.1124]	[0.0000]	[0.0000]	[0.0000]	[0.0010]	

Table 4

Maximum Likelihood Estimates of the CIR-GARCH Models.

Models	κ	θ	a_0	a_1	β_1	φ	Log-L
Panel B							
CIR-NGARCH	0.0003	4.7516	0.0003	0.0343	0.0490	-0.2243	8697.10
	[0.2846]	[0.0567]	[0.0000]	[0.0000]	[0.0000]	[0.0002]	
CIR-QGARCH	0.0002	5.1698	0.0003	0.0359	0.0500	-0.0009	8695.10
	[0.3558]	[0.1024]	[0.0000]	[0.0000]	[0.0000]	[0.0006]	

This table displays the maximum likelihood estimates of the Discrete Vasicek and CIR model, the CIR-GARCH with symmetric mean, and the CIR-AGARCH, CIR-NGARCH, CIR-QGARCH, CIR-TARCH, CIR-VGARCH and the CIR-GJR-GARCH models with asymmetric mean. The parameter estimates with asymptotic t-statistics in parentheses are presented for each model. The maximized log likelihood (Log-L) for the models are show to compare the explanatory power of these models. The daily data set is obtained from the Federal Reserve H.15 database and consists of the three and six months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms. The Parameters of the models are estimated according to the following econometric specifications.

$$r_t - r_{t-1} = +\kappa\left(\theta - r_{t-1}\right) + \varepsilon_t$$

$$E\left(\varepsilon_t \mid \Phi_{t-1}\right) = 0, \quad E\left(\varepsilon_t^2 \mid \Phi_{t-1}\right) = \sigma_t^2 = h_t r_{t-1}$$

CIR-GARCH : $h_t = a_0 + a_1 \varepsilon_{t-1}{}^2 + \beta_1 h_{t-1}$

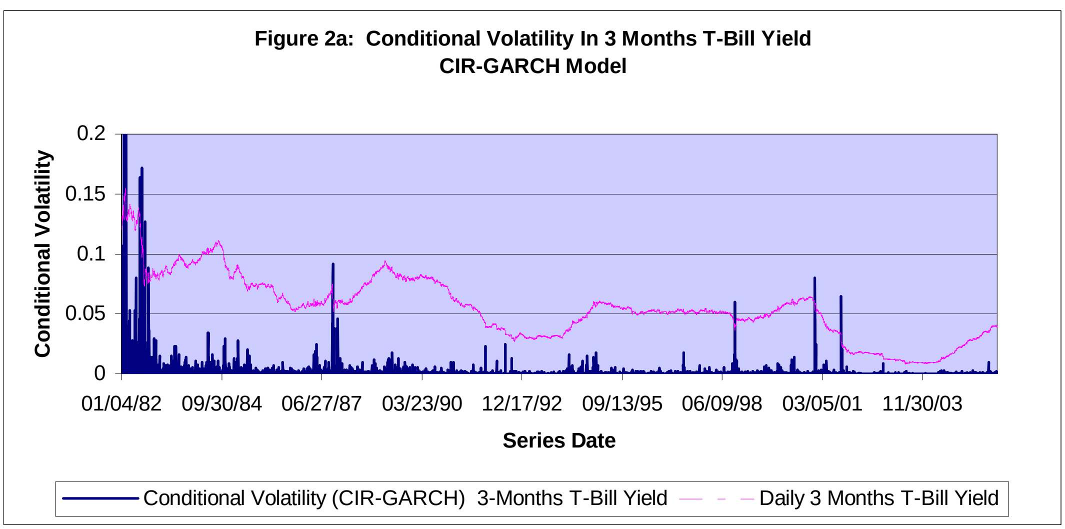
CIR-AGARCH: $h_t = a_0 + a_1\left(\varepsilon_{t-1} + \theta\right)^2 + \beta_1 h_{t-1}$

CIR-NGARCH : $h_t = a_0 + a_1\left(\varepsilon_{t-1} + \theta\sqrt{h_{t-1}}\right)^2 + \beta_1 h_{t-1}$

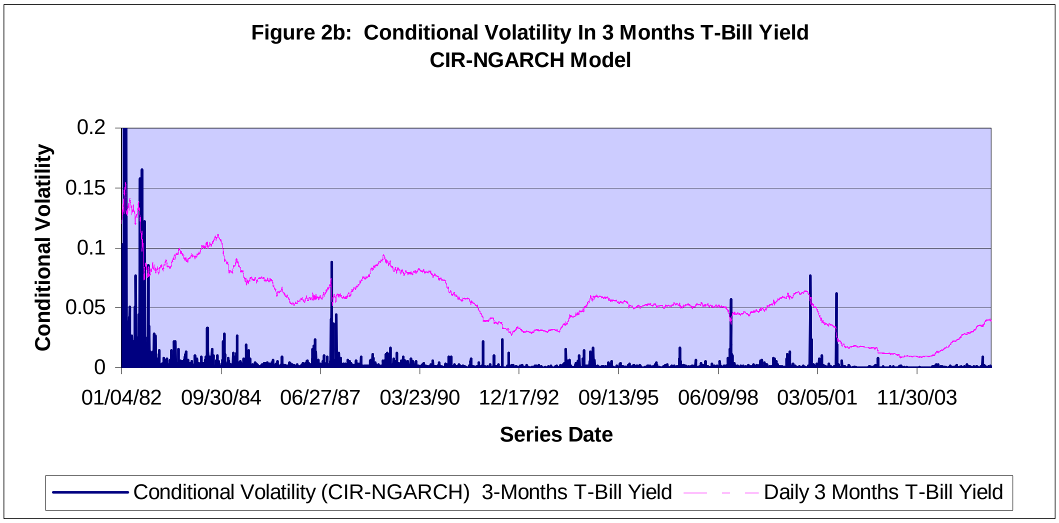
CIR-QGARCH: $h_t = a_0 + a_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} + \theta \varepsilon_{t-1}$

The estimates for the 3-month and the 6-months data are presented in Panel A, and Panel B respectively. The estimates of the coefficient of the speed of adjustment k, appears to suggest a weak mean reversion for both the 3-months and the 6-months yields. Similar evidence is also documented in BHK (1996)[8]. The coefficient of the long-run mean θ, for all GARCH specifications are statistically significant and range between 4.48% in the CIR-NGARCH model to 10.55% in the CIR-GARCH model for the 3-months T-bill yields. And 4.75% in the CIR-NGARCH to 13.91% in the CIR-GARCH model for the 6-months T-bill series. In addition, all the ARCH and GARCH coefficient for each model are statistically significant. Fig 2 graphs the evolution of the 3 months T-bill yield along with the conditional volatility estimated from the CIR-GARCH and the CIR-NGARCH models. Notice the sharp increase in volatility around periods with significant movement in the yields.

Figure 2: Conditional Second Moments of 3-months T-Bill yield. Time series plot of the predicted conditional volatility from the CIR-GARCH and CIR-NGARCH model.



[8] In BHK, κ is presented as β. Similar results can also be found in Bali (2000)



Since the all GARCH augmented CIR models subsume the CIR model, the likelihood ratio (LR) test would be a reliable method to compare the goodness-of-fit for these models relative to the benchmark CIR model. Results shown in Table 5 suggest that the CIR-NGARCH specification provides a slightly superior fit over the CIR-Level, the CIR-GARCH, and CIR-AGARCH and CIR-QGARCH models. The same pattern is also evident in the estimate of the 6-months T-bill series. Table 6 presents the Akaike (AIC) and the Bayesian Information Criteria (BIC) test of goodness-of-fit of each model. In both the AIC and the BIC, the likelihood is adjusted downwards by the number of parameters in each model. The results obtained from the information criterion test reinforce the findings in Table 5. For both the 3-months and the 6-months data, the CIR-GARCH, CIR-NGARCH, CIR-QGARCH, and CIR-AGARCH uphold their superiority of fit over the CIR-Levels only model. In addition the CIR-NGARCH possesses the lowest Akaike and Bayesian information criteria statistics. Thus highlighting an superior fit to the data that other GARCH specification estimated in this research. This result is

also consistent with Bali (2000)[9] test on 3-, 6-, and 12-months T-bill yields. In his research he also shows that that CIR-NGARCH specification provides the best fit to the data over every other model estimated in his paper using the likelihood ratio and forecasting test.

Table 5

Empirical Performance of Level-GARCH models

Likelihood Ratio Tests

	Likelihood Ratios	
	Panel A: 3-Months	Panel B: 6 Months
Interest Models		
CIR-GARCH	2060.5	1524.6
CIR-AGARCH	2081.5	1531.0
CIR-NGARCH	2082.5	1535.2
CIR-QGARCH	2082.3	1531.3

This table displays the likelihood ratio (LR) statistics for comparing the performance of the CIR-GARCH with symmetric mean, and the CIR-AGARCH, CIR-NGARCH, CIR-QGARCH models with asymmetric mean against the CIR-Levels model. The null hypotheses tested is that $a_0 = \sigma^2$, and $a_1 = \beta = \varphi = 0$. The LR statistics is calculated as LR=-2(Log-*L** - Log-*L*) where Log-*L is* the value of the log likelihood under the null hypothesis and Log-*L** is the value on the alternative. The daily data set is obtained from the Federal Reserve H.15 database and consists of the three and six months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms. The Critical values with one and two degrees of freedom at the 5% level of significance are $\chi(1,0.05)=3.84$ and $\chi(2,0.05)=5.99$ respectively

Despite the superiority of CIR-NGARCH over other level-GARCH and level models, a Kolmogorov-Smirnov test of normality reveals that significant excess kurtosis still remains in the residuals obtained from the model. The kurtosis estimated from the univariate test conducted on the residuals is 48.24 (not shown in the tables). This finding

[9] Bali, argues that asymmetric models, outperform symmetric models, because positive interest rate shocks cause higher volatility than negative interest rate shocks and that volatility is sensitive to stochastic volatility factors than to the level of the interest rate. Das (2002) sees this in a different light. He argues that volatilities are sensitive to levels of interest rate because, the results from his regime switching models show that when interest rates are higher, volatility is higher.

provides the crucial motivation for the econometrician to further explore whether the addition of jump dynamic to the model can resolve the problem. Having selected a benchmark level-GARCH model, I now focus my attention on the 3-months T-bill yields.

Table 6

Empirical Performance of Level-GARCH models (Information Criteria Tests)

	Panel A: 3-Months		Panel B: 6 Months	
Interest Models	BIC	AIC	BIC	AIC
Vasicek	-12833	-12834	-13350	-13370
CIR-GARCH	-17539	-17573	-17335	-17369
CIR-AGARCH	-17551	-17592	-17338	-17378
CIR-NGARCH	-17552	-17593	-17342	-17382
CIR-QGARCH	-17552	-17592	-17338	-17378

This table displays the Akaike and Bayesian Information Criteria for comparing the performance of the Vasicek, the CIR-GARCH with symmetric mean, and the CIR-AGARCH, CIR-NGARCH, CIR-QGARCH, CIR-TARCH, CIR-VGARCH and the CIR-GJR-GARCH models with asymmetric mean against the CIR-Levels model. The information criteria are adjusted for the difference in the number of parameters in each model. The daily data set is obtained from the Federal Reserve H.15 database and consists of the three months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms.

c. CIR-NGARCH-Jump Effect.

Naturally, a reliable starting point for augmenting the CIR-GARCH model specifications with jump dynamics is to incorporate jump coefficients into the CIR-NGARCH framework. Another good reason why the NGARCH-jump framework might be suitable for modeling the evolution of the short rate is its successful application to modeling the evolution of equity returns. Duan, Ritchken and Sun (2004) estimated a special case of NGARCH-jump model and conclude that the model specification significantly improved the fit of the historical time series of the S&P 500, in addition to explaining a significant proportion of the volatility smile noticed in options.

Estimating jump models is a daunting undertaking, because the addition of jumps to a smoothly evolving process creates discontinuities that most programs (Matlab, in this case) often find hard to accommodate in their optimization algorithm. As in any non-linear dynamic models, convergence of the model in question is above all driven by choices of the initial values in the estimation. This sometimes opens the result to estimation bias and local optimization problems. To guard against this, I followed a three-stage estimation step. First, I select starting values based on proximity to likely values of the designated coefficients using the CIR-GARCH and CIR-NGARCH model as the benchmark model. The jump models are estimated and the results are re-fed into the system to see if there is significant increase in the log-likelihood or in the estimates of the coefficients. If I suspect a significant change in these values, then the estimates are once again re-fed into the model and adjustments are made for any deviations. The model is re-estimated until there is no significant change in the log-likelihood and estimates of the coefficients.

Table 7 reports the maximum likelihood estimates for the coefficients, the asymptotic *p*-value and the log-likelihood for the CIR-GARCH-jump and the CIR-NGARCH-jump model under the assumption of constant jump intensity. I also include the estimates of the Vasicek-GARCH-jump model of Das(2002) for expository purposes[10]. The results show that there is a significant upshot from the introduction of jumps on the estimates of coefficients of the models. For example, the magnitude of the coefficient of the speed of adjustment κ changes from 0.0004 in the CIR-NGARCH

[10] Since Das (2002) employed the Vasicek-ARCH framework for Fed Funds Rates, I included the estimates of his model for readers interested in the comparing the performance the model when applied to the 3-month T-bill yields.

model to, 0.000414 in the Vasicek-GARCH-jump model, 0.000315 in the CIR-GARCH-jump model, and 0.000324 for the CIR-NGARCH-Jump model. Thus, it appears that the introduction of jumps into the model slightly dampens the speed at which the short rate reverts back to its long-run mean. One plausible reason for this could be that the addition of jump coefficients into the model allows the process to easily capture outliers in its evolution without necessarily requiring an increase in the speed of reversal back to its long-run mean. In addition, the coefficient of the long-run mean θ for the all the jump models are statistically significant and slightly higher than the estimate for the CIR-NGARCH model. In particular the estimate of θ are 3.94% for the Vasicek-GARCH-jump, 4.91% for the CIR-GARCH-jump and 4.73% for the CIR-NGARCH-jump model. The value of θ obtained from the CIR-GARCH-jump and CIR-NGARCH-jump are much higher than those obtained from the CIR-NGARCH only model and more realistic than the estimate of the CIR-GARCH model.

Next, I draw attention to the estimates of the jump coefficients. Table 7 shows that the average jump size μ is approximately 1.5 basis points and statistically significant in all models. Also, under the assumption of constant jump intensity, the parameter estimate of the expected number of jumps λ, is also statistically significant in all models. For the CIR-GARCH-jump model, the estimated value of λ are 0.0770. The estimates from the Vasicek-GARCH-jump and the CIR-NGARCH-jump are likewise in close proximity and are 0.0741 and 0.0679. Taking together the results point to the fact that jumps are inherent in the evolution of the short-term interest rate. Furthermore, we can also deduce from that on the average we can expect a jump in the 3-month T-bill yield every 13-14 days. I also estimated the average daily probability of at least a jump

occurring within that trading day, by taking the average of $P(n(t) \geq 1 | \Phi_{t-1}) = 1 - P(n(t) = 0 | \Phi_{t-1})$, which is readily available from the model estimation. These average daily probabilities of jumps are 0.074, 0.0671, and 0.0674 from the Vasicek-GARCH-jump, CIR-GARCH-jump and the CIR-NGARCH-jump model respectively. In subsequent sections, I relax the restriction in the intensity process by parametizing the ex-ante jump intensity to be conditional on ex-post observables of the data.

The most significant effect of introducing jumps coefficient into the level-GARCH specification is on the coefficients that captures conditional volatility. For example; across all three models, (Vasicek-GARCH-jump, CIR-GARCH-jump and the CIR-NGARCH-jump), Table 7 shows that jump introduction have significant consequence for the conditional volatility specification of the dynamics of the short rate. In particular the introduction of jumps tempers the persistence of conditional volatility by allowing the short rate process to evolve in tandem with the smoothly arriving information flow process.

Table 7

Maximum Likelihood Estimates of the Level-GARCH-Jump Model

3 Months T-Bill Yields

	Panel A	**Panel B**	**Panel C**
Models	Vasicek-GARCH Jump	CIR-GARCH-Jump	CIR-NGARCH-Jump
κ	4.14E-04	3.115E-04	3.247E-04
	[0.0153]	[0.0433]	[0.0445]
θ	(3.9421)	4.9089	4.7355
	0.0000	0.0000	[0.0000]
a_0	8.77E-06	2.20E-05	2.19E-05
	[0.0000]	[0.0000]	[0.0000]
a_1	0.0783	0.0121	0.0121
	[0.0000]	[0.0000]	[0.0000]
β_1	0.8929	0.8487	0.8489
	[0.0000]	[0.0000]	[0.0000]
φ			2.84E-05
			[0.49747]
μ_j	{0.0148}	0.0149	0.0148
	[0.0062]	[0.0244]	[0.0284]
δ	0.1140	0.1156	0.1185
	[0.0000]	[0.0000]	[0.0000]
λ	(0.0741)	0.0770	0.0679
	[0.0000]	[0.0000]	[0.0000]
Log L	9416.60	9430.00	9480.70

This table displays the maximum likelihood estimates of the CIR-NGARCH-JUMP model. The parameter estimates with the p-values in parentheses are presented for each model. The maximized log likelihood (Log-L) for the models are shown to compare the explanatory power of these models. The daily data set is obtained from the Federal Reserve H.15 database and consists of the three months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms. The Parameters of the models are estimated according to the following econometric specifications.

Model: $r_t = r_{t-1} + k(\theta - \frac{\lambda\mu_j}{k} - r_{t-1})\Delta t + \sqrt{h_t} r_{t-1}\Delta z_t + \sum_{i=1}^{n(t)} J(\mu_j, \delta^2)$

$h_t = a_0 + a_1\left(\varepsilon_{t-1} + \varphi\sqrt{h_{t-1}}\right)^2 + \beta_1 h_{t-1}$ and $z_t \sim NID(0,1)$

Where $a_0 > 0,\ 0 \le a_1 < 1, 0 \le a_2 < 1, a_1 + \beta_1 < 1,$ and $\psi > 0$

CIR Model: $\sigma_t^2 = h_t r_{t-1}$

Vasicek Model: $\sigma_t^2 = h_t$

Though, the coefficient of the constant a_0 and a_1 remain statistically significant in all the jump models and across all GARCH specifications, their magnitudes suffer a significant reduction in the jump models. For example, the estimate of the ARCH coefficient a_0 is reduced from 0.0003 in the CIR-NGARCH model, to approximately 0.00002 in the CIR-GARCH and NGARCH-jump models and 8.77E-06 in the Vasicek-GARCH-jump model. Similar trend can also be found in the estimate for the ARCH coefficient a_1. Its value is reduced from 0.0490 and 0.0479 in the CIR-GARCH and CIR-NGARCH respectively, to approximately 0.0121 in CIR-GARCH-jump and CIR-NGARCH-jump models.

In contrast the estimates of β_1 which also significant in all three jump models increases from 0.0492 and 0.0479 in the CIR-GARCH and CIR-NGARCH model respectively approximately 0.849 in both the CIR-GARCH-jump and the CIR-NGARCH-jump models. For the Vasicek-GARCH-jump model, the estimate of β_1 is 0.8929. Lastly, the coefficient that captures asymmetry in NGARCH model specification φ was found to be statistically insignificant.

In summary, these results suggest that the inclusion of jumps in these level-GARCH specifications eliminates the need for the GARCH process to capture the arrival of large discrete shocks to the levels, by means of a particularly high ARCH constant coefficient. In particular the conditional volatility in each time period under the jump specification is profoundly dependent on volatility in the immediate past period and not on some distance discrete shocks whose effect can easily be capture within the framework of the jump coefficients.

Table 8	
Empirical Performance of Level-GARCH-Jump models	
Likelihood Ratio Tests	
	Likelihood Ratios
Interest Models	
CIR-GARCH-Jump	1277.4
CIR-NGARCH-Jump	1357.4

This table displays the likelihood ratio statistics for comparing the performance of the CIR-GARCH, and CIR-GARCH-Jump, and the CIR-NGARCH and CIR-NGARCH Jump Model. The null hypothesis tested is $\lambda = \mu_j = \delta_j = 0$. The LR statistics is calculated as LR=-2(Log-*L** - Log-*L*) where Log-*L* is the value of the log likelihood under the null hypothesis and Log-*L** is the value on the alternative The daily data set is obtained from the Federal Reserve H.15 database and consists of the three months U.S. Treasury bill yields. The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms. The Critical values with one and two degrees of freedom at the 5% level of significance are $\chi(1, 0.05) = 3.84$ and $\chi(2, 0.05) = 5.99$ respectively

Table 9		
Empirical Performance of Level-GARCH-Jump models		
Information Criteria Tests		
	AIC	BIC
Interest Models		
Vasicek-GARCH-Jump	-18815	-18755
CIR-GARCH-Jump	-18843	-18783
CIR-NGARCH-Jump	-18943	-18883

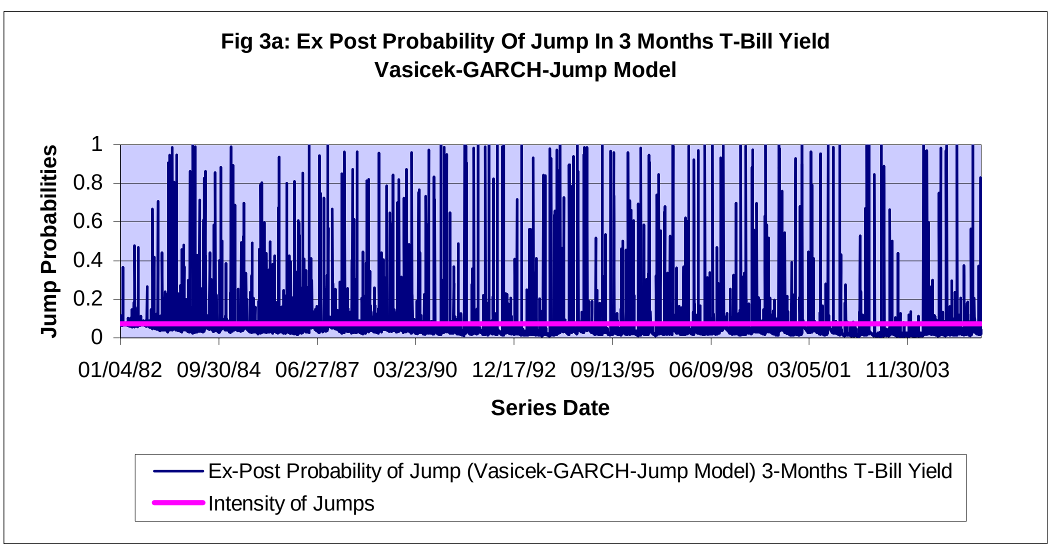
This table displays the Akaike and Bayesian Information Criteria for comparing the performance of CIR-NGARCH and the CIR-NGARCH-Jump models. The information criteria are adjusted for the difference in the number of parameters in each model. The daily data set is obtained from the Federal Reserve H.15 database and consists of the three months U.S. Treasury bill yields. The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms.

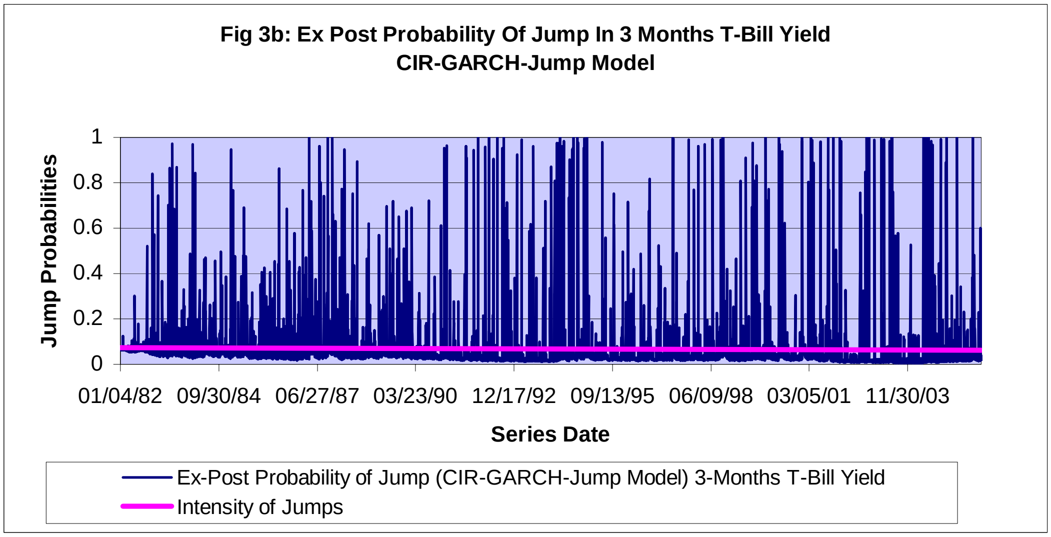
Next, I investigate the ability of the jump models present in Table 7 to capture relevant features of the data. First, I compare the suitability-of-fit of the CIR-GARCH-jump and the CIR-NGARCH-jump models against the, CIR-GARCH and CIR-NGARCH only model using the likelihood ratio test in Table 8. I then present the information criteria results in Tables 9. The result overwhelmingly supports the fact that jump enhanced Level-GARCH models supersedes the Level-GARCH only models. In Table 8,

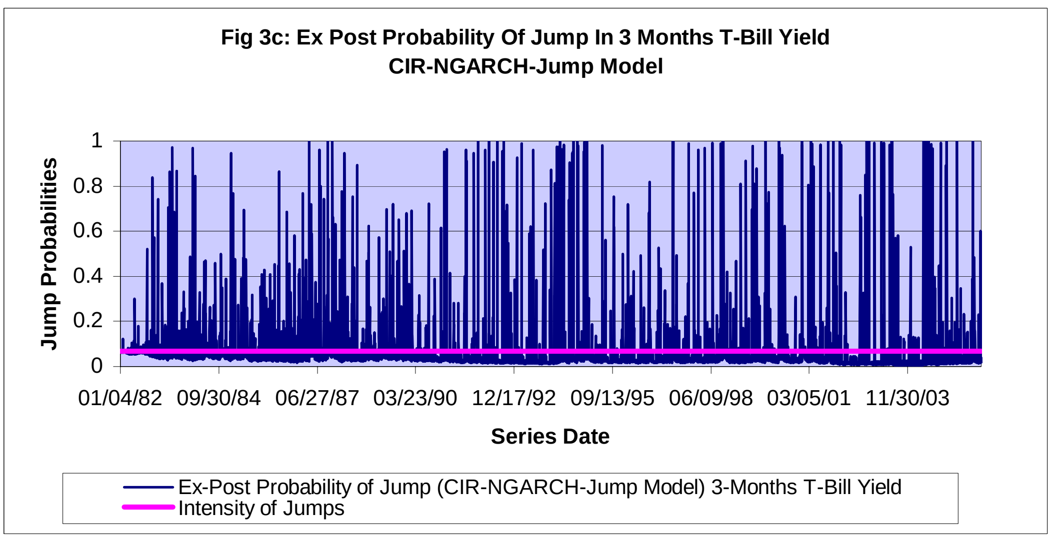
the likelihood ratio tests provide strong evidence that jump augmented models are more appropriate for modeling the salient characteristics of the T-bill yields. Furthermore, after controlling for the increased number of parameters in the jump models, by means of the information criterion tests, the result continues to point towards the superiority of jump models over level-GARCH models. In addition I can also deduce from the information criterion test that amongst the three jump models estimated in this subsection, the CIR-NGARCH-jump model appears to have the best fit to the data.

Fig 3a-3c graphs the ex post probability of jumps, along with the ex post assessment of the arrival jump for each trading day. Notice that the ex post probability of jumps on many trading days are clustered and are as high as 1, and in conflict the estimates of the constant jump intensity coefficient λ which implies that jumps will only arrive on every 13-14 consecutive trading days. This result highlights a possible source of misspecification in the constant intensity models and provides the motivation for the introduction of state dependencies in the jump dynamics.

Figure 3: Ex Post Probabilities of Jump in 3-months T-Bill Yield. Time series plot of the ex post probability of jump implied by the constant intensity jump augmented models.







d. Autoregressive Conditional Jump Intensity

In this subsection, explore the role of state dependencies in the arrival of jumps. It is possible that jump intensity can vary over time and can be clustered around specific new events such as change in macroeconomic conditions, or the incident of significant economic shocks such as war, terrorism and natural disasters. For example, the Federal

Reserve Bank of Sans Francisco Economic Letter mentioned that the spread on junk bonds skyrocketed by 200 basis points in the period following the September 11th attack on the World Trade Center. By following a conditional intensity process, the econometrician can allow the expected arrival rate of jump to be state dependent, thus provide a better description of instantaneous market condition. Das (2002) argued that it is possible that jump size distribution is positively skewed at low levels of Feds Funds rate and negatively skewed at high levels of Feds Funds rate; thus, the mean jump size of his model depends on the level of the Federal Funds rate. Eraker (2004) on the contrary suggest that the arrival of jumps might be more probable in high volatility periods than in low volatility periods. Maheu and McCurdy (2004) see the arrival of jumps in a different light and model the intensity as an autoregressive process.

Following Maheu and McCurdy (2004), I extend the CIR-NGARCH-jump model by incorporating autoregressive jump intensity coefficients that governs the likelihood of jumps occurring in each interval of time. The expectation of the number of jumps λ_t is now conditional on the information set Φ_{t-1} and assumed to be governed by the dynamic process parametized as:

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \gamma\xi_{t-1} \tag{34}$$

where,

$$\begin{aligned} \xi_{t-1} &\equiv \mathrm{E}\left[n_{t-1} \mid \Phi_{t-1}\right] - \lambda_{t-1} \\ &= \sum_{j=0}^{\infty} jP\left(n_{t-1} = j \mid \Phi_{t-1}\right) - \lambda_{t-1} \end{aligned} \tag{35}$$

λ_t is the conditional jump intensity at time t and is a function of past period conditional intensities and an innovation ξ_{t-1} to the econometricians forecast of n_{t-1} (the number of jumps in past period) as the information set is updated. The stationarity of the

innovation process $\left(\mathrm{E}\left[\xi_{t-1} \mid \Phi_{t-1}=0\right]\right)$ guarantees the existence of an unconditional mean of λ_t that is equal to:

$$\mathrm{E}\left[\lambda_t\right]=\frac{\lambda_0}{1-\rho} \tag{36}$$

We can obtain the conditional jump filter, $P\left(n_t=j \mid \Phi_{t-1}\right)$ which is the ex post distribution for the number of jumps by integrating out the number of jumps in terms of the observables following the specification below.

$$f(r_t \mid \Phi_{t-1})=\sum_{j=0}^{\infty} P(n(t)=j \mid \Phi_{t-1}) f(r_t \mid n(t)=j, \Phi_{t-1}) \tag{37}$$

$$P\left(n(t)=j \mid \Phi_{t-1}\right)=\frac{P\left(n(t)=j \mid \Phi_{t-1}\right) f(r_t \mid n(t)=j, \Phi_{t-1})}{f(r_t \mid \Phi_{t-1})} \tag{38}$$

Where,

$$f(r_t \mid n(t)=j, \Phi_{t-1})=\frac{1}{\sqrt{2\pi\left(\sigma_t^2+j\delta_j^2\right)}} \exp\left(-\frac{\left(r_t-r_{t-1}-k(\theta-r_{t-1})-\lambda\mu_j+j\mu_j\right)^2}{2\left(\sigma_t^2+j\delta_j^2\right)}\right) \tag{39}$$

Thus, the redefined the unconditional variance of under the assumption of time-varying jump intensity process now becomes:

$$Var(r_t)=\frac{\theta a_0}{1-a_1\varphi-\beta-a_1}+\frac{\lambda_0\left[\mu_j^2+\delta_j^2\right]}{1-\rho} \tag{40}$$

Notice that, a likelihood ratio test can also be constructed to test model misspecification for the case of the constant intensity such that $\lambda_t=\lambda$ and $\rho=\gamma=0$. Also notice, that the conditional density function is now slightly modified to accommodate the state dependencies of λ_t as shown below:

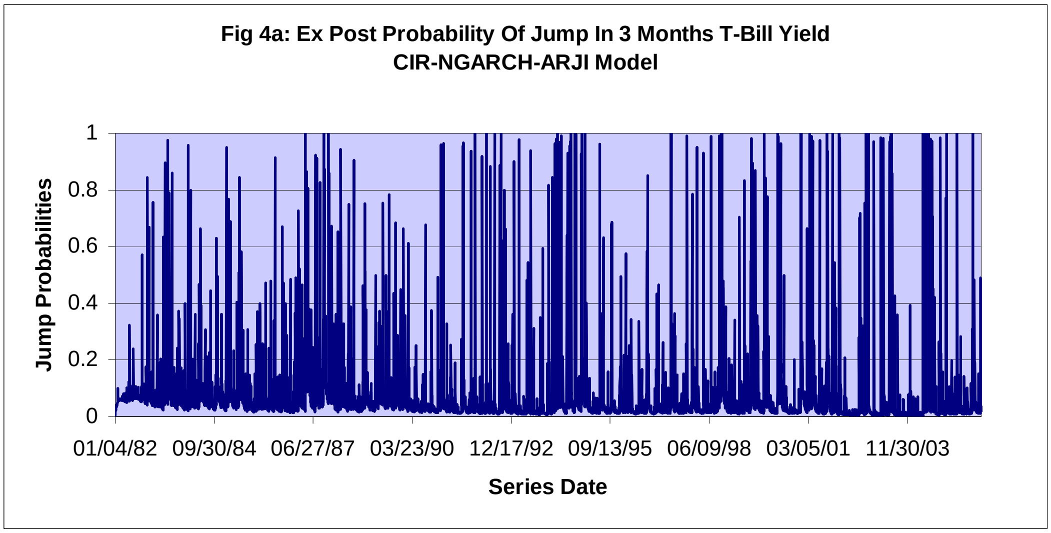
$$L(\Theta; r_1, \ldots r_T) = \sum_{t=2}^{T} \ln\left[f(r_t \mid \Phi_{t-1}) \right]$$

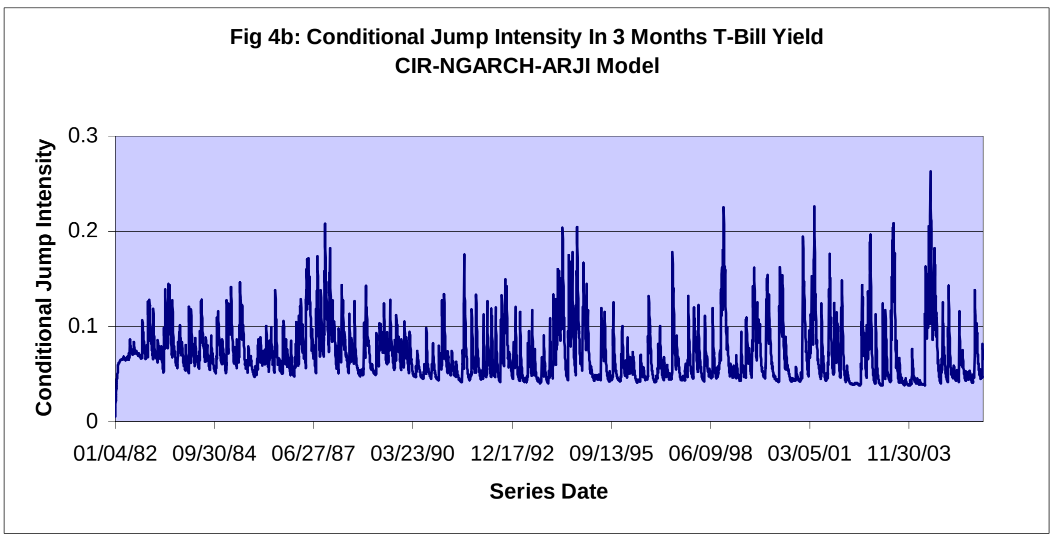
$$f(r_t \mid \Phi_{t-1}) = \sum_{j=0}^{\infty} \frac{\exp(-\lambda_t)\lambda_t^j}{j!} \cdot \frac{1}{\sqrt{2\pi\left(\sigma_t^2 + j\delta_j^2\right)}} \exp\left(-\frac{\left(r_t - r_{t-1} - k(\theta - r_{t-1}) - \lambda_t \mu_j + j\mu_j\right)^2}{2\left(\sigma_t^2 + j\delta_j^2\right)} \right) \quad (41)$$

In Table 10, I report the parameter estimates and *p*-values from the estimation of the CIR-NGARCH with autoregressive jump intensity (CIR-NGARCH-ARJI) model following the specification of equation (34). I also provide the log-likelihood, the likelihood ratio test and the information criteria test result at the bottom of the table to adjudge the improvement in the fit of the model over the constant intensity CIR-NGARCH-jump model. The results support the fact that that jumps are time varying and reflect instantaneous market conditions. The coefficients of all the autoregressive jump intensity parameters are significant at the 95% confidence level. In addition the log likelihood shows a slightly significant improvement in performance of model over the constant jump intensity models. By eliminating the restriction on the jump intensity coefficients, the model is better suited to track the evolution of volatility and the arrival of discrete economic shocks induced by unexpected information flow into the market.

The coefficient that captures the asymmetry in the volatility still remains insignificant in the CIR-NGARCH-ARJI model. A plausible reason for this phenomenon could be that the asymmetric effect captured in the CIR-NGARCH model might be due to the adjustment of the conditional variance process for large discrete shocks to the evolution of the short rate. Thus, when jump coefficients are introduced into the model, the importance of the asymmetric GARCH parameters diminishes completely.

Figure 4: Conditional Intensities of Jumps and Ex Post Probabilities of Jumps in 3-months T-Bill Yields. Time series plot of the conditional jump intensities and ex post probability implied by the CIR-NGARCH-ARJI model.





To surmise, the results of the estimation of the constant intensity and the autoregressive intensity model of the short rate robustly support the fact that jump intensities are time varying and are reflection of instantaneous market conditions. Fig 4a and 4b graph the ex-post probabilities of jumps and conditional intensities estimated from the CIR-NGARCH-ARJI model. Notice how closely the probability of jumps closely

tracks the conditional arrival of jumps.

Table 10

Maximum Likelihood Estimates of the CIR-NGARCH-ARJI Model

3 Months T-Bill Yields

Parameters	Estimates	Parameters	Estimates
κ	2.83E-04 [0.0670]	μ_j	0.0112 [0.0252]
θ	4.6376 [0.0000]	δ	0.1222 [0.0000]
a_0	2.07E-05 [0.0000]	λ	0.0052 [0.0188]
a_1	0.0099 [0.0000]	ρ	0.9279 [0.0000]
β_1	0.8654 [0.0000]	γ	0.0754 [0.0121]
φ	0.0001 [0.4927]		
Log L	9491.40	AIC	-18965.00
LR	21.4 [0.00]	BIC	-18905.00

This table displays the maximum likelihood estimates of the CIR-NGARCH-Autoregressive Conditional Jump Intensity model. The results are presented in this order. The parameters estimates are in topmost row, followed by the *P*-values are in lowest row for each coefficient. The maximized log likelihood (Log-L) for the model is shown to compare the explanatory power of the model. The likelihood ratio test, nest the CIR-NGARCH-Jump on the CIR-NGARCH-ARJI. The null tested is $\lambda_t = \lambda$ and $\rho = \gamma = 0$ **.** The daily data set is obtained from the Federal Reserve H.15 database and consists of the three months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms.

$$\text{Model}: \; r_t = r_{t-1} + k(\theta - \frac{\lambda\mu_j}{k} - r_{t-1})\Delta t + \sqrt{h_t}r_{t-1}\Delta z_t + \sum_{i=1}^{n(t)} J(\mu_j, \delta^2)$$

$$h_t = a_0 + a_1\left(\varepsilon_{t-1} + \varphi\sqrt{h_{t-1}}\right)^2 + \beta_1 h_{t-1}$$

$$\textit{Where } a_0 > 0,\; 0 \le a_1 < 1, 0 \le a_2 < 1, a_1 + \beta_1 < 1, \text{ and } \psi > 0$$

and

$$z_t \sim NID(0,1)$$

$$\sigma_t^2 = h_t r_{t-1}$$

$$\lambda_t = \lambda_0 + \rho\lambda_{t-1} + \gamma\xi_t$$

For example, the mean of the ex post probability of at least a jump per day implied by the CIR-NGARCH-ARJI model is 0.0708, which is in close proximity to the estimated mean jump intensity of 0.0713. The average kurtosis generated by the CIR-NGARCH-ARJI model is 24.45 (not reported in the tables but available upon request) which 24 units less than that generated value by the CIR-NGARCH model.

2.6 Specification

In this section, I proceed with a set of specification test to gauge the adequacy of jump augmented models and well as their superiority over Levels-GARCH models. In the first stage, I explore the features of the standardized residuals generated by the models. Following, Scruggs and Glabadanidis (2003), Maheu and McCurdy (2004), BHK, I analyzed the statistical properties of the squared standardized residuals generated by the jump augmented models. Table 11 presents the Ljung-Box (1978) test for serial correlation in the squared standardized residuals for one, two and three lags respectively. The results show no evidence of serial correlation remaining in the data.

Next, I applied the sign and size bias test introduced by Engle and Ng (1993). These tests examines if the squared normalized residuals can be predicted by some variables observed in the past that are not included in the volatility specification of the model. If these variables can predict the squared normalized residuals, the volatility structures employed in the jump models is misspecified. I conduct the sign and the size bias test jointly following the regression model specified below.

Table 11

Residual-Based Diagnostic of Level-GARCH-Jump Models

Models:		Vasicek GARCH Jump	CIR GARCH Jump	CIR NGARCH Jump	CIR NGARCH ARJI
Series					
	$Q^2(1)$	0.3680	0.2657	0.2149	0.2151
		[0.544]	[0.644]	[0.6430]	[0.6430]
	$Q^2(2)$	2.9837	3.3637	3.2679	3.2694
		0.2250	[0.186]	[0.1950]	[0.195]
	$Q^2(3)$	6.1026	5.6765	5.4660	5.4681
		0.1070	[0.1500]	[0.1410]	[0.1410]

This table reports the Ljung-Box portmanteau test, for serial correlation in the squared standardized residuals with one,two and three lags respectively. The residuals are derived from the Level-GARCH-jump models and are standardized as shown below.

Standardized Residuals :
$$\widehat{z}_t = \left[\widehat{z}_{r,t} \right] = \left(h_t^{-\frac{1}{2}} \cdot \widehat{\varepsilon}_t \right)$$

Q-Statistics:
$$Q^2(q) = N(N+2)\sum_{1}^{q} \frac{r(i;\hat{v}_t^2)}{(N-i)}$$

The Q-statistics are presented for each lag. The significance probability are shown in square brackets.The daily data set is obtained from the Federal Reserve H.15 database and consists of the three months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms.

$$v_t^2 = a + b_1 S_{t-1}^- + b_2 S_{t-1}^- \varepsilon_{t-1} + b_3 S_{t-1}^+ \varepsilon_{t-1} + e_t \tag{42}$$

where the sign indicators

$$\begin{aligned} S_{t-1}^- &= I\left(\varepsilon_{1,t} < 0\right) \\ S_{t-1}^+ &= I\left(\varepsilon_{1,t} > 0\right) \end{aligned} \tag{43}$$

The sign bias, negative size bias, and positive size bias test statistics are the t-statistics for the coefficients of the regression respectively. The joint test of all the coefficients is the Lagrange Multiplier test for adding the three variables in the variance equation under the null that the specified GARCH process for each model is adequate. If the volatility model is accurate, then all the coefficients of the regression in equation (42)

$\left(b_1 = b_2 = b_3 = 0\right)$ and e_t is i.i.d. Both the t-statistics and the LM test have standard limiting distributions. In particular the LM test is chi-square distributed and is estimated by taking the product of the unadjusted R^2 and the number of observations. The results of this test can be found in Table 12.

For all the models, the negative sign bias and positive size bias tests do not reject the null. On the contrary, the negative size bias test rejects the null for all four models. This suggests that large negative values of ε_{t-1} shocks to the short rate affects volatility much more that small values. The joint test of the null is rejected in all the models. Taking all these finding into perspective, I can surmise that the result points to evidence of asymmetry in the evolution of the T-bill yield that is not accounted for in any of the models. This raises the question of the appropriateness of the GARCH and NGARCH framework when modeling the evolution of the short rate with the level-GARCH-jumps configuration and creates another motivation for future research into the dynamics of the short-rate.

2.7 Conclusions

I investigate the suitability of a new class of jump-GARCH augmented Cox, Ingersoll and Ross (1985) model to model the dynamics of the U.S. short-term interest rate by focusing on the ability of the model to capture the salient characteristic of the short rate such as leverage effect and the leptokurtosis. Using the 3-months Treasury bill yield, I examined whether the Vasicek-GARCH-jump, CIR-GARCH-jump, CIR-NGARCH-jump, CIR-NGARCH-Autoregressive jump model can rightfully model the evolution of the 3-months T-bill yield.

Table 12

Sign and Size Bias Test of Levels-GARCH-Jump Models

Models:		**Vasicek GARCH Jump**	**CIR GARCH Jump**	**CIR NGARCH Jump**	**CIR NGARCH ARJI**
	b_1	0.9937	0.0684	0.0862	0.0576
		[0.544]	[0.9454]	[0.9313]	[0.9541]
	b_2	-2.5578	-2.9541	-2.9416	-2.9610
		[0.0106]	[0.0031]	[0.0033]	[0.0031]
	b_3	1.0018	1.3505	1.3530	1.3419
		[0.3165]	[0.1769]	[0.1761]	[0.1797]
$LM = T \cdot R^2$		11.14 [0.01]	11.09 [0.01]	11.13 [0.01]	10.99 [0.01]

This table reports the sign and size bias test to examine the specification of each jump augmented level-GARCH model. Econometric specification for the test is show below:

$$v_t^2 = a + b_1 S_{t-1}^- + b_2 S_{t-1}^- \varepsilon_{t-1} + b_3 S_{t-1}^+ \varepsilon_{t-1} + e_t$$

The null tested is that: $\left(b_1 = b_2 = b_3 = 0\right)$

The significance probability are shown in square brackets.The daily data set is obtained from the Federal Reserve H.15 database and consists of the three months U.S. Treasury bill yields. Yields are on actively traded non-inflation-indexed issues adjusted to constant maturities and are quoted on investment basis The sample period is from 01/04/1982 to 12/15/2005. There are 5987 observations for each series in the data set. The rates are expressed in annualized percentage terms.

I find that the CIR-NGARCH with autoregressive Jump model supersede other GARCH augmentations to the CIR model in terms of the ability to describe the time-series of the U.S. short-term interest rate. In addition, I also show that the yield on short-term default-free bond responds asymmetrically to information arrival. The negative size bias test reveals that the squared standardize residuals observed from the NGARCH-ARJI model along with Vasicek-GARCH-jump, CIR-GARCH-jump, and CIR-NGARCH-jump, are sensitive to high level of negative innovation to the 3-months T-bill yield. Sadly asymmetry coefficients in the CIR-NGARCH-jump specifications were found to be insignificant in the estimated models. The implies that while evidence of leverage does

effect exist in the time series of the 3-month T-bill yield, when jumps are introduced into the CIR-NGARCH model, the NGARCH specification fails to adequately account for the evolution of the conditional volatility of the short-rate. CIR-NGARCH-ARJI model along with Vasicek-GARCH-jump, CIR-GARCH-jump, and CIR-NGARCH-jump all fail to account for this effect. Therefore all the models fail to adequately explain the time-varying dynamics of the 3-months T-bill yield.

I also find that jumps in short-term interest rate are time varying in way that has not been highlighted in existing literatures. I document evidence of state dependencies in the arrival of jumps in the 3-months T-Bill yield. I find that jump arrival is positively related to the level of uncertainty in the market and is more likely to arrive if there was a jump in the immediate past period. Lastly, I also find that jumps have implications for conditional volatility. The introduction of jumps parameters into the model severely dampens the GARCH effect in the dynamics of the short-rate. The estimates of the GARCH coefficient obtained from the jump enhanced level-GARCH model for the 3-months T-bill yields is significantly lower than those obtained in the level-GARCH models.

Chapter 3

Jumps and the Comovement of Equity and Bond Return

3.1 Introduction

The fact that bond and equities market move together is well documented in the literatures. This comovement has also been revealed to fluctuate considerably over short horizons, together with periods of negative correlation as mentioned in Scruggs and Glabadanidis (2003), Gulko, (2002) and Li (2002). A plausible rationalization for the time-varying comovement between equity and bond market is the theoretical posit of asset pricing theory. Since the asset prices represent the discounted value of all future benefits streams accruing to that asset, then both equity and bond prices and their return should vary in tandem with the change in the value of the discounting parameter. Although the influence of nominal interest rate on bond return is lucid, its effect on equity return is unclear. For example, if all increase in the discount rate through inflation and real activity is reflected in the dividend, then the equity prices (thus, return) will be unaffected by changes in nominal interest rate. Furthermore, Ilmanen (2003) argued that in period of higher inflation expectation, the changes in the discount rate might overshadow the changes in dividend, such that the effect of increase in nominal interest rate on equity prices will be negative.

While financial markets tend to move together, in periods of escalating risk, rational investors seek safer investments. The anticipation or realization of adverse outcome in one market motivates rational savers to seek indemnity against imminent or future losses. Barro (2005) suggested that increase in the demand for safe asset during global conflict or economic turmoil could be the motivating factor for the sharp decline in real interest

rates during such periods. Periods where an increase in risk in one market is followed by an increase in demand for safe asset is described as a flight-to-quality episode in financial literatures. Connolly, Stivers and Sun (2005) delineate flight-to-quality as an incident of the decoupling between two financial markets that are closely related when there is a crisis in one or both markets. As extension of this assertion to the equity-bond relationship, I describe a flight-to-quality episode as a period where a change in a set of information variables that proxy financial distress in equity(bond) market leads to a decrease in the correlations between the return on equities and the bond market indices. As a contrast to flight-to-quality, a contagion period will then be characterized as a significant increase in correlations amongst markets in period of major decline in one or more market (Forbes and Rigobon, 2002).

In this essay, my objectives are as follows: I apply the Multivariate-GARCH-jump method to scrutinize the time-varying comovement between equities and bond return in period of significant uncertainty. In particular, I examine whether the time varying correlation between equities and the bond market is conditional on a set of information variables and market conditions in a multivariate GARCH framework. Taking that equity and bond returns are positively correlated over time, I study the impact of jump intensities on the time-varying correlation between the equity and bond markets.

This essay contributes to the literature in two distinct ways. First, it distinguishes itself from previous studies by explicitly modeling the relationship between equity and bonds using the MGARCH-jump structure. By incorporating jumps in a MGARCH framework, the econometric model is better equipped to account for leptokurtosis that are driven by local jump shocks, and jump shocks that originates from other markets or

assets. In addition, the inclusion of correlated jump shocks will also provide the econometrician a better description of the comovement of equity and bonds under normal and unusual news events. Secondly, in a slight deviation from existing literatures, I study the issue of flight-to-quality by analyzing the contemporaneous relationship between the arrival of jumps, and other proxies of market uncertainty, on the time-varying correlation between equity and bonds.

There are a few reasons why public policy makers would be interested in a study such as this. For example, since changes in the equity-bond relation represents a shift in the investment opportunity set, public policy makers can garner immense information about economic agent's perception of future economic conditions by examining the equity-bond relation. In addition, this study also sheds light on the how economic shocks are propagated between financial markets. Do equity and bond markets respond in similar manner to the same discrete economic shocks? Are shocks propagated only through the diffusion process from the equity (bond) market to bond (equity) market? Can discrete economic shocks pertaining to the equity (bond) market create simultaneous reaction in both equity and bond markets, or are the shocks propagated from one market to the other. All these are interesting questions which this study attempts to provide answers to.

For private and institutional investors, the study of the issue of flight-to-quality will provide important clarification about investors' behavior in normal and tumultuous market conditions. Since investors require a lucid description of the correlation between financial assets when pricing securities, managing risks, and rebalancing their portfolio, the findings of this study, would be of immense contribution in illuminating one of the

many black-boxes in financial investment.

Multivariate GARCH (MGARCH) models are veritable framework for examining the covariance matrix of asset returns. Prominent amongst these MGARCH models are the VECH (VEC) model of Bollerslev, Engle and Wooldridge (1998), the constant correlation (CCOR) model of Bollerslev (1990), the factor ARCH (FARCH) model of Engle, Ng, and Rothschild (1990), the BEKK model of Engle and Kroner (1995), the General Dynamic Covariance (GDC) of Kroner and Ng (1998), and the dynamic conditional correlation multivariate GARCH (DCCMGARCH) of Engle, (2002). Many researchers have combined jumps with various univariate GARCH processes to examine the volatility of equity and bond markets. For example, Bates (1996) found evidence of jumps in stock indices, Maheu and McCurdy (2004), found similar evidence of time dependence in jump intensities for both individual stock and indices using an autoregressive conditional jump intensity parameterization. For literatures presenting evidence of jumps in short-term interest rate, see Das (2002), Andersen, Benzoni and Lund (1997), Attari (2000) and Pazzesi (2005). Recently, Chan (2003 and 2004) scrutinized the existence of correlated jump in the foreign exchange market using a bivariate jump model on daily data of German Mark against the British Pound and Japanese Yen against the US dollar. He finds that foreign currency return correlations are driven by both normal innovations and simultaneous jumps innovations.

Since the evidence that equity and bond returns exhibit jump phenomenon have been corroborated in a considerable number of literatures, the challenge for the econometrician now becomes how this documented incidence of leptokurtosis can be captured in the framework of a tractable multivariate econometric model. Even a seemingly small

discontinuous change due to the arrival of information that creates large infrequent shock in one market can have severe impact on its covariance structure with other markets. A new and promising technique introduced Chan (2003&2004) explores the role of bivariate jump dynamics on the comovement of returns in the foreign exchange market. This insight by Chan (2003&2004) provides a reliable framework for my investigation of the time-varying volatility linkage between the bonds and the equity markets.

Following Chan (2004) I estimate an augmented Multivariate GARCH-Jump model of equity and bond index return using NYSE/AMEX/NASDAQ value-weighted return and a portfolio of 1-,2-,3-,5-,7- and 10-years constant maturity Treasury bill returns. The estimation yields the time-varying correlation between equity and T-bond returns, their independent and correlated jump intensities, and their conditional second moments. Next, I test whether the time-varying correlation is conditional on: the arrival of jumps in the bond and equity market, the level of short-term nominal interest rate, the return shocks to both the bond and equity market indices, and a measure of market expectations of near-term volatility.

I uncover a number of important results regarding the time-varying comovement between equity and bond returns. First, I find that jump-augmented multivariate GARCH model provides a better description of the matrix of conditional second moments of equity and bonds over the sample period. Jumps have significant consequence on the comovement of the equity and bond market. In particular, when jumps coefficients are introduced into the model, the conditional covariance between equity and bond will now be driven, not only by the covariance between the normal shocks, but also by the independent and correlated jump shocks.

Secondly, jump-augmented multivariate GARCH model provides an interesting perspective of the equity-bond relations. The introduction of jump coefficients alleviates the serial correlation problems which GARCH models are prone to. The effect of non-lasting shocks is easily accounted for, thus alleviating the GARCH parameters' burden of carrying instantaneous shocks for longer period. There are many reasons why jump-GARCH models are well suited for modeling the time series of financial assets. For example, Maheu and McCurdy (2004) posit that news impact resulting jump innovations can have a different feedback on expected volatility than news impact related to normal innovations. Significant news information associated with jump may be quickly incorporated into current prices and have smaller effect on expected volatility. I find that when jumps are introduced into the BEKK multivariate GARCH model, GARCH effect is significantly reduced.

Thirdly, though equity and bonds, historically exhibit unconditional correlation, their conditional correlation fluctuates considerably on a day-to-day basis over the sample period. In particular, the implied correlation between equity and bonds is sensitive to the arrival of normal and unusual news to either the equity or the bond market. Conditional correlations reduce with the anticipation or realization of adverse conditions in either the equities or bond market. Investors observed market conditions and respond by flying to quality when the state of affairs in equity market becomes significantly more risky than in the Treasuries' market.

3.2 Review of Literatures

A sizeable number of literatures have examined the conditional comovement between equity and bond return in a multivariate GARCH framework with promising

results. For example, Scruggs and Glabadanidis (2003) tested a variant of Merton's ICAPM, in which excess returns on an equity index and a long-term government bond portfolio proxy for risk factors. Using interest rate related variables such as short-term T-bill yields and term spread as a proxy for interest rate risk factor they developed an asymmetric dynamic covariance model to examine the manner in which shocks and volatility are transmitted between stock and bonds. They find that the volatility in the equities market is asymmetrically affected by both equity and bond return. In addition, they also revealed that, while in general, the conditional correlation between bonds and stock return is positive, they are period in time when this correlation was negative (late 1950s to early 1960s). Goeji and Maquering (2004) find similar evidence of asymmetric effect in the conditional heteroscedasticity in the covariance between equity and bond market returns. Fleming, Kirby and Ostdiek (1998) examine the nature of volatility linkage between the equity, bond and money markets and found strong evidence of linkages amongst the three. Connolly, Stivers and Sun (2005) examined whether time variation in the comovement of daily equity and Treasury bonds return can be linked to measures of stock market uncertainty, and found evidence of flight-to-quality. In particular, they find that bond returns tend to be low (high) relative to stock returns during days when implied volatility increases (decreases) substantially and during days when stock turnover is unexpectedly high (low). Gulko (2002) finds evidence in favor of decoupling of stock and bonds in period of extreme stock market volatility. David and Veronesi (2004) show that uncertainty about macroeconomic factors such as expected inflation possess significant ability to predict the conditional covariance and correlation of stock and bond returns. Gebhardt, Hvidkjaer and Swaminathan (2005), examined the

interaction between the momentum in the return of equities and corporate bonds. They find significant evidence of momentum spillover from equities to investment grade corporate bonds of the same firms. Kim, Moshirian and Wu (2006) examined whether, the time-varying correlation between government bonds and equity return over the past decade has been affected by the implementation of the European Monetary Union (EMU) in a bivariate EGARCH model. They find that the introduction of EMU has "Granger" caused the equities market to splinter from the bond market within Europe but not outside. Addona and Kind (2006) find that volatility of real interest rate increases the correlation between equities and bond returns.

a. Multivariate GARCH Models

For exposition purposes, I summarize the comprehensive survey of Multivariate GARCH literatures conducted by Bauwens, Laurent and Rombouts (2006)[11]. Consider a vector stochastic process $\{Y_t\}$ defined as:

$$Y_t = \mu + \varepsilon_t \tag{44}$$

Where Y_t is a $N \times 1$ vector consisting of equity return and bond return $(i.e.\ N = 2)$ with a constant mean μ, and a random error vector ε_t. Assume that,

$$\varepsilon_t = H_t^{1/2} z_t, \tag{45}$$

$$z_t \quad iid,\ \mathrm{E}(z_t) = 0,\ Var(z_t) = I_N \tag{46}$$

[11] The summary I present in this essay is restricted to areas that are linked to my essay. A comprehensive survey of important development in Multivariate GARCH literatures and models are available in Bauwens, Laurent and Rombouts (2006).

H_t is a positive definite conditional variance matrix of (Y_t) such that

$$Var(Y_t \mid \Phi_{t-1}) = Var_{t-1}(\varepsilon_t) = H_t^{1/2} Var_{t-1}(z_t)\left(H_t^{1/2}\right)' \tag{47}$$

where Φ_{t-1} is the information available at time $t-1$ and $\Sigma = \mathrm{E}(H_t)$.

A generalization of H_t presented in Bollerslev, Engle, and Wooldridge (1988) is defined as $H_t = (h_{ijt})$ which is a linear function of the lagged squared errors, cross products of errors and lagged values of each element of H_t. This definition, popularly referred to as the VEC (p,q) model is shown below as[12]:

$$h_t = c + \sum_{j=1}^{q} A_j \eta_{t-j} + \sum_{j=1}^{p} G_j h_{t-j} \tag{48}$$

where

$$h_t = vech(H_t) \tag{49}$$

$$\eta_t = vech\left(\varepsilon_t \varepsilon_t'\right) \tag{50}$$

c is a $N^* \times 1$ vector of parameters of the order $[N^* = N(N+1/2)]$ and $vech(.)$ is an operator that stacks the lower triangular portion of a $N \times N$ matrix as a $N^* \times 1$ vector. A_j and G_j are square matrices of order $N^* \times N^*$.

Following the bivariate nature of this research, consider a bivariate VEC(1,1) model of equity and bond index return defined as:

[12] VEC acronym is used interchangeably here, first as an acronym for the VECH model of Bollerslev et al (1988), and second as an operator that stacks a $(m \times n)$ matrix into a $(mn \times 1)$ vector. It has no similarity with Vector Error Correction Model.

$$h_t = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon^2_{2,t-1} \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} \tag{51}$$

We note that the conditional variance of equity return is a function of not only its own lag, but also of the lagged conditional variance of the bond return as well as the lagged conditional covariance between equity return and bond return. In a matrix format, H_t can be expressed as:

$$H_t = C + \left(I_N \otimes \varepsilon'_{t-1}\right)\tilde{A}\left(I_N \otimes \varepsilon_{t-1}\right) + E_{t-2}\left(I_N \otimes \varepsilon'_{t-1}\right)\tilde{G}\left(I_N \otimes \varepsilon_{t-1}\right) \tag{52}$$

Hence a sufficient condition that H_t will be positive definite is that $C \geq 0, A \geq 0, G \geq 0$ with at least one strict inequality.

One shortcoming of the VEC model specification is the number of parameters to estimate. For example, for the bivariate VEC (1,1) specified above, the number of parameters to be estimated is 21. To overcome this challenge, other researchers have imposed simplifying assumptions on the structure of H_t. Bollerslev, Engle, and Wooldridge (1988) reduced the number of parameters to be estimated from 21 to 9 by suggesting a diagonal VEC (DVEC) in which the A_j and G_j matrices are assumed to be diagonal. In this structure, the variance will depend only on own past squared errors, and covariance its own past cross-products of errors. Nevertheless, there still exists a challenge in guaranteeing that H_t will be positive definite in the VEC representation. Engle and Kroner (1995) (Here after BEKK) proposed a parametization (BEKK (p,q,K) model) that easily imposes the positive definiteness of H_t, and remains well-accepted in multivariate GARCH literatures till today. The BEKK (p,q,K) model is defined as:

$$H_t = C^{*\prime} C^* + \sum_{k=1}^{K} \sum_{j=1}^{q} A_{jk}^* \varepsilon_{t-j} \varepsilon_{t-j}' A_{jk}^{*\prime} + \sum_{k=1}^{K} \sum_{j=1}^{p} G_{jk}^* H_{t-j} G_{jk}^* \tag{53}$$

Where C^*, A_{jk}^* and G_{jk}^* are $(N \times N)$ matrices but C^* is upper triangular. The model assumes the conditional variance matrix of Y_t is determined by the outer product matrices of the vector of innovations in the previous period. The positive definiteness of H_t is guaranteed as long as $H_0 \geq 0$. For example, a bivariate form of the BEKK (p,q,K) can be presented in matrices form as:

$$\begin{aligned}
\begin{bmatrix} h_{11,t} & h_{21,t} \\ h_{12,t} & h_{22,t} \end{bmatrix} &= \begin{bmatrix} c_{11}^* & 0 \\ c_{21}^* & c_{22}^* \end{bmatrix} \begin{bmatrix} c_{11}^* & c_{21}^* \\ 0 & c_{22}^* \end{bmatrix} \\
&+ \begin{bmatrix} a_{11}^* & a_{12}^* \\ a_{21}^* & a_{22}^* \end{bmatrix}' \begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11}^* & a_{12}^* \\ a_{21}^* & a_{22}^* \end{bmatrix} \\
&+ \begin{bmatrix} g_{11}^* & g_{12}^* \\ g_{21}^* & g_{22}^* \end{bmatrix}' \begin{bmatrix} h_{11,t-1} & h_{21,t-1} \\ h_{12,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} g_{11}^* & g_{12}^* \\ g_{21}^* & g_{22}^* \end{bmatrix}
\end{aligned} \tag{54}$$

For $K = 1$, the model requires the estimation of 11 parameters as against the 21 in the bivariate VEC model. We can expand equation (54) to obtain the linear form of the bivariate BEKK model as shown below:

$$h_{11,t} = c_{11} + a_{11}^{*2}\varepsilon_{1,t-1}^2 + 2a_{11}^* a_{21}^* \varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{21}^{*2}\varepsilon_{2,t-1}^2 + g_{11}^{*2} h_{11,t-1} + 2g_{11}^* g_{21}^* h_{21,t-1} + g_{21}^{*2} h_{22,t-1} \tag{55}$$

$$\begin{aligned}
h_{21,t} &= c_{21} + a_{11}^* a_{12}^* \varepsilon_{1,t-1}^2 + \left(a_{11}^* a_{22}^* + a_{12}^* a_{21}^*\right)\varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{22}^* a_{21}^* \varepsilon_{2,t-1}^2 + g_{11}^* g_{12}^* h_{11,t-1} \\
&\quad + \left(g_{11}^* g_{22}^* + g_{21}^* g_{21}^*\right) h_{21,t-1} + g_{22}^* g_{21}^* h_{22,t-1}
\end{aligned} \tag{56}$$

$$h_{22,t} = c_{22} + a_{22}^{*2}\varepsilon_{2,t-1}^2 + 2a_{11}^* a_{21}^* \varepsilon_{1,t-1}\varepsilon_{2,t-1} + a_{12}^{*2}\varepsilon_{1,t-1}^2 + g_{22}^{*2} h_{22,t-1} + 2g_{11}^* g_{21}^* h_{21,t-1} + g_{21}^{*2} h_{11,t-1} \tag{57}$$

The stationarity of the BEKK (1, 1, 1) model is guaranteed under the structure,

$$H_t = \Omega + A^{*\prime}\varepsilon_{t-1}\varepsilon'_{t-1}A^* + G^{*\prime}H_{t-1}G^* \tag{58}$$

where $\Omega = C^{*\prime}C^*$. H_t can also be represented under the properties of the *vec* operator as

$$vec\left(H_t\right) = vec\ \left(\Omega\right) + \left(A^{*\prime} \otimes A^*\right)vec\left(\varepsilon_{t-1}\varepsilon'_{t-1}\right) + \left(G^{*\prime} \otimes G^*\right)vec\left(H_{t-1}\right) \tag{59}$$

where $vec(.)$ is an operator that stacks a $(m \times n)$ matrix into a $(mn \times 1)$ vector. H_t is covariance stationary if the eigen values of $\left(A^* \otimes A^*\right) + \left(G^* \otimes G\right)$ is less than 1 in the modulus, such that

$$vec\left(\Sigma\right) = \mathrm{E}\left(vec\ H_t\right) = \left(I_{N^2} - \left(A^{*\prime} \otimes A^*\right)' - \left(G^{*\prime} \otimes G\right)'\right)^{-1} vec\left(\Omega\right). \tag{60}$$

Furthermore imposing a diagonal structure on A^*_{jk} and G^*_{jk} (diagonal BEKK model) reduces the number of parameters to 7 as against 9 in the DVEC model.

Kroner and Ng (1998) provides a comprehensive summary of the strength and weaknesses of the 4 dominant multivariate GARCH (the VECH model of Bollerslev, Engle, and Wooldridge (1988), the constant correlation model of Bollerslev (1990), the FARCH model of Engle, Ng, Rothschild (1990), and the BEKK model of Engle and Kroner (1995)). They also introduced a General Dynamic Covariance (GDC) model that nests many of the existing models as special cases and show that the choice of multivariate GARCH models can lead to substantially different conclusions in application when forecasting the dynamics of the variance matrices. For example, in comparison to other models, the constant correlation model does not permit shocks in one asset to directly affect the other, since the model is assumed to be driven by a time-invariant correlation. Lastly, Kroner and Ng surmise by showing that all these models are misspecified in the sense that none of these models capture the well-documented

incidence of leverage effect in financial data.

3.3 Model Specification

I apply a modification of Chan (2004) model to investigate the covariance between equity and bond return and focus on the nature of the volatility linkage between the fixed income market and equities market. To shed light on the primary motivation for this study, consider an augmentation to Equation (44), and an extended version of Chan (2003).

$$Y_t = \alpha + \beta \cdot X_{t-1} + \varepsilon_t + q_t \tag{61}$$[13]

where

$$Y_t = \begin{bmatrix} \Delta s_t \\ \Delta b_t \end{bmatrix}, X_t = \begin{bmatrix} \Delta s_{t-1}, \Delta b_{t-1} \\ \Delta b_{t-1}, \Delta s_{t-1} \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{bt} \end{bmatrix} \tag{62}$$

$$\varepsilon_t = H_t^{1/2} z_t, \tag{63}$$

$$z_t \quad iid,\ \mathrm{E}(z_t) = 0,\ \ Var(z_t) = I_N,\ H_t^{1/2} \bullet H_t^{1/2\prime} = H_t \tag{64}$$

Where Y_t and X_t is a 2×1 vector consisting of equity return and bond return of constant mean α, a random diffusion error vector ε_t which, follows the specification in equations (44) - (45). Let a jump component q_t follow the framework of Chan (2003&2004), which defines an independent Poisson process embodying a random jump $J(\mu_i, \delta_i^2)$ size drawn from an independent normal distribution, and whose arrival is governed by a Poisson arrival frequency parameter $n_{it} \in \{0,1,2,....\}$ over the

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[13] Equities and Bonds may be endogenous in the economy and their interaction might lead to a wrong conclusion if this problem is not accurately accounted for. One way to mitigate this problem is the selection of an augmented VAR-GARCH modeling approach.

interval $(t, t-1)$.

a. Jump Component

The jump component q_t is given as:

$$q_t = \begin{bmatrix} q_{st} \\ q_{bt} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n_{1t}} J(\mu_s, \delta_s^2) \ -\lambda_{s,t}\mu_s \\ \sum_{j=1}^{n_{2t}} J(\mu_b, \delta_r^2) \ -\lambda_{b,t}\mu_b \end{bmatrix} \tag{65}$$

where

$$q_{i,t} = \sum_{i=1}^{n(t)} J(\mu_i, \delta_i^2) \ -\lambda_{i,t}\mu_i, \qquad J(\mu \ , \delta_i^2) \sim NID(\mu_i, \delta_i^2) \tag{66}$$

This model follows the univariate adaptation of Maheu and McCurdy (2004). The jump innovation q_t is the sum of the stochastic jumps $J(\mu_i, \delta_i^2)$ arriving at a time interval $\Delta t = 1$ and adjusted by $\mathrm{E}\left[\sum_{i=1}^{n_{1t}} J(\mu_i, \delta_i^2)|\Phi_{\text{t-1}}\right] = \lambda_i$, such that $\mathrm{E}\left[q_t \mid \Phi_{t-1}\right] = 0$. q_t is bivariate normal with zero mean and variance covariance matrix $\Theta_{ijk,t}$. λ_i is the jump intensity per period.

Since each jump size is governed by a normal distribution with a constant mean and variance, I assume these mean and variance parameters remain the same across time but differ between equity returns and bond returns. Following Chan (2003), I also specify that two discrete counting variables $n_{s,t}$ and $n_{b,t}$ governing the arrival of jumps are constructed via three independent Poisson variables, namely $n_{s,t}^*, n_{b,t}^*, n_{sb,t}^*$ such that the correlated jump counters are defined as,

$$n_{s,t} = n_{s,t}^* + n_{sb,t}^* \text{ and } n_{b,t} = n_{b,t}^* + n_{sb,t}^* \tag{67}$$

By definition, $n_{s,t}$ is a function of independently generated jump $n_{s,t}^*$ and

correlated jump $n^*_{sb,t}$. When $n^*_{sb,t} = 0$, then there is no jump propagation from the one market to the other. The joint probability density function for the three independent Poisson variables is:

$$P\left(n^*_{s,t} = i, n^*_{b,t} = j, n^*_{sb,t} = k \mid \Phi_{t-1}\right) = \frac{\exp(-\lambda_{s,t})\lambda^i_{s,t}}{i!} \cdot \frac{\exp(-\lambda_{b,t})\lambda^j_{b,t}}{j!} \cdot \frac{\exp(-\lambda_{sb,t})\lambda^k_{sb,t}}{k!} \quad (68)$$

b. Autoregressive Conditional Jump Intensity (ARJI)

It is possible that jumps intensity can vary over time and can be clustered around specific new events such as change in macroeconomic conditions or the event with significant economic shocks such war, terrorism and natural disasters or an inherent part of the market condition or how arrival of information is imputed in equities and bond return. For example, Das (2002) models the arrival of jump intensity to depend on the day of the week. He argues that jumps may be more likely to occur on Monday since the release of pent up information over the weekend may drive up the possibility of a large change in interest rates, or on Wednesdays or Thursdays because of option expiration or on Fridays when last minute trade may create excess volatility.

Eraker (2004) allows jump intensity to depend on volatility. Following similar argument as Das (2002), Eraker suggested that it is plausible that jumps are more likely in periods of high volatility than period of low volatility. McCurdy and Maheu (2004) on the other hand parametize the jump intensity to be a function of past period jump intensities and a martingale intensity residual, which represent the change in the econometrician's conditional forecast of the number of jumps per period as the information set is updated. I explore the specification of Maheu and McCurdy (2004) by

parametizing the jump intensity as follows:

$$\lambda_{i,t} = \lambda_{i0} + \rho_i \lambda_{i,t-1} + \gamma_i \xi_{i,t-1} \tag{69}$$

Where

$$\begin{aligned} \xi_{i,t-1} &\equiv \mathrm{E}\left[n_{i,t-1} \mid \Phi_{t-1} \right] - \lambda_{i,t-1} \\ &= \sum_{j=0}^{\infty} jP\left(n_{i,t-1} = j \mid \Phi_{t-1} \right) - \lambda_{i,t-1} \end{aligned} \tag{70}$$

The conditional jump intensity at time t is related to past period conditional intensities and as well the innovation to the econometrician's forecast of n_{t-1} as the information set is updated. We can obtain our conditional jump filter $P\left(n_t^* = j \mid \Phi_{t-1}\right)$, which is the ex post distribution for the number jumps by integrating out the number of jumps in terms of the observables, from the conditional density of Y_t (in (73)) given the j number of jumps occur, and the information set Φ_{t-1} as shown in equation (71-73)

$$f(Y_t \mid \Phi_{t-1}) = \sum_{j=0}^{\infty} P(n_t^* = j \mid \Phi_{t-1}) f(Y_t \mid X_t, n_t^* = j, \Phi_{t-1}) \tag{71}$$

$$P\left(n_t^* = j \mid \Phi_{t-1}\right) = \frac{P\left(n_t^* = j \mid \Phi_{t-1}\right) f(Y_t \mid X_t, n_t^* = j, \Phi_{t-1})}{f(Y_t \mid \Phi_{t-1})}, j = 0,1,2,... \tag{72}$$

Furthermore, Chan (2003) and Maheu and McCurdy (2004) also show that, giving i and j jumps per period, we can readily derived the density of Y_t conditional on k correlated jump and I present as shown below:

$$f(Y_t \mid n_{sb,t}^* = k, \Phi_{t-1}) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} f(r_t \mid n_{s,t}^* = i, n_{b,t}^* = j, n_{sb,t}^* = k, \Phi_{t-1}) \tag{73}$$

where

$$f(Y_t \mid X_{t-1}, n_{s,t}^* = i, n_{b,t}^* = j, n_{sb,t}^* = k, \Phi_{t-1}) = \frac{1}{2\pi^{N/2}} \left| H_{ijk,t} \right|^{-\frac{1}{2}} \exp\left[\varepsilon_{ijk,t}' H_{ijk,t}^{-1} \varepsilon_{ijk,t} \right] \tag{74}$$

and $\varepsilon_{ijk,t}$, is the normal innovation term consisting of diffusion and jump error and is shown below as:

$$\begin{aligned}\varepsilon_{ijk,t} &= Y_t - \alpha - \beta \cdot X_{t-1} - q_{ijk,t} \\ &= \begin{bmatrix} \Delta s_t - \alpha_s - \beta_s \cdot \Delta s_{t-1} - \beta_{sb} \cdot \Delta b_{t-1} - (i+k)\mu_s + (\lambda_{s,t} + \lambda_{sb,t})\mu_s \\ \Delta b_t - \alpha_b - \beta_b \cdot \Delta b_{t-1} - \beta_{bs} \cdot \Delta s_{t-1} - (j+k)\mu_r + (\lambda_{b,t} + \lambda_{sb,t})\mu_r \end{bmatrix}\end{aligned} \tag{75}$$

Under the assumption of a normal distribution, $H_{ijk,t}$ can be separated into two parts: the variance-covariance matrix for the normal innovations $\tilde{H}_t$ (the dynamics of which, I will present in the next subsection), and the variance covariance matrix for the jump components $\Theta_{ijk,t}$. Furthermore, under the assumption that the jump correlation is constant across contemporaneous equation and zero across time: The variance covariance matrix for the jump components conditional upon i and j will then be:

$$\Theta_{ijk,t} = \begin{bmatrix} (i+k)\delta_s^2 & \rho_{sb}\sqrt{(i+k)(j+k)\delta_s\delta_b} \\ \rho_{sb}\sqrt{(i+k)(j+k)\delta_s\delta_b} & (j+k)\delta_b^2 \end{bmatrix} \tag{76}$$

c. Time-Varying Volatility Component

Estimating multivariate-GARCH models often becomes impractical as number of assets in the return matrix increases. For example, a 20-asset VECH model will have 630 parameters. Similarly a 20-asset BEKK (p,q,K) will have 1010 parameters to estimate. Even with success in estimation, the result obtained in these analyses is often devoid of a tractable economic value. The large numbers of parameters to estimate in both BEKK and VECH often eliminate its applicability to large asset scenarios that are common in the financial or portfolio analysis.

For our case, I sidestep this problem by modeling $\tilde{H}_t$, to follow bivariate BEKK (p, q, K) model as specified below:

$$\tilde{H}_t = C^{*\prime}C^* + \sum_{k=1}^{K}\sum_{j=1}^{q} A_k^* \varepsilon_{t-j}\varepsilon_{t-j}' A_k^{*\prime} + \sum_{k=1}^{K}\sum_{j=1}^{p} G_k^* H_{t-j} G_k^* \tag{77}$$

where A_k^* and G_k^* are diagonal matrices. In all the number of parameters to be estimated for $\tilde{H}_t$ in the $p = 1, q = 1, N = 2,$ and $K = 1$ framework will be 11, rather 21 in the original VEC(p,q) model. Thus, the variance covariance matrix $H_{ijk,t}$ will always be positive definite as along as $\tilde{H}_t$ is positive definite. Since by construction, $\Theta_{ijk,t}$ is guaranteed to be positive definite, $H_{ijk,t}$ as the sum of two positive definite matrix will be positive definite as well.

3.4 Estimation.

Two popular methods of estimating models including jump dynamics using the maximum likelihood approach are; (1) use a mixture of Poisson-normal distributions and (2) use a discrete mixture of a discrete sum of two normal distributions using the Bernoulli distribution as an approximation of the Poisson process (Das 2002). The Bernoulli approximation is easier and quicker to estimate at the expense of accuracy. On the other hand, the Poisson-normal distribution, which is more appropriate, is more daunting to estimate and often requires careful selection of the point of truncation in some cases to allow for convergence. I define the Poisson-normal density function for the estimation of (60) and construct a log-likelihood function defined as:

$$\left\{ L(\Theta; Y_1, \ldots Y_T) = \sum_{t=1}^{T} \ln\left[Q(Y_t \mid \Phi_{t-1}) \right] \right\} \tag{78}$$

with

$$Q(Y_t \mid \Phi_{t-1}) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} f\left(Y_t \mid X_{t-1}, n_{s,t}^{*}=i, n_{b,t}^{*}=j, n_{sb,t}^{*}=k, \Phi_{t\text{-}1}\right) \times P\left(n_{s,t}^{*}=i, n_{b,t}^{*}=j, n_{sb,t}^{*}=k \mid \Phi_{t-1}\right) \tag{79}$$

Estimating jump models can be a daunting challenge, because the addition of jumps to a smoothly evolving process creates discontinuities that most programs often find hard to accommodate in their optimization algorithm. In addition, the estimation of the conditional density in terms of the observables requires that we integrate out the number of jumps for each realization of the time series through an infinite summation of densities. This requires a tremendous amount of computing resource and time for each run of the model. For example, estimating the model using 2499 observations of equity and bond returns takes about seven hours on a Pentium-IV class computer, with 64-bit processor and 1-Gigabit of RAM.

As in any non-linear dynamic models, convergence of the model in question will primarily be driven by choices of the initial values in the estimation. This sometimes opens the result to estimation bias and local optimization problems. To guard against this incident, I select the starting the values of the MGARCH-Jump model from a prior estimation of the MGARCH-only and Bivariate-jump with constant covariance matrix-only model.

3.5 Data Description

This essay is based on equities, bonds, short-term interest rate, daily Chicago Board Options Exchange's (CBOE) Volatility Index (VIX) data from January, 1995 to

December, 2004. For the equity market index return data, I obtained daily observation of the returns on the value-weighted NYSE/AMEX/NASDAQ index from CRSP, which is readily available on Fama-French website. Next, I obtain daily observation of the Federal Funds rates published by the Federal Reserve Bank and available in the Federal Reserve H.15 database. To obtain the bond index returns, I analyze the 1-, 2-, 3-, 5-, 7-and 10-years U.S. Treasury bonds constant maturity yields also available on the Federal Reserve website. Following, Connolly, Stivers and Sun (2005), and Jones, Lamount and Lumsdaine (1998), I construct the implied return on a portfolio of 1-,2-,3-,5-,7-and 10-years U.S. Treasury bonds from their constant maturity yields. Finally, I obtained the Chicago Board Options Exchange's (CBOE) Volatility Index (VIX)[14]. The VIX is a generally accepted measure of market expectations of near-term volatility conveyed by S&P 500 stock index option prices.

For each observation, a corresponding Feds Fund rate, NYSE/AMEX/NASDAQ return, Bond index return and VIX are matched for that trading day. Where an observation is missing from any series, the observation for that day is deleted. Luckily, majority of these missing observations are days in which the Fed Funds and Treasuries market are closed due to Federal Government holidays and the stock market is open. In all there are 2,499 daily observations for each data series. I report the descriptive statistics pertaining to my series in Table 1. Returns are reported using raw returns, likewise, the Fed Fund rates are also in raw values.

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[14] VIX is constructed by using the Black-Scholes option pricing model to calculate implied volatilities for eight different OEX option series so that, at any given time, it represents the implied volatility of a hypothetical at-the-money OEX option with exactly 30 days to expiration. These are combined to create an overall measure of the market's expectations for near term volatility.

Table 1

Summary Statistics

Panel A: Sample Moments

	Equity	Bond	Term Bond	Term Bond	Funds Rates	Federal Funds	VIX
Mean	0.0005	-0.0001	-0.0001	0.0000	0.0415	0.0008	0.2153
Std. Dev	0.0112	0.0224	0.0248	0.0211	0.0196	0.0498	0.0642
Min	-0.0663	-0.1024	-0.1355	-0.0930	0.0086	-0.4131	0.1036
Max	0.0532	0.1040	0.1387	0.0798	0.0780	0.8655	0.4574
Skewness	-0.1087	-0.1800	-0.2417	-0.1484	-0.5816	3.0361	0.7422
Kurtosis	3.1721	1.9536	3.3093	1.1240	-1.3579	52.9612	0.5984

Panel B: Correlation Matrix

Equity	1.0000						
Bond	0.0035	1.0000					
Short-Term Bond	0.0112	0.9795	1.0000				
Long-Term Bond	-0.0056	0.9715	0.9038	1.0000			
Federal Funds Rates	0.0106	0.0177	0.0192	0.0150	1.0000		
Change in Federal Fu	0.0273	0.0279	-0.0053	0.0655	0.0423	1.0000	
VIX	-0.1380	-0.0071	-0.0081	-0.0056	-0.0954	-0.0270	1.0000

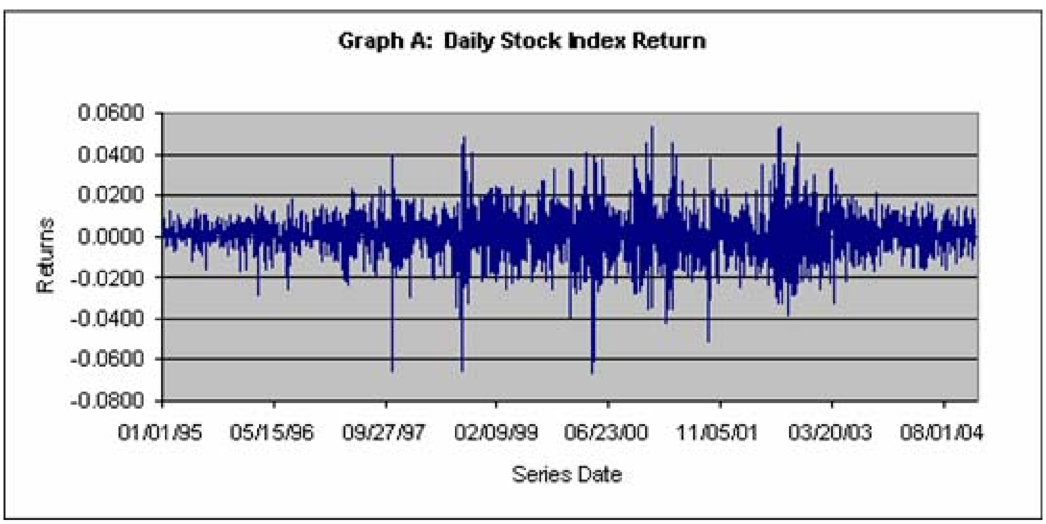
Equity Return consist of daily NYSE/AMEX/NASDAQ return from CRSP and is readily available on Fama-French Website. Bond return is the implied return on a portfolio of 1-,2-,3-,5-,7-, and 10 years constant maturity yield Treasury Bonds quoted on investment basis. The Short-Term Bonds is the implied return on a portfolio of 1-,2-, and 3 years constant maturity yield Treasury Bonds, while the Long-Term Bonds is the implied return on a portfolio of 5-,7-,and 10 years constant maturity yield Treasry Bonds. These yields along with the daily Federal Fund rates are available in the Federal Reserve H.15 database on the Federal Reserve Website. VIX is the Chicago Board Options Exchange's volatility index in annualized percentage, standard deviation units. All values are in raw terms. Std. Dev. denotes standard deviations. Panel B report the correlation matrix between these

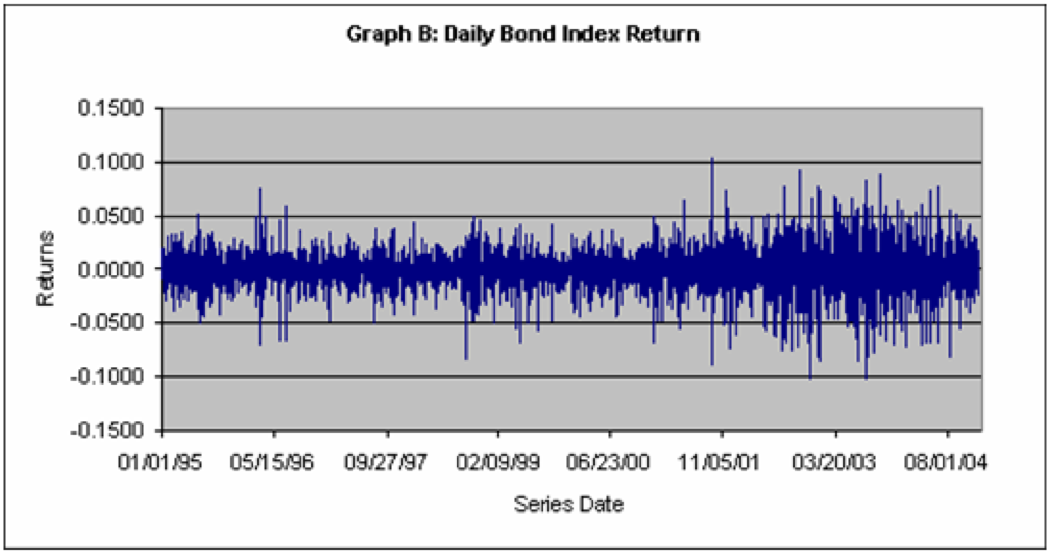
http://www.cboe.com/micro/vix/faq.aspx

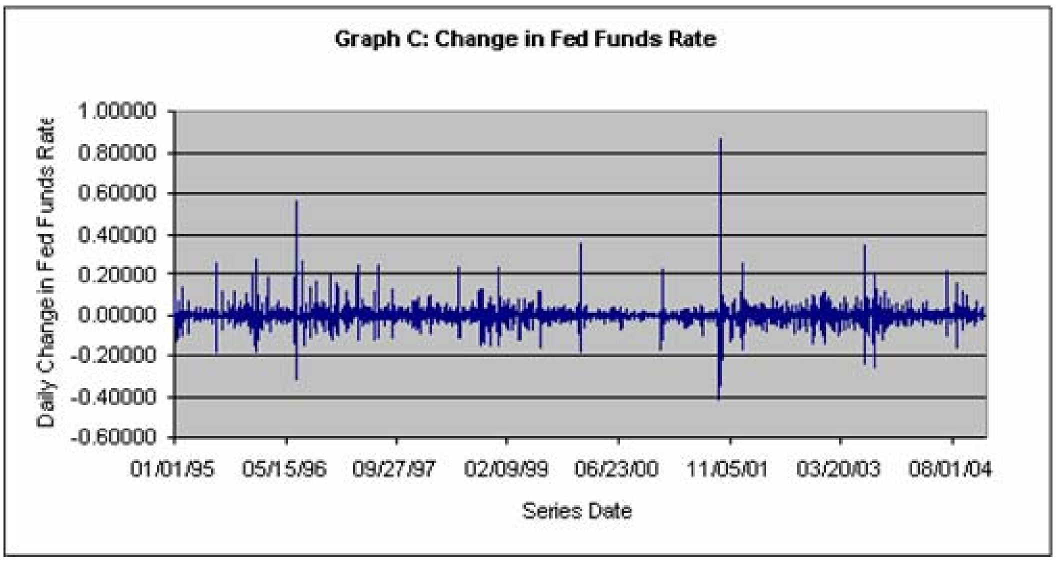
In Panel A, I report the univariate statistics for the entire sample period, while the in Panel B, I report the unconditional correlation between the variables. In general, the unconditional correlation between stock and bonds returns and Fed Fund rate are all positive. The correlation between stock and bond is 0.0035, the correlation between bond and Fed Fund rate is 0.0177 and the correlation between stock and Fed Funds is 0.0106. On the other hand, the unconditional correlation between VIX and stock, bonds, and Fed Fund rates are all negative.

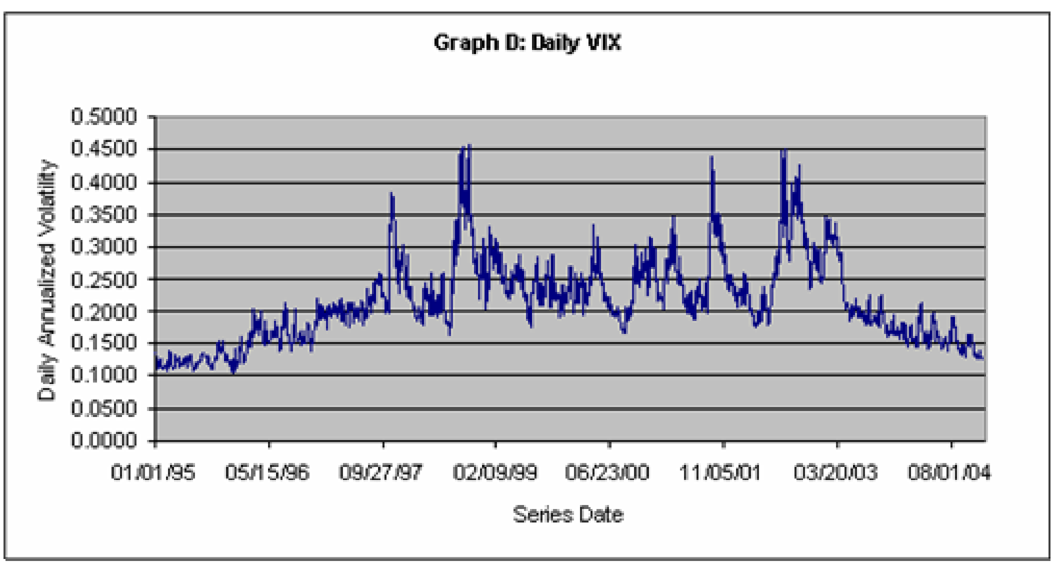
Fig 1, Graph A and B shows the time plot of daily stock and bond returns, while Graph C and D presents the daily change in the Federal Funds rate and daily VIX.

Figure 1: Time Series of Equity and Bond Index Returns, Fed Funds Rate, and the VIX.




Graph B: Daily Bond Index Return
0.1500
0.1000
0.0500
0.0000
-0.0500
-0.1000
-0.1500
Returns
01/01/95
05/15/96
09/27/97
02/09/99
06/23/00
11/05/01
03/20/03
08/01/04
Series Date


Graph C: Change in Fed Funds Rate
1.00000
0.80000
0.60000
0.40000
0.20000
0.00000
-0.20000
-0.40000
-0.60000
Daily Change in Fed Funds Rate
01/01/95
05/15/96
09/27/97
02/09/99
06/23/00
11/05/01
03/20/03
08/01/04
Series Date


Graph D: Daily VIX
0.5000
0.4500
0.4000
0.3500
0.3000
0.2500
0.2000
0.1500
0.1000
0.0500
0.0000
Daily Annualized Volatility
01/01/95
05/15/96
09/27/97
02/09/99
06/23/00
11/05/01
03/20/03
08/01/04
Series Date

3.6 Results.

a. MGARJI Model.

Here, I discuss the result of the estimation of the multivariate-GARCH-Jump model with autoregressive jump intensity (MGARJI) applied on the equity and bond index return data. Table 2 reports the maximum likelihood parameter estimates, asymptotic *p*-values and the Log-Likelihood of the BEKK-multivariate GARCH (MGARCH) only model, multivariate GARCH with constant jump intensity (MGARCH-J) model, and multivariate GARCH with autoregressive jump intensity (MGARCH-ARJI) model for the equity and bond index return series. The specifications for all the models are presented in the Table.

The MGARCH only model requires the estimation of 17 parameters, which includes 6 mean equation coefficients and 11 MGARCH coefficients. The MGARCH-J model introduces 8 new parameters to the MGARCH-only model to capture the dynamics of the jumps in the data. All together, the MGARCH-J has 25 parameters. Lastly, the MGARCH-ARJI model integrates 6 more coefficients to the MGARCH-J model. The 6 new coefficients capture the autoregressive dynamics of the jump intensities. It includes 3 coefficients to capture the persistence of jump intensities and 3 coefficients to capture the adjustment of the econometrician forecast of the number of jumps in past period as the information set is updated.

The likelihood ratio test reported at the bottom of the table lends credence to the superiority of the MGARCH-ARJI model over the MGARCH-J and the MGARCH-only model. Thus, the discussion in this section will focus only on the results obtained from the MGARCH-ARJI model.

The estimation of the MGARCH-ARJI produces 31 parameters. In all, 24 of the 31 coefficients are significant at the 5% confidence level. For intuitive purposes, I highlight the coefficients and their t-stats (in square brackets) in this subsection. For the mean equation, the estimate of the constant term for the equity returns ($\alpha_s = 0.0005, [2.15]$) is significant, the same coefficient for the bond return ($\alpha_b = 0.0001, [0.38]$) is insignificant. In addition, the estimate of the autoregressor for equity return $(\beta_s = 0.0261, [1.31])$ is insignificant, which is in contrast to the significant result obtained for the estimate of the autoregressor for bond return $(\beta_b = -0.2629, [-7.86])$. Another interesting result is impact of the cross-regressors. The estimates of the past period bond return's feedback into equity return $(\beta_{bs} = 0.0093, [0.87])$ is insignificant, while the past period equity return's feedback into present period bond return $(\beta_{sb} = -0.4429, [-24.95])$ is negative and significant. Jones, Lamont and Lumsdaine (1998) and Connolly, Stivers and Sun (2005) both reported evidence of serial dependence in the bond index return. The result obtained here suggest that equity returns appears to follow a random walk, while in contrast bond return appears to be serially dependent on both own lagged return and lagged equity return in its evolution.

Next, I discuss the parameter estimates of the MGARCH coefficients of the model. One weakness of the BEKK method of modeling conditional volatility is the difficulty in deriving an intuitive economic interpretation of the each parameters estimates of the model. This is due to the fact that most of the BEKK parameters are introduced into the MGARCH components of the model in a quadratic form. However, we can garner immense information by evaluating the overall implication of the results of

the model using graphical analyses. Nevertheless, Table 2 shows that 8 of the 11 parameters of the MGARCH components of the model are significant and all fall within the expected region to guarantee covariance stationarity. Time plot of the conditional variances, covariance and correlations are provided in Figure 2. These graphs show evidence of time variation in the variance and covariance of equity and bond returns. Though the unconditional covariance between equity and bond is positive, Graph C shows that the conditional comovement between equity and bond returns varies considerably over the sample period.

This movement would have implications for the implied daily conditional correlations between equity and bonds as shown in Figure 3. Graph A. It is no surprise that the time plot of daily implied correlations between equity and bond appears to be extremely volatile. Since our estimates of implied daily conditional correlations was obtained by deriving the daily volatilities weighted estimate of the covariance. As volatility and co-volatilities evolve, we would expect their estimated products to exhibit significant variation over time. This enables the econometrician to fully grasp the nature of the instantaneous inter-relationships between the equity and the bond markets. It is not uncommon for the financial press to report the incidence of a rally in bond (equity) market with a simultaneous sink in the equity (bond) market. A recent example is the new release on the 16th of June, 2006. The Cable News Network (CNN) Money magazine cites a rally in the equities' market as the bond markets sinks due investor's expectation of another rates hike by the Feds.

Table 2

MGARCH-ARJI Parameter Estimates

	MGARCH	MGARCH-J	MGARCH-ARJI
α_s	0.0005 [0.0000]	0.0005 [0.0000]	0.0005 [0.0000]
α_b	0.0001 [0.35318]	0.0001 [0.35318]	0.0001 [0.35318]
β_s	0.0261 [0.19167]	0.0261 [0.19167]	0.0261 [0.19167]
β_b	-0.2629 [0.0000]	-0.2629 [0.0000]	-0.2629 [0.0000]
β_{bs}	0.0093 [0.19167]	0.0093 [0.19167]	0.0093 [0.19167]
β_{sb}	-0.4429 [0.0000]	-0.4429 [0.0000]	-0.4429 [0.0000]
c_{11}	0.0011 [0.0000]	0.0004 [0.00028]	0.0010 [0.0000]
c_{21}	0.0006 [0.03388]	0.0006 [0.02232]	0.0003 [0.00000]
c_{22}	0.0018 [0.00000]	0.0009 [0.00014]	0.0036 [0.00000]
a_{11}	0.2993 [0.00000]	0.7380 [0.00000]	0.4772 [0.00020]
a_{12}	0.0146 [0.36573]	0.0125 0.2198	1.11E-06 [0.27660]
a_{21}	0.0032 [0.34918]	0.0041 [0.20227]	9.37E-06 [0.49823]
a_{22}	0.1944 [0.00000]	0.2934 [0.00000]	0.5656 [0.00000]
g_{11}	0.9519 [0.00000]	0.4847 [0.00000]	0.2724 [0.00000]
g_{12}	0.0020 [0.44376]	0.0028 [0.46565]	1.11E-06 [0.44171]

Table 2 Continued

MGARCH-ARJI Parameter Estimates

	MGARCH	MGARCH-J	MGARCH-ARJI
g_{21}	0.0016 [0.19157]	0.0013 [0.41201]	9.37E-06 [0.14018]
g_{22}	0.9761 [0.00110]	0.9304 [0.00000]	0.1402 [0.00000]
μ_s		0.0017 [0.03939]	0.0034 [0.00000]
μ_b		0.0018 [0.17279]	0.0018 [0.00000]
δ_s^2		8.89E-09 [0.44240]	1.06E-07 [0.87912]
ρ_{sb}		-0.0208 [0.38340]	-0.0217 [0.00000]
δ_b^2		1.97E-09 [0.28670]	1.22E-08 [0.00000]
λ_s		0.0154 [0.00000]	0.0376 [0.00000]
λ_b		0.0082 [0.03938]	0.0276 [0.00000]
λ_{sb}		0.0199 [0.25552]	0.0262 [0.00000]
ϕ_s			0.1540 [0.00170]
ϕ_b			0.1030 [0.00000]
ϕ_{sb}			0.1568 [0.00000]
γ_s			0.2470 [0.00000]
γ_b			0.2227 [0.00010]

Table 2 Continued

MGARCH-ARJI Parameter Estimates

	MGARCH	MGARCH-J	MGARCH-ARJI
γ_{sb}			0.4321 [0.00000]
Log L	14444.00	16343.00	17738.00
LR		3798.00 [0.00]	2790.00 [0.00]

This table displays the maximum likelihood estimates of the multivariate-GARCH, multivariate-GARCH with constant jump intensity and the multivariate-GARCH with autoregressive jump intensity. The parameters estimates with the *P*-values are presented for each estimates. The maximized log likelihood (Log L) for the models and the likelihood ratio statistics (LR) are shows to compare the explanatory power of each models. It compares the explanatory each model with the model in the preceding column. Equity Return(s_t) consist of daily NYSE/AMEX/NASDAQ return from CRSP and is readily available on Fama-French Website. Bond return (b_t) is the implied return on a portfolio of 1-,2-,3-,5-,7-, and 10 years constant maturity yield Treasury Bonds quoted on investment basis. These yields along with the daily Federal Fund rates are available in the Federal Reserve H.15 database on the Federal Reserve Website. The sample period is from 01-03-1995 to 12-31-2004. In all, there are 2499 observations in this data set. All values are in raw terms.. The parameters of the models are estimated according to the following econometric specification

$$Y_t = \begin{bmatrix} \Delta s_t \\ \Delta b_t \end{bmatrix}, X_t = \begin{bmatrix} \Delta s_t, \Delta b_t \\ \Delta b_t, \Delta s_t \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{bt} \end{bmatrix}, q_t = \begin{bmatrix} q_{st} \\ q_{bt} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n_{1t}} J(\mu_s, \delta_s^2) \ \text{-}\lambda_{s,t}\mu_s \\ \sum_{j=1}^{n_{2t}} J(\mu_b, \delta_b^2) \ \text{-}\lambda_{b,t}\mu_b \end{bmatrix}$$

$$\tilde{H}_t = C^{*\prime}C^* + \sum_{k=1}^{K}\sum_{j=1}^{q} A_k^* \varepsilon_{t-j}\varepsilon_{t-j}' A_k^{*\prime} + \sum_{k=1}^{K}\sum_{j=1}^{p} G_k^* H_{t-j} G_k^*$$

$$\lambda_{i,t} = \lambda_{i0} + \rho_i \lambda_{i,t-1} + \gamma_i \xi_{i,t-1}$$

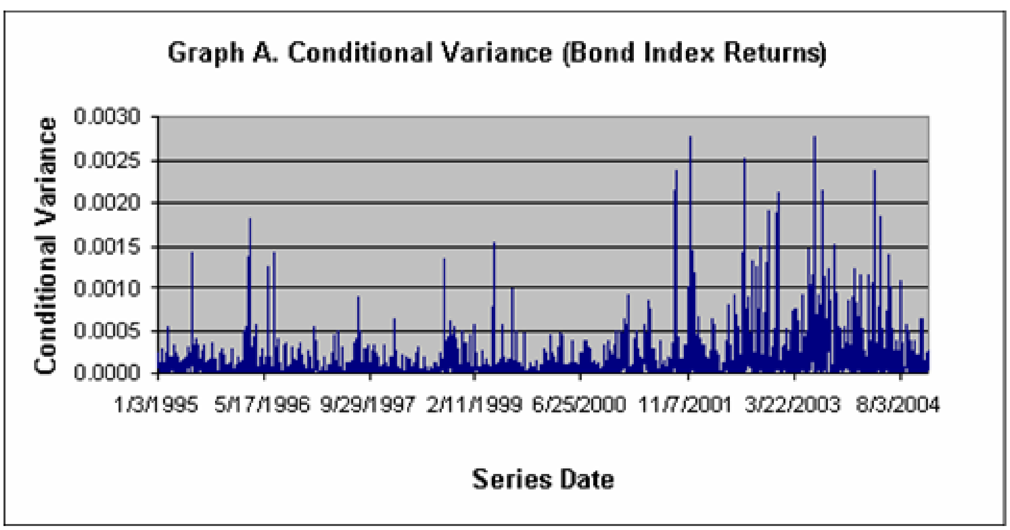
$$\begin{aligned} &MGARCH\ Model &&: \quad q_t = 0 \\ &MGARCH - J &&: \quad q_t \neq 0,\ \phi_i = \gamma_i = 0 \\ &MGARCH - ARJI &&: \quad q_t \neq 0,\ \phi_i, \gamma_i \neq 0 \end{aligned}$$

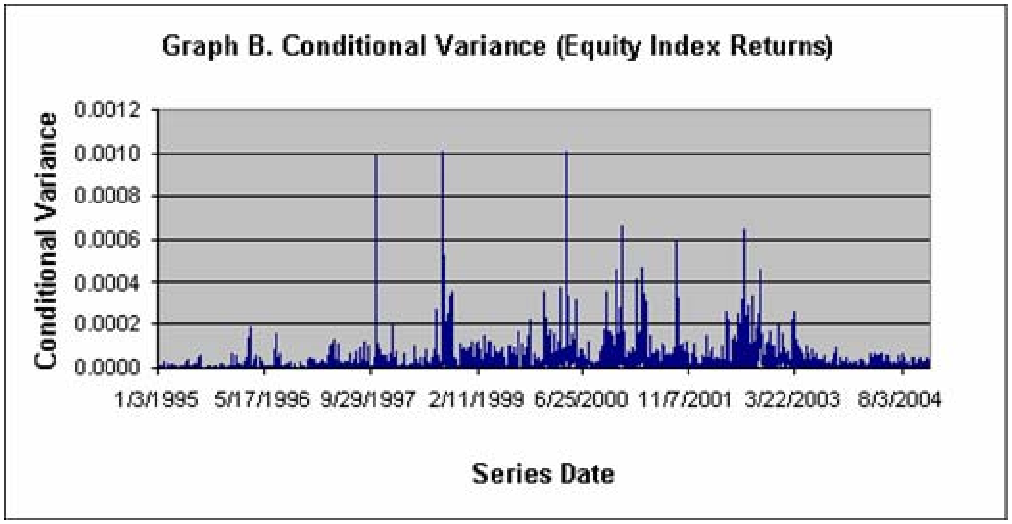
I also report the parameters estimates, and the *p*-value of the jump coefficients in the model. The mean jump size μ_s for the equity return which has a value of -0.0034 is negative and significant. This result supports also finds support in Maheu and McCurdy (2004). In their research, they applied a univariate generalized autoregressive conditional heteroscedasticity and jump intensity (GARJI) model to the DJIA, the NASDAQ 100 and to the CBOE Technology index and found the coefficient of the mean jump size to be

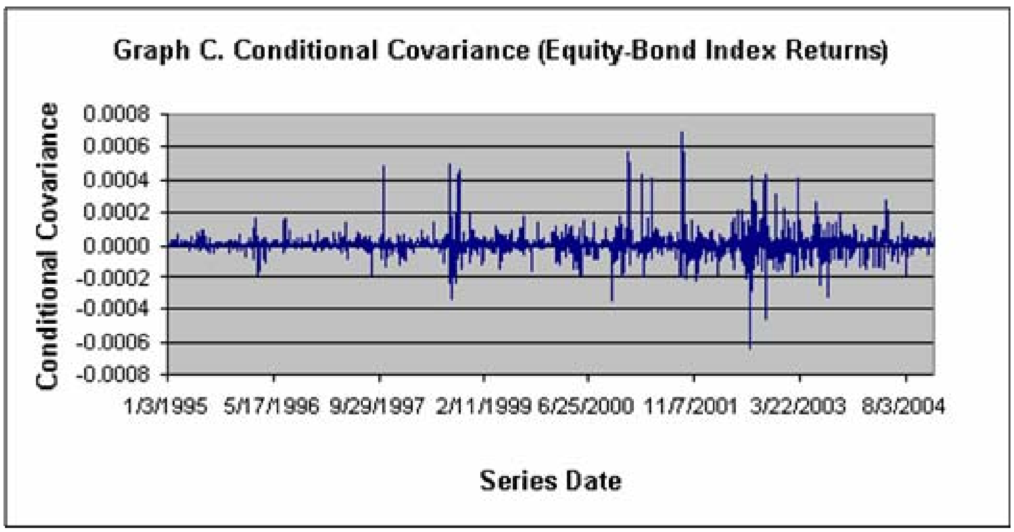
negative and significant for all these indices. On the other hand, mean jump size μ_b for bond return has a value of 0.0018 and is positive and significant. Taking together, the results leans to the fact that on average, equity returns appears to have negative jump size in returns and while on the average, bond return is characterized by a positive jump size.

For the autoregressive jump intensity parameter estimates, the results reveal striking evidence of persistent autoregressive jump intensity in both equity and bond returns and as well as the existence of a correlated jump intensity. All the coefficients are statistically significant. On the average we can expect the arrival of jumps every 16 and 17 trading days for equity and bond returns respectively. In addition we can also expect to have a correlated jump between these assets every 22 trading days. One possible reason for the 22 trading day interval for the arrival of correlated jumps might be the arrival of monthly macroeconomic news information.

Figure 2: Conditional Second Moments of Equity and Bond Index Returns. Time series plots of the conditional volatilities and cross-volatilities of equity and bonds.







Bates (2000), Eraker (2004) and Maheu and McCurdy (2004) all found evidence of state dependencies in the arrival of jumps in equity returns. Chan (2004) finds evidence of time variation in jump arrival in foreign currencies' return. I extend these findings, by highlighting the presence of state-dependencies in the arrival of jumps in both equity and bond returns in a multivariate setting.

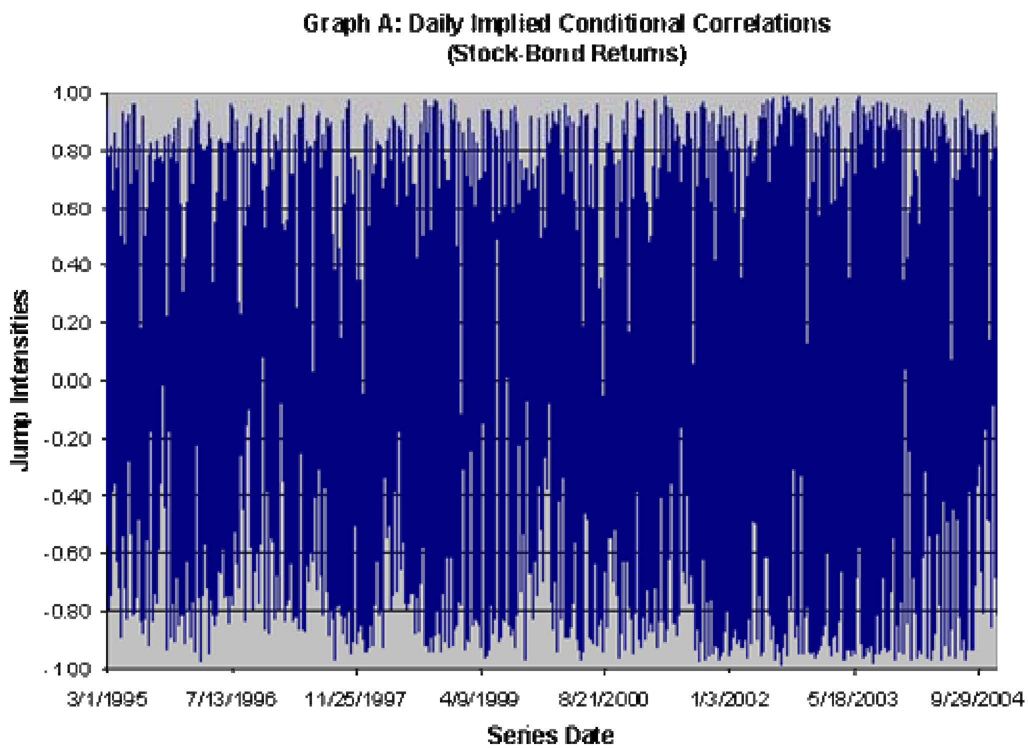
In Figure 3, Graphs B – D, I present the time plots of conditional jump intensities in bond and equity returns and their correlated jump intensities. Notice, that all the

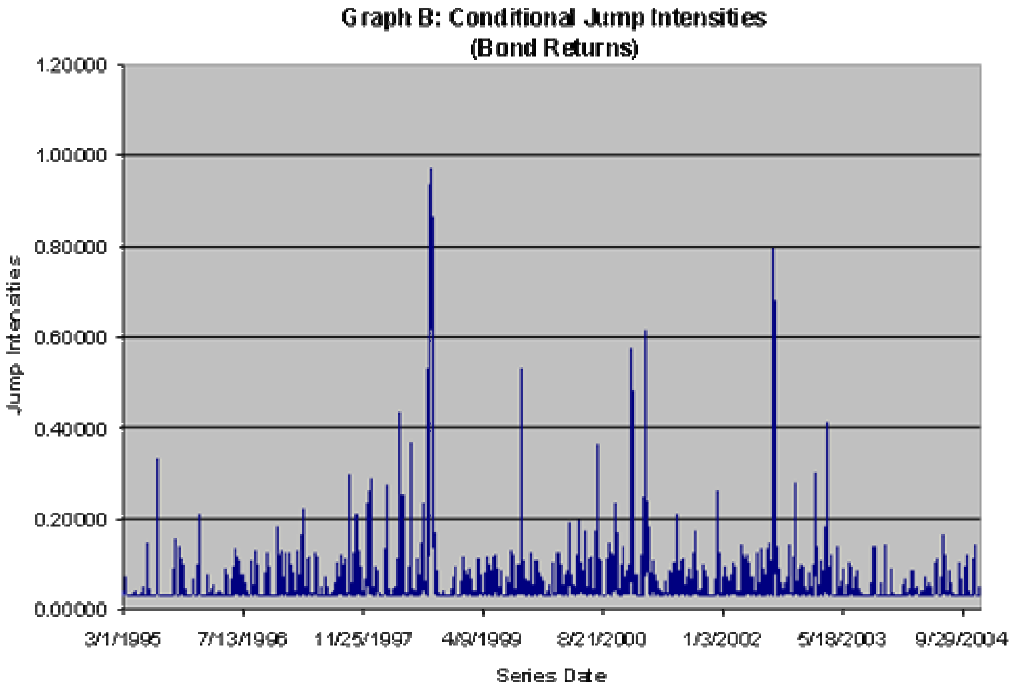
intensities appears to move in tandem. For example, on the July 31st, 2002 the estimated jump intensity for bond return was 0.79, the expected intensities for equity returns was 0.64, while the expected cross intensities was 0.64. These results imply that investors' expectation at least a jump in both markets within a 2-trading day window. A Lexis-Nexis search for news articles in periods surrounding this date highlights the prevailing uncertainty in the financial market surrounding that day. For example, on the 30th of July, the New York Times reported a 5% gain in the three major indices in the previous trading day, with the DJIA having the highest four day gains days since 1933. The estimated implied correlation for that trading day was 0.7084. In stark contrast, on the 31st of July, the London Financial Times reported deterioration in consumers' confidence in the economic recovery, sending the equities market plummeting and creating a rally in the treasury bonds market as investors weary of weak consumer spending and overall corporate profitability shift funds into the treasuries market. The estimated implied correlation for that trading day was -0.9926.

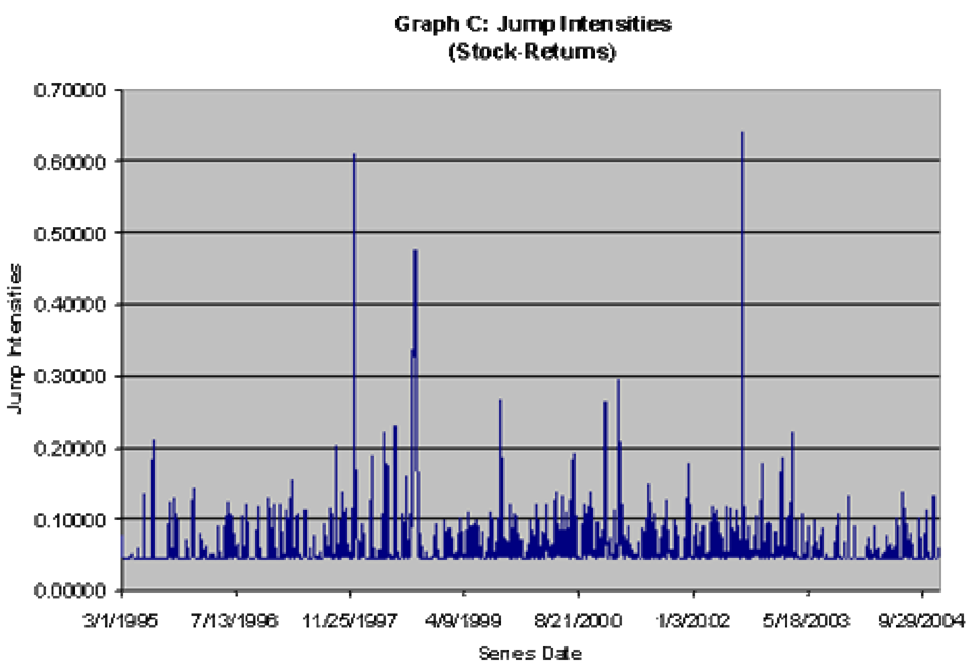
Evident news information such as this lends credence to the existence of correlated jump arrival between equity and bond markets. An implication of this is the likelihood that in period of high volatility, shocks might be propagated to either markets concurrently, or the arrival of periodic macroeconomic information such a monthly economic report can cause both markets to jump simultaneously. Furthermore, the estimates of the mean jump size coefficients and the significance of the jump correlation coefficient, $\rho_{sb} = -0.0217$ also suggest that on the average, equities' and Treasury bonds' markets respond in different manner to the arrival of news information that might induce concurrent jumps.

Lastly, the coefficient for jump variance in the model for both equities and bonds are insignificant. This could be due to the specification of the model since assumption of an unconditional mean jump size imposes a restriction on the variability of the magnitudes for each arrival instance. Taking all these results into account, it suffice to say that jump-augmented MGARCH model appears to track market dynamics fairly well.

Figure 3: Implied Correlations and Conditional Jump Intensities of Equity and Bond Index Returns. Time series plots of the implied conditional volatilities and Conditional jump intensities of equity and bonds return.




Graph B: Conditional Jump Intensities
(Bond Returns)
Jump Intensities
1.20000
1.00000
0.80000
0.60000
0.40000
0.20000
0.00000
3/1/1995
7/13/1996
11/25/1997
4/9/1999
8/21/2000
1/3/2002
5/18/2003
9/29/2004
Series Date


Graph C: Jump Intensities
(Stock-Returns)
Jump Intensities
0.70000
0.60000
0.50000
0.40000
0.30000
0.20000
0.10000
0.00000
3/1/1995
7/13/1996
11/25/1997
4/9/1999
8/21/2000
1/3/2002
5/18/2003
9/29/2004
Series Date

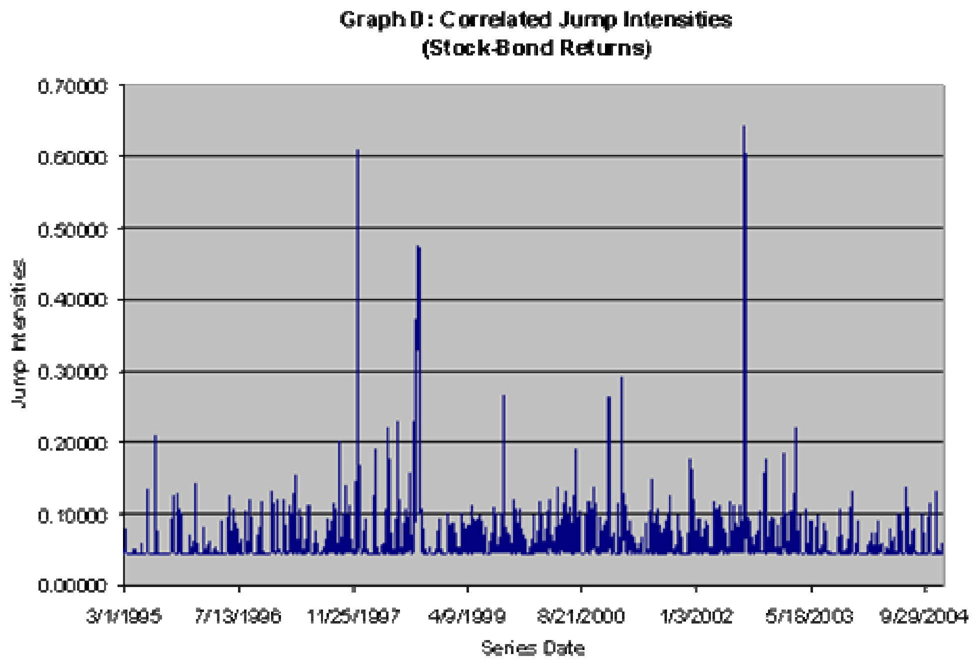


Table 3 reports some descriptive statistics for the jump components and conditional second moments. The sample average for conditional jump intensity for the bond and equities returns and for their correlated intensities are very close. However they significantly differ from their estimates under the assumption of unconditional jump intensity in the MGARCH-J model. The sample average of the implied correlations between bond and equities is a modest 0.02, which is slightly greater than the unconditional correlation shown in Table 1.

Table 3

Summary Statistics for Conditional Jump Intesities and Conditional Second Moments of Returns

	Mean	Std. Dev	Min	Max	Skewness	Kurtosis
$\widehat{\lambda}_{b,t}$	0.0562	0.0762	0.0262	0.9688	7.1751	63.1349
$\widehat{\lambda}_{s,t}$	0.0454	0.0435	0.0276	0.5698	7.1352	64.4554
$\widehat{\lambda}_{sb,t}$	0.0598	0.0430	0.0376	0.6425	7.3344	70.8990
$\widehat{h}_{b,t}$	0.0001	0.0002	1.36E-05	0.0028	4.8008	31.8878
$\widehat{h}_{s,t}$	3.21E-05	0.0001	1.05E-06	0.0010	6.8617	72.0424
$\widehat{h}_{sb,t}$	1.10E-06	0.0001	-6.40E-04	0.0007	1.3128	23.9477
$\widehat{\rho}_{sb,t}$	0.0274	0.6272	-0.9926	0.9862	-0.0628	-1.4112

Table 3 reports the summary statistics of sample average of the predicted values of conditional and correlated jump intensities in equity and bond returns. And the sample average of the conditional variance, covariance and correlations of stock and bond returns Std. Dev, denotes standard deviations.The Federal Fund rates are available in the Federal Reserve H.15 database on the Federal Reserve Website. VIX is the Chicago Board Options Exchange's volatility index in annualized percentage, standard deviation units. All values are in raw terms. The estimates of the conditional and correlated intensities were obtained from the MGARCH-ARJI model. The sample period is from 01-03-1995 to 12-31-2004. In all, there are 2499 observations in this data set.

b. Dynamics of Shock Propagation (News Impact Surfaces)

Kroner and Ng (1998) introduced a multivariate generalization of the "news impact curve" of Engle and Ng (1993) and called it the "news impact surface". This multivariate generalization graphs the conditional variance and covariance against shocks to both the bond and equity returns, while holding the past conditional variances and covariance, constant at their unconditional sample mean levels. In general, the news impact surface examines the effect of a unit increase in the standardized shocks to equity and bonds returns on the conditional variance, and covariance between equity and bonds, holding the past period conditional variance and covariance constant at their unconditional

sample mean. Figure 4, shows the news impact surfaces for the conditional bond variance, conditional equity variance, and conditional bond-equity covariance generated by the MGARCH-only (Panel A.), MGARCH-J (Panel B.), and the MGARCH-ARJI model (Panel C). Since all these models assumes the effect of shocks on the second moments to be symmetric, it quite natural for us to notice, that all the surface respond symmetrically to induced shocks.

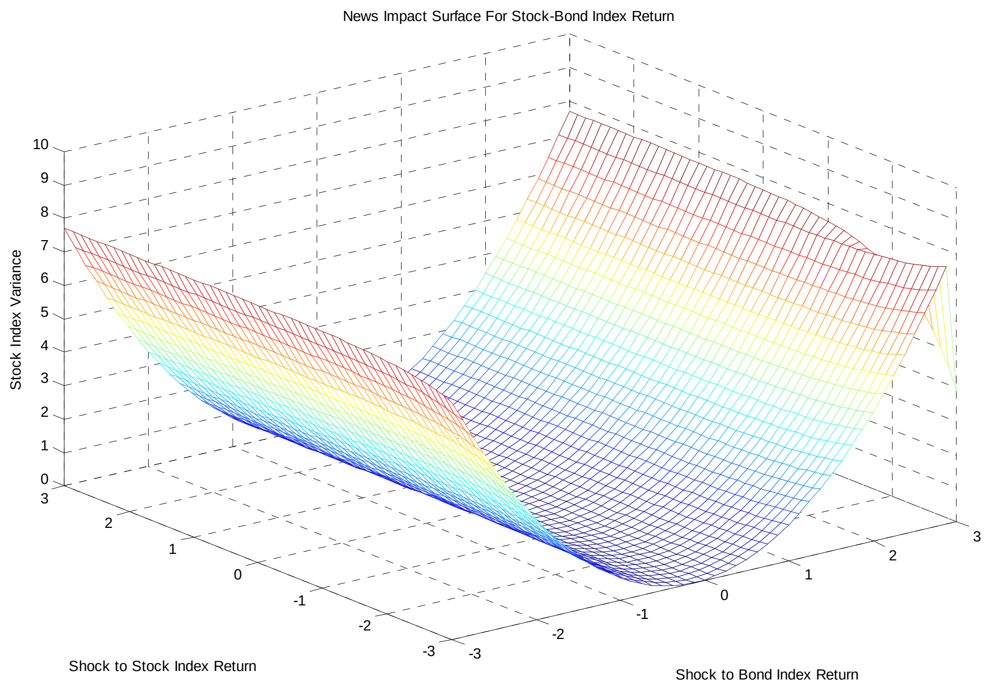
Also, notice that as we progress from Panel A (MGARCH-only) to Panel C. (MGARCH-ARJI), the impact of induced shocks to the conditional second moments appears to decrease steadily. For example the shape of the news impact surface for the conditional variance of equity returns decreases in curvature as we move from Panel A to Panel C. One reason for the dampening effect which the introduction of jump parameters has on the MGARCH coefficients is the ability of the former to mitigate the persistence of serial correlation in the evolution of the conditional second moments, thus alleviating the MGARCH parameters burden of carrying instantaneous shocks for longer period.

Also, consider the evolution in the curvature of the conditional variance surface for bond returns from Panel A to Panel C. The news impact surface evolves from a quasi-convex surface in Panel A to a slightly convex surface in Panel C. The surface in the MGARCH-only model appears to suggest that irrespective of the magnitude of shocks in the bond market, extreme shocks in the equities market appear to dampen the level of volatility in the bond market. This is rarely the case. Though, markets might move in different direction, it seems highly implausible that shocks in equity market might reduce the volatility in the bond markets as traders rebalance their portfolios to either cash-in on the gains or mitigate the loss in one market.

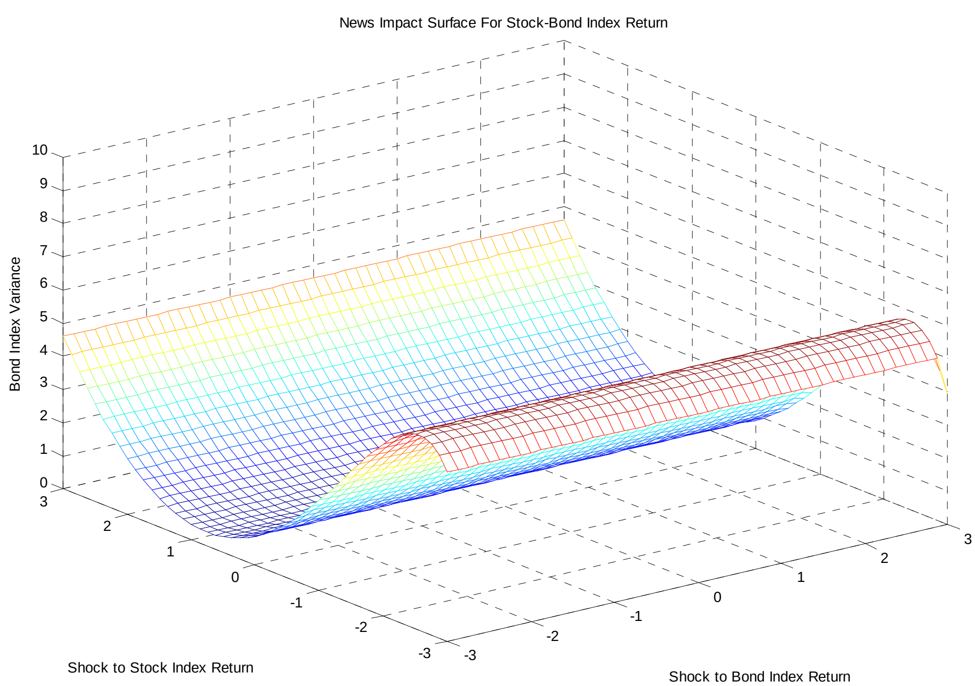
Figure 4: News Impact Surfaces. News impact surfaces for the conditional variance for equity and bond returns and conditional covariance produced by the MGARCH-only, the MGARCH-J and the MGARCH-ARJI model.

Panel A. MGARCH-only

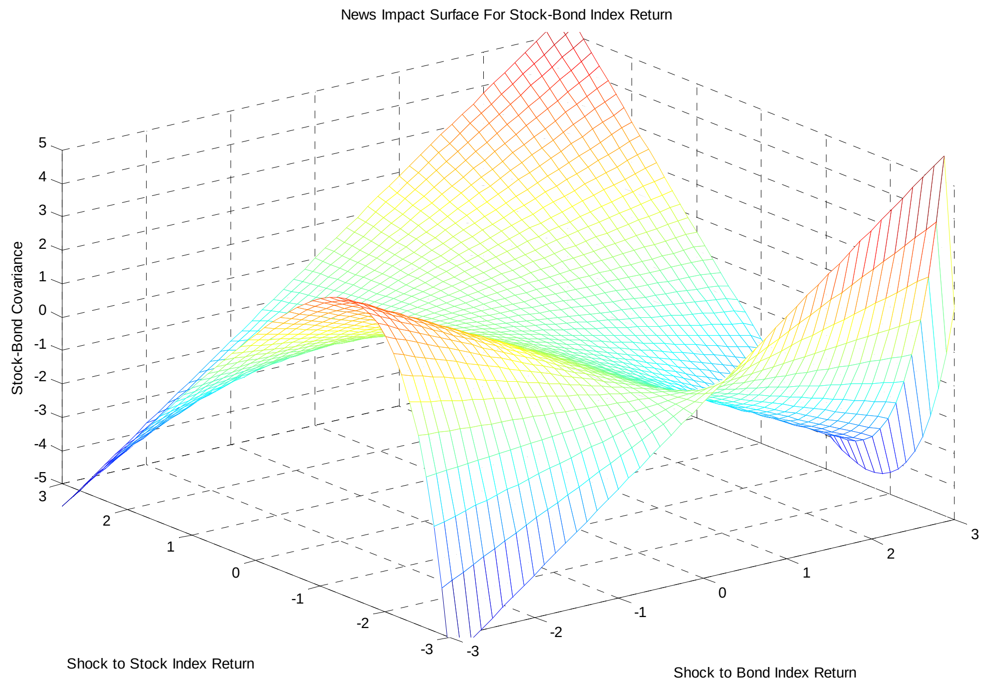
Graph I. Equity Variance



Graph II. Bond Variance

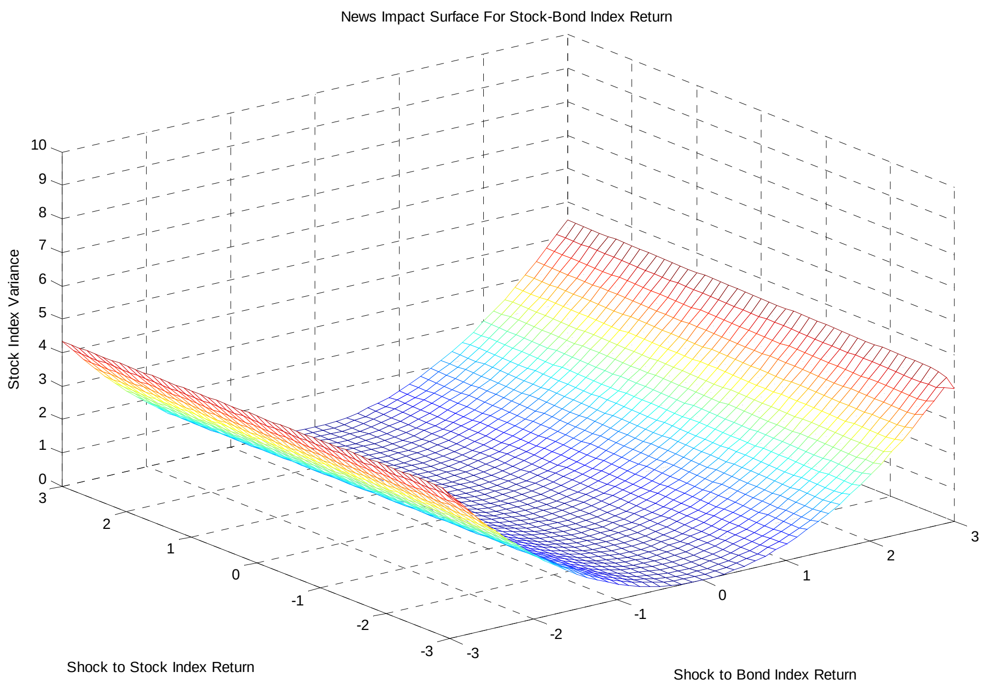


Graph III. Equity-Bond Covariance.

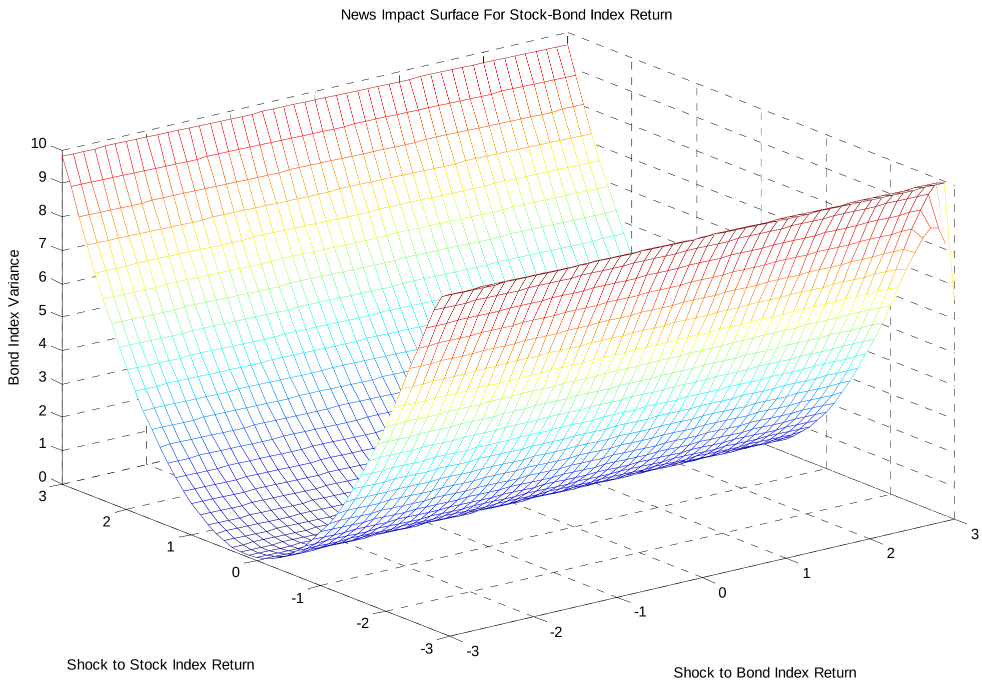


Panel B. MGARCH-J

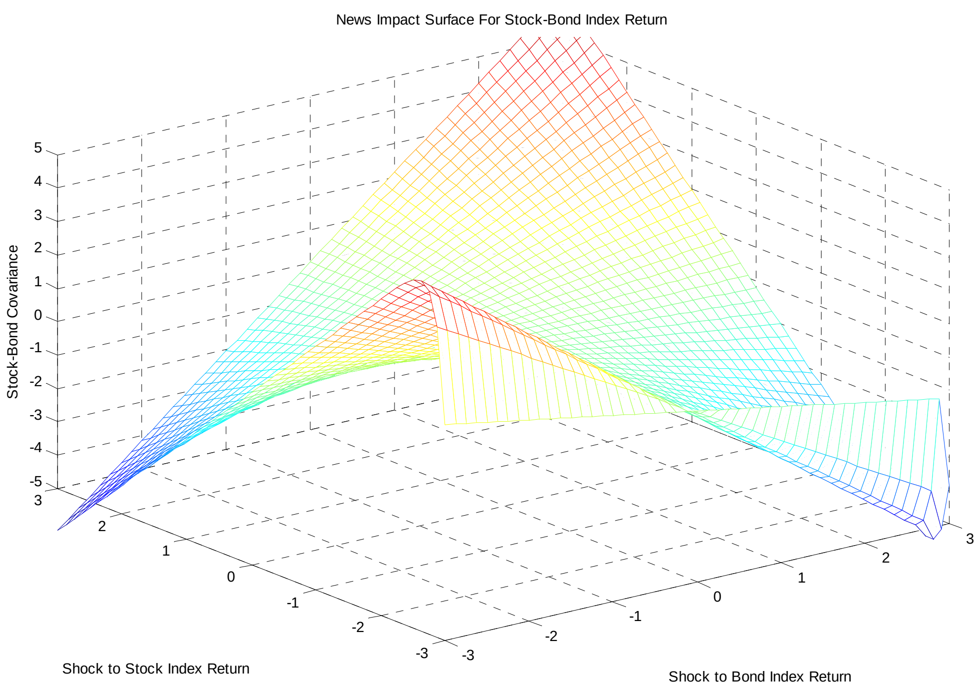
Graph I. Equity Variance



Graph II. Bond Variance

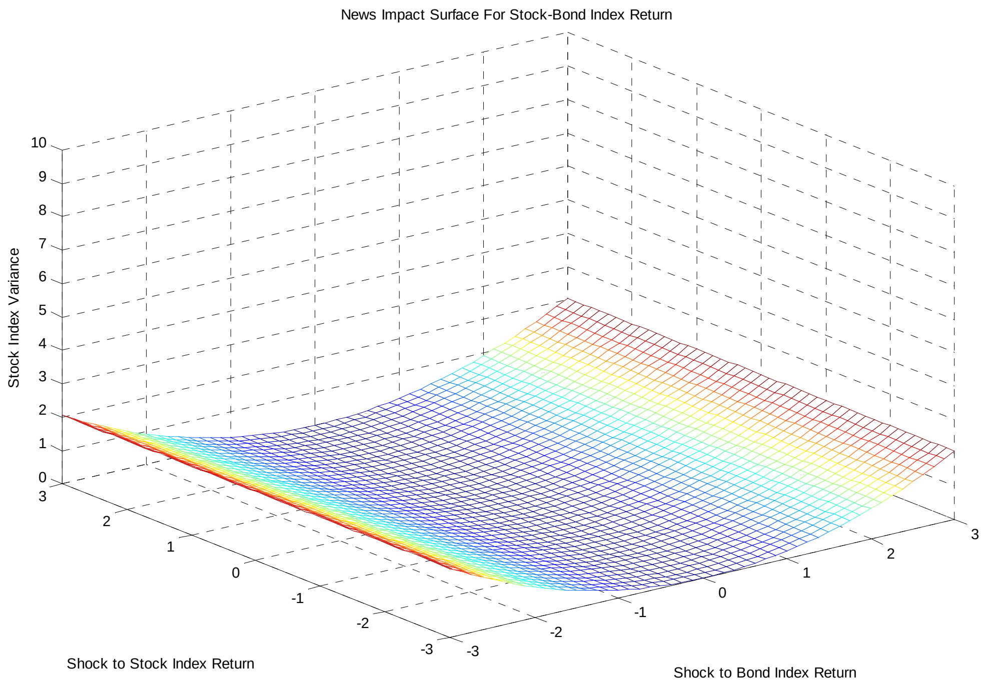


Graph III. Equity-Bond Covariance

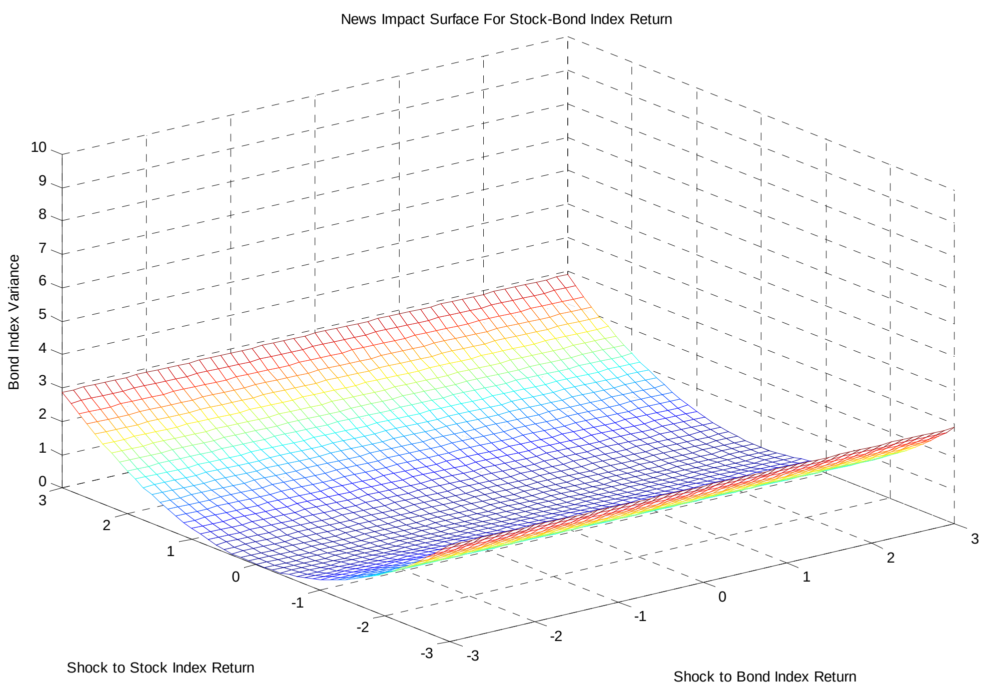


Panel C: MGARCH-ARJI

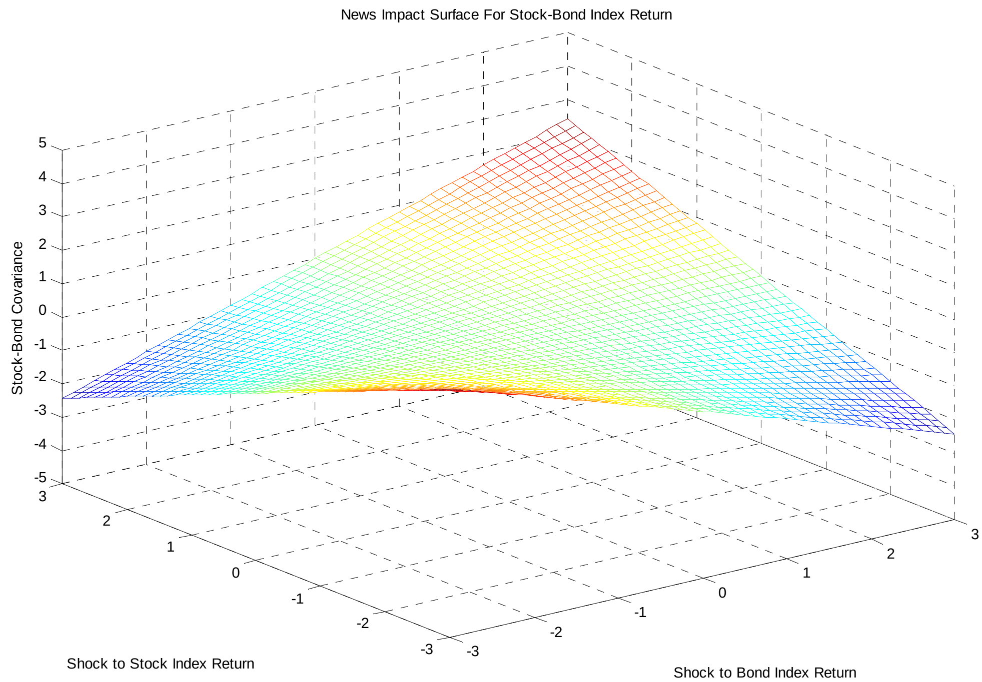
Graph I. Equity Variance



Graph II. Bond Variance



<u>Graph III. Equity-Bond Covariance</u>



c. Comovement in the Stock, Bonds, Money Markets and CBOE

Table 4 shows the sample correlations between the conditional jump intensities of equity and bonds, their correlated intensities, the Federal Fund rates and the VIX. The results show a positive correlation between the VIX and all 3 conditional intensities. The VIX is which is a widely accepted measure of investors' view of the "riskiness" of the market represents the “investors’ fear gauge” that dramatically increases during period of financial turmoil or an increase in investors expectation of adverse outcome in the near future.[15] The positive correlation pattern between the VIX and the estimates of the conditional jump intensity also lends credence to the assertion that increased intensity of jumps is also associated with high level of uncertainty in financial markets.

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[15] More information about the VIX can be found on the website of Chicago Board Option Exchange at http://www.cboe.com/micro/vix/introduction.aspx

Table 4

Sample Correlations

	Federal Funds Rates (FFR)	VIX	Conditional Jump Intensity (Equity)	Conditional Jump Intensity (Bond)	Correlated Jump Intensity
Federal Funds Rates (FFR)	1.0000				
VIX	-0.0954	1.0000			
Conditional Jump Intensity (Equity) $(\widehat{\lambda}_{s,t})$	0.0573	0.2986	1.0000		
Conditional Jump Intensity (Bond) $(\widehat{\lambda}_{b,t})$	0.0735	0.3059	0.9418	1.0000	
Correlated Jump Intensity $(\widehat{\lambda}_{sb,t})$	0.0633	0.2990	0.9629	0.9839	1.0000

This table reports the sample corelations between the Federal Funds Rate, the VIX, and the conditional and correlated intensities between equity and bond returns. The Federal Fund rates are available in the Federal Reserve H.15 database on the Federal Reserve Website. VIX is the Chicago Board Options Exchange's volatility index in annualized percentage, standard deviation units. All values are in raw terms. The estimates of the conditional and correlated intensities were obtained from the MGARCH-ARJI model.

3.7 Flight-to-Quality or Contagion.

In this subsection, I address the notion of flight-to-quality between the equity and bond markets following the classification mentioned in Baur and Lucey (2006). Expectation or realization of adverse outcomes motivates rational investors to seek insurance against impending or future losses. An episode such as this is characterized by the flow of funds from a distressed market into safer investments. Increase in demand for

safer investments pushes up the price of safe assets, thus creating a rally in the safe asset's market.

A reliable yardstick to track, in a study of flight-to-quality between equity and bonds is the evolution of conditional correlation of the returns of these two assets (Baur and Lucey, 2006). The incidence of a negative correlation between equity and bond that is characterized by a fall in equity (bond) market with a corresponding rally in the bond (equity) market is a peculiar characteristic of a flight-to-quality episode. On the other hand, contagion as described in Forbes and Rigobon (2002) is the incidence of a significant increase in the correlation between equity and bond, that is characterized by a fall in equity (bond) market with a corresponding fall in the bond (equity) market

To investigate the flight-to-quality linkage between equity and bonds, I recover the estimate of the implied daily conditional correlation derived from the H_t matrix in our MGARCH-ARJI model as defined below:

$$\widehat{\rho}_{sb,t} = \left[\frac{\widehat{h}_{sb,t}}{\widehat{h}_{s,t}^{\frac{1}{2}} \otimes \widehat{h}_{b,t}^{\frac{1}{2}}} \right] \tag{80}$$

I then explore the time-varying dynamics of the conditional correlation as a function of a set of information variables that tracks conditions in both equity and bonds market. I do not explicitly attempt to capture the incidence of flight-to-quality around specific episode since a more accurate measure of investors' movement into safer markets will be a proxy for the flow of funds; rather I study the impact of measures of market uncertainty on the correlation between equity and bond by assessing their time varying effect on the evolution of the conditional comovement between equity and bonds.

Since the flow of funds into safe market is likely to create a rally in that market and a decline in the unsafe market.

To explore the notion of flight-to-quality, I regress the implied conditional correlation estimated $\widehat{\rho}_{sb,t}$ from the MGARCH-ARJI on its own lagged and a set of information variables L_t following the specification below.

$$\widehat{\rho}_{sb,t} = \alpha + \beta \widehat{\rho}_{sb,t-1} + \psi L_{t-1} \tag{81}$$

$$\text{Where } L_t = \left[\Delta s_t^{+}, \Delta b_t^{+}, \widehat{\lambda}_{s,t}, \widehat{\lambda}_{b,t}, \widehat{\lambda}_{sb,t}, FFR_t, VIX_t \right]$$

Δs_t^{+} and Δb_t^{+} represents the standardized shocks to equities and bond index return respectively . The hypothesis is as follows: if the implied conditional correlation is a negative function of Δs_t^{+}, then there is contagion, on the other hand if conditional correlation is positively related to Δs_t^{+} then we see evidence of flight-to-quality. The same logic applies to Δb_t^{+}. Baur and Lucey (2006) found that time variation in the standardized shocks have little or no effect on the conditional correlations between stock and bonds that was obtained using the DCC model. Analogous to their results, I also find no significant relations between the lag standardized shocks to equity and bond returns, and the conditional correlation between equity and bond, and thus do not present the results in the table.

Next, I focus my attention on the impact of jump intensities and correlated jump intensity of equity and bond returns $\widehat{\lambda}_{i,t}$, the Federal Funds rate, and the VIX on the dynamics of the implied conditional correlations of equity and bond return. Bates (2000), Eraker (2004), Maheu and McCurdy (2004), Das (2002), all present evidence of higher

level of jump intensity during periods of increased uncertainty in various financial markets. Therefore, we can exemplify periods of high uncertainty in financial market by a significant increase in jump intensity. Since Table 4, also highlights the strong sample correlation between the intensities and the VIX, I further distinguish a flight-to-quality episode as associated with a significant increase in jump intensity or in the VIX that is followed by a reduction in the implied correlation between the bond and equity index return. As an upshot of this assertion, the hypothesis predicts the sign for the coefficient of the $\widehat{\lambda}_{i,t}$ and VIX will be negative if there is flight-to-quality and positive if there is contagion.

While the effect of change in the short-term interest rate on bond price is obvious, its impact on equity returns is ambiguous. Although, it is possible to argue that an increase in the Fed Funds rates might signal the market's participant desire for higher compensation for an expected increase in liquidity and inflation risk. However, since my focus is on the market conditions and not the underlying factors driving such conditions, the lack of clarity with respect to the predicted sign of the interest rate variable, will not necessarily weaken the implications of the result of my regressions. It is noteworthy to mention here that since all the 4 measures of market volatility ($\widehat{\lambda}_{i,t}$ and VIX) are strongly correlated I estimated 4 separate regressions to eliminate multi-collinearity problems in the regression.

Table 5 present the estimates of the linear regression of the conditional correlations on the set of information variables. The *p*-values are in square brackets. In EQ1, I regress the conditional correlation on its own lagged and the lag of the Federal Funds rate (FFR) and the VIX. The estimated coefficients shows that the past period

correlation, Federal Fund rates and VIX all have negative impact on current correlations. The lagged conditional correlation and the VIX are significant at 5% critical value, while the significance of the Federal Fund rate can only be accepted at the 10% significance criteria. One plausible rationalization for the negative sign of the lag correlation could be market reversal after period of sell-offs by traders flying to quality.

The fact that the coefficient of VIX is negative and significant is not surprising. Connolly, Stivers and Sun (2005), find forward-looking correlations to vary negatively and significantly with VIX. They show that for a greater than 25% increase in the VIX, the 22-trading-day mean correlation between equities and the 10 year bond would be expected to drop by as much as 100%. Analogous to their result, I find that a 10% increase in the daily VIX, will reduce the implied daily correlation between equities and bond return by 2.3%.

Table 5				
Flight-To-Quality Analysis: Conditional Correlations (Equity and Bond)				
	EQ1	**EQ2**	**EQ3**	**EQ4**
Intercept	0.1777 [0.0009]	0.0998 [0.0030]	0.0447 [0.0043]	0.0534 [0.0034]
$\hat{\rho}_{sb,t-1}$	-0.0612 [0.0023]	-0.0603 [0.0029]	-0.0592 [0.0032]	-0.0594 [0.0030]
FFR_{t-1}	-1.1164 [0.0826]			
VIX_{t-1}	-0.0047 [0.0158]			
$\hat{\lambda}_{s,t-1}$		-0.5591 [0.0463]		
$\hat{\lambda}_{b,t-1}$			-0.2768 [0.0931]	
$\hat{\lambda}_{sb,t-1}$				-0.5343 [0.0645]
F-test for $H_0:\beta=0$	5.3588	5.7609	5.3749	5.5754
p-value	[0.0003]	[0.0006]	[0.0011]	[0.0008]

This table reports the estimates of the linear regression of the conditional correlations on the set of information variables using the econometric model specified below. FFR_t is the Federal Fund rates and are available in the Federal Reserve H.15 database on the Federal Reserve Website. VIX_t is the Chicago Board Options Exchange's volatility index in annualized percentage, standard deviation units. FFR_t and VIX_t are both in raw value.

$$\hat{\rho}_{sb,t} = \alpha + \beta\hat{\rho}_{sb,t-1} + \psi L_{t-1}$$

$$L_t = \left[\hat{\lambda}_{s,t}, \hat{\lambda}_{b,t}, \hat{\lambda}_{sb,t}, FFR_t, VIX_t\right]$$

The first variable in the L matrix is the model estimates of the conditional jump intensities in equity returns.The second variable is the model estimates of the conditional jump intensity in bond returns. The third is the model estimates of the correlated jump intensity between equity and bond return. All three intensities were obtained from the MGARCH-ARJI model. The sample period is from 01-03-1995 to 12-31-2004. In all, there are 2499 observations in this data set. The *P* values are in square brackets.

EQ2 shows that coefficient of conditional jump intensities of equity returns is significant and negatively related to the implied correlations. In economic terms, a 10% increase in ex post number of jumps in equity returns, is expected to generate a 5.5%

reduction in the conditional correlations. In similarity, EQ3 and EQ4 shows that conditional jump intensities in bond returns and the correlated intensities have a negative and weakly significant impact on implied conditional correlations. In economic terms, we can expect a 10% increase in the ex post conditional jumps intensity of bond returns to generate a 2.6% reduction in correlation, while a 10% increase in the number of expected correlated jumps to reduce the conditional correlation by as much as 5.2%. In summary, the results imply that the conditional correlations between equity and bond return is twice as sensitive to jumps in equity returns, and the correlated jumps, than jumps in bond returns.

The F-test of the null $(H_0 = \alpha = \beta = \psi = 0)$ for the regression is rejected for all the models. Thus, providing some support to the explanatory power of the regression coefficients. Taking all these results together, over all implication of the Table 5 is consistent with the idea that conditional correlation between equity and bonds can be partly explained by the arrival of jumps in the equities' and bonds' markets, and investors expectation of near-term volatility. Conditional correlations reduces with the expectation or realization of adverse or tumultuous outcome in either the equities or bond market. Investors observed these market conditions and respond by flying to quality when conditions in the equity market become adverse or significantly more risky than in the Treasuries' market.

3.8 Specifications and Robustness.

In this section, I proceed with a set of specification and robustness test to gauge the adequacy of the model. In the first stage, I explore the features of the standardized

residuals generated by the model. Following, Scruggs and Glabadanidis (2003), I analyzed the statistical properties of the standardized residuals and the standardized products of residuals generated by the MGARCH-ARJI model. Table 6 presents the Ljung-Box portmanteau test for serial correlation in the standard residuals and standardized product of the residuals for one, two and three lags respectively. I find no evidence of serial correlation in the standardized equity returns, and evidence of serial correlation in the standardized bond index return. The Q-statistics for the 1, 2, and 3 lags of equity returns all indicate the absence of serial correlation. For the bond return, the Q-statistics shows evidence of serial correlation in its residual patterns.

The fact that Treasury bond return is prone to persistent serial correlation should be considered in the light that investors consider treasury bonds as hedge against volatilities in their equity holdings as suggested by Merton (1973). As a consequence, we expect to observe strong serial correlation patterns as investors continually rebalance their portfolio of safe bonds in responses to conditions in the equity markets. Moving on to the standardized product of residuals, the Q-statistics of the cross-residuals of equity and bonds shows no effect of serial correlation for all the three lags examined. Consequently, one can safely surmise that the MGARCH-ARJI model provides a satisfactory description of the comovement of equity and bond returns.

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Table 6

Residual-Based Diagnostics for the MGARCH-ARJI Model

	Bond	Equity	Equity-Bond
$Q(1)$	497.5900 [0.0000]	1.6848 [0.1940]	
$Q(2)$	506.8200 [0.0000]	4.6470 [0.0980]	
$Q(3)$	508.2100 [0.0000]	5.5882 [0.1330]	
$Q_{sb}(1)$			0.0238 [0.8770]
$Q_{sb}(2)$			0.1298 [0.9370]
$Q_{sb}(3)$			0.1320 [0.9880]

standardized residuals and cross standardized residuals with one,two and three lags respectively. The residuals are derived from the MGARCH-ARJI model and are standardized as shown below.

Standardized Residuals : $\widehat{z}_t = \left[\widehat{z}_{s,t} \ \widehat{z}_{b,t}\right] = \left(\widehat{H}_t^{-\frac{1}{2}} \cdot \widehat{\varepsilon}_t\right)$

Standard Product of Residuals : $\widehat{z}_{sb,t} = \left[\widehat{z}_{s,t} \cdot \widehat{z}_{b,t}\right] = \dfrac{\left(\widehat{\varepsilon}_{s,t} \cdot \widehat{\varepsilon}_{b,t}\right)}{h_{sb,t}}$

Q-Statistics: $Q(q) = N(N+2)\sum_{1}^{q} \dfrac{r(i;\hat{v}_t^2)}{(N-i)}$

The Q-statistics are presented for each lag. The significance probability are shown in square brackets.

Tabel 7

MGARCH-ARJI Parameter Estimates

	Equity-Short-Term Bond	Equity-Long-Term Bond
α_s	0.0005 [0.01618]	0.00048 [0.01617]
α_b	-0.00003 [0.47586]	0.00005 [0.44711]
β_s	0.02600 [0.09768]	0.02602 [0.09750]
β_b	-0.34641 [0.00000]	-0.30517 [0.00000]
β_{bs}	0.00229 {0.4005}	0.00551 [0.29168]
β_{sb}	-0.45039 [0.00000]	-0.44649 [0.00000]
c_{11}	0.00181 [0.00000]	0.00178 [0.00000]
c_{21}	0.0004 [0.005789]	0.00033 [0.00000]
c_{22}	0.00346 [0.00000]	0.00488 [0.00000]
a_{11}	0.67063 [0.00000]	0.41925 [0.00000]
a_{12}	3.65E-07 [0.27660]	2.01E-06 [0.38637]
a_{21}	3.82E-07 [0.36996]	2.76E-06 [0.07016]
a_{22}	0.81076 [0.00000]	0.58731 [0.00000]
g_{11}	0.2974 [0.00000]	0.49048 [0.00000]
g_{12}	1.11E-06 [0.34380]	1.39E-06 [0.40515]

Table 7 Continued

MGARCH-ARJI Parameter Estimates

	Equity-Short-Term Bond	Equity-Long-Term Bond
g_{21}	9.94E-06	5.73E-07
	[0.63919]	[0.41028]
g_{22}	0.065523	0.045991
	[0.00000]	[0.00000]
μ_s	-0.00299	-0.00294
	[0.00000]	[0.00000]
μ_b	0.00243	0.00323
	[0.00000]	[0.00000]
δ_s^2	6.31E-08	8.10E-08
	[0.63985]	[0.54662]
ρ_{sb}	-0.017253	-0.01164
	[0.39975]	[0.00000]
δ_b^2	8.83E-09	7.10E-08
	[0.23400]	[0.77995]
λ_s	0.04339	0.03674
	[0.00000]	[0.00000]
λ_b	0.021464	0.02941
	[0.00000]	[0.00000]
λ_{sb}	0.01893	0.037117
	[0.00000]	[0.0000]
ϕ_s	0.09847	0.11517
	[0.00000]	[0.00000]
ϕ_b	0.13409	0.18623
	[0.36081]	[0.00000]
ϕ_{sb}	0.17565	0.27379
	[0.36092]	[0.00000]
γ_s	-0.14912	0.26107
	[0.10382]	[0.00000]
γ_b	0.20995	0.25737
	[0.42042]	[0.00000]

Table 7 Continued

MGARCH-ARJI Parameter Estimates

	Equity-Short-Term Bond	Equity-Long-Term Bond
γ_{sb}	0.29783 [0.43913]	0.06956 [0.00000]
Log L	18528.00	16852.00
LR	3640.00 [0.0000]	1380.00 [0.000]

This table displays the maximum likelihood estimates of the multivariate-GARCH with autoregressive jump intensity model. The parameters estimates with asymptoptic standard errors in parentheses are presented for each estimates. The maximized log likelihood (lgl) for the models and the likelihood ratio statistics (LR) are shows to compare the explanatory power of each models. Equity Return consist of daily NYSE/AMEX/NASDAQ return from CRSP and is readily available on Fama-French Website. Short-Term and Long-Term bond return is the implied return on a portfolio of 1-,2-,3-, and 5-,7-, and 10 years constant maturity yield Treasury Bonds respectively. These yields along with the daily Federal Fund rates are available in the Federal Reserve H.15 database on the Federal Reserve Website. The sample period is from 01-03-1995 to 12-31-2004. In all, there are 2499 observations in this data set. All values are in raw terms. The parameters of the models are estimated according to the following econometric specifications.

$$Y_t = \begin{bmatrix} \Delta s_t \\ \Delta b_t \end{bmatrix}, X_t = \begin{bmatrix} \Delta s_t, \Delta b_t \\ \Delta b_t, \Delta s_t \end{bmatrix}, \varepsilon_t = \begin{bmatrix} \varepsilon_{st} \\ \varepsilon_{bt} \end{bmatrix}, q_t = \begin{bmatrix} q_{st} \\ q_{bt} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n_{1t}} J(\mu_s, \delta_s^2) \ -\lambda_{s,t}\mu_s \\ \sum_{j=1}^{n_{2t}} J(\mu_b, \delta_b^2) \ -\lambda_{b,t}\mu_b \end{bmatrix}$$

$$\tilde{H}_t = C^{*\prime}C^* + \sum_{k=1}^{K}\sum_{j=1}^{q} A_k^* \varepsilon_{t-j}\varepsilon_{t-j}' A_k^{*\prime} + \sum_{k=1}^{K}\sum_{j=1}^{p} G_k^* H_{t-j} G_k^*$$

$$\lambda_{i,t} = \lambda_{i0} + \rho_i \lambda_{i,t-1} + \gamma_i \xi_{i,t-1}$$

$$\begin{array}{lcl} MGARCH\ Model & : & q_t = 0 \\ MGARCH-J & : & q_t \neq 0,\ \phi_i = \gamma_i = 0 \\ MGARCH-ARJI & : & q_t \neq 0,\ \phi_i, \gamma_i \neq 0 \end{array}$$

In the second stage, I split the bond data into short-term and long-term bond returns and match each trading day with the corresponding equity index return. Short-term bond returns is the return on a portfolio of 1-, 2-, and 3-year constant maturity yield Treasury bonds. Long-term bonds is the return on a portfolio of 5-, 7-, and 10-years constant maturity yield Treasury bonds. I repeat the estimation of the MGARCH-ARJI model and flight-to-quality test.

Table 8

Flight-To-Quality Analysis: Conditional Correlations (Equity and Short-Term Bond)

	EQ1	EQ2	EQ3	EQ4
Intercept	0.1320	0.0351	0.0163	0.0148
	[0.0033]	[0.0985]	[0.3665]	[0.5117]
$\widehat{\rho}_{sb,t-1}$	-0.0053	-0.0027	-0.0031	-0.0030
	[0.7913]	[0.894]	[0.87911]	[0.8808]
FFR_{t-1}	-1.0012			
	[0.0625]			
VIX_{t-1}	-0.0032			
	[0.0517]			
$\widehat{\lambda}_{s,t}$		-0.2966		
		[0.4632]		
$\widehat{\lambda}_{b,t}$			0.1343	
			[0.7191]	
$\widehat{\lambda}_{sb,t}$				0.1831
				[0.7349]
F-test for $H_0: \beta=0$	2.7116	1.5861	1.4495	1.4446
p-value	[0.0286]	[0.1907]	[0.2265]	[0.2279]

This table reports the estimates of the linear regression of the conditional correlations on the set of information variables using the econometric model specified below. FFRt is the Federal Fund rates and are available in the Federal Reserve H.15 database on the Federal Reserve Website. VIXt is the Chicago Board Options Exchange's volatility index in annualized percentage, standard deviation units. FFRt and VIXt are both in raw value.

$$\widehat{\rho}_{sb,t} = \alpha + \beta \widehat{\rho}_{sb,t-1} + \psi L_{t-1}$$

$$L_t = \left[\widehat{\lambda}_{s,t}, \widehat{\lambda}_{b,t}, \widehat{\lambda}_{sb,t}, FFR_t, VIX_t \right]$$

The first variable in the L matrix is the model estimates of the conditional jump intensities in equity returns.The second variable is the model estimates of the conditional jump intensity in bond returns. The third is the model estimates of the correlated jump intensity between equity and bond return. All three intensities were obtained from the MGARCH-ARJI model. The sample period is from 01-03-1995 to 12-31-2004. In all, there are 2499

Table 9

Flight-To-Quality Analysis: Conditional Correlations (Equity and Long-Term Bond)

	EQ1	EQ2	EQ3	EQ4
Intercept	0.1321	0.0442	0.0544	0.1048
	[0.0015]	[0.0014]	[0.0051]	[0.2055]
$\widehat{\rho}_{sb,t-1}$	0.0764	0.0777	0.0778	0.0787
	[0.0001]	[0.0001]	[0.0001]	[0.0001]
FFR_{t-1}	-0.5789			
	[0.2452]			
VIX_{t-1}	-0.0039			
	[0.010]			
$\widehat{\lambda}_{s,t}$		-0.3490		
		[0.0347]		
$\widehat{\lambda}_{b,t}$			-0.5557	
			[0.0661]	
$\widehat{\lambda}_{sb,t}$				-1.4695
				[0.3228]
F-test for $H_0:\beta=0$	7.5418	9.0447	8.6811	7.8726
p-value	[0.0003]	[0.0006]	[0.0001]	[0.0000]

This table reports the estimates of the linear regression of the conditional correlations on the set of information variables using the econometric model specified below. FFR_t is the Federal Fund rates and are available in the Federal Reserve H.15 database on the Federal Reserve Website. VIX_t is the Chicago Board Options Exchange's volatility index in annualized percentage, standard deviation units. FFR_t and VIX_t are both in raw value.

$$\widehat{\rho}_{sb,t} = \alpha + \beta\widehat{\rho}_{sb,t-1} + \psi L_{t-1}$$

$$L_t = \left[\widehat{\lambda}_{s,t}, \widehat{\lambda}_{b,t}, \widehat{\lambda}_{sb,t}, FFR_t, VIX_t \right]$$

The first variable in the L matrix is the model estimates of the conditional jump intensities in equity returns.The second variable is the model estimates of the conditional jump intensity in bond returns. The third is the model estimates of the correlated jump intensity between equity and bond return. All three intensities were obtained from the MGARCH-ARJI model. The sample period is from 01-03-1995 to 12-31-2004. In all, there are 2499 observations in this data set. The *P*-values are in square brackets.

Table 7 presents the parameters estimates of the MGARCH-ARJI for the equity and short-term bonds, and equity and long-term bond. Except for the insignificance of the coefficient of the autoregressive jump intensity in the equity and short-term bond

model, the results do not appear to be qualitatively different from the estimates of the equity and full-bond return model. Furthermore, the results of likelihood ratio statistics (shown at the bottom of the Table 7) which is in sync with likelihood ratio statistics obtained in equity and full bond model also leans towards the MGARCH-ARJI model rather than MGARCH with constant jump intensity model.

In Table 8 and 9, I repeat the conditional correlation regression in Table 5 for the equities and short- and long-term bonds respectively. The results for the long-term bonds closely match that of the full bond sample. The VIX and the jump intensity per period for stock return have a significant and negative relationship to the conditional correlations. The jump intensity per period for bond return also has negative but significantly weaker relation to the implied correlations. The correlated jump intensity was however found to be insignificantly related the implied correlation between equity and long-term bonds.

For the short-term bonds, the conditional correlation regression fails to capture any relations between the jump intensities and correlations. One plausible explanation for this would be the fact that short-term bond often tends to have low correlations with equity returns even in periods of low volatility regimes. Yields on short-term bonds also have been recorded to track short-term interest rate more closely than equity returns, thus making it less sensitive than long-term bonds to shocks in the equity markets.

3.9 Conclusions.

Understanding and predicting the temporal comovement of the asset returns have long been of interest to financial economists. The challenge for the econometrician is the

design of a tractable econometric model that provides a realistic description of the time-varying correlation of financial assets over time. This essay examines the conditional comovement of equity and bond returns using a jump-augmented multivariate-GARCH model with the objective of providing an insight into the following phenomena. First, I examined the impact of jumps on the covariance between equity and bond returns. In particular, I analyze the response of the volatility and covolatility of equity and bond returns to the arrival of discrete economic shocks. Are shocks propagated only through a smooth diffusion process from the equity (bond) market to bond (equity) market or can discrete economic shocks pertaining to the equity (bond) market create simultaneous reaction in both equity and bond markets.

Second, I study the issue of flight-to-quality by focusing on the relation between time varying equity-bond correlations and, the magnitude of return shocks in the equity and bond markets, the arrival of jumps, a measure of investors' view of the "riskiness" of the equity market, and the level of Federal Funds rate.

I find that jump-augmented multivariate GARCH model provides an enhanced depiction of the matrix of conditional second-order moments of equity and bonds over the sample period. Jumps have significant implications for the comovement of the equity and bond returns. When jumps coefficients are introduced into the model, the conditional correlation between equity and bond becomes function of not only the covariance between the normal shocks, but also the arrival of independent and correlated jump shocks. Furthermore, the introduction of jump coefficients alleviates the serial correlation problems which GARCH models are prone to. The effect of non-lasting shocks is easily accounted for, thus minimizing the GARCH parameters' burden of carrying

instantaneous shocks for longer periods.

I also find that, though equity and bonds, historically show evidence of positive unconditional correlation, their conditional correlation fluctuates considerably on a day-to-day basis over the sample period. The implied correlation estimated from the jump-augmented MGARCH model is sensitive to the arrival of unusual news to either the equity or the bond market. Conditional correlations reduce with the anticipation or realization of unfavorable conditions in either the equities or bond market. Investors observed these markets conditions and respond by flying to quality when the need to indemnify against losses in equities' market arise.

An interesting extension to this research is to examine the linkage of the equity, bond and money markets using similar econometric framework. Financial markets closely monitor the activities of the Federal Reserve Bank because every meeting of the Federal Open market Committee (FOMC) concludes with the Feds informing their open market desk a target range for the borrowing rates for commercial banks. A strand of the literatures argues that the Feds monetary policy could be in response to condition in the equities market. In December 1996, then Chairman of the Federal Reserve Bank (Mr. Alan Greenspan) appears to suggest the Feds growing concern with the stock market boom of the 1990s in a speech delivered at the American Enterprise Institute. He raised the possibility that the burgeoning equities market might be as result of "irrational exuberance." This seemingly mundane comment, tucked in a lingering speech, shook the financial world and is argued in the literature as one of the leading cause of crash of equities market toward end of the last decade.

Does the Feds intervene when equities prices deviate from the fundamentals and is it

worthwhile for the Fed to do so? What are the implications of the Feds intervention on the equity-bond relations? All these are interesting questions deserving of further exploration by researchers. Rigobon and Sack (2001) presents compelling evidence as to why the Feds would be inclined to intervene in the financial market when equities prices deviate from the fundamentals. They argued that as of year 2000, 32.5% of the $35.7 trillion of the financial wealth of U.S. household is held in equities. Consequently, equities price movements have significant impact on household wealth and are therefore likely to be an important determinant of monetary policy. Piazzesi (2005) also developed a high frequency policy rule by implying that the Federal Open Market Committee reacts to information contained in the yield curve. All these, taken together in one account provide a compelling motivation for future empirical study into the linkages amongst financial markets.

Chapter 4

Overall Conclusions

Volatilities is almost certainly the most evident characteristic of financial markets and vast amount of literatures have highlighted the effect of the time variation in volatilities and co-volatilities on the price of financial asset, portfolio selection and risk management. As financial markets evolve, it is imperative that financial economist continue to examine the dynamics of volatilities in financial assets and its impact on many crucial financial and economic decisions. The goal of this dissertation is to provide a new perceptive of volatilities in financial data through the eye of jump-augmented stochastic volatility models in both a univariate and a multivariate framework.

In chapter one, I highlighted the evolution of stochastic volatility models from the primeval Bachelier (1900) Brownian motion model to the recent multivariate-GARCH-jump model of Chan (2003), and discuss the motivation for my research.

In chapter two, I study the impact of normal and jump innovations on the intertemporal levels and, the volatility of short-term nominal interest rate using a new class of jump-GARCH augmented Cox, Ingersoll and Ross (1985) model.

In chapter three, I apply a multivariate-GARCH (MGARCH) jump model to investigate the contemporaneous comovement between equity and bond returns. I also explore the issue of flight-to-quality by probing to see if the time-varying correlation between equities and the bond market is conditional on a set of information variables and market conditions.

In chapter two, I find that the CIR-NGARCH-Jump model supplants other GARCH extension to the fundamental CIR model in the ability to describe the dynamics

of 3-months T-bill yield. In addition, I also find that yield on 3-month T-bills responds unevenly to information arrival. The negative size bias test reveals that the squared standardize residuals obtained from the CIR-NGARCH-ARJI model along with those obtained from Vasicek-GARCH-jump, CIR-GARCH-jump, and CIR-NGARCH-jump, are susceptible to high level of negative innovation to the 3-months T-bill yield. Surprisingly the coefficient that captures the incidence of asymmetry in the CIR-NGARCH-jump models was found to be insignificant. This implies that the CIR-NGARCH-ARJI model along with Vasicek-GARCH-jump, CIR-GARCH-jump, and CIR-NGARCH-jump all fail to completely describe the time-varying dynamics of the 3-month T-bill yield.

In chapter three, I find that jump-augmented multivariate GARCH model provide a superior description of the comovement of equity and bonds. The conditional correlation of equity and bonds depends not only on the covariance between normal shocks to equity and bond returns, but also on discrete independent and correlated jump shocks. I also find that, the conditional correlation between equity and bonds fluctuates considerably on a daily basis over the sample period. The implied correlation estimated from the jump-augmented MGARCH model reacts to the arrival of jump inducing information in both the equities and the bond market, and to investors' perception of near-term volatilities in the equity market. The time varying correlations between equities and bond is a reflection of investors' reaction to anticipated and/or realized conditions that is deemed to be unfavorable, in either the equities or bond market. Investors observed these market conditions and take indemnifying action accordingly.

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Appendix: Calculating Implied Bond Return.

To calculate the implied returns on each bond using yield data: consider a hypothetical x-yrs bond that pays a coupon= $YTM_t(\%)$ and matures on date $t+x$. Given that the yield on a hypothetical time t is $YTM_t(\%)$ and the yield at time $(t+j)$ where j is holding period days) changes to $YTM_{t+j}(\%)$. Since the coupon=the YTM, this bond will trade at par of \$1000. At time $(t+j)$, this same bond trades at different prices since $YTM_{t+j}(\%)$ is different from $YTM_t(\%)$. To determine the new price at $(t+j)$, I calculate the NPV price of the this hypothetical bond on the close at $(t+j)$, where the future cash flows are determined by the coupon yield from Friday, the life of the bond is now $t+\left(\frac{j}{365}\right)$ years , and the discount rate that is used to calculate the bond's PV is from the closing yield at time $(t+j)$.

Next, we calculate the accrued interest on the bond, which is the $YTM_t \bullet \left(\frac{j}{365}\right)$. Under the assumption that holding period are based on settlement days and not trade days. We calculate holding period returns as:

Total implied returns=capital gains + accrued interest.

For the portfolio of bonds, I estimated the return on a portfolio consisting of a unit of each bond maturity.

Vita

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